FACULATY OF ENGINEERING AND TECHNOLOGY

DEPARTMENT OF ELECTRICAL AND COMPUTER ENGINEERING

Basic Electrical Engineering Lab (ENEE 2101)

Report of Experiment 7

“Second Order Circuits”

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Abstract:

This experiment examines the behavior of second-order RLC circuits in series and parallel configurations, focusing on their responses to step and sinusoidal inputs.

Theory:

The process of determining the natural or step responses of a series RLC circuit follows the same steps as for a parallel RLC circuit, since both types of circuits are governed by differential equations with the same structure.

We start by adding up the voltages around the closed loop on the circuit shown

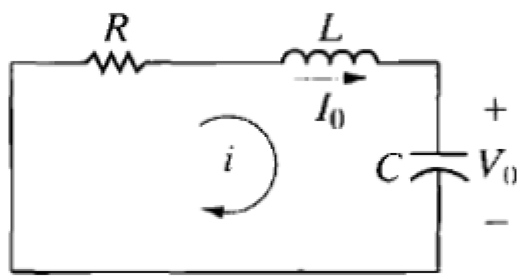


Figure 1:RLC circuit

The total voltage drop across each component in the loop:

$$Ri + L\frac{di}{dt} + \frac{1}{C}\int_0^t i\,d\tau + V_0 = 0.$$

We now differentiate the above equation with respect to t to get the following equation:

$$R\frac{di}{dt} + L\frac{d^2i}{dt^2} + \frac{i}{C} = 0.$$

After rearranging the equation, we get:

$$\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{i}{LC} = 0.$$

The characteristic equation for a series RLC circuit can be expressed as follows:

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0.$$

Since this is a quadratic equation, it will have two distinct solutions, as shown in the following equation:

$$s_{1,2} = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}.$$

Assuming that $\boldsymbol{\alpha = \frac{R}{2L}}$ $\mathbf{Rad}/\boldsymbol{s}$ and $\boldsymbol{\omega_0 = \frac{1}{LC}\ Rad/s^2}$

While α is the neper frequency for RLC circuits, ω_0 is the resonant radian frequency.

As a result, the equation will be:

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}.$$

The response will be overdamped when $\boldsymbol{{\omega_0}^2 < \alpha^2}$

The response will be underdamped when $\boldsymbol{{\omega_0}^2 > \alpha^2}$

The response will be critically damped when $\boldsymbol{\omega^2 = \alpha^2}$

Therefore, the three possible solutions for the current are as follows:

$$i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} \text{ (overdamped)},$$
$$i(t) = B_1 e^{-\alpha t} \cos \omega_d t + B_2 e^{-\alpha t} \sin \omega_d t \text{ (underdamped)},$$
$$i(t) = D_1 t e^{-\alpha t} + D_2 e^{-\alpha t} \text{ (critically damped)}.$$

Figure 2: current equations

The three possible solutions for ***v* care** as follows:

$$v_C = V_f + A_1' e^{s_1 t} + A_2' e^{s_2 t} \text{ (overdamped)},$$
$$v_C = V_f + B_1' e^{-\alpha t} \cos \omega_d t + B_2' e^{-\alpha t} \sin \omega_d t \text{ (underdamped)},$$
$$v_C = V_f + D_1' t e^{-\alpha t} + D_2' e^{-\alpha t} \text{ (critically damped)},$$

Figure 3: voltage equations

Where *Vf* if the final value of *Vc*

<u>Step response of second-order parallel RLC circuit</u>

These equations is a helpful equation to calculate Decay time constant, Damping Coefficient, Damped radian frequency in underdamped response case:

Decay time constant $\tau = \frac{t_b - t_a}{ln\left(\frac{V_a - V_{o(\infty)}}{V_b - V_{o(\infty)}}\right)}$ ..

Damping Coefficient $\alpha = \frac{1}{\tau}$ ....

Damped radian frequency $\omega_d = \frac{2\pi}{t_b - t_a}$ ....

Figure 4:Parallel RLC circuit Equations

Procedure:

