Frequency Response

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Frequency Response.

$G(j\omega) = M_G \angle \phi_G$, where.

M_G : the magnitude of $G(j\omega)$

ϕ_G : the angle of $G(j\omega)$

$M_G \angle \phi_G$: The frequency response of the system &

$$G(j\omega) = G(s)\Big|_{s \to j\omega}.$$

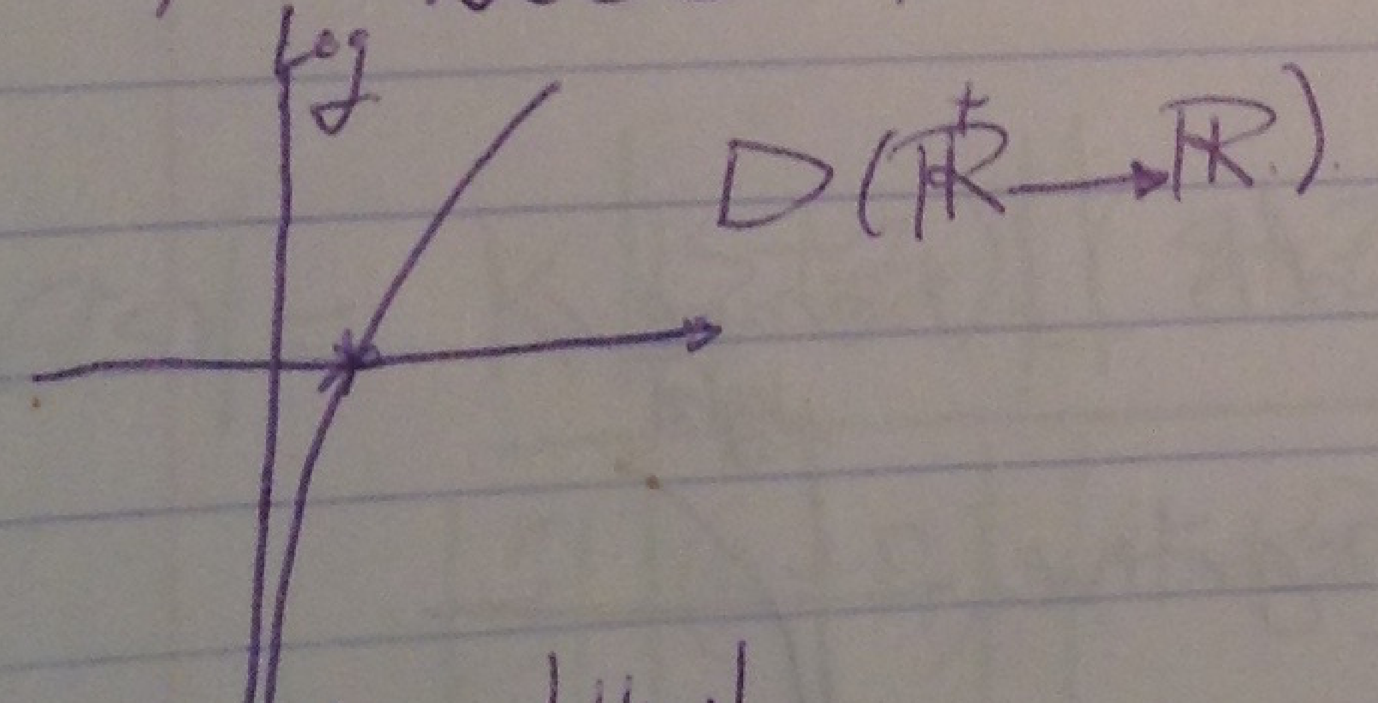
$$y(t) = \sum y_n(t),$$

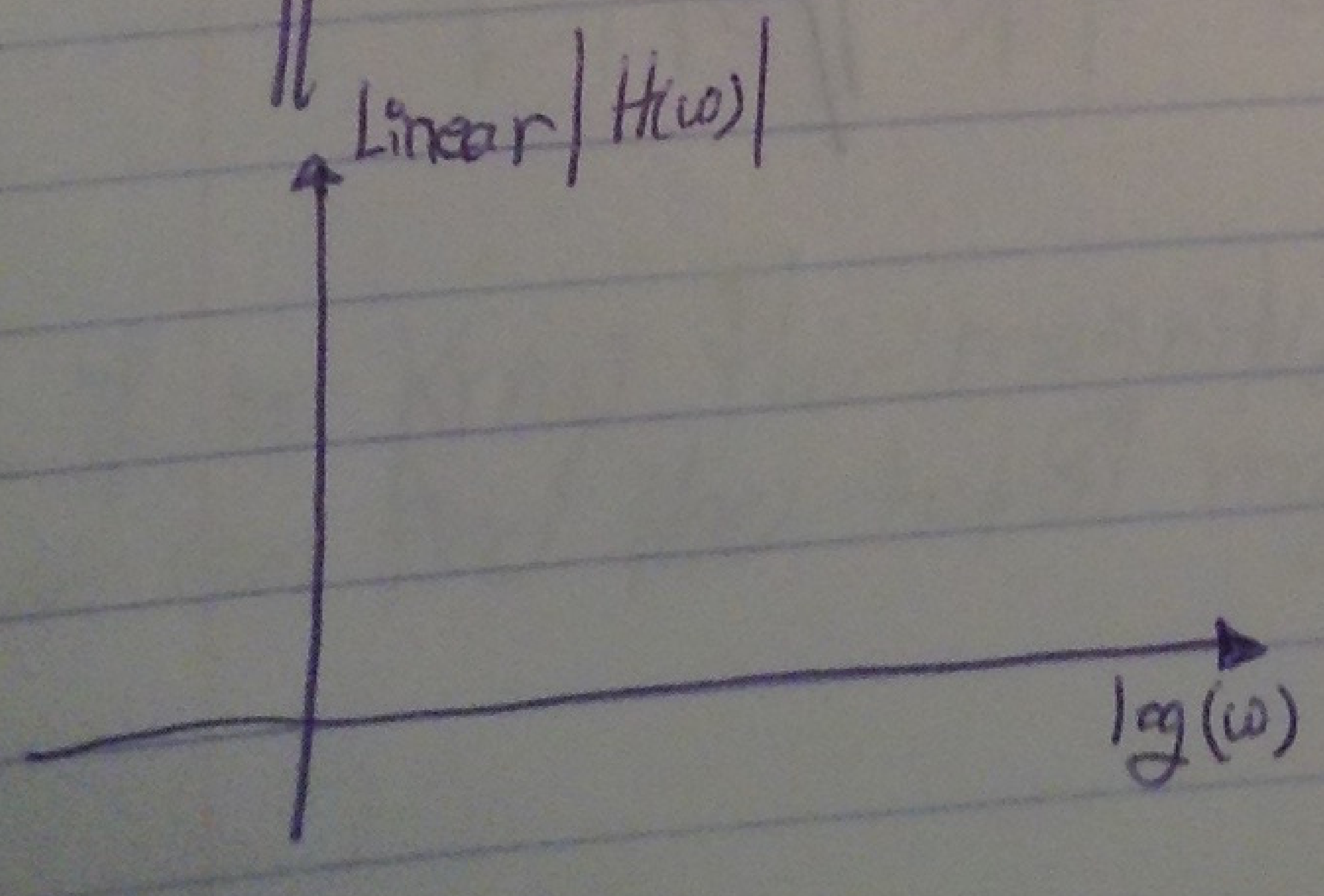
$$y_n = X_n |H(\omega)| \cos(\omega_n t + I + \measuredangle H(\omega))\Big|_{n\omega_0}$$

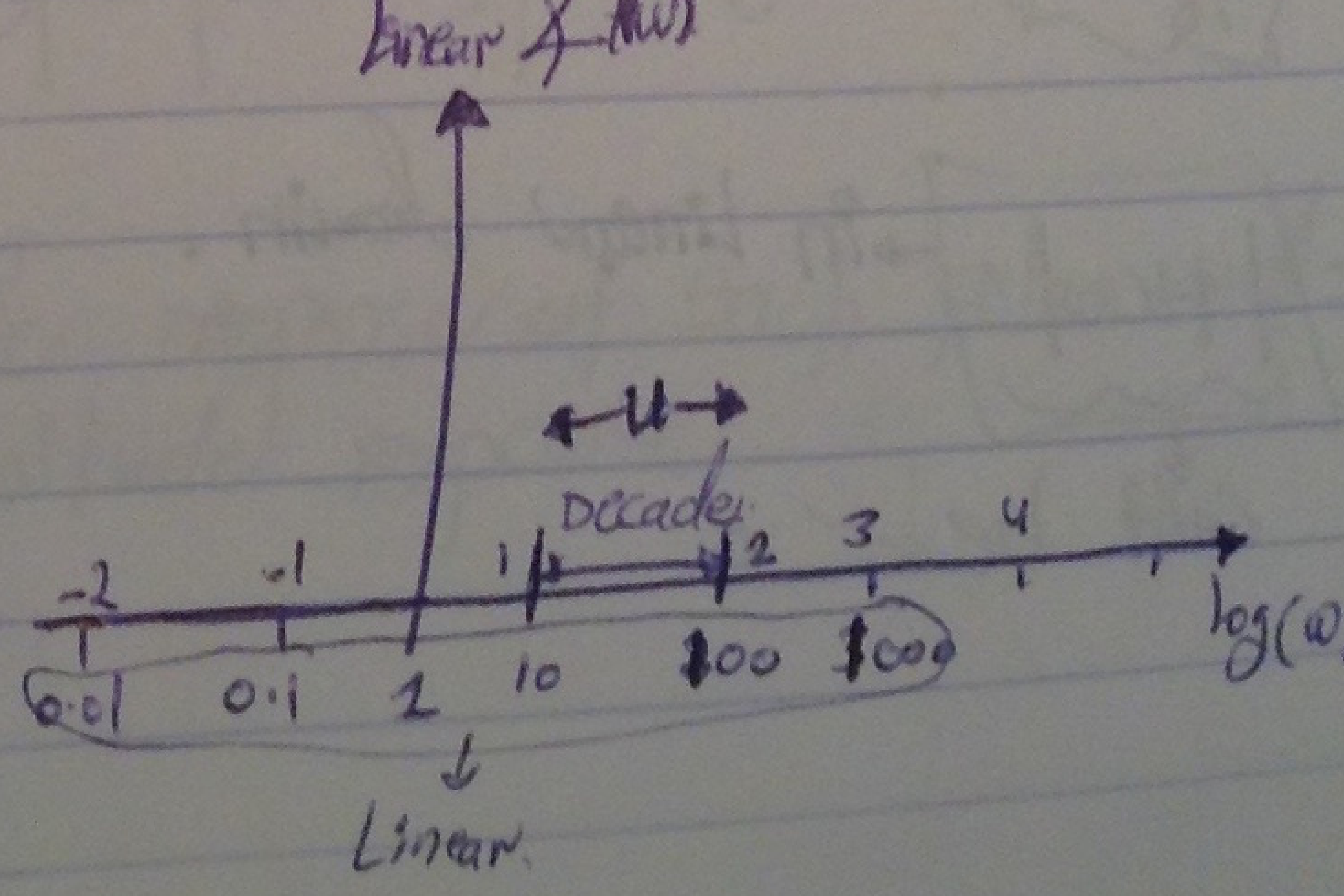
-Bode Plots.

function of frequency (Depend of f).

