

Chapter 8

Interval Estimation

Learning Objectives

1. Know how to construct and interpret an interval estimate of a population mean and / or a population proportion.
2. Understand and be able to compute the margin of error.
3. Learn about the t distribution and its use in constructing an interval estimate when σ is unknown for a population mean.
4. Be able to determine the size of a simple random sample necessary to estimate a population mean and/or a population proportion with a specified margin of error.
5. Know the definition of the following terms:

confidence interval	margin of error
confidence coefficient	degrees of freedom
confidence level	

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Solutions:

1. a. $\sigma_{\bar{x}} = \sigma / \sqrt{n} = 5 / \sqrt{40} = .79$
 b. At 95%, $z\sigma / \sqrt{n} = 1.96(5 / \sqrt{40}) = 1.55$
2. a. $32 \pm 1.645 (6 / \sqrt{50})$
 32 ± 1.4 or 30.6 to 33.4
 b. $32 \pm 1.96 (6 / \sqrt{50})$
 32 ± 1.66 or 30.34 to 33.66
 c. $32 \pm 2.576 (6 / \sqrt{50})$
 32 ± 2.19 or 29.81 to 34.19
3. a. $80 \pm 1.96 (15 / \sqrt{60})$
 80 ± 3.8 or 76.2 to 83.8
 b. $80 \pm 1.96 (15 / \sqrt{120})$
 80 ± 2.68 or 77.32 to 82.68
 c. Larger sample provides a smaller margin of error.
4. Sample mean $\bar{x} = \frac{160+152}{2} = 156$
 Margin of Error = $160 - 156 = 4$
 $1.96(\sigma / \sqrt{n}) = 4$
 $\sqrt{n} = 1.96\sigma / 4 = 1.96(15) / 4 = 7.35$
 $n = (7.35)^2 = 54$
5. a. $1.96\sigma / \sqrt{n} = 1.96(5 / \sqrt{49}) = 1.40$
 b. 24.80 ± 1.40 or 23.40 to 26.20
6. $\bar{x} \pm z_{.025}(\sigma / \sqrt{n})$
 $8.5 \pm 1.96(3.5 / \sqrt{300})$
 $8.5 \pm .4$ or 8.1 to 8.9

7. $z_{.025}(\sigma / \sqrt{n}) = 1.96(4000 / \sqrt{60}) = 1012$

A larger sample size would be needed to reduce the margin of error. Section 8.3 can be used to show that the sample size would need to be increased to $n = 246$.

$$1.96(4000 / \sqrt{n}) = 500$$

Solving for n , shows $n = 246$

8. a. Since n is small, an assumption that the population is at least approximately normal is required.

b. $z_{.025}(\sigma / \sqrt{n}) = 1.96(5 / \sqrt{10}) = 3.1$

c. $z_{.005}(\sigma / \sqrt{n}) = 2.576(5 / \sqrt{10}) = 4.1$

9. $\bar{x} \pm z_{.025}(\sigma / \sqrt{n})$

$$3.37 \pm 1.96(.28 / \sqrt{120})$$

$$3.37 \pm .05 \text{ or } 3.32 \text{ to } 3.42$$

10. a. $\bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$

$$119,155 \pm 1.645(30,000 / \sqrt{80})$$

$$119,155 \pm 5517 \text{ or } \$113,638 \text{ to } \$124,672$$

b. $119,155 \pm 1.96(30,000 / \sqrt{80})$

$$119,155 \pm 6574 \text{ or } \$112,581 \text{ to } \$125,729$$

c. $119,155 \pm 2.576(30,000 / \sqrt{80})$

$$119,155 \pm 8640 \text{ or } \$110,515 \text{ to } \$127,795$$

d. The confidence interval gets wider as we increase our confidence level. We need a wider interval to be more confident that it will contain the population mean.

11. a. .025

b. $1 - .10 = .90$

c. .05

d. .01

e. $1 - 2(.025) = .95$

f. $1 - 2(.05) = .90$

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12. a. 2.179

b. -1.676

c. 2.457

d. Use .05 column, -1.708 and 1.708

e. Use .025 column, -2.014 and 2.014

13. a. $\bar{x} = \frac{\sum x_i}{n} = \frac{80}{8} = 10$

b.

