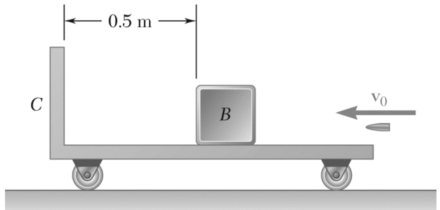
CHAPTER 14

PROBLEM 14.1



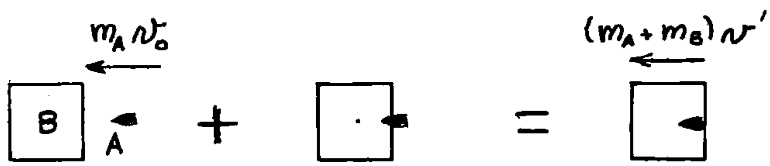
A 30-g bullet is fired with a horizontal velocity of 450 m/s and becomes embedded in block *B* which has a mass of 3 kg. After the impact, block *B* slides on 30-kg carrier *C* until it impacts the end of the carrier. Knowing the impact between *B* and *C* is perfectly plastic and the coefficient of kinetic friction between *B* and *C* is 0.2, determine (*a*) the velocity of the bullet and *B* after the first impact, (*b*) the final velocity of the carrier.

SOLUTION

For convenience, label the bullet as particle *A* of the system of three particles *A*, *B*, and *C*.

(*a*) *Impact between A and B*: Use conservation of linear momentum of *A* and *B*. Assume that the time period is so short that any impulse due to the friction force between *B* and *C* may be neglected.

$$\Sigma m\overline{\mathbf{v}}_1 + \Sigma \mathbf{Imp}_{1\to 2} = \Sigma m\overline{\mathbf{v}}_2$$



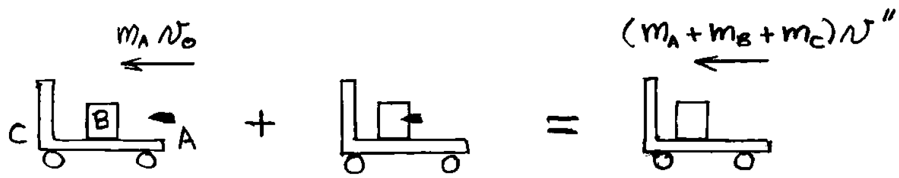
Components $\leftarrow$: $\quad m_A v_0 + 0 = (m_A + m_B)v'$

$$v' = \frac{m_A v_0}{m_A + m_B} = \frac{(30\times 10^{-3}\text{ kg})(450\text{ m/s})}{(30\times 10^{-3}\text{ kg} + 3\text{ kg}} = 4.4554\text{ m/s}$$

$\mathbf{v}' = 4.46$ m/s $\longleftarrow$ ◀

(*b*) *Final velocity of the carrier*: Particles *A*, *B*, and *C* have the same velocity v'' to the left. Use conservation of linear momentum of all three particles. The friction forces between *B* and *C* are internal forces. Neglect friction at the wheels of the carrier.

$$\Sigma m\overline{\mathbf{v}}_2 + \Sigma \mathbf{Imp}_{2\to 3} = \Sigma m\overline{\mathbf{v}}_3$$



Components $\leftarrow$: $\quad (m_A + m_B)v' + 0 = (m_A + m_B + m_C)v$

$$v'' = \frac{(m_A + m_B)v'}{m_A + m_B + m_C} = \frac{m_A v_0}{m_A + m_B + m_C}$$

$$= \frac{(30\times 10^{-3}\text{ kg})(450\text{ m/s})}{30\times 10^{-3}\text{ kg} + 3\text{ kg} + 30\text{ kg}} = 0.4087\text{ m/s}$$

$\mathbf{v}'' = 0.409$ m/s $\longleftarrow$ ◀

PROBLEM 14.2

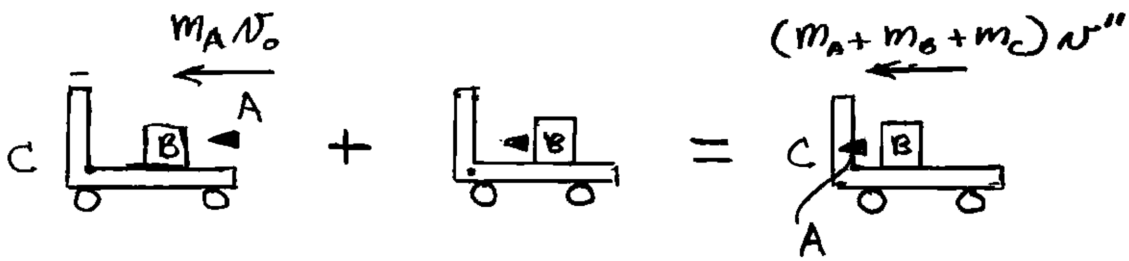
A 30-g bullet is fired with a horizontal velocity of 450 m/s through 3-kg block *B* and becomes embedded in carrier *C* which has a mass of 30 kg. After the impact, block *B* slides 0.3 m on *C* before coming to rest relative to the carrier. Knowing the coefficient of kinetic friction between *B* and *C* is 0.2, determine (*a*) the velocity of the bullet immediately after passing through *B*, (*b*) the final velocity of the carrier.

SOLUTION

For convenience, label the bullet as particle *A* of the system of three particles *A*, *B*, and *C*.

(*b*) *Final velocity of carrier*: Use conservation momentum for all three particles, since the impact forces and the friction force between *B* and *C* are internal forces of the system.

$$\Sigma m\bar{\mathbf{v}}_1 + \Sigma \mathbf{Imp}_{1\to 2} = \Sigma m\bar{\mathbf{v}}_2$$



Components $\longleftarrow$: $\quad m_A v_0 + 0 = (m_A + m_B + m_C)v''$

$$v'' = \frac{m_A v_0}{m_A + m_B + m_C} = \frac{(0.030 \text{ kg})(450 \text{ m/s})}{33.03 \text{ kg}} = 0.40872 \text{ m/s}$$

$$\mathbf{v}'' = 0.409 \text{ m/s} \longleftarrow \blacktriangleleft$$

(*a*) *Velocity v_A of the bullet*:

The sequence of events described is broken into the following states and processes. The symbols for velocities of *A*, *B*, and *C* at the various states are given in the following table:

State	Symbol for velocity			Process
	A	**B**	**C**	
(1)	v_0	0	0	Initial state
(2)	v_A	v_B	0	$1 \longrightarrow 2$: Bullet passes through block
(3)	v_{AC}	v_B	v_{AC}	$2 \longrightarrow 3$: Bullet impacts end of carrier
(4)	v''	v''	v''	$3 \longrightarrow 4$: Block slides to rest relative to carrier

For process 1 $\longrightarrow$ 2 apply conservation of momentum.

$$m_A v_0 = m_A v_A + m_B v_B \tag{1}$$

For process 2 $\longrightarrow$ 3 apply conservation of momentum to A and C.

$$m_A v_A = (m_A + m_C) v_{AC} \tag{2}$$

For process 3 $\longrightarrow$ 4 apply conservation of momentum to A, B, and C.

$$(m_A + m_C) v_{AC} + m_B v_B = (m_A + m_B + m_C) v'' \tag{3}$$

For process 3 $\longrightarrow$ 4 apply the principle of work and energy, since the work $U_{3\to 4}$ of the friction force may be calculated.

Normal force: $N = W_B = m_B g = (3 \text{ kg})(9.81 \text{ m/s}) = 29.43 \text{ N}$

Friction force: $F_f = \mu_k N = (0.2)(29.43) = 5.886 \text{ N}$

Work: $U_{3\to 4} = -F_f d = -(5.886 \text{ N})(0.3 \text{ m}) = -1.7658 \text{ J}$

Principle of work and energy: $T_{AC} + T_B + U_{3\to 4} = T''$ (4)

where
$$T_{AC} = \frac{1}{2}(m_A + m_C) v_{AC}^2$$

$$T_B = \frac{1}{2} m_B v_B^2$$

$$T'' = \frac{1}{2}(m_A + m_B + m_C)(v'')^2$$

Applying the numerical data gives

$$(0.030)(450) = 0.030 v_A + 3 v_B \tag*{(1)$'$}$$

$$0.030 v_A = 30.03 v_{AC} \tag*{(2)$'$}$$

$$30.03 v_{AC} + 3 v_B = (33.03)(0.40872) \tag*{(3)$'$}$$

$$\frac{1}{2}(30.03) v_{AC}^2 + \frac{1}{2}(3) v_B^2 - 1.7658 = \frac{1}{2}(33.03)(0.40872)^2 \tag*{(4)$'$}$$

From Eq. (3)$'$,
$$v_{AC} = \frac{(33.03)(0.40872) - 3v_B}{30.03} = 0.44955 - 0.0999\, v_B$$

Substituting into Eq. (4)$'$ gives

$$(15.015)(0.44955 - 0.0999 v_B)^2 + 1.5 v_B^2 - 1.7658 = 2.7586$$

which reduces to the quadratic equation

$$1.64985 v_B^2 - 1.34865 v_B - 1.48995 = 0$$

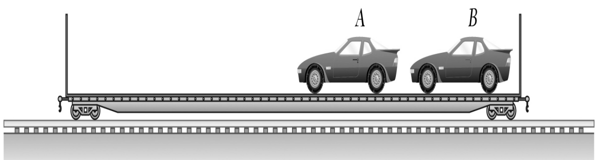
PROBLEM 14.2 (Continued)

Solving, $v_B = 1.44319 \quad \text{and} \quad -0.62575$

$v_B = 1.44319 \text{ m/s}$

Using Eq. (1)′ with numerical data, $13.5 = 0.030 v_A + (3)(1.44319)$

$\mathbf{v}_A = 306 \text{ m/s} \longleftarrow$ ◀



PROBLEM 14.3

Car A weighing 4000 lb and car B weighing 3700 lb are at rest on a 22-ton flatcar which is also at rest. Cars A and B then accelerate and quickly reach constant speeds relative to the flatcar of 7 ft/s and 3.5 ft/s, respectively, before decelerating to a stop at the opposite end of the flatcar. Neglecting friction and rolling resistance, determine the velocity of the flatcar when the cars are moving at constant speeds.

SOLUTION

The masses are $m_A = \dfrac{4000}{32.2} = 124.2$ slugs, $m_B = \dfrac{3700}{32.2} = 114.9$ slugs, and $m_F = \dfrac{(22)(2000)}{32.2} = 1366.5$ slugs

Let v_A, v_B, and v_F be the sought after velocities in ft/s, positive to the right.

Initial values: $(v_A)_0 = (v_B)_0 = (v_F)_0 = 0.$

Initial momentum of system: $m_A(v_A)_0 + m_B(v_B)_0 + m_F(v_F)_0 = 0.$

There are no horizontal external forces acting during the time period under consideration. Momentum is conserved.

$$0 = m_A v_A + m_B v_B + m_F v_F$$

$$124.2 v_A + 114.9 v_B + 1366.5 v_F = 0 \qquad (1)$$

The relative velocities are given as

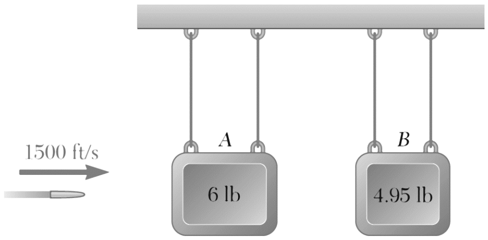
$$v_{A/F} = v_A - v_F = -7 \text{ ft/s} \qquad (2)$$

$$v_{B/F} = v_B - v_F = -3.5 \text{ ft/s} \qquad (3)$$

Solving (1), (2), and (3) simultaneously,

$$v_A = -6.208 \text{ ft/s}, \; v_B = -2.708 \text{ ft/s}, \; v_F = 0.7919 \text{ ft/s}$$

$\mathbf{v}_F = 0.792$ ft/s $\longrightarrow$ ◀



PROBLEM 14.4

A bullet is fired with a horizontal velocity of 1500 ft/s through a 6-lb block *A* and becomes embedded in a 4.95-lb block *B*. Knowing that blocks *A* and *B* start moving with velocities of 5 ft/s and 9 ft/s, respectively, determine (*a*) the weight of the bullet, (*b*) its velocity as it travels from block *A* to block *B*.

SOLUTION

The masses are m for the bullet and m_A and m_B for the blocks.

(*a*) The bullet passes through block *A* and embeds in block *B*. Momentum is conserved.

Initial momentum: $mv_0 + m_A(0) + m_B(0) = mv_0$

Final momentum: $mv_B + m_A v_A + m_B v_B$

Equating, $mv_0 = mv_B + m_A v_A + m_B v_B$

$$m = \frac{m_A v_A + m_B v_B}{v_0 - v_B} = \frac{(6)(5) + (4.95)(9)}{1500 - 9} = 0.0500 \text{ lb}$$

$$m = 0.800 \text{ oz} \blacktriangleleft$$

(*b*) The bullet passes through block *A*. Momentum is conserved.

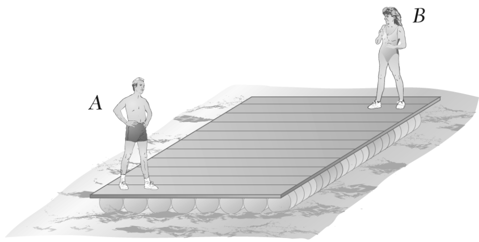
Initial momentum: $mv_0 + m_A(0) = mv_0$

Final momentum: $mv_1 + m_A v_A$

Equating, $mv_0 = mv_1 + m_A v_A$

$$v_1 = \frac{mv_0 - m_A v_A}{m} = \frac{(0.0500)(1500) - (6)(5)}{0.0500} = 900 \text{ ft/s}$$

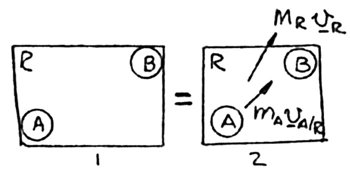
$$\mathbf{v}_1 = 900 \text{ ft/s} \longrightarrow \blacktriangleleft$$



PROBLEM 14.5

Two swimmers A and B, of weight 190 lb and 125 lb, respectively, are at diagonally opposite corners of a floating raft when they realize that the raft has broken away from its anchor. Swimmer A immediately starts walking toward B at a speed of 2 ft/s relative to the raft. Knowing that the raft weighs 300 lb, determine (a) the speed of the raft if B does not move, (b) the speed with which B must walk toward A if the raft is not to move.

SOLUTION



(a) The system consists of A and B and the raft R.

Momentum is conserved.

$$(\Sigma m\mathbf{v})_1 = (\Sigma m\mathbf{v})_2$$

$$0 = m_A\mathbf{v}_A + m_B\mathbf{v}_B + m_R\mathbf{v}_R \qquad (1)$$

$$\mathbf{v}_A = \mathbf{v}_{A/R} + \mathbf{v}_R \qquad \mathbf{v}_B - \mathbf{v}_{B/R} + \mathbf{v}_R \qquad v_{B/R} = 0$$

$$\mathbf{v}_A = 2\text{ ft/s}\ {}_A\!\nearrow^{B} + \mathbf{v}_R \qquad \mathbf{v}_B = \mathbf{v}_R$$

$$0 = m_A[2\ {}_A\!\nearrow^{B} + \mathbf{v}_R] + m_B\mathbf{v}_R + m_R\mathbf{v}_e$$

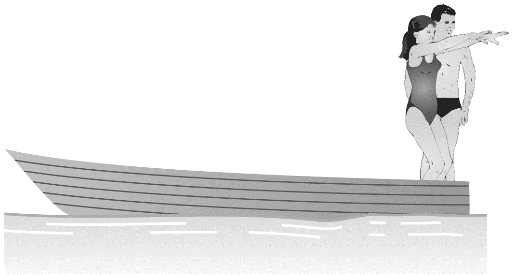
$$\mathbf{v}_R = \frac{-2m_A}{(m_A + m_B + m_R)} = \frac{-(2\text{ ft/s})(190\text{ lb})}{(190\text{ lb} + 125\text{ lb} + 300\text{ lb})} \qquad v_R = 0.618\text{ ft/s} \blacktriangleleft$$

(b) From Eq. (1),

$$0 = m_A v_A + m_B v_B + 0 \qquad (v_R = 0)$$

$$v_B = -\frac{m_A v_A}{m_B} \qquad v_A = v_{A/R} + \cancel{v}_R^{\,0} = 2\text{ ft/s}$$

$$v_B = -\frac{(2\text{ ft/s})(190\text{ lb})}{(125\text{ lb})} = 3.04\text{ ft/s} \qquad v_B = 3.04\text{ ft/s} \blacktriangleleft$$



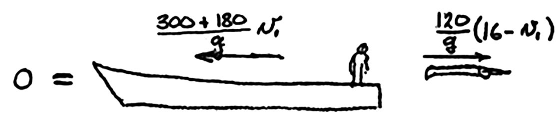
PROBLEM 14.6

A 180-lb man and a 120-lb woman stand side by side at the same end of a 300-lb boat, ready to dive, each with a 16-ft/s velocity relative to the boat. Determine the velocity of the boat after they have both dived, if (*a*) the woman dives first, (*b*) the man dives first.

SOLUTION

(*a*) Woman dives first.

Conservation of momentum:



$$\frac{120}{g}(16 - v_1) - \frac{300 + 180}{g} v_1 = 0$$

$$v_1 = \frac{(120)(16)}{600} = 3.20 \text{ ft/s} \longleftarrow$$

Man dives next. Conservation of momentum:

$$-\frac{300 + 180}{g} v_1 = -\frac{300}{g} v_2 + \frac{180}{g}(16 - v_2)$$

$$v_2 = \frac{480 v_1 + (180)(16)}{480} = 9.20 \text{ ft/s} \qquad \mathbf{v}_2 = 9.20 \text{ ft/s} \longleftarrow \blacktriangleleft$$

(*b*) Man dives first.

Conservation of momentum:

$$\frac{180}{g}(16 - v_1') - \frac{300 + 120}{g} v_1' = 0$$

$$v_1' = \frac{(180)(16)}{600} = 4.80 \text{ ft/s}$$

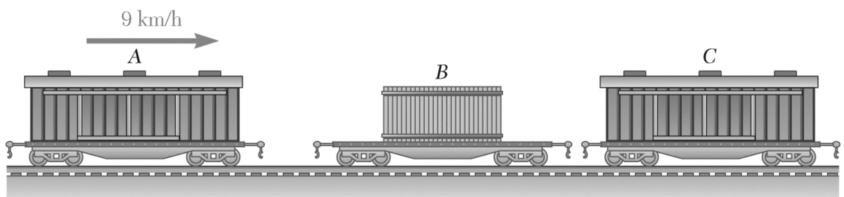
Woman dives next. Conservation of momentum:

$$-\frac{300 + 120}{g} v_1' = -\frac{300}{g} v_2' + \frac{120}{g}(16 - v_2')$$

$$v_2' = \frac{420 v_1' + (120)(16)}{420} = 9.37 \text{ ft/s} \qquad \mathbf{v}_2' = 9.37 \text{ ft/s} \longleftarrow \blacktriangleleft$$

PROBLEM 14.7

A 40-Mg boxcar *A* is moving in a railroad switchyard with a velocity of 9 km/h toward cars *B* and *C*, which are both at rest with their brakes off at a short distance from each other. Car *B* is a 25-Mg flatcar supporting a 30-Mg container, and car *C* is a 35-Mg boxcar. As the cars hit each other they get automatically and tightly coupled. Determine the velocity of car *A* immediately after each of the two couplings, assuming that the container (*a*) does not slide on the flatcar, (*b*) slides after the first coupling but hits a stop before the second coupling occurs, (*c*) slides and hits the stop only after the second coupling has occurred.



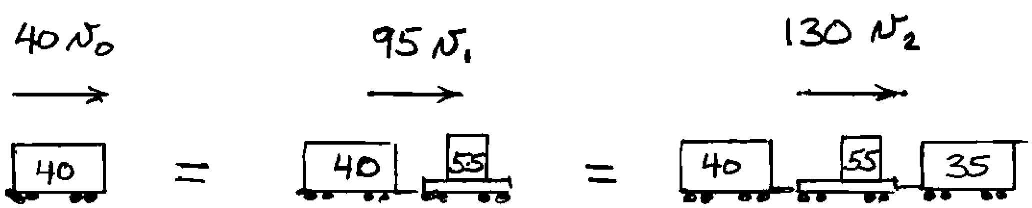
SOLUTION

Each term of the conservation of momentum equation is mass times velocity. As long as the same units are used in all terms, any unit may be used for mass and for velocity. We use Mg for mass and km/h for velocity and apply conservation of momentum.

Note: Only moving masses are shown in the diagrams.

Initial momentum: $m_A v_0 = (40)(9) = 360$

(*a*) Container does not slide

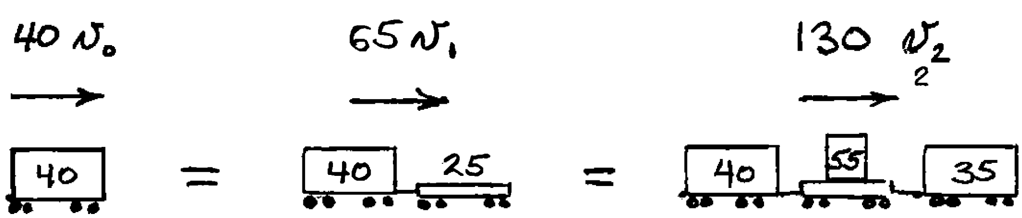


$$360 = 95v_1 = 130v_2$$

$\mathbf{v}_1 = 3.79$ km/h $\longrightarrow$ ◀

$\mathbf{v}_2 = 2.77$ km/h $\longrightarrow$ ◀

(*b*) Container slides after 1st coupling, stops before 2nd



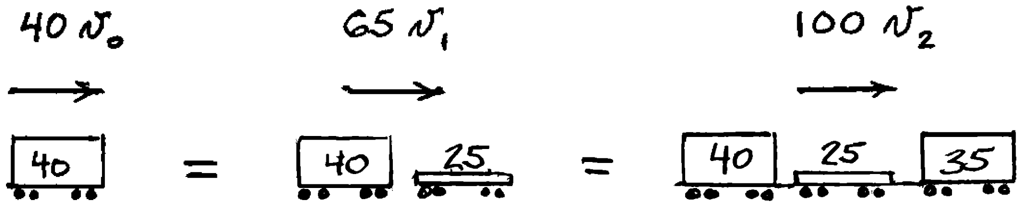
$$360 = 65v_1 = 130v_2$$

$\mathbf{v}_1 = 5.54$ km/h $\longrightarrow$ ◀

$\mathbf{v}_2 = 2.77$ km/h $\longrightarrow$ ◀

PROBLEM 14.7 (Continued)

(*c*) Container slides and stops only after 2nd coupling

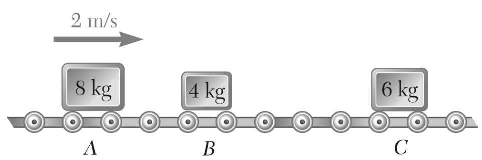


$360 = 65v_1 = 100v_2$

$\mathbf{v}_1 = 5.54$ km/h $\longrightarrow$ ◀

$\mathbf{v}_2 = 3.60$ km/h $\longrightarrow$ ◀

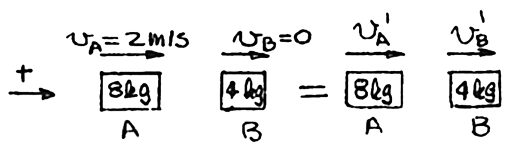
PROBLEM 14.8



Packages in an automobile parts supply house are transported to the loading dock by pushing them along on a roller track with very little friction. At the instant shown, packages B and C are at rest and package A has a velocity of 2 m/s. Knowing that the coefficient of restitution between the packages is 0.3, determine (a) the velocity of package C after A hits B and B hits C, (b) the velocity of A after it hits B for the second time.

