

4) $\sum_{n=1}^{\infty} \frac{(3x-2)^n}{n}$

10.7

$$\lim_{n \rightarrow \infty} \frac{(3x-2)^{n+1}}{n+1} \cdot \frac{n}{(3x-2)^n} = \lim_{n \rightarrow \infty} \frac{|3x-2|^{n+1}}{n+1} \cdot \frac{n}{(3x-2)^n} = |3x-2| < 1$$

$$-1 < 3x-2 < 1$$

$$1 < 3x < 3$$

$$\frac{1}{3} < x < 1 \rightarrow \text{conv Abs}$$

$$\frac{1}{3} \leq x < 1 \rightarrow \text{conv cond}$$

$$\text{at } x = \frac{1}{3} \leftarrow \text{conv cond}$$

$$R = \frac{1}{3}$$

$$\frac{1}{3} \quad 1$$

$$x = \frac{1}{3} \rightarrow \sum_{n=1}^{\infty} (-1)^n \frac{1}{n} \rightarrow \text{conv AST}$$

$$x = 1 \rightarrow \sum_{n=1}^{\infty} \frac{1}{n} \rightarrow \text{div P-series}$$

12) $\sum_{n=0}^{\infty} \frac{3^n x^n}{n!}$

$$\lim_{n \rightarrow \infty} \frac{3^{n+1} x^{n+1}}{(n+1)!} \cdot \frac{n!}{3^n x^n} = \lim_{n \rightarrow \infty} \frac{3|x|}{n+1} = 0 < 1 \text{ for all } x$$

$$R = \infty$$

conv Abs for all x

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14) $\sum_{n=1}^{\infty} \frac{(x-1)^n}{3^n n^2}$

$$\lim_{n \rightarrow \infty} \frac{(x-1)^{n+1}}{3^{n+1} (n+1)^2} \cdot \frac{3^n n^2}{(x-1)^n} = \lim_{n \rightarrow \infty} \frac{|x-1| n^2}{3 (n+1)^2} = \frac{1}{3} |x-1| < 1$$

$$-1 < \frac{1}{3} (x-1) < 1$$

$$-3 < (x-1) < 3$$

$$-2 \leq x \leq 4 \rightarrow \text{conv Abs}$$

$$R = 3$$

$$x = -2 \rightarrow \sum_{n=1}^{\infty} \frac{(-3)^n}{3^n n^2} = \sum_{n=1}^{\infty} (-1)^n \frac{1}{n^2} \rightarrow \text{conv AST + Abs}$$

$$x = 4 \rightarrow \sum_{n=1}^{\infty} \frac{1}{n^2} \rightarrow \text{conv}$$

$$\sum_{n=1}^{\infty} \left(1 + \frac{1}{n}\right)^n x^n$$

23) $\lim_{n \rightarrow \infty} \frac{\left(1 + \frac{1}{n+1}\right)^{n+1} x^{n+1}}{\left(1 + \frac{1}{n}\right)^n x^n} = |x| \lim_{n \rightarrow \infty} \frac{\left(1 + \frac{1}{n+1}\right)^{n+1}}{\left(1 + \frac{1}{n}\right)^n} = \frac{|x| e}{e} = |x| < 1$
 $-1 < x < 1$ Abs Conv

$x = -1 \rightarrow \sum_{n=1}^{\infty} (-1)^n \left(1 + \frac{1}{n}\right)^n$ $R = 1$

$\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n = e$ div by n^{th} term test

$x = 1 \rightarrow \sum_{n=1}^{\infty} \left(1 + \frac{1}{n}\right)^n \rightarrow$ div by n^{th} term test

29) $\sum_{n=2}^{\infty} \frac{x^n}{n(\ln n)^2}$

$\lim_{n \rightarrow \infty} \frac{x^{n+1}}{(n+1)(\ln(n+1))^2} \cdot \frac{n(\ln n)^2}{x^n} = |x| \lim_{n \rightarrow \infty} \frac{n}{n+1} \cdot \frac{(\ln n)^2}{(\ln(n+1))^2} = |x| < 1$
 $-1 \leq x \leq 1 \rightarrow \text{Conv Abs}$

$x = -1 \rightarrow \sum_{n=2}^{\infty} (-1)^n \frac{1}{n(\ln n)^2} \rightarrow \text{Conv}$

$R = 1$

$\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$

$\int_2^{\infty} \frac{1}{x(\ln x)^2} dx$ $u = \ln x$
 $du = \frac{dx}{x}$

$\lim_{b \rightarrow \infty} \left. \frac{-1}{\ln x} \right|_2^b = \lim_{b \rightarrow \infty} \left[\frac{-1}{\ln b} + \frac{1}{\ln 2} \right] = \frac{1}{\ln 2}$ Conv

$x = 1 \rightarrow \sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2} \rightarrow \text{Conv}$