x_i	$(x_i - \bar{x})$	$(x_i - \bar{x})^2$
10	0	0
8	-2	4
12	2	4
15	5	25
13	3	9
11	1	1
6	-4	16
5	-5	25
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$$s = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n-1}} = \sqrt{\frac{84}{7}} = 3.464$$

c. $t_{.025}(s/\sqrt{n}) = 2.365(3.464/\sqrt{8}) = 2.9$

d. $\bar{x} \pm t_{.025}(s/\sqrt{n})$

$$10 \pm 2.9 \text{ or } 7.1 \text{ to } 12.9$$

14. $\bar{x} \pm t_{\alpha/2}(s/\sqrt{n}) \quad df = 53$

a. $22.5 \pm 1.674(4.4/\sqrt{54})$

$$22.5 \pm 1 \text{ or } 21.5 \text{ to } 23.5$$

b. $22.5 \pm 2.006(4.4/\sqrt{54})$

$$22.5 \pm 1.2 \text{ or } 21.3 \text{ to } 23.7$$

c. $22.5 \pm 2.672(4.4/\sqrt{54})$

$$22.5 \pm 1.6 \text{ or } 20.9 \text{ to } 24.1$$

d. As the confidence level increases, there is a larger margin of error and a wider confidence interval.

15. $\bar{x} \pm t_{\alpha/2}(s/\sqrt{n})$

90% confidence $df = 64$ $t_{.05} = 1.669$

$19.5 \pm 1.669(5.2/\sqrt{65})$

19.5 ± 1.08 or 18.42 to 20.58

95% confidence $df = 64$ $t_{.025} = 1.998$

$19.5 \pm 1.998(5.2/\sqrt{65})$

19.5 ± 1.29 or 18.21 to 20.79

16. a. $t_{.025}(s/\sqrt{n})$ $df = 99$ $t_{.025} = 1.984$

$1.984(8.5/\sqrt{100}) = 1.69$

b. $\bar{x} \pm t_{.025}(s/\sqrt{n})$

49 ± 1.69 or 47.31 to 50.69

- c. At 95% confidence, the population mean flying time for Continental pilots is between 47.31 and 50.69 hours per month. This is clearly more flying time than the 36 hours for United pilots. With the greater flying time, Continental will use fewer pilots and have lower labor costs. United will require relatively more pilots and can be expected to have higher labor costs.

17. Using Minitab or Excel, $\bar{x} = 6.34$ and $s = 2.163$

$\bar{x} \pm t_{.025}(s/\sqrt{n})$ $df = 49$ $t_{.025} = 2.010$

$6.34 \pm 2.010(2.163/\sqrt{50})$

$6.34 \pm .61$ or 5.73 to 6.95

18. Using Minitab or Excel, $\bar{x} = 3.8$ and $s = 2.257$

a. $\bar{x} = 3.8$ minutes

b. $t_{.025}(s/\sqrt{n})$ $df = 29$ $t_{.025} = 2.045$

$2.045(2.257/\sqrt{30}) = .84$

c. $\bar{x} \pm t_{.025}(s/\sqrt{n})$

$3.8 \pm .84$ or 2.96 to 4.64

- d. There is a modest positive skewness in this data set. This can be expected to exist in the population. While the above results are acceptable, considering a larger sample next time would be a good strategy.

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19. a. $t_{.025}(s/\sqrt{n}) \quad df = 599$

Use ∞ df row, $t_{.025} = 1.96$

$$1.96 (175/\sqrt{600}) = 14$$

b. $\bar{x} \pm t_{.025}(s/\sqrt{n})$

$$649 \pm 14 \text{ or } 635 \text{ to } 663$$

- c. At 95% confidence, the population mean is between \$635 and \$663. This is slightly above the prior year's \$632 level, so holiday spending is increasing.

The point estimate of the slight increase is \$649 - \$632 = \$17 or 2.7% per household.

20. $\bar{x} = \Sigma x_i / n = 22$ minutes

$$s = \sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n-1}} = 1.12 \text{ minutes}$$

$$\bar{x} \pm t_{.025}(s/\sqrt{n}) \quad df = 19$$

$$22.00 \pm 2.093 (1.12/\sqrt{20})$$

$$22.00 \pm .52 \text{ or } 21.48 \text{ to } 22.52 \text{ minutes}$$

21. $\bar{x} = \frac{\Sigma x_i}{n} = \frac{2600}{20} = 130$ liters of alcoholic beverages

$$s = \sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n-1}} = \sqrt{\frac{81244}{20-1}} = 65.39$$

$$t_{.025} = 2.093 \quad df = 19$$

$$95\% \text{ confidence interval: } \bar{x} \pm t_{.025}(s/\sqrt{n})$$

$$130 \pm 2.093 (65.39/\sqrt{20})$$

$$130 \pm 30.60 \quad \text{or } 99.40 \text{ to } 160.60 \text{ liters per year}$$