Part A: Step response of second-order Series RLC circuit

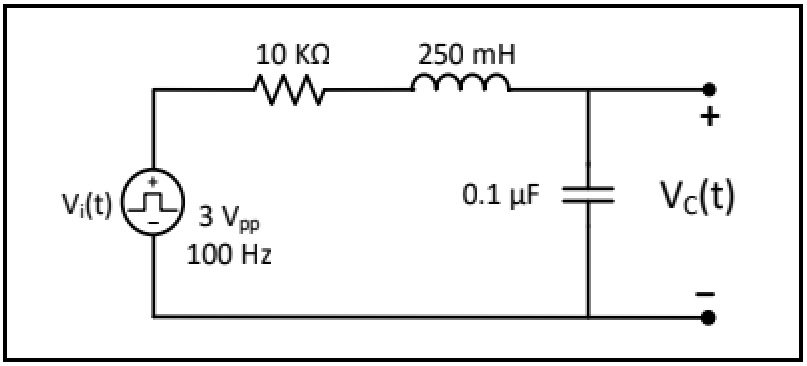


Figure 7.5

Figure 5: Part A circuit

To examine the step response of a second-order series RLC circuit, begin by connecting the circuit as shown in the provided diagram. Set up the function generator to output a square wave with a peak-to-peak voltage of 3 V, a frequency of 100 Hz, and a DC offset of 1.5 V. Use a decade box to adjust the resistor R as needed. Next, use a digital multimeter (DMM) to measure the DC resistance of the inductor and record this value in the data sheet. Connect Channel 2 (CH2) of the oscilloscope across the capacitor. Gradually adjust the variable resistor within its range (10 kΩ to 1 kΩ) while observing the changes in the output voltage $V_c(t)$ across the capacitor.

- ***Case A: Over damped response***

Set the variable resistor to 10 kΩ.

- ***Case B: Critically damped response***

Set the variable resistor to the critical resistance.

- ***Case C: Under damped response***

Set the variable resistor to 500Ω.

After take a picture of resulting waveform we must use cursors to find the values $V_a, t_a, V_b, t_a, V(\infty)$. Then record the results in table 7.1.

Part B: Step response of second-order parallel RLC circuit

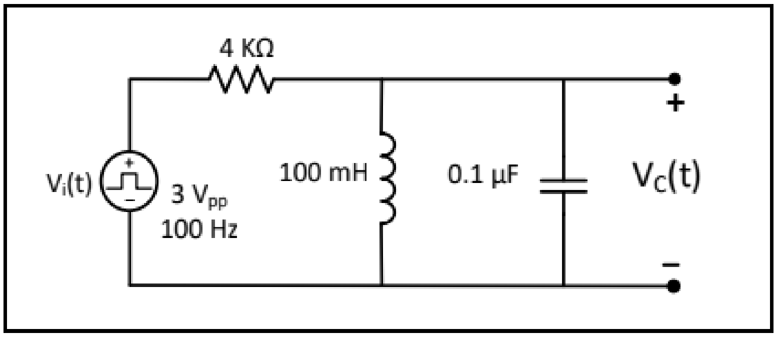


Figure 7.6

Figure 6: Part B circuit

In this procedure for studying the step response of a second-order parallel RLC circuit, Keep the function generator settings the same: an output square wave with a peak-to-peak voltage of 3 V, a frequency of 100 Hz, and a DC offset of 1.5 V. Use a decade box to adjust the resistor R as needed. Next, use a digital multimeter (DMM) to measure the DC resistance of the inductor and record this value in the data sheet. Connect Channel 2 (CH2) of the oscilloscope across the capacitor. Adjust the variable resistor (set to 4 kΩ initially) over its entire range, and observe how the output waveform $V_c(t)$ across the capacitor changes.

- ***Case A: Under damped response***

Set the variable resistor to 4 kΩ.

- ***Case B: Critically damped response***

Set the variable resistor to the critical resistance.

- ***Case C: Over damped response***

Set the variable resistor to 150Ω.

After adjusting the variable resistor and observing the resulting waveform for each setting, capture a picture of the waveform in each case. Use the oscilloscope's cursors to measure the following values $V_a, t_a, V_b, t_a, V(\infty)$. Then record the results in table 7.1.

Data, calculations, and analysis of results

Part A: Step response of second-order Series RLC circuit

Use a digital multimeter (DMM) to measure the resistance of the inductor.

R inductor (Ω)	12.3 Ω

Table 1: inductor resistance

- ***Case A: Over damped response***

Set the variable resistor to 10 kΩ.

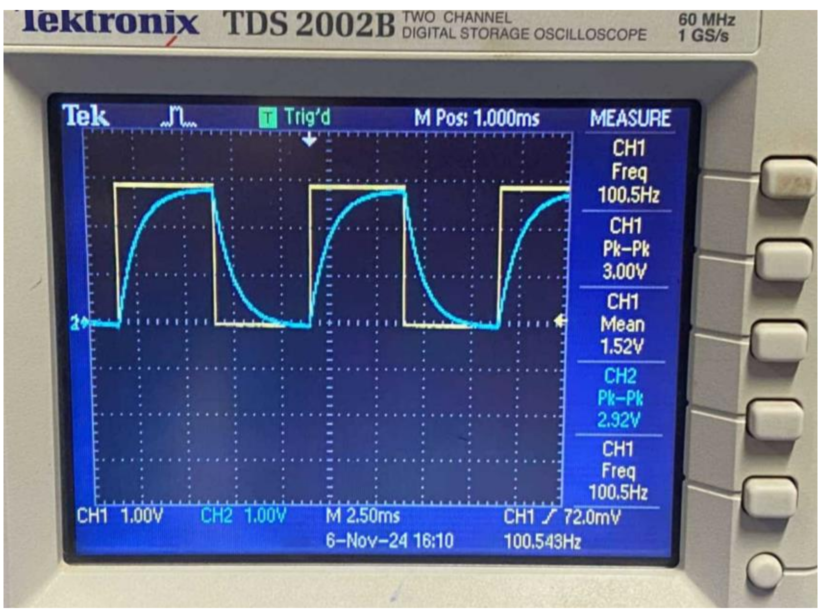


Figure 7: Overdamped response

- ***Case B: Critically damped response***

Calculating R critical using this equation $R_{cr} = \sqrt{\frac{4L}{C}}$ after calculate it we get $R_{cr} = 3.16k\Omega$

