Atomic and Nuclear Physics

Introductory experiments
Dualism of wave and particle

**LD
Physics
Leaflets**

P6.1.5.1

Diffraction of electrons in a polycrystalline lattice (Debye-Scherrer diffraction)

Objects of the experiment

■ Determination of wavelength of the electrons

■ Verification of the de Broglie's equation

■ Determination of lattice plane spacings of graphite

Principles

Louis de Broglie suggested in 1924 that particles could have wave properties in addition to their familiar particle properties. He hypothesized that the wavelength of the particle is inversely proportional to its momentum:

$$\lambda = \frac{h}{p} \qquad\qquad (I)$$

λ: wavelength

h: Planck's constant

p: momentum

His conjecture was confirmed by the experiments of Clinton Davisson and Lester Germer on the diffraction of electrons at crystalline Nickel structures in 1927.

In the present experiment the wave character of electrons is demonstrated by their diffraction at a polycrystalline graphite lattice (Debye-Scherrer diffraction). In contrast to the experiment of Davisson and Germer where electron diffraction is observed in reflection this setup uses a transmission diffraction type similar to the one used by G.P. Thomson in 1928.

From the electrons emitted by the hot cathode a small beam is singled out through a pin diagram. After passing through a focusing electron-optical system the electrons are incident as sharply limited monochromatic beam on a polycrystalline graphite foil. The atoms of the graphite can be regarded as a space lattice which acts as a diffracting grating for the electrons. On the fluorescent screen appears a diffraction pattern of two concentric rings which are centred around the indiffracted electron beam (Fig. 1). The diameter of the concentric rings changes with the wavelength λ and thus with the accelerating voltage U as can be seen by the following considerations:
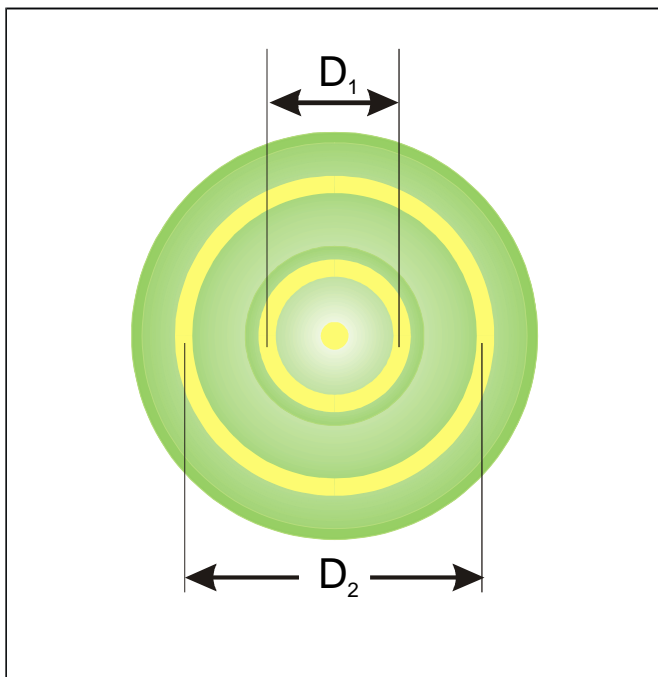

Fig. 1: Schematic representation of the observed ring pattern due to the diffraction of electrons on graphite. Two rings with diameters D_1 and D_2 are observed corresponding to the lattice plane spacings d_1 and d_2 (Fig. 3).

From energy equation for the electrons accelerated by the voltage U

$$e \cdot U = \frac{1}{2} m \cdot v^2 = \frac{p^2}{2 \cdot m} \qquad (II)$$

U: accelerating voltage

e: electron charge

m: mass of the particle

v: velocity of the particle

the momentum p can be derived as

$$p = m \cdot v = \sqrt{2 \cdot e \cdot m \cdot U} \qquad (III)$$

Substituting equation (III) in equation (I) gives for the wavelength:

$$\lambda = \frac{h}{\sqrt{2 \cdot m \cdot e \cdot U}} \qquad (IV)$$

In 1913, H. W. and W. L. Bragg realized that the regular arrangement of atoms in a single crystal can be understood as an array of lattice elements on parallel lattice planes. When we expose such a crystal lattice to monochromatic x-rays or mono-energetic electrons, and, additionally assuming that those have a wave nature, then each element in a lattice plane acts as a "scattering point", at which a spherical wavelet forms. According to Huygens' principle, these spherical wavelets are superposed to create a "reflected" wave front. In this model, the wavelength λ remains unchanged with respect to the "incident" wave front, and the radiation directions which are perpendicular to the two wave fronts fulfil the condition "angle of incidence = angle of reflection".