22. a. $\bar{x} = 3.35$ So 3.35% is a point estimate of the mean return for the population.

b. $\bar{x} \pm t_{.025}(s/\sqrt{n}) \quad t_{.025} = 2.064 \quad df = 24$

From the sample, $s = 2.29$

$$3.35 \pm 2.064 (2.29/\sqrt{25})$$

95% Confidence Interval: 3.35 \pm .95 or 2.40% to 4.30%

$$23. \quad n = \frac{z_{.025}^2 \sigma^2}{E^2} = \frac{(1.96)^2 (40)^2}{10^2} = 61.47 \quad \text{Use } n = 62$$

$$24. \quad \text{a. Planning value of } \sigma = \text{Range}/4 = 36/4 = 9$$

$$\text{b. } n = \frac{z_{.025}^2 \sigma^2}{E^2} = \frac{(1.96)^2 (9)^2}{3^2} = 34.57 \quad \text{Use } n = 35$$

$$\text{c. } n = \frac{(1.96)^2 (9)^2}{2^2} = 77.79 \quad \text{Use } n = 78$$

$$25. \quad n = \frac{(1.96)^2 (6.84)^2}{(1.5)^2} = 79.88 \quad \text{Use } n = 80$$

$$n = \frac{(1.645)^2 (6.84)^2}{2^2} = 31.65 \quad \text{Use } n = 32$$

$$26. \quad \text{a. } n = \frac{z_{.025}^2 \sigma^2}{E^2} = \frac{(1.96)^2 (.15)^2}{(.07)^2} = 17.64 \quad \text{Use } 18.$$

If the normality assumption for the population appears questionable, this should be adjusted upward.

$$\text{b. } n = \frac{(1.96)^2 (.15)^2}{(.05)^2} = 34.6 \quad \text{Use } 35$$

$$\text{c. } n = \frac{(1.96)^2 (.15)^2}{(.03)^2} = 96.04 \quad \text{Use } 97$$

For reporting purposes, the newspaper might decide to round up to a sample size of 100.

$$27. \quad \text{Planning value } \sigma = \frac{45,000 - 30,000}{4} = 3750$$

$$\text{a. } n = \frac{z_{.025}^2 \sigma^2}{E^2} = \frac{(1.96)^2 (3750)^2}{(500)^2} = 216.09 \quad \text{Use } n = 217$$

$$\text{b. } n = \frac{(1.96)^2 (3750)^2}{(200)^2} = 1350.56 \quad \text{Use } n = 1351$$

$$\text{c. } n = \frac{(1.96)^2 (3750)^2}{(100)^2} = 5402.25 \quad \text{Use } n = 5403$$

d. Sampling 5403 college graduates to obtain the \$100 margin of error would be viewed as too expensive and too much effort by most researchers.

$$28. \quad \text{a. } n = \frac{z_{\alpha/2}^2 \sigma^2}{E^2} = \frac{(1.645)^2 (1100)^2}{(100)^2} = 327.43 \quad \text{Use } n = 328$$

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b. $n = \frac{(1.96)^2 (1100)^2}{(100)^2} = 464.83$ Use $n = 465$

c. $n = \frac{(2.576)^2 (1100)^2}{(100)^2} = 802.93$ Use $n = 803$

- d. The sample size gets larger as the confidence is increased. We would not recommend 99% confidence. The sample size must be increased by $137 = 465 - 328$ to go from 90% to 95%. This may be reasonable. However, increasing the sample size by $338 = 803 - 465$ to go from 95% to 99% would probably be viewed as too expensive and time consuming for the 4% gain in confidence.

29. a. $n = \frac{(1.96)^2 (6.25)^2}{2^2} = 37.52$ Use $n = 38$

b. $n = \frac{(1.96)^2 (6.25)^2}{1^2} = 150.06$ Use $n = 151$

30. Planning value $\sigma = \frac{60 - 5}{4} = 13.75$

$$n = \frac{z_{.025}^2 \sigma^2}{E^2} = \frac{(1.96)^2 (13.75)^2}{(3)^2} = 80.70 \quad \text{Use } n = 81$$