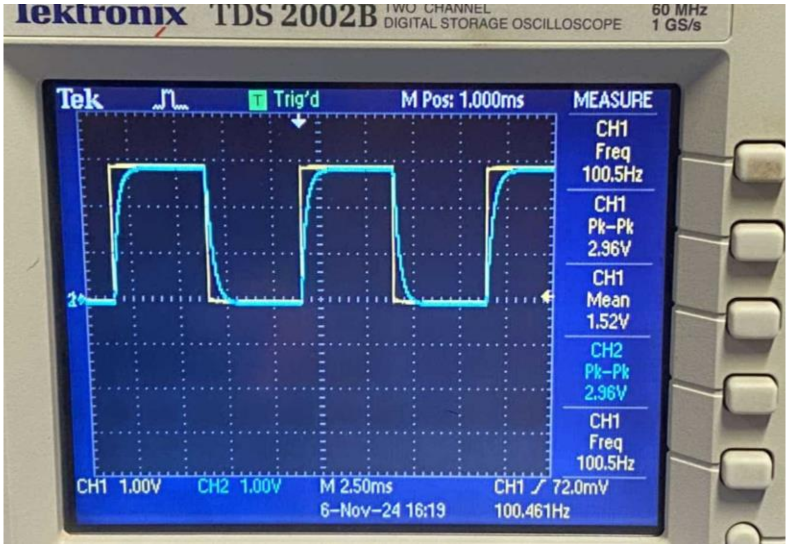


Figure 8: Critical response

As shown in the figures above, the overdamped curve rises more slowly than the critically damped one, as critical damping allows the system to reach its maximum value more quickly.

- ***Case C: Under damped response***

Set the variable resistor to 500Ω.