Constructive interference arises in the neighbouring rays reflected at the individual lattice planes when their path differences $\Delta = \Delta_1 + \Delta_2 = 2 \cdot d \cdot \sin \vartheta$ are integer multiples of the wavelength λ (Fig. 2):

$$2 \cdot d \cdot \sin \vartheta = n \cdot \lambda \qquad n = 1, 2, 3, \ldots \qquad (V)$$

d: lattice plane spacing

ϑ: diffraction angle

This is the so called 'Bragg condition' and the corresponding diffraction angle ϑ is known as the glancing angle.

In this experiment a polycrystalline material is used as diffraction object. This corresponds to a large number of small single crystallites which are irregularly arranged in space. As a result there are always some crystals where the Bragg condition is satisfied for a given direction of incidence and wavelength. The reflections produced by these crystallites lie on a cones whose common axis is given by the direction of incidence. Concentric circles thus appear on a screen located perpendicularly to this axis. The lattice planes which are important for the electron diffraction pattern obtained with this setup possess the lattice plane spacings (Fig. 3):

$d_1 = 2.13 \cdot 10^{-10}$ m

$d_2 = 1.23 \cdot 10^{-10}$ m

From Fig. 4 we can deduce the relationship

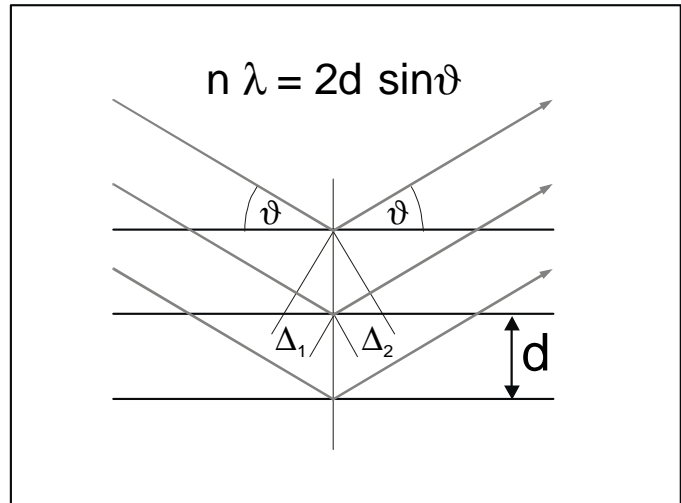$$\tan 2 \cdot \vartheta = \frac{D}{2 \cdot L} \qquad (VI)$$

Fig. 2: Schematic representation of the Bragg condition.

If we approximate $\tan 2 \cdot \vartheta = \sin 2 \cdot \vartheta = 2 \cdot \sin \vartheta$ for small angles we obtain

$$2 \cdot \sin \vartheta = \frac{D}{2 \cdot L} \qquad (VII)$$

The substitution of equation (VII) in (V) leads in first order diffraction (n = 1) to

$$\lambda = d \cdot \frac{D}{2 \cdot L} \qquad (VIII)$$

D: ring diameter

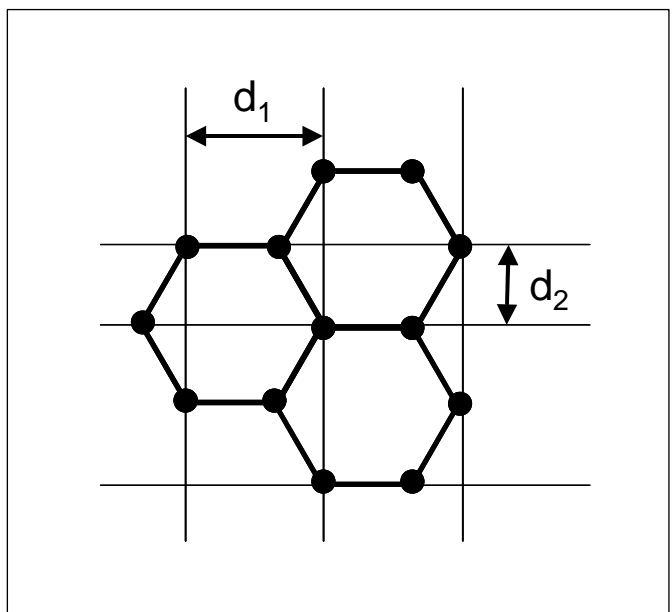L: distance between graphite and screen

d: lattice plane spacing

Fig. 3 Lattice plane spacings in graphite:
$d_1 = 2.13 \cdot 10^{-10}$ m
$d_2 = 1.23 \cdot 10^{-10}$ m

