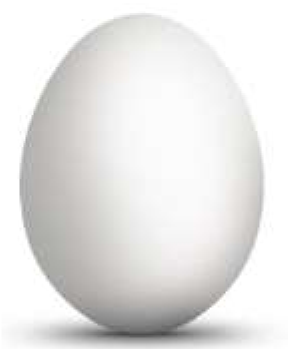
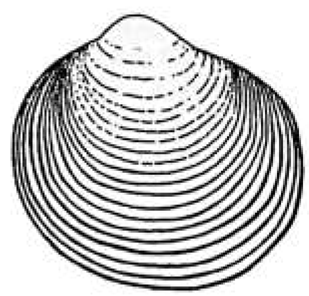
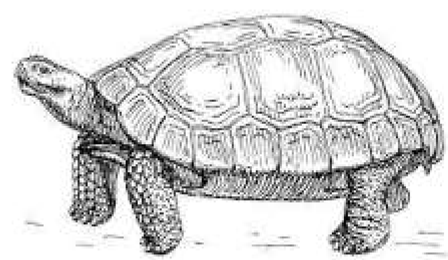
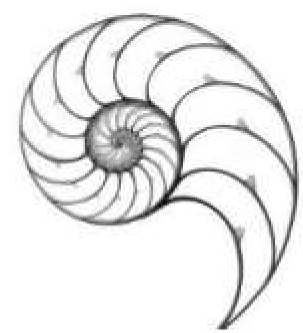
SHELLS AND ARCHES

Section 6.3

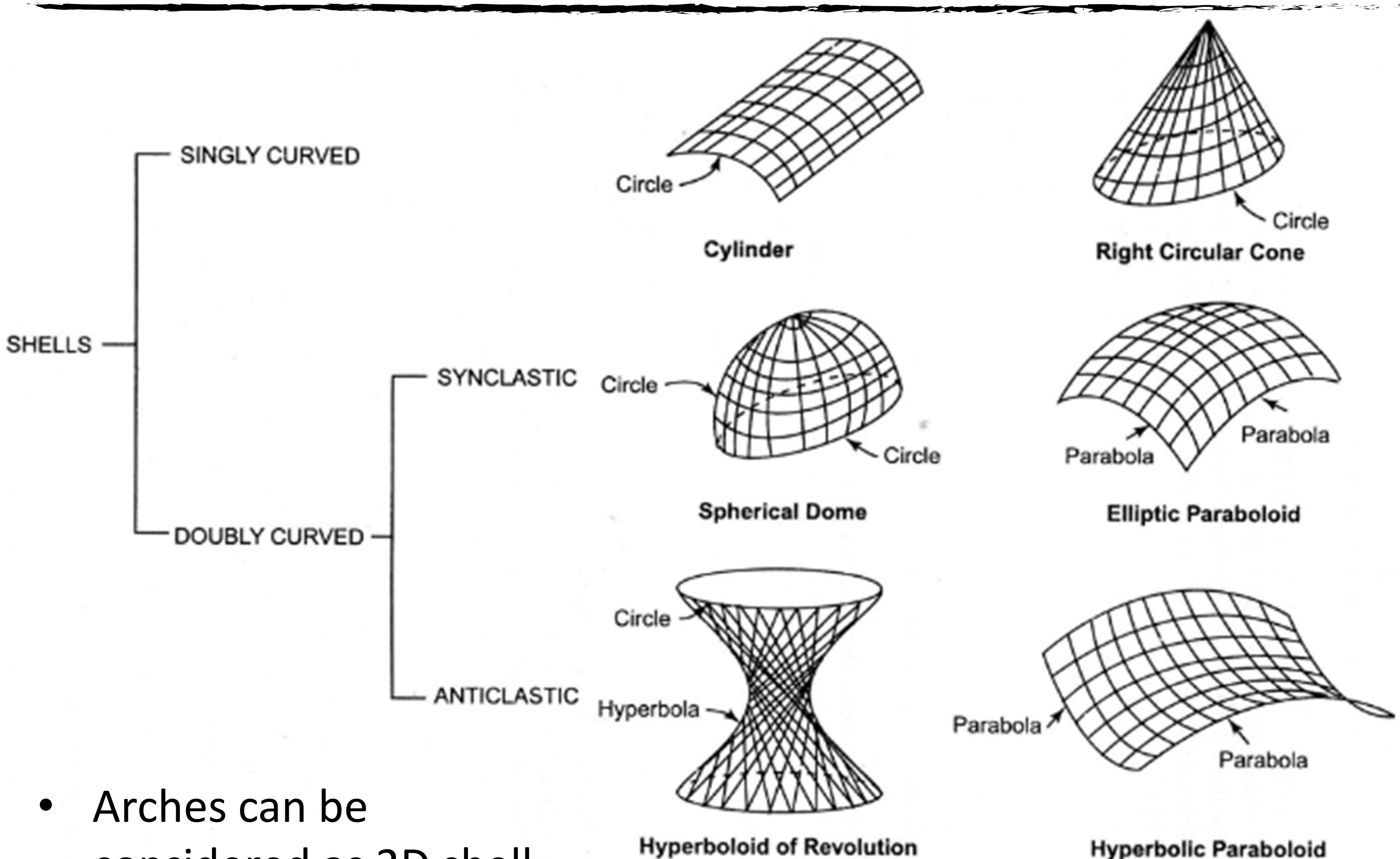
Introduction

- A shell structure is a thin curved membrane or slab usually of reinforced concrete that functions both as structure and covering. Shell possesses strength and rigidity due to its thin nature and curved form.
- Interestingly, many shell shapes are found in nature. Biological structures are never square or angular but are curved, and composed of shells and tubes.