SOLUTION

(a) Packages A and B:



Total momentum conserved:

$$m_A v_A + m_B v_B = m_A v_A' + m_B v_B'$$
$$(8\text{ kg})(2\text{ m/s}) + 0 = (8\text{ kg})v_A' + (4\text{ kg})v_B'$$
$$4 = 2v_A' + v_B' \qquad (1)$$

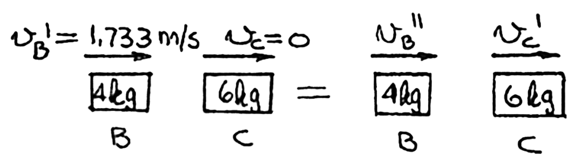
Relative velocities.

$$(v_A - v_B)e = (v_B' - v_A')$$
$$(2)(0.3) = v_B' - v_A' \qquad (2)$$

Solving Equations (1) and (2) simultaneously,

$$v_A' = 1.133\text{ m/s} \longrightarrow$$
$$v_B' = 1.733\text{ m/s} \longrightarrow$$

Packages B and C:



$$\xrightarrow{+} m_B v_B' + m_C v_C = m_B v_B'' + m_C v_C'$$

$$(4\text{ kg})(1.733\text{ m/s}) + 0 = 4v_B'' + 6v_c'$$

$$6.932 = 4v_B'' + 6v_C' \qquad (3)$$

PROBLEM 14.8 (Continued)

Relative velocities:

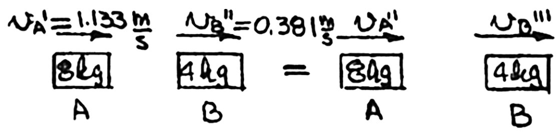
$$(v_B' - v_C)e = v_C' - v_B''$$

$$(1.733)(0.3) = 0.5199 = v_C' - v_B'' \qquad (4)$$

Solving equations (3) and (4) simultaneously,

$$\mathbf{v}_C' = 0.901 \text{ m/s} \longrightarrow \blacktriangleleft$$

(*b*) Packages *A* and *B* (second time),



Total momentum conserved:

$$(8)(1.133) + (4)(0.381) = 8v_A'' + 4v_B'''$$

$$10.588 = 8v_A'' + 4v_B'' \qquad (5)$$

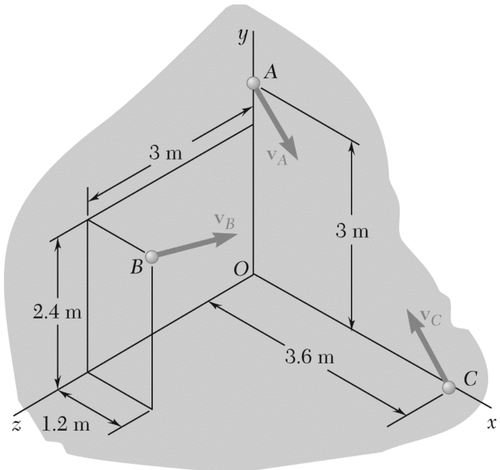
Relative velocities:

$$(v_A' - v_B'')e = v_B''' - v_A''$$

$$(1.133 - 0.381)(0.3) = 0.2256 = v_B''' - v_A'' \qquad (6)$$

Solving (5) and (6) simultaneously,

$$v_A'' = 0.807 \text{ m/s} \qquad \mathbf{v}_A'' = 0.807 \text{ m/s} \longrightarrow \blacktriangleleft$$



PROBLEM 14.9

A system consists of three particles A, B, and C. We know that $m_A = 3$ kg, $m_B = 2$ kg, and $m_C = 4$ kg and that the velocities of the particles expressed in m/s are, respectively, $\mathbf{v}_A = 4\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$, $\mathbf{v}_B = 4\mathbf{i} + 3\mathbf{j}$, and $\mathbf{v}_C = -2\mathbf{i} + 4\mathbf{j} + 2\mathbf{k}$. Determine the angular momentum $\mathbf{H}_O$ of the system about O.

SOLUTION

Linear momentum of each particle expressed in kg·m/s.

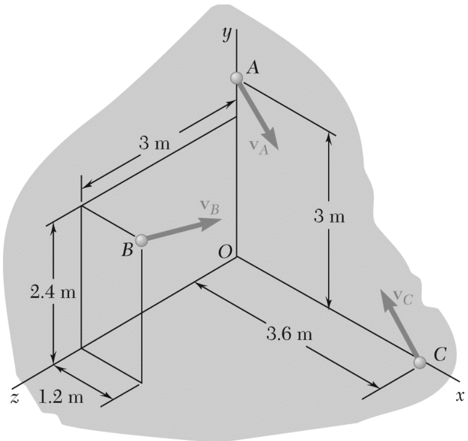
$$m_A\mathbf{v}_A = 12\mathbf{i} + 6\mathbf{j} + 6\mathbf{k}$$
$$m_B\mathbf{v}_B = 8\mathbf{i} + 6\mathbf{j}$$
$$m_C\mathbf{v}_C = -8\mathbf{i} + 16\mathbf{j} + 8\mathbf{k}$$

Position vectors, (meters): $\mathbf{r}_A = 3\mathbf{j}$, $\mathbf{r}_B = 1.2\mathbf{i} + 2.4\mathbf{j} + 3\mathbf{k}$, $\mathbf{r}_C = 3.6\mathbf{i}$

Angular momentum about O, (kg·m^2/s).

$$\begin{aligned}\mathbf{H}_O &= \mathbf{r}_A \times (m_A\mathbf{v}_A) + \mathbf{r}_B \times (m_B\mathbf{v}_B) + \mathbf{r}_C \times (m_C\mathbf{v}_C)\\ &= \begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 0 & 3 & 0\\ 12 & 6 & 6\end{vmatrix} + \begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 1.2 & 2.4 & 3\\ 8 & 6 & 0\end{vmatrix} + \begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 3.6 & 0 & 0\\ -8 & 16 & 8\end{vmatrix}\\ &= (18\mathbf{i} - 36\mathbf{k}) + (-18\mathbf{i} + 24\mathbf{j} - 12\mathbf{k}) + (-28.8\mathbf{j} + 57.6\mathbf{k})\\ &= 0\mathbf{i} - 4.8\mathbf{j} + 9.6\mathbf{k}\end{aligned}$$

$$\mathbf{H}_O = -(4.80\ \text{kg}\cdot\text{m}^2/\text{s})\mathbf{j} + (9.60\ \text{kg}\cdot\text{m}^2/\text{s})\,\mathbf{k} \blacktriangleleft$$



PROBLEM 14.10

For the system of particles of Problem 14.9, determine (*a*) the position vector $\bar{\mathbf{r}}$ of the mass center *G* of the system, (*b*) the linear momentum $m\bar{\mathbf{v}}$ of the system, (*c*) the angular momentum $\mathbf{H}_G$ of the system about *G*. Also verify that the answers to this problem and to problem 14.9 satisfy the equation given in Problem 14.27.

PROBLEM 14.9 A system consists of three particles *A*, *B*, and *C*. We know that $m_A = 3$ kg, $m_B = 2$ kg, and $m_C = 4$ kg and that the velocities of the particles expressed in m/s are, respectively, $\mathbf{v}_A = 4\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$, $\mathbf{v}_B = 4\mathbf{i} + 3\mathbf{j}$, and $\mathbf{v}_C = -2\mathbf{i} + 4\mathbf{j} + 2\mathbf{k}$. Determine the angular momentum $\mathbf{H}_O$ of the system about *O*.

SOLUTION

Position vectors, (meters): $\mathbf{r}_A = 3\mathbf{j}$, $\mathbf{r}_B = 1.2\mathbf{i} + 2.4\mathbf{j} + 3\mathbf{k}$, $\mathbf{r}_C = 3.6\mathbf{i}$

(*a*) Mass center: $$(m_A + m_B + m_C)\bar{\mathbf{r}} = m_A\mathbf{r}_A + m_B\mathbf{r}_B + m_C\mathbf{r}_C$$

$$9\bar{\mathbf{r}} = (3)(3\mathbf{j}) + (2)(1.2\mathbf{i} + 2.4\mathbf{j} + 3\mathbf{k}) + (4)(3.6\mathbf{i})$$
$$\bar{\mathbf{r}} = 1.86667\mathbf{i} + 1.53333\mathbf{j} + 0.66667\mathbf{k}$$

$$\bar{\mathbf{r}} = (1.867\text{ m})\mathbf{i} + (1.533\text{ m})\mathbf{j} + (0.667\text{ m})\mathbf{k} \blacktriangleleft$$

Linear momentum of each particle, (kg·m²/s).

$$m_A\mathbf{v}_A = 12\mathbf{i} + 6\mathbf{j} + 6\mathbf{k}$$
$$m_B\mathbf{v}_B = 8\mathbf{i} + 6\mathbf{j}$$
$$m_C\mathbf{v}_C = -8\mathbf{i} + 16\mathbf{j} + 8\mathbf{k}$$

(*b*) Linear momentum of the system, (kg·m/s.)

$$m\bar{\mathbf{v}} = m_A\mathbf{v}_A + m_B\mathbf{v}_B + m_C\mathbf{v}_C = 12\mathbf{i} + 28\mathbf{j} + 14\mathbf{k}$$

$$m\bar{\mathbf{v}} = (12.00\text{ kg}\cdot\text{m/s})\mathbf{i} + (28.0\text{ kg}\cdot\text{m/s})\mathbf{j} + (14.00\text{ kg}\cdot\text{m/s})\mathbf{k} \blacktriangleleft$$

Position vectors relative to the mass center, (meters).

$$\mathbf{r}'_A = \mathbf{r}_A - \bar{\mathbf{r}} = -1.86667\mathbf{i} + 1.46667\mathbf{j} - 0.66667\mathbf{k}$$
$$\mathbf{r}'_B = \mathbf{r}_B - \bar{\mathbf{r}} = -0.66667\mathbf{i} + 0.86667\mathbf{j} + 2.33333\mathbf{k}$$
$$\mathbf{r}'_C = \mathbf{r}_C - \bar{\mathbf{r}} = 1.73333\mathbf{i} - 1.53333\mathbf{j} - 0.66667\mathbf{k}$$

(*c*) Angular momentum about G, $(\text{kg}\cdot\text{m}^2/\text{s})$.

$$\mathbf{H}_G = \mathbf{r}'_A \times m_A\mathbf{v}_A + \mathbf{r}'_B \times m_B\mathbf{v}_B + \mathbf{r}'_C \times m_C\mathbf{v}_C$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1.86667 & 1.46667 & -0.66667 \\ 12 & 6 & 6 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.66667 & 0.86667 & 2.33333 \\ 8 & 6 & 0 \end{vmatrix}$$

$$+ \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1.73333 & -1.53333 & -0.66667 \\ -8 & 16 & 8 \end{vmatrix}$$

$$= (12.8\mathbf{i} + 3.2\mathbf{j} - 28.8\mathbf{k}) + (-14\mathbf{i} + 18.6667\mathbf{j} - 10.9333\mathbf{k})$$

$$+ (-1.6\mathbf{i} - 8.5333\mathbf{j} + 15.4667\mathbf{k})$$

$$= -2.8\mathbf{i} + 13.3333\mathbf{j} - 24.2667\mathbf{k}$$

$$\mathbf{H}_G = -(2.80\ \text{kg}\cdot\text{m}^2/\text{s})\mathbf{i} + (13.33\ \text{kg}\cdot\text{m}^2/\text{s})\mathbf{j} - (24.3\ \text{kg}\cdot\text{m}^2/\text{s})\mathbf{k} \blacktriangleleft$$

$$\bar{\mathbf{r}} \times m\bar{\mathbf{v}} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1.86667 & 1.53333 & 0.66667 \\ 12 & 28 & 14 \end{vmatrix}$$

$$= (2.8\ \text{kg}\cdot\text{m}^2/\text{s})\mathbf{i} - (18.1333\ \text{kg}\cdot\text{m}^2/\text{s})\mathbf{j} + (33.8667\ \text{kg}\cdot\text{m}^2/\text{s})\mathbf{k}$$

$$\mathbf{H}_G + \bar{\mathbf{r}} \times m\bar{\mathbf{v}} = -(4.8\ \text{kg}\cdot\text{m}^2/\text{s})\mathbf{j} + (9.6\ \text{kg}\cdot\text{m}^2/\text{s})\mathbf{k}$$

Angular momentum about O.

$$\mathbf{H}_O = \mathbf{r}_A \times (m_A\mathbf{v}_A) + \mathbf{r}_B \times (m_B\mathbf{v}_B) + \mathbf{r}_C \times (m_C\mathbf{v}_C)$$

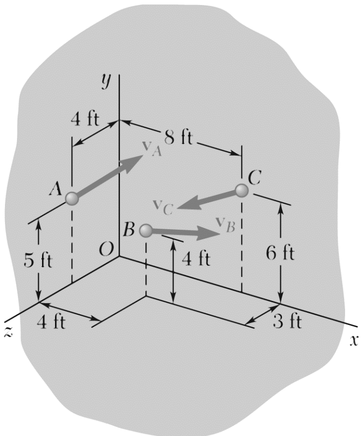
$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 3 & 0 \\ 12 & 6 & 6 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1.2 & 2.4 & 3 \\ 8 & 6 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3.6 & 0 & 0 \\ -8 & 16 & 8 \end{vmatrix}$$

$$= (18\mathbf{i} - 36\mathbf{k}) + (-18\mathbf{i} + 24\mathbf{j} - 12\mathbf{k}) + (-28.8\mathbf{j} + 57.6\mathbf{k})$$

$$= -(4.8\ \text{kg}\cdot\text{m}^2/\text{s})\mathbf{j} + (9.6\ \text{kg}\cdot\text{m}^2/\text{s})\mathbf{k}$$

Note that

$$\mathbf{H}_O = \mathbf{H}_G + \bar{\mathbf{r}} \times m\bar{\mathbf{v}}$$



PROBLEM 14.11

A system consists of three particles A, B, and C. We know that $W_A = 5\,\text{lb}$, $W_B = 4\,\text{lb}$, and $W_C = 3\,\text{lb}$, and that the velocities of the particles expressed in ft/s are, respectively, $\mathbf{v}_A = 2\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$, $\mathbf{v}_B = v_x\mathbf{i} + v_y\mathbf{j} + v_z\mathbf{k}$, and $\mathbf{v}_C = -3\mathbf{i} - 2\mathbf{j} + \mathbf{k}$. Determine (*a*) the components v_x and v_z of the velocity of particle B for which the angular momentum $\mathbf{H}_O$ of the system about O is parallel to the x axis, (*b*) the value of $\mathbf{H}_O$.

SOLUTION

$$\mathbf{H}_O = \Sigma \mathbf{r}_i \times m\mathbf{v}_i = \Sigma m_i \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x_i & y_i & z_i \\ (v_i)_x & (v_i)_y & (v_i)_z \end{vmatrix}$$

$$= \frac{5}{g}\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 5 & 4 \\ 2 & 3 & -2 \end{vmatrix} + \frac{4}{g}\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 4 & 3 \\ v_x & 2 & v_z \end{vmatrix} + \frac{3}{g}\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 8 & 6 & 0 \\ -3 & -2 & 1 \end{vmatrix}$$

$$= \frac{1}{g}[5(-10-12) + 4(4v_z - 6) + 3(6-0)]\mathbf{i}$$

$$+ \frac{1}{g}[5(8-0) + 4(3v_x - 4v_z) + 3(0-8)]\mathbf{j}$$

$$+ \frac{1}{g}[5(0-10) + 4(8-4v_x) + 3(-16+18)]\mathbf{k}$$

$$\mathbf{H}_O = \frac{1}{g}[(16v_z - 116)\mathbf{i} + (12v_z - 16v_z + 16)\mathbf{j} + (-16v_x - 12)\mathbf{k}] \qquad (1)$$

(*a*) For $\mathbf{H}_O$ to be parallel to the x axis, we must have $H_y = H_z = 0$:

$$H_z = 0: \quad -16v_x - 12 = 0 \qquad v_x = -0.75 \text{ ft/s} \blacktriangleleft$$

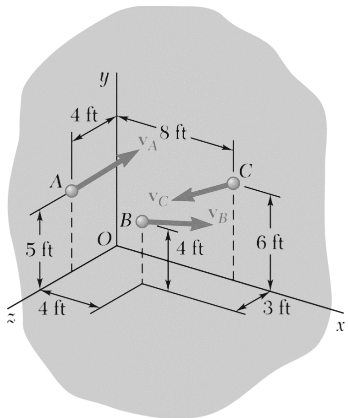
$$H_y = 0: \quad 12(-0.75) - 16v_z + 16 = 0 \qquad v_z = 0.4375 \text{ ft/s} \blacktriangleleft$$

(*b*) Substitute into Eq. (1):

$$\mathbf{H}_O = \frac{1}{g}(16v_z - 116)\mathbf{i} = \frac{1}{g}[16(0.4375) - 116]\mathbf{i} = -\frac{109.0}{32.2}\mathbf{i}$$

$$\mathbf{H}_O = -(3.39 \text{ ft} \cdot \text{lb} \cdot \text{s})\mathbf{i} \blacktriangleleft$$

PROBLEM 14.12



For the system of particles of Problem 14.11, determine (*a*) the components v_x and v_z of the velocity of particle *B* for which the angular momentum $\mathbf{H}_O$ of the system about *O* is parallel to the *z* axis, (*b*) the value of $\mathbf{H}_O$.

PROBLEM 14.11 A system consists of three particles *A*, *B*, and *C*. We know that $W_A = 5\,\text{lb}$, $W_B = 4\,\text{lb}$, and $W_C = 3\,\text{lb}$ and that the velocities of the particles expressed in ft/s are, respectively, $\mathbf{v}_A = 2\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$, $\mathbf{v}_B = v_x\mathbf{i} + v_y\mathbf{j} + v_z\mathbf{k}$, and $\mathbf{v}_C = -3\mathbf{i} - 2\mathbf{j} + \mathbf{k}$. Determine (*a*) the components v_x and v_z of the velocity of particle *B* for which the angular momentum $\mathbf{H}_O$ of the system about *O* is parallel to the *x* axis, (*b*) the value of $\mathbf{H}_O$.

SOLUTION

$$\mathbf{H}_O = \Sigma \mathbf{r}_i \times m\mathbf{v}_i = \Sigma m_i \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x_i & y_i & z_i \\ (v_i)_x & (v_i)_y & (v_i)_z \end{vmatrix}$$

$$= \frac{5}{g}\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 5 & 4 \\ 2 & 3 & -2 \end{vmatrix} + \frac{4}{g}\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 4 & 3 \\ v_x & 2 & v_z \end{vmatrix} + \frac{3}{g}\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 8 & 6 & 0 \\ -3 & -2 & 1 \end{vmatrix}$$

$$= \frac{1}{g}[5(-10-12) + 4(4v_z - 6) + 3(6-0)]\mathbf{i}$$

$$+ \frac{1}{g}[5(8-0) + 4(3v_x - 4v_z) + 3(0-8)]\mathbf{j}$$

$$+ \frac{1}{g}[5(0-10) + 4(8-4v_x) + 3(-16+18)]\mathbf{k}$$

$$\mathbf{H}_O = \frac{1}{g}[(16v_z - 116)\mathbf{i} + (12v_z - 16v_z + 16)\mathbf{j} + (-16v_x - 12)\mathbf{k}] \quad (1)$$

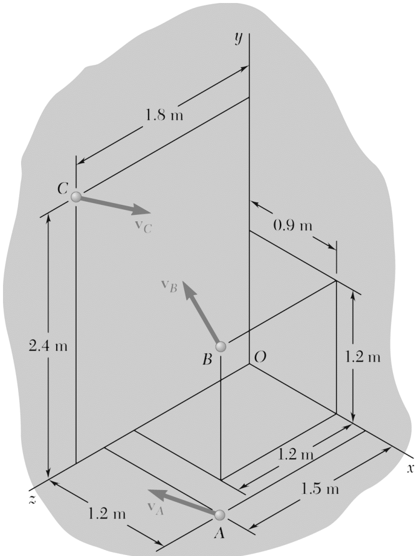
(*a*) For $\mathbf{H}_O$ to be parallel to the *z* axis, we must have $H_x = H_y = 0$:

$$H_x = 0: \quad 16v_z - 116 = 0 \qquad v_z = 7.25 \text{ ft/s} \blacktriangleleft$$

$$H_y = 0: \quad 12v_x - 16(7.25) + 16 = 0 \qquad v_x = 8.33 \text{ ft/s} \blacktriangleleft$$

(*b*) Substituting into Eq. (1):

$$\mathbf{H}_O = \frac{1}{32.2}[-16(8.33) - 12]\mathbf{k} \qquad \mathbf{H}_O = -(4.51\ \text{ft}\cdot\text{lb}\cdot\text{s})\mathbf{k} \blacktriangleleft$$



PROBLEM 14.13

A system consists of three particles A, B, and C. We know that $m_A = 3$ kg, $m_B = 4$ kg, and $m_C = 5$ kg, and that the velocities of the particles expressed in m/s are, respectively, $\mathbf{v}_A = -4\mathbf{i} + 4\mathbf{j} + 6\mathbf{k}$, $\mathbf{v}_B = -6\mathbf{i} + 8\mathbf{j} + 4\mathbf{k}$, and $\mathbf{v}_C = 2\mathbf{i} - 6\mathbf{j} - 4\mathbf{k}$. Determine the angular momentum $\mathbf{H}_O$ of the system about O.