Apparatus

1 Electron diffraction tube......................................555 626
1 Tube stand 555 600
1 High-voltage power supply 10 kV 521 70
1 Precision vernier callipers.................................311 54
1 Safety Connection Lead 25 cm red 500 611
1 Safety Connection Lead 50 cm red 500 621
1 Safety Connection Lead 100 cm red 500 641
1 Safety Connection Lead 100 cm blue.................500 642
2 Safety Connection Lead 100 cm black 500 644

Due to equation (IV) the wavelength λ is determined by the accelerating voltage U. Combining the equation (IV) and equation (VIII) shows that the diameters D_1 and D_2 of the concentric rings change with the accelerating voltage U:

$$D = k \cdot \frac{1}{\sqrt{U}} \qquad (IX)$$

with

$$k = \frac{2 \cdot L \cdot h}{d \cdot \sqrt{2 \cdot m \cdot e}} \qquad (X)$$

Measuring Diameters D_1 and D_2 as function of the accelerating voltage U allows thus to determine the lattice plane spacings d_1 and d_2.

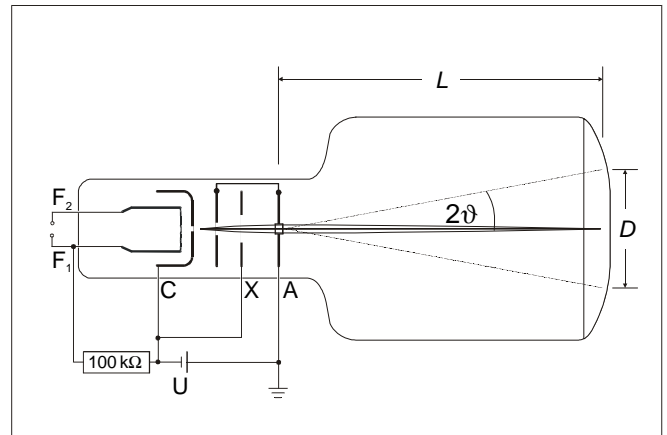
Fig. 4: Schematic sketch for determining the diffraction angle.
L = 13.5 cm (distance between graphite foil and screen),
D: diameter of a diffraction ring observed on the screen
ϑ: diffraction angle
For meaning of F_1, F_2, C, X and A see Fig. 5.

Setup

The experimental setup (wiring diagram) is shown in Fig. 5.

- Connect the cathode heating sockets F1 and F2 of the tube stand to the output on the back of the high-voltage power supply 10 kV.

- Connect the sockets C (cathode cap) and X (focussing electrode) of the tube stand to the negative pole.

- Connect the socket A (anode) to the positive pole of the 5 kV/2 mA output of the high-voltage power supply 10 kV.

- Ground the positive pole on the high-voltage power supply 10 kV.

Safety notes

When the electron diffraction tube is operated at high voltages over 5 kV, X-rays are generated.

■ Do not operate the electron diffraction tube with high voltages over 5 keV.

The connection of the electron diffraction tube with grounded anode given in this instruction sheet requires a high-voltage enduring voltage source for the cathode heating.

■ Use the high-voltage power supply 10 kV (521 70) for supplying the electron diffraction tube with power.

Danger of implosion: the electron diffraction tube is a high-vacuum tube made of thin-walled glass.

■ Do not expose the electron diffraction tube to mechanical stress, and connect it only if it is mounted in the tube stand.

■ Treat the contact pins in the pin base with care, do not bend them, and be careful when inserting them in the tube stand.