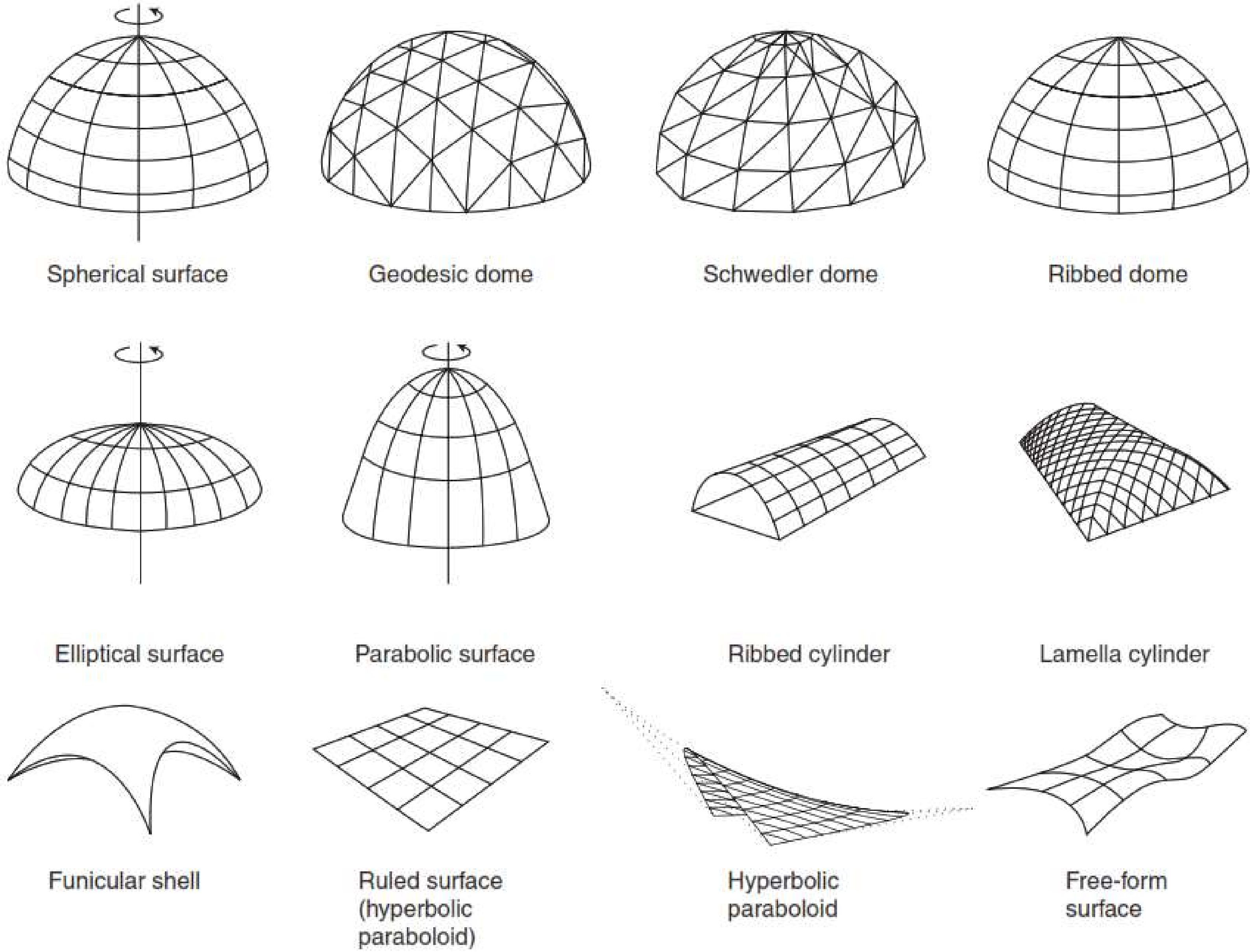




Classification of shells



- Arches can be considered as 2D shell

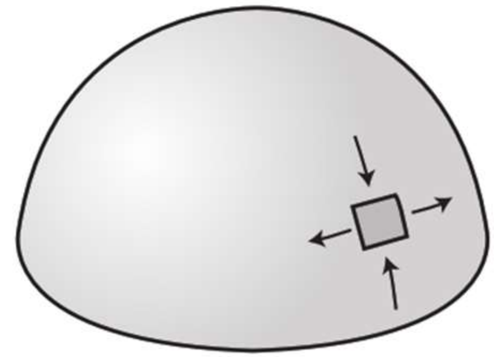
Examples of different surface shapes



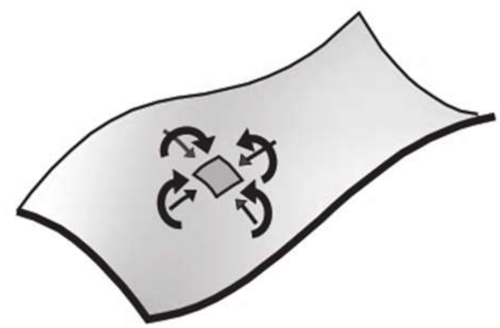
Structural Efficiency of Shells

Why shells are Structurally Efficient?

- The structural action of shells depends strongly on the ***shape or form*** in relation to the loading condition present.
- With shell action, primary internal forces that are developed in response to loadings lie within the plane of the surface and are in tension or compression.
- In-plane forces of this type are normally called membrane forces. Corresponding in-plane stresses are called membrane stresses. Significant bending is normally not present in shell structures that carry loads by membrane action.



(a) Thin shell structure: Only in-plane tension or compression membrane forces are developed.



(b) Complex shape that does not exhibit membrane action. Large bending moments are developed. Increased thicknesses or supporting framing systems are consequently needed.

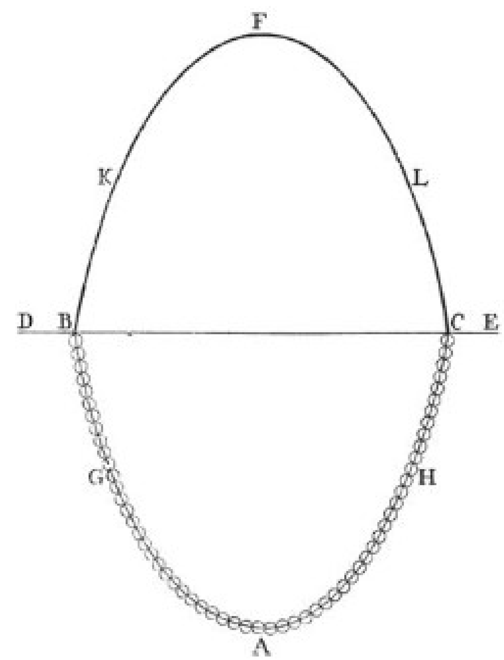
Structural Efficiency of Shells

- In structures exhibiting membrane or shell action, loads applied to shell surfaces are carried to the ground by the development of compressive, tensile, and shear stresses acting in the in-plane direction of the surface, therefore structural thicknesses can be made very thin. The thinness of the surface does not allow the development of appreciable bending resistance.
- Thin shell structures are uniquely suited to carrying distributed loads and find wide applications as roofs of buildings. They are, however, unsuited to carrying concentrated loads.
- As a consequence of carrying loads by in-plane forces (primarily tension and compression), Shell structures made of reinforced concrete can be very thin and still span great distances. Span-thickness ratios of 400 or 500 are not uncommon; for example, 8 cm thicknesses are possible for domes spanning 30 to 38 m.

Finding shape: The Concept

- The challenge of shell design is to find the appropriate form for the given problem. And the joy of shell design is that an infinite number of structural forms are waiting to be discovered. Even highly constrained boundary conditions can still lead to a vastly rich landscape of forms to explore.

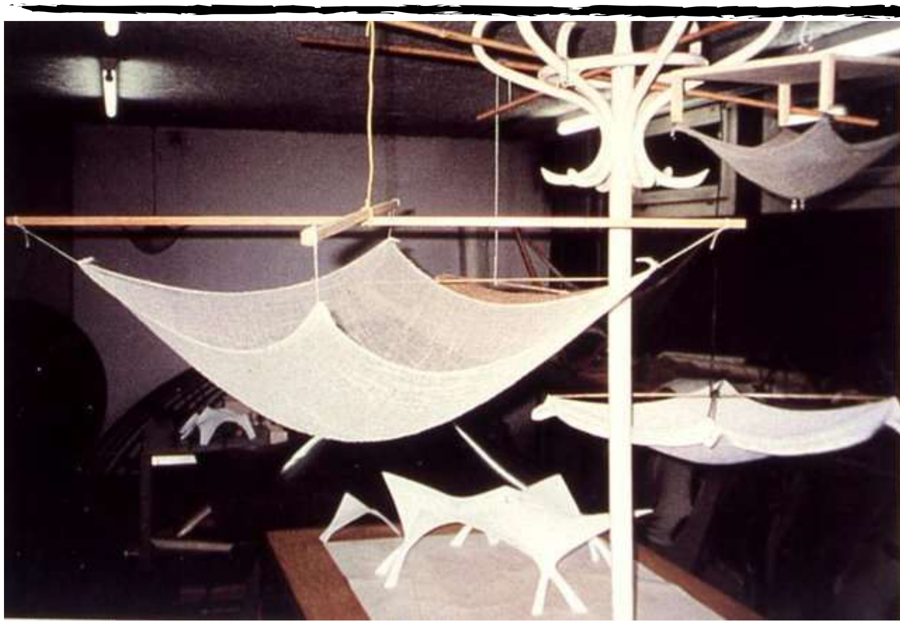
- The earliest example of structural form finding of shells was formulated by Robert Hooke (1635–1703). The idea is simple: invert the shape of the hanging chain, which by definition is in pure tension and free of bending, to obtain the equivalent arch that acts in pure compression (‘Hooke’s law of inversion’).



Finding shape: The Concept

- The form of the ideal arch will depend on the applied loading. For a chain of constant weight per unit length, the shape of a hanging chain acting under self-weight is a catenary. But if the load is uniformly distributed horizontally, the ideal arch would take the form of a parabola, and the chain would take different geometries according to the loading.
- Three-dimensional funicular systems are considerably more complex because of the multiple load paths that are possible. Unlike the two-dimensional arch, the three-dimensional shell can carry a wide range of different loadings through membrane behavior without introducing bending.
- However, a three-dimensional model of intersecting chains could be created. This hanging model could be used to design a discrete shell, in which elements are connected at nodes, or the model could be used to help define a continuous surface.

Finding shape: The Concept



- Physical models by Isler

- Antoni Gaudi, designed the Sagrada Familia from a series of hanging chains

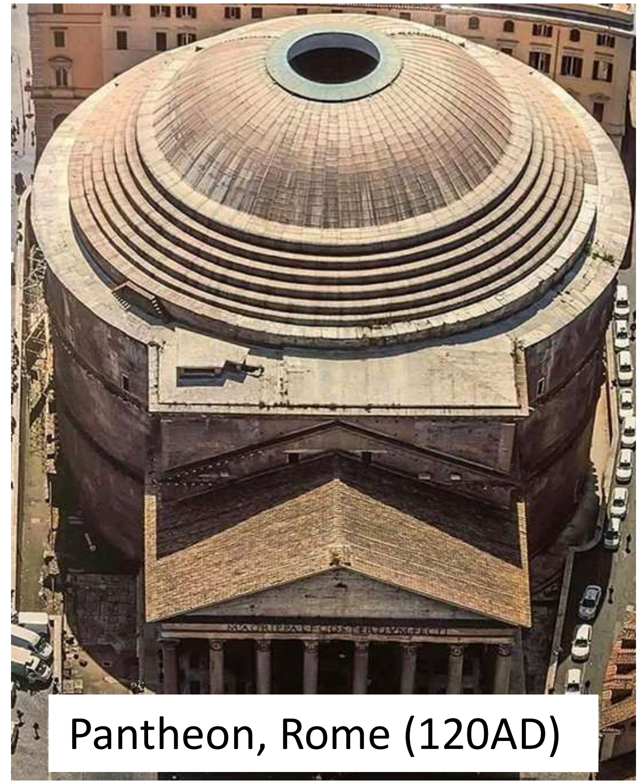
Finding shape: The Concept

Hanging-chain model of the Multihalle in Mannheim

Model material shall be carefully selected

Spherical shells - Domes

- A dome is the simplest type of synclastic shell. It is used as a roof and ceiling, usually to cover long spans.
- From the architectural element point of view dome resembles the hollow hemisphere.
- Dome has been used in various ways in Classical European Architecture, Islamic Architecture and in the Modern Architecture as well.