SOLUTION

Linear momentum of each particle, $(\text{kg} \cdot \text{m/s})$:

$$m_A \mathbf{v}_A = -12\mathbf{i} + 12\mathbf{j} + 18\mathbf{k}$$
$$m_B \mathbf{v}_B = -24\mathbf{i} + 32\mathbf{j} + 16\mathbf{k}$$
$$m_C \mathbf{v}_C = 10\mathbf{i} - 30\mathbf{j} - 20\mathbf{k}$$

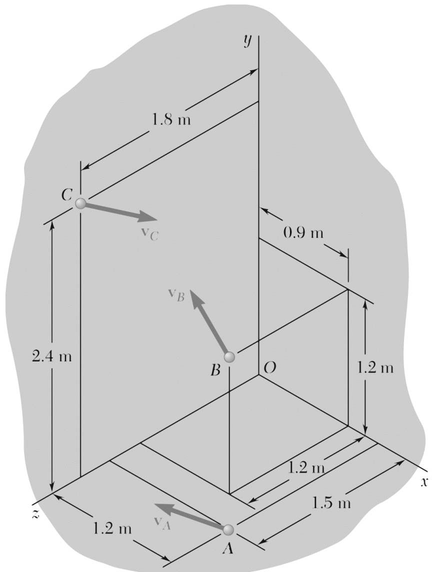
Position vectors, (meters):

$$\mathbf{r}_A = 1.2\mathbf{i} + 1.5\mathbf{k}, \quad \mathbf{r}_B = 0.9\mathbf{i} + 1.2\mathbf{j} + 1.2\mathbf{k}, \quad \mathbf{r}_C = 2.4\mathbf{j} + 1.8\mathbf{k}$$

Angular momentum about O, $(\text{kg} \cdot \text{m}^2/\text{s})$:

$$\begin{aligned}
\mathbf{H}_O &= \mathbf{r}_A \times m_A \mathbf{v}_A + \mathbf{r}_B \times m_B \mathbf{v}_B + \mathbf{r}_C \times m_C \mathbf{v}_C \\
&= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1.2 & 0 & 1.5 \\ -12 & 12 & 18 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.9 & 1.2 & 1.2 \\ -24 & 32 & 16 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 2.4 & 1.8 \\ 10 & -30 & -20 \end{vmatrix} \\
&= (-18\mathbf{i} - 39.6\mathbf{j} + 14.4\mathbf{k}) + (-19.2\mathbf{i} - 43.2\mathbf{j} + 57.6\mathbf{k}) + (6\mathbf{i} + 18\mathbf{j} - 24\mathbf{k}) \\
&= -31.2\mathbf{i} - 64.8\mathbf{j} + 48.0\mathbf{k}
\end{aligned}$$

$$\mathbf{H}_O = -(31.2 \text{ kg} \cdot \text{m}^2/\text{s})\mathbf{i} - (64.8 \text{ kg} \cdot \text{m}^2/\text{s})\mathbf{j} + (48.0 \text{ kg} \cdot \text{m}^2/\text{s})\mathbf{k} \blacktriangleleft$$



PROBLEM 14.14

For the system of particles of Problem 14.13, determine (*a*) the position vector $\bar{\mathbf{r}}$ of the mass center G of the system, (*b*) the linear momentum $m\bar{\mathbf{v}}$ of the system, (*c*) the angular momentum $\mathbf{H}_G$ of the system about G. Also verify that the answers to this problem and to Problem 14.13 satisfy the equation given in Problem 14.27.

PROBLEM 14.13 A system consists of three particles A, B, and C. We know that $m_A = 3$ kg, $m_B = 4$ kg, and $m_C = 5$ kg and that the velocities of the particles expressed in m/s are, respectively, $\mathbf{v}_A = -4\mathbf{i} + 4\mathbf{j} + 6\mathbf{k}$, $\mathbf{v}_B = -6\mathbf{i} + 8\mathbf{j} + 4\mathbf{k}$, and $\mathbf{v}_C = 2\mathbf{i} - 6\mathbf{j} - 4\mathbf{k}$. Determine the angular momentum $\mathbf{H}_O$ of the system about O.

SOLUTION

Position vectors, (meters):

$$\mathbf{r}_A = 1.2\mathbf{i} + 1.5\mathbf{k}, \quad \mathbf{r}_B = 0.9\mathbf{i} + 1.2\mathbf{j} + 1.2\mathbf{k}, \quad \mathbf{r}_C = 2.4\mathbf{j} + 1.8\mathbf{k}$$

(*a*) Mass center:

$$(m_A + m_B + m_C)\bar{\mathbf{r}} = m_A\mathbf{r}_A + m_B\mathbf{r}_B + m_C\mathbf{r}_C$$

$$12\bar{\mathbf{r}} = (3)(1.2\mathbf{i} + 1.5\mathbf{k}) + (4)(0.9\mathbf{i} + 1.2\mathbf{j} + 1.2\mathbf{k}) + (5)(2.4\mathbf{j} + 1.8\mathbf{k})$$

$$\bar{\mathbf{r}} = 0.6\mathbf{i} + 1.4\mathbf{j} + 1.525\mathbf{k}$$

$$\bar{\mathbf{r}} = (0.600 \text{ m})\mathbf{i} + (1.400 \text{ m})\mathbf{j} + (1.525 \text{ m})\mathbf{k} \blacktriangleleft$$

Linear momentum of each particle, (kg · m/s):

$$m_A\mathbf{v}_A = -12\mathbf{i} + 12\mathbf{j} + 18\mathbf{k}$$
$$m_B\mathbf{v}_B = -24\mathbf{i} + 32\mathbf{j} + 16\mathbf{k}$$
$$m_C\mathbf{v}_C = 10\mathbf{i} - 30\mathbf{j} - 20\mathbf{k}$$

(*b*) Linear momentum of the system, (kg · m/s):

$$m\bar{\mathbf{v}} = m_A\mathbf{v}_A + m_B\mathbf{v}_B + m_C\mathbf{v}_C = -26\mathbf{i} + 14\mathbf{j} + 14\mathbf{k}$$

$$m\bar{\mathbf{v}} = -(26.0 \text{ kg}\cdot\text{m/s})\mathbf{i} + (14.00 \text{ kg}\cdot\text{m/s})\mathbf{j} + (14.00 \text{ kg}\cdot\text{m/s})\mathbf{k} \blacktriangleleft$$

Position vectors relative to the mass center, (meters).

$$\mathbf{r}'_A = \mathbf{r}_A - \bar{\mathbf{r}} = 0.6\mathbf{i} - 1.4\mathbf{j} - 0.025\mathbf{k}$$
$$\mathbf{r}'_B = \mathbf{r}_B - \bar{\mathbf{r}} = 0.3\mathbf{i} - 0.2\mathbf{j} - 0.325\mathbf{k}$$
$$\mathbf{r}'_C = \mathbf{r}_C - \bar{\mathbf{r}} = -0.6\mathbf{i} + 1.0\mathbf{j} + 0.275\mathbf{k}$$

(*c*) Angular momentum about G, $(\mathrm{kg \cdot m^2/s})$:

$$\mathbf{H}_G = \mathbf{r}'_A \times m_A \mathbf{v}_A + \mathbf{r}'_B \times m_B \mathbf{v}_B + \mathbf{r}'_C \times m_C \mathbf{v}_C$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.6 & -1.4 & -0.025 \\ -12 & 12 & 18 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.3 & -0.2 & -0.325 \\ -24 & 32 & 16 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.6 & 1.0 & 0.275 \\ 10 & -30 & -20 \end{vmatrix}$$

$$= (-24.9\mathbf{i} - 10.5\mathbf{j} - 9.6\mathbf{k}) + (7.2\mathbf{i} + 3.0\mathbf{j} + 4.8\mathbf{k}) + (-11.75\mathbf{i} - 9.25\mathbf{j} + 8.0\mathbf{k})$$

$$= -29.45\mathbf{i} - 16.75\mathbf{j} + 3.2\mathbf{k}$$

$$\mathbf{H}_G = -(29.5\ \mathrm{kg \cdot m^2/s})\mathbf{i} - (16.75\ \mathrm{kg \cdot m^2/s})\mathbf{j} + (3.20\ \mathrm{kg \cdot m^2/s})\mathbf{k} \blacktriangleleft$$

$$\bar{\mathbf{r}} \times m\bar{\mathbf{v}} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.6 & 1.4 & 1.525 \\ -26 & 14 & 14 \end{vmatrix} = -1.75\mathbf{i} - 48.05\mathbf{j} + 44.8\mathbf{k}$$

$$\mathbf{H}_G + \bar{\mathbf{r}} \times m\mathbf{v} = -(31.2\ \mathrm{kg \cdot m^2/s})\mathbf{i} - (64.8\ \mathrm{kg \cdot m^2/s})\mathbf{j} + (48.0\ \mathrm{kg \cdot m^2/s})\mathbf{k}$$

Angular momentum about O, $(\mathrm{kg \cdot m^2/s})$:

$$\mathbf{H}_O = \mathbf{r}_A \times m_A \mathbf{v}_A + \mathbf{r}_B \times m_B \mathbf{v}_B + \mathbf{r}_C \times m_C \mathbf{v}_C$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1.2 & 0 & 1.5 \\ -12 & 12 & 18 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.9 & 1.2 & 1.2 \\ -24 & 32 & 16 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 2.4 & 1.8 \\ 10 & -30 & -20 \end{vmatrix}$$

$$= (-18\mathbf{i} - 39.6\mathbf{j} + 14.4\mathbf{k}) + (-19.2\mathbf{i} - 43.2\mathbf{j} + 57.6\mathbf{k}) + (6\mathbf{i} + 18\mathbf{j} - 24\mathbf{k})$$

$$= -(31.2\ \mathrm{kg \cdot m^2/s})\mathbf{i} - (64.8\ \mathrm{kg \cdot m^2/s})\mathbf{j} + (48.0\ \mathrm{kg \cdot m^2/s})\mathbf{k}$$

Note that $\mathbf{H}_O = \mathbf{H}_G + \bar{\mathbf{r}} \times m\bar{\mathbf{v}}$.

PROBLEM 14.15

A 13-kg projectile is passing through the origin O with a velocity $\mathbf{v}_0 = (35 \text{ m/s})\mathbf{i}$ when it explodes into two fragments A and B, of mass 5 kg and 8 kg, respectively. Knowing that 3 s later the position of fragment A is (90 m, 7 m, –14 m), determine the position of fragment B at the same instant. Assume $a_y = -g = -9.81 \text{ m/s}^2$ and neglect air resistance.

SOLUTION

Motion of mass center:

It moves as if projectile had not exploded.

$$\begin{aligned}\overline{\mathbf{r}} &= v_0 t\mathbf{i} - \frac{1}{2}gt^2\mathbf{j}\\ &= (35 \text{ m/s})(3 \text{ s})\mathbf{i} - \frac{1}{2}(9.81 \text{ m/s}^2)(3 \text{ s})^2\mathbf{j}\\ &= (105 \text{ m})\mathbf{i} - (44.145 \text{ m})\mathbf{j}\end{aligned}$$

Equation (14.12):

$$\begin{aligned}m\overline{\mathbf{r}} &= \Sigma m_i \mathbf{r}_i:\\ m\overline{\mathbf{r}} &= m_A\mathbf{r}_A + m_B\mathbf{r}_B\\ 13(105\mathbf{i} - 44.145\mathbf{j}) &= 5(90\mathbf{i} + 7\mathbf{j} - 14\mathbf{k}) + 8\mathbf{r}_B\\ 8\mathbf{r}_B &= (13\times 105 - 5\times 90)\mathbf{i}\\ &\quad + (-13\times 44.145 - 5\times 7)\mathbf{j} + (5\times 14)\mathbf{k}\\ &= 915\mathbf{i} - 608.89\mathbf{j} + 70\mathbf{k}\end{aligned}$$

$$\mathbf{r}_B = (114.4 \text{ m})\mathbf{i} - (76.1 \text{ m})\mathbf{j} + (8.75 \text{ m})\mathbf{k} \quad ◀$$

PROBLEM 14.16

A 300-kg space vehicle traveling with a velocity $\mathbf{v}_0 = (360 \text{ m/s})\mathbf{i}$ passes through the origin O at $t = 0$. Explosive charges then separate the vehicle into three parts A, B, and C, with mass, respectively, 150 kg, 100 kg, and 50 kg. Knowing that at $t = 4$ s, the positions of parts A and B are observed to be A (1170 m, –290 m, –585 m) and B (1975 m, 365 m, 800 m), determine the corresponding position of part C. Neglect the effect of gravity.

SOLUTION

Motion of mass center:

Since there is no external force,

$$\bar{\mathbf{r}} = \mathbf{v}_0 t = (360 \text{ m/s})\mathbf{i}\,(4 \text{ s}) = (1440 \text{ m})\mathbf{i}$$

Equation (14.12):

$$m\bar{\mathbf{r}} = \Sigma m_i \mathbf{r}_i:$$

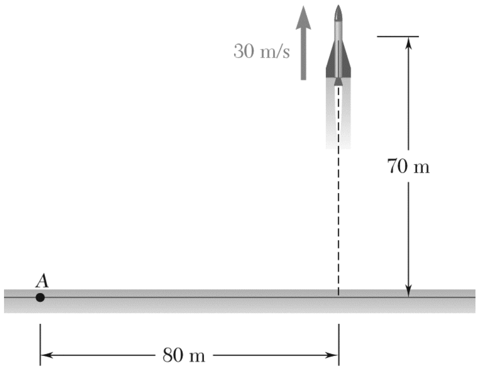
$$\begin{aligned}(300)(1440\mathbf{i}) = {} & (150)(1170\mathbf{i} - 290\mathbf{j} - 585\mathbf{k}) \\ & + (100)(1975\mathbf{i} + 365\mathbf{j} + 800\mathbf{k}) \\ & + (50)\mathbf{r}_C\end{aligned}$$

$$\begin{aligned}50\mathbf{r}_C = {} & (300\times 1440 - 150\times 1170 - 100\times 1975)\mathbf{i} \\ & + (150\times 290 - 100\times 365)\mathbf{j} + (150\times 585 - 100\times 800)\mathbf{k}\end{aligned}$$

$$= 59{,}000\mathbf{i} + 7{,}000\mathbf{j} + 7{,}750\mathbf{k}$$

$$\mathbf{r}_C = (1180 \text{ m})\mathbf{i} + (140 \text{ m})\mathbf{j} + (155 \text{ m})\mathbf{k} \blacktriangleleft$$

PROBLEM 14.17



A 2-kg model rocket is launched vertically and reaches an altitude of 70 m with a speed of 30 m/s at the end of powered flight, time $t = 0$. As the rocket approaches its maximum altitude it explodes into two parts of masses $m_A = 0.7$ kg and $m_B = 1.3$ kg. Part A is observed to strike the ground 80 m west of the launch point at $t = 6$ s. Determine the position of part B at that time.

SOLUTION

Choose a planar coordinate system having coordinates x and y with the origin at the launch point on the ground and the x-axis pointing east and the y-axis vertically upward.

Let subscript E refer to the point where the explosion occurs, and A and B refer to the fragments A and B.
Let t be the time elapsed after the explosition.

Motion of the mass center:

$$\bar{x} = x_E + (\bar{v}_x)t = 0$$

$$\bar{y} = y_E + (\bar{v}_y)_0^t - \frac{1}{2}gt^2$$

where $y_E = 70$ m and $(\bar{v}_y)_0 = 30$ m/s

At $t = 6$ s, $\bar{x} = 0$

$$\bar{y} = 70 + (30)(6) - \frac{1}{2}(9.81)(6)^2 = 73.42 \text{ m}$$

Definition of mass center:

$$m_A\bar{x} = m_A x_A + m_B x_B$$

$$0 = (0.7 \text{ kg})(-80 \text{ m}) + (1.3 \text{ kg})x_B$$

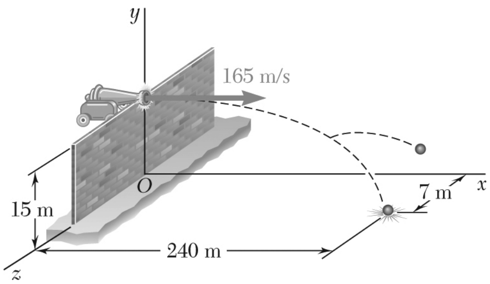
$$x_B = 43.1 \text{ m}$$

$$m\bar{y} = m_A y_A + m_B y_B$$

$$(2 \text{ kg})(73.42 \text{ m}) = (0.7 \text{ kg})(0) + (1.3 \text{ kg})y_B$$

$$y_B = 113.0 \text{ m}$$

Position of part B: 43.1 m (east), 113.0 m (up) ◀



PROBLEM 14.18

An 18-kg cannonball and a 12-kg cannonball are chained together and fired horizontally with a velocity of 165 m/s from the top of a 15-m wall. The chain breaks during the flight of the cannonballs and the 12-kg cannonball strikes the ground at $t = 1.5$ s, at a distance of 240 m from the foot of the wall, and 7 m to the right of the line of fire. Determine the position of the other cannonball at that instant. Neglect the resistance of the air.

SOLUTION

Let subscript A refer to the 12-kg cannonball and B to the 18-kg cannonball.

The motion of the mass center of A and B is uniform in the x-direction, uniformly accelerated with acceleration $-g = -9.81\ \text{m/s}^2$ in the y-direction, and zero in the z-direction.

$$\bar{x} = (v_0)_x t = (165\ \text{m/s})(1.5\ \text{s}) = 247.5\ \text{m}$$

$$\bar{y} = \bar{y}_0 + (\bar{v}_0)_y t - \frac{1}{2}gt^2$$

$$= 15\ \text{m} + 0 - \frac{1}{2}(9.81\ \text{m/s}^2)(1.55)^2 = 3.964\ \text{m}$$

$$\bar{z} = 0$$

$$\bar{\mathbf{r}} = (247.5\ \text{m})\mathbf{i} + (3.964\ \text{m})\mathbf{j}$$

Definition of mass center:

$$m\bar{\mathbf{r}} = m_A\mathbf{r}_A + m_B\mathbf{r}_B$$

Data: $m_A = 12\ \text{kg}, \quad m_B = 18\ \text{kg}, \quad m = m_A + m_B = 30\ \text{kg}$

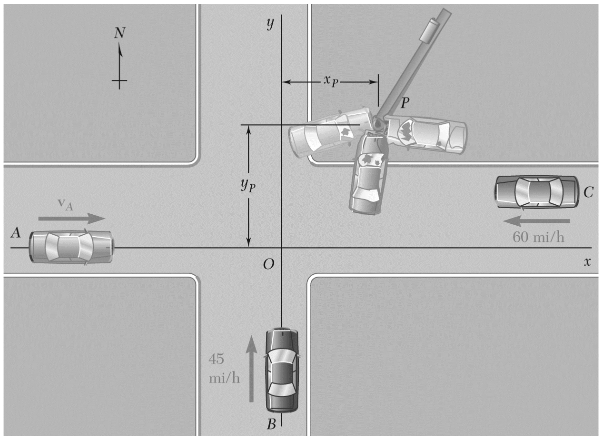
$$t = 1.5\ \text{s}, \quad x_A = 240\ \text{m}, \quad y_A = 0, \quad z_A = 7\ \text{m}$$

$$(30)(247.5\mathbf{i} + 3.964\ \mathbf{j}) = (12)(240\mathbf{i} + \mathbf{7k}) + (18)(x_B\mathbf{i} + y_B\mathbf{j} + z_B\mathbf{k})$$

$\mathbf{i}$: $(30)(247.5) = (12)(240) + 18\,x_B$ $\qquad x_B = 253\ \text{m}$ ◄

$\mathbf{j}$: $(30)(3.964) = (12)(0) + 18\,y_B$ $\qquad y_B = 6.61\ \text{m}$ ◄

$\mathbf{k}$: $(30)(0) = (12)(7) + 18\,z_B$ $\qquad z_B = -4.67\ \text{m}$ ◄



PROBLEM 14.19

Car A was traveling east at high speed when it collided at Point O with car B, which was traveling north at 45 mi/h. Car C, which was traveling west at 60 mi/h, was 32 ft east and 10 ft north of Point O at the time of the collision. Because the pavement was wet, the driver of car C could not prevent his car from sliding into the other two cars, and the three cars, stuck together, kept sliding until they hit the utility pole P. Knowing that the weights of cars A, B, and C are, respectively, 3000 lb, 2600 lb, and 2400 lb, and neglecting the forces exerted on the cars by the wet pavement, solve the problems indicated.

Knowing that the speed of car A was 75 mi/h and that the time elapsed from the first collision to the stop at P was 2.4 s, determine the coordinates of the utility pole P.

SOLUTION

Let t be the time elapsed since the first collision. No external forces in the xy plane act on the system consisting of cars A, B, and C during the impacts with one another. The mass center of the system moves at the velocity it had before the collision.

Setting the origin at O, we can find the initial mass center $\bar{\mathbf{r}}_0$: at the moment of the first collision:

$$(m_A + m_B + m_C)(\bar{x}_0\mathbf{i} + \bar{y}_0\mathbf{j}) = m_A(0) + m_B(0) + m_C(x_C\mathbf{i} + y_C\mathbf{j})$$

$$\bar{x}_0 = 0.3x_C = (0.3)(32) = 9.6 \text{ ft}, \quad \bar{y}_0 = 0.3y_C = (0.3)(10) = 3 \text{ ft}$$

Given velocities:

$$\mathbf{v}_A = (75 \text{ mi/h})\mathbf{i} = (110 \text{ ft/s})\mathbf{i}, \quad \mathbf{v}_B = (45 \text{ mi/h})\mathbf{i} = (66 \text{ ft/s})\mathbf{j}, \quad \mathbf{v}_C = (60 \text{ mi/h})\mathbf{i} = (88 \text{ ft/s})\mathbf{i}$$

Velocity of mass center:

$$(m_A + m_B + m_C)\bar{\mathbf{v}} = m_A\mathbf{v}_A + m_B\mathbf{v}_B + m_C\mathbf{v}_C$$

$$\bar{\mathbf{v}} = 0.375\mathbf{v}_A + 0.325\mathbf{v}_B + 0.3\mathbf{v}_C$$

Since the collided cars hit the pole at

$$\mathbf{r}_P = x_P\mathbf{i} + y_P\mathbf{j}$$

$$\mathbf{x}_P\mathbf{i} + y_P\mathbf{j} = \bar{x}_0\mathbf{i} + \bar{y}_0\mathbf{j} + \bar{\mathbf{v}}t \qquad \text{Resolve into components.}$$

$$x: \quad x_P = \bar{x}_0 + 0.375v_A t_P - 0.3v_C t_P \tag{1}$$

$$y: \quad y_P = \bar{y}_0 + 0.325v_B t_P \tag{2}$$

PROBLEM 14.19 (Continued)

Data: $t_P = 2.4\text{ s}$

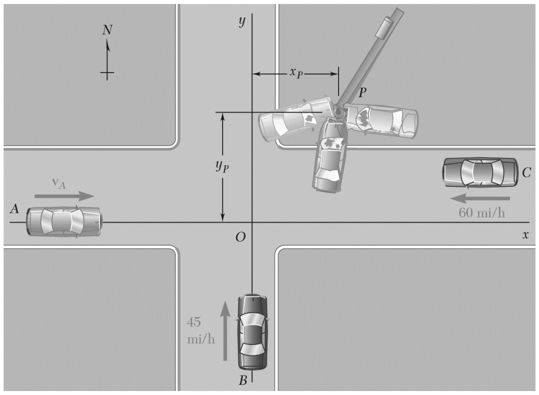
From (1), $x_P = 9.6 + (0.375)(110)(2.4) - (0.3)(88)(2.4) = 45.240$

From (2), $y_P = 3.0 + (0.325)(66)(2.4) = 54.480\text{ ft}$

$x_P = 45.2\text{ ft}$ ◀

$y_P = 54.5\text{ ft}$ ◀

PROBLEM 14.20



Car A was traveling east at high speed when it collided at Point O with car B, which was traveling north at 45 mi/h. Car C, which was traveling west at 60 mi/h, was 32 ft east and 10 ft north of Point O at the time of the collision. Because the pavement was wet, the driver of car C could not prevent his car from sliding into the other two cars, and the three cars, stuck together, kept sliding until they hit the utility pole P. Knowing that the weights of cars A, B, and C are, respectively, 3000 lb, 2600 lb, and 2400 lb, and neglecting the forces exerted on the cars by the wet pavement, solve the problems indicated. Knowing that the coordinates of the utility pole are $x_P = 46$ ft and $y_P = 59$ ft, determine (*a*) the time elapsed from the first collision to the stop at P, (*b*) the speed of car A.