The electron diffraction tube may be destroyed by voltages or currents that are too high:

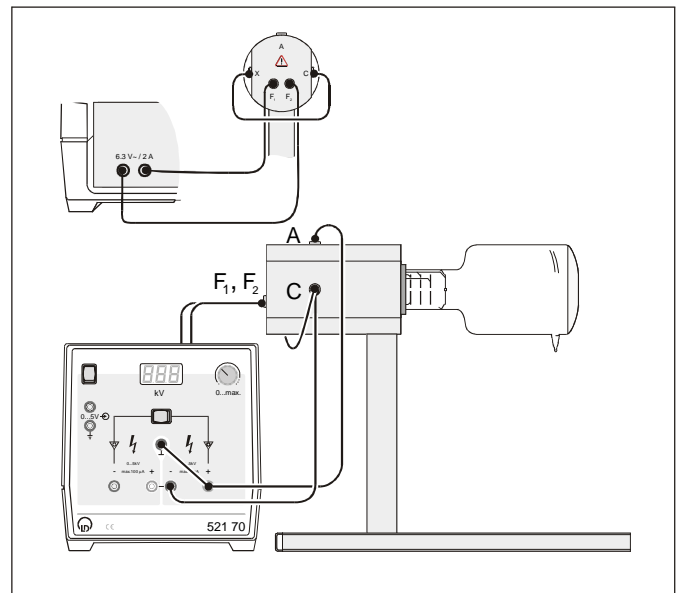■ Keep to the operating parameters given in the section on technical data.

Fig. 5: Experimental setup (wiring diagram) for observing the electron diffraction on graphite. Pin connection:
F_1, F_2: sockets for cathode heating
C: cathode cap
X: focusing electrode
A: anode (with polycrystalline graphite foil see Fig. 4)

Carrying out the experiment

- Apply an accelerating voltage $U \leq 5$ kV and observe the diffraction pattern.

Hint: The direction of the electron beam can be influenced by means of a magnet which can be clamped on the neck of tube near the electron focusing system. To illuminate an another spot of the sample an adjustment of the magnet might be necessary if at least two diffraction rings cannot be seen perfectly in the diffraction pattern.

- Vary the accelerating voltage U between 3 kV and 5 kV in step of 0.5 kV and measure the diameter D_1 and D_2 of the diffraction rings on the screen (Fig. 1).

- Measure the distance between the graphite foil and the screen.

Measuring example

Table 1: Measured diameters D_1 and D_2 (average of 5 measurements) of the concentric diffraction rings as function of the accelerating voltage U.

$\dfrac{U}{kV}$	$\dfrac{D_1}{cm}$	$\dfrac{D_2}{cm}$
3.0	3.30	5.25
3.5	2.83	4.88
4.0	2.66	4.58
4.5	2.40	4.35
5.0	2.33	4.12

Distance between graphite foil and screen: L = 13.5 cm

Evaluation and results

a) Determination of wavelength of the electrons

From the measured values for D_1 and D_2 and the lattice plane spacings d_1 and d_2 the wavelength can be determined using equation (VIII). The result for D_1 and D_2 is summarized in Table 2 and Table 3, respectively.

Note: Rewriting equation (VIII) as

$$d = \lambda \cdot \frac{2 \cdot L}{D}$$

shows that the diameter D of the rings (Fig. 1) is inversely proportional to the lattice plane spacings d (Fig. 2). This information is necessary for the evaluation of the wavelength from the lattice plane spacings (here assumed as known) according equation (VIII). The lattice plane parameters are derived directly in part c) using equations (IX) and (X).

The dominant error in the measurement is the determination of the ring diameters D_1 and D_2. For an accuracy of reading about 2 mm the error is approximately 5% for the outer ring and approximately 10% for the inner ring.

Table 2: Measured diameter D_1 of the concentric diffraction rings as function of the accelerating voltage U. The wavelengths λ_1 and $\lambda_{1,theory}$ are determined by equation (VIII); and equation (IV), respectively.

$\dfrac{U}{kV}$	$\dfrac{D_1}{cm}$	$\dfrac{\lambda_1}{pm}$	$\dfrac{\lambda_{1,theroy}}{pm}$
3.0	3.30	22.9	22.4
3.5	2.83	21.1	20.7
4.0	2.66	19.4	19.4
4.5	2.40	18.5	18.3
5.0	2.33	17.6	17.3

Table 3: Measured diameter D_2 of the concentric diffraction rings as function of the accelerating voltage U. The wavelengths λ_2 and $\lambda_{2,theory}$ are determined by equation (VIII); and equation (IV), respectively.