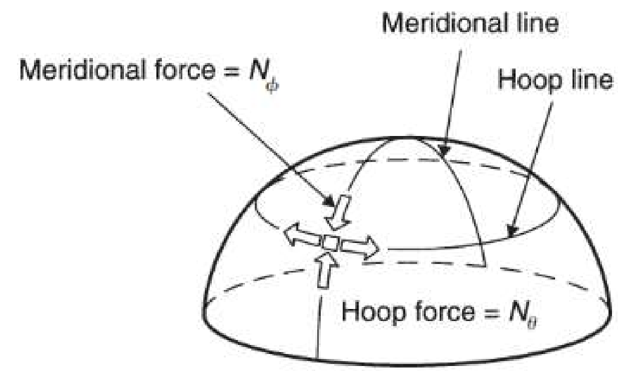
Pantheon, Rome (120AD)

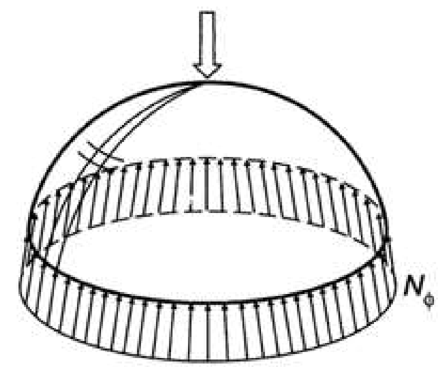
Types of Forces in Spherical Shells

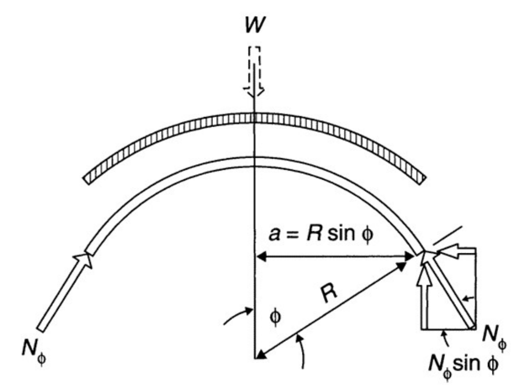
- Subjected to a vertical load, the thin spherical shell surface acts as a membrane and two in-plane perpendicular forces develop. These are the **meridional** force and the **hoop** force.

- **The meridional forces** are always in compression, indicating that the dome acts similar to a funicular arch in this direction.

- The sum of the vertical components of the in-plane meridional forces at a specific section equal to the sum of the applied force in that section.



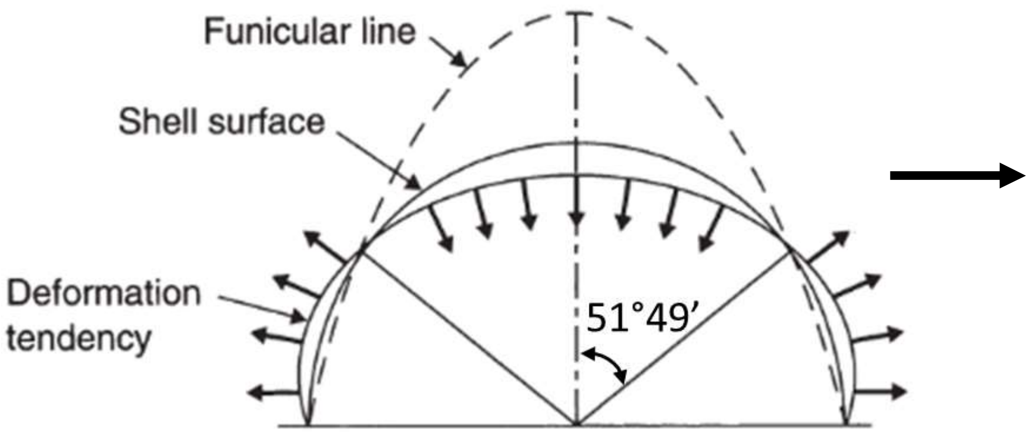


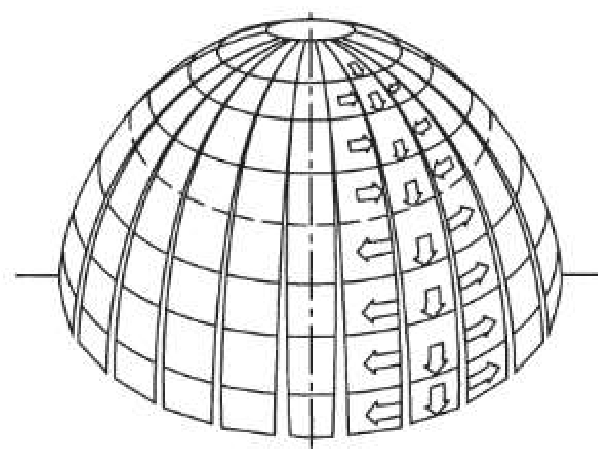


Types of Forces in Spherical Shells

- **Hoop force**. Based on the deflected shape, the hoop forces are at compression on the top of the dome and at tension on the bottom.

- In a semi-spherical dome, the transition occur at an angle of 51°49’, as measured from the perpendicular. from the perpendicular. Shells cut off above this angle develop compression stresses only in their surfaces, whereas deeper shells can develop tension stresses in the hoop.



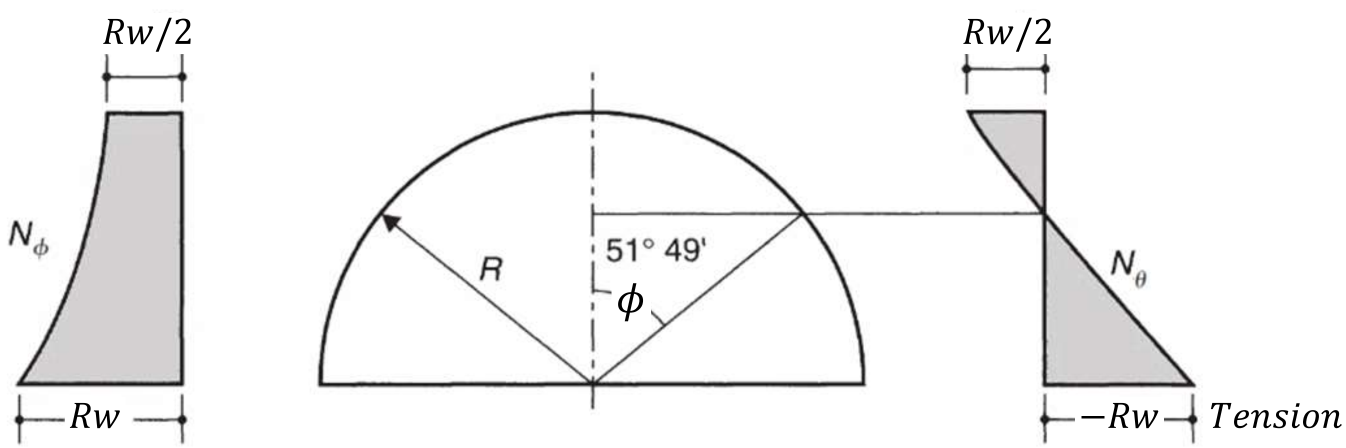


Deflected shape of a semi-spherical dome under UDL

Types of Forces in Spherical Shells

- **Distribution of Forces**. The figure below shows the distribution of meridional and hoop forces.

$$N_\phi = \frac{Rw}{1 + \cos\phi} \qquad N_\theta = Rw\left(-\frac{1}{1 + \cos\phi} + \cos\phi\right)$$

