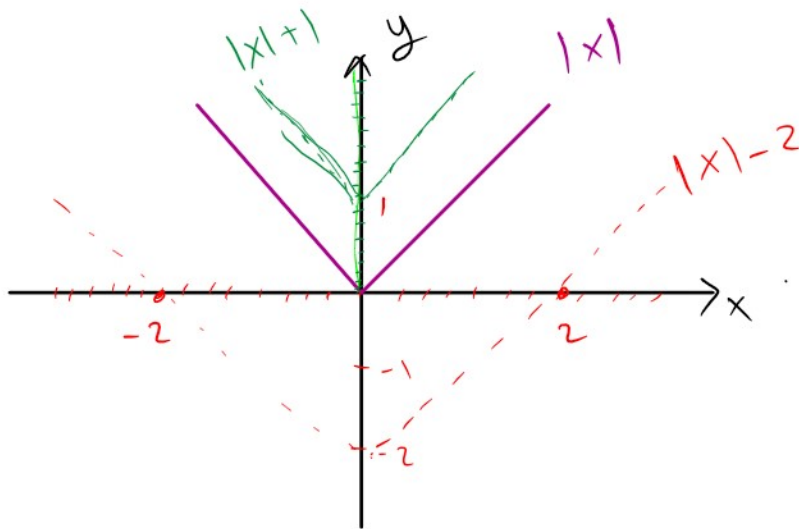


Exp $f(x) = |x| = \sqrt{x^2}$

$$D(f) = (-\infty, \infty) = \mathbb{R}$$

$$R(f) = [0, \infty)$$



Find D, R

x axis

y axis

x axis: $|x|$ y axis: $|y|$

$$|-3| = 3$$

$$|-2| = 2$$

$$|0| = 0$$

$$|7| = 7$$

$$|10| = 10$$

Exp $y = |x| + 1$
 $= |x| - 2$

Exp $f(x) = \frac{1}{x}$

Find D, R

$D(f)$: $x \neq 0$ (all real numbers except 0)

$$D = \mathbb{R} \setminus \{0\}$$

$$R = (-\infty, 0) \cup (0, \infty)$$

$$D = \mathbb{R} \setminus \{0\} = (-\infty, 0) \cup (0, \infty) \quad \checkmark$$

$$y = \frac{1}{x}$$

$R =$ x کے لیے y کے ممکنہ تمام قیمتوں کا مجموعہ

$$f(1) = \frac{1}{1} = 1 \quad \checkmark$$

$$f(-1) = \frac{1}{-1} = -1 \quad \checkmark$$

$$f(2) = \frac{1}{2} \quad \checkmark$$

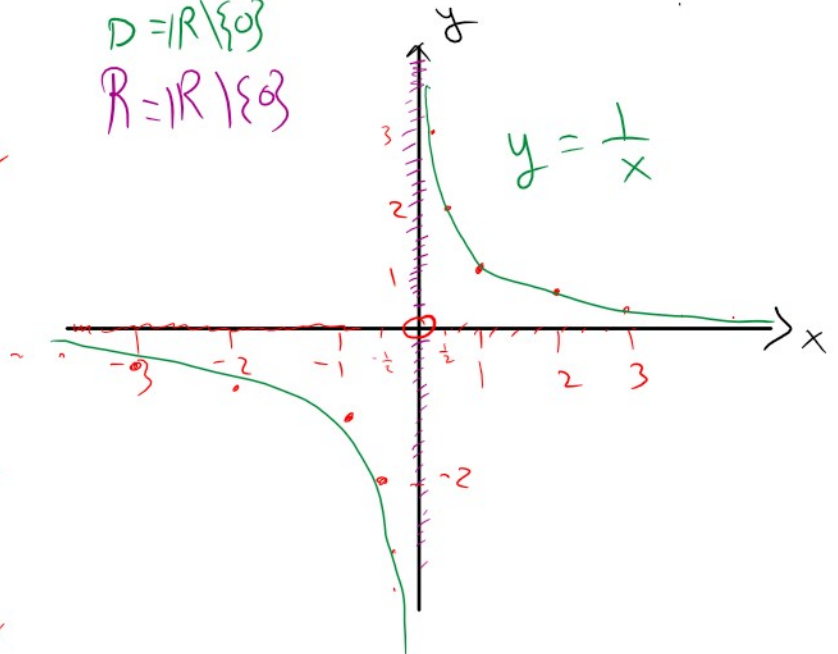
$$f(-2) = -\frac{1}{2} \quad \checkmark$$