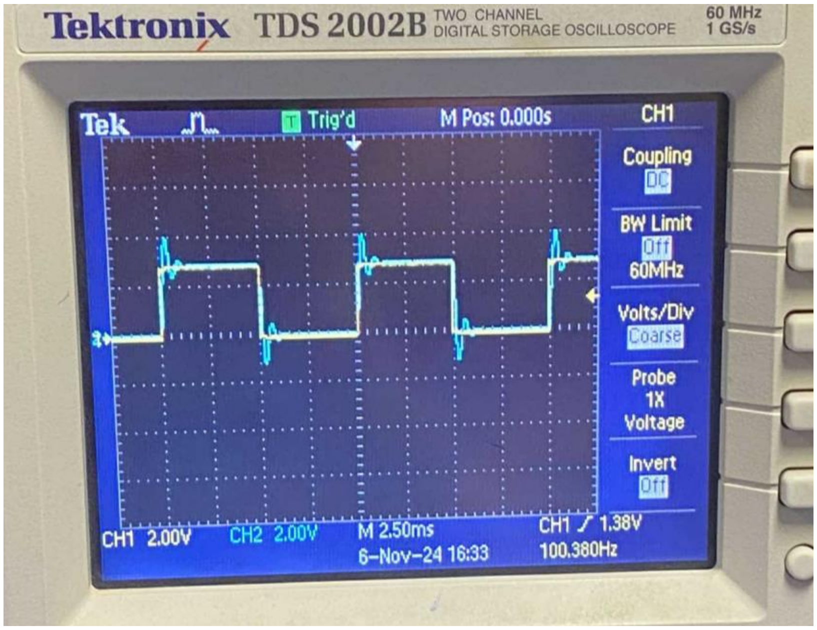


Figure 9: Underdamped response

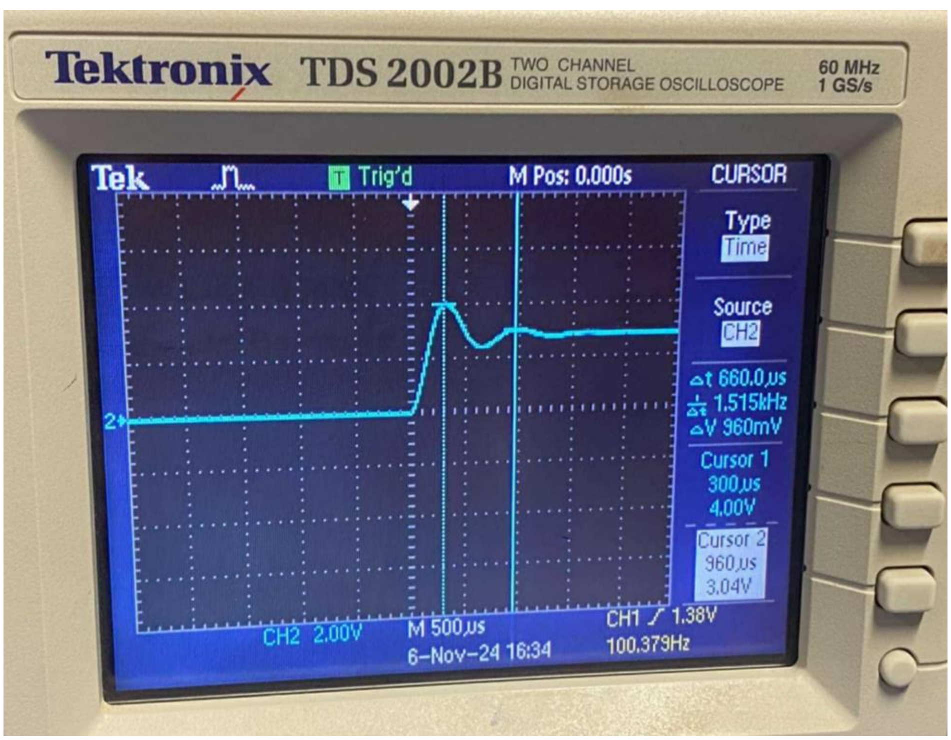


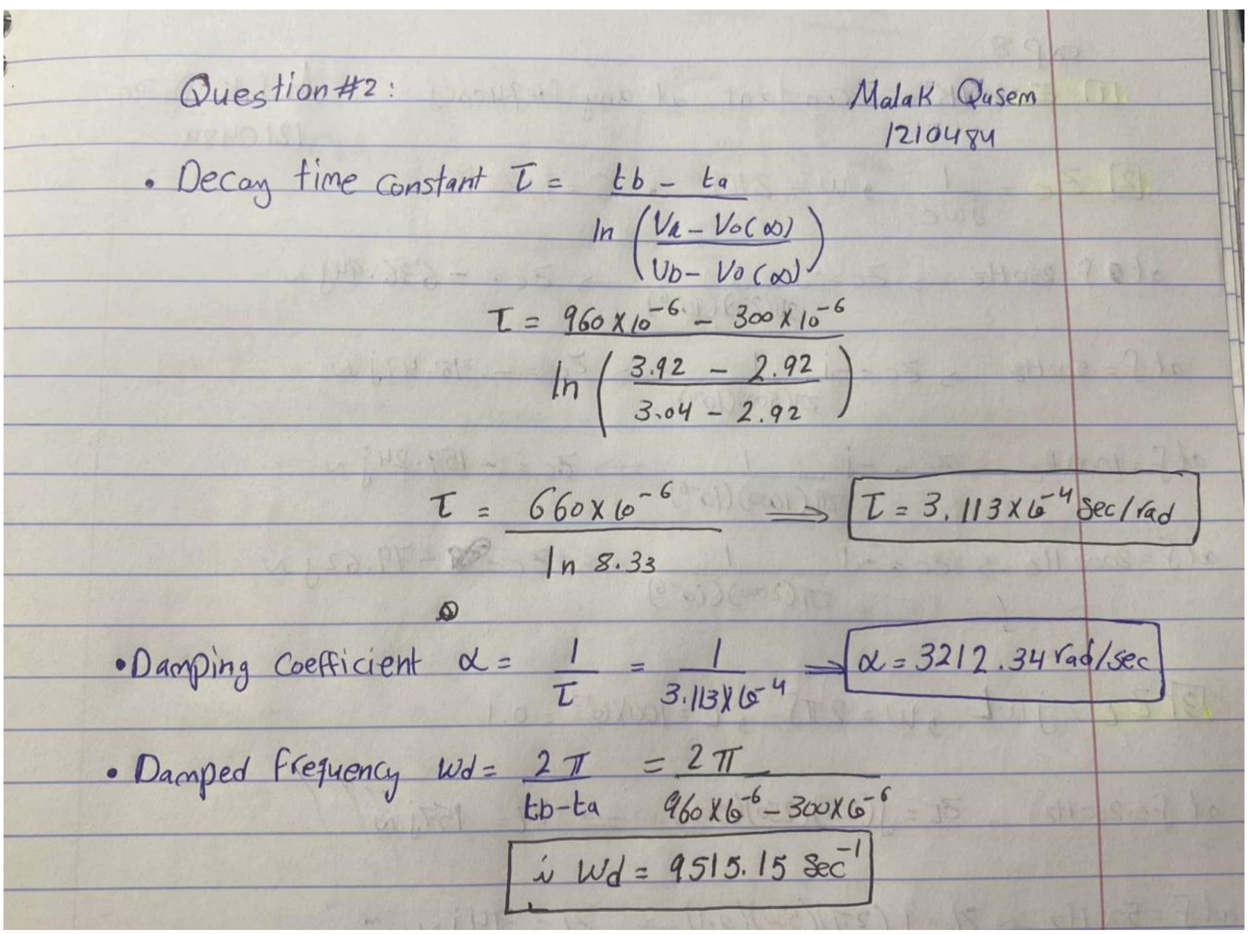
Figure 10: Underdamped response in detailed

The voltage (Va) and time (ta) at the greatest peak and the voltage (Vb) and time (tb) at the lowest peak are measured in an underdamped response. The system's oscillatory nature is captured by these values. The voltage value V (∞) at which the system reaches equilibrium without varying, is measured in a Critical damped response.