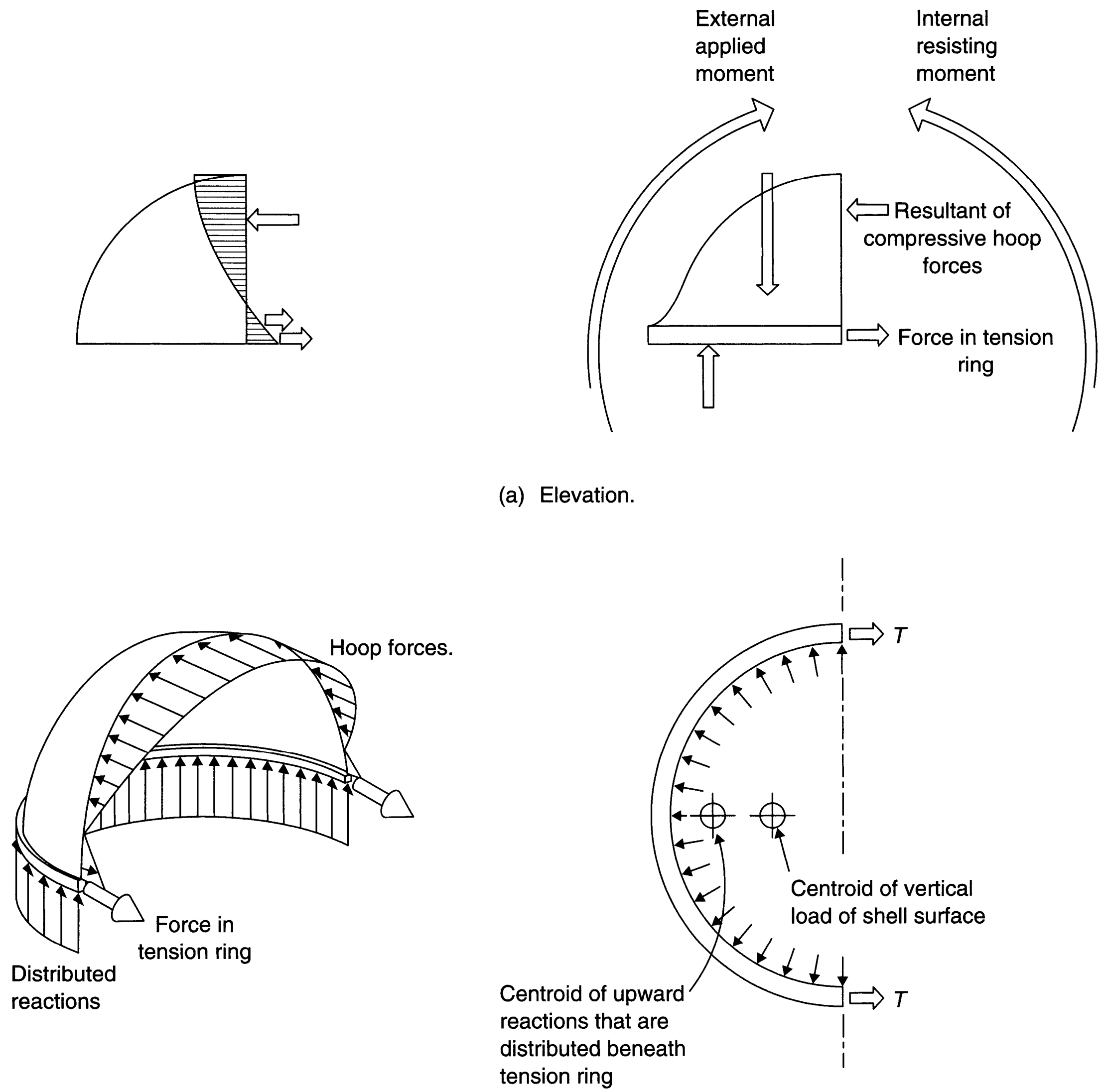


(a) Meridional forces (b) Hoop forces

Forces distribution in a spherical dome under a uniformly distributed load

Types of Forces in Spherical Shells

- Alternative approach to understand the overall behavior of a dome-and-ring assembly
- Note that concentrated loads should be avoided on shells as failure would occur if the shell surface could offer no bending resistance



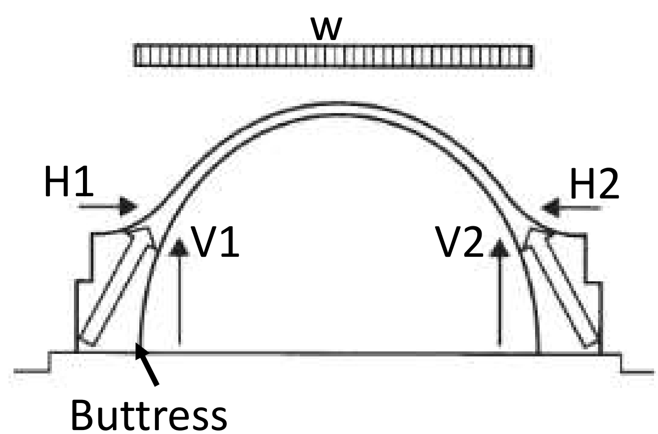
(b) Free-body diagram.

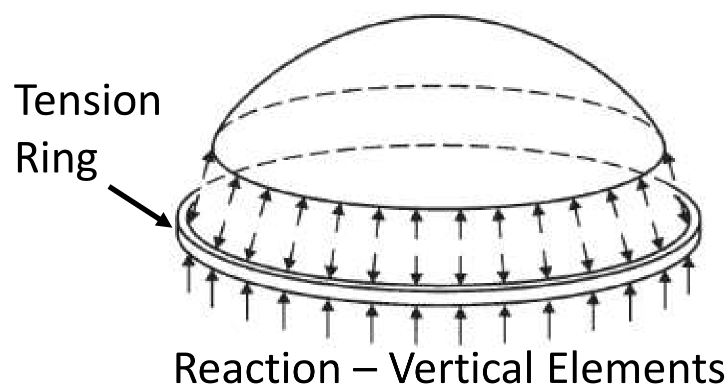
(c) Plan.

Support Conditions

- A key design factor in a shell of revolution is the type of boundary or support conditions. Special mechanisms are required to absorb the horizontal thrusts caused by meridional in-plane forces at the shell’s lower edge.

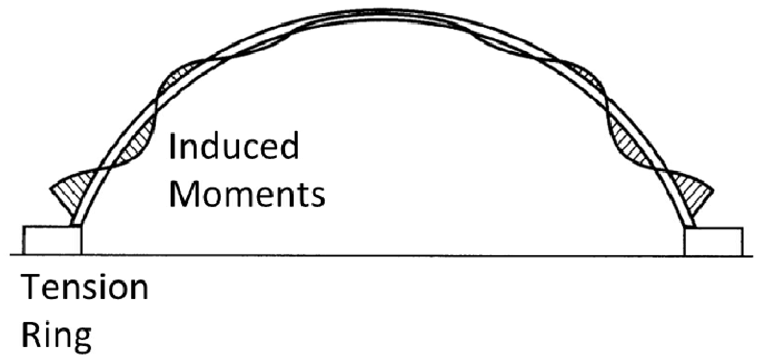
- In the case of a dome, this can be achieved through
  - A circular buttress system that can resist the huge thrust.
  - A planar circular ring, known as a tension ring, can be placed around the dome's base to restrain the outward components of the meridional forces, and only downward loads will be transformed to the vertical elements

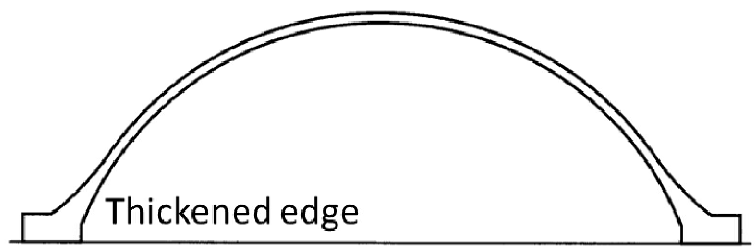


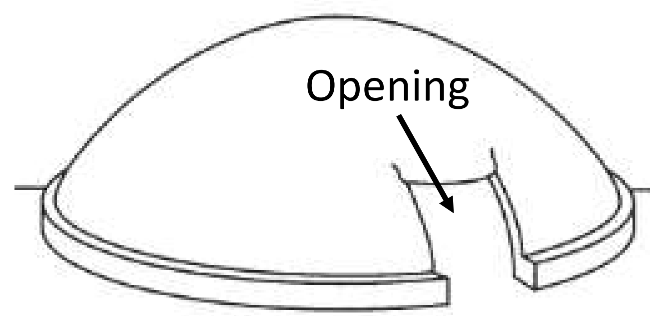


Support Condition

- The use of a tension ring does introduce bending at the intersection between the ring and the shell surface. However, this bending quickly dissipates within the shell, leaving most of the shell's surface unaffected.
- To resist the induced moments, the shell edge can be stiffened and reinforced to accommodate.
- The tension ring must be continuous around the lower perimeter of the dome; otherwise, it will be ineffective.



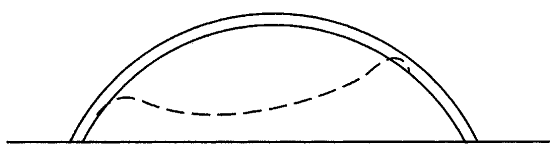




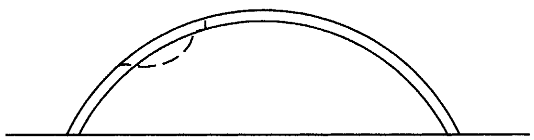
Discontinuous Tension Ring

Buckling of Domes

- A key concern in dome design is preventing premature and catastrophic failure due to buckling or surface instability. There are two recognized modes of buckling: snap-through buckling and local buckling.