$$f\left(\frac{1}{2}\right) = \frac{1}{\frac{1}{2}} = 2 \quad \checkmark$$

$$f\left(-\frac{1}{2}\right) = \frac{1}{-\frac{1}{2}} = -2 \quad \checkmark$$

$$D = \mathbb{R} \setminus \{0\}$$

$$R = \mathbb{R} \setminus \{0\}$$



Floor function

Greatest Integer function

$$\lfloor -4 \rfloor = -4$$

$$\lfloor -4.1 \rfloor = -5$$

$$\lfloor -4.5 \rfloor = -5$$

$$\lfloor -4.9 \rfloor = -5$$

$$\lfloor 4 \rfloor = 4$$

$$\lfloor 4.1 \rfloor = 4$$

$$\lfloor 4.2 \rfloor = 4$$

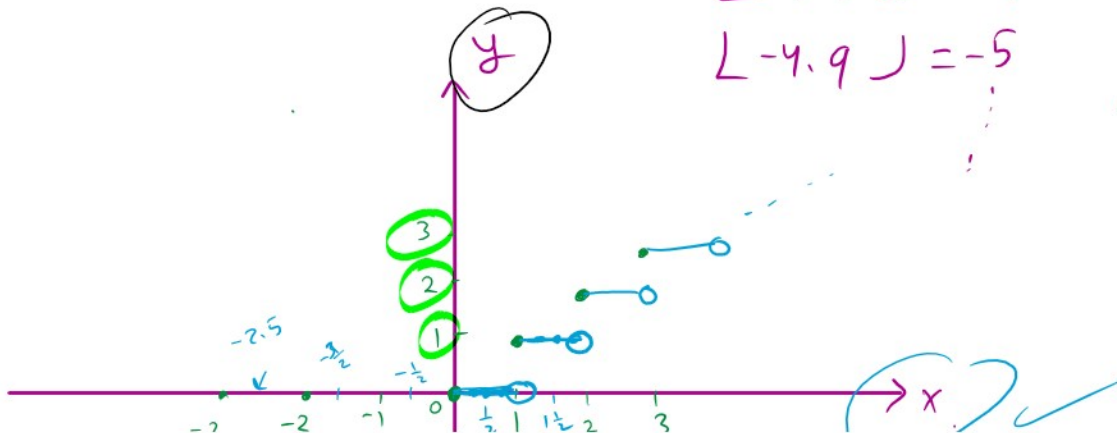
$$\lfloor 4.5 \rfloor = 4$$

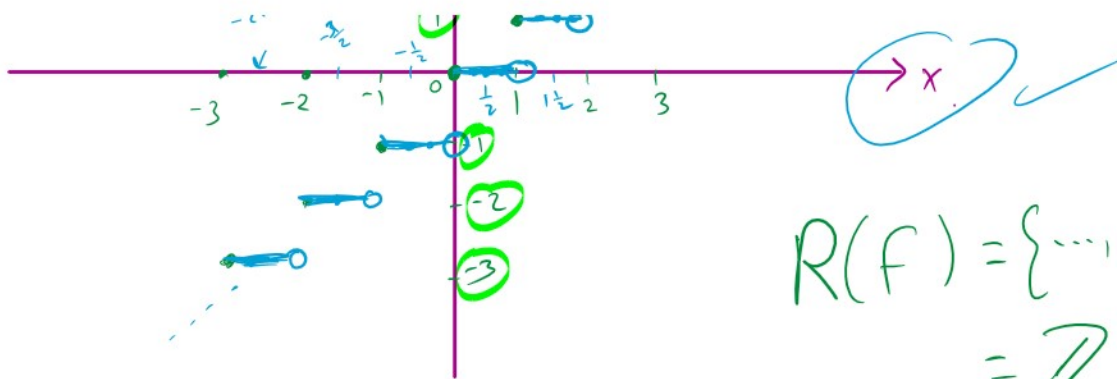
$$\lfloor 4.8 \rfloor = 4$$

$$\lfloor 4.9 \rfloor = 4$$

Exp $f(x) = \lfloor x \rfloor$

$$D = \mathbb{R} = (-\infty, \infty) \quad \checkmark$$





$$R(f) = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$$

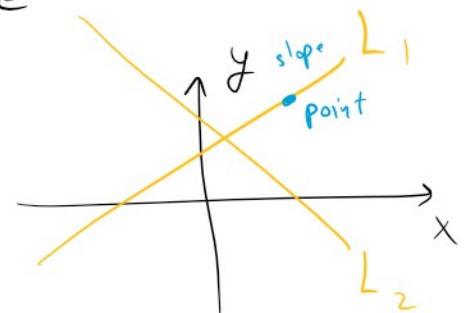
$$= \mathbb{Z}$$

$$= \text{Integers}$$

line equation (Tangent line) ↖ المماس

- we need two points
- or one point and slope

$$y - y_0 = m(x - x_0)$$



$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Exp Find tangent line through the points

$(\overset{x_1}{1}, \overset{y_1}{2})$, $(\overset{x_2}{3}, \overset{y_2}{7})$

$\underset{x_0}{1}$ $\underset{y_0}{2}$

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{7 - 2}{3 - 1} = \frac{5}{2}$$