$\dfrac{U}{kV}$	$\dfrac{D_2}{cm}$	$\dfrac{\lambda_2}{pm}$	$\dfrac{\lambda_{2,theroy}}{pm}$
3.0	5.25	22.6	22.4
3.5	4.88	21.0	20.7
4.0	4.58	19.7	19.4
4.5	4.35	18.6	18.3
5.0	4.12	17.5	17.3

b) Verification of the de Broglie's equation

The de Broglie relation (equation (I)) can be verified using

$e = 1.6021 \cdot 10^{-19}$ C

$m = 9.1091 \cdot 10^{-31}$ kg

$h = 6.6256 \cdot 10^{-34}$ J·s

in equation (IV). The results for the wavelengths determined by equation (IV) are $\lambda_{1,theroy}$ and $\lambda_{2,theory}$. They are listed for the diameters D_1 and D_2 in Table 2 and Table 3, respectively. The values λ_1 and λ_2 determined from the diffraction pattern agree quite well with the theoretical values $\lambda_{1,theroy}$ and $\lambda_{2,theory}$ due to the de Broglie relation.

c) Determination of lattice plane spacings of graphite

In Fig. 6 the ring diameters D_1 and D_2 are plotted versus $1/\sqrt{U}$. The slopes k_1 and k_2 are determined by linear fits through the origin according equation (IX) to the experimental data:

$k_1 = 1{,}578 \cdot m \sqrt{V}$

$k_2 = 2.729 \cdot m \sqrt{V}$

Resolving equation (X) for the lattice plane spacing d

$$d = \frac{2 \cdot L \cdot h}{k \cdot \sqrt{2 \cdot m \cdot e}}$$
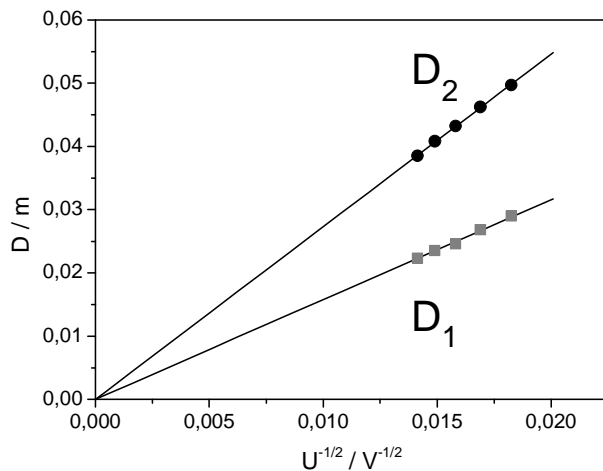
Fig. 6: Ring diameters D_1 and D_2 as function of $1/\sqrt{U}$. The solid lines correspond to the linear fits with the slopes $k_1 = 1.578 \text{ m} \sqrt{V}$ and $k_2 = 2.729 \text{ m} \sqrt{V}$, respectively.

gives

$d_1 = 2.10 \cdot 10^{-10} \text{ m}$

$d_2 = 1.21 \cdot 10^{-10} \text{ m}$

which is within the error limits in accordance of the parameters depicted in Fig. 3.

Supplementary information

After the experiment of Davisson and Germer further experiments with particle wave effects due to particles confirmed the de Broglie relation and thus the wave-particle dualism. In 1930, for instance, O. Stern and I. Esterman succeeded in demonstrating the diffraction of hydrogen molecules and in 1931 they diffracted Helium atoms using a Lithium Fluoride crystal.

Experimental results which can be described by quantum theory only have the Planck constant h in their basic formula. In this experiment, for instance, the Planck's constant can be determined from equation (X) if the lattice spacings d_1 and d_2 of graphite are assumed to be known e.g. from x-ray structure analysis:

$$h = \frac{d \cdot k \cdot \sqrt{2 \cdot m \cdot e}}{2 \cdot L}$$

Using the values k_1 and k_2 obtained by the linear fit to experimental data (Fig. 6) gives

d_1: $h = 6.724 \cdot 10^{-34} \text{ J} \cdot \text{s}$

d_2: $h = 6.717 \cdot 10^{-34} \text{ J} \cdot \text{s}$

Literature: $h = 6.6256 \cdot 10^{-34} \text{ J} \cdot \text{s}$