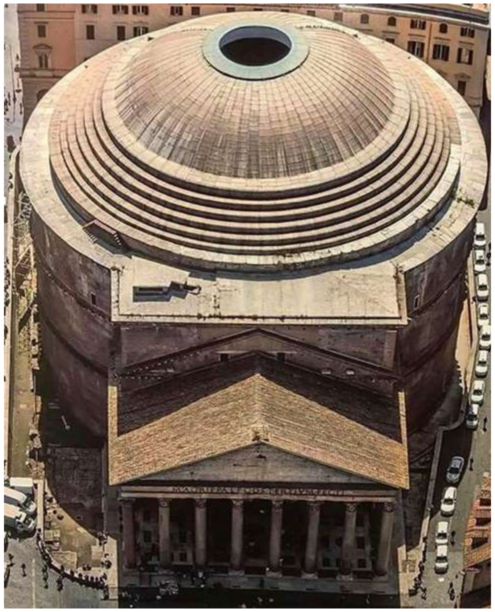
(a) Snap-through buckling: The entire shell can buckle inward under external loads. Shells with flatter curvatures are particularly vulnerable to this type of buckling. Increasing the shell's curvature decreases the likelihood of snap-through buckling.

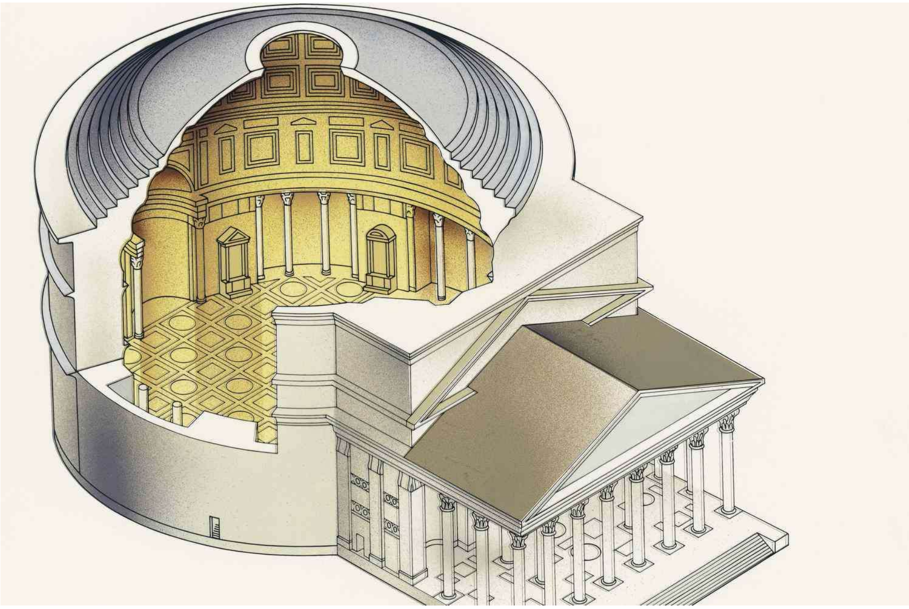


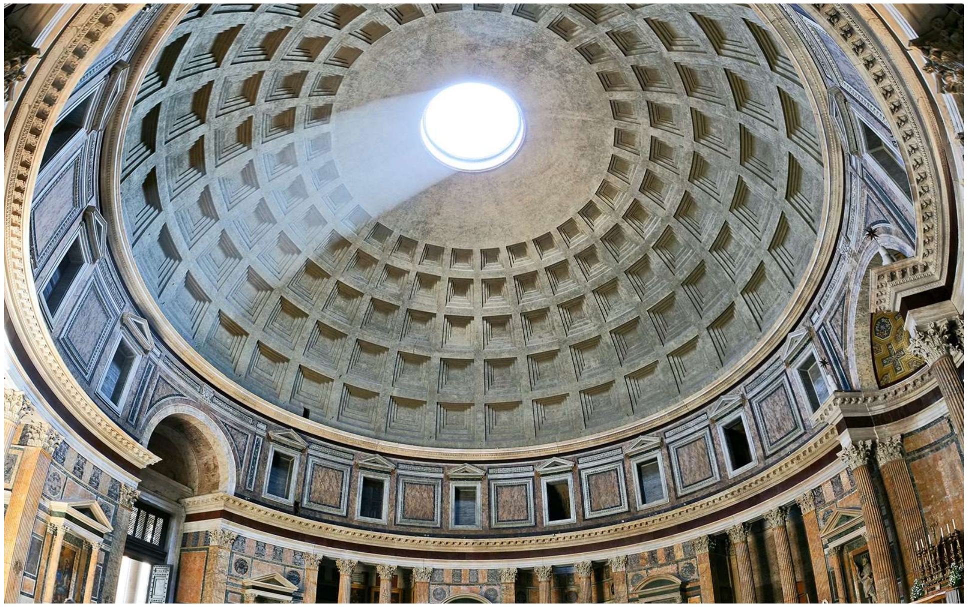
(b) Local buckling: Instead of the entire shell buckling, a section of the shell may buckle inward. Similar to snap-through buckling, sharper shell curvatures reduce the likelihood of local buckling.

Historical Domes - The Pantheon

The dome was constructed during the reign of Emperor Hadrian (AD 75–138) and is still in use today. With a diameter of 43.30 meters, it remained the largest dome in the world until modern times and was the first to be built using concrete.





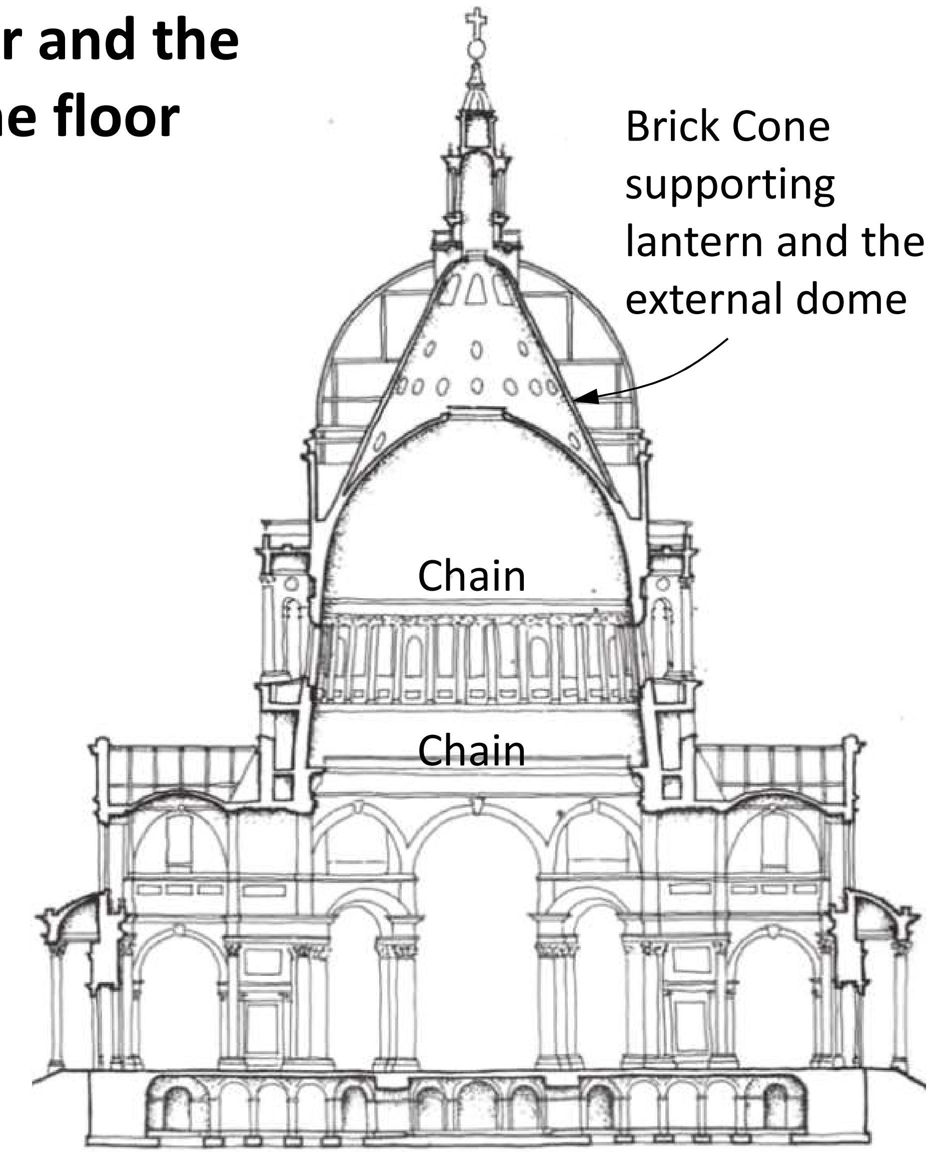


St. Peter’s Dome - Rome

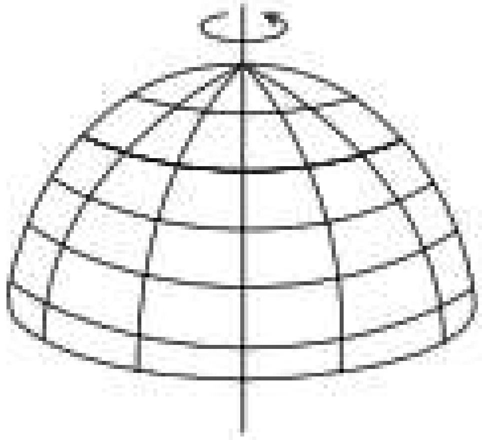
Completed in 1590, the dome is 42.52 m in diameter, and its apex is 136.6 m above floor level.

