# 10.6 Part 1

Wednesday, March 31, 2021 12:47 PM

$\sum a_n \rightarrow ??$

$n^{th}$ partial sum Test $\Rightarrow S_n$ (conv./Div)

$\lim_{n\to\infty} S_n = L \Rightarrow \sum a_n = L$

$\lim_{n\to\infty} S_n$ div $\Rightarrow \sum a_n$ div

---

$n^{th}$ term test (only for div)

$\Rightarrow \lim_{n\to\infty} a_n \neq 0 \Rightarrow \sum a_n$ div

$\lim_{n\to\infty} a_n$ DNE $\Rightarrow \sum a_n$ div

---

Geometric Series $\begin{cases} \to \text{conv.} \\ \to \text{div} \end{cases}$

(conv. /Div)

$\sum_{n=1}^{\infty} 2^n = 2 + 2^2 + 2^3 + 2^4 + 2^5 + \dots$

$a=2$, $r = 2 > 1 \Rightarrow$ div by $n^{th}$ term test

geom.

$\lim_{n\to\infty} 2^n = \infty \neq 0$

$\sum_{n=1}^{\infty} \frac{1}{2^n} = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots$ geometric conv.

$a = \frac{1}{2}$, $r = \frac{1}{2} \in (-1,1)$

$\frac{\frac{1}{2}}{1-\frac{1}{2}} = 1$

$\sum_{n=1}^{\infty} \frac{1}{2^n} = 1$

---

RT: (conv./Div)

$\sum a_n \Rightarrow \lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \rho$

$\rho > 1 \Rightarrow \sum a_n$ div

$\rho < 1 \Rightarrow \sum a_n$ conv.

$\rho = 1 \Rightarrow$ Test fail

---

Root Test (conv. /Div)

$\lim_{n\to\infty} \sqrt[n]{a_n} = \rho$

---

DCT conv./div.

Conv.

$< \sum a_n <$ $\Rightarrow \sum a_n$ conv.

div

$< \sum a_n \Rightarrow \sum a_n$ div.

$\bigcirc < \sum a_n \Rightarrow \sum a_n$ div.

**LCT** (conv./Div) $\sum a_n \Rightarrow$ find $b_n$ s.t $\sum b_n$ known.

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \begin{cases} 0 & \text{and } \sum b_n \text{ conv.} \Rightarrow \sum a_n \text{ conv} \\ \infty & \text{and } \sum b_n \text{ div} \Rightarrow \sum a_n \text{ div} \\ c>0 & \text{both conv. or both Div. } \sum a_n, \sum b_n \end{cases}$$

**IT** (conv./div.) $\sum_{n=1}^{\infty} a_n$ , $a_n$ cont. on $[1,\infty)$ $\quad a_n = f(x)$

$$\int_1^{\infty} f(x)\,dx = \begin{cases} \text{conv.} \Rightarrow \sum a_n \text{ conv.} \\ \text{div} \Rightarrow \sum a_n \text{ div.} \end{cases}$$

**10.6 AST**

$\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} (-1)^{n+1} u_n$ ... *

$a_n = (-1)^{n+1} u_n$

$u_2 > 0$

$a_2 = -u_2 < 0$

$a_3 = u_3 > 0$

$a_4 = -u_4 < 0$

$= u_1 - u_2 + u_3 - u_4 + \cdots$

$(-1)^n$ ✓

$(-1)^{n+1}$ ✓

$(-1)^{n-1}$ ✓

**AST**: $\sum (-1)^{n+1} u_n$ conv. if

$u_n = |a_n|$

1. $u_n > 0 \ \forall n$ and
2. $u_n \downarrow$ for large $n$ $\quad u_{n+1} \le u_n$ and
3. $\lim_{n\to\infty} u_n = 0$ " if not $\Rightarrow \sum (-1)^{n+1} u_n$ div by $n^{th}$ term test "

**Exp** Check Conv./Div for

1. $\sum^{\infty} (-1)^{n+1}$ ... Alternating Harmonic Series conv.