Va	ta	Vb	tb	V (∞)
3.92V	300µs	3.04V	960µs	2.92V

Table 2: 7.1 Table

Calculate decay-envelope time constant (τ), the damping coefficient (α), and damped frequency (ω_d):



Question#2:

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• Decay time Constant $\tau = \dfrac{tb - ta}{\ln\left(\dfrac{Va - Vo(\infty)}{Vb - Vo(\infty)}\right)}$

$$\tau = \frac{960\times10^{-6} - 300\times10^{-6}}{\ln\left(\dfrac{3.92 - 2.92}{3.04 - 2.92}\right)}$$

$$\tau = \frac{660\times10^{-6}}{\ln 8.33} \Longrightarrow \boxed{\tau = 3.113\times10^{-4}\ \text{sec/rad}}$$

• Damping Coefficient $\alpha = \dfrac{1}{\tau} = \dfrac{1}{3.113\times10^{-4}} \Longrightarrow \boxed{\alpha = 3212.34\ \text{rad/sec}}$

• Damped frequency $Wd = \dfrac{2\pi}{tb - ta} = \dfrac{2\pi}{960\times10^{-6} - 300\times10^{-6}}$

$$\boxed{\therefore\ Wd = 9515.15\ \text{sec}^{-1}}$$

Figure 11: Question #2

Part B: Step response of second-order parallel RLC circuit

Use a digital multimeter (DMM) to measure the resistance of the inductor.

R inductor (Ω)	12.3 Ω

Table 3: Inductor resistance

- ***Case A: Under damped response***

Set the variable resistor to 4 kΩ.