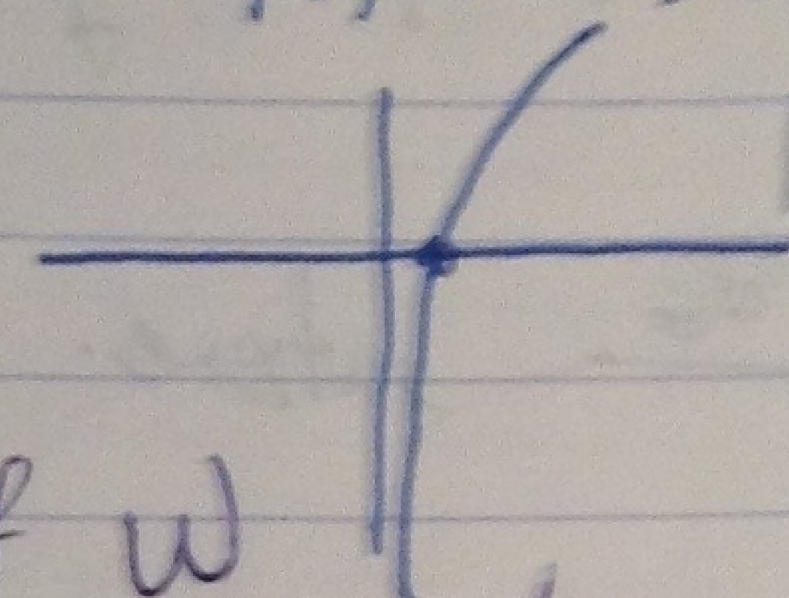
Bode plots: spectral representation using semi log refrence





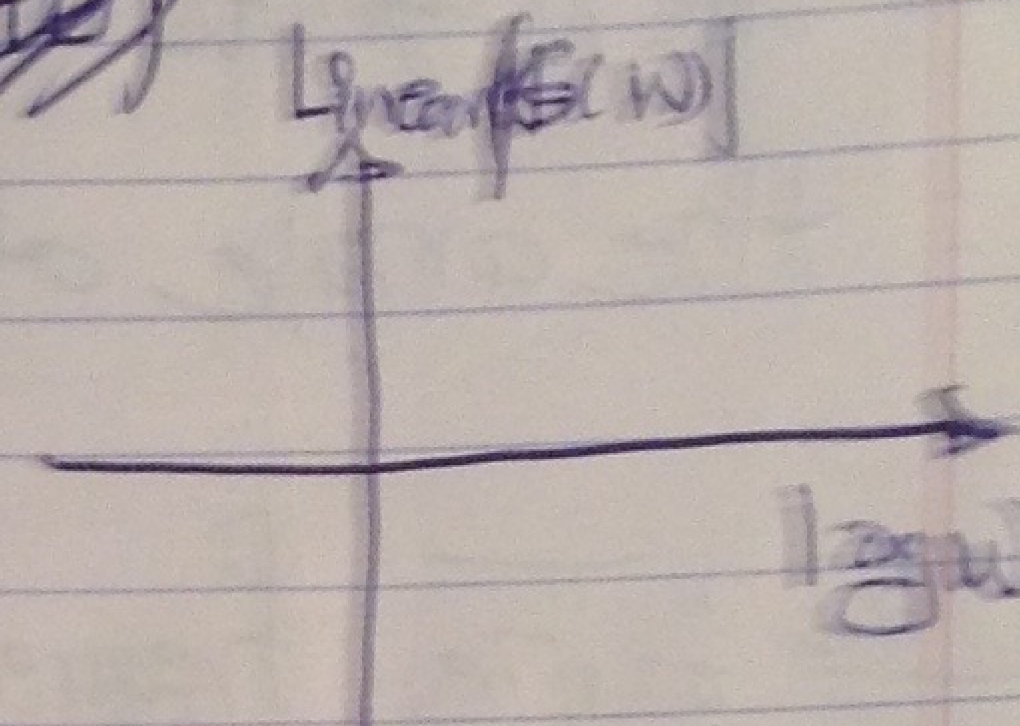


- Bode plots: is single side representation because we have positive domain. since $\mathbb{R} \to \mathbb{R}^+$

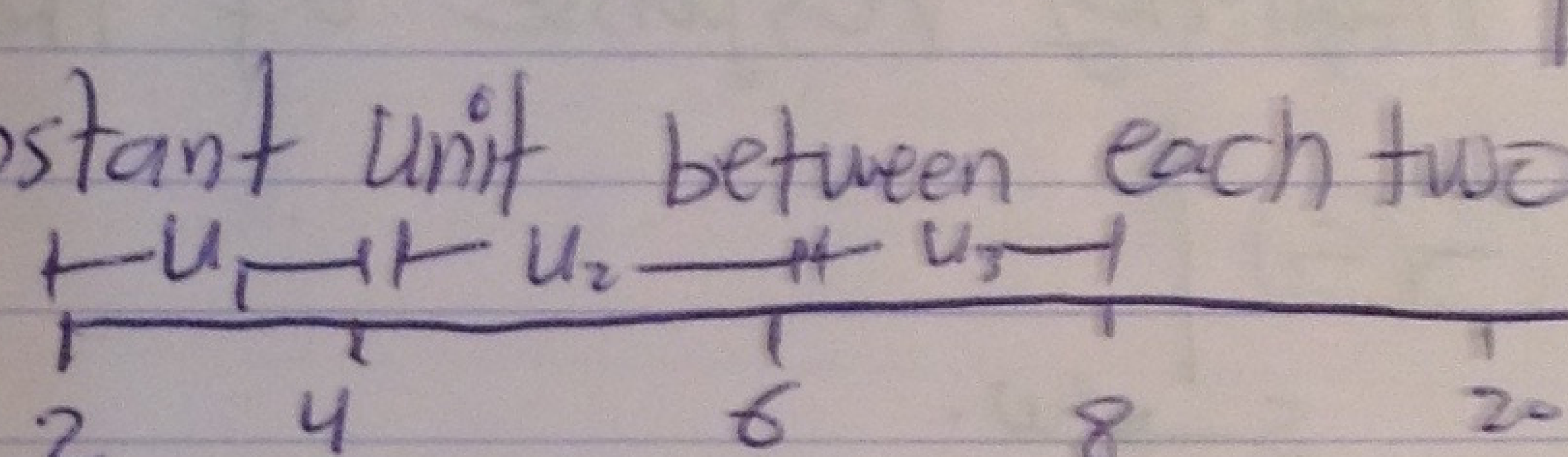


Semilog: one axis is $\underbrace{\text{Logarithm}}_{\text{the axis of } \omega}$

and the other is $\underbrace{\text{Linear}}_{\text{the axis of magnitude}}$

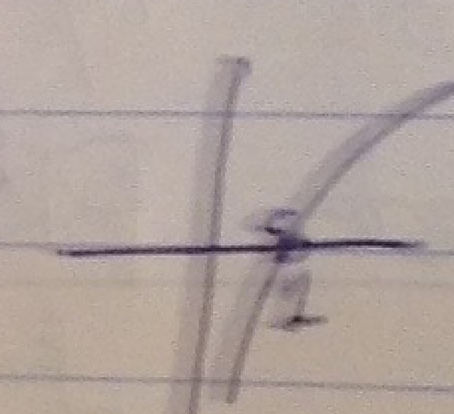


Linear: have a constant unit between each two points on its domain.



$u_1 = u_2 = u_3$

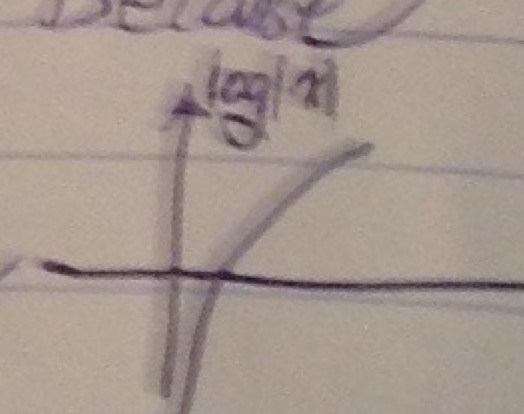
In logarith refrence we not have a constant unit.



- the origion for linear scale at $-\infty$
- // // // logarithmics // 1

- We can say 1db, 2db, -1db, -2db, because

$1db = 20 \log|x|$ → may be positive or negative



$\underbrace{|H(\omega)|_{db}}_{\text{can be} < 0} \neq \boxed{|H(\omega)|}$ → in Linear domain.

can't be < 0

L.T.I

1. pole-zero form
2. time-constant form.

pole-zero form $= \dfrac{\bar{K}\,\Pi(s+z_i)\,\Pi\, s^2+2\xi_{zi}\,\omega_{nzi}\,s+\omega^2_{nzi}}{\Pi(s+P_i)\,\Pi\,(s^2+2\xi_{Pi}\,\omega_{nPi}\,s+\omega^2_{nPi})}$

time-constant form $= K \dfrac{\Pi_i\,(1+\tau_{zi}s)\,\Pi_i\left(\frac{s^2}{\omega^2_{nzi}}+\frac{2\xi_{zi}}{\omega_{nzi}}s+1\right)}{s^n\,\Pi_i\,(1+\tau_{Pi}s)\,\Pi\left(\frac{s^2}{\omega^2_{npi}}+\frac{2\xi_{pi}}{\omega_{npi}}+1\right)}$

static gain

- Consider the following transfer function

$$G(s) = \frac{K(s+z_1)(s+z_2)(s+z_3)\cdots(s+z_n)}{s^m(s+P_1)(s+P_2)(s+P_3)\cdots(s+P_n)}$$

$$|G(s)| = \frac{K\,|(s+z_1)|\,|(s+z_2)|\,|(s+z_3)|\cdots|(s+z_n)|}{|s^m|\,|(s+P_1)|\,|(s+P_2)|\,|(s+P_3)|\cdots|(s+P_n)|}$$

- If we know the magnitude response of each pole and zero term we can find the total magnitude response.

- the magnitude can be calculate in dB. (decibles) Vs $\log\omega$.

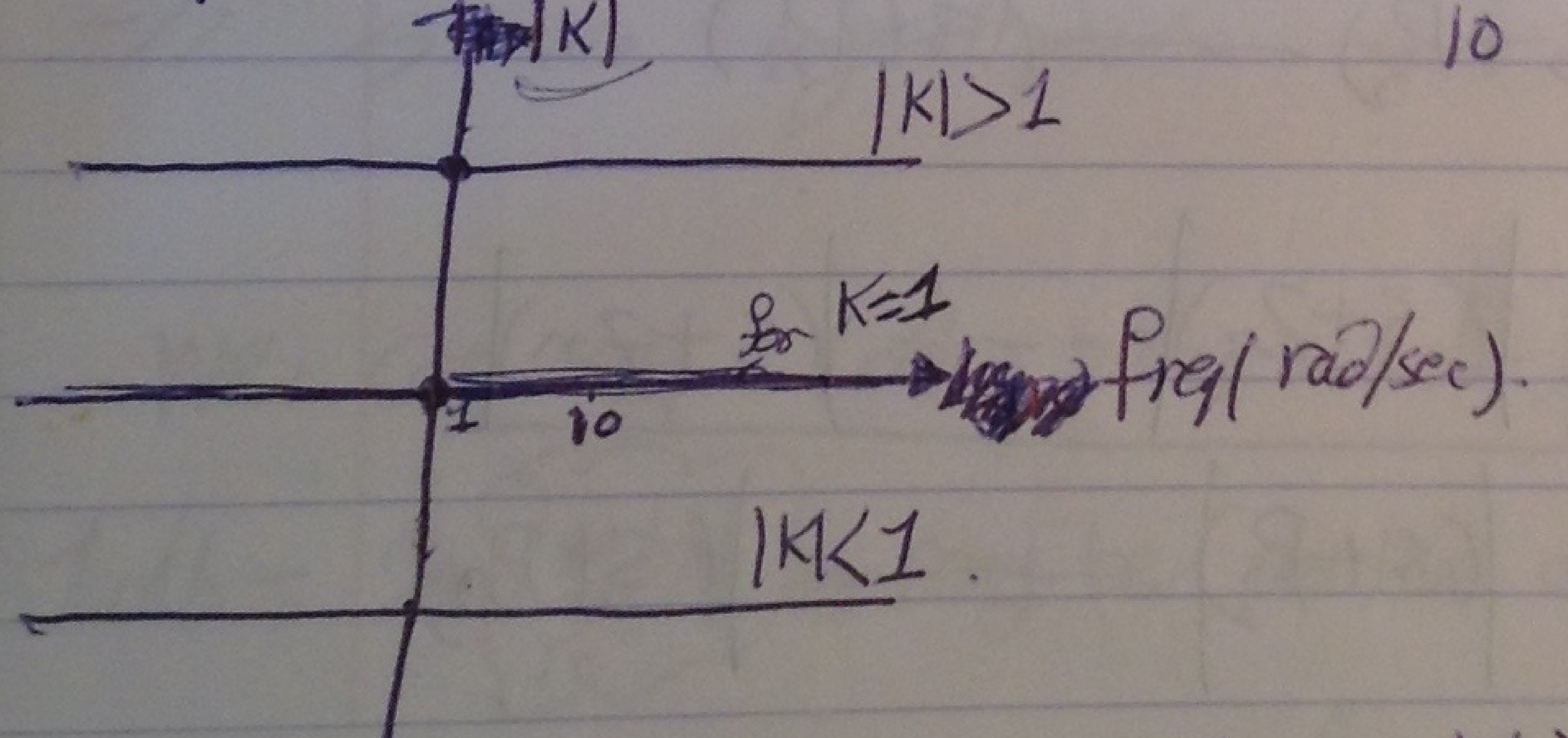
since $1\ dB = 20 \log \hat{M}$ (magnitude)

$$20\log|G(j\omega)| = 20\log K + 20\log|(s+z_1)| + 20\log|(s+z_2)| + \dots - 20\log|(s+z_n)|$$
$$-20\log|(s+p_1)| - 20\log|(s+p_2)| - 20\log|(s+p_3)| + \dots - 20\log|(s+p_n)|$$