SOLUTION

Let t be the time elapsed since the first collision. No external forces in the xy plane act on the system consisting of cars A, B, and C during the impacts with one another. The mass center of the system moves at the velocity it had before the collision.

Setting the origin at O, we can find the initial mass center $\overline{\mathbf{r}}_0$: at the moment of the first collision:

$$(m_A + m_B + m_C)(\overline{x}_0\mathbf{i} + \overline{y}_0\mathbf{j}) = m_A(0) + m_B(0) + m_C(x_C\mathbf{i} + y_C\mathbf{j})$$

$$\overline{x}_0 = 0.3x_C = (0.3)(32) = 9.6 \text{ ft}, \quad \overline{y}_0 = 0.3y_C = (0.3)(10) = 3 \text{ ft}$$

Given velocities:

$$\mathbf{v}_A = v_A\mathbf{i}, \quad \mathbf{v}_B = (45 \text{ mi/h})\mathbf{i} = (66 \text{ ft/s})\mathbf{j}, \quad \mathbf{v}_C = (60 \text{ mi/h})\mathbf{i} = (88 \text{ ft/s})\mathbf{i}$$

Velocity of mass center:

$$(m_A + m_B + m_C)\mathbf{v} = m_A\mathbf{v}_A + m_B\mathbf{v}_B + m_C\mathbf{v}_C$$

$$\overline{\mathbf{v}} = 0.375\mathbf{v}_A + 0.325\mathbf{v}_B + 0.3\mathbf{v}_C$$

Since the collided cars hit the pole at

$$\mathbf{r}_P = x_P\mathbf{i} + y_P\mathbf{j}$$

$$x_P\mathbf{i} + y_P\mathbf{j} = \overline{x}_0\mathbf{i} + \overline{y}_0\mathbf{j} + \overline{\mathbf{v}}t \qquad \text{Resolve into components.}$$

$$x: \quad x_P = \overline{x}_0 + 0.375v_At_P - 0.3v_Ct_P \tag{1}$$

$$y: \quad y_P = \overline{y}_0 + 0.325v_Bt_P \tag{2}$$

PROBLEM 14.20 (Continued)

Data: $x_P = 59\text{ ft},\quad y_P = 46\text{ ft}$

(*a*) From (2), $46 = 3 + (0.325)(66)t_P$

$t_P = 2.0047\text{ s}$ $\qquad t_P = 2.00\text{ s}$ ◀

(*b*) From (1), $59 = 9.6 + (0.375)v_A(2.0047) - (0.3)(88)(2.0047)$

$v_A = 136.11\text{ ft/s}$ $\qquad v_A = 92.8\text{ mi/h}$ ◀

PROBLEM 14.21

An expert archer demonstrates his ability by hitting tennis balls thrown by an assistant. A 2-oz tennis ball has a velocity of (32 ft/s)**i** – (7 ft/s)**j** and is 33 ft above the ground when it is hit by a 1.2-oz arrow traveling with a velocity of (165 ft/s)**j** + (230 ft/s)**k** where **j** is directed upwards. Determine the position *P* where the ball and arrow will hit the ground, relative to Point *O* located directly under the point of impact.

SOLUTION

Assume that the ball and arrow move together after the hit.

Conservation of momentum of ball and arrow during the hit.

$$m_A = \frac{1.2/16}{32.2} = 2.3292\times 10^{-3} \text{ slug} \qquad m_B = \frac{2/16}{32.2} = 3.8820\times 10^{-3} \text{ slug}$$

$$m_A\mathbf{v}_A + m_B\mathbf{v}_B = (m_A + m_B)\overline{\mathbf{v}}$$

$$(2.3292\times 10^{-3})(165\mathbf{j} + 230\mathbf{k}) + (3.8820\times 10^{-3})(32\mathbf{i} - 7\mathbf{j}) = (2.3292\times 10^{-3} + 3.8820\times 10^{-3})\overline{\mathbf{v}}$$

$$\overline{\mathbf{v}} = (20.0 \text{ ft/s})\mathbf{i} + (57.5 \text{ ft/s})\mathbf{j} + (86.25 \text{ ft/s})\mathbf{k}$$

After the hit, the ball and arrow move as a projectile.

Vertical motion:

$$y = y_0 + (v_y)_0 t - \frac{1}{2}gt^2$$

$$y = 33 + 57.5t - \frac{1}{2}(32.2)t^2$$

$y = 0$ at ground.

$$-16.1t^2 + 57.5t + 33 = 0 \qquad \text{Solve for } t.$$

After rejecting the negative root, $t = 4.0745$ s

Horizontal motion:

$$x = x_0 + (v_x)_0 t$$

$$z = z_0 + (v_2)_0 t$$

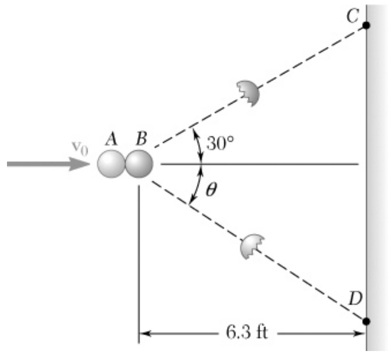
$$x = 0 + (20)(4.07448)$$

$$= 81.490 \text{ ft}$$

$$z = 0 + (86.25)(4.07448)$$

$$= 351.42 \text{ ft}$$

$$\mathbf{r}_P = (81.5 \text{ ft})\mathbf{i} + (351 \text{ ft})\mathbf{k} \blacktriangleleft$$



PROBLEM 14.22

Two spheres, each of mass m, can slide freely on a frictionless, horizontal surface. Sphere A is moving at a speed $v_0 = 16$ ft/s when it strikes sphere B which is at rest and the impact causes sphere B to break into two pieces, each of mass $m/2$. Knowing that 0.7 s after the collision one piece reaches Point C and 0.9 s after the collision the other piece reaches Point D, determine (*a*) the velocity of sphere A after the collision, (*b*) the angle θ and the speeds of the two pieces after the collision.

SOLUTION

Velocities of pieces C and D after impact and fracture.

$$(v'_C)_x = \frac{x_C}{t_C} = \frac{6.3}{0.7} = 9 \text{ ft/s}, \qquad (v'_C)_y = 9 \tan 30° \text{ ft/s}$$

$$(v'_D)_x = \frac{x_D}{t_D} = \frac{6.3}{0.9} = 7 \text{ ft/s}, \qquad (v'_D)_y = -7 \tan\theta \text{ ft/s}$$

Assume that during the impact the impulse between spheres A and B is directed along the x-axis. Then, the y component of momentum of sphere A is conserved.

$$0 = m(v'_A)_y$$

Conservation of momentum of system:

$$\xrightarrow{+}: \quad m_A v_0 + m_B(0) = m_A v'_A + m_C(v'_C)_x + m_D(v'_D)_x$$

$$m(16) + 0 = mv'_A + \frac{m}{2}(9) + \frac{m}{2}(7)$$

(*a*) $\mathbf{v}'_A = 8.00 \text{ ft/s} \longrightarrow$ ◀

$$+\uparrow: \quad m_A(0) + m_B(0) = m_A(v'_A)_y + m_C(v'_C)_y + m_D(v'_D)_y$$

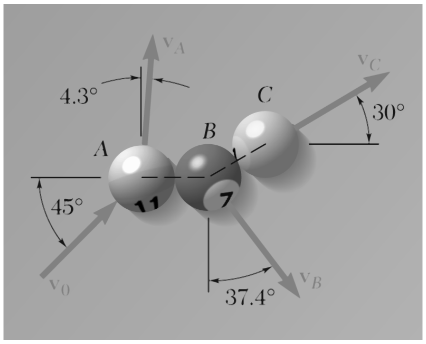
$$0 + 0 = 0 + \frac{m}{2}(9\tan 30°) - \frac{m}{2}(7\tan\theta)$$

(*b*) $\tan\theta = \frac{9}{7}\tan 30° = 0.7423$ $\qquad \theta = 36.6°$ ◀

$$v_C = \sqrt{(v_C)_x^2 + (v_C)_y^2} = \sqrt{(9)^2 + (9\tan 30°)^2}$$ $v_C = 10.39$ ft/s ◀

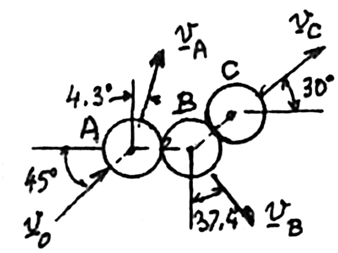
$$v_D = \sqrt{(v_D)_x^2 + (v_D)_y^2} = \sqrt{(7)^2 + (7\tan 36.6°)^2}$$ $v_D = 8.72$ ft/s ◀

PROBLEM 14.23



In a game of pool, ball A is moving with a velocity $\mathbf{v}_0$ when it strikes balls B and C which are at rest and aligned as shown. Knowing that after the collision the three balls move in the directions indicated, and that $v_0 = 12$ ft/s and $v_C = 6.29$ ft/s, determine the magnitude of the velocity of (*a*) ball A, (*b*) ball B.

SOLUTION



Conservation of linear momentum. In x direction:

$$m(12\text{ ft/s})\cos 45° = mv_A \sin 4.3° + mv_B \sin 37.4° + m(6.29)\cos 30°$$

$$0.07498v_A + 0.60738v_B = 3.0380 \tag{1}$$

In y direction:

$$m(12\text{ ft/s})\sin 45° = mv_A \cos 4.3° - mv_B \cos 37.4° + m(6.29)\sin 30°$$

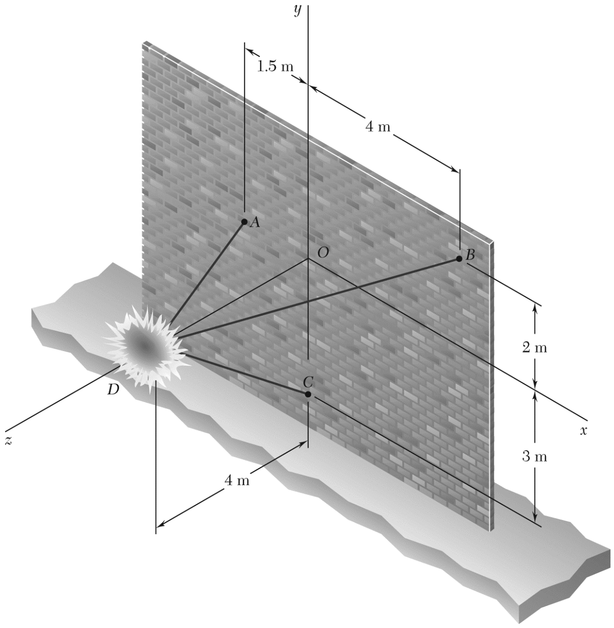
$$0.99719v_A - 0.79441v_B = 5.3403 \tag{2}$$

(*a*) Multiply (1) by 0.79441, (2) by 0.60738, and add:

$$0.66524v_A = 5.5670 \qquad v_A = 8.50 \text{ ft/s} \blacktriangleleft$$

(*b*) Multiply (1) by 0.99719, (2) by –0.07498, and add:

$$0.66524v_B = 2.6290 \qquad v_B = 3.95 \text{ ft/s} \blacktriangleleft$$



PROBLEM 14.24

A 6-kg shell moving with a velocity $\mathbf{v}_0 = (12\text{ m/s})\mathbf{i} - (9\text{ m/s})\mathbf{j} - (360\text{ m/s})\mathbf{k}$ explodes at Point D into three fragments A, B, and C of mass, respectively, 3 kg, 2 kg, and 1 kg. Knowing that the fragments hit the vertical wall at the points indicated, determine the speed of each fragment immediately after the explosion. Assume that elevation changes due to gravity may be neglected.

SOLUTION

Position vectors (m): $\mathbf{r}_D = 4\mathbf{k}$

$$\mathbf{r}_A = -1.5\mathbf{i} \qquad \mathbf{r}_{A/D} = -1.5\mathbf{i} - 4\mathbf{k} \qquad r_{A/D} = 4.272$$

$$\mathbf{r}_B = 4\mathbf{i} + 2\mathbf{j} \qquad \mathbf{r}_{B/D} = 4\mathbf{i} + 2\mathbf{j} - 4\mathbf{k} \qquad r_{B/D} = 6$$

$$\mathbf{r}_C = -3\mathbf{j} \qquad \mathbf{r}_{C/D} = -3\mathbf{j} - 4\mathbf{k} \qquad r_{C/D} = 5$$

Unit vectors:

Along $\mathbf{r}_{A/D}$, $\quad \boldsymbol{\lambda}_A = \dfrac{1}{4.272}(-1.5\mathbf{i} - 4\mathbf{k})$

Along $\mathbf{r}_{B/D}$, $\quad \boldsymbol{\lambda}_B = \dfrac{1}{6}(4\mathbf{i} + 2\mathbf{j} - 4\mathbf{k})$

Along $\mathbf{r}_{C/D}$, $\quad \boldsymbol{\lambda}_C = \dfrac{1}{5}(-3\mathbf{j} - 4\mathbf{k})$

Assume that elevation changes due to gravity may be neglected. Then, the velocity vectors after the explosion have the directions of the unit vectors.

$$\mathbf{v}_A = v_A \boldsymbol{\lambda}_A$$
$$\mathbf{v}_B = v_B \boldsymbol{\lambda}_B$$
$$\mathbf{v}_C = v_C \boldsymbol{\lambda}_C$$

Conservation of momentum: $\quad m\mathbf{v}_0 = m_A\mathbf{v}_A + m_B\mathbf{v}_B + m_C\mathbf{v}_C$

$$6(12\mathbf{i} - 9\mathbf{j} - 360\mathbf{k}) = 3\left(\frac{v_A}{4.272}\right)(-1.5\mathbf{i} - 4\mathbf{k}) + 2\left(\frac{v_B}{6}\right)(4\mathbf{i} + 2\mathbf{j} - 4\mathbf{k})$$
$$+1\left(\frac{v_C}{5}\right)(-3\mathbf{j} - 4\mathbf{k})$$

PROBLEM 14.24 (Continued)

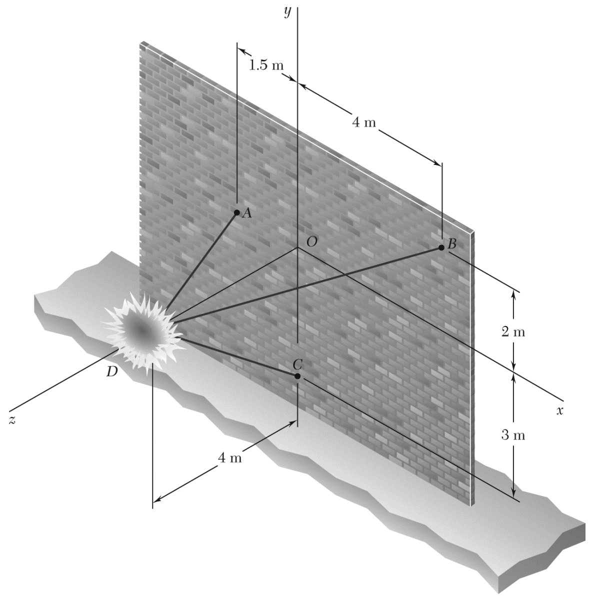
Resolve into components.

$$72 = -1.0534v_A + 1.3333v_B$$
$$-54 = 0.66667v_B - 0.60000v_C$$
$$-2160 = -2.8090v_A - 1.3333v_B - 0.80000v_C$$

Solving, $v_A = 431 \text{ m/s}$ ◀

$v_B = 395 \text{ m/s}$ ◀

$v_C = 528 \text{ m/s}$ ◀



PROBLEM 14.25

A 6-kg shell moving with a velocity $\mathbf{v}_0 = (12 \text{ m/s})\mathbf{i} - (9 \text{ m/s})\mathbf{j} - (360 \text{ m/s})\mathbf{k}$ explodes at Point *D* into three fragments *A*, *B*, and *C* of mass, respectively, 2 kg, 1 kg, and 3 kg. Knowing that the fragments hit the vertical wall at the points indicated, determine the speed of each fragment immediately after the explosion. Assume that elevation changes due to gravity may be neglected.

SOLUTION

Position vectors (m): $\mathbf{r}_D = 4\mathbf{k}$

$\mathbf{r}_A = -1.5\mathbf{i}$	$\mathbf{r}_{A/D} = -1.5\mathbf{i} - 4\mathbf{k}$	$r_{A/D} = 4.272$
$\mathbf{r}_B = 4\mathbf{i} + 2\mathbf{j}$	$\mathbf{r}_{B/D} = 4\mathbf{i} + 2\mathbf{j} - 4\mathbf{k}$	$r_{B/D} = 6$
$\mathbf{r}_C = -3\mathbf{j}$	$\mathbf{r}_{C/D} = -3\mathbf{j} - 4\mathbf{k}$	$r_{C/D} = 5$

Unit vectors:

Along $\mathbf{r}_{A/D}$, $\boldsymbol{\lambda}_A = \dfrac{1}{4.272}(-1.5\mathbf{i} - 4\mathbf{k})$

Along $\mathbf{r}_{B/D}$, $\boldsymbol{\lambda}_B = \dfrac{1}{6}(4\mathbf{i} + 2\mathbf{j} - 4\mathbf{k})$

Along $\mathbf{r}_{C/D}$, $\boldsymbol{\lambda}_C = \dfrac{1}{5}(-3\mathbf{j} - 4\mathbf{k})$

Assume that elevation changes due to gravity may be neglected. Then the velocity vectors after the explosion have the directions of the unit vectors.

$$\mathbf{v}_A = v_A \boldsymbol{\lambda}_A$$
$$\mathbf{v}_B = v_B \boldsymbol{\lambda}_B$$
$$\mathbf{v}_C = v_C \boldsymbol{\lambda}_C$$

Conservation of momentum: $m\mathbf{v}_0 = m_A \mathbf{v}_A + m_B \mathbf{v}_B + m_C \mathbf{v}_C$

Resolve into components.

$$6(12\mathbf{i} - 9\mathbf{j} - 360\mathbf{k}) = 2\left(\frac{v_A}{4.272}\right)(-1.5\mathbf{i} - 4\mathbf{k}) + 1\left(\frac{v_B}{6}\right)(4\mathbf{i} + 2\mathbf{j} - 4\mathbf{k})$$
$$+3\left(\frac{v_C}{5}\right)(3\mathbf{j} - 4\mathbf{k})$$

PROBLEM 14.25 (Continued)

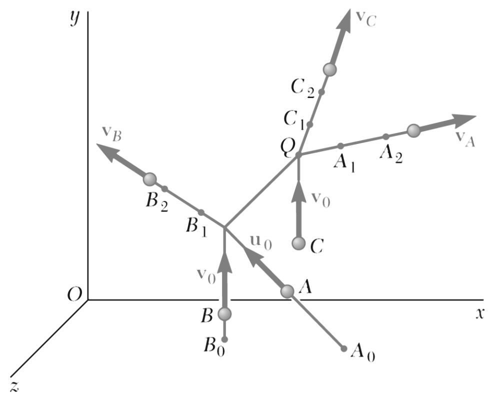
$$72 = -0.70225 v_A + 0.66667 v_B$$
$$-54 = 0.33333 v_B - 1.8000 v_C$$
$$-2160 = -1.8727 v_A - 0.66667 v_B - 2.40000 v_C$$

Solving, $v_A = 646 \text{ m/s}$ ◀

$v_B = 789 \text{ m/s}$ ◀

$v_C = 176 \text{ m/s}$ ◀

PROBLEM 14.26



In a scattering experiment, an alpha particle A is projected with the velocity $\mathbf{u}_0 = -(600 \text{ m/s})\mathbf{i} + (750 \text{ m/s})\mathbf{j} - (800 \text{ m/s})\mathbf{k}$ into a stream of oxygen nuclei moving with a common velocity $\mathbf{v}_0 = (600 \text{ m/s})\mathbf{j}$. After colliding successively with nuclei B and C, particle A is observed to move along the path defined by the Points $A_1(280, 240, 120)$and $A_2(360, 320, 160)$, while nuclei B and C are observed to move along paths defined, respectively, by $B_1(147, 220, 130)$, $B_2(114, 290, 120)$,and by $C_1(240, 232, 90)$ and $C_2(240, 280, 75)$. All paths are along straight lines and all coordinates are expressed in millimeters. Knowing that the mass of an oxygen nucleus is four times that of an alpha particle, determine the speed of each of the three particles after the collisions.

SOLUTION

Position vectors (mm): $\overrightarrow{A_1A_2} = 80\mathbf{i} + 80\mathbf{j} + 40\mathbf{k} \qquad (A_1A_2) = 120$

$$\overrightarrow{B_1B_2} = -33\mathbf{i} + 70\mathbf{j} - 10\mathbf{k} \qquad (B_1B_2) = 78.032$$

$$\overrightarrow{C_1C_2} = 48\mathbf{j} - 15\mathbf{k} \qquad (C_1C_2) = 50.289$$

Unit vectors: Along A_1A_2, $\quad \lambda_A = 0.66667\mathbf{i} + 0.66667\mathbf{j} + 0.33333\mathbf{k}$

Along B_1B_2, $\quad \lambda_B = -0.42290\mathbf{i} + 0.89707\mathbf{j} - 0.12815\mathbf{k}$

Along C_1C_2, $\quad \lambda_C = 0.95448\mathbf{j} - 0.29828\mathbf{k}$

Velocity vectors after the collisions:

$$\mathbf{v}_A = v_A\lambda_A$$
$$\mathbf{v}_B = v_B\lambda_B$$
$$\mathbf{v}_C = v_C\lambda_C$$

Conservation of momentum:

$$m\mathbf{u}_0 + 4m\mathbf{v}_0 + 4m\mathbf{v}_0 = m\mathbf{v}_A + 4m\mathbf{v}_B + 4m\mathbf{v}_C$$

Divide by m and substitute data.

$$(-600\mathbf{i} + 750\mathbf{j} - 800\mathbf{k}) + 2400\mathbf{j} + 2400\mathbf{j} = v_A\lambda_A + 4v_B\lambda_B + 4v_C\lambda_C$$

Resolving into components,

$$\mathbf{i}: \; -600 = 0.66667v_A - 1.69160v_B$$
$$\mathbf{j}: \; 5550 = 0.66667v_A + 3.58828v_B + 3.81792v_C$$
$$\mathbf{k}: \; -800 = 0.33333v_A - 0.51260v_B - 1.19312v_C$$

PROBLEM 14.26 (Continued)

Solving the three equations simultaneously,

$$v_A = 919.26 \text{ m/s}$$
$$v_B = 716.98 \text{ m/s}$$
$$v_C = 619.30 \text{ m/s}$$

$v_A = 919 \text{ m/s}$ ◀

$v_B = 717 \text{ m/s}$ ◀

$v_C = 619 \text{ m/s}$ ◀

PROBLEM 14.27

Derive the relation

$$\mathbf{H}_O = \overline{\mathbf{r}} \times m\overline{\mathbf{v}} + \mathbf{H}_G$$

between the angular momenta $\mathbf{H}_O$ and $\mathbf{H}_G$ defined in Eqs. (14.7) and (14.24), respectively. The vectors $\overline{\mathbf{r}}$ and $\overline{\mathbf{v}}$ define, respectively, the position and velocity of the mass center G of the system of particles relative to the newtonian frame of reference $Oxyz$, and m represents the total mass of the system.