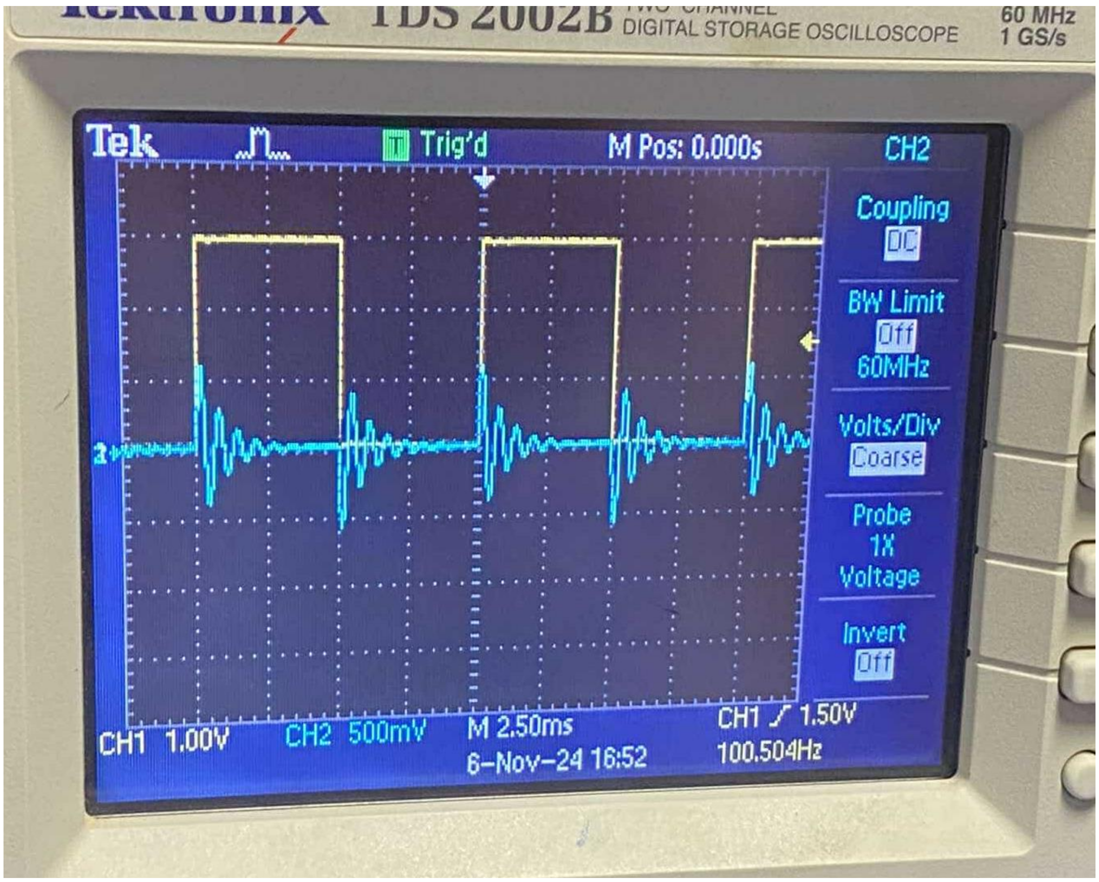


Figure 12: Underdamped response

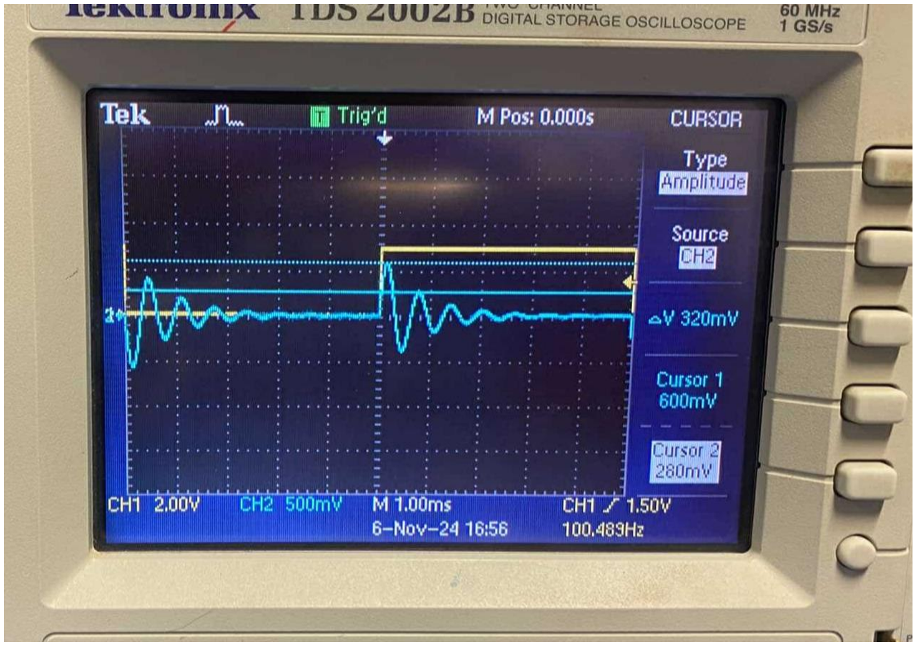
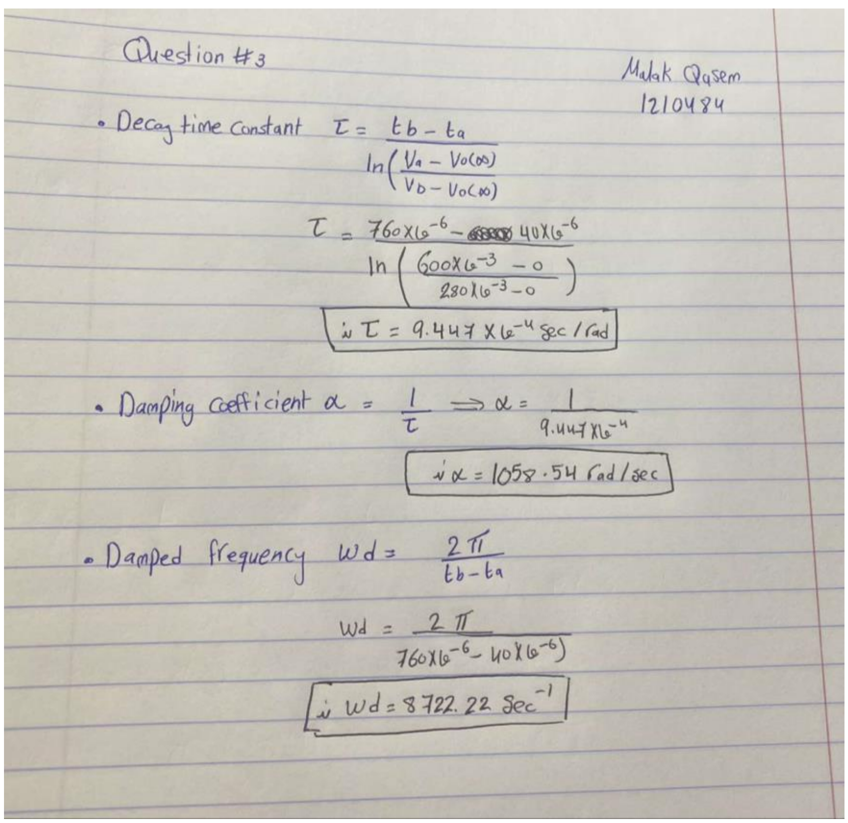


Figure 13: Underdamped response in detailed

Calculate decay-envelope time constant (τ), the damping coefficient (α), and damped frequency (ω_d):



