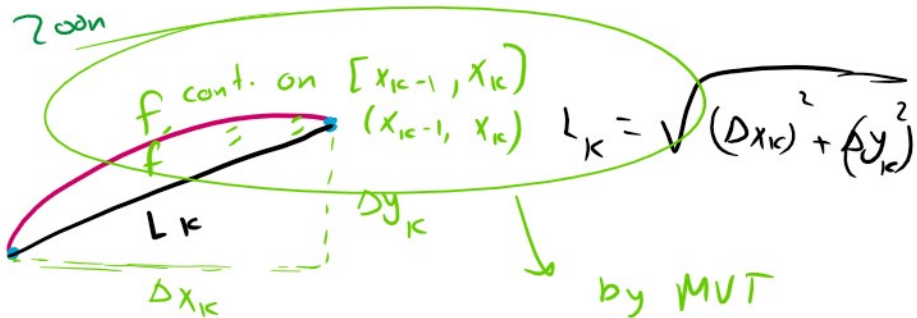
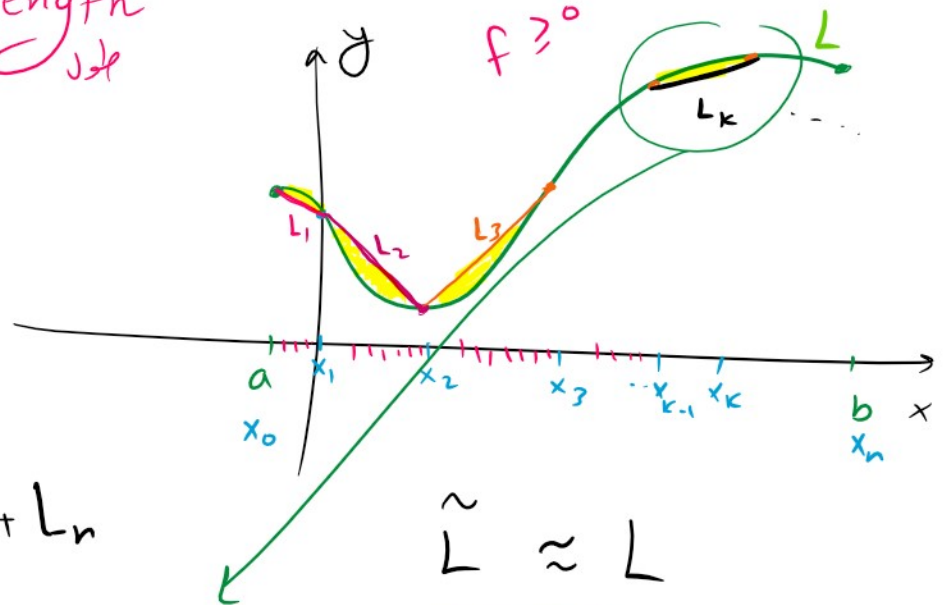


Arc length
قوس

L : True length

$$\tilde{L} = \sum_{k=1}^n K_k$$

$$= L_1 + L_2 + \dots + L_n$$



by MVT
 $\exists c_k \in (x_{k-1}, x_k)$

$$\text{s.t. } \hat{f}(c_k) = \frac{\Delta y_k}{\Delta x_k}$$

$$L_k = \sqrt{(\Delta x_k)^2 \left[1 + \left(\frac{\Delta y_k}{\Delta x_k} \right)^2 \right]}$$

$$\tilde{L} = \sum_{k=1}^n L_k = \sum_{k=1}^n \sqrt{\left(1 + \left(\frac{\Delta y_k}{\Delta x_k} \right)^2 \right)} \Delta x_k$$

$$= \sum_{k=1}^n \sqrt{1 + f'(c_k)^2} \Delta x_k$$

$$= \sum_{k=1}^n \sqrt{1 + (\hat{f}(c_k))^2} \Delta x_k$$

$$\lim_{n \rightarrow \infty} \hat{L} = \lim_{n \rightarrow \infty} \sum_{k=1}^n \sqrt{1 + (\hat{f}(c_k))^2} \Delta x_k$$