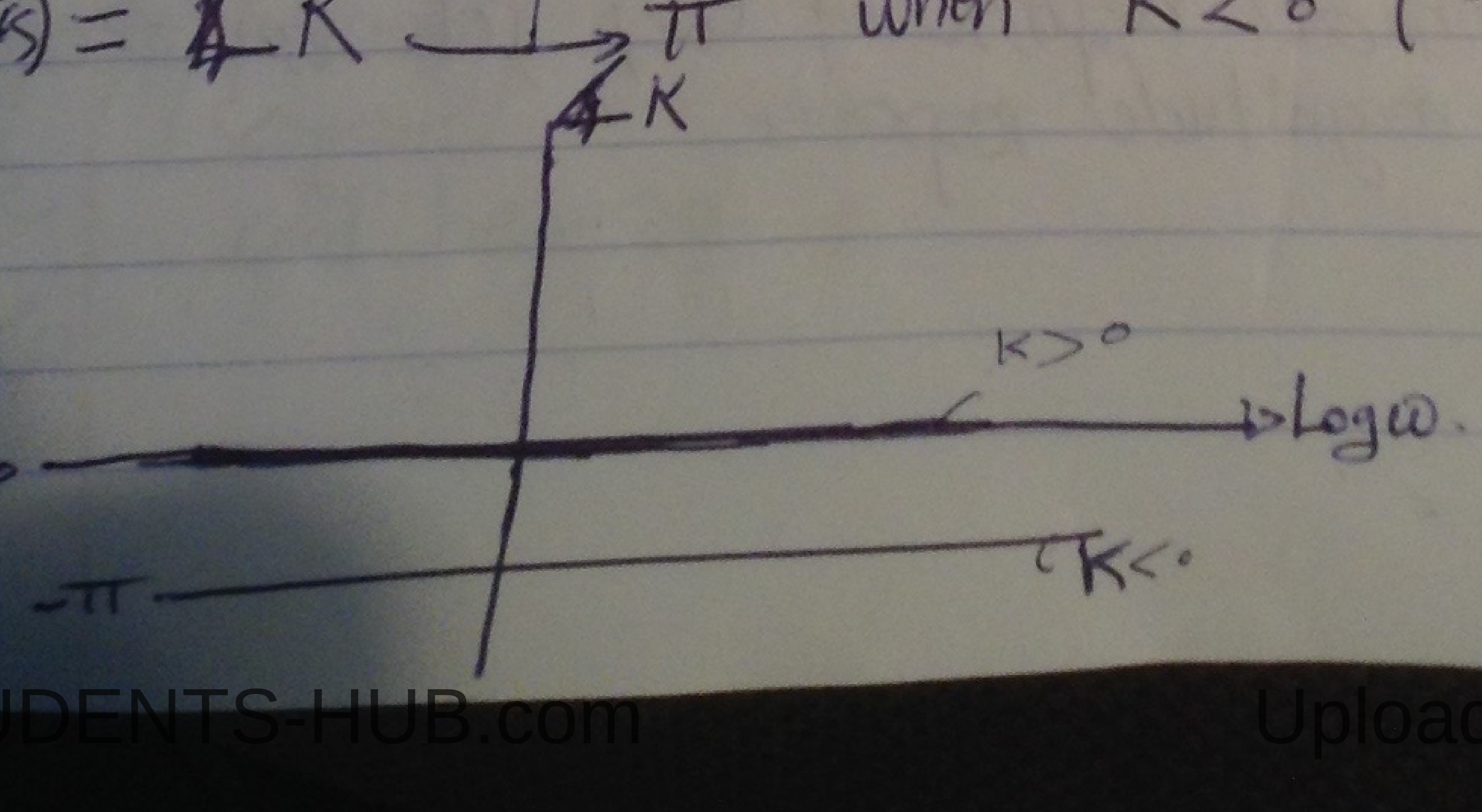
so if we can knew the response of each term, the algebric sum would yield the total response in dB.

$\boxed{\text{III}}$ Bode plots for $G(s)=K$.

$|G(s)| = |K| \longrightarrow$ in $db = 20\log_{10}|K|$ $\rightarrow$ for $|K|>0$, $\rightarrow$ for $|K|<0$



$\angle G(s) = \angle K$ $\rightarrow$ 0 when $K>0$; $\rightarrow$ π when $K<0$ $(-K = K\angle 180)$

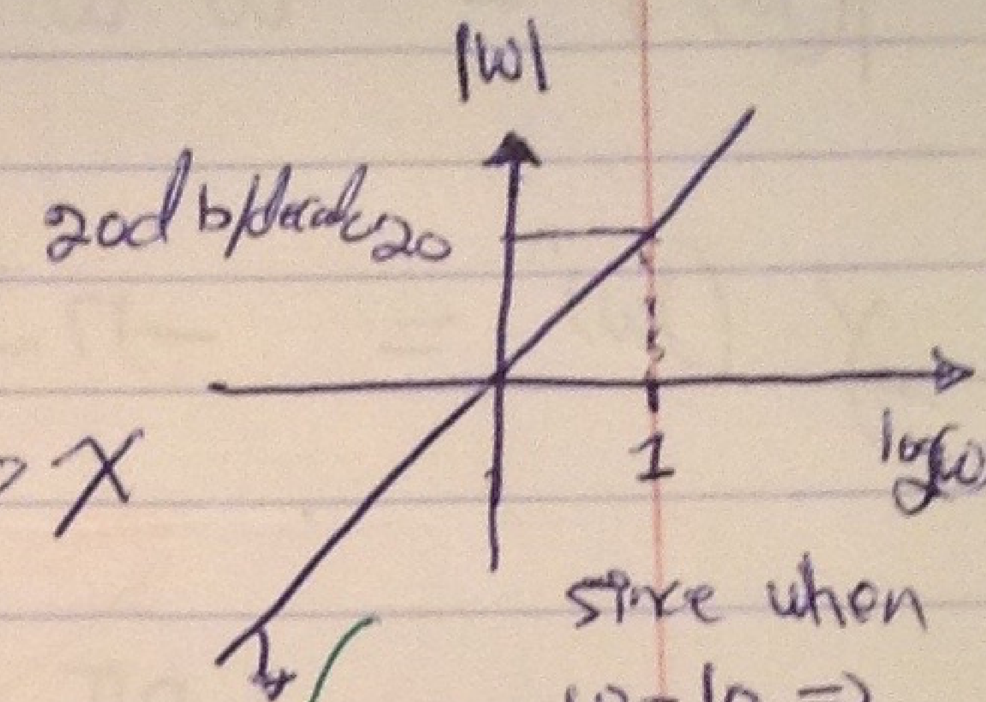


For $G(s) = s^n$.

$G(j\omega) = (j\omega)^n$.

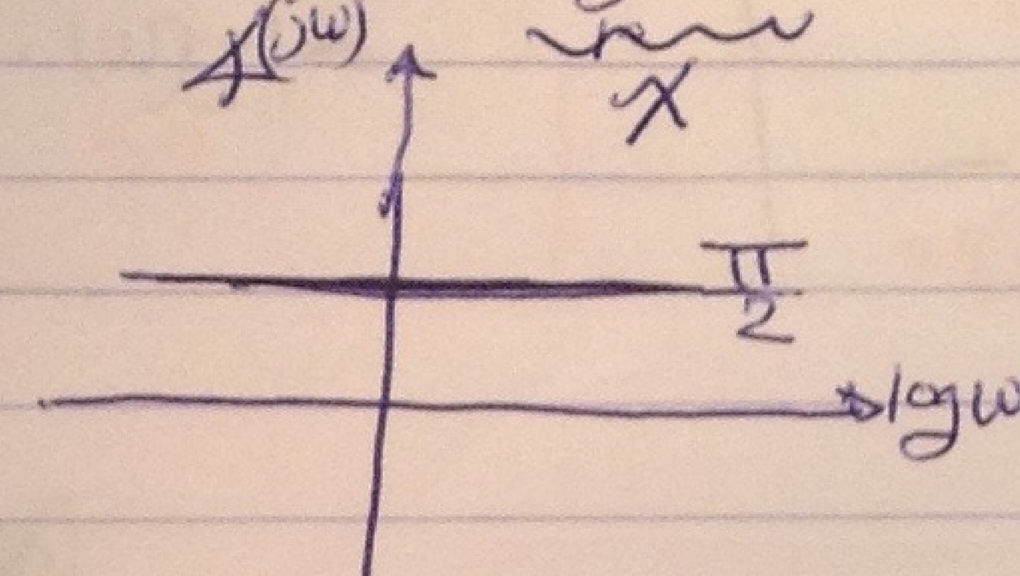
when $\boxed{n=1}$ (zero at origion).

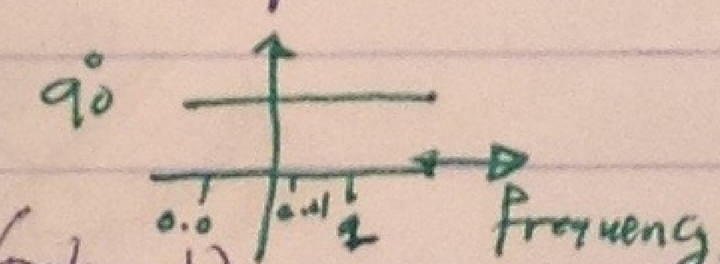
$|G(j\omega)| = |j\omega| = \omega \xrightarrow{\text{in db}} 20\log\underbrace{|\omega|}_{x} = 20x$