32) $\sum_{n=1}^{\infty} \frac{(3x+1)^{n+1}}{2n+2}$

10.7

$$\lim_{n \rightarrow \infty} \frac{(3x+1)^{n+1}}{2n+2} \cdot \frac{2n+2}{(3x+1)^{n+1}} = |3x+1| \lim_{n \rightarrow \infty} \frac{2n+2}{2n+4} = |3x+1| < 1$$

$$x = -\frac{2}{3} \rightarrow \sum_{n=1}^{\infty} (-1)^n \frac{1}{2n+2} \rightarrow \text{conv AST}$$

$$-1 < 3x+1 < 1$$

$$-2 < 3x < 0$$

$$-\frac{2}{3} < x < 0 \rightarrow \text{conv Abs}$$

$$\sum_{n=1}^{\infty} \frac{1}{2n+2} \quad \sum_{n=1}^{\infty} \frac{1}{n}$$

$$R = \frac{1}{3}$$

$$\lim_{n \rightarrow \infty} \frac{1}{2n+2} \cdot n = \frac{1}{2} > 0 \text{ div}$$

$$\text{conv cond at } x = -\frac{2}{3}$$

$$\text{Interval of conv } -\frac{2}{3} \leq x < 0$$

$$x=0 \rightarrow \sum_{n=1}^{\infty} \frac{1}{2n+2} \rightarrow \text{div}$$

40) $\sum_{n=1}^{\infty} \left(\frac{n}{n+1}\right)^{n^2} x^n$

$$\lim_{n \rightarrow \infty} \sqrt[n]{\frac{n}{n+1}} |x| = \frac{|x|}{e} < 1$$

$$-1 < \frac{x}{e} < 1$$

$$-e < x < e$$

$$R = \frac{e - (-e)}{2} \Rightarrow R = e$$

$$\sum_{n=0}^{\infty} (\ln x)^n$$

46) $\lim_{n \rightarrow \infty} \frac{(\ln x)^{n+1}}{(\ln x)^n}$

$$\lim_{n \rightarrow \infty} |\ln x| = |\ln x| < 1$$

$$x = e^{-1} \rightarrow \sum_{n=0}^{\infty} (\ln e^{-1})^n = \sum_{n=0}^{\infty} (-1)^n \text{ div}$$

$$x = e \rightarrow \sum_{n=0}^{\infty} (\ln e)^n = \sum_{n=0}^{\infty} 1^n \text{ div}$$

$$-1 < \ln x < 1$$

$$e^{-1} < x < e \rightarrow \text{conv Abs}$$

$$\text{Sum} = \frac{1}{1 - \ln x}$$

$$49) \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n (x-3)^n$$

$$\lim_{n \rightarrow \infty} \frac{\left(\frac{1}{2}\right)^{n+1} (x-3)^{n+1}}{\left(\frac{1}{2}\right)^n (x-3)^n} = \lim_{n \rightarrow \infty} \frac{1}{2} |x-3| = \frac{1}{2} |x-3| < 1$$

$$-1 < \frac{1}{2}(x-3) < 1$$

$$-2 < x-3 < 2$$

$$1 < x < 5 \rightarrow \text{Conv Abs}$$

$$R = 2$$

$$\text{Sum} = \frac{1}{1 + \frac{x-3}{2}} = \frac{2}{x-1}$$

$$x=1 \rightarrow \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n (-2)^n = \sum_{n=0}^{\infty} 1^n \text{ div}$$

$$x=5 \rightarrow \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n 2^n = \sum_{n=0}^{\infty} (-1)^n \text{ div}$$

$$11) \sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n!}$$

$$\lim_{n \rightarrow \infty} \frac{x^{n+1}}{(n+1)!} \cdot \frac{n!}{x^n} = \lim_{n \rightarrow \infty} \frac{|x|}{n+1} = 0 < 1 \text{ for all } x \text{ Conv Abs } R = \infty$$

$$24) \sum_{n=1}^{\infty} (\ln n) x^n$$

$$\lim_{n \rightarrow \infty} \frac{\ln(n+1) x^{n+1}}{\ln(n) x^n} = |x| \lim_{n \rightarrow \infty} \frac{\ln(n+1)}{\ln n} = |x| < 1$$

$$-1 < x < 1 \rightarrow \text{Conv Abs}$$

$$R = 1$$

$$x=-1 \rightarrow \sum_{n=1}^{\infty} (\ln n) (-1)^n \rightarrow \text{div by } n^{\text{th}} \text{ term test}$$

$$x=1 \rightarrow \sum_{n=1}^{\infty} (\ln n) \rightarrow \text{div by } n^{\text{th}} \text{ term test}$$

$$41) \sum_{n=0}^{\infty} 3^n x^n$$

$$\lim_{n \rightarrow \infty} \frac{3^{n+1} x^{n+1}}{3^n x^n} = |x| \lim_{n \rightarrow \infty} 3 = 3|x| < 1$$

$$-1 < 3x < 1$$

$$-\frac{1}{3} < x < \frac{1}{3} \rightarrow \text{Conv Abs}$$

$$R = \frac{\frac{1}{3} - (-\frac{1}{3})}{2} \Rightarrow R = \frac{1}{3}$$

$$x = -\frac{1}{3} \rightarrow \sum_{n=0}^{\infty} \left(\frac{1}{3}\right)^n 3^n = \sum_{n=0}^{\infty} (-1)^n \text{ div}$$

$$x = \frac{1}{3} \rightarrow \sum_{n=0}^{\infty} \left(\frac{1}{3}\right)^n 3^n = \sum_{n=0}^{\infty} 1^n \text{ div}$$

$$\sum_{n=0}^{\infty} (3x)^n$$

$$\text{Sum} = \frac{1}{1-3x} \quad -\frac{1}{3} < x < \frac{1}{3}$$