SOLUTION

From Eq. (14.7),

$$\begin{aligned}
\mathbf{H}_O &= \sum_{i=1}^{n} (\mathbf{r}_i \times m_i \mathbf{v}_i) \\
&= \sum_{i=1}^{n} \left[\left(\overline{\mathbf{r}} + \mathbf{r}_i' \right) \times m_i \mathbf{v}_i \right] \\
&= \overline{\mathbf{r}} \times \sum_{i=1}^{n} (m_i v_i) + \sum_{i=1}^{n} \left(r_i' \times m_i \mathbf{v}_i \right) \\
&= \overline{\mathbf{r}} \times m\overline{\mathbf{v}} + \mathbf{H}_G
\end{aligned}$$

PROBLEM 14.28

Show that Eq. (14.23) may be derived directly from Eq. (14.11) by substituting for $\mathbf{H}_O$ the expression given in Problem 14.27.

SOLUTION

From Eq. (14.7),

$$\mathbf{H}_O = \sum_{i=1}^{n} (\mathbf{r}_i \times m_i \mathbf{v}_i)$$

$$= \sum_{i=1}^{n} \left[(\overline{\mathbf{r}} + \mathbf{r}_i') \times m_i \mathbf{v}_i \right]$$

$$= \overline{\mathbf{r}} \times \sum_{i=1}^{n} (m_i v_i) + \sum_{i=1}^{n} (r_i' \times m_i \mathbf{v}_i)$$

$$= \overline{\mathbf{r}} \times m\overline{\mathbf{v}} + \mathbf{H}_G$$

Differentiating,

$$\dot{\mathbf{H}}_O = \dot{\overline{\mathbf{r}}} \times m\overline{\mathbf{v}} + \overline{\mathbf{r}} \times m\dot{\overline{\mathbf{v}}} + \dot{\mathbf{H}}_G$$

Using Eq. (14.11),

$$\Sigma \mathbf{M}_O = \dot{\overline{\mathbf{r}}} \times m\overline{\mathbf{v}} + \overline{\mathbf{r}} \times m\dot{\overline{\mathbf{v}}} + \dot{\mathbf{H}}_G$$

$$= \overline{\mathbf{v}} \times m\overline{\mathbf{v}} + \overline{\mathbf{r}} \times m\overline{\mathbf{a}} + \dot{\mathbf{H}}_G$$

$$= 0 + \overline{\mathbf{r}} \times \left(\sum_{i=1}^{n} \mathbf{F}_i \right) + \dot{\mathbf{H}}_G \qquad (1)$$

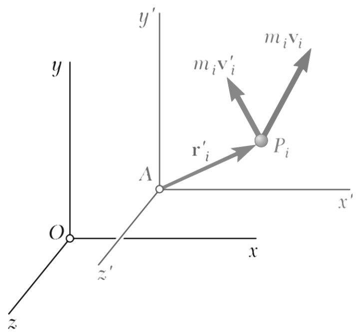
But

$$\sum_{i=1}^{n} \mathbf{M}_O = \sum_{i=1}^{n} \mathbf{M}_G + \sum_{i=1}^{n} \overline{\mathbf{r}} \times \left(\sum_{i=1}^{n} F_i \right)$$

Subtracting $\overline{\mathbf{r}} \times \left(\Sigma_{i=1}^{n} F_i \right)$ from each side of Eq. (1) gives

$$\Sigma \mathbf{M}_G = \dot{\mathbf{H}}_G$$

PROBLEM 14.29



Consider the frame of reference $Ax'y'z'$ in translation with respect to the newtonian frame of reference $Oxyz$. We define the angular momentum $\mathbf{H}'_A$ of a system of n particles about A as the sum

$$\mathbf{H}'_A = \sum_{i=1}^{n} \mathbf{r}'_i \times m_i \mathbf{v}'_i \tag{1}$$

of the moments about A of the momenta $m_i\mathbf{v}'_i$ of the particles in their motion relative to the frame $Ax'y'z'$. Denoting by $\mathbf{H}_A$ the sum

$$\mathbf{H}_A = \sum_{i=1}^{n} \mathbf{r}'_i \times m_i \mathbf{v}_i \tag{2}$$

of the moments about A of the momenta $m_i\mathbf{v}_i$ of the particles in their motion relative to the newtonian frame $Oxyz$, show that $\mathbf{H}_A = \mathbf{H}'_A$ at a given instant if, and only if, one of the following conditions is satisfied at that instant: (*a*) A has zero velocity with respect to the frame $Oxyz$, (*b*) A coincides with the mass center G of the system, (*c*) the velocity $\mathbf{v}_A$ relative to $Oxyz$ is directed along the line AG.

SOLUTION

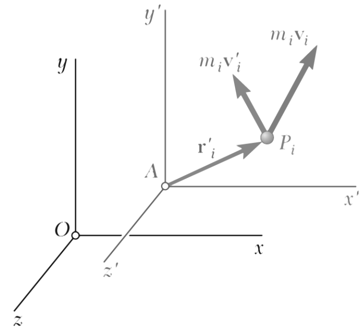
$$\mathbf{v}_i = \mathbf{v}_A + \mathbf{v}'_i$$

$$\begin{aligned}
\mathbf{H}_A &= \sum_{i=1}^{n} \mathbf{r}'_i \times m_i \mathbf{v}_i \\
&= \sum_{i=1}^{n} \mathbf{r}'_i \times m_i (\mathbf{v}_A + \mathbf{v}'_i) \\
&= \sum_{i=1}^{n} (\mathbf{r}'_i \times m_i \mathbf{v}_A) + \sum_{i=1}^{n} \mathbf{r}'_i \times m_i \mathbf{v}_i \\
&= \sum_{i=1}^{n} (m_i \mathbf{r}'_i) \times \mathbf{v}_A + \mathbf{H}'_A \\
&= \sum_{i=1}^{n} m_i (\mathbf{r}_i - \mathbf{r}_A) \times \mathbf{v}_A + \mathbf{H}'_A \\
&= m(\overline{\mathbf{r}} - \mathbf{r}_A) \times \mathbf{v}_A + \mathbf{H}'_A
\end{aligned}$$

$$\mathbf{H}_A = \mathbf{H}'_A \quad \text{if, and only if,} \qquad m(\mathbf{r} - \mathbf{r}_A) \times \mathbf{v}_A = 0$$

This condition is satisfied if

	(*a*)	$\mathbf{v}_A = 0$	Point A has zero velocity.
or	(*b*)	$\overline{\mathbf{r}} = \mathbf{r}_A$	Point A coincides with the mass center.
or	(*c*)	$\mathbf{v}_A$ is parallel to $\overline{\mathbf{r}} - \mathbf{r}_A$.	Velocity $\mathbf{v}_A$ is directed along line AG.



PROBLEM 14.30

Show that the relation $\Sigma\mathbf{M}_A = \dot{\mathbf{H}}'_A$, where $\mathbf{H}'_A$ is defined by Eq. (1) of Problem 14.29 and where $\Sigma\mathbf{M}_A$ represents the sum of the moments about A of the external forces acting on the system of particles, is valid if, and only if, one of the following conditions is satisfied: (*a*) the frame $Ax'y'z'$ is itself a newtonian frame of reference, (*b*) A coincides with the mass center G, (*c*) the acceleration $\mathbf{a}_A$ of A relative to $Oxyz$ is directed along the line AG.

SOLUTION

From equation (1),
$$\mathbf{H}'_A = \sum_{i=1}^{n}(\mathbf{r}'_i \times m_i\mathbf{v}'_i)$$
$$\mathbf{H}'_A = \sum_{i=1}^{n}[(\mathbf{r}_i - \mathbf{r}_A)\times m_i(\mathbf{v}_i - \mathbf{v}_A)]$$

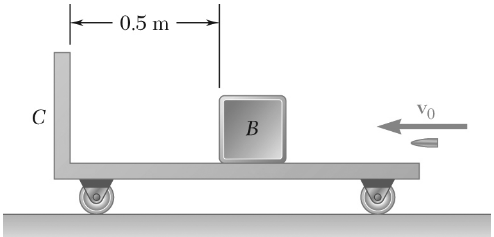
Differentiate with respect to time.
$$\dot{\mathbf{H}}'_A = \sum_{i=1}^{n}[(\dot{\mathbf{r}}_i - \dot{\mathbf{r}}_A)\times m_i(\mathbf{v}_i - \mathbf{v}_A)] + \sum_{i=1}^{n}[(\mathbf{r}_i - \mathbf{r}_A)\times m_i(\dot{\mathbf{v}}_i - \dot{\mathbf{v}}_A)]$$

But
$$\dot{\mathbf{r}}_i = \mathbf{v}_i$$
$$\dot{\mathbf{v}}_i = \mathbf{a}_i$$
$$\dot{\mathbf{r}}_A = \mathbf{v}_A$$
and
$$\dot{\mathbf{v}}_A = \mathbf{a}_A$$

Hence,
$$\dot{\mathbf{H}}'_A = 0 + \sum_{i=1}^{n}[(\mathbf{r}_i - \mathbf{r}_A)\times m_i(\mathbf{a}_i - \mathbf{a}_A)]$$
$$= \sum_{i=1}^{n}[(\mathbf{r}_i - \mathbf{r}_A)\times(\mathbf{F}_i - m_i\mathbf{a}_A)]$$
$$= \sum_{i=1}^{n}[(\mathbf{r}_i - \mathbf{r}_A)\times\mathbf{F}_i] - \sum_{i=1}^{n}[m_i(\mathbf{r}_i - \mathbf{r}_A)]\times\mathbf{a}_A$$
$$= \mathbf{M}_A - m(\overline{\mathbf{r}} - r_A)\times\mathbf{a}_A$$
$$\dot{\mathbf{H}}'_A = M_A \quad \text{if, and only if,} \quad m(\overline{\mathbf{r}} - \mathbf{r}_A)\times\mathbf{a}_A = 0$$

This condition is satisfied if

	(*a*)	$\mathbf{a}_A = 0$	The frame is newtonian.
or	(*b*)	$\overline{\mathbf{r}} = \mathbf{r}_A$	Point A coincides with the mass center.
or	(*c*)	$\mathbf{a}_A$ is parallel to $\overline{\mathbf{r}} - \mathbf{r}_A$.	Acceleration $\mathbf{a}_A$ is directed along line AG.



PROBLEM 14.31

Determine the energy lost due to friction and the impacts for Problem 14.1.

PROBLEM 14.1 A 30-g bullet is fired with a horizontal velocity of 450 m/s and becomes embedded in block B which has a mass of 3 kg. After the impact, block B slides on 30-kg carrier C until it impacts the end of the carrier. Knowing the impact between B and C is perfectly plastic and the coefficient of kinetic friction between B and C is 0.2, determine (*a*) the velocity of the bullet and B after the first impact, (*b*) the final velocity of the carrier.

SOLUTION

From the solution to Problem 4.1 the velocity of A and B after the first impact is $v' = 4.4554$ m/s and the velocity common to A, B, and C after the sliding of block B and bullet A relative to the carrier C has ceased in $v'' = 0.4087$ m/s.

Friction loss due to sliding:

Normal force:
$$N = W_A + W_B = (m_A + m_B)g$$
$$= (0.030 \text{ kg} + 3 \text{ kg})(9.81 \text{ m/s}) = 29.724 \text{ N}$$

Friction force:
$$F_f = \mu_k N = (0.2)(29.724) = 5.945 \text{ N}$$

Relative sliding distance: Assume $d = 0.5$ m.

Energy loss due to friction:
$$F_f d = (5.945)(0.5) \qquad F_f d = 2.97 \text{ J} \blacktriangleleft$$

Kinetic energy of block with embedded bullet immediately after first impact:

$$T'_{AB} = \frac{1}{2}(m_A + m_B)(v')^2 = \frac{1}{2}(3.03 \text{ kg})(4.4554 \text{ m/s})^2 = 30.07 \text{ J}$$

Final kinetic energy of A, B, and C together

$$T''_{ABC} = \frac{1}{2}(m_A + m_B + m_C)(v'')^2 = \frac{1}{2}(33.03 \text{ kg})(0.4087 \text{ m/s})^2 = 2.76 \text{ J}$$

Loss due to friction and stopping impact:
$$T'_{AB} - T''_{ABC} = 30.07 - 2.76 = 27.31 \text{ J}$$

Since $27.31 \text{ J} \geq 2.97 \text{ J}$, the block slides 0.5 m relative to the carrier as assumed above.

PROBLEM 14.31 (Continued)

Impact loss due to AB impacting the carrier:

$$27.31 - 2.97 = 24.34 \qquad \text{Loss} = 24.3 \text{ J} \blacktriangleleft$$

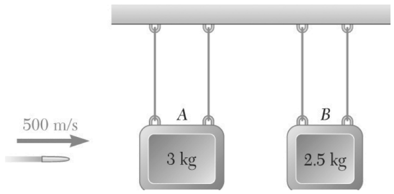
Initial kinetic energy of system ABC.

$$T_0 = \frac{1}{2} m_A v_0^2 = \frac{1}{2}(0.030 \text{ kg})(450 \text{ m/s})^2 = 3037.5 \text{ J}$$

Impact loss at first impact:

$$T_0 - T'_{AB} = 3037.5 - 30.07 \qquad \text{Loss} = 3007 \text{ J} \blacktriangleleft$$

PROBLEM 14.32



In Problem 14.4, determine the energy lost as the bullet (*a*) passes through block *A*, (*b*) becomes embedded in block *B*.

SOLUTION

The masses are *m* for the bullet and m_A and m_B for the blocks.

The bullet passes through block *A* and embeds in block *B*. Momentum is conserved.

Initial momentum: $mv_0 + m_A(0) + m_B(0) = mv_0$

Final momentum: $mv_B + m_A v_A + m_B v_B$

Equating, $mv_0 = mv_B + m_A v_A + m_B v_B$

$$m = \frac{m_A v_A + m_B v_B}{v_0 - v_B} = \frac{(6)(5) + (4.95)(9)}{1500 - 9} = 0.0500 \text{ lb}$$

The bullet passes through block *A*. Momentum is conserved.

Initial momentum: $mv_0 + m_A(0) = mv_0$

Final momentum: $mv_1 + m_A v_A$

Equating, $mv_0 = mv_1 + m_A v_A$

$$v_1 = \frac{mv_0 - m_A v_A}{m} = \frac{(0.0500)(1500) - (6)(5)}{0.0500} = 900 \text{ ft/s}$$

The masses are:

$$m = \frac{0.05}{32.2} = 1.5528 \times 10^{-3} \text{ lb} \cdot \text{s}^2/\text{ft}$$

$$m_A = \frac{6}{32.2} = 0.18633 \text{ lb} \cdot \text{s}^2/\text{ft}$$

$$m_B = \frac{4.95}{32.2} = 0.153727 \text{ lb} \cdot \text{s}^2/\text{ft}$$

(*a*) Bullet passes through block *A*. Kinetic energies:

Before: $$T_0 = \frac{1}{2}mv_0^2 = \frac{1}{2}(1.5528 \times 10^{-3})(1500)^2 = 1746.9 \text{ ft} \cdot \text{lb}$$

After: $$T_1 = \frac{1}{2}mv_1^2 + \frac{1}{2}m_A v_A^2 = \frac{1}{2}(1.5528 \times 10^{-3})(900)^2 + \frac{1}{2}(0.18633)(5)^2 = 631.2 \text{ ft} \cdot \text{lb}$$

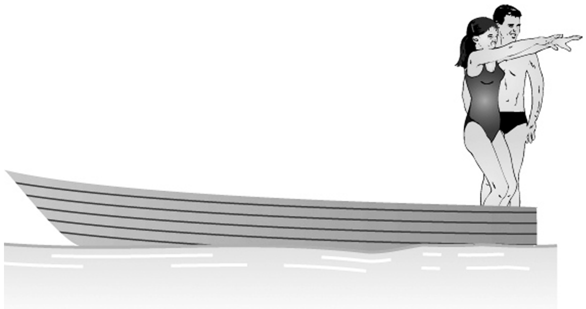
Lost: $T_0 - T_1 = 1746.9 - 631.2 = 1115.7 \text{ ft} \cdot \text{lb}$ energy lost $= 1116 \text{ ft} \cdot \text{lb}$ ◀

(*b*) Bullet becomes embedded in block *B*. Kinetic energies:

Before: $$T_2 = \frac{1}{2}mv_1^2 = \frac{1}{2}(1.5528 \times 10^{-3})(900)^2 = 628.9 \text{ ft} \cdot \text{lb}$$

After: $$T_3 = \frac{1}{2}(m + m_B)v_B^2 = \frac{1}{2}(0.15528)(9)^2 = 6.29 \text{ ft} \cdot \text{lb}$$

Lost: $T_2 - T_3 = 628.9 - 6.29 = 622.6 \text{ ft} \cdot \text{lb}$ energy lost $= 623 \text{ ft} \cdot \text{lb}$ ◀

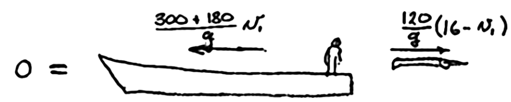


PROBLEM 14.33

In Problem 14.6, determine the work done by the woman and by the man as each dives from the boat, assuming that the woman dives first.

SOLUTION

Woman dives first.



Conservation of momentum:

$$\frac{120}{g}(16-v_1)-\frac{300+180}{g}v_1=0$$

$$v_1=\frac{(120)(16)}{600}=3.20\text{ ft/s}\ \longleftarrow$$

$$16-v_1=12.80\text{ ft/s}\ \longrightarrow$$

Kinetic energy before dive: $\quad T_0=0$

Kinetic energy after dive:

$$T_1=\frac{1}{2}\frac{300+180}{32.2}(3.20)^2+\frac{1}{2}\frac{120}{32.2}(12.80)^2$$
$$=381.61\text{ ft}\cdot\text{lb}$$

Work of woman: $\quad T_1-T_0=381.61\text{ ft}\cdot\text{lb}$ $\qquad T_1-T_0=382\text{ ft}\cdot\text{lb}$ ◀

Man dives next. Conservation of momentum:

$$-\frac{300+180}{g}v_1=-\frac{300}{g}v_2+\frac{180}{g}(16-v_2)$$

$$v_2=\frac{480v_1+(180)(16)}{480}=9.20\text{ ft/s}\ \longleftarrow$$

$$16-9.20=6.80\text{ ft/s}\ \longrightarrow$$

PROBLEM 14.33 (Continued)

Kinetic energy before dive:

$$T_1' = \frac{1}{2}\frac{300+180}{32.2}(3.20)^2$$
$$= 76.323 \text{ ft} \cdot \text{lb}$$

Kinetic energy after dive:

$$T_2' = \frac{1}{2}\frac{300}{32.2}(9.20)^2 + \frac{1}{2}\frac{180}{32.2}(6.80)^2$$
$$= 523.53 \text{ ft} \cdot \text{lb}$$

Work of man: $T_2' - T_1' = 447.2 \text{ ft} \cdot \text{lb}$ $\quad T_2' - T_1' = 447 \text{ ft} \cdot \text{lb}$ ◀

PROBLEM 14.34

Determine the energy lost as a result of the series of collisions described in Problem 14.8.

PROBLEM 14.8 Packages in an automobile parts supply house are transported to the loading dock by pushing them along on a roller track with very little friction. At the instant shown, packages *B* and *C* are at rest and package *A* has a velocity of 2 m/s. Knowing that the coefficient of restitution between the packages is 0.3, determine (*a*) the velocity of package *C* after *A* hits *B* and *B* hits *C*, (*b*) the velocity of *A* after it hits *B* for the second time.

SOLUTION

From the solution to Problem 14.8

$$v_A = 2 \text{ m/s}, \quad v_B = v_C = 0,$$
$$v_A' = 1.133 \text{ m/s}, \quad v_B' = 1.733 \text{ m/s}, \quad v_B'' = 0.382 \text{ m/s}$$
$$v_B' = 0.901 \text{ m/s} \quad v_A'' = 0.807 \text{ m/s}, \quad v_B'' = 1.033 \text{ m/s}$$
$$m_A = 8 \text{ kg}, \quad m_B = 4 \text{ kg}, \quad m_C = 6 \text{ kg}$$

A hits *B*:

$$T_1 = \frac{1}{2}m_A v_A^2 = \frac{1}{2}(8 \text{ kg})(2 \text{ m/s})^2 = 16 \text{ J}$$
$$T_2 = \frac{1}{2}m_A(v_A')^2 + \frac{1}{2}m_B(v_B')^2$$
$$T = \frac{1}{2}(8 \text{ kg})(1.133 \text{ m/s})^2 + \frac{1}{2}(4 \text{ kg})(1.733 \text{ m/s})^2 = 11.14 \text{ J}$$

Loss $= T_1 - T_2$: Loss $= 4.86$ J ◀

B hits *C*:

$$T_3 = \frac{1}{2}m_B(v_B')^2 = \frac{1}{2}(4 \text{ kg})(1.733)^2 = 6.007 \text{ J}$$
$$T_4 = \frac{1}{2}m_B(v_B'')^2 + \frac{1}{2}m_C(v_C')^2$$
$$= \frac{1}{2}(4 \text{ kg})(0.382 \text{ m/s})^2 + \frac{1}{2}(6 \text{ kg})(0.901 \text{ m/s})^2 = 2.727 \text{ J}$$

Loss $= T_3 - T_4$: Loss $= 3.28$ J ◀

A hits *B* again:

$$T_5 = \frac{1}{2}m_A(v_A')^2 + \frac{1}{2}m_B(v_B'')^2$$
$$= \frac{1}{2}(8 \text{ kg})(1.33 \text{ m/s})^2 + \frac{1}{2}(4 \text{ kg})(0.382)^2 = 5.427 \text{ J}$$
$$T_6 = \frac{1}{2}m_A(v_A'')^2 + \frac{1}{2}m_B(v_B''')^2$$
$$= \frac{1}{2}(8 \text{ kg})(0.807 \text{ m/s})^2 + \frac{1}{2}(4 \text{ kg})(1.033 \text{ m/s})^2 = 4.739 \text{ J}$$

Loss $= T_5 - T_6$: Loss $= 0.688$ J

PROBLEM 14.35

Two automobiles A and B, of mass m_A and m_B, respectively, are traveling in opposite directions when they collide head on. The impact is assumed perfectly plastic, and it is further assumed that the energy absorbed by each automobile is equal to its loss of kinetic energy with respect to a moving frame of reference attached to the mass center of the two-vehicle system. Denoting by E_A and E_B, respectively, the energy absorbed by automobile A and by automobile B, (a) show that $E_A/E_B = m_B/m_A$, that is, the amount of energy absorbed by each vehicle is inversely proportional to its mass, (b) compute E_A and E_B, knowing that $m_A = 1600$ kg and $m_B = 900$ kg and that the speeds of A and B are, respectively, 90 km/h and 60 km/h.