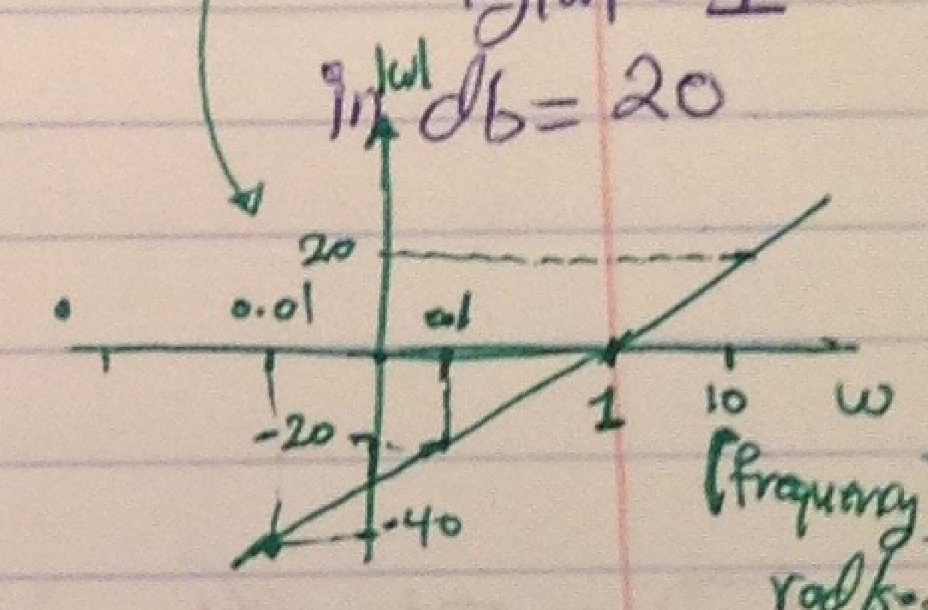


since when
$\omega = 10 \Rightarrow$
$\log|\omega| = 1$
in db $= 20$

$\measuredangle(j\omega) = j = +\frac{\pi}{2}$

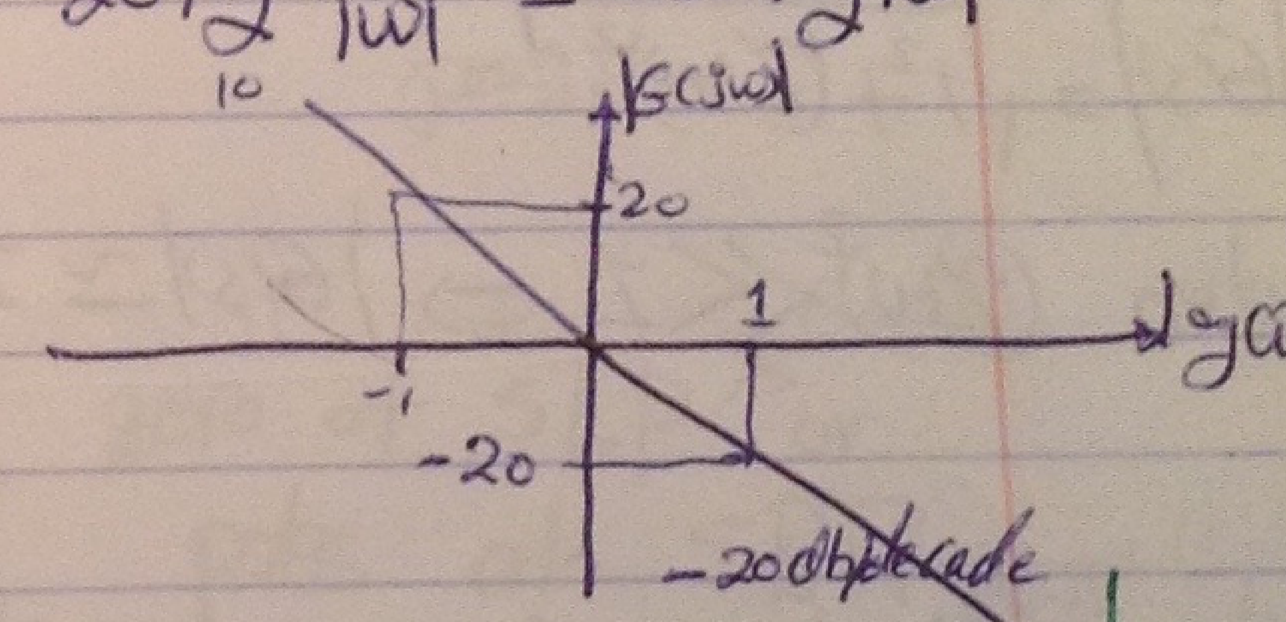


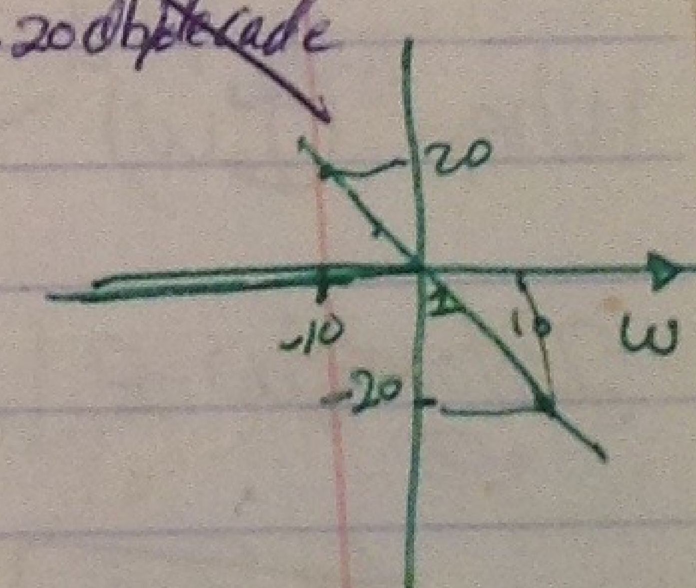




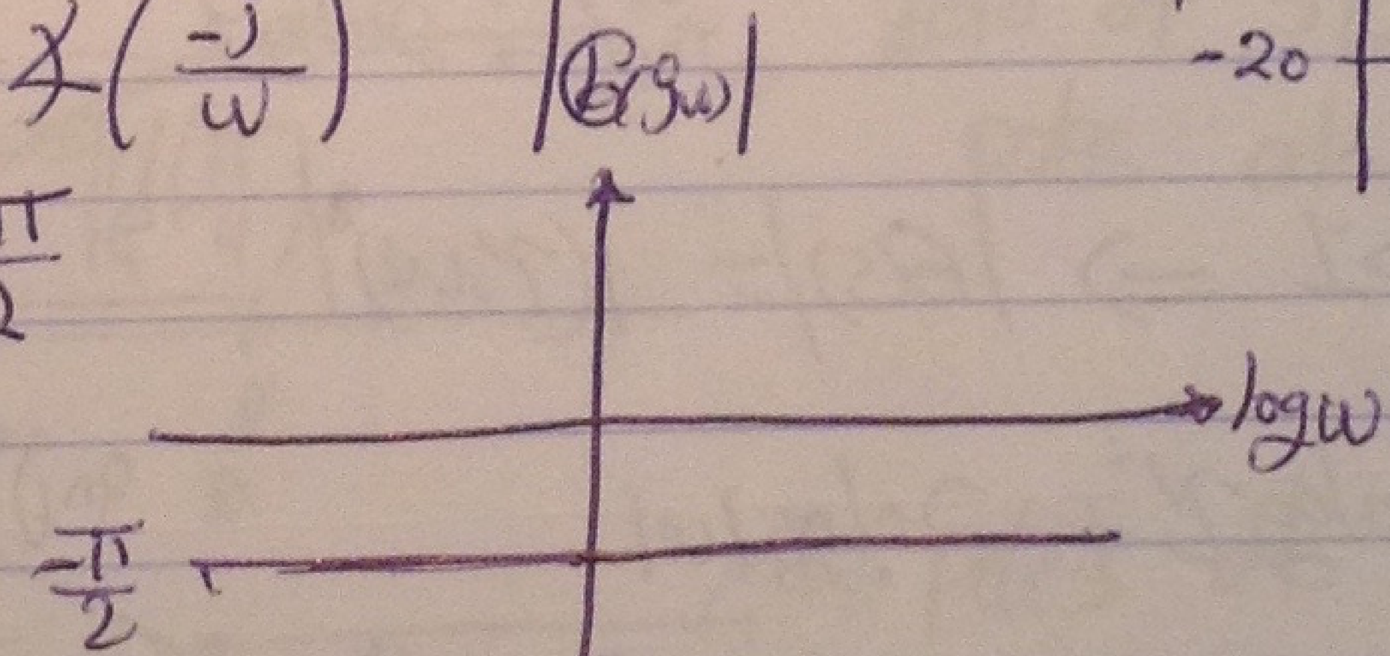
when $\boxed{n = -1}$ (pole at origion).

$|G(j\omega)| = |(j\omega)^{-1}| = \left|\frac{1}{j\omega}\right| = \frac{1}{|\omega|} \xrightarrow{\text{in db}} 20\log\frac{1}{|\omega|} = -20\overbrace{\log|\omega|}^{x}$





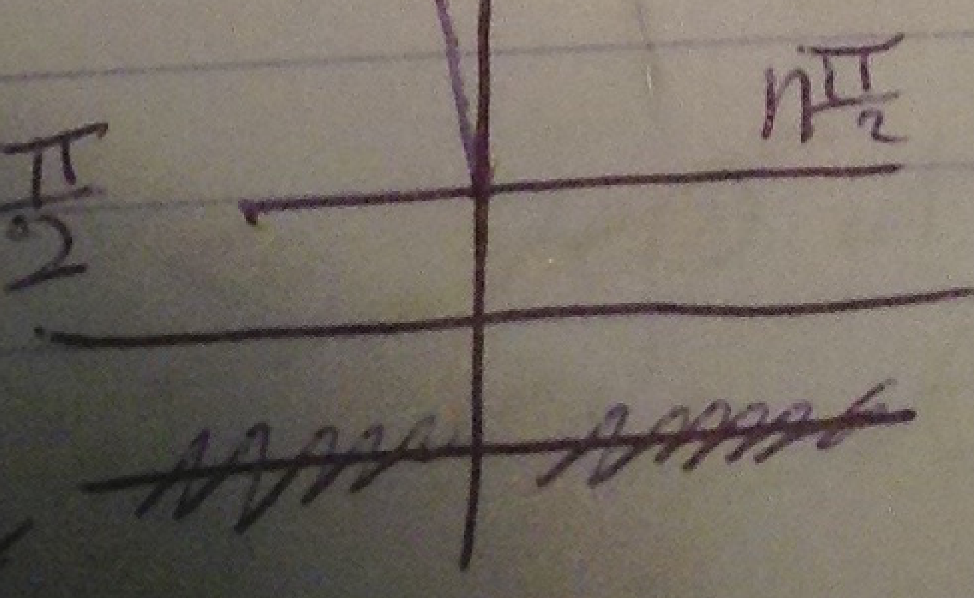
$\measuredangle(j\omega)^{-1} = \measuredangle\left(\frac{1}{j\omega}\right) = \measuredangle\left(\frac{-j}{\omega}\right)$
$= \frac{-\pi}{2}$

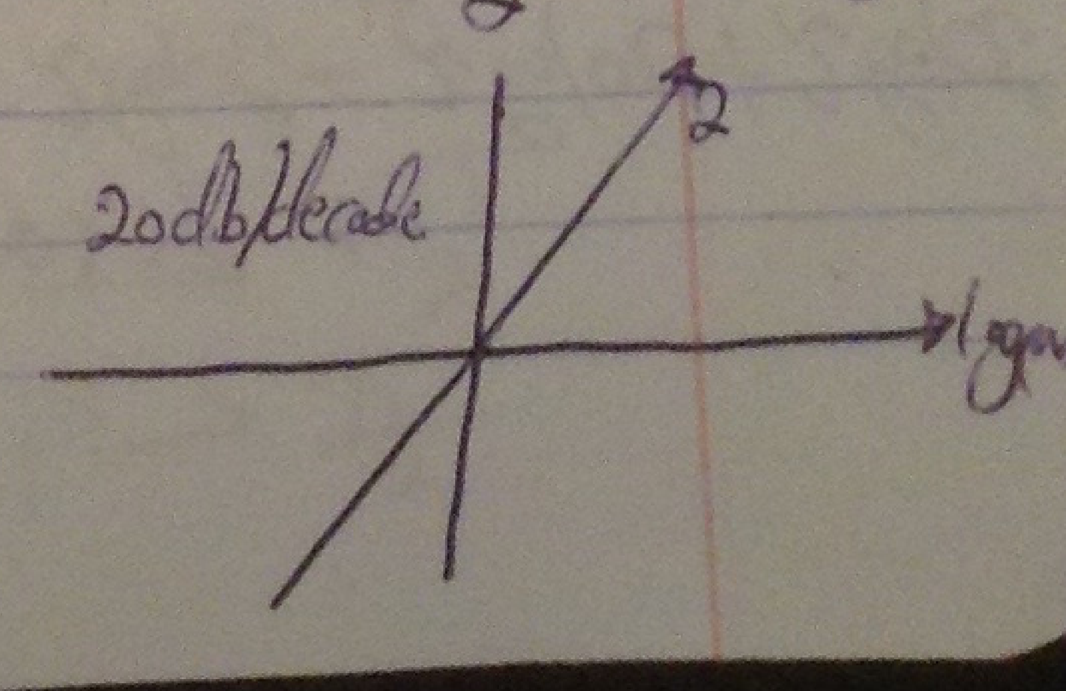


when $\boxed{n > 1}$

$|G(j\omega)^n| = |(j\omega)^n| = |j\omega||j\omega||j\omega||j\omega| \cdots |j\omega| = \omega^n \xrightarrow{\text{in db}} 20\log|\omega|^n = 20n\log|\omega|$