① $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$ Alternating Harmonic Series

$a_n = (-1)^{n+1} \frac{1}{n} \Rightarrow \boxed{u_n = |a_n| = \frac{1}{n}}$

Apply AST Test

① $u_n > 0$ for all $n$ ✓

② $u_n \downarrow$ for all

③ $\lim_{n\to\infty} u_n = \lim_{n\to\infty} \frac{1}{n} = 0$

Hence, $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n}$ conv. by AST

But Harmonic Series $\sum_{n=1}^{\infty} \frac{1}{n}$ div − p series $p=1$

---

② $\sum_{n=1}^{\infty} (-1)^n (0.2)^n$ Alternation $(-1)^n$

$a_n = (-1)^n (0.2)^n \Rightarrow u_n = (0.2)^n = \left(\frac{2}{10}\right)^n = \left(\frac{1}{5}\right)^n = \frac{1}{5^n}$

Apply AST

① $u_n > 0$ ✓ for all $n$

② $u_n \downarrow$ for all $n$ ✓

③ $\lim_{n\to\infty} u_n = \lim_{n\to\infty} \frac{1}{5^n} = \lim_{n\to\infty} (0.2)^n = 0$ by 10.1 $\Rightarrow \frac{1}{5} = 5^{-1}$

Hence, $\sum (-1)^n (0.2)^n$ conv. by AST

$\sum_{n=1}^{\infty} (-1)^n (0.2)^n = -(0.2) + (0.2)^2 - (0.2)^3 + (0.2)^4 + \dots = \frac{a}{1-r} = \frac{-0.2}{1 - -0.2} = \frac{-0.2}{1.2}$

$a = -0.2$, $r = -0.2 \in (-1,1) \Rightarrow$ conv. geometric

$\Rightarrow$

$= \frac{-2}{12} = \frac{-1}{6}$ ✓

$= -0.16666667$

$\Rightarrow$

$= -0.16666667$

③ $\sum_{n=1}^{\infty} (-1)^n n$   Alternating $\Rightarrow$ Apply AST

$a_n \Rightarrow u_n = n$

$u_1 = 1, \; u_2 = 2, \; u_3 = 3 \ldots$

$\boxed{1}$ $u_n > 0$ ✓

$\boxed{2}$ $u_n$ is not decreasing

$\boxed{3}$ $\lim_{n\to\infty} u_n = \lim_{n\to\infty} n = \infty \neq 0 \Rightarrow$

$\sum (-1)^n n$ div by $n^{th}$ term test

④ $\sum_{n=3}^{\infty} (-1)^n \frac{2n}{3n-4}$   Alternating $\Rightarrow$ Apply AST

$u_n = \frac{2n}{3n-4}$

$\boxed{1}$ ✓

$\boxed{2}$

$\rightarrow \boxed{3}$ $\lim_{n\to\infty} u_n = \lim_{n\to\infty} \frac{2n}{3n-4} = \frac{2}{3} \neq 0$ ✓

Hence, $\sum (-1)^n \frac{2n}{3n-4}$ div. by $n^{th}$ term test

AST $\sum a_n$

$\boxed{1}$ ✓

$\boxed{2}$ ✓ $\Rightarrow \sum a_n$ conv.

$\boxed{2}$ ✓

$\boxed{3}$ ✓

$\boxed{1}$

$\boxed{2}$ $\Longrightarrow$ $\sum a_n$ div by $n^{th}$ term test

$\boxed{3}$ X

$\boxed{1}$ ✓

$\boxed{2}$ X $\Rightarrow$ we can't say $\sum a_n$ conv. / $\sum a_n$ div

$\boxed{3}$ ✓ Test Fail

---

Def ( Conv. Abs. )

When $\sum a_n$ conv. Abs?

$\sum a_n$ conv. Abs if $\sum |a_n|$ conv.

$\sum u_n$

---

Exp Check if $\sum a_n$ conv. Abs ?!

① $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n^2}$ $u_n$ $\boxed{1}$✓ $\boxed{2}$✓ $\boxed{3}$✓ → conv. by AST → $a_n$

$\Rightarrow \sum |a_n| = \sum_{n=1}^{\infty} \frac{1}{n^2}$ → conv. p-series

$n=1$ Un ① by AST $a_n$ Conv. p-series

$\Rightarrow \sum (-1)^{n+1} \frac{1}{n^2}$ conv. Abs.

Th If $\sum |a_n|$ conv. $\Rightarrow \sum a_n$ conv.

means if $\sum a_n$ conv. Abs

$(\sum |a_n|$ conv$)$

then $\sum a_n$ conv.

②

 