31. a. $\bar{p} = 100/400 = .25$

b. $\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = \sqrt{\frac{.25(.75)}{400}} = .0217$

c. $\bar{p} \pm z_{.025} \sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$

$$.25 \pm 1.96 (.0217)$$

$$.25 \pm .0424 \text{ or } .2076 \text{ to } .2924$$

32. a. $.70 \pm 1.645 \sqrt{\frac{.70(.30)}{800}}$

$$.70 \pm .0267 \text{ or } .6733 \text{ to } .7267$$

b. $.70 \pm 1.96 \sqrt{\frac{.70(.30)}{800}}$

$$.70 \pm .0318 \text{ or } .6682 \text{ to } .7318$$

33. $n = \frac{z_{.025}^2 P^*(1-P^*)}{E^2} = \frac{(1.96)^2 (.35)(.65)}{(.05)^2} = 349.59$ Use $n = 350$

34. Use planning value $p^* = .50$

$$n = \frac{(1.96)^2 (.50)(.50)}{(.03)^2} = 1067.11 \quad \text{Use } n = 1068$$

35. a. $\bar{p} = 281/611 = .4599 \quad (46\%)$

b. $z_{.05} \sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 1.645 \sqrt{\frac{.4599(1-.4599)}{611}} = .0332$

c. $\bar{p} \pm .0332$

$$.4599 \pm .0332 \text{ or } .4267 \text{ to } .4931$$

36. a. $\bar{p} = 46/200 = .23$

b. $\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = \sqrt{\frac{.23(1-.23)}{200}} = .0298$

$$\bar{p} \pm z_{.025} \sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

$$.23 \pm 1.96(.0298)$$

$$.23 \pm .0584 \quad \text{or } .1716 \text{ to } .2884$$

37. a. $\bar{p} = 473/1100 = .43$

b. $z_{.025} \sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 1.96 \sqrt{\frac{.43(1-.43)}{1100}} = .0293$

c. $\bar{p} \pm .0293$

$$.43 \pm .0293 \text{ or } .4007 \text{ to } .4593$$

- d. With roughly 40% to 46% of employees surveyed indicating strong dissatisfaction and with the high cost of finding successors, employers should take steps to improve employee satisfaction. The survey suggested employers may anticipate high employee turnover costs if employee dissatisfaction remains at the current level.

38. a. $\bar{p} = 29/162 = .1790$

b. $\bar{p} = 104/162 = .6420$

$$\text{Margin of error} = 1.96 \sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 1.96 \sqrt{\frac{(.642)(.358)}{162}} = .0738$$

$$\text{Confidence interval: } .6420 \pm .0738 \text{ or } .5682 \text{ to } .7158$$

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c. $n = \frac{1.96^2 (.642)(.358)}{(.05)^2} = 353.18$ Use $n = 354$

39. a. $n = \frac{z_{.025}^2 p^*(1-p^*)}{E^2} = \frac{(1.96)^2 (.156)(1-.156)}{(.03)^2} = 562$

b. $n = \frac{z_{.005}^2 p^*(1-p^*)}{E^2} = \frac{(2.576)^2 (.156)(1-.156)}{(.03)^2} = 970.77$ Use 971

40. $z_{.025} \sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 1.96 \sqrt{\frac{.86(1-.86)}{650}} = .0267$

$$\bar{p} \pm .0267$$

$$.86 \pm .0267 \text{ or } .8333 \text{ to } .8867$$

41. Margin of error = $z_{.025} \sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 1.96 \sqrt{\frac{.09(1-.09)}{1400}} = .0150$

$$.09 \pm .0150 \text{ or } .075 \text{ to } .1050$$

42. a. $\sqrt{\frac{p^*(1-p^*)}{n}} = \sqrt{\frac{.50(1-.50)}{491}} = .0226$

$$z_{.025} \sqrt{\frac{p^*(1-p^*)}{n}} = 1.96(.0226) = .0442$$

b. $n = \frac{z_{.025}^2 p^*(1-p^*)}{E^2}$

September $n = \frac{1.96^2 (.50)(1-.50)}{.04^2} = 600.25$ Use 601

October $n = \frac{1.96^2 (.50)(1-.50)}{.03^2} = 1067.11$ Use 1068

November $n = \frac{1.96^2 (.50)(1-.50)}{.02^2} = 2401$

Pre-Election $n = \frac{1.96^2 (.50)(1-.50)}{.01^2} = 9604$

43. a. Margin of Error = $z_{\alpha/2} \sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 1.96 \sqrt{\frac{(.53)(.47)}{1500}} = .0253$

$$95\% \text{ Confidence Interval: } .53 \pm .0253 \text{ or } .5047 \text{ to } .5553$$

b. Margin of Error = $1.96 \sqrt{\frac{(.31)(.69)}{1500}} = .0234$

95% Confidence Interval: $.31 \pm .0234$ or .2866 to .3334

c. Margin of Error = $1.96 \sqrt{\frac{(.05)(.95)}{1500}} = .0110$

95% Confidence Interval: $.05 \pm .0110$ or .039 to .061

- d. The margin of error decreases as \bar{p} gets smaller. If the margin of error for all of the interval estimates must be less than a given value (say .03), an estimate of the largest proportion should be used as a planning value. Using $p^* = .50$ as a planning value guarantees that the margin of error for all the interval estimates will be small enough.