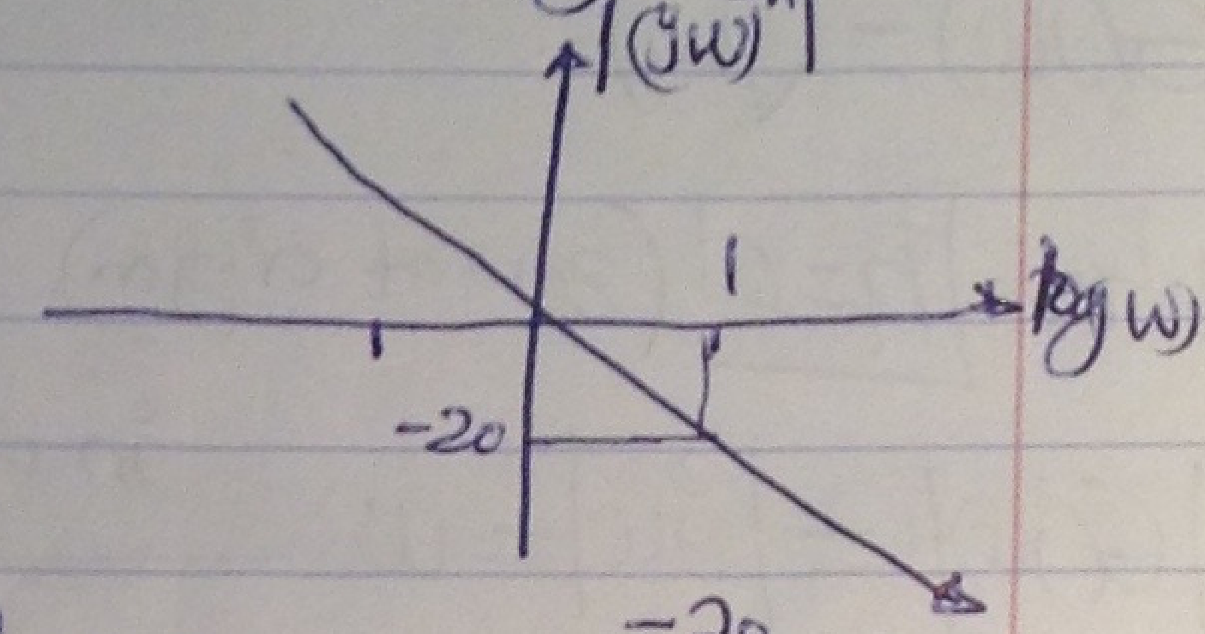
$\measuredangle((j\omega)^n) = n\frac{\pi}{2}$



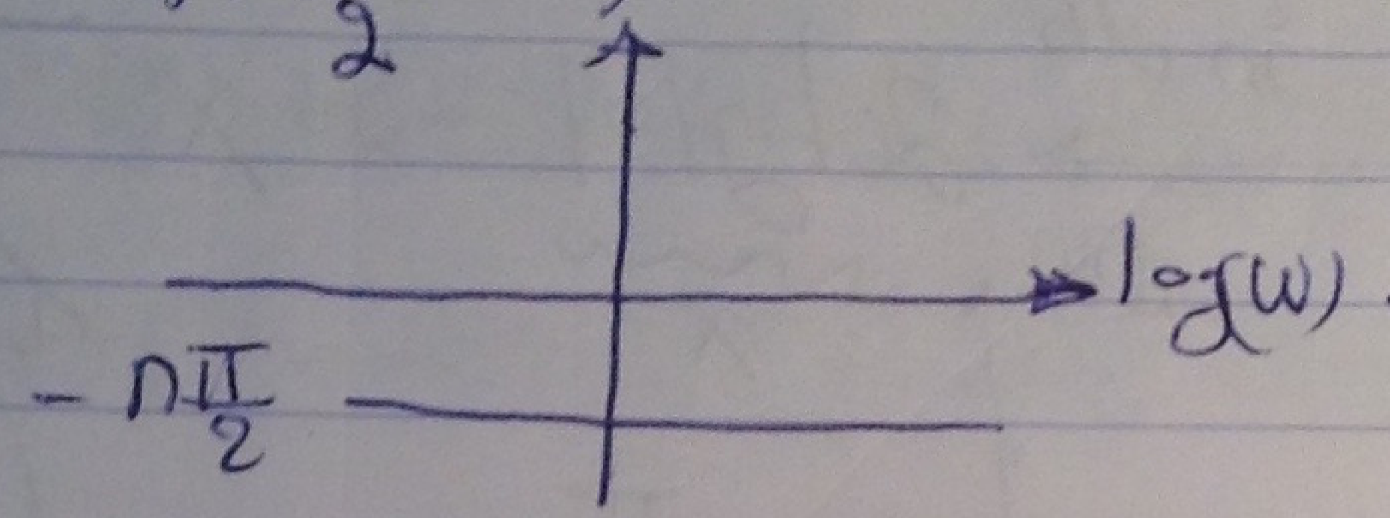


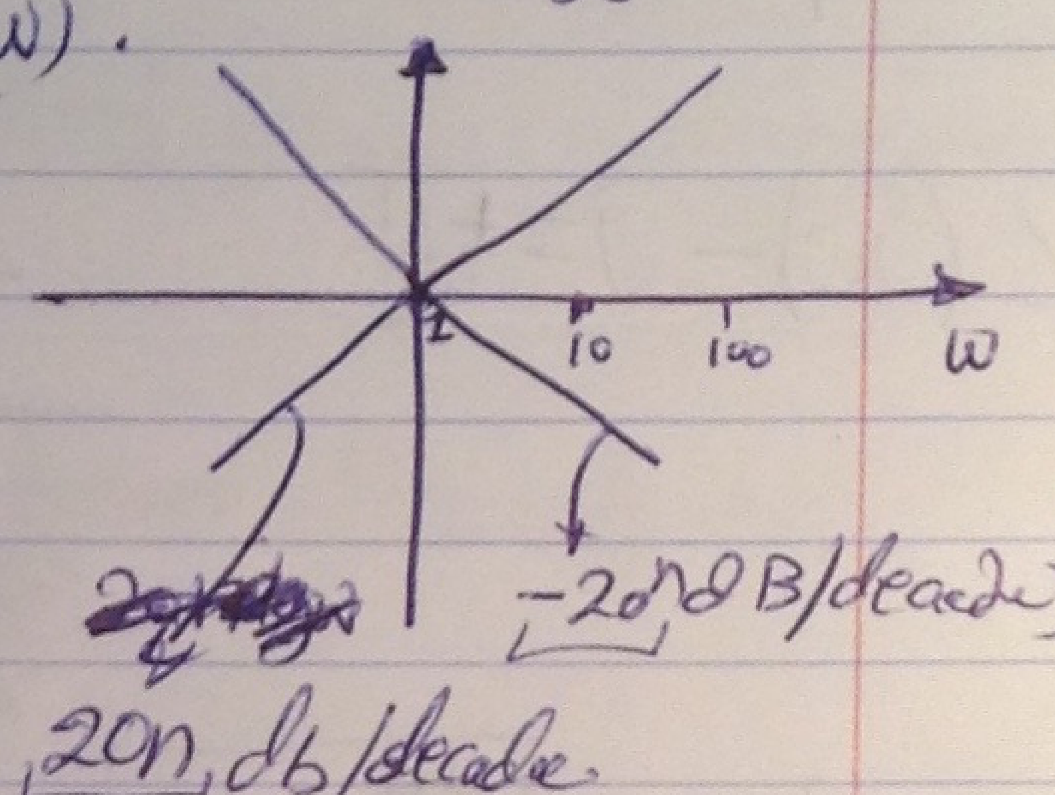
when $n<1$

$$|(j\omega)^{-n}| = \frac{1}{\omega}\cdot\frac{1}{\omega}\cdot\frac{1}{\omega} = \omega^{-n} \xrightarrow{\text{in db}} -20n\log|\omega|$$