$$y - y_0 = m(x - x_0)$$

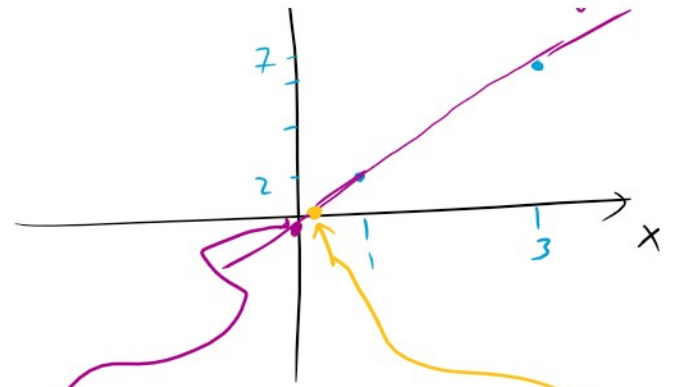
$$y - 2 = \frac{5}{2}(x - 1)$$



$$0 = \frac{5}{2}(x-1) \\ = \frac{5}{2}x - \frac{5}{2}$$

$$y = \frac{5}{2}x - \left(\frac{5}{2}\right) + 2$$

$\swarrow$
2.5



x-intercept $\Rightarrow y=0$

$$0 = \frac{5}{2}x - \frac{1}{2}$$

$$0 = 5x - 1$$

$$5x = 1$$

$$x = \frac{1}{5}$$

y-intercept $\Rightarrow x=0$

$$y = \frac{5}{2}(0) - \frac{1}{2}$$

$$= 0 - \frac{1}{2}$$

$$= -\frac{1}{2}$$

Ex $y = \sqrt{x}$

$$D = [0, \infty)$$

$$R = [0, \infty)$$

Find D, R

$$x \in D$$

$$y \in D$$

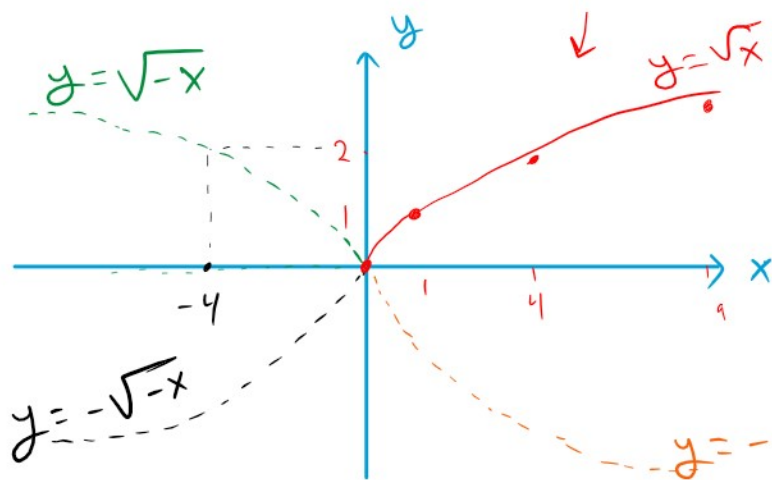
$$\begin{array}{l} y(0) = 0 \\ y(1) = 1 \\ y(4) = 2 \\ y(9) = 3 \end{array}$$