Question #3

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- Decay time constant $\tau = \dfrac{t_b - t_a}{\ln\left(\dfrac{V_a - V_o(\infty)}{V_b - V_o(\infty)}\right)}$

$$\tau = \frac{760\times10^{-6} - 40\times10^{-6}}{\ln\left(\dfrac{600\times10^{-3} - 0}{280\times10^{-3} - 0}\right)}$$

$$\boxed{\therefore \tau = 9.447\times10^{-4}\ \text{sec/rad}}$$

- Damping coefficient $\alpha = \dfrac{1}{\tau} \Rightarrow \alpha = \dfrac{1}{9.447\times10^{-4}}$

$$\boxed{\therefore \alpha = 1058.54\ \text{rad/sec}}$$

- Damped frequency $w_d = \dfrac{2\pi}{t_b - t_a}$

$$w_d = \frac{2\pi}{760\times10^{-6} - 40\times10^{-6}}$$

$$\boxed{\therefore w_d = 8722.22\ \text{sec}^{-1}}$$

Figure 14: Question #3

- ***Case B: Critically damped response***

Calculating R critical using this equation $R_{cr} = \sqrt{\frac{4L}{C}}$ after calculate it we get $R_{cr} = 500\Omega$

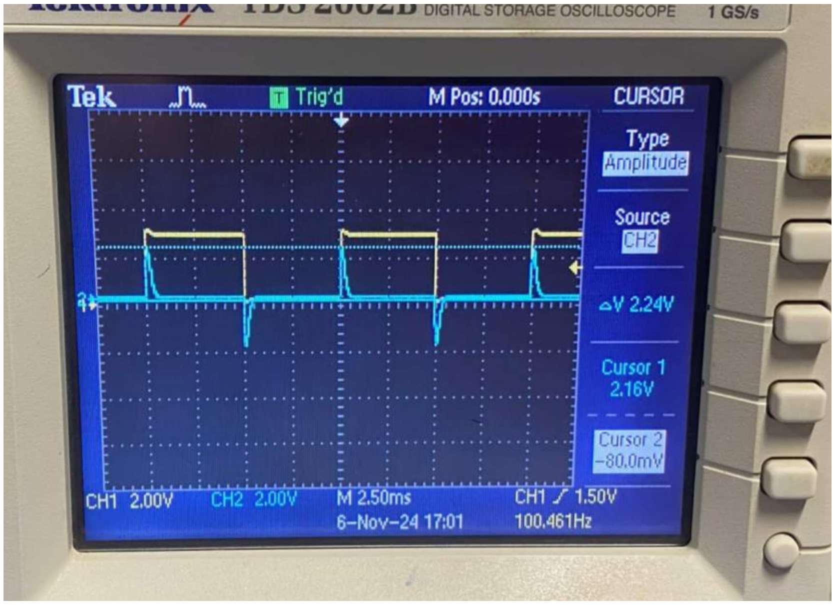


Figure 15: Critical response

- ***Case C: Over damped response***

Set the variable resistor to 150Ω.

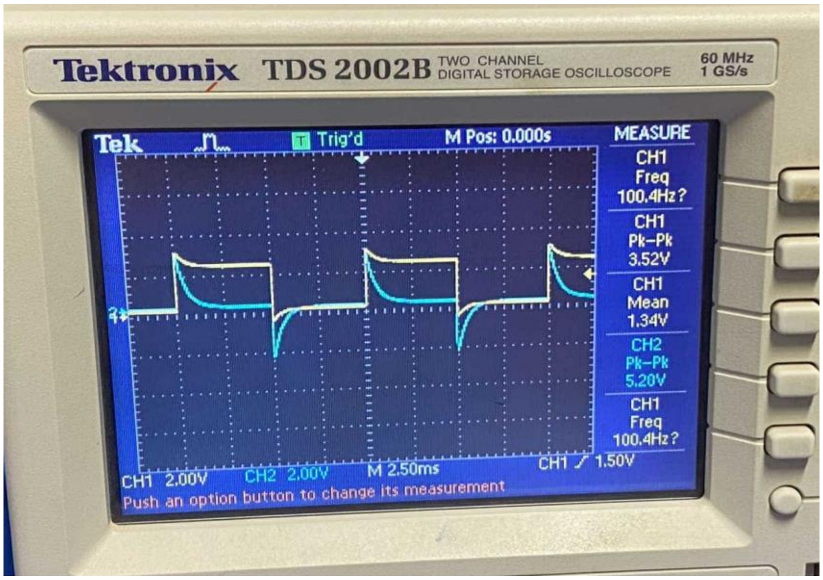


Figure 16: Overdamped response

The voltage (Va) and time (ta) at the greatest peak and the voltage (Vb) and time (tb) at the lowest peak are measured in an underdamped response. The system's oscillatory nature is captured by these values. The voltage value V(∞) at which the system reaches equilibrium without varying, is measured in a Critical damped response.

