0.7 Algebraic Fractions

Def • Rational functions have the form

$$R(x) = \frac{f(x)}{g(x)} \ , \ g(x) \neq 0 \quad \text{where } f \text{ and } g \text{ are polynomials}$$

- f is the numerator
  g is the denominator
- If f and g are more than polynomials, then $\frac{f}{g}$ is called algebraic fraction

Exp (Simplifying fractions)

Simplify the following fractions:

[1] $\dfrac{x-3y}{3x-9y} = \dfrac{\cancel{x-3y}}{3\cancel{(x-3y)}} = \dfrac{1}{3}$

[2] $\dfrac{x^2-5x+6}{9-x^2} = \dfrac{(x-3)(x-2)}{(3-x)(3+x)} = \dfrac{-\cancel{(3-x)}(x-2)}{\cancel{(3-x)}(3+x)} = \dfrac{2-x}{3+x}$

[3] $\dfrac{3x^2-14x+8}{x^2-16} = \dfrac{(3x-2)\cancel{(x-4)}}{\cancel{(x-4)}(x+4)} = \dfrac{3x-2}{x+4}$

Exp (Multiplying Fractions)

Perform the following operations and simplify

[1] $\dfrac{6x^3}{8y^3} \cdot \dfrac{16x}{9y^3} \cdot \dfrac{15y^4}{x^3} = \dfrac{6\cancel{x^3} \cdot 16x \cdot 15\cancel{y^4}^{\,y}}{8\cancel{y^3} \cdot 9\cancel{y^3}_{\,y^2} \cdot \cancel{x^3}} = \dfrac{x \cdot \cancel{6}^{\,3} \cdot \cancel{16}^{\,4} \cdot \cancel{15}^{\,5}}{y \cdot \cancel{8}_{\,2} \cdot \cancel{9}_{\,3}}$

$= \dfrac{20x}{y}$

$\boxed{35}$

$\boxed{2}$ $\frac{4x^2}{5y} \cdot \frac{10x}{y^2} \cdot \frac{y}{8x^2} = \frac{\cancel{4}\cancel{x^2} \cdot \cancel{10}^{2} x \cdot \cancel{y}}{\cancel{8}_{2}\cancel{x^2} \cdot \cancel{5}\cancel{y} \cdot y^2} = \frac{x}{y^2}$

$\boxed{3}$ $\frac{-4x+8}{3x+6} \cdot \frac{2x+4}{4x+12} = \frac{(-4x+8)(2x+4)}{(3x+6)(4x+12)}$

$= \frac{-\cancel{4}(x-2)\ (2)\cancel{(x+2)}}{3\cancel{(x+2)}\ \cancel{(4)}(x+3)} = \frac{-2(x-2)}{3(x+3)}$

$\boxed{4}$ $(x^2-9) \cdot \frac{2x+1}{x-3}$

$= \frac{x^2-9}{1} \cdot \frac{2x+1}{x-3} = \frac{(x^2-9)(2x+1)}{x-3} = \frac{\cancel{(x-3)}(x+3)(2x+1)}{\cancel{(x-3)}}$

$= (x+3)(2x+1)$

Exp (Dividing Fractions)

Perform the following operations and simplify

① $\frac{16ac^2}{7bd} \div \frac{4a}{14b^2d} = \frac{16ac^2}{7bd} \cdot \frac{14b^2d}{4a} = \frac{\cancel{16}^{4}\cancel{a}c^2 \cdot \cancel{14}^{2} b^{\cancel{2}}\cancel{d}}{\cancel{7}\cancel{b}\cancel{d} \cdot \cancel{4}\cancel{a}}$

$= 8bc^2$

$\boxed{2}$ $\frac{6x^2-6}{x^2+3x+2} \div \frac{x-1}{x^2+4x+4} = \frac{6(x^2-1)}{(x+1)(x+2)} \cdot \frac{x^2+4x+4}{(x-1)}$

$= \frac{6\cancel{(x-1)}\cancel{(x+1)}\ \cancel{(x+2)}(x+2)}{\cancel{(x+1)}\ \cancel{(x+2)}\ \cancel{(x-1)}}$

$= 6(x+2)$

$\boxed{3}$ $(2x^2-4x) \div 2x$

$\cancel{2x}(x-2) \cdot \frac{1}{\cancel{2x}} = x-2$

Exp (Adding or subtracting fractions)

Perform the following operations and simplify

[1] $\frac{2x}{x^2-x-2} - \frac{x+2}{x^2-x-2} = \frac{2x-(x+2)}{x^2-x-2}$

$= \frac{2x-x-2}{(x-2)(x+1)} = \frac{\cancel{(x-2)}}{\cancel{(x-2)}(x+1)} = \frac{1}{x+1}$

[2] $\frac{3x}{a^2} + \frac{4}{ax}$

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$\frac{3x\cdot x}{a^2\cdot x} + \frac{4\cdot a}{ax\cdot a} = \frac{3x^2}{a^2x} + \frac{4a}{a^2x} = \frac{3x^2+4a}{a^2x}$

a^2x is called

LCD: Least common denominator

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[3] $\frac{y-3}{(y-5)^2} - \frac{y-2}{y^2-4y-5} = \frac{y-3}{(y-5)^2} - \frac{y-2}{(y-5)(y+1)}$

$= \frac{(y-3)\cdot(y+1)}{(y-5)^2\cdot(y+1)} - \frac{(y-2)\cdot(y-5)}{(y-5)(y+1)\cdot(y-5)}$

$= \frac{(y-3)(y+1)}{(y-5)^2(y+1)} - \frac{(y-2)(y-5)}{(y-5)^2(y+1)} = \frac{(y-3)(y+1)-(y-2)(y-5)}{(y-5)^2(y+1)}$