$$y = \sqrt{-x}$$

$\uparrow$ y

$\downarrow$ $y = \sqrt{x}$

1 Ex $y = -\sqrt{x}$



Exp $y = -\sqrt{x}$ ✓

Exp $y = \sqrt{-x} \Rightarrow D = (-\infty, 0]$

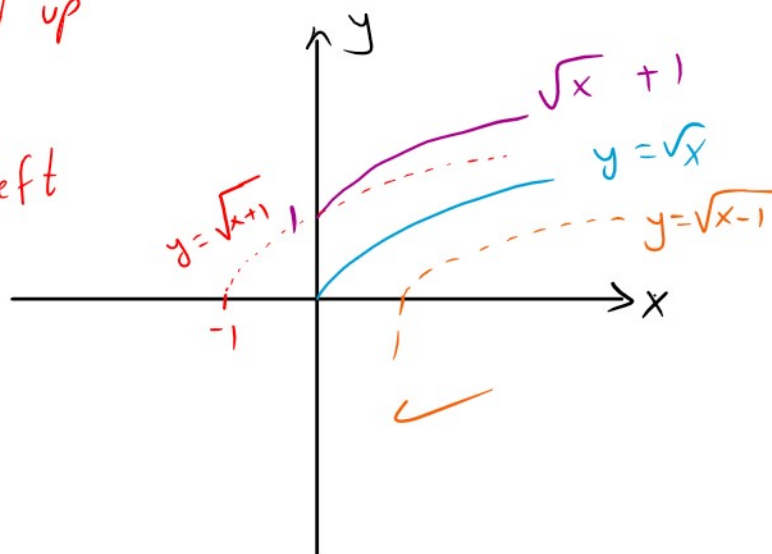
Exp $y = -\sqrt{-x}$

$y = \sqrt{-3}$

Exp ① $y = \sqrt{x} + 1$ shift up

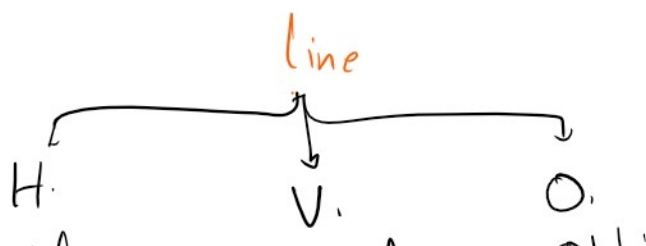
② $y = \sqrt{x+1}$ shift left

D: $x+1 \geq 0$
 $x \geq -1$
 $[-1, \infty)$
 ✓



③ $y = \sqrt{x-1}$

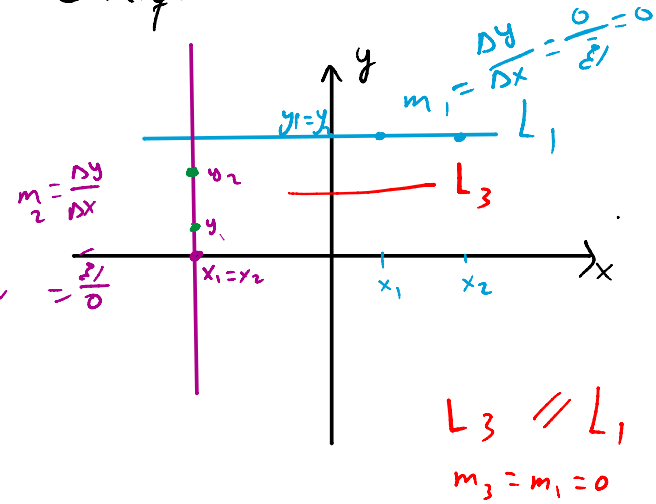
D: $x-1 \geq 0$
 $x \geq 1$
 $[1, \infty)$



H.
Horizontal
 $m_1 = 0$

V.
Vertical
 m_2 undefined

O.
Oblique



Two lines L_1, L_2

$L_1 \parallel L_2$ iff $m_1 = m_2$ parallel

$L_1 \perp L_2$ iff $m_1 m_2 = -1$
perpendicular (normal), (orthogonal)

Exp $L_1: y = 2x + 1$

$L_2: y = 4 - \frac{1}{2}x$

Is $L_1 \perp L_2$

$m_1 = 2$
 $m_2 = -\frac{1}{2}$ $\Rightarrow m_1 m_2 = (2) \left(-\frac{1}{2}\right) = -1$

Yes

Absolute Value (1) $|x| = a \Rightarrow x = a$ or $x = -a$

Exp $|x| = 5 \Rightarrow x = 5$ or $x = -5$
 $\{-5, 5\}$

Exp $|2x-1| = 3 \Rightarrow 2x-1 = 3 \Rightarrow 2x = 4 \Rightarrow x = 2$
or $2x-1 = -3 \Rightarrow 2x = -2 \Rightarrow x = -1$
 $\{-1, 2\}$

Exp $|1-4x| = -3$

(2) $|x| \leq a \Rightarrow -a \leq x \leq a$

Exp $|2x-4| \leq 8$

$-8 \leq 2x-4 \leq 8$

$-4 \leq 2x \leq 12$

$-2 \leq x \leq 6$

$\Rightarrow x \in [-2, 6]$

$|2(-1)-4| = |-2-4| = |-6|$
 $= 6 \leq 8$

(-1)