$$L = \int_a^b \sqrt{1 + (\hat{f}(x))^2} dx$$

Def

f cont. $[a, b]$ ✓
 f' cont. $[a, b]$ ✓

$$\frac{df}{dx} = \frac{dy}{dx}$$

length for the curve $y=f(x)$ is *

If f or f' not cont. on $[a, b] \Rightarrow$

$$L = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2} dy$$

Find length of H

Exp Find length of the curve

$$f(x) = \frac{x^3}{12} + \frac{1}{x}, \quad 1 \leq x \leq 4$$

$$L = \int_a^b \sqrt{1 + (f'(x))^2} dx$$

$$= \int_1^4 \sqrt{1 + \left(\frac{x^2}{4} - \frac{1}{x^2}\right)^2} dx$$

$$= \int_1^4 \sqrt{1 + \frac{x^4}{16} - 2 \left(\frac{x^2}{4}\right) \left(\frac{1}{x^2}\right) + \left(\frac{-1}{x^2}\right)^2} dx$$

$$= \int_1^4 \sqrt{1 + \frac{x^4}{16} - \frac{1}{2} + \frac{1}{x^4}} dx$$

$$= \int_1^4 \sqrt{\frac{x^4}{16} + \frac{1}{2} + \frac{1}{x^4}} dx$$

$x=0$??

f cont. on $[1, 4]$

$$f' = \frac{x^2}{4} - \frac{1}{x^2} \text{ cont. } [1, 4]$$

$$= \int_1^4 \sqrt{\left(\frac{x^2}{4} + \frac{1}{x^2}\right)} dx$$

$$= \int_1^4 \left(\frac{x^2}{4} + \frac{1}{x^2}\right) dx = \int_1^4 \left(\frac{x^2}{2} + x^{-2}\right) dx$$

$$= \frac{x^3}{3} - \frac{1}{x} \Big|_1^4 = \dots = 6$$

② Find arc length of $y = \left(\frac{x}{2}\right)^{\frac{2}{3}}$
from $x=0$ to $x=2$

$$y = \sqrt[3]{\left(\frac{x}{2}\right)^2} \quad [0, 2] \quad \text{cont.}$$

$$\dot{y} = \frac{x}{3} \left(\frac{x}{2}\right)^{-\frac{1}{3}} \left(\frac{1}{2}\right)$$

$$= \frac{1}{3} \sqrt[3]{\frac{1}{x/2}} \quad \text{discont at } x=0$$

$$y = \left(\frac{x}{2}\right)^{\frac{2}{3}}$$

$$\dot{y} = \frac{x}{2}$$

$$\left[x = 2 \quad y^{\frac{3}{2}} \right] [0, 1]$$

$$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$$

$$= \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2} dy$$

$$x=0 \Rightarrow y=0$$

$$x=0 \Rightarrow y=0$$

$$x=2 \Rightarrow y=1$$

$$= \int_0^1 \sqrt{1+9y} \, dy$$

$$u = 1 + 9y$$

$$du = 9 \, dy \Rightarrow dy = \frac{du}{9}$$

$$= \int_1^{10} \sqrt{u} \, \frac{du}{9}$$

when $y=0 \Rightarrow u=1$
 $y=1 \Rightarrow u=10$

$$= \frac{1}{9} \int_1^{10} u^{\frac{1}{2}} \, du = \frac{1}{9} \left[\frac{u^{\frac{3}{2}}}{\frac{3}{2}} \right]_1^{10} = \dots \approx 2.27$$

$$= \frac{2}{27} (10\sqrt{10} - 1)$$

$$\boxed{x = 2y^{\frac{3}{2}}} \quad [0,1]$$

x cont. on $[0,1]$

$$\left(\frac{dx}{dy} \right) = \dot{x} = 2 \cdot \frac{3}{2} y^{\frac{1}{2}} = 3\sqrt{y} \quad \text{cont. on } [0,1]$$

$$\left(\frac{dx}{dy} \right)^2 = 9y$$

Exp Find length of

$$y = \int_0^x \sqrt{\cos 2t} \, dt$$

from $x=0$ to $x = \frac{\pi}{4}$

$$L = \int_a^b \sqrt{1 + (f'(x))^2} \, dx$$

$$\dot{y} = \sqrt{\cos 2x}$$

$$(\dot{y})^2 = \cos 2x$$

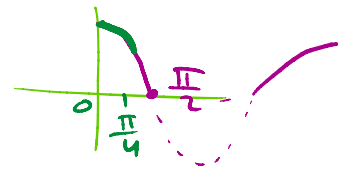
$$= \int_0^{\frac{\pi}{4}} \sqrt{1 + \cos 2x} \, dx$$