SOLUTION

Velocity of mass center:

$$(m_A + m_B)\overline{\mathbf{v}} = m_A \mathbf{v}_A + m_B \mathbf{v}_B$$

$$\overline{\mathbf{v}} = \frac{m_A \mathbf{v}_A + m_B \mathbf{v}_B}{m_A + m_B}$$

Velocities relative to the mass center:

$$\mathbf{v}'_A = \mathbf{v}_A - \overline{\mathbf{v}} = \mathbf{v}_A - \frac{m_A \mathbf{v}_A + m_B \mathbf{v}_B}{m_A + m_B} = \frac{m_B(\mathbf{v}_A + \mathbf{v}_B)}{m_A + m_B}$$

$$\mathbf{v}'_B = \mathbf{v}_B - \overline{\mathbf{v}} = \mathbf{v}_B - \frac{m_A \mathbf{v}_A + m_B \mathbf{v}_B}{m_A + m_B} = \frac{m_A(\mathbf{v}_A + \mathbf{v}_B)}{m_A + m_B}$$

Energies:

$$E_A = \frac{1}{2} m_A \mathbf{v}'_A \cdot \mathbf{v}'_A = \frac{m_A m_B^2 (\mathbf{v}_A + \mathbf{v}_B)\cdot(\mathbf{v}_A + \mathbf{v}_B)}{2(m_A + m_B)^2}$$

$$E_B = \frac{1}{2} m_B \mathbf{v}'_B \cdot \mathbf{v}'_B = \frac{m_A^2 m_B (\mathbf{v}_A + \mathbf{v}_B)\cdot(\mathbf{v}_A + \mathbf{v}_B)}{2(m_A + m_B)^2}$$

(a) Ratio:

$$\frac{E_A}{E_B} = \frac{m_B}{m_A} \blacktriangleleft$$

(b)

$$\mathbf{v}_A = 90 \text{ km/h} = 25 \text{ m/s} \longrightarrow$$

$$\mathbf{v}_B = 60 \text{ km/h} = 16.667 \text{ m/s} \longleftarrow$$

$$\mathbf{v}_A + \mathbf{v}_B = 41.667 \text{ m/s} \longrightarrow$$

$$E_A = \frac{(1600)(900)^2(41.667)^2}{(2)(2500)^2} = 180.0 \times 10^3 \text{ J} \qquad E_A = 180.0 \text{ kJ} \blacktriangleleft$$

$$E_B = \frac{(1600)^2(900)(41.667)^2}{(2)(2500)^2} = 320 \times 10^3 \text{ J} \qquad E_B = 320 \text{ kJ} \blacktriangleleft$$

PROBLEM 14.36

It is assumed that each of the two automobiles involved in the collision described in Problem 14.35 had been designed to safely withstand a test in which it crashed into a solid, immovable wall at the speed v_0. The severity of the collision of Problem 14.35 may then be measured for each vehicle by the ratio of the energy it absorbed in the collision to the energy it absorbed in the test. On that basis, show that the collision described in Problem 14.35 is $(m_A/m_B)^2$ times more severe for automobile B than for automobile A.

SOLUTION

Velocity of mass center:

$$(m_A + m_B)\overline{\mathbf{v}} = m_A\mathbf{v}_A + m_B\mathbf{v}_B$$

$$\overline{\mathbf{v}} = \frac{m_A\mathbf{v}_A + m_B\mathbf{v}_B}{m_A + m_B}$$

Velocities relative to the mass center:

$$\mathbf{v}_A' = \mathbf{v}_A - \overline{\mathbf{v}} = \mathbf{v}_A - \frac{m_A\mathbf{v}_A + m_B\mathbf{v}_B}{m_A + m_B} = \frac{m_B(\mathbf{v}_A + \mathbf{v}_B)}{m_A + m_B}$$

$$\mathbf{v}_B' = \mathbf{v}_B - \overline{\mathbf{v}} = \mathbf{v}_B - \frac{m_A\mathbf{v}_A + m_B\mathbf{v}_B}{m_A + m_B} = \frac{m_A(\mathbf{v}_A + \mathbf{v}_B)}{m_A + m_B}$$

Energies:

$$E_A = \frac{1}{2}m_A\mathbf{v}_A'\cdot\mathbf{v}_A' = \frac{m_A m_B^2(\mathbf{v}_A + \mathbf{v}_B)\cdot(\mathbf{v}_A + \mathbf{v}_B)}{2(m_A + m_B)^2}$$

$$E_B = \frac{1}{2}m_B\mathbf{v}_B'\cdot\mathbf{v}_B' = \frac{m_A^2 m_B(\mathbf{v}_A + \mathbf{v}_B)\cdot(\mathbf{v}_A + \mathbf{v}_B)}{2(m_A + m_B)^2}$$

Energies from tests:

$$(E_A)_0 = \frac{1}{2}m_A v_0^2, \quad (E_B)_0 = \frac{1}{2}m_B v_0^2$$

Severities:

$$S_A = \frac{E_A}{(E_A)_0} = \frac{m_B^2(\mathbf{v}_A + \mathbf{v}_B)\cdot(\mathbf{v}_A + \mathbf{v}_B)}{(m_A + m_B)^2 v_0^2}$$

$$S_B = \frac{E_B}{(E_B)_0} = \frac{m_A^2(\mathbf{v}_A + \mathbf{v}_B)\cdot(\mathbf{v}_A + \mathbf{v}_B)}{(m_A + m_B)^2 v_0^2}$$

Ratio:

$$\frac{S_A}{S_B} = \frac{m_B^2}{m_A^2} \blacktriangleleft$$

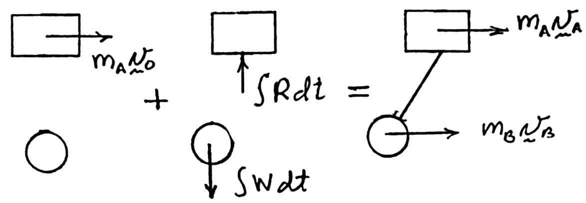
PROBLEM 14.37

Solve Sample Problem 14.4, assuming that cart A is given an initial horizontal velocity $\mathbf{v}_0$ while ball B is at rest.

SOLUTION

(a) Velocity of B at maximum elevation: At maximum elevation, ball B is at rest relative to cart A. $\mathbf{v}_B = \mathbf{v}_A$

Use impulse-momentum principle.



x components:

$$m_A v_0 + 0 = m_A v_A + m_B v_B$$
$$= (m_A + m_B) v_B$$

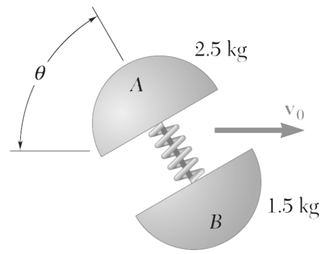
$$v_B = \frac{m_A v_0}{m_A + m_B} \longrightarrow \blacktriangleleft$$

(b) Conservation of energy:

$$T_1 = \frac{1}{2} m_A v_0^2, \quad V_1 = 0$$
$$T_2 = \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2$$
$$= \frac{1}{2}(m_A + m_B) v_B^2$$
$$= \frac{m_A^2 v_0^2}{2(m_A + m_B)}$$
$$V_2 = m_B g h$$
$$T_2 + V_2 = T_1 + V_1$$
$$\frac{m_A^2 v_0^2}{2(m_A + m_B)} + m_B g h = \frac{1}{2} m_A v_0^2$$

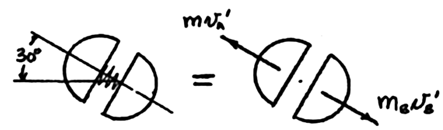
$$h = \frac{1}{2 m_B g}\left[m_A v_0^2 - \frac{m_A^2 v_0^2}{m_A + m_B} \right] \qquad h = \frac{m_A}{m_A + m_B} \frac{v_0^2}{2g} \blacktriangleleft$$

PROBLEM 14.38



Two hemispheres are held together by a cord which maintains a spring under compression (the spring is not attached to the hemispheres). The potential energy of the compressed spring is 120 J and the assembly has an initial velocity $\mathbf{v}_0$ of magnitude $v_0 = 8$ m/s. Knowing that the cord is severed when $\theta = 30°$, causing the hemispheres to fly apart, determine the resulting velocity of each hemisphere.

SOLUTION



Use a frame of reference moving with the mass center.

Conservation of momentum:

$$0 = -m_A v_A' + m_B v_B'$$

$$v_A' = \frac{m_B}{m_A} v_B'$$

Conservation of energy:

$$V = \frac{1}{2} m_A (v_A')^2 + \frac{1}{2} m_B (v_B')^2$$

$$= \frac{1}{2} m_A \left(\frac{m_B}{m_A} v_B' \right)^2 + \frac{1}{2} m_B (v_B')^2$$

$$= \frac{m_B (m_A + m_B)}{2m_A} (v_B')^2$$

$$v_B' = \sqrt{\frac{2m_A V}{m_B (m_A + m_B)}}$$

Data: $m_A = 2.5$ kg $\quad m_B = 1.5$ kg

$$V = 120 \text{ J}$$

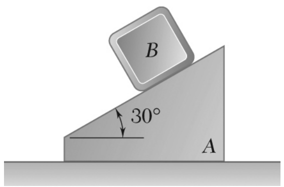
$$v_B' = \sqrt{\frac{(2)(2.5)(120)}{(1.5)(4.0)}} = 10 \qquad v_B' = 10 \text{ m/s} \nwarrow 30°$$

$$v_A' = \frac{1.5}{2.5}(10) = 6 \qquad v_A' = 6 \text{ m/s} \searrow 30°$$

Velocities of A and B.

$$\mathbf{v}_A = [8 \text{ m/s} \longrightarrow] + [6 \text{ m/s} \searrow 30°] \qquad \mathbf{v}_A = 4.11 \text{ m/s} \measuredangle 46.9° \blacktriangleleft$$

$$\mathbf{v}_B = [8 \text{ m/s} \longrightarrow] + [10 \text{ m/s} \nwarrow 30°] \qquad \mathbf{v}_B = 17.39 \text{ m/s} \nwarrow 16.7° \blacktriangleleft$$

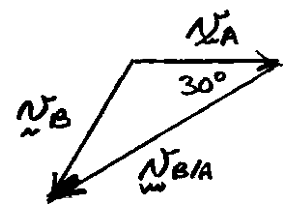


PROBLEM 14.39

A 15-lb block B starts from rest and slides on the 25-lb wedge A, which is supported by a horizontal surface. Neglecting friction, determine (*a*) the velocity of B relative to A after it has slid 3 ft down the inclined surface of the wedge, (*b*) the corresponding velocity of A.

SOLUTION

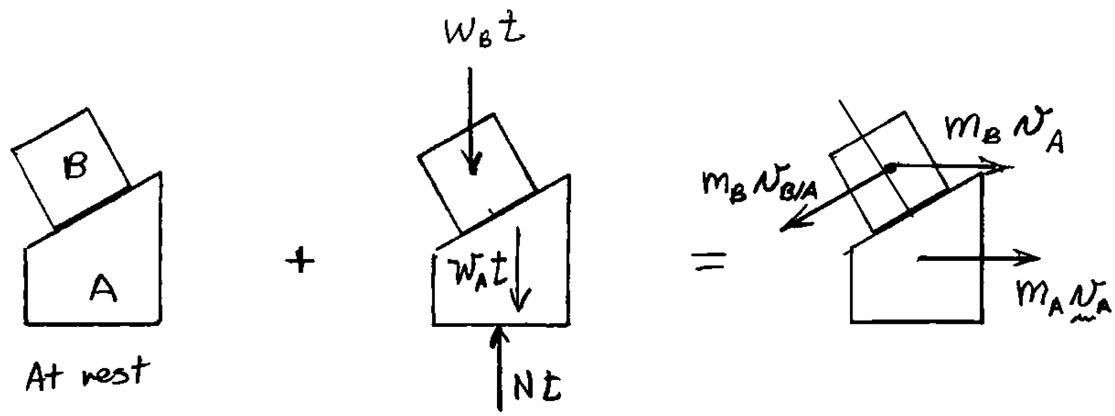
Kinematics: $\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$



Law of cosines: $$v_B^2 = v_A^2 + v_{B/A}^2 - 2v_A v_{B/A}\cos 30° \quad (1)$$

Principle of impulse and momentum:

$$\Sigma m\mathbf{v}_0 + \Sigma \mathbf{F}t = \Sigma m\mathbf{v}$$



Components $\longrightarrow$: $$0 + 0 = m_A v_A + m_B(v_A - v_{B/A}\cos 30°)$$

$$v_A = \frac{m_B v_{B/A}\cos 30°}{m_A + m_B} = \frac{15\cos 30°}{25+15} v_{B/A}$$
$$= 0.32476\, v_{B/A}$$

From Eq. (1) $$v_B^2 = (0.32476)^2 v_{B/A}^2 + v_{B/A}^2 - (2)(0.32476)\cos 30° v_{B/A}^2$$
$$= 0.54297\, v_{B/A}^2$$

Principle of conservation of energy: $$T_0 + V_0 = T_1 + V_1$$

$$0 + 0 = \frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2 - W_B d\sin 30°$$

$$\frac{1}{2}\frac{W_A}{g}(0.32476 v_{B/A})^2 + \frac{1}{2}\frac{W_B}{g}(0.54297)\, v_{B/A}^2 = W_B d\sin 30°$$

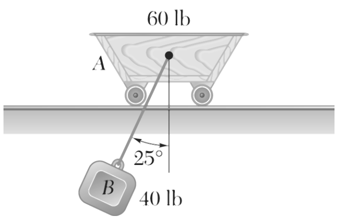
PROBLEM 14.39 (Continued)

$$\left[\frac{1}{2}\frac{25}{32.2}(0.32476)^2+\frac{1}{2}\frac{15}{32.2}(0.54297)\right]v_{B/A}^2=(15)(3)\sin 30°$$

$$0.16741 v_{B/A}^2 = 22.5$$

(*a*) $\mathbf{v}_{B/A} = 11.59 \text{ ft/s} \measuredangle 30°$ ◀

(b) $v_A = (0.32476)(11.59)$ $\mathbf{v}_A = 3.76 \text{ ft/s} \longrightarrow$ ◀



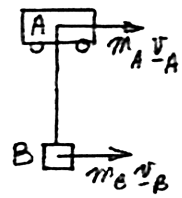
PROBLEM 14.40

A 40-lb block B is suspended from a 6-ft cord attached to a 60-lb cart A, which may roll freely on a frictionless, horizontal track. If the system is released from rest in the position shown, determine the velocities of A and B as B passes directly under A.

SOLUTION

Conservation of linear momentum:

Since block and cart are initially at rest,



$$\mathbf{L}_0 = 0$$

Thus, as B passes under A,

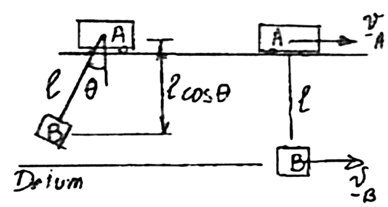
$$\mathbf{L} = m_A \mathbf{v}_A + m_B \mathbf{v}_B = 0$$

$$\xrightarrow{+} \quad m_A v_A + m_B v_B = 0$$

$$v_A = -\frac{m_B}{m_A} v_B \tag{1}$$

Conservation of energy:

Initially,



$$T_0 = 0$$

$$V_0 = m_B g l(1 - \cos\theta)$$

As B passes under A,

$$T = \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2$$

$$V = 0$$

Thus,

$$T_0 + V_0 = T + V: \quad m_B g l(1 - \cos\theta) = \frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2$$

Substituting for v_A from (1) and multiplying by 2:

$$2m_B g l(1 - \cos\theta) = m_A \left(\frac{m_B^2}{m_A^2} v_B^2 \right) + m_B v_B^2$$

$$= \left(\frac{m_B^2}{m_A} + m_B \right) v_B^2 = m_B \frac{m_B + m_A}{m_A} v_B^2$$

$$v_B = \sqrt{\frac{2m_A}{m_A + m_B} g l(1 - \cos\theta)} \tag{2}$$

PROBLEM 14.40 (Continued)

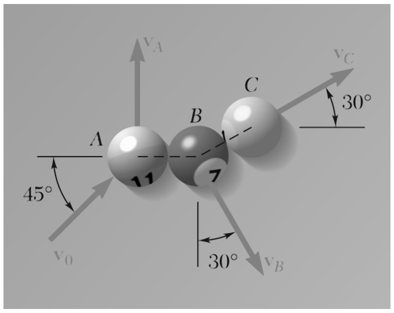
Given data: $w_A = 60 \text{ lb} \quad w_B = 40 \text{ lb}, \quad l = 6 \text{ ft}$

$$g = 32.2 \text{ ft/s}^2 \quad \theta = 25°$$

$$\frac{2m_A}{m_A + m_B} = \frac{2w_A}{w_A + w_B} = \frac{(2)(60)}{60+40} = 1.2$$

From Eq. (2), $v_B = \sqrt{(1.2)(32.2)(6)(1-\cos 25°)}$ $\qquad \mathbf{v}_B = 4.66 \text{ ft/s} \longrightarrow$ ◀

From Eq. (1), $v_A = -\frac{w_B}{w_A} v_B = -\frac{40}{60}(4.66)$ $\qquad \mathbf{v}_A = 3.11 \text{ ft/s} \longleftarrow$ ◀



PROBLEM 14.41

In a game of pool, ball A is moving with a velocity $\mathbf{v}_0$ of magnitude $v_0 = 15$ ft/s when it strikes balls B and C, which are at rest and aligned as shown. Knowing that after the collision the three balls move in the directions indicated and assuming frictionless surfaces and perfectly elastic impact (i.e., conservation of energy), determine the magnitudes of the velocities $\mathbf{v}_A$, $\mathbf{v}_B$, and $\mathbf{v}_C$.

SOLUTION

Velocity vectors: $\mathbf{v}_0 = v_0(\cos 45°\,\mathbf{i} + \sin 45°\,\mathbf{j})$ $\quad v_0 = 15$ ft/s

$$\mathbf{v}_A = v_A\mathbf{j}$$
$$\mathbf{v}_B = v_B(\sin 30°\,\mathbf{i} - \cos 30°\,\mathbf{j})$$
$$\mathbf{v}_C = v_C(\cos 30°\,\mathbf{i} + \sin 30°\,\mathbf{j})$$

Conservation of momentum:

$$m\mathbf{v}_0 = m\mathbf{v}_A + m\mathbf{v}_B + m\mathbf{v}_C$$

Divide by m and resolve into components.

$$\mathbf{i}:\quad v_0\cos 45° = v_B\sin 30° + v_C\cos 30°$$

$$\mathbf{j}:\quad v_0\sin 45° = v_A - v_B\cos 30° + v_C\sin 30°$$

Solving for v_B and v_C,

$$v_B = -0.25882v_0 + 0.86603v_A$$
$$v_C = 0.96593v_0 - 0.5v_A$$

Conservation of energy: $\dfrac{1}{2}mv_0^2 = \dfrac{1}{2}mv_A^2 + \dfrac{1}{2}mv_B^2 + \dfrac{1}{2}mv_C^2$

Divide by $\frac{1}{2}m$ and substitute for v_B and v_C.

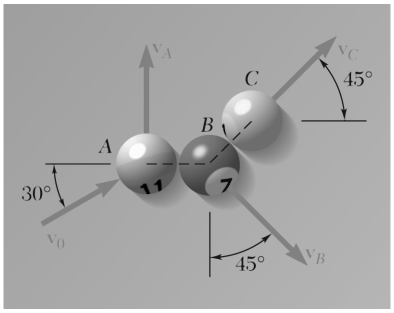
$$\begin{aligned} v_0^2 &= v_A^2 + (-0.25882\,v_0 + 0.86603v_A)^2 \\ &\quad + (0.96593\,v_0 - 0.5v_A)^2 \\ &= 2v_A^2 + v_0^2 - 1.41422\,v_0v_A \end{aligned}$$

$v_A = 0.70711\,v_0 = 10.61$ ft/s $\qquad v_A = 10.61$ ft/s ◀

$v_B = 0.35355\,v_0 = 5.30$ ft/s $\qquad v_B = 5.30$ ft/s ◀

$v_C = 0.61237\,v_0 = 9.19$ ft/s $\qquad v_C = 9.19$ ft/s ◀

PROBLEM 14.42



In a game of pool, ball A is moving with a velocity $\mathbf{v}_0$ of magnitude $v_0 = 15$ ft/s when it strikes balls B and C, which are at rest and aligned as shown. Knowing that after the collision the three balls move in the directions indicated and assuming frictionless surfaces and perfectly elastic impact (i.e., conservation of energy), determine the magnitudes of the velocities $\mathbf{v}_A$, $\mathbf{v}_B$, and $\mathbf{v}_C$.

SOLUTION

Velocity vectors:

$$\mathbf{v}_0 = v_0(\cos 30° \mathbf{i} + \sin 30° \mathbf{j}) \qquad v_0 = 15 \text{ ft/s}$$

$$\mathbf{v}_A = v_A \mathbf{j}$$

$$\mathbf{v}_B = v_B(\sin 45° \mathbf{i} - \cos 45° \mathbf{j})$$

$$\mathbf{v}_C = v_C(\cos 45° \mathbf{i} + \sin 45° \mathbf{j})$$

Conservation of momentum:

$$m\mathbf{v}_0 = m\mathbf{v}_A + m\mathbf{v}_B + m\mathbf{v}_C$$

Divide by m and resolve into components.

$$\mathbf{i}: \quad v_0 \cos 30° = v_B \sin 45° + v_C \cos 45°$$

$$\mathbf{j}: \quad v_0 \sin 30° = v_A - v_B \cos 45° + v_C \sin 45°$$

Solving for v_B and v_C,

$$v_B = 0.25882 v_0 + 0.70711 v_A$$

$$v_C = 0.96593 v_0 - 0.70711 v_A$$

Conservation of energy: $$\frac{1}{2}mv_0^2 = \frac{1}{2}mv_A^2 + \frac{1}{2}mv_B^2 + \frac{1}{2}mv_C^2$$

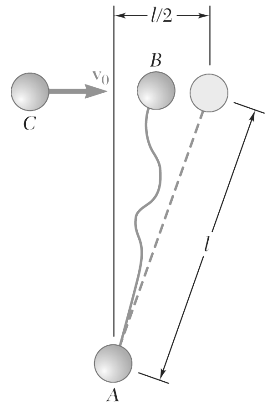
Divide by m and substitute for v_B and v_C.

$$\begin{aligned} v_0^2 &= v_A^2 + (0.25882 v_0 + 0.70711 v_A)^2 \\ &\quad + (0.96593 v_0 - 0.70711 v_A)^2 \\ &= v_0^2 - v_0 v_A + 2v_A^2 \end{aligned}$$

$$v_A = 0.5 v_0 = 7.500 \text{ ft/s} \qquad v_A = 7.50 \text{ ft/s} \blacktriangleleft$$

$$v_B = 0.61237 v_0 = 9.1856 \text{ ft/s} \qquad v_B = 9.19 \text{ ft/s} \blacktriangleleft$$

$$v_C = 0.61237 v_0 = 9.1856 \text{ ft/s} \qquad v_C = 9.19 \text{ ft/s} \blacktriangleleft$$



PROBLEM 14.43

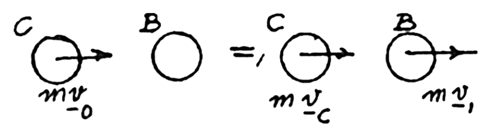
Three spheres, each of mass m, can slide freely on a frictionless, horizontal surface. Spheres A and B are attached to an inextensible, inelastic cord of length l and are at rest in the position shown when sphere B is struck squarely by sphere C, which is moving to the right with a velocity $\mathbf{v}_0$. Knowing that the cord is slack when sphere B is struck by sphere C and assuming perfectly elastic impact between B and C, determine (a) the velocity of each sphere immediately after the cord becomes taut, (b) the fraction of the initial kinetic energy of the system which is dissipated when the cord becomes taut.