$LCD = (y-5)^2(y+1)$

$= \frac{\cancel{y^2}+y-3y-3-(\cancel{y^2}-5y-2y+10)}{(y-5)^2(y+1)} = \frac{5y-13}{(y-5)^2(y+1)}$

4) $\dfrac{x}{x+1} - x + 1 = \dfrac{x}{x+1} + \dfrac{1-x}{1}$

$= \dfrac{x}{x+1} + \dfrac{(1-x)(x+1)}{1\cdot(x+1)} = \dfrac{x}{x+1} + \dfrac{(1-x)(1+x)}{x+1}$

Note that LCD $= x+1$

$= \dfrac{x+1-x^2}{x+1} = \dfrac{-x^2+x+1}{x+1}$

5) $\dfrac{x}{x^2-4} + \dfrac{4}{x^2-x-2} - \dfrac{x-2}{x^2+3x+2}$

$= \dfrac{x}{(x-2)(x+2)} + \dfrac{4}{(x-2)(x+1)} - \dfrac{x-2}{(x+1)(x+2)}$

$= \dfrac{x\,(x+1)}{(x-2)(x+2)(x+1)} + \dfrac{4\,(x+2)}{(x-2)(x+1)(x+2)} - \dfrac{(x-2)\,(x-2)}{(x+1)(x+2)(x-2)}$

Note that LCD $= (x+1)(x+2)(x-2)$

$= \dfrac{\cancel{x^2} + x + 4x + 8 - (\cancel{x^2} - 4x + 4)}{(x-2)(x+2)(x+1)}$

$= \dfrac{9x + 4}{(x-2)(x+2)(x+1)}$

Def A fraction expression that contains one or more fractions in its numerator or denominator is called complex fraction

Exp $\dfrac{\frac{1}{2} - \frac{5}{x}}{7 + \frac{2}{xy}}$ is complex fraction

Exp (Complex Fractions)

Simplify the following complex fractions

(1) $\dfrac{x+y}{\frac{1}{x}+\frac{1}{y}} = \dfrac{x+y}{\frac{1\cdot y}{x\cdot y}+\frac{1\cdot x}{y\cdot x}} = \dfrac{x+y}{\frac{y}{xy}+\frac{x}{xy}}$ LCD = xy

$= \dfrac{x+y}{\frac{y+x}{xy}} = \cancel{(x+y)}\cdot\dfrac{xy}{\cancel{(y+x)}} = xy$

(2) $\dfrac{x^{-3}+x^{2}y^{-3}}{(xy)^{-2}} = \dfrac{\frac{1}{x^3}+\frac{x^2}{y^3}}{\frac{1}{(xy)^2}} = \dfrac{\frac{1\cdot y^3}{x^3\cdot y^3}+\frac{x^2\cdot x^3}{y^3\cdot x^3}}{\frac{1}{x^2y^2}}$

LCD = x^3y^3

$= \dfrac{\frac{y^3}{x^3y^3}+\frac{x^5}{x^3y^3}}{\frac{1}{x^2y^2}} = \dfrac{\frac{y^3+x^5}{x^3y^3}}{\frac{1}{x^2y^2}} = \dfrac{y^3+x^5}{xy}$

Exp Simplify $(2^{-2}-3^{-1})^{-1}$ so that only positive exponents remain and simplify

$\left(\frac{1}{2^2}-\frac{1}{3^1}\right)^{-1} = \left(\frac{1}{4}-\frac{1}{3}\right)^{-1} = \left(\frac{1\cdot 3}{4\cdot 3}-\frac{1\cdot 4}{3\cdot 4}\right)^{-1}$

LCD = 12

$= \left(\frac{3}{12}-\frac{4}{12}\right)^{-1} = \left(\frac{-1}{12}\right)^{-1} = \frac{12}{-1} = -12$

Exp (Rationalizing Denominators)

Rationalize the denominator of

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(1) $$\frac{1-\sqrt{x}}{1+\sqrt{x}} = \frac{1-\sqrt{x}}{1+\sqrt{x}} \cdot \frac{1-\sqrt{x}}{1-\sqrt{x}}$$

$$= \frac{(1-\sqrt{x})^2}{1-x} = \frac{1-2\sqrt{x}+x}{1-x}$$

(2) $$\frac{1}{\sqrt{x}-2} = \frac{1}{\sqrt{x}-2} \cdot \frac{\sqrt{x}+2}{\sqrt{x}+2} = \frac{\sqrt{x}+2}{x-4}$$

(3) $$\frac{3+\sqrt{x}}{\sqrt{x}+\sqrt{5}} = \frac{3+\sqrt{x}}{\sqrt{x}+\sqrt{5}} \cdot \frac{\sqrt{x}-\sqrt{5}}{\sqrt{x}-\sqrt{5}} = \frac{(3+\sqrt{x})(\sqrt{x}-\sqrt{5})}{x-5}$$

$$= \frac{3\sqrt{x}-3\sqrt{5}+x-\sqrt{5}\sqrt{x}}{x-5} = \frac{3\sqrt{x}-3\sqrt{5}+x-\sqrt{5x}}{x-5}$$

Exp (Rationalizing Numerator)

Rationalize the numerator of $\frac{2+\sqrt{x}}{\sqrt{x}}$

$$\frac{2+\sqrt{x}}{\sqrt{x}} \cdot \frac{2-\sqrt{x}}{2-\sqrt{x}} = \frac{4-x}{\sqrt{x}(2-\sqrt{x})} = \frac{4-x}{2\sqrt{x}-x}$$