44. a. Margin of error: $z_{.025} \frac{\sigma}{\sqrt{n}} = 1.96 \frac{15}{\sqrt{54}} = 4.00$

- b. Confidence interval: $\bar{x} \pm$ margin of error

33.77 ± 4.00 or \$29.77 to \$37.77

45. a. $\bar{x} \pm t_{.025} (s / \sqrt{n})$ $df = 63$ $t_{.025} = 1.998$

$252.45 \pm 1.998 (74.50 / \sqrt{64})$

252.45 ± 18.61 or \$233.84 to \$271.06

- b. Yes. the lower limit for the population mean at Niagara Falls is \$233.84 which is greater than \$215.60.

46. a. $t_{.025} (s / \sqrt{n})$ $df = 99$ $t_{.025} = 1.984$

$1.984 (4980 / \sqrt{100}) = 998$

- b. $\bar{x} \pm 998$

25467 ± 998 or \$24,479 to \$26,455

c. $3672(\$25,467) = \$93,514,824$

- d. *Harry Potter* beat *Lost World* by $\$93.5 - 72.1 = \21.4 million. This is a $21.4/72.1(100) = 30\%$ increase in the first weekend. The words “shatter the record” are justified.

47. a. From the sample of 30 stocks, we find $\bar{x} = 21.9$ and $s = 14.86$

A point estimate of the mean P/E ratio for NYSE stocks on January 19, 2004 is 21.9.

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$$\text{Margin of error} = t_{.025} \left(\frac{s}{\sqrt{n}} \right) = 2.045 \left(\frac{14.86}{\sqrt{30}} \right) = 5.5$$

95% Confidence Interval: 21.9 ± 5.5 or 16.4 to 27.4

- b. The point estimate is greater than 20 but the 95% confidence interval goes down to 16.4. So we would be hesitant to conclude that the population mean P/E ratio was greater than 20. Perhaps taking a larger sample would be in order.
- c. From the sample of 30 stocks, we find $\bar{p} = 21/30 = .70$
A point estimate of the proportion of NYSE stocks paying dividends is .70.

With $n\bar{p} = 30(.7) = 21$ and $n(1 - \bar{p}) = 30(.3) = 9$, we would be justified in using a normal distribution to construct a confidence interval. A 95% confidence interval is

$$\bar{p} \pm z_{.025} \sqrt{\frac{\bar{p}(1 - \bar{p})}{n}}$$

$$.7 \pm 1.96 \sqrt{\frac{(.7)(.3)}{30}}$$

$.7 \pm .16$ or .54 to .86

While the sample size is large enough to use the normal distribution approximation, the sample size is not large enough to provide much precision. The margin of error is larger than most people would like.

48.

Variable	N	Mean	StDev	SE Mean	95.0% CI
Time	150	14.000	3.838	0.313	(13.381, 14.619)

- a. $\bar{x} = 14$ minutes
- b. 13.381 to 14.619
- c. 7.5 hours = $7.5(60) = 450$ minutes per day

An average of $450/14 = 32$ reservations per day if no idle time. Assuming perhaps 15% idle time or time on something other than reservations, this could be reduced to 27 reservations per day.

- d. For large airlines, there are many telephone calls such as these per day. Using the online reservations would reduce the telephone reservation staff and payroll. Adding in a reduction in total benefit costs, a change to online reservations could provide a sizeable cost reduction for the airline.

