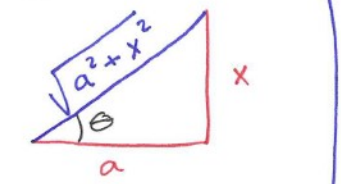
8.3

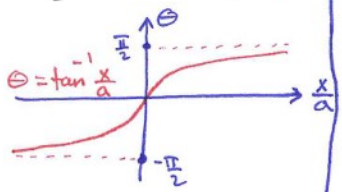
Saturday, June 5, 2021 7:28 AM

8.3	~~14, 24~~	10, 18, 26, 28, 32, 36, 45, 46

14



$x = a\tan\theta$ requires $\theta = \tan^{-1}\frac{x}{a}$ with $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$ ♥1



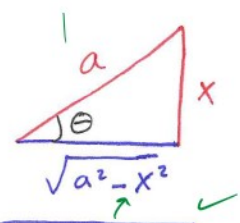
$\sqrt{a^2+x^2} = \sqrt{a^2+a^2\tan^2\theta}$

$= a\sqrt{1+\tan^2\theta}$

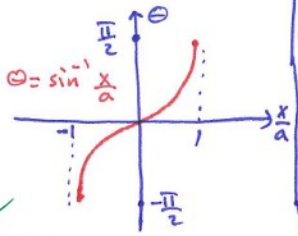
$= a\sqrt{\sec^2\theta}$

$= a|\sec\theta|$ ♥1

$= a\sec\theta$



$x = a\sin\theta$ requires $\theta = \sin^{-1}\frac{x}{a}$ with $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$ ♥2



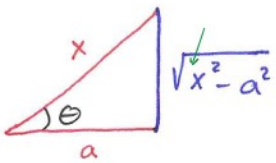
$\sqrt{a^2-x^2} = \sqrt{a^2-a^2\sin^2\theta}$

$= a\sqrt{1-\sin^2\theta}$

$= a\sqrt{\cos^2\theta}$

$= a|\cos\theta|$ ♥2

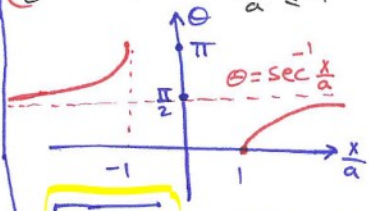
$= a\cos\theta$



$x = a\sec\theta$ requires $\theta = \sec^{-1}\frac{x}{a}$ with

$0 \le \theta < \frac{\pi}{2}$ if $\frac{x}{a} \ge 1$ ♥3

$\frac{\pi}{2} < \theta \le \pi$ if $\frac{x}{a} \le -1$



$\sqrt{x^2-a^2} = \sqrt{a^2\sec^2\theta - a^2}$

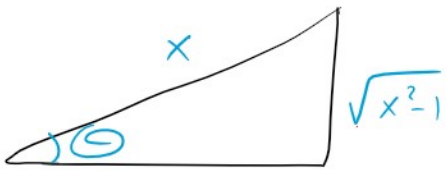
$= a\sqrt{\sec^2\theta - 1}$

$= a\sqrt{\tan^2\theta}$

$= a|\tan\theta|$ ♥3

$= a\tan\theta$

14 $\int \frac{2\,dx}{x^3\sqrt{x^2-1}} = \int \frac{2\ \sec\theta\tan\theta\,d\theta}{\sec^3\theta\ \tan\theta}$



$= 2\int \frac{1}{\sec^2\theta}\,d\theta = 2\int \cos^2\theta\,d\theta$

$= 2\int \frac{1+\cos 2\theta}{2}\,d\theta = \int (1+\cos 2\theta)\,d\theta$

$= \theta + \frac{\sin 2\theta}{2} + C$

$x = \sec\theta$

$dx = \sec\theta\tan\theta\,d\theta$

$\sqrt{x^2-1} = \tan\theta$

$$= \theta + \frac{\sin 2\theta}{2} + C$$

$$= \theta + \frac{1}{2} \cancel{2}\sin\theta\cos\theta + C$$

$$= \theta + \sin\theta\cos\theta + C$$

$$= \sec^{-1}x + \frac{\sqrt{x^2-1}}{x}\cdot\frac{1}{x} + C$$

$\sec^{-1}x = \sec^{-1}(\sec\theta)$

$\sec^{-1}x = \theta$

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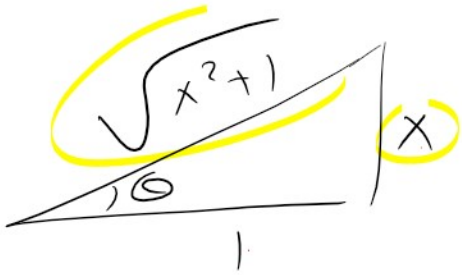
(18) $\int \frac{dx}{x^2\sqrt{x^2+1}}$



$x = \tan\theta$

$dx = \sec^2\theta\, d\theta$

$\sqrt{x^2+1} = \sec\theta$

$$\int \frac{\sec^2\theta\, d\theta}{\tan^2\theta \sec\theta} = \int \frac{\sec\theta\, d\theta}{\tan^2\theta}$$

$$= \int \frac{\frac{1}{\cos\theta}}{\frac{\sin^2\theta}{\cos^2\theta}} d\theta = \int \frac{\cos\theta\, d\theta}{\sin^2\theta}$$

$u = \sin\theta$

$du = \cos\theta\, d\theta$

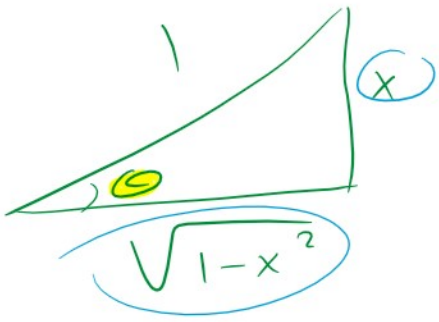
$$= \int \frac{du}{u^2} = \int u^{-2}du = \frac{u^{-1}}{-1} + C$$

$$= \frac{-1}{u} + C = \frac{-1}{\sin\theta} + C$$

$$= \frac{-1}{\frac{\sqrt{x^2+1}}{x}} + C = -\frac{x}{\sqrt{x^2+1}} + C$$

$$= \frac{-1}{\frac{x}{\sqrt{x^2+1}}} + C = -\frac{\sqrt{x^2+1}}{x}$$

28) $\int \frac{\sqrt{1-x^2}}{x^4}\,dx$



$x = \sin\theta$

$dx = \cos\theta\, d\theta$

$\sqrt{1-x^2} = \cos\theta$

$$\int \frac{\cos\theta \cos\theta\, d\theta}{\sin^4\theta}$$

$$\int \frac{\cos^2\theta}{\sin^2\theta}\frac{1}{\sin^2\theta}\,d\theta$$

$$\int \cot^2\theta \csc^2\theta\, d\theta$$

$u = \cot\theta$

$du = -\csc^2\theta\, d\theta$

$$\int u^2 (-du)$$

$$-\frac{u^3}{3} + C = -\frac{1}{3}\cot^3\theta + C$$

$$= -\frac{1}{3}\left(\frac{\sqrt{1-x^2}}{x}\right)^3 + C$$