$$\angle(j\omega)^{-n} = -n\frac{\pi}{2}$$



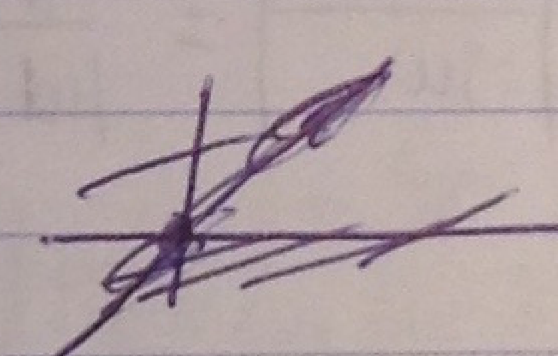


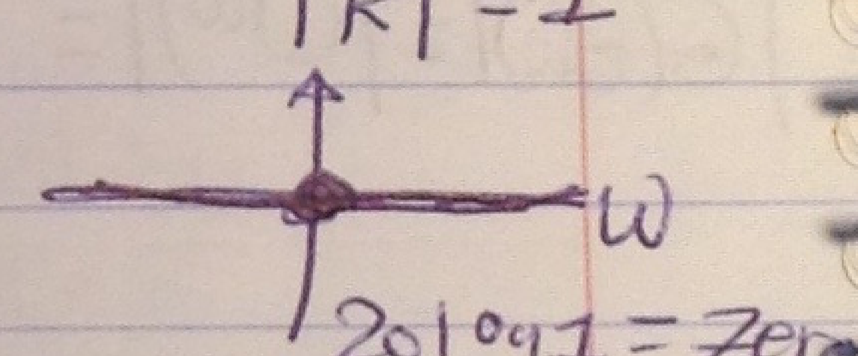
- For $G(s) = (1 + j\tau\omega)^n$

$$|G(s)| = \sqrt{1^2 + (\tau\omega)^2}$$

when $(\tau\omega)^2 \ll 1 \Rightarrow |G(s)| \simeq 1$

in db = 20 log 1

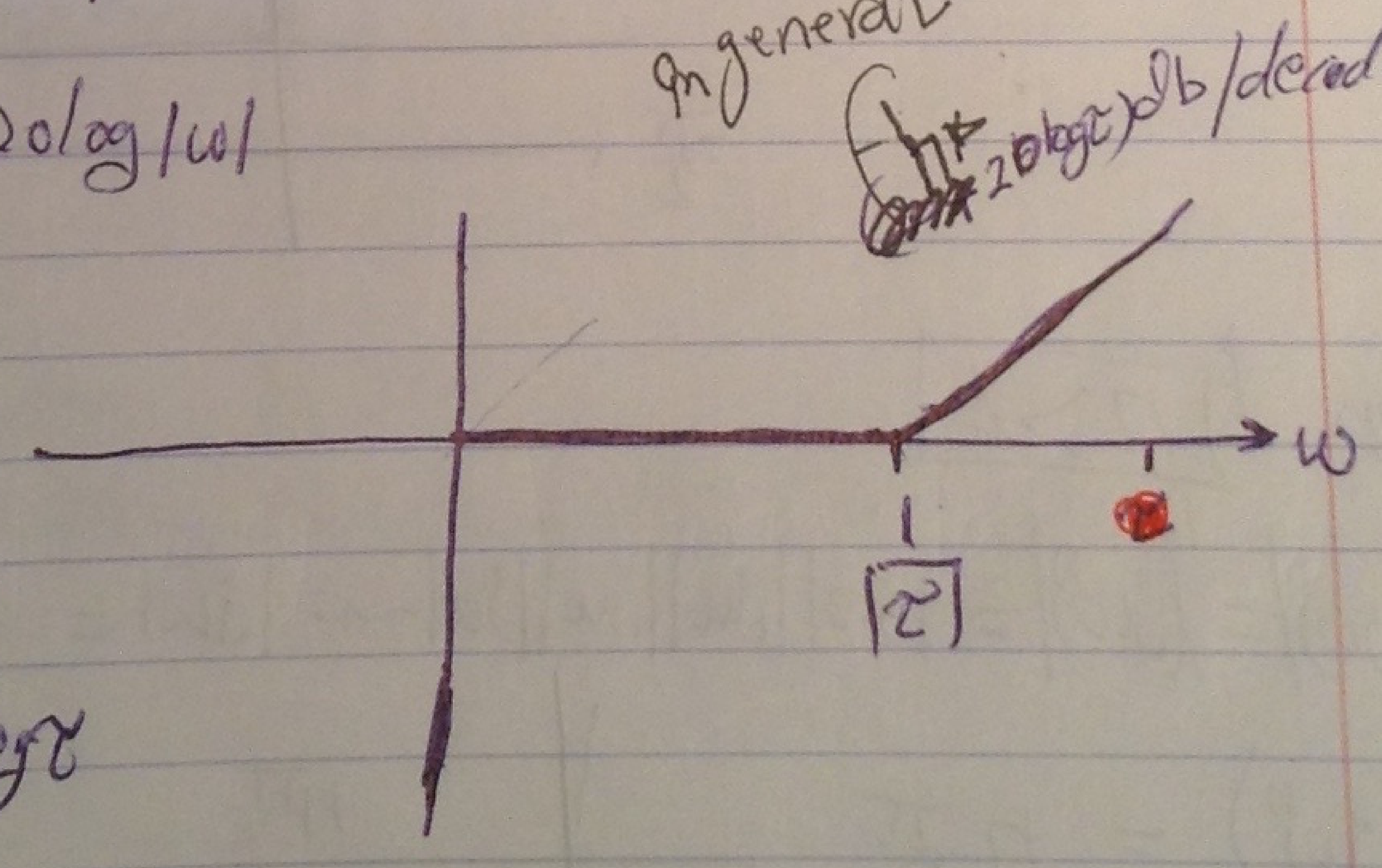




When $(\tau\omega)^2 \gg 1 \Rightarrow |G(s)| = |(\tau\omega)| \xrightarrow{\text{in dB}}$

$$20\log\tau\omega = 20\log\tau + 20\log|\omega|$$

in general

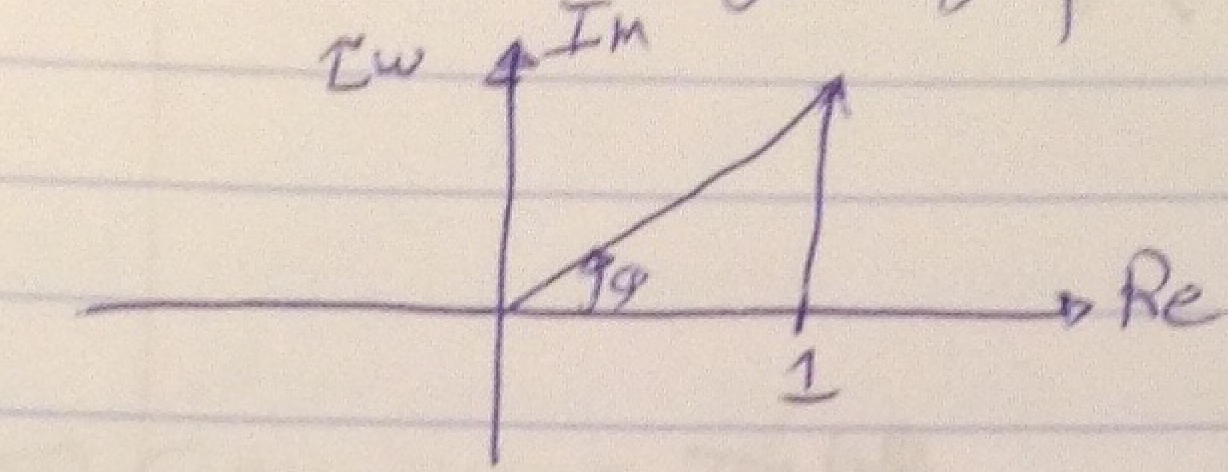


when $\boxed{\omega = \frac{1}{|\tau|}}$

$$20\log\tau + 20\log|\tfrac{1}{\tau}|$$

$$20\log\tau + 20\log\tau^{-1} = 20\log\tau - 20\log\tau$$
$$= \text{Zero}$$

To find the angle graph.



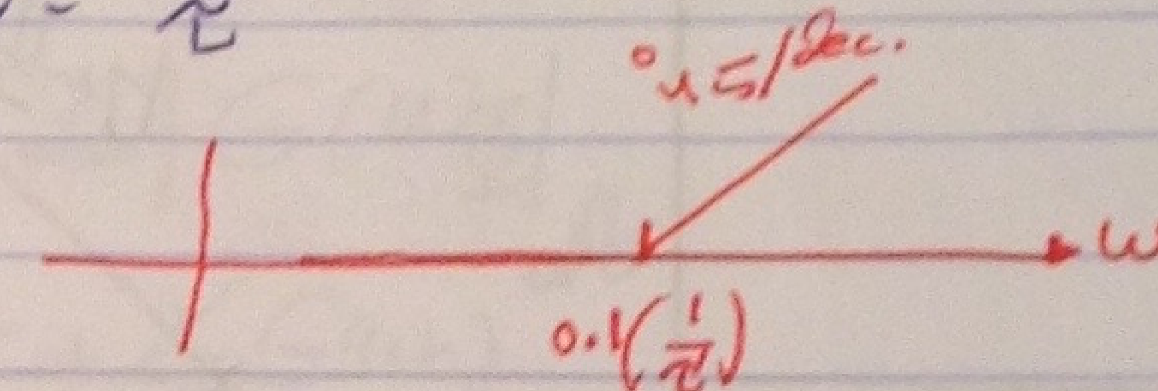
$\tan(\varphi) = (\tau\omega)$, at $\omega = \frac{1}{\tau}$

$\tan(\varphi) = 1$

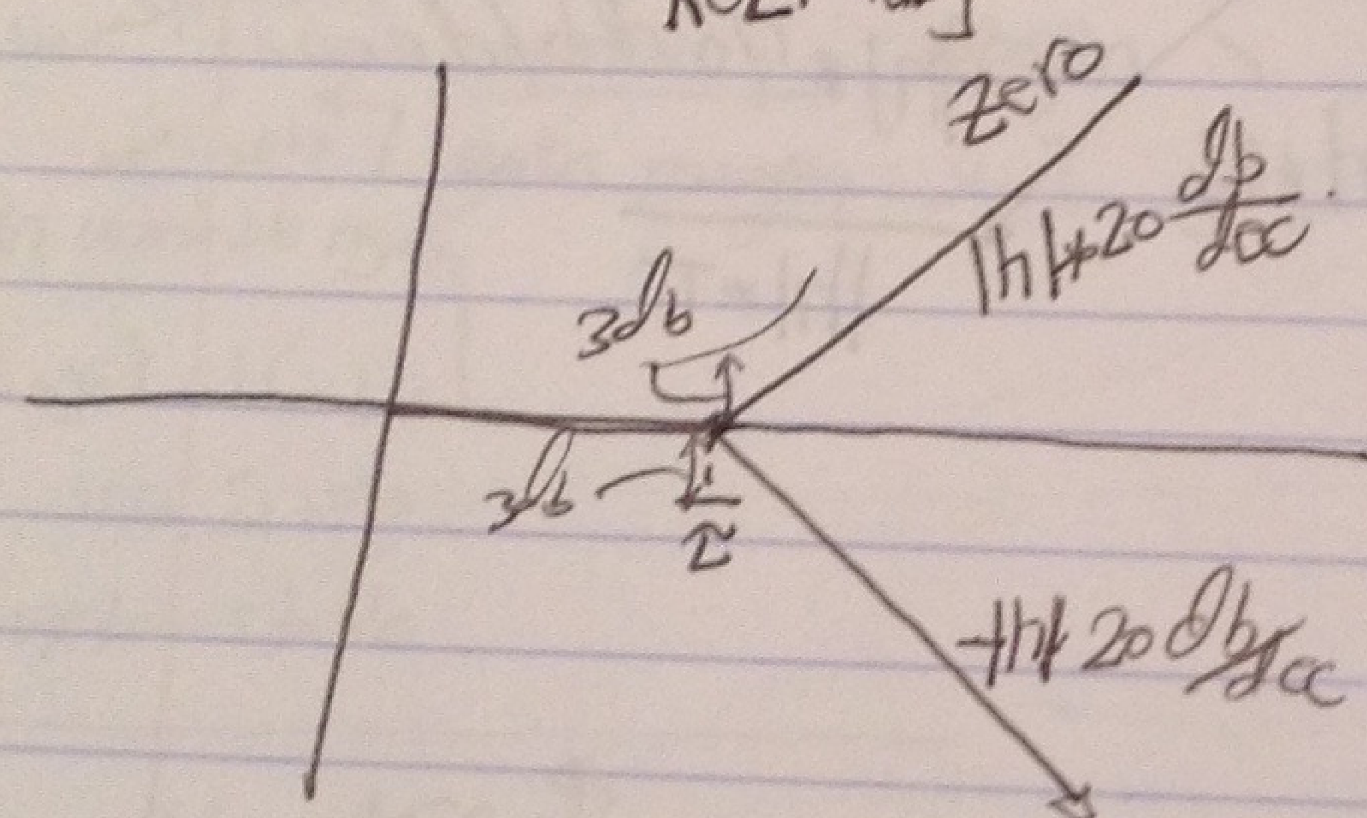
so

$\varphi = \pi/4 = \tan^{-1}(1)$

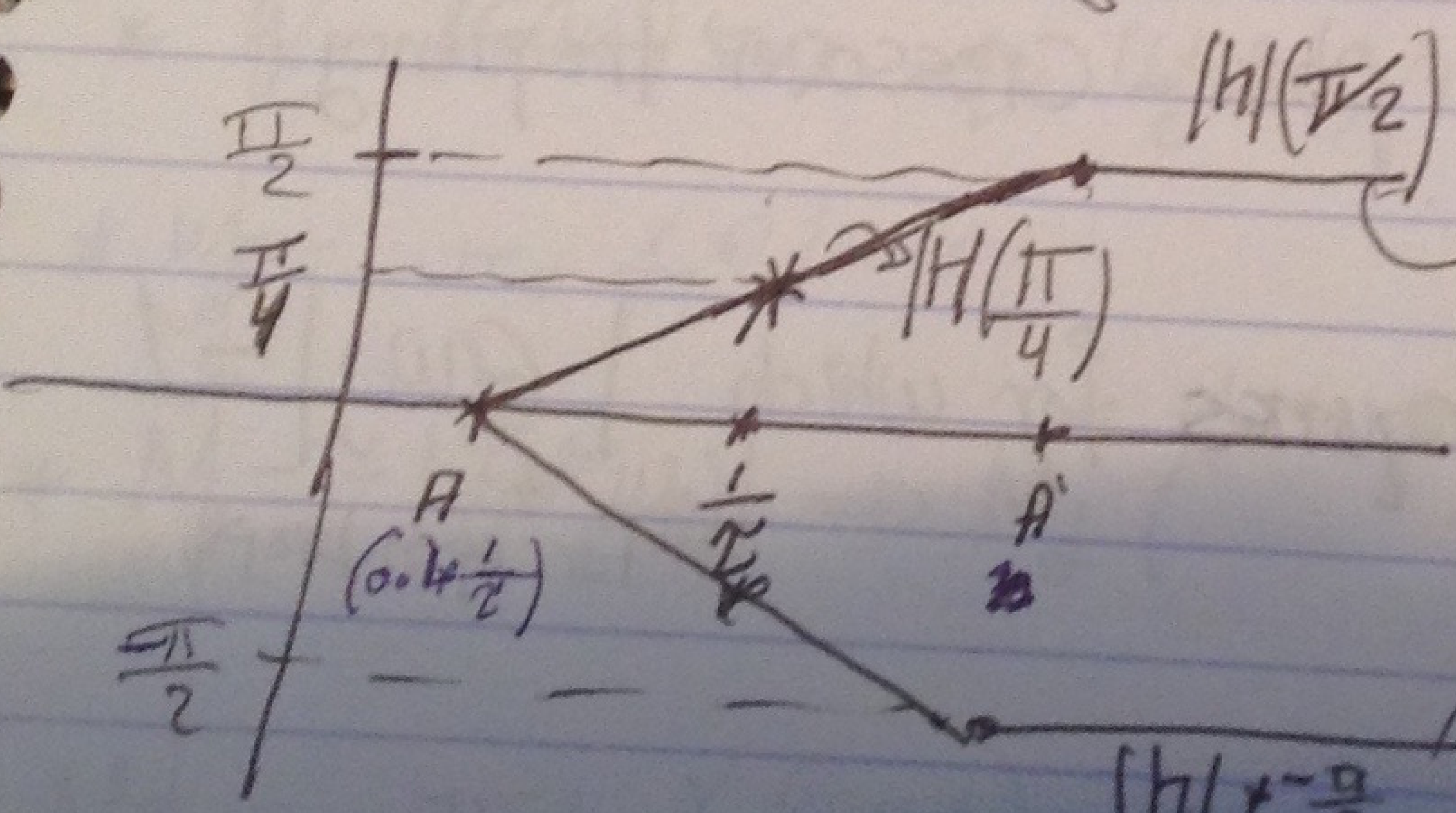
so



$$\tau = \frac{-1}{Re[poles]}$$



* single side representation ⇒ phase positive.