SOLUTION

(a) Determination of velocities.

Impact of C and B.

Conservation of momentum:



$$m\mathbf{v}_0 = m\mathbf{v}_C + m\mathbf{v}_1$$

$$v_C + v_1 = v_0 \qquad (1)$$

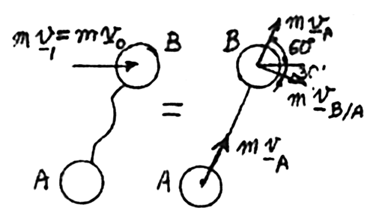
Conservation of energy (perfectly elastic impact):

$$\frac{1}{2}mv_0^2 = \frac{1}{2}mv_C^2 + \frac{1}{2}mv_1^2 \quad v_C^2 + v_1^2 = v_0^2 \qquad (2)$$

Square Eq. (1): $\quad v_C^2 + 2v_C v_1 + v_1^2 = v_0^2$

Subtract Eq. (2): $\quad 2v_C v_1 = 0$

$v_1 = 0$ corresponds to initial conditions and should be eliminated. Therefore, $\quad \mathbf{v}_C = 0$ ◀



From Eq. (1): $\quad v_1 = v_0$

Cord AB becomes taut:

Because cord is inextensible, component of $\mathbf{v}_B$ along AB must be equal to $\mathbf{v}_A$.

Conservation of momentum:

$$m\mathbf{v}_0 = 2m\mathbf{v}_A + m\mathbf{v}_{B/A}$$

$+\uparrow$ y comp: $\quad 0 = 2mv_A \sin 60° - mv_{B/A} \sin 30°$

$$v_{B/A} = 2\sqrt{3}v_A \qquad (3)$$

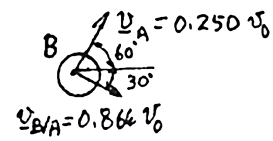
$\xrightarrow{+}$ x comp: $\quad mv_0 = 2mv_A \cos 60° + mv_{B/A} \cos 30°$

Dividing by m and substituting for $v_{B/A}$ from Eq. (3):

$$v_0 = 2v_A(0.5) + (2\sqrt{3}v_A)(\sqrt{3/2})$$

$$v_0 = 4v_A \qquad v_A = 0.250v_0 \qquad\qquad \mathbf{v}_A = 0.250v_0 \angle 60^\circ \blacktriangleleft$$

Carrying into Eq. (3): $\quad v_{B/A} = 2\sqrt{3}(0.250v_0) = 0.866v_0$



Thus, $\quad \mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$

$$= 0.250v_0 \angle 60^\circ + 0.866v_0 \searrow 30^\circ$$

$$\mathbf{v}_B = (0.250v_0 \cos 60^\circ + 0.866v_0 \cos 30^\circ)\mathbf{i} + (0.250v_0 \sin 60^\circ - 0.866v_0 \sin 30^\circ)\mathbf{j}$$

$$\mathbf{v}_B = 0.875v_0\mathbf{i} - 0.2165\mathbf{j}$$

$$\mathbf{v}_B = 0.90139v_0 \searrow 13.90^\circ \qquad\qquad \mathbf{v}_B = 0.901v_0 \searrow 13.9^\circ \blacktriangleleft$$

(*b*) Fraction of kinetic energy lost:

$$T_0 = \frac{1}{2}mv_0^2$$

$$T_{\text{final}} = \frac{1}{2}mv_A^2 + \frac{1}{2}mv_B^2 + \frac{1}{2}mv_C^2$$

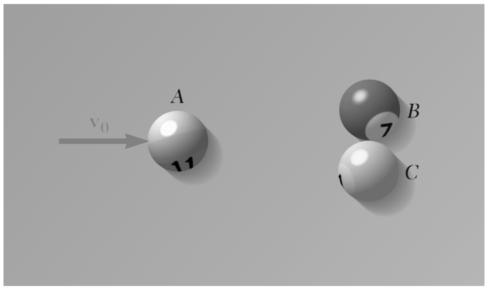
$$= \frac{1}{2}m(0.250v_0)^2 + \frac{1}{2}(0.90139v_0)^2 + \frac{1}{2}m(0)$$

$$= \frac{1}{2}m(0.875)v_0^2$$

$$\text{Kinetic energy lost } = T_0 - T_{\text{final}} = \frac{1}{2}m(1-0.875)v_0^2 = \frac{1}{2}\cdot\frac{1}{8}mv_0^2$$

$$\text{Fraction of kinetic energy lost} = \frac{1}{8} \blacktriangleleft$$

PROBLEM 14.44

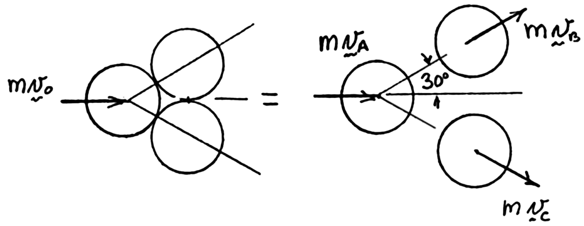


In a game of pool, ball A is moving with the velocity $\mathbf{v}_0 = v_0\mathbf{i}$ when it strikes balls B and C, which are at rest side by side. Assuming frictionless surfaces and perfectly elastic impact (i.e., conservation of energy), determine the final velocity of each ball, assuming that the path of A is (a) perfectly centered and that A strikes B and C simultaneously, (b) not perfectly centered and that A strikes B slightly before it strikes C.

SOLUTION

(a) A strikes B and C simultaneously:

During the impact, the contact impulses make $30°$ angles with the velocity $\mathbf{v}_0$.



Thus, $$\mathbf{v}_B = v_B(\cos 30°\mathbf{i} + \sin 30°\mathbf{j})$$
$$\mathbf{v}_C = v_C(\cos 30°\mathbf{i} - \sin 30°\mathbf{j})$$

By symmetry, $$\mathbf{v}_A = v_A\mathbf{i}$$

Conservation of momentum: $$m\mathbf{v}_0 = m\mathbf{v}_A + m\mathbf{v}_B + m\mathbf{v}_C$$

y component: $$0 = 0 + mv_B \sin 30° - mv_C \sin 30° \qquad v_C = v_B$$

x component: $$mv_0 = mv_A + mv_B \cos 30° + mv_C \cos 30°$$

$$v_B + v_C = \frac{v_0 - v_A}{\cos 30°} = \frac{2}{\sqrt{3}}(v_0 - v_A)$$

$$v_B = v_C = \frac{v_0 - v_A}{\sqrt{3}}$$

Conservation of energy: $$\frac{1}{2}mv_0^2 = \frac{1}{2}mv_A^2 + \frac{1}{2}mv_B^2 + \frac{1}{2}mv_C^2$$

$$v_0^2 = v_A^2 + \frac{2}{3}(v_0 - v_A)^2$$

$$v_0^2 - v_A^2 = (v_0 - v_A)(v_0 + v_A) = \frac{2}{3}(v_0 - v_A)^2$$

$$v_0 + v_A = \frac{2}{3}(v_0 - v_A) \qquad \frac{1}{3}v_0 = -\frac{5}{3}v_A \qquad v_A = -\frac{1}{5}v_0$$

$$v_B = v_C = \frac{6}{5\sqrt{3}}v_0 = \frac{2\sqrt{3}}{5}v_0$$

$\mathbf{v}_A = 0.200v_0 \longleftarrow$ ◀

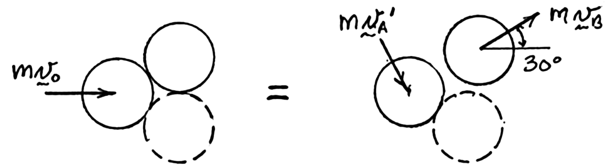
$\mathbf{v}_B = 0.693v_0 \measuredangle\, 30°$ ◀

$\mathbf{v}_C = 0.693v_0 \searrow 30°$ ◀

(*b*) *A* strikes *B* before it strikes *C*:

First impact: *A* strikes *B*.

During the impact, the contact impulse makes a $30°$ angle with the velocity $\mathbf{v}_0$.



Thus, $\mathbf{v}_B = v_B(\cos 30°\mathbf{i} + \sin 30°\mathbf{j})$

Conservation of momentum: $m\mathbf{v}_0 = m\mathbf{v}_A + m\mathbf{v}_B$

y component: $0 = m(v_A')_y + mv_B \sin 30° \qquad (v_A')_y = -v_B \sin 30°$

x component: $v_0 = m(v_A')_x + mv_B \cos 30° \qquad (v_A')_x = v_0 - v_B \cos 30°$

Conservation of energy:

$$\frac{1}{2}mv_0^2 = \frac{1}{2}m(v_A')_x^2 + \frac{1}{2}m(v_A')_y^2 + \frac{1}{2}mv_B^2$$

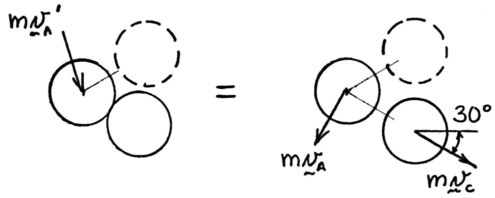
$$= \frac{1}{2}m(v_0 - v_B\cos 30°)^2 + \frac{1}{2}(v_B \sin 30°)^2 + \frac{1}{2}v_B^2$$

$$= \frac{1}{2}m\left(v_0^2 - 2v_0v_B + v_B^2\cos^2 30° + v_B^2 \sin^2 30° + v_B^2\right)$$

$$v_B = v_0 \cos 30° = \frac{\sqrt{3}}{2}v_0, \quad (v_A')_x = v_0 \sin^2 30° = \frac{1}{4}v_0,$$

$$(v_A')_y = -v_0 \cos 30° \sin 30° = -\frac{\sqrt{3}}{4}v_0$$

Second impact: A strikes C.
During the impact, the contact impulse makes a $30°$ angle with the velocity $\mathbf{v}_0$.



Thus, $$\mathbf{v}_C = v_C(\cos 30°\mathbf{i} - \sin 30°\mathbf{j})$$

Conservation of momentum: $$m\mathbf{v}_A' = m\mathbf{v}_A + m\mathbf{v}_C$$

x component: $$m(v_A')_x = m(v_A)_x + mv_C \cos 30°,$$

$$(v_A)_x = (v_A')_x - v_C \cos 30° = \frac{1}{4}v_0 - v_C \cos 30°$$

y component: $$m(v_A')_y = m(v_A)_y - mv_C \sin 30°$$

$$(v_A)_y = (v_A')_y + v_C \sin 30° = -\frac{\sqrt{3}}{4}v_0 + v_C \sin 30°$$

Conservation of energy:

$$\frac{1}{2}m(v_A')_x^2 + \frac{1}{2}m(v_A')_y^2 = \frac{1}{2}m(v_A)_x^2 + \frac{1}{2}m(v_A)_y^2 + \frac{1}{2}mv_C^2$$

$$\frac{1}{2}m\left[\frac{1}{16}v_0^2 + \frac{3}{16}v_0^2\right] = \frac{1}{2}m\left[\left(\frac{1}{4}v_0 - v_C\cos 30°\right)^2 + \left(-\frac{\sqrt{3}}{4}v_0 + v_C \sin 30°\right)^2 + v_C^2\right]$$

$$= \frac{1}{2}m\left[\frac{1}{16}v_0^2 - \frac{1}{2}v_0v_C\cos 30° + v_C^2\cos^2 30° + \frac{3}{16}v_0^2 - \frac{\sqrt{3}}{2}v_0v_C\sin 30° + v_C^2\sin^2 30° + v_C^2\right]$$

$$0 = -v_0v_C\left(\frac{1}{2}\cos 30° + \frac{\sqrt{3}}{2}\sin 30°\right) + 2v_C^2$$

$$v_C = v_0\left(\frac{1}{4}\cos 30° + \frac{\sqrt{3}}{4}\sin 30°\right) = \frac{\sqrt{3}}{4}v_0$$

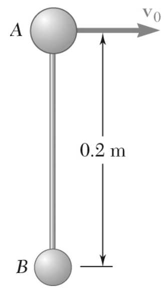
$$(v_A)_x = \frac{1}{4}v_0 - \frac{\sqrt{3}}{4}v_0\cos 30° = -\frac{1}{8}v_0$$

$$(v_A)_y = -\frac{\sqrt{3}}{4}v_0 + \frac{\sqrt{3}}{4}v_0 \sin 30° = -\frac{\sqrt{3}}{8}v_0$$

$\mathbf{v}_A = 0.250v_0 \measuredangle 60°$ ◀

$\mathbf{v}_B = 0.866v_0 \measuredangle 30°$ ◀

$\mathbf{v}_C = 0.433v_0 \measuredangle 30°$ ◀



PROBLEM 14.45

Two small spheres A and B, of mass 2.5 kg and 1 kg, respectively, are connected by a rigid rod of negligible weight. The two spheres are resting on a horizontal, frictionless surface when A is suddenly given the velocity $\mathbf{v}_0 = (3.5 \text{ m/s})\mathbf{i}$. Determine (*a*) the linear momentum of the system and its angular momentum about its mass center G, (*b*) the velocities of A and B after the rod AB has rotated through 180°.

SOLUTION

<u>Position of mass center</u>:

$$\bar{y} = \sum \frac{m_i y_i}{m_i} = \frac{2.5(0) + 1(0.2)}{2.5 + 1} = 0.057143 \text{ m}$$

(*a*) *Linear and angular momentum*:

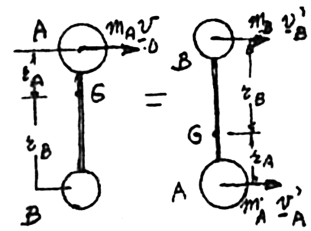
$$L = m_A \mathbf{v}_0 = 2.5 \text{ kg}(3.5 \text{ m/s})\mathbf{i} = (8.75 \text{ kg} \cdot \text{m/s})\mathbf{i}$$

$$\mathbf{L} = (8.75 \text{ kg} \cdot \text{m/s})\mathbf{i} \blacktriangleleft$$

$$\mathbf{H}_G = \overrightarrow{GA} \times m_A \mathbf{v}_0 = (0.05714285 \text{ m})\mathbf{j} \times (8.75 \text{ kg} \cdot \text{m/s})\mathbf{i}$$
$$= -(0.50000 \text{ kg} \cdot \text{m}^2/\text{s})\mathbf{k}$$

$$\mathbf{H}_G = -(0.500 \text{ kg} \cdot \text{m}^2/\text{s})\mathbf{k} \blacktriangleleft$$

(*b*) *Velocities of A and B after 180° rotation*



Conservation of linear momentum:

$$m_A v_0 = m_A v_A' + m_B v_B'$$
$$(2.5)(3.5) = (2.5)v_A' + (1.0)v_B'$$

$$2.55 v_A' + v_B' = 8.75 \qquad (1)$$

Conservation of angular momentum about G':

$$+\circlearrowright: \quad r_A m_A v_0 = -r_A m_A v_A' + r_B m_B v_B'$$

$$r_B = 0.20 - r_A = 0.14286 \text{ m}$$
$$(0.057143)(2.5)(3.5) = -(0.057143)(2.5)v_A' + (0.14286)(1.0)v_B'$$

Dividing by 0.057143: $\quad -2.5 v_A' + \dfrac{0.14286}{0.057143} v_B' = 8.75 \qquad (2)$

Add Eqs. (1) and (2): $\quad 3.5 v_B' = 17.5 \quad v_B' = +5.00 \text{ m/s}$

From Eq. (1): $\quad 2.5 v_A' + (5) = 8.75 \quad v_A' = +1.50 \text{ m/s}$

$$\mathbf{v}_A' = (1.50 \text{ m/s})\mathbf{i}; \quad \mathbf{v}_B' = (5.00 \text{ m/s})\mathbf{i} \blacktriangleleft$$

PROBLEM 14.46

A 900-lb space vehicle traveling with a velocity $\mathbf{v}_0 = (1500\text{ ft/s})\mathbf{k}$ passes through the origin O. Explosive charges then separate the vehicle into three parts A, B, and C, with masses of 150 lb, 300 lb, and 450 lb, respectively. Knowing that shortly thereafter the positions of the three parts are, respectively, A(250, 250, 2250), B (600, 1300, 3200), and C (–475, –950, 1900), where the coordinates are expressed in ft, that the velocity of B is $\mathbf{v}_B = (500\text{ ft/s})\mathbf{i} + (1100\text{ ft/s})\mathbf{j} + (2100\text{ ft/s})\mathbf{k}$, and that the x component of the velocity of C is -400 ft/s, determine the velocity of part A.

SOLUTION

Position vectors (ft):

$$\mathbf{r}_A = 250\mathbf{i} + 250\mathbf{j} + 2250\mathbf{k}$$
$$\mathbf{r}_B = 600\mathbf{i} + 1300\mathbf{j} + 3200\mathbf{k}$$
$$\mathbf{r}_C = -475\mathbf{i} - 950\mathbf{j} + 1900\mathbf{k}$$

Since there are no external forces, linear momentum is conserved.

$$(m_A + m_B + m_C)\mathbf{v}_0 = m_A\mathbf{v}_A + m_B\mathbf{v}_B + m_C\mathbf{v}_C$$

$$\mathbf{v}_A = \frac{m_A + m_B + m_C}{m_A}\mathbf{v}_0 - \frac{m_B}{m_A}\mathbf{v}_B - \frac{m_C}{m_A}\mathbf{v}_C = 6\mathbf{v}_0 - 2\mathbf{v}_B - 3\mathbf{v}_C \qquad (1)$$

$$= (6)(1500\mathbf{k}) - (2)(500\mathbf{i} + 1100\mathbf{j} + 2100\mathbf{k}) - (3)[-400\mathbf{i} + (v_C)_y\mathbf{j} + (v_C)_z\mathbf{k}]$$

$$= -3(v_C)_y\mathbf{j} - 3(v_C)_z\mathbf{k} + 200\mathbf{i} - 2200\mathbf{j} + 4800\mathbf{k}$$

$$(v_A)_x = 200, \quad (v_C)_y = -3(v_C)_y - 2200, \quad (v_A)_z = -3(v_C)_z + 4800$$

Conservation of angular momentum about O:

$$(\mathbf{H}_O)_2 = (\mathbf{H}_O)_1$$

Since the vehicle passes through the origin, $(\mathbf{H}_O)_1 = 0.$

$$(\mathbf{H}_O)_2 = \mathbf{r}_A \times (m_A\mathbf{v}_A) + \mathbf{r}_B \times (m_B\mathbf{v}_B) + \mathbf{r}_C \times (m_C\mathbf{v}_C) = 0$$

Divide by m_A.

$$\begin{aligned}
\mathbf{r}_A \times \mathbf{v}_A + \frac{m_B}{m_A}\mathbf{r}_B \times \mathbf{v}_B + \frac{m_C}{m_A}\mathbf{r}_C \times \mathbf{v}_C &= \mathbf{r}_A \times \mathbf{v}_A + 2\mathbf{r}_B \times \mathbf{v}_B + 3\mathbf{r}_C \times \mathbf{v}_C \\
&= \mathbf{r}_A \times (6\mathbf{v}_0 - 2\mathbf{v}_B - 3\mathbf{v}_C) + 2\mathbf{r}_B \times \mathbf{v}_B + 3\mathbf{r}_C \times \mathbf{v}_C \\
&= 3(\mathbf{r}_C - \mathbf{r}_A) \times \mathbf{v}_C + 6\mathbf{r}_A \times \mathbf{v}_0 + 2(\mathbf{r}_B - \mathbf{r}_A) \times \mathbf{v}_B \\
&= (-2175\mathbf{i} - 3600\mathbf{j} - 1050\mathbf{k}) \times \mathbf{v}_C + (1500\mathbf{i} + 1500\mathbf{j} + 13500\mathbf{k}) \times \mathbf{v}_0 \\
&\quad + (700\mathbf{i} + 2100\mathbf{j} + 1900\mathbf{k}) \times \mathbf{v}_B = 0
\end{aligned}$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -2175 & -3600 & -1050 \\ (v_C)_x & (v_C)_y & (v_C)_z \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1500 & 1500 & 13500 \\ 0 & 0 & 1500 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 700 & 2100 & 1900 \\ 500 & 1100 & 2100 \end{vmatrix} = 0$$

Resolve into components.

i: $1050(v_C)_y - 3600(v_C)_z + 2,250,000 + 2,320,000 = 0$ (2)

j: $2175(v_C)_z - 1050(v_C)_x - 2,250,000 - 520,000 = 0$ (3)

k: $3600(v_C)_x - 2175(v_C)_y + 0 - 280,000 = 0$ (4)

Set $(v_C)_x = -400$ ft/s

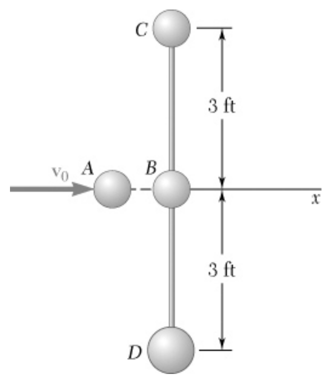
From Eq. (4), $(v_C)_y = -790.80$ ft/s

From Eq. (3), $(v_C)_z = 1080.5$ ft/s

From Eq. (1), $\mathbf{v}_A = (6)(1500\mathbf{k}) - (2)(500\mathbf{i} + 1100\mathbf{j} + 2100\mathbf{k}) - (3)[-400\mathbf{i} + (790.80)\mathbf{j} + (1080.5)\mathbf{k}]$

$= 200\mathbf{i} + 172.4\mathbf{j} + 1558.6\mathbf{k}$

$\mathbf{v}_A = (200 \text{ ft/s})\mathbf{i} + (172 \text{ ft/s})\mathbf{j} + (1560 \text{ ft/s})\mathbf{k}$ ◀

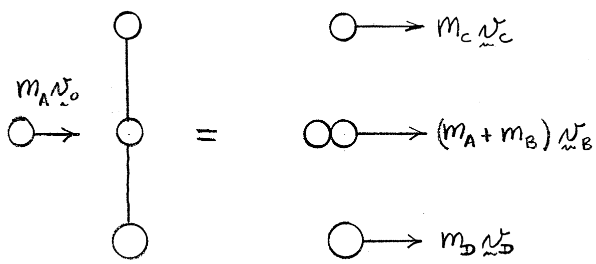


PROBLEM 14.47

Four small disks A, B, C, and D can slide freely on a frictionless horizontal surface. Disks B, C, and D are connected by light rods and are at rest in the position shown when disk B is struck squarely by disk A, which is moving to the right with a velocity $\mathbf{v}_0 = (38.5 \text{ ft/s})\mathbf{i}$. The weights of the disks are $W_A = W_B = W_C = 15$ lb, and $W_D = 30$ lb. Knowing that the velocities of the disks immediately after the impact are $\mathbf{v}_A = \mathbf{v}_B = (8.25 \text{ ft/s})\mathbf{i}$, $\mathbf{v}_C = v_C\,\mathbf{i}$, and $\mathbf{v}_D = v_D\,\mathbf{i}$, determine (*a*) the speeds v_C and v_D, (*b*) the fraction of the initial kinetic energy of the system which is dissipated during the collision.

