

8.2	36, 47, 63	9, 11, 19, 21, 28, 33, 38, 45, 51, 67, 69
-----	------------	---

Product of sin and cos

$$\int \sin mx \sin nx \, dx \checkmark$$

$$\int \sin mx \cos nx \, dx \checkmark$$

$$\int \cos mx \cos nx \, dx \checkmark$$

$$\sin mx \sin nx = \frac{1}{2} [\cos(m-n)x - \cos(m+n)x]$$

$$\sin mx \cos nx = \frac{1}{2} [\sin(m-n)x + \sin(m+n)x]$$

$$\cos mx \cos nx = \frac{1}{2} [\cos(m-n)x + \cos(m+n)x]$$

$$(38) \int \sec^4 x \tan^2 x \, dx$$

$$\int \sec^2 x \frac{\tan^2 x}{u^2} \frac{\sec^2 x \, dx}{du}$$

$$1 + \tan^2 x = \sec^2 x$$

$$u = \tan x$$

$$du = \sec^2 x \, dx$$

$$\int (1 + \tan^2 x) \tan^2 x \sec^2 x \, dx = \int (1 + u^2) u^2 \, du = \int (u^2 + u^4) \, du$$

$$= \frac{u^3}{3} + \frac{u^5}{5} + C = \frac{1}{3} \tan^3 x + \frac{1}{5} \tan^5 x + C$$

$$(51) \int \sin^3 x \cos^2 x \, dx = \frac{1}{2} \int [\sin(3-2)x + \sin(3+2)x] \, dx$$

$$= \frac{1}{2} \int (\sin x + \sin 5x) \, dx$$

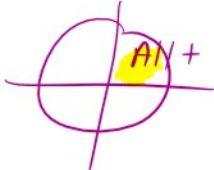
$$= \frac{1}{2} \left(-\cos x - \frac{\cos 5x}{5} \right) + C$$

$$= -\frac{1}{2} \cos x - \frac{1}{10} \cos 5x + C$$

(69) Find length of the curve $y = \ln(\sec x)$, $0 \leq x \leq \frac{\pi}{4}$

$$L = \int_0^{\frac{\pi}{4}} \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$$

$$y' = \frac{dy}{dx} = \frac{\sec x \tan x}{\sec x} = \tan x$$

$$(y')^2 = \tan^2 x$$


$$= \int_0^{\frac{\pi}{4}} \sqrt{1 + \tan^2 x} dx = \int_0^{\frac{\pi}{4}} \sqrt{\sec^2 x} dx = \int_0^{\frac{\pi}{4}} |\sec x| dx$$

$$= \int_0^{\frac{\pi}{4}} \sec x dx = \ln|\sec x + \tan x| \Big|_0^{\frac{\pi}{4}}$$

$$= \ln|\sec \frac{\pi}{4} + \tan \frac{\pi}{4}| - \ln|\sec 0 + \tan 0|$$

$$= \ln|\sqrt{2} + 1| - \ln|1 + 0|$$

$$= \ln(\sqrt{2} + 1) - (\ln 1) \rightarrow 0$$

$$= \ln(\sqrt{2} + 1)$$

$$\cos \frac{\pi}{4} = \frac{1}{\sqrt{2}}$$

Q21) $\int 8 \cos^3 2\theta \sin 2\theta d\theta$

$$16 \sin^2 2\theta \cos 2\theta d\theta$$

$$\sin^2 2\theta + \cos^2 2\theta = 1$$

$$\cos^2 2\theta = 1 - \sin^2 2\theta$$

$$\int \textcircled{8} \cos^2 2\theta \sin 2\theta \cos 2\theta d\theta \quad \left| \begin{array}{l} u = \sin 2\theta \\ du = 2 \cos 2\theta d\theta \end{array} \right.$$

$$\textcircled{4} \int (1 - \sin^2 2\theta) \sin 2\theta \textcircled{2} \cos 2\theta d\theta \quad \underline{du} = 2 \cos 2\theta d\theta$$

$$4 \int (1 - u^2) u du$$

$$4 \int (u - u^3) du = 4 \left(\frac{u^2}{2} - \frac{u^4}{4} \right) + C$$

$$= 2u^2 - u^4 + \textcircled{4C} \rightarrow D$$

~~100~~

$$= 2 \sin^2 2\theta - \sin^4 2\theta + D$$

$$= \sin^2 2\theta \left[2 - \sin^2 2\theta \right] + D$$

~~cos 2θ~~

$$= \sin^2 2\theta \left[1 + \underbrace{1 - \sin^2 2\theta}_{\cos^2 2\theta} \right] + D$$

$$= \sin^2 2\theta \left[1 + \cos^2 2\theta \right]$$

$$= \left(\frac{1 - \cos 2\theta}{2} \right) (1 + \cos^2 2\theta)$$

$$= \frac{1}{2} (1 - \cos 2\theta) (1 + \cos^2 2\theta)$$

$$\cos 2\theta = 1 - 2\sin^2 \theta$$

$$= 2\cos^2 \theta - 1$$

$$= \cos^2 \theta - \sin^2 \theta$$

$$\sin^2 2\theta = \frac{1 - \cos 2\theta}{2}$$

$$- \cos 2\theta$$

$$- \cos^4 2\theta$$