Dome of St. Paul's Cathedral

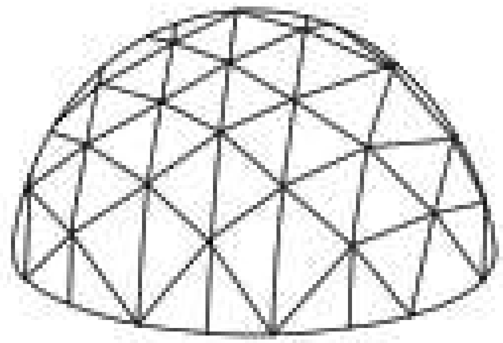
The dome is 112 ft in diameter and the cross on top is 365 ft above the floor



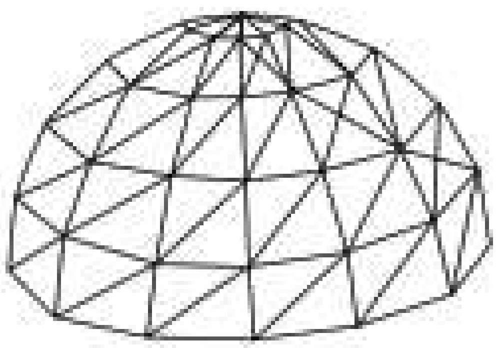
Modern Domes



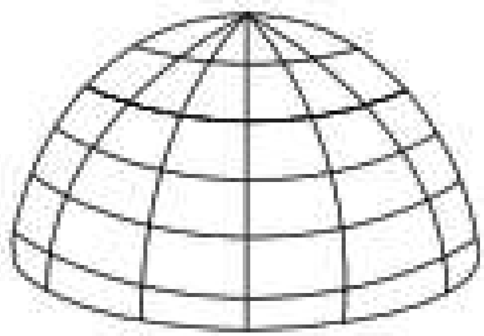
Spherical surface



Geodesic dome



Schwedler dome

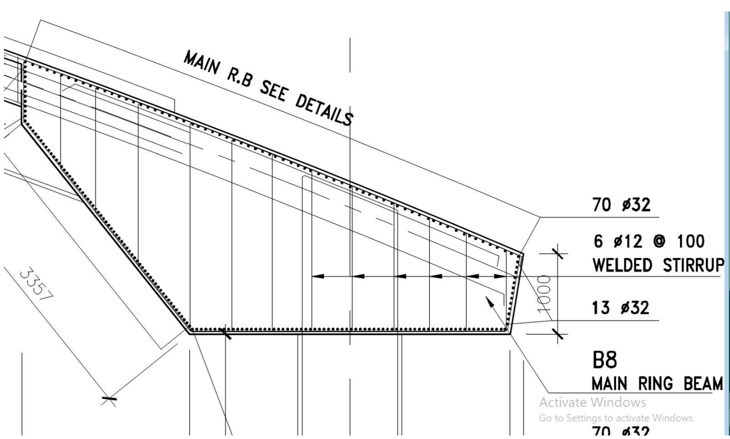


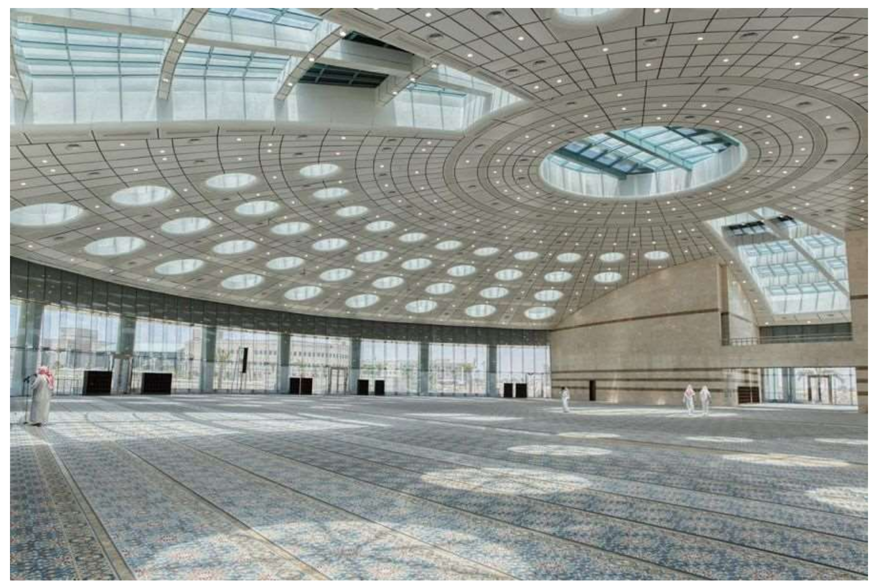
Ribbed dome

Domes in Architecture

University of Tabuk Mosque

The roof is made of a ribbed reinforced concrete dome about 90 m in diameter. The ribs are reinforced concrete beams 30X60 m and toped by 10 cm slab, and supported by irregular huge main Ring beam which is supported by 1.3 m diameter columns.

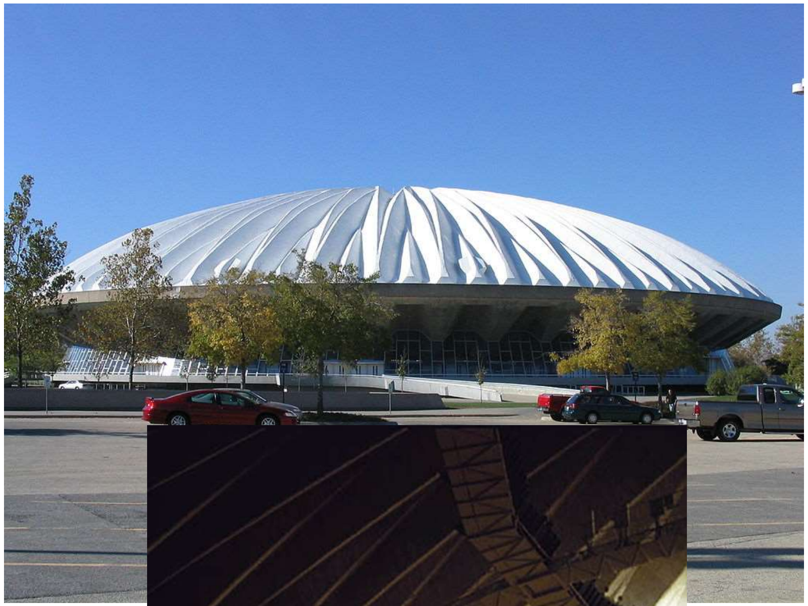


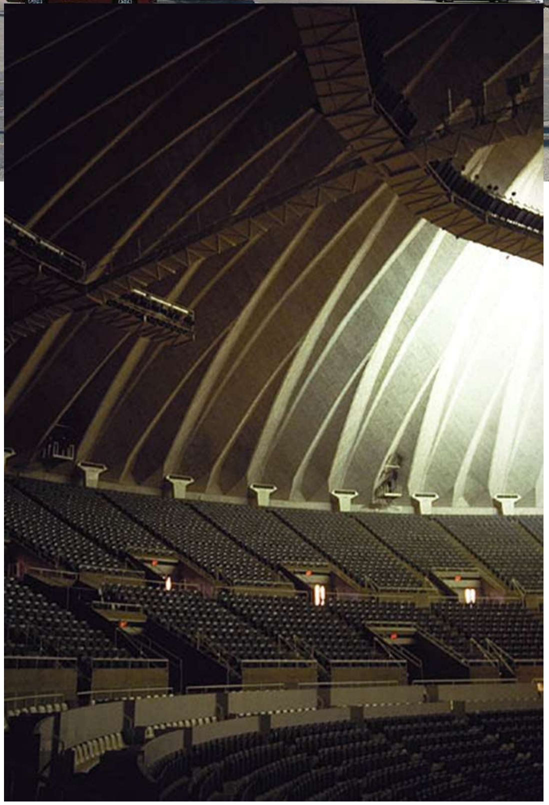


Domes in Architecture

Assembly Hall, University of Illinois

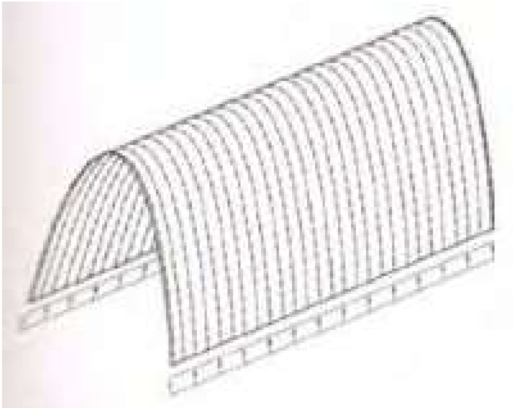
120 m diameter ribbed reinforced concrete dome. The ribs are folded-plate construction. Dome was cast on falsework, then the ring beam on which it rests was prestressed by wrapping it with steel wire under high tension until the dome became self-supporting.

