Chapter 7.1, Problem 16E

Problem

Let F and G be functions from the set of all real numbers to itself. Define new functions $F - G$: $\mathbf{R} \to \mathbf{R}$ and $G - F$: $\mathbf{R} \to \mathbf{R}$ as follows: For all $x \in \mathbf{R}$,

$(F - G)(x) = F(x) - G(x)$

$(G - F)(x) = G(x) - F(x)$

Does $F - G = G - F$? Explain.

Step-by-step solution

Step 1 of 1

Consider the functions F and G are defined from real numbers to itself.

Define the product $F-G:\mathbb{R}\to\mathbb{R}$ and $G-F:\mathbb{R}\to\mathbb{R}$ as,

$$(F-G)(x)=F(x)\cdot G(x)$$

$$(G-F)(x)=G(x)\cdot F(x)$$

The objective does $F-G=G-F$, explain.

We know that the subtraction of real numbers does not obey the Abelian property.

That is,

$$F(x)-G(x)\neq G(x)-F(x)$$

$$(F-G)(x)\neq(G-F)(x),\ \text{for all } x\in\mathbb{R}$$

Therefore, $\boxed{F-G\neq G-F}$