49. a. Using a computer, $\bar{x} = 49.8$ minutes
- b. Using a computer, $s = 15.99$ minutes

c. $\bar{x} \pm t_{.025}(s/\sqrt{n}) \quad df = 199 \quad t_{.025} \approx 1.96$

$$49.8 \pm 1.96 (15.99/\sqrt{200})$$

$$49.8 \pm 2.22 \text{ or } 47.58 \text{ to } 52.02$$

50. $n = \frac{(2.33)^2(2.6)^2}{1^2} = 36.7 \quad \text{Use } n = 37$

51. $n = \frac{(1.96)^2(8)^2}{2^2} = 61.47 \quad \text{Use } n = 62$

$$n = \frac{(2.576)^2(8)^2}{2^2} = 106.17 \quad \text{Use } n = 107$$

52. $n = \frac{(1.96)^2(675)^2}{100^2} = 175.03 \quad \text{Use } n = 176$

53. a. $\bar{p} \pm 1.96 \sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$

$$.47 \pm 1.96 \sqrt{\frac{(.47)(.53)}{450}}$$

$$.47 \pm .0461 \text{ or } .4239 \text{ to } .5161$$

b. $.47 \pm 2.576 \sqrt{\frac{(.47)(.53)}{450}}$

$$.47 \pm .0606 \text{ or } .4094 \text{ to } .5306$$

c. The margin of error becomes larger.

54. a. $\bar{p} = 388/1250 = .31$

$$\bar{p} \pm 1.96 \sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

$$.31 \pm 1.96 \sqrt{\frac{(.31)(.69)}{1250}}$$

$$.31 \pm .0256 \text{ or } .2844 \text{ to } .3356$$

b. $.82 \pm 1.96 \sqrt{\frac{(.82)(.18)}{1250}}$

$$.82 \pm .0213 \text{ or } .7987 \text{ to } .8413$$

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- c. The margin of error is larger in part (a) because \bar{p} is closer to $1/2$. The closer \bar{p} is to $1/2$, the larger the margin of error becomes.

55. a. $\bar{p} = .74$

$$\text{Margin of error} = z_{.025} \sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 1.96 \sqrt{\frac{(.74)(.26)}{1677}} = .02$$

$$95\% \text{ Confidence Interval: } .74 \pm .02 \text{ or } .72 \text{ to } .76$$

b. $\bar{p} = .48$

$$\text{Margin of error} = z_{.005} \sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 2.576 \sqrt{\frac{(.48)(.52)}{1677}} = .03$$

$$95\% \text{ Confidence Interval: } .48 \pm .03 \text{ or } .45 \text{ to } .51$$

- c. The margin of error is larger in part b for two reasons. With $\bar{p} = .48$, the estimate of the standard error is larger. And $z_{.005} = 2.576$ is larger than $z_{.025} = 1.96$

56. a. $\bar{p} = 455/550 = .8273$

b. $\text{Margin of error} = 1.96 \sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 1.96 \sqrt{\frac{.8273(1-.8273)}{550}} = .0316$

$$95\% \text{ Confidence interval: } .8273 \pm .0316 \text{ or } .7957 \text{ to } .8589$$

57. a. $n = \frac{(1.96)^2 (.3)(.7)}{(.02)^2} = 2016.84$ Use $n = 2017$

b. $\bar{p} = 520/2017 = .2578$

c. $\bar{p} \pm 1.96 \sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$

$$.2578 \pm 1.96 \sqrt{\frac{(.2578)(.7422)}{2017}}$$

$$.2578 \pm .0191 \text{ or } .2387 \text{ to } .2769$$

58. a. $n = \frac{(2.33)^2 (.70)(.30)}{(.03)^2} = 1266.74$ Use $n = 1267$

b. $n = \frac{(2.33)^2 (.50)(.50)}{(.03)^2} = 1508.03$ Use $n = 1509$

59. a. $\bar{p} = 110/200 = .55$

$$0.55 \pm 1.96 \sqrt{\frac{(.55)(.45)}{200}}$$

$$.55 \pm .0689 \text{ or } .4811 \text{ to } .6189$$

b. $n = \frac{(1.96)^2 (.55)(.45)}{(.05)^2} = 380.32 \quad \text{Use } n = 381$

60. a. $\bar{p} = 618/1993 = .3101$

b. $\bar{p} \pm 1.96 \sqrt{\frac{\bar{p}(1-\bar{p})}{1993}}$

$$.3101 \pm 1.96 \sqrt{\frac{(.3101)(.6899)}{1993}}$$

$$.3101 \pm .0203 \text{ or } .2898 \text{ to } .3304$$

c. $n = \frac{z^2 p^* (1-p^*)}{E^2}$

$$z = \frac{(1.96)^2 (.3101)(.6899)}{(.01)^2} = 8218.64 \quad \text{Use } n = 8219$$

No; the sample appears unnecessarily large. The .02 margin of error reported in part (b) should provide adequate precision.