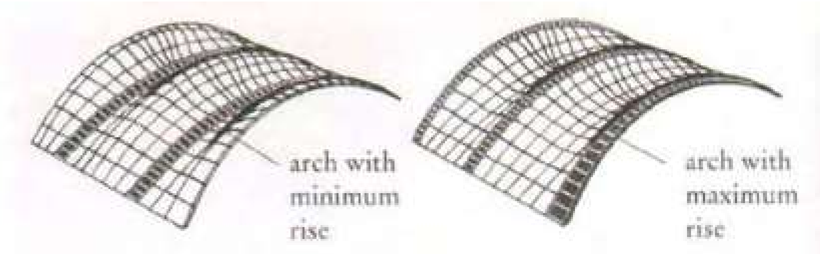




Vault Structures

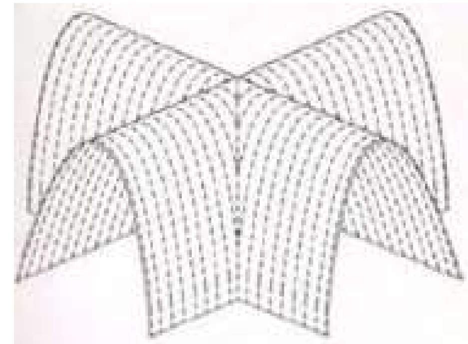
- A vault can be created with a series of parallel arches between which a secondary structure is inserted. If the arches are brought together to the point of touching, making the secondary structure unnecessary, the result will be a vault (more precisely, a barrel vault).
- The use of modern materials like reinforced concrete, sheet metal, and composite materials permits the construction of sufficiently stable vaults by corrugating the surface leading to significant savings on material.



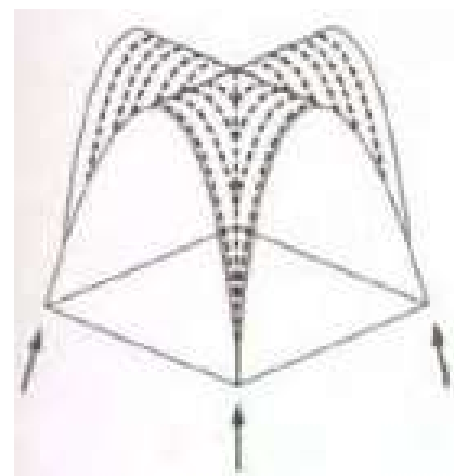


Vault Structures

- By crossing two perpendicular barrel vaults and removing the portions blocking the passage below the point of intersection at the top, a new structural figure is obtained, known as the groin vault (or cross vault).



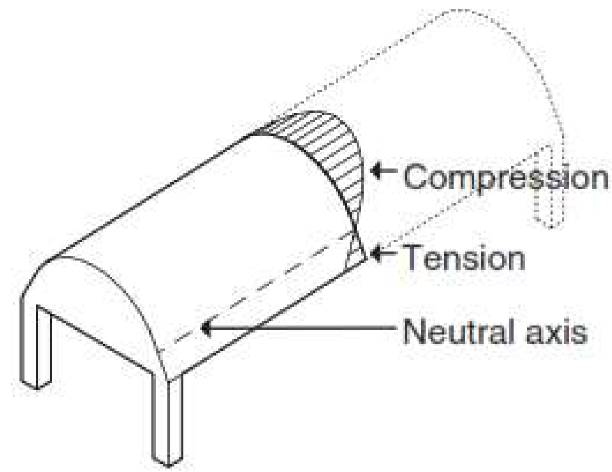
- All the elements discharge their thrust to the ground at just four points. This means that a groin vault, unlike a barrel vault (which must be supported by a continuous structure) can be placed on single-point supports.
- Groin vaults are usually much more stable than barrel vaults. This is because the two vault elements can cooperate to stabilize each other.



Groin Vault

Cylindrical Shells

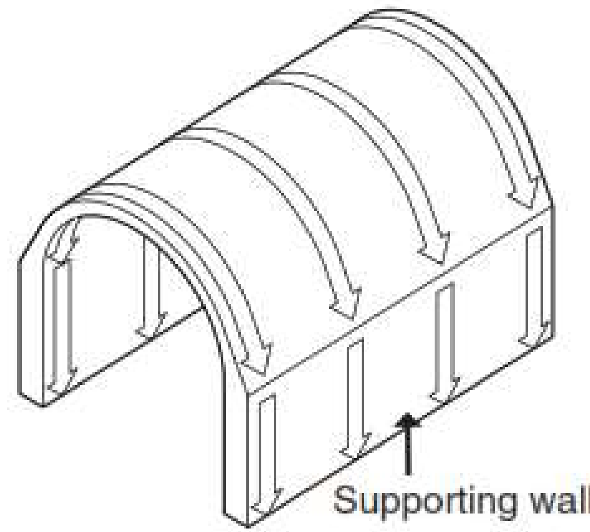
- Cylindrical shells are curved structures that lie horizontally, extending lengthwise over a considerable distance in comparison to the thickness of their cross-sectional surface.
- Cylindrical shells function similarly to beams, with bending stresses developing along the surface in the direction of the span. These stresses vary linearly from the top to the bottom of the overall effective cross-section.

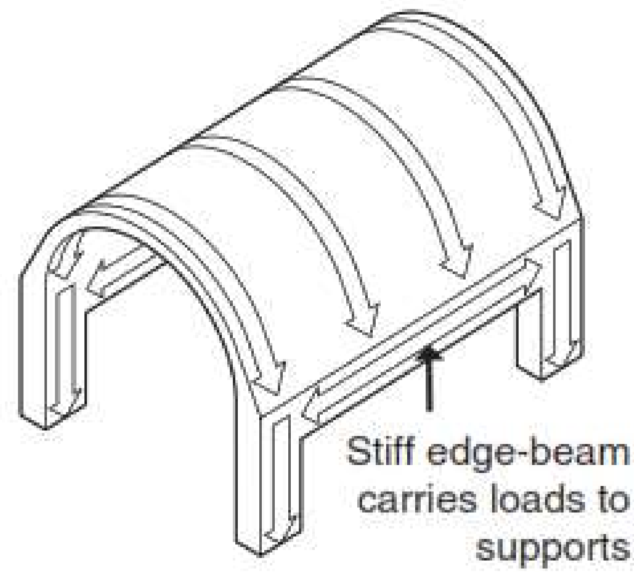


- The purpose of this form is to take advantage of its cross-section, which has a large moment of inertia. This allows for achieving larger spans, supporting greater loads, and/or constructing a much thinner structure than would typically be possible.

Barrel vault

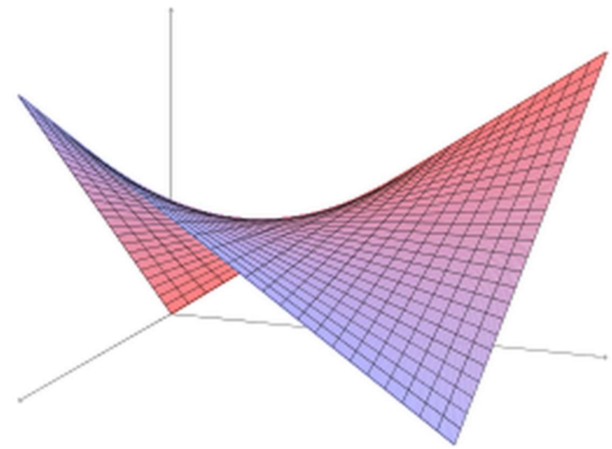
- Barrel vault (the semicircular vault), although it shares a similar overall geometry with the cylindrical shell, is fundamentally supported in a different manner along its two longitudinal edges.
- A barrel vault carries loads in an arch-like manner across its transverse section, with compression stresses following the arched profile. Bending stresses are also typically present, but they act only across the thickness of the vault surface.