$$\cos 2x = 2\cos^2 x - 1$$

$$= \int_0^{\frac{\pi}{4}} \sqrt{1 + 2\cos^2 x - 1} \, dx$$

$$= \int_0^{\frac{\pi}{4}} \sqrt{2\cos^2 x} \, dx = \sqrt{2} \int_0^{\frac{\pi}{4}} |\cos x| \, dx$$

$$= \sqrt{2} \int_0^{\frac{\pi}{4}} \cos x \, dx$$



$$= \sqrt{2} \sin x \Big|_0^{\frac{\pi}{4}} = \sqrt{2} \left[\sin \frac{\pi}{4} - \sin 0 \right]$$

$$= \sqrt{2} \left[\frac{1}{\sqrt{2}} - 0 \right]$$

$$= 1$$

Q4 Find length of this curve

$$x = \frac{y^{\frac{3}{2}}}{3} - y^{\frac{1}{2}}, \quad y=1 \text{ to } y=9$$

d

$$x = \frac{\sqrt{y^3}}{3} - \sqrt{y} \text{ cont. on } [1, 9]$$

$$L = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2} dy$$

$$x = \frac{\sqrt{y^3}}{3} - \sqrt{y} \text{ cont. on } [1, 9]$$

$$x' = \frac{1}{2} \cdot \frac{1}{3} y^{\frac{1}{2}} - \frac{1}{2\sqrt{y}}$$

$$= \frac{1}{2} \sqrt{y} - \frac{1}{2\sqrt{y}}$$

$$\text{Cont. on } [1, 9] \\ = \frac{1}{2} (\sqrt{y} - \frac{1}{\sqrt{y}})$$

$$= \int_1^9 \sqrt{1 + \left(\frac{1}{2}\sqrt{y} - \frac{1}{2\sqrt{y}}\right)^2} dy$$

$$= \int_1^9 \sqrt{1 + \frac{1}{4} (\sqrt{y} - \frac{1}{\sqrt{y}})^2} dy$$

$$= \int_1^9 \sqrt{1 + \frac{1}{4} (y - 2 + \frac{1}{y})} dy$$

$$= \int_1^9 \sqrt{1 + \frac{y}{4} - \frac{1}{2} + \frac{1}{4y}} dy$$

$$= \int_1^9 \sqrt{\frac{y}{4} + \frac{1}{2} + \frac{1}{4y}} dy$$

$$= \int_1^9 \sqrt{\frac{1}{4} (y + 2 + \frac{1}{y})} dy$$

$$= \int_1^9 \sqrt{\frac{1}{4} \left(\sqrt{y} + \frac{1}{\sqrt{y}} \right)^2} dy$$

$$= \frac{1}{2} \int_1^9 \left(\sqrt{y} + \frac{1}{\sqrt{y}} \right) dy$$

$$= \frac{1}{2} \int_1^9 \left(y^{\frac{1}{2}} + y^{-\frac{1}{2}} \right) dy$$

$$= \frac{1}{2} \left(\frac{2}{3} y^{\frac{3}{2}} + 2 y^{\frac{1}{2}} \right) \Big|_1^9 = \dots = \frac{32}{3}$$

Q Find the curve passes through (1,1) whose length

$$L = \int_1^4 \sqrt{1 + \frac{1}{4x}} dx$$

How many curves are there?

$$L = \int_a^b \sqrt{1 + (f'(x))^2} dx$$

we need to find $f(x)$

$$(f'(x))^2 = \frac{1}{4x}$$

$$(f'(x))^2 = \frac{1}{4x}$$

$$f'(x) = \sqrt{\frac{1}{4x}} = \frac{1}{2\sqrt{x}}$$

$$f(x) = \int f'(x) dx = \int \frac{1}{2\sqrt{x}} = \sqrt{x} + c$$

← *ثابت التكامل*

$$\left. \begin{array}{l} f(1) = \sqrt{1} + c \\ 1 = 1 + c \\ \boxed{0 = c} \end{array} \right\} \Rightarrow$$

$$\boxed{f(x) = \sqrt{x}}$$

unique
since it is the
only curve passes
(1,1)