→ zero at SLP $(s+1)^1$ or poles at SRP $\frac{1}{(s-1)^1} = (s-1)^{-1}$

→ poles at SLP or $\frac{1}{(s+1)^1} = (s+1)^{-1}$

zero at SRP $(s-1)^1$

on the semi left plane.

When $h=1$, $\tau>0$, zero at SLP $\to \pi/2$

$h=-1$, $\tau>0$, poles at SLP $\to -\frac{\pi}{2}$

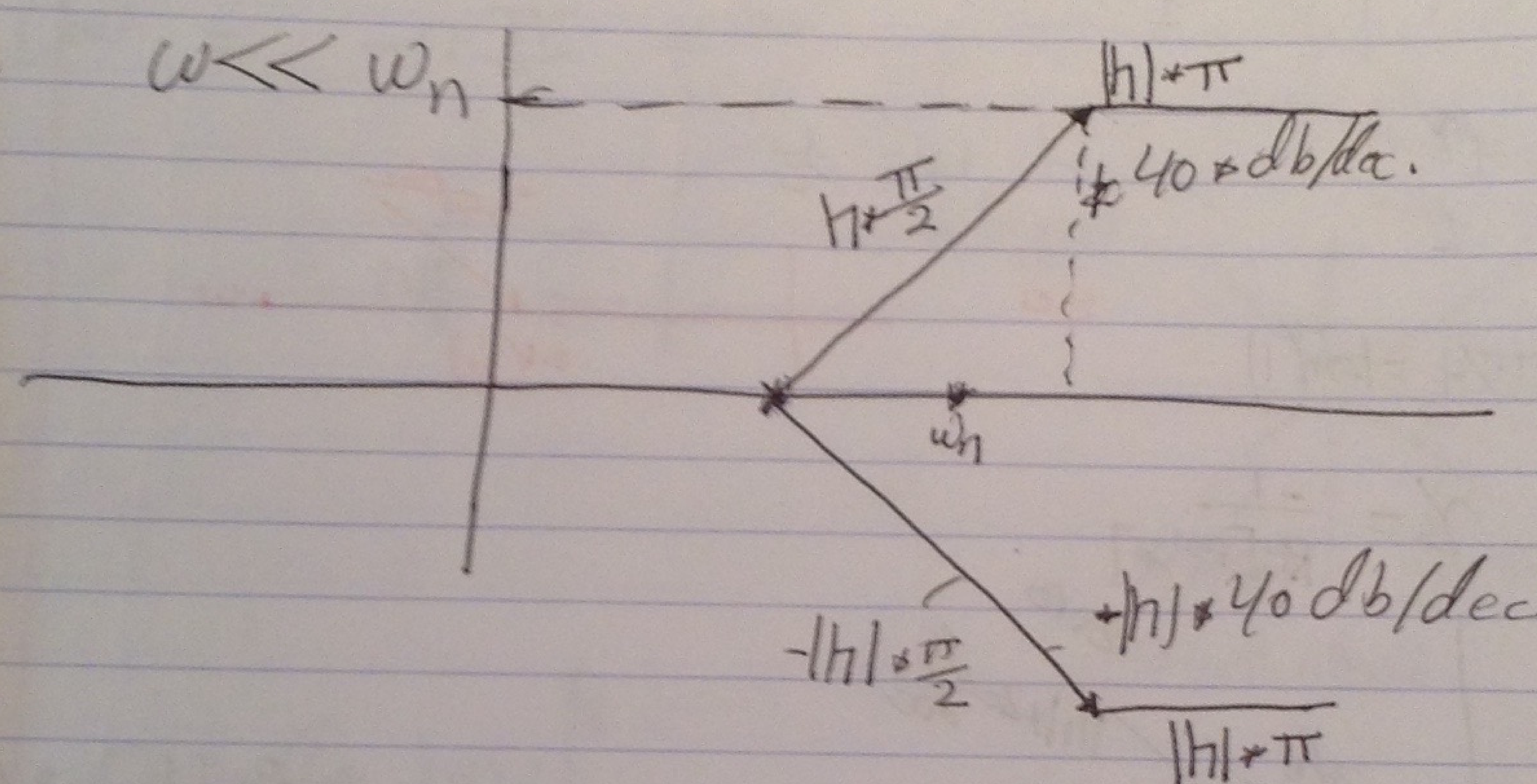
When $h=1$, $\tau<0$, zero at SRP $\to +\frac{\pi}{2}$

$h=-1$, $\tau<0$, poles at SRP $\to \frac{\pi}{2}$

$$G(s) = \left(\frac{s^2}{\omega_n^2} + \frac{2\zeta}{\omega_n} s + 1\right)$$

double real poles

when $\omega \ll \omega_n$



<u>Control</u>

$\boxed{1}$ Gain crossover frequency phase crossover frequency.

W_c : gain " " " (frequency for which $|G(\omega_c)| = 1$ for open loop.

$|G(\omega_c)|_{db} = 0$ gain crossover frequency.

W_ϕ : phase cross over frequency : frequency for which

$\measuredangle G(\omega_\phi) = -\pi$

* Draw the Bode plot (refer to).

$\boxed{2}$ Gain margin ($M_g = \dfrac{1}{|G(\omega_\Phi)|}$

$M_{g_{db}} = -|G(\omega_\Phi)|_{db}$

phase margin $= -\pi + \measuredangle G(\omega_c)$.

إذا كانت فوق ال $-\pi$ بكون في منطقة ال stability أما إذا كانت تحت ال $-\pi$ بكون unstable

phase margin

- نستخدم ال gain crossover frequency ونقوم بحساب ال phase margin

- ونستخدم ال phase crossover frequency ونقوم بحساب ال gain margin

إذا أكبر من واحد بكون in unstable region

إذا كان أقل من واحد بكون ال system stable.

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* $\omega_c = |G(\omega_c)| = 0 \Rightarrow \omega_c$: gain cross over frequency.

* $\omega_\Phi = -\pi + \measuredangle \Phi \; / \; |G(\omega_\Phi)| = 0$

* $M_g = -|G(\omega_\Phi)| \Rightarrow M_g$ (نستعمل ال phase cross over freq لما نجد ال gain margin)

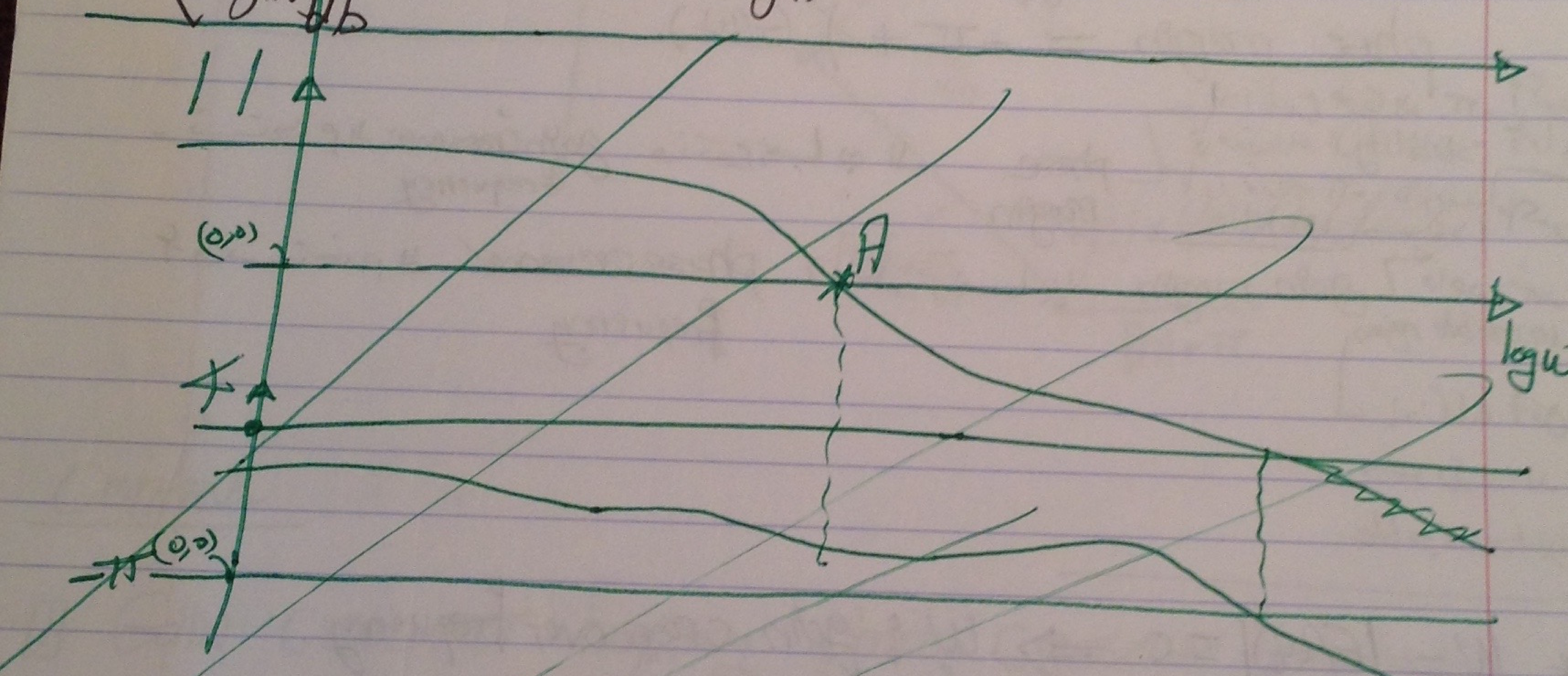
* $M_\Phi = -\pi + \measuredangle G(\omega_c)$ (نستعمل ال gain cross over frequency لما نجد ال phase margin)