Hyperbolic Paraboloid Shells

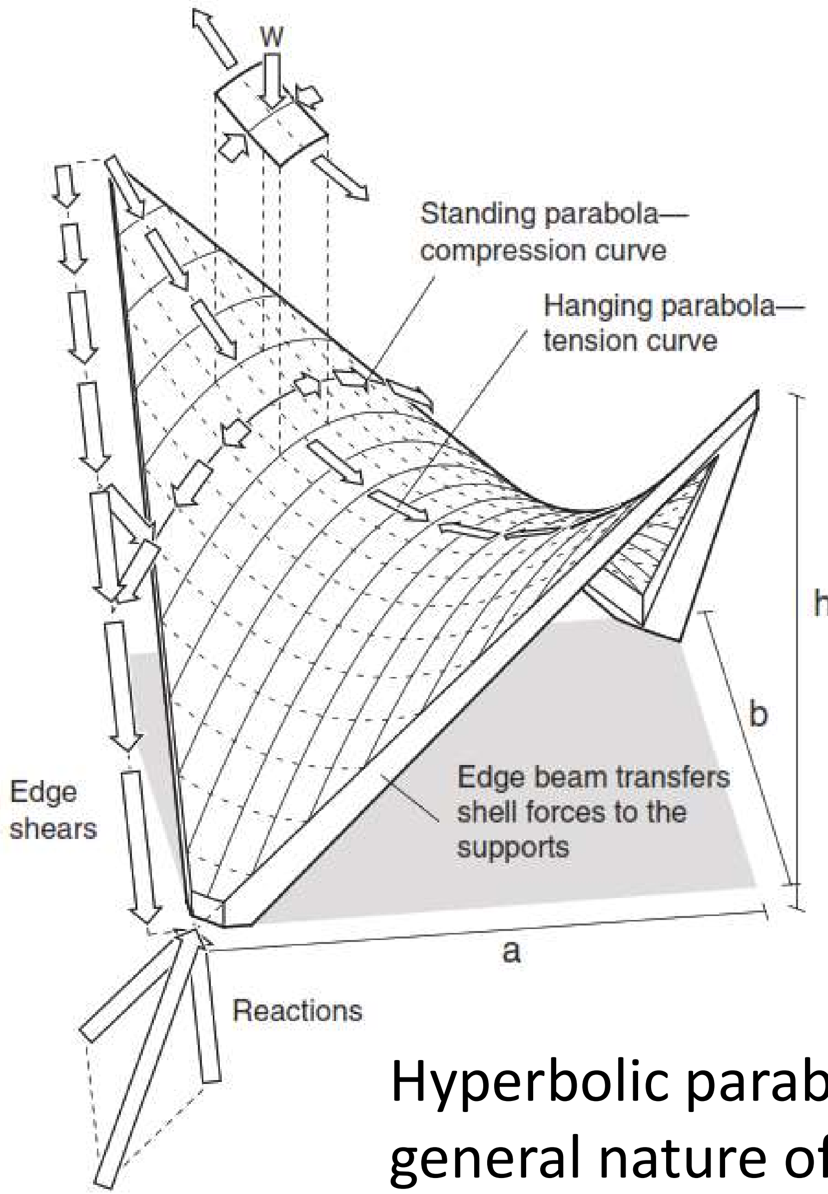
- Hyperbolic Paraboloids resemble the shape of a saddle formed by the combination of concave and convex surfaces. They are doubly curved surfaces with negative Gaussian curvature.



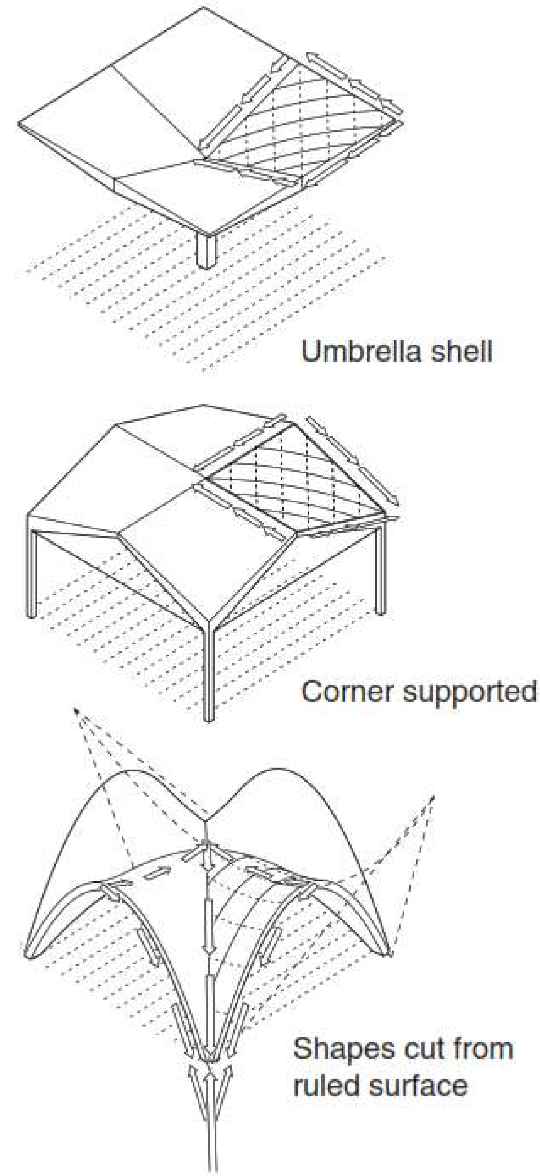
- This type of shell structure can be built to what appears to be the ultimate in the lightness of construction, minimum reinforcing, and ease of moving forms.

- If the edge conditions can offer restraint (i.e., foundations or stiff edge beams), an arch-like action will exist in regions of convex curvature and a cable-like action in regions of concave curvature. The stress field in the plate is thus compressive in one direction and tensile in the perpendicular direction, each of which is at 45° to the original straight-line generators.

Hyperbolic Paraboloid Shells



Hyperbolic paraboloid - The general nature of the forces



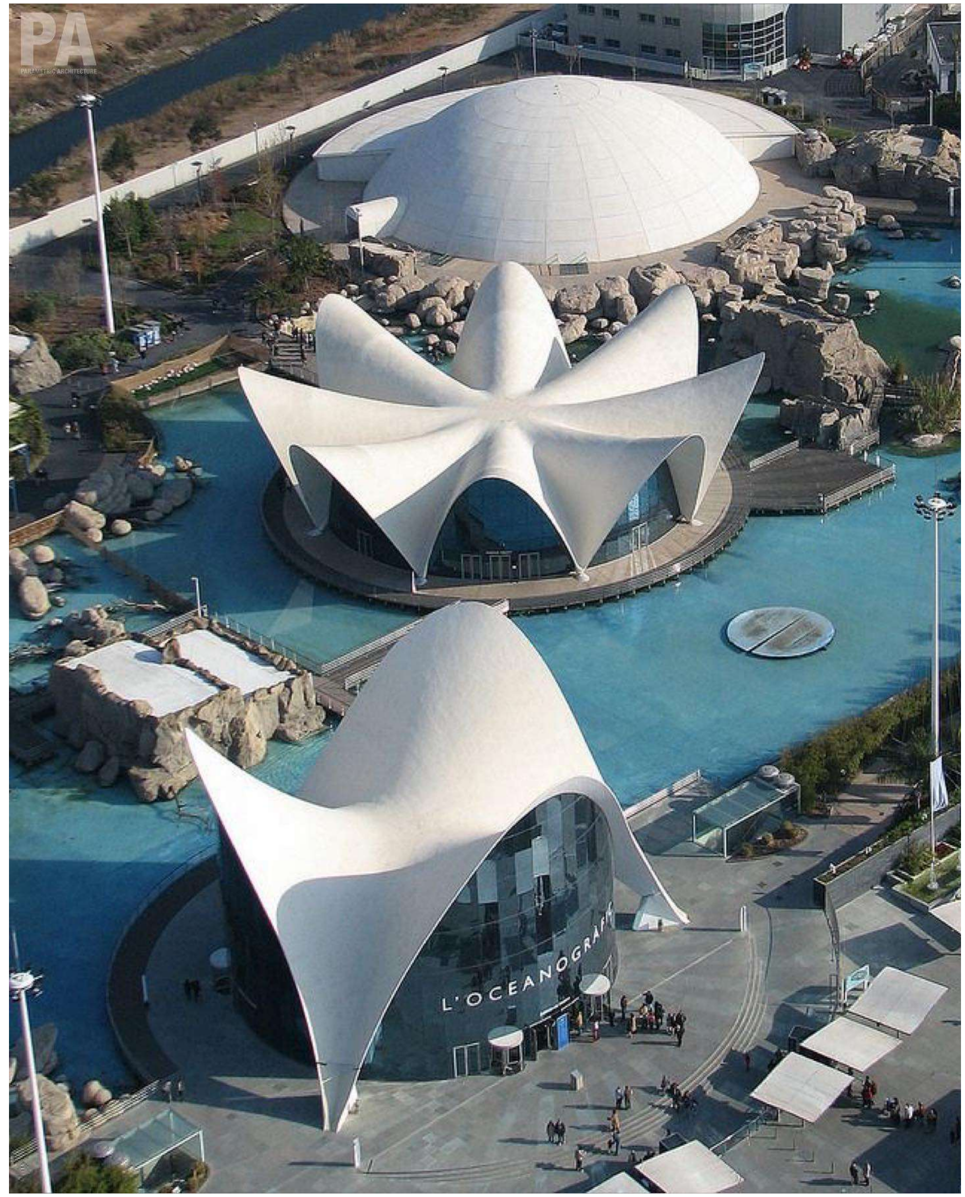
Application of Shells in Architecture

<u>the L´Oceanogràfic in Valencia</u>

Hyperbolic parabola structures of the L´Oceanogràfic in Valencia, Spain designed by Felix Candela.

Candela used thin-shell reinforced concrete to create his signature hyperbolic parabola structures.

L'Oceanographic was his final project, which was completed in 1997.



Bacardí Factory in Cuautitlán, Mexico

Félix Candela

The factory roof consisted of three adjacent hyperbolic paraboloid groined vaults 4cm. thick and 26 m square in plan with 2.5 m. overhangs on each side.

