1 Lyapunov Stability theorem for Linear and non-linears Systems

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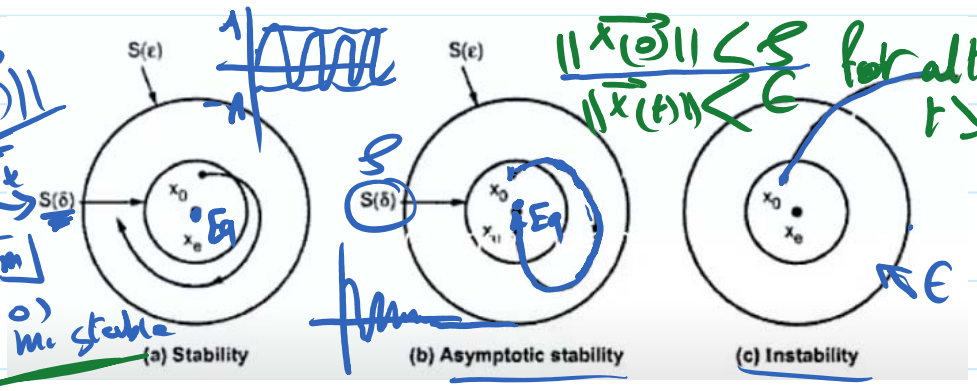
* Assume $\vec{x}(0)$ represent the vector of IC for given Sys. with astable equilibrium point if any arbitrary number $\epsilon$ that exist some the number $\delta$ such that whenever:

* $\|\vec{x}(0)\| < \delta$ then $\|\vec{x}(t)\| < \epsilon$ for all $t > 0$ "stability Condition"

- if $\lim_{t \to \infty} \|\vec{x}(t)\| = 0$, then the Sys. is said Asy. Stable.




$\dot{x}(t) = f(x, u, t) \Rightarrow \dot{x}(t) = Ax + Bu$ (internal Stability)

nonlinear

$u(t) = 0$

$x(0) =$

* According to Lyapnuov theory, one can check the Stability of the Sys by finding some Scalar Fun of the Vector $\vec{x}$ denoted by $v(x)$, which has Continuous first partial derivative and satisfy

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the following Conditions :-

① $V(\vec{x}) > 0$ for all values of $\vec{x}(t) \neq \vec{0}$

② $\dot{V}(\vec{x}) = \frac{\partial V}{\partial x}\dot{\vec{x}} \leq 0$ forall values of $\vec{x}(t) \neq \vec{0}$

$\frac{\partial V}{\partial x}\frac{dx}{dt} \leq 0$

③ $\vec{V}(0) = 0$

if you can find such fun satisfied the previous Condition in the case the Sys. is Asy. stable

but if the Fun. $V(\vec{x})$ doesn't satisfy the previous Condition in this case you can't know if the Sys is Asy. stable / m.stable / unstable.

## * Study the Stability of Non linear Sys. by L.V.

Ex:- Study the stability of the following Sys.

$\dot{x}_1 = x_2$

$\dot{x}_2 = -x_1 - e^{-t} x_2$

} Non linear (*) t Varient Sys

$\dot{x} = f(x, u, t)$, let $V(x) = x_1^2 + x_2^2$

① $V(\vec{x}) > 0$ for all value of $\vec{x}(t) \neq \vec{0}$

$\vec{V}(x) > 0$ (ox)

$\frac{\partial V}{\partial x_1} = 2x_1$

$\frac{\partial V}{\partial x_2} = 2x_2$

② $\dot{V}(x) = \frac{\partial V}{\partial x}\dot{x} \leq 0$ =

$\dot{V}(x) = 2x_1\dot{x}_1 + 2x_2\dot{x}_2$ Sub Eq(*) in (**)

$\dot{V}(x) = 2x_1x_2 + 2x_2(-x_1 - e^{-t}x_2) \leq 0$ ??

$\dot{V}(x) = -2x_2^2 e^{-t} < 0$ satisfied

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$x_1 = 0$
$x_2 = 0$

③ $V(0) = 0 \Rightarrow$ $V(x) = x_1^2 + x_2^2$
$V(0) = (0)^2 + (0)^2$ satisfied

$V(x)$ is Lyapunov fun, Thus the Sys is Asy. Stable

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Ex: Check the stability $\dot{x}_1 = x_2$

$\dot{x}_2 = -(2 + \cos(x_1)) + \sin(x_1)$

let $V(x) = a x_1^2 + b x_2^2$ where $a, b > 0$

③ $V(0) = 0$ for $x_1 = 0, x_2 = 0$ satisfied

① $V(\vec{x}) > 0$ for all $\vec{x(t)} \neq \vec{0}$
satisfied

② $\frac{\partial V}{\partial x} \dot{x} = 2a x_1 \dot{x}_1 + 2b x_2 \dot{x}_2$ $\frac{\partial V}{\partial x} \dot{x} \leq 0$

$= 2a x_1 x_2 - 2b x_2^2 (2 + \cos x_1) + 2b x_2 \sin x_1$

$V(x)$ is not Lyapunov fun. So we can't determine anything

let try $V(x) = a + \cos x_1 + b x_2^2$ for $\begin{cases} a > 1 \\ b > 0 \end{cases}$