Stability Theorem

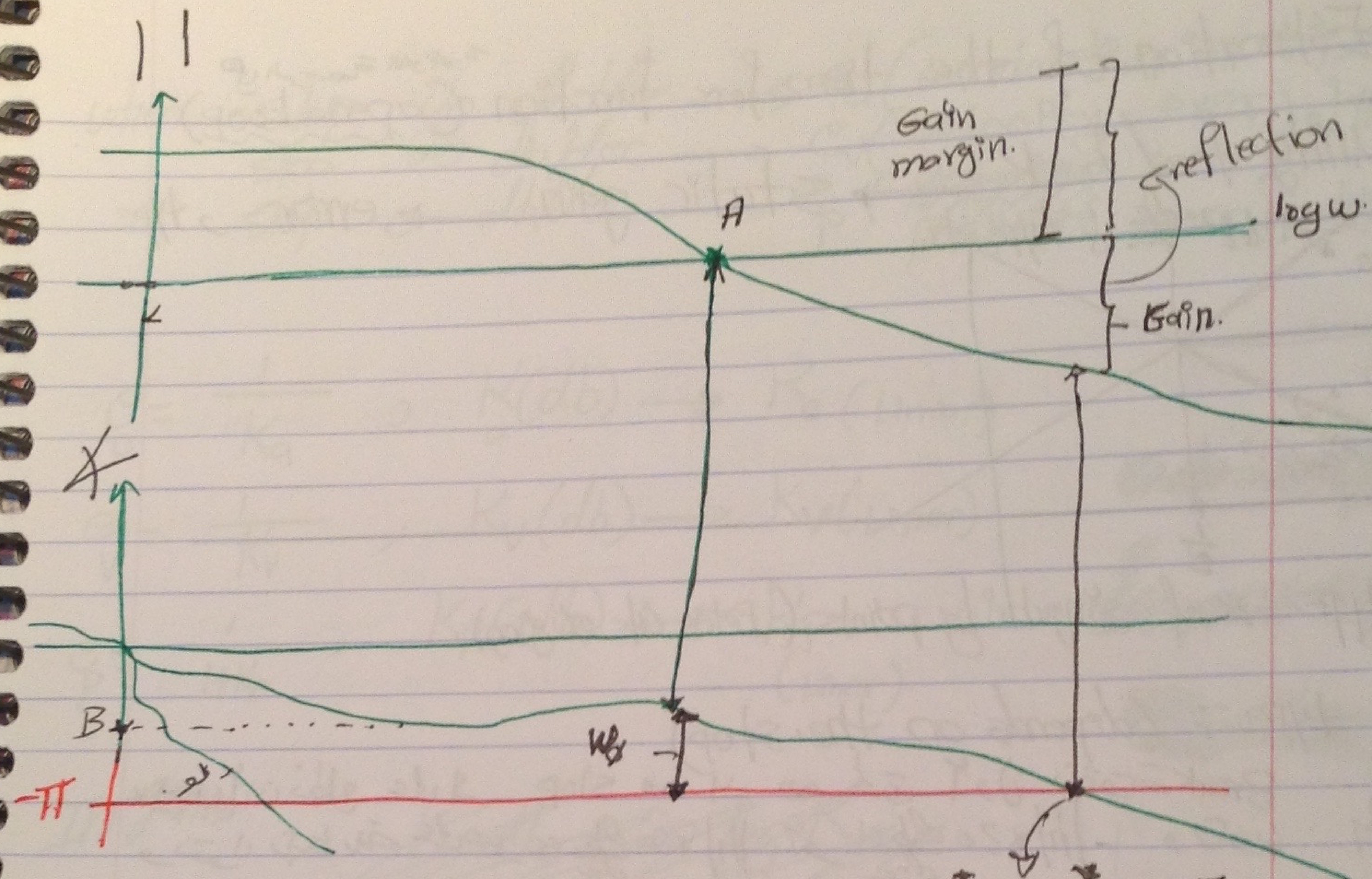
Given a L.T.I system with frequency response (open-Loop) the system is marginally stable $\Longleftrightarrow$ the phase margin is positive or the gain margin in db positive $\rightarrow$ ($M_g > 1$).

* الرسمة تكون لل open loop (والقوس ال

ال system بكون marginally stable اذا كان phase margin > 0 أو (gain margin)$_{db} > 0$



$|G(\omega_c)|_{db} = 0$ at poin A



$|G(\omega_c)| = 0$ @ point A & $\angle G(\omega_\phi) = -\pi$

phas margin @ point B

Phase margin positive or above $-\pi$

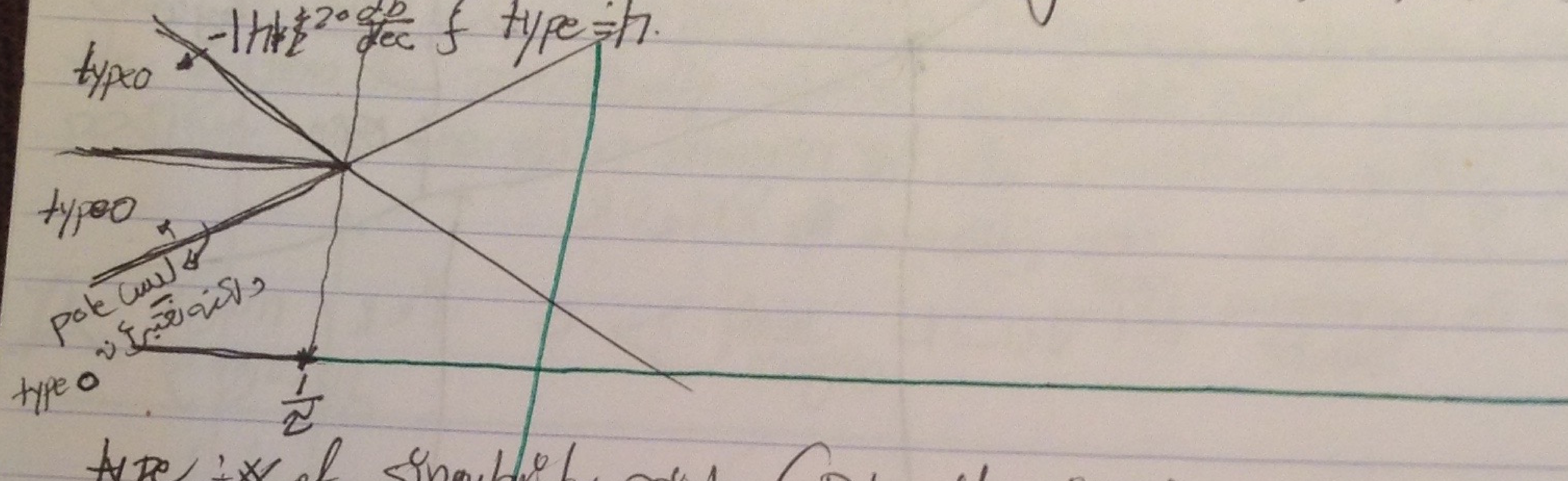
or gain margin in db > 0

so the system is <u>Marginally</u> stable

هي الرسمة الموجودة

- Estimation of the transfer function (open-Loop)

unity feed back ⟶ static gain ⟶ error , type



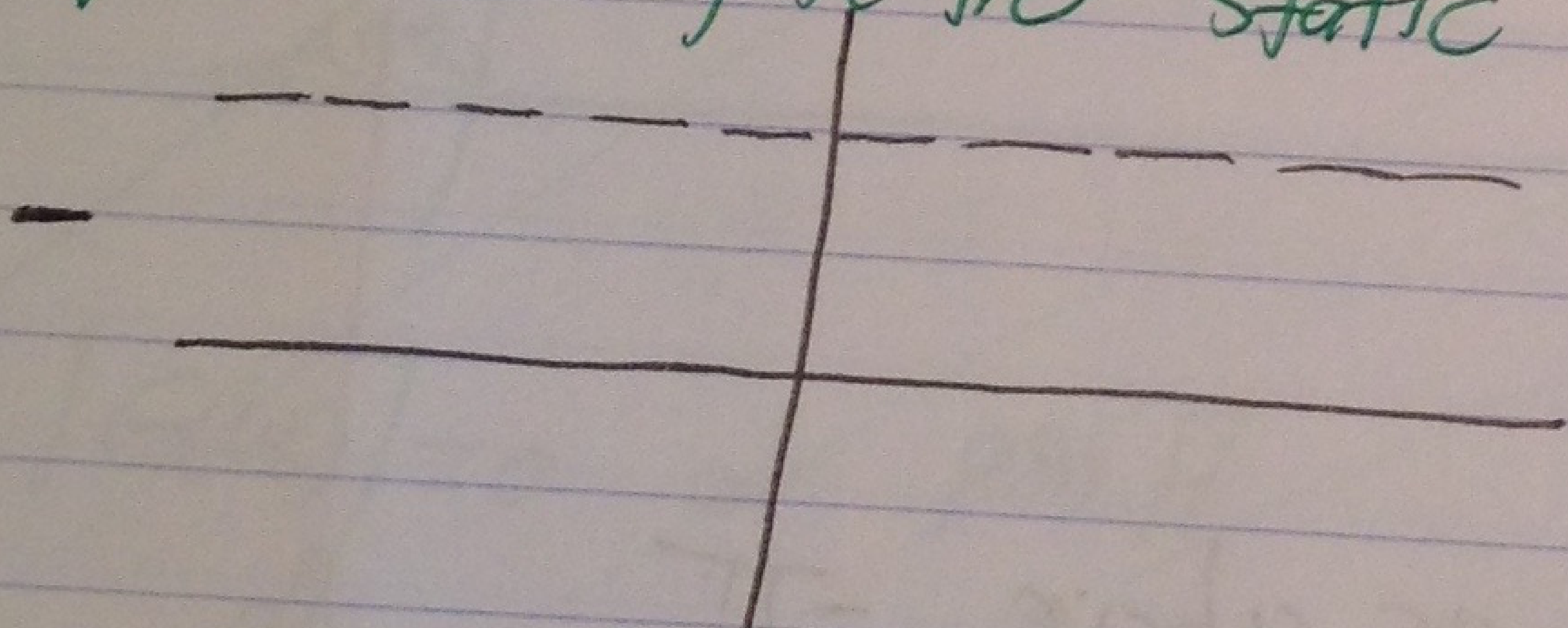
type = # of singularity points (poles at origin).

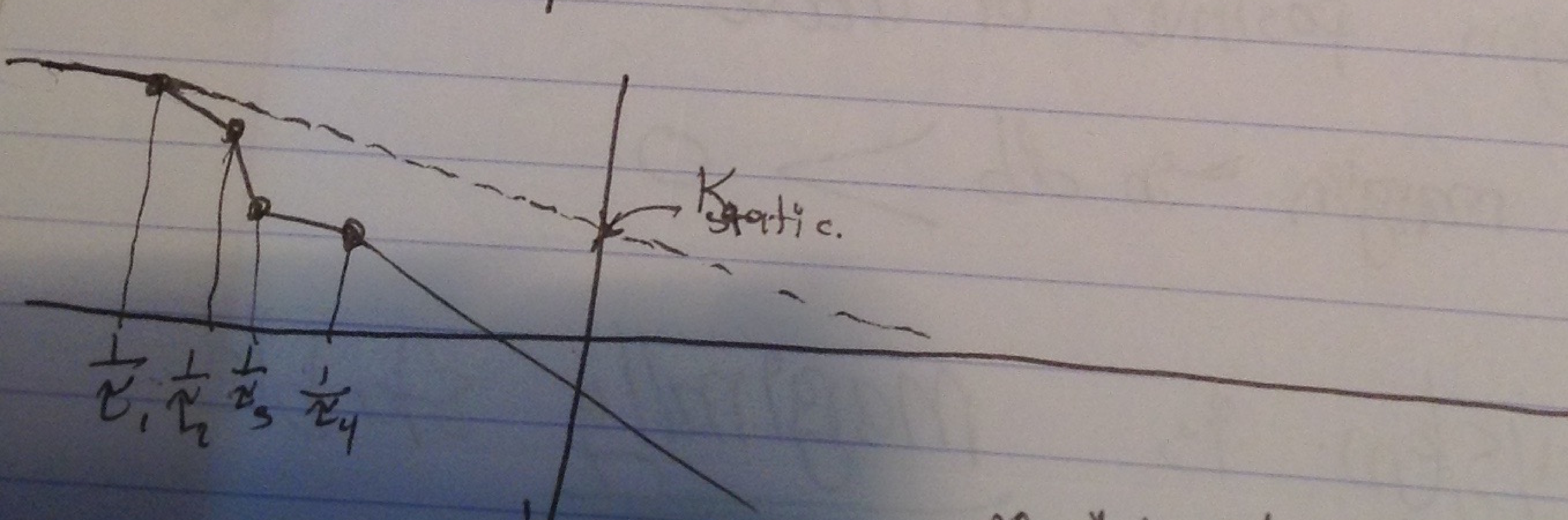
* type : (depends on the stop)

* يجب أن نطلع على ال slop من ال ∞ حتى أول نقطة Break ونقول بداية عليها $|h| * 20 \frac{dB}{dec}$ أو $-|h| * 20 \frac{dB}{dec}$ وتكون h هي

type. أ

* How to compute the static gain.





* نقوم بمد الخط الذي جاي من ال -∞
ونرى أين يقطع ||