(3) $|x| \geq a \Rightarrow x \geq a$ or $x \leq -a$

Exp $|1-2x| \geq 5$

Exp $|1-2x| \geq 5$

$$1-2x \geq 5$$

$$-2x \geq 4$$

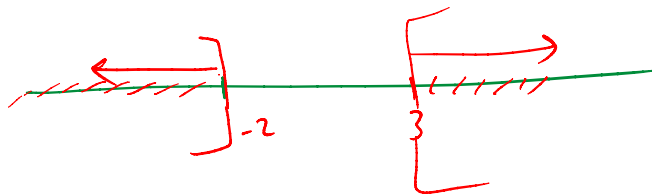
$$x \leq -2$$

or

$$1-2x \leq -5$$

$$-2x \leq -6$$

$$x \geq 3$$



$$(-\infty, -2] \cup [3, \infty)$$

Exp

$$|x-1| > 2$$

$$x-1 > 2$$

$$x > 3$$

or

$$x-1 < -2$$

$$x < -1$$



$$(-\infty, -1) \cup (3, \infty)$$

Exp $|1-x| > 2$

$$1-x > 2$$

$$-x > 1$$

$$x < -1$$

or

$$1-x < -2$$

$$-x < -3$$

$$x > 3$$

$$x < -1$$

$$x > 3$$



Ex

$$|x| > 5$$

$$x = -6$$

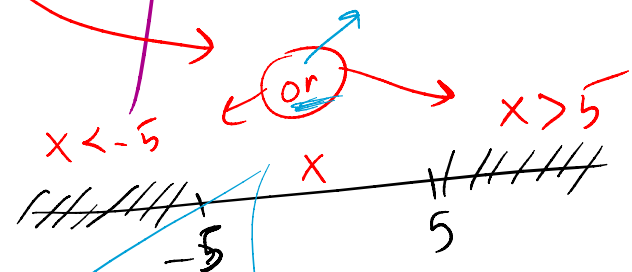
$$= 7$$

$$7 > 5$$

$$-6 < -5$$

$$x > 3$$

$$x \leq -2$$



$$(-\infty, -5) \cup (5, \infty)$$

$$x < -5 \text{ and } x > 5$$

Factorization

$$x^2 - a^2 = (x - a)(x + a)$$

$$x^2 - 3x + 2 = 0$$

$$(x - 1)(x - 2) = 0$$

$$x_2 = 1$$

$$x_1 = 2$$

$$x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

$$= \frac{3 \pm \sqrt{9 - 4(1)(2)}}{2(1)}$$

$$= 3 \pm \sqrt{9 - 8}$$

$$AX^2 + BX + C = 0$$

$$(x_2 = 1, x_1 = 2)$$

$$A=1, B=-3, C=2$$

$$\begin{aligned} &= \frac{3 \pm \sqrt{9-8}}{2} \\ &= \frac{3 \pm \sqrt{1}}{2} \\ &= \frac{3 \pm 1}{2} \end{aligned}$$