Va	ta	Vb	tb	V (∞)
600mV	40μs	280mV	760μs	0V

Table 4: 7.2 Table

The critically damped response represents the circuit's quickest decay time without oscillating, while the overdamped response also avoids oscillation but decays at a slower rate.

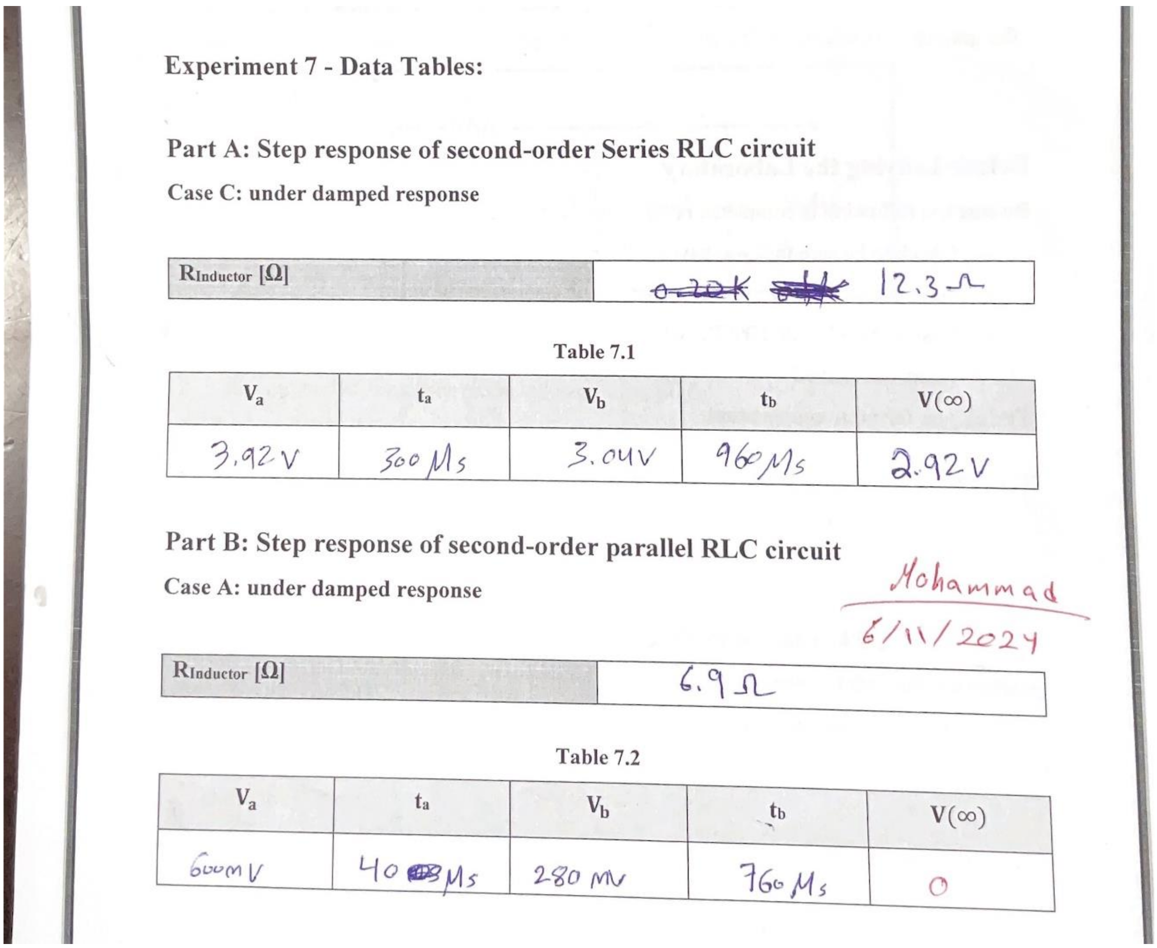
Conclusion:

In conclusion the experiment provided clarity on the natural response of series and parallel RLC circuits, specifically analyzing the three damping cases: **overdamped**, **underdamped**, and **critical damped** responses. The practical analysis allowed for an understanding of the decay time constant, damping coefficient, and damped radian frequency, demonstrating how these factors influence the circuit's response. This Practical method helped clarify theoretical concepts by showing how damping conditions impact RLC circuit behavior and stability.

References:

-ENEE2101-Manual

-Data taken in the laboratory



Experiment 7 - Data Tables:

Part A: Step response of second-order Series RLC circuit

Case C: under damped response

$R_{Inductor}$ [Ω]	~~0.22K~~ ~~[illegible]~~ 12.3 Ω

Table 7.1

V_a	t_a	V_b	t_b	V(∞)
3.92 V	300 μs	3.04 V	960 μs	2.92 V

Part B: Step response of second-order parallel RLC circuit

Case A: under damped response

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6/11/2024

$R_{Inductor}$ [Ω]	6.9 Ω

Table 7.2

V_a	t_a	V_b	t_b	V(∞)
600 mV	40 μs	280 mV	760 μs	0

Figure 17: Data sheet