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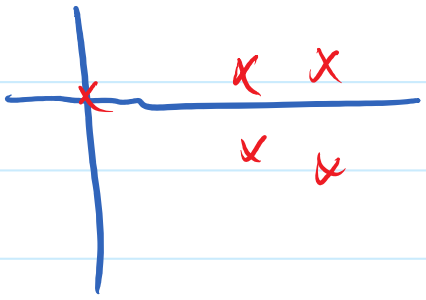
# Lyapnow Stability for LTI

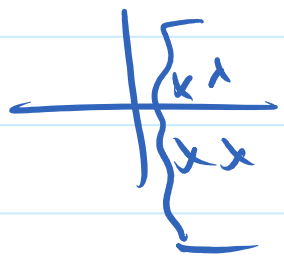
- A Sys is stable in the sense of Lyapnow if it is able to find lyapunov fun.

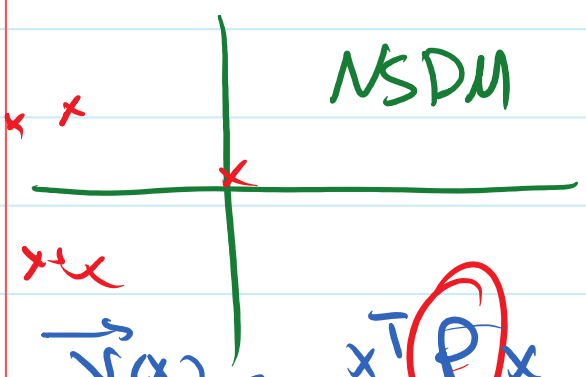
* for all LTI Sys $\dot{x}(t) = Ax(t)$, $\vec{x}(0) = \vec{x_0}$
  the lyapunov fun $\vec{V}(x) = x^T P x > 0$
  P: symetric definite matrix $P = P^T > 0$
  (Positive)

PSDM (semi)

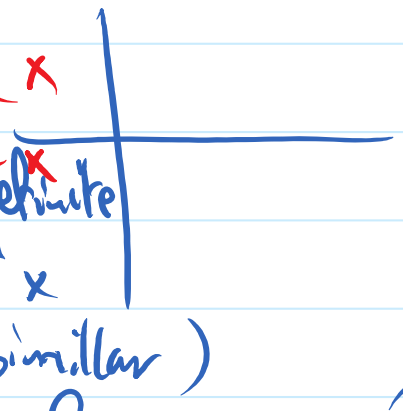


PDM






NDM
Negative definite matrix



$\rightarrow Px^2$ (similar)

$\vec{V}(x) = x^T P x$, Lyapnow fun or not??

(1) $V(0) = 0$ satisfied

(2) $V(x) > 0$ for all $\vec{x}(t) \neq \vec{0}$

$\dot{V}(x) = \dot{\vec{x}}^T P \vec{x} + \vec{x}^T P \dot{\vec{x}}$ — (1) $\dot{\vec{x}} = A\vec{x}$

$\dot{V}(x) = (A\vec{x})^T P \vec{x} + \vec{x}^T P (A\vec{x})$

$= \vec{x}^T A^T P \vec{x} + \vec{x}^T P A \vec{x} = \vec{x}^T (A^T P + PA) x$

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$\dot{V}(x) = \dot{x}^T (A^T P + PA) x$

$Q$ — $\Sigma x$ PDM

$< 0$ ??

ib =

$$A^T P + PA = -Q$$ (ii)

$Q = Q^T > 0$ is any symmetric positive definite matrix

let suppose and Q matrix which is PDM and if you find P matrix is PDM in this case the second Condition is satisfied

$\Sigma x$: $\dot{x} = \begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix} x$ Check the Stability

$|\lambda I - A| =$ ⟵ Lyapunov.

$V(x) = x^T P x$ ① $V(0) = 0$ ② $V(x) > 0$

$\dot{V}(x) = x^T (A^T P + PA) x \leq ?$

$= -Q$

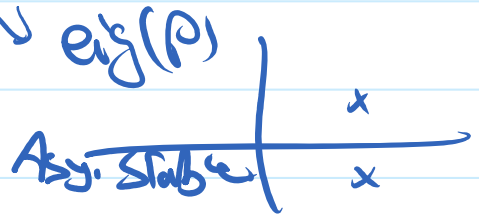
let $Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ ⟵ $(\lambda I - Q) = \lambda_{1,2} = 1$ PDM

$P = \begin{bmatrix} P_{11} & P_{12} \\ P_{12} & P_{22} \end{bmatrix}$

$\begin{bmatrix} 0 & 2 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} P_{11} & P_{12} \\ P_{12} & P_{22} \end{bmatrix} + \begin{bmatrix} P_{11} & P_{12} \\ P_{12} & P_{22} \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

$P = \begin{bmatrix} & \\ & \end{bmatrix}$ ⟶ eig(P)

if P PDM then the Asy. Stable



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