$$x_1 = \frac{3+1}{2} = \frac{4}{2} = 2$$

$$x_2 = \frac{3-1}{2} = \frac{2}{2} = 1$$

معادلة الدائرة نصف قطرها r ومركزها (x_0, y_0)

Center (x_0, y_0) , radius r

circle equation

$$(x - x_0)^2 + (y - y_0)^2 = r^2$$

Exp Find the center and radius for the circle

$$x^2 + 2x + y^2 = 4y$$

$$x^2 + 2x + \boxed{1} + y^2 - 4y + \boxed{4} = \boxed{1} + \boxed{4}$$

$$(x+1)^2 + (y-2)^2 = 5 \rightarrow (\sqrt{5})^2$$

$$x^2 + 2x + \boxed{1}$$

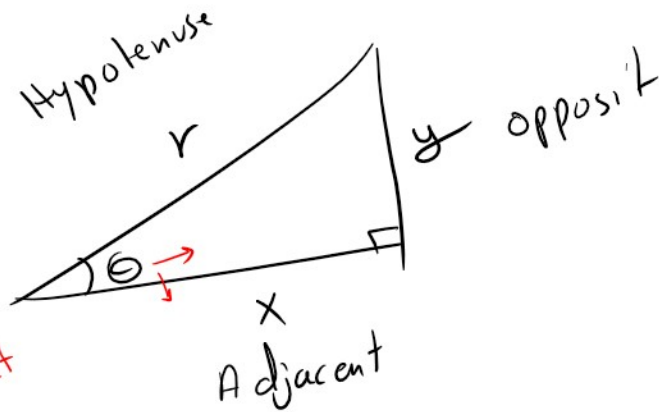
$$(x+1)^2 + (y-2)^2 = (\sqrt{5})^2$$

$$(-1, 2)$$

$$(x+1)^2 + (y-2)^2 = (r)^2$$

center $(x_0, y_0) = (-1, 2)$
radius = $\sqrt{5}$

$$(a+b)^2 = a^2 + 2ab + b^2$$



$$\sin \theta = \frac{y}{r}$$

$$\cos \theta = \frac{x}{r}$$

$$\tan \theta = \frac{y}{x} = \frac{\sin \theta}{\cos \theta}$$

co-secant

$$\csc \theta = \frac{1}{\sin \theta} = \frac{r}{y}$$

secant

$$\sec \theta = \frac{1}{\cos \theta} = \frac{r}{x}$$

co-tan

$$\cot \theta = \frac{1}{\tan \theta} = \frac{x}{y} = \frac{\cos \theta}{\sin \theta}$$

$$x^2 + y^2 = r^2$$

$$\frac{x^2}{r^2} + \frac{y^2}{r^2} = 1$$

$$\cos^2 \theta + \sin^2 \theta = 1$$