8.2

Saturday, June 5, 2021 7:28 AM

8.2	36, 47, 63	9, 11, 19, 21, 28, 33, 38, 45, 51, 67, 69

L.N

Product of sin and cos

$\int \sin mx \sin nx \, dx$

$\int \sin mx \cos nx \, dx$

$\int \cos mx \cos nx \, dx$

$\sin mx \sin nx = \frac{1}{2}\left[\cos(m-n)x - \cos(m+n)x\right]$

$\sin mx \cos nx = \frac{1}{2}\left[\sin(m-n)x + \sin(m+n)x\right]$

$\cos mx \cos nx = \frac{1}{2}\left[\cos(m-n)x + \cos(m+n)x\right]$

$1 + \tan^2 x = \sec^2 x$

(38) $\int \sec^4 x \tan^2 x \, dx$

$\int \sec^2 x \tan^2 x \sec^2 x \, dx$

u^2 du

$u = \tan x$

$du = \sec^2 x \, dx$

$\int (1+\tan^2 x)\tan^2 x \sec^2 x \, dx = \int (1+u^2)u^2 \, du = \int (u^2+u^4)\, du$

$= \frac{u^3}{3} + \frac{u^5}{5} + C = \frac{1}{3}\tan^3 x + \frac{1}{5}\tan^5 x + C$

(51) $\int \sin^3 3x \cos^2 2x \, dx = \frac{1}{2}\int \left[\sin(3-2)x + \sin(3+2)x\right] dx$

$= \frac{1}{2}\int (\sin x + \sin 5x)\, dx$

$= \frac{1}{2}\left(-\cos x - \frac{\cos 5x}{5}\right) + C$

$= -\frac{1}{2}\cos x - \frac{1}{10}\cos 5x + c$

(69) Find length of the curve $y = \ln(\sec x)$, $0 \le x \le \frac{\pi}{4}$

$$L = \int_0^{\frac{\pi}{4}} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$y' = \frac{dy}{dx} = \frac{\sec x \tan x}{\sec x} = \tan x$$

$$(y')^2 = \tan^2 x$$

All +

$$= \int_0^{\frac{\pi}{4}} \sqrt{1 + \tan^2 x}\, dx = \int_0^{\frac{\pi}{4}} \sqrt{\sec^2 x}\, dx = \int_0^{\frac{\pi}{4}} |\sec x|\, dx$$

$$= \int_0^{\frac{\pi}{4}} \sec x\, dx = \ln|\sec x + \tan x| \Big|_0^{\frac{\pi}{4}}$$

$$= \ln\left|\sec\tfrac{\pi}{4} + \tan\tfrac{\pi}{4}\right| - \ln|\sec 0 + \tan 0|$$

$\cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

$$= \ln|\sqrt{2} + 1| - \ln|1 + 0|$$

$$= \ln(\sqrt{2} + 1) - \ln 0 \to 0$$

$$= \ln(\sqrt{2} + 1)$$

(Q21) $\int 8\cos^3 2\theta \sin 2\theta\, d\theta$

$\sin^2 2\theta + \cos^2 2\theta = 1$

$\cos^2 2\theta = 1 - \sin^2 2\theta$

(Q) $\cos^2 2\theta \sin 2\theta \cos 2\theta\, d\theta$

$\int (8)\ \cos^2 2\theta \sin 2\theta \cos 2\theta \, d\theta$

$u = \sin 2\theta$

$du = 2\cos 2\theta \, d\theta$

$(4)\int (1 - \sin^2 2\theta) \sin 2\theta \, (2\cos 2\theta \, d\theta)$

$4\int (1 - u^2)\, u \, du$

$4\int (u - u^3)\, du = 4\left(\frac{u^2}{2} - \frac{u^4}{4}\right) + C$

$= 2u^2 - u^4 + 4C$

$= 2\sin^2 2\theta - \sin^4 2\theta + D$

$= \sin^2 2\theta \left[2 - \sin^2 2\theta\right] + D$

$= \sin^2 2\theta \left[1 + 1 - \sin^2 2\theta\right] + D$

$1 - \sin^2 2\theta = \cos^2 2\theta$

$= \sin^2 2\theta \left[1 + \cos^2 2\theta\right]$

$= \left(\frac{1 - \cos 2\theta}{2}\right)\left(1 + \cos^2 2\theta\right)$

$\cos 2\theta = 1 - 2\sin^2 2\theta$

$= 2\cos^2\theta - 1$

$= \cos^2\theta - \sin^2\theta$

$\sin^2 2\theta = \frac{1 - \cos 2\theta}{2}$

$= \frac{1}{2}(1 - \cos 2\theta)\left(1 + \cos^2 2\theta\right)$

$\frac{4}{-\cos 2\theta}$ ----- 0