SOLUTION

There are no external forces. Momentum is conserved.



(*a*) Moments about D +:

$$3m_A v_0 = 6m_C v_C + 3(m_A + m_B)v_B$$

$$v_C = \frac{3m_A}{6m_C}v_0 - \frac{3(m_A + m_B)}{6m_C}v_B = (0.5)(38.5) - (8.25) = 11$$

$v_C = 11.00$ ft/s ◀

Moments about C +:

$$3m_A v_0 = 3(m_A + m_B)v_B + 6m_D v_D$$

$$v_D = \frac{3m_A v_0}{6m_D} - \frac{3(m_A + m_B)}{6m_D}v_B = (0.25)(38.5) - (0.5)(8.25) = 5.5 \text{ ft/s}$$

$v_D = 5.50$ ft/s ◀

(*b*) Initial kinetic energy:

$$T_1 = \frac{1}{2}\frac{W_A}{g}v_0^2 = \frac{1}{2}\frac{15}{32.2}(38.5)^2 = 345.24 \text{ ft}\cdot\text{lb}$$

Final kinetic energy:

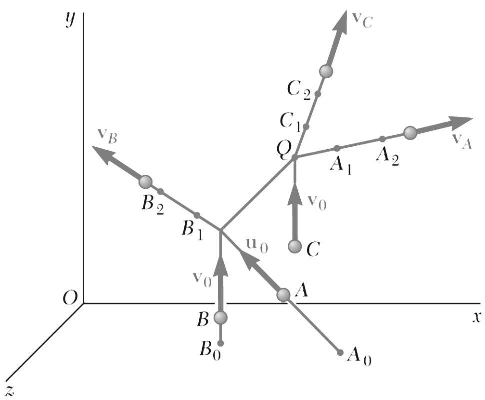
$$T_2 = \frac{1}{2}\frac{W_A + W_B}{g}v_B^2 + \frac{1}{2}\frac{W_C}{g}v_C^2 + \frac{1}{2}\frac{W_D}{g}v_D^2$$

$$= \frac{1}{2}\frac{30}{32.2}(8.25)^2 + \frac{1}{2}\frac{15}{32.2}(11.00)^2 + \frac{1}{2}\frac{30}{32.2}(5.50)^2 = 73.98 \text{ ft}\cdot\text{lb}$$

Energy lost: $345.24 - 73.98 = 271.26 \text{ ft}\cdot\text{lb}$

Fraction of energy lost $= \dfrac{271.26}{345.24} = 0.786$

$\dfrac{(T_1 - T_2)}{T_1} = 0.786$ ◀



PROBLEM 14.48

In the scattering experiment of Problem 14.26, it is known that the alpha particle is projected from $A_0(300, 0, 300)$ and that it collides with the oxygen nucleus C at $Q(240, 200, 100)$, where all coordinates are expressed in millimeters. Determine the coordinates of Point B_0 where the original path of nucleus B intersects the zx plane. (*Hint:* Express that the angular momentum of the three particles about Q is conserved.)

PROBLEM 14.26 In a scattering experiment, an alpha particle A is projected with the velocity $\mathbf{u}_0 = -(600\text{ m/s})\mathbf{i} + (750\text{ m/s})\mathbf{j} - (800\text{ m/s})\mathbf{k}$ into a stream of oxygen nuclei moving with a common velocity $\mathbf{v}_0 = (600\text{ m/s})\mathbf{j}$. After colliding successively with nuclei B and C, particle A is observed to move along the path defined by the Points $A_1(280, 240, 120)$ and $A_2(360, 320, 160)$, while nuclei B and C are observed to move along paths defined, respectively, by $B_1(147, 220, 130)$, $B_2(114, 290, 120)$, and by $C_1(240, 232, 90)$ and $C_2(240, 280, 75)$. All paths are along straight lines and all coordinates are expressed in millimeters. Knowing that the mass of an oxygen nucleus is four times that of an alpha particle, determine the speed of each of the three particles after the collisions.

SOLUTION

Conservation of angular momentum about Q:

$$\overrightarrow{QA}_0 \times (m\mathbf{u}_0) + \overrightarrow{QB}_0 \times (4m\mathbf{v}_0) + \overrightarrow{QC}_0 \times (4m\mathbf{v}_0) = \overrightarrow{QA}_1 \times (m\mathbf{v}_A) + \overrightarrow{QB}_1 \times (4m\mathbf{v}_B) + \overrightarrow{QC}_1 \times (4m\mathbf{v}_C)$$

$$\overrightarrow{QA}_0 \times (m\mathbf{u}_0) + \overrightarrow{QB}_0 \times (4m\mathbf{v}_0) + 0 = 0 + \overrightarrow{QB}_1 \times (4m\mathbf{v}_B) + 0 \qquad (1)$$

where
$$\overrightarrow{QA}_0 = \mathbf{r}_{A_0} - \mathbf{r}_Q = (300\mathbf{i} + 300\mathbf{k}) - (240\mathbf{i} + 200\mathbf{j} + 100\mathbf{k})$$
$$= (60\text{ mm})\mathbf{i} - (200\text{ mm})\mathbf{j} + (200\text{ mm})\mathbf{k}$$
$$\overrightarrow{QB}_0 = (\Delta x)\mathbf{i} + (\Delta y)\mathbf{j} + (\Delta z)\mathbf{k}$$
$$\overrightarrow{QB}_1 = \mathbf{r}_{B_1} - \mathbf{r}_Q = (147\mathbf{i} + 220\mathbf{j} + 130\mathbf{k}) - (240\mathbf{i} + 200\mathbf{j} + 100\mathbf{k})$$
$$-(93\text{ mm})\mathbf{i} + (20\text{ mm})\mathbf{j} + (30\text{ mm})\mathbf{k}$$
$$\mathbf{u}_0 = -(600\text{ m/s})\mathbf{i} + (750\text{ m/s})\mathbf{j} - (800\text{ m/s})\mathbf{k} \qquad \mathbf{v}_0 = (600\text{ m/s})\mathbf{j}$$

and from the solution to Problem 14.26,
$$\mathbf{v}_B = v_B\boldsymbol{\lambda}_B = (716.98)(-0.42290\mathbf{i} + 0.89707\mathbf{j} - 0.12815\mathbf{k})$$
$$= -(303.21\text{ m/s})\mathbf{i} + (643.18\text{ m/s})\mathbf{j} - (91.88\text{ m/s})\mathbf{k}$$

Calculating each term and dividing by *m*,

$$\overrightarrow{QA}_0 \times \mathbf{u}_0 \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 60 & -200 & 200 \\ -600 & 750 & -800 \end{vmatrix} = 10,000\mathbf{i} - 72,000\mathbf{j} - 75,000\mathbf{k}$$

$$\begin{aligned} \overrightarrow{QB}_0 \times (4\mathbf{v}_0) &= [(\Delta x)\mathbf{i} + (\Delta y)\mathbf{j} + (\Delta z)\mathbf{k}] \times (2400\mathbf{j}) \\ &= -2400(\Delta z)\mathbf{i} + 2400(\Delta x)\mathbf{k} \end{aligned}$$

$$\overrightarrow{QB}_1 \times (4\mathbf{v}_B) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -93 & 20 & 30 \\ -1212.84 & 2572.72 & -367.52 \end{vmatrix} = -84,532\mathbf{i} - 70,565\mathbf{j} - 215,006\mathbf{k}$$

Collect terms and resolve into components.

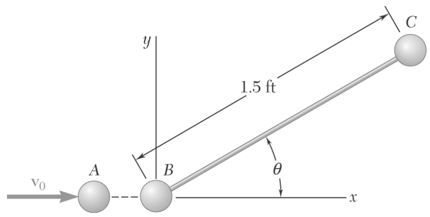
$$\mathbf{i}: \quad 10,000 - 2400(\Delta z) = -84,532 \qquad \Delta z = 39.388 \text{ mm}$$

$$\mathbf{k}: \quad -75,000 + 2400(\Delta x) = -215,006 \qquad \Delta x = -58.336 \text{ mm}$$

Coordinates: $x_{B_0} = x_Q + \Delta x = 240 - 58.336$ $\qquad x_{B_0} = 181.7 \text{ mm}$ ◀

$y_{B_0} = 0$ ◀

$z_{B_0} = z_Q + \Delta z = 100 + 39.388$ $\qquad z_{B_0} = 139.4 \text{ mm}$ ◀



PROBLEM 14.49

Three identical small spheres, each of weight 2 lb, can slide freely on a horizontal frictionless surface. Spheres *B* and *C* are connected by a light rod and are at rest in the position shown when sphere *B* is struck squarely by sphere *A* which is moving to the right with a velocity $\mathbf{v}_0 = (8\text{ ft/s})\mathbf{i}$. Knowing that $\theta = 45°$ and that the velocities of spheres *A* and *B* immediately after the impact are $\mathbf{v}_A = 0$ and $\mathbf{v}_B = (6\text{ ft/s})\mathbf{i} + (\mathbf{v}_B)_y\ \mathbf{j}$, determine $(v_B)_y$ and the velocity of *C* immediately after impact.

SOLUTION

Let m be the mass of one ball.

Conservation of linear momentum: $(\Sigma m\mathbf{v}) = (\Sigma m\mathbf{v})_0$

$$m\mathbf{v}_A + m\mathbf{v}_B + m\mathbf{v}_C = m(\mathbf{v}_A)_0 + (m\mathbf{v}_B)_0 + (m\mathbf{v}_C)_0$$

Dividing by m and applying numerical data,

$$0 + [(6\text{ ft/s})\mathbf{i} + (v_B)_y\mathbf{j}] + [(v_C)_x\mathbf{i} + (v_C)_y\mathbf{j}] = (8\text{ ft/s})\mathbf{i} + 0 + 0$$

Components:

$$x:\quad 6 + (v_C)_x = 8 \qquad\qquad (v_C)_x = 2\text{ ft/s}$$

$$y:\quad (v_B)_y + (v_C)_y = 0 \qquad\qquad (1)$$

Conservation of angular momentum about O:

$$\Sigma[\mathbf{r} \times (m\mathbf{v})] = \Sigma[\mathbf{r} \times (m\mathbf{v}_0)]$$

where $\mathbf{r}_A = 0, \qquad \mathbf{r}_B = 0, \qquad \mathbf{r}_C = (1.5\text{ ft})(\cos 45°\mathbf{i} + \sin 45°\mathbf{j})$

$$(1.5)(\cos 45°\mathbf{i} + \sin 45°\mathbf{j}) \times [m(v_C)_x\mathbf{i} + m(v_C)_y\mathbf{j}] = 0$$

Since their cross product is zero, the two vectors are parallel.

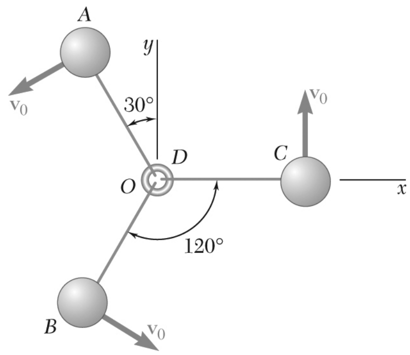
$$(v_C)_y = (v_C)_x \tan 45° = 2\tan 45° = 2\text{ ft/s}$$

From (1), $(v_B)_y = -2$ ft/s

$$(v_B)_y = -2.00\text{ ft/s} \blacktriangleleft$$

$$\mathbf{v}_C = (2.00\text{ ft/s})\mathbf{i} + (2.00\text{ ft/s})\mathbf{j} \blacktriangleleft$$

PROBLEM 14.50



Three small spheres A, B, and C, each of mass m, are connected to a small ring D of negligible mass by means of three inextensible, inelastic cords of length l. The spheres can slide freely on a frictionless horizontal surface and are rotating initially at a speed v_0 about ring D which is at rest. Suddenly the cord CD breaks. After the other two cords have again become taut, determine (a) the speed of ring D, (b) the relative speed at which spheres A and B rotate about D, (c) the fraction of the original energy of spheres A and B which is dissipated when cords AD and BD again became taut.

SOLUTION

Let the system consist of spheres A and B.

State 1: Instant cord DC breaks.

$$m(\mathbf{v}_A)_1 = mv_0\left(-\frac{\sqrt{3}}{2}\mathbf{i} - \frac{1}{2}\mathbf{j}\right)$$

$$m(\mathbf{v}_B)_1 = mv_0\left(\frac{\sqrt{3}}{2}\mathbf{i} - \frac{1}{2}\mathbf{j}\right)$$

$$\mathbf{L}_1 = m(\mathbf{v}_A)_1 + m(\mathbf{v}_B)_1 = -mv_0\mathbf{j}$$

$$\overline{\mathbf{v}} = \frac{\mathbf{L}_1}{2m} = -\frac{1}{2}v_0\mathbf{j}$$

Mass center lies at Point G midway between balls A and B.

$$(\mathbf{H}_G)_1 = \frac{\sqrt{3}}{2}l\mathbf{j}\times(m\mathbf{v}_A)_1 + -\frac{\sqrt{3}}{2}l\mathbf{j}\times(m\mathbf{v}_B)_1$$

$$= \frac{3}{2}lmv_0\mathbf{k}$$

$$T_1 = \frac{1}{2}mv_0^2 + \frac{1}{2}mv_0^2 = mv_0^2$$

State 2: The cord is taut. Conservation of linear momentum:

(a) $$\mathbf{v}_D = \overline{\mathbf{v}} = -\frac{1}{2}v_0\mathbf{j}$$ $v_D = 0.500v_0$ ◀

Let $$(\mathbf{v}_A)_2 = \overline{\mathbf{v}} + \mathbf{u}_A \quad \text{and} \quad \mathbf{v}_B = \overline{\mathbf{v}} + \mathbf{u}_B$$

$$\mathbf{L}_2 = 2m\overline{\mathbf{v}} + m\mathbf{u}_A + m\mathbf{u}_B = \mathbf{L}_1$$

$$\mathbf{u}_B = -\mathbf{u}_A \qquad u_B = u_A$$

$$(\mathbf{H}_G)_2 = lmu_A\mathbf{k} + lmu_B\mathbf{k} = 2lmu_A\mathbf{k}$$

PROBLEM 14.50 (Continued)

(*b*) Conservation of angular momentum:

$$(\mathbf{H}_G)_2 = (\mathbf{H}_G)_1$$

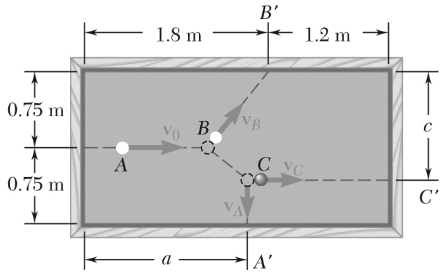
$$2lmu_A\mathbf{k} = \frac{3}{2}lmv_0\mathbf{k} \qquad u_A = u_B = \frac{3}{4}v_0 \qquad u = 0.750v_0 \blacktriangleleft$$

$$T_2 = \frac{1}{2}(2m)\overline{v}^2 + \frac{1}{2}mu_A^2 + \frac{1}{2}mu_B^2$$

$$= \frac{1}{2}mv_0^2\left(\frac{1}{2} + \frac{9}{16} + \frac{9}{16}\right) = \frac{13}{16}mv_0^2$$

(*c*) Fraction of energy lost: $\dfrac{T_1 - T_2}{T_1} = \dfrac{1 - \frac{13}{16}}{1} = \dfrac{3}{16}$ $\qquad \dfrac{T_1 - T_2}{T_1} = 0.1875 \blacktriangleleft$

PROBLEM 14.51



In a game of billiards, ball A is given an initial velocity $\mathbf{v}_0$ along the longitudinal axis of the table. It hits ball B and then ball C, which are both at rest. Balls A and C are observed to hit the sides of the table squarely at A' and C', respectively, and ball B is observed to hit the side obliquely at B'. Knowing that $v_0 = 4$ m/s, $v_A = 1.92$ m/s, and $a = 1.65$ m, determine (*a*) the velocities $\mathbf{v}_B$ and $\mathbf{v}_C$ of balls B and C, (*b*) the Point C' where ball C hits the side of the table. Assume frictionless surfaces and perfectly elastic impacts (that is, conservation of energy).

SOLUTION

Velocities in m/s. Lengths in meters. Assume masses are 1.0 for each ball.

Before impacts: $(\mathbf{v}_A)_0 = v_0\mathbf{i} = 4\mathbf{i}, \qquad (\mathbf{v}_B)_0 = (\mathbf{v}_C)_0 = 0$

After impacts: $\mathbf{v}_A = -1.92\mathbf{j}, \qquad \mathbf{v}_B = (v_B)_x\mathbf{i} + (v_B)_y\mathbf{j}, \qquad \mathbf{v}_C = v_C\mathbf{i}$

Conservation of linear momentum: $\mathbf{v}_0 = \mathbf{v}_A + \mathbf{v}_B + \mathbf{v}_C$

$$\mathbf{i}: \quad 4 = 0 + (v_B)_x + v_C \qquad (v_B)_x = 4 - v_C$$

$$\mathbf{j}: \quad 0 = -1.92 + (v_B)_y + 0 \qquad (v_B)_y = 1.92$$

Conservation of energy: $\frac{1}{2}v_0^2 = \frac{1}{2}v_A^2 + \frac{1}{2}v_B^2 + \frac{1}{2}v_C^2$

$$\frac{1}{2}(4)^2 = \frac{1}{2}(1.92)^2 + \frac{1}{2}(1.92)^2 + \frac{1}{2}(4 - v_C)^2 + \frac{1}{2}v_C^2$$

$$v_C^2 - 4v_C + 3.6864 = 0$$

$$v_C = \frac{4 \pm \sqrt{(4)^2 - (4)(3.6864)}}{2} = 2 \pm 0.56 = 2.56 \qquad \text{or} \qquad 1.44$$

Conservation of angular momentum about B':

$$0.75v_0 = (1.8 - a)v_A + cv_C$$

$$cv_C = (0.75)(4) - (1.8 - 1.65)(1.92) = 2.712$$

$$c = \frac{2.712}{v_C}$$

If $v_C = 1.44$, $\qquad c = 1.8833$ off the table. Reject.

If $v_C = 2.56$, $\qquad c = 1.059$

PROBLEM 14.51 (Continued)

Then, $(v_B)_x = 4 - 2.56 = 1.44, \qquad \mathbf{v}_B = 1.44\mathbf{i} + 1.92\mathbf{j}$

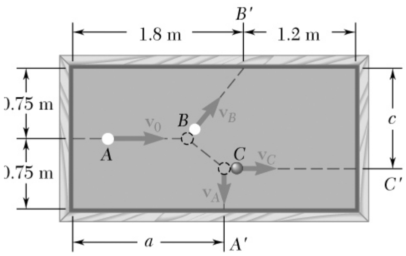
Summary.

(*a*) $\mathbf{v}_B = 2.40 \text{ m/s} \measuredangle 53.1° \blacktriangleleft$

$\mathbf{v}_C = 2.56 \text{ m/s} \longrightarrow \blacktriangleleft$

(*b*) $c = 1.059 \text{ m} \blacktriangleleft$

PROBLEM 14.52



For the game of billiards of Problem 14.51, it is now assumed that $v_0 = 5$ m/s, $v_C = 3.2$ m/s, and $c = 1.22$ m. Determine (*a*) the velocities $\mathbf{v}_A$ and $\mathbf{v}_B$ of balls *A* and *B*, (*b*) the Point A' where ball *A* hits the side of the table.

PROBLEM 14.51 In a game of billiards, ball *A* is given an initial velocity $\mathbf{v}_0$ along the longitudinal axis of the table. It hits ball *B* and then ball *C*, which are both at rest. Balls *A* and *C* are observed to hit the sides of the table squarely at A' and C', respectively, and ball *B* is observed to hit the side obliquely at B'. Knowing that $v_0 = 4$ m/s, $v_A = 1.92$ m/s, and $a = 1.65$ m, determine (*a*) the velocities $\mathbf{v}_B$ and $\mathbf{v}_C$ of balls *B* and *C*, (*b*) the Point C' where ball *C* hits the side of the table. Assume frictionless surfaces and perfectly elastic impacts (that is, conservation of energy).

SOLUTION

Velocities in m/s. Lengths in meters. Assume masses are 1.0 for each ball.

Before impacts: $(\mathbf{v}_A)_0 = v_0\mathbf{i} = 5\mathbf{i}, \qquad (\mathbf{v}_B)_0 = (\mathbf{v}_C)_0 = 0$

After impacts: $\mathbf{v}_A = -v_A\mathbf{j}, \qquad \mathbf{v}_B = (v_B)_x\mathbf{i} + (v_B)_y\mathbf{j}, \qquad \mathbf{v}_C = 3.2\mathbf{i}$

Conservation of linear momentum: $\mathbf{v}_0 = \mathbf{v}_A + \mathbf{v}_B + \mathbf{v}_C$

$$\mathbf{i}: \quad 5 = 0 + (v_B)_x + 3.2 \qquad (v_B)_x = 1.8$$

$$\mathbf{j}: \quad 0 = -v_A + (v_B)_y + 0 \qquad (v_B)_y = v_A$$

Conservation of energy: $\frac{1}{2}v_0^2 = \frac{1}{2}v_A^2 + \frac{1}{2}v_B^2 + \frac{1}{2}v_C^2$

$$\frac{1}{2}(5)^2 = \frac{1}{2}(v_A)^2 + \frac{1}{2}(1.8)^2 + \frac{1}{2}(v_A)^2 + \frac{1}{2}(3.2)^2$$

(*a*) $v_A^2 = 11.52 \qquad v_A = 2.4$ $\qquad \mathbf{v}_A = 2.40 \text{ m/s} \downarrow$ ◀

$(v_B)_y = 2.4 \qquad \mathbf{v}_B = 1.8\mathbf{i} + 2.4\mathbf{j}$ $\qquad \mathbf{v}_B = 3.00 \text{ m/s} \measuredangle 53.1°$ ◀

Conservation of angular momentum about B':

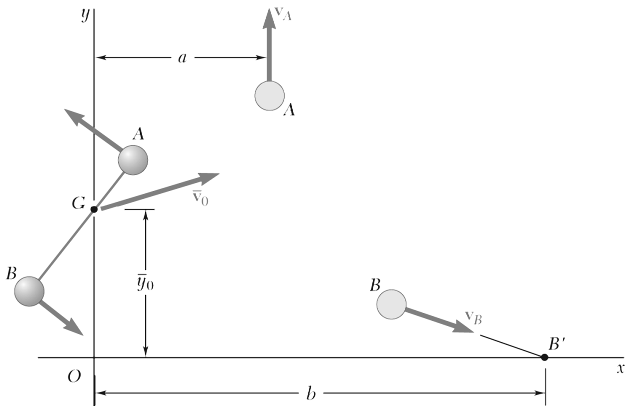
$$0.75v_0 = (1.8 - a)v_A + cv_C$$

$$av_A = 1.8v_A + cv_C - 0.75v_0$$

$$= (1.8)(2.4) + (1.22)(3.2) - (0.75)(5) = 4.474$$

(*b*) $$a = \frac{4.474}{v_A} = \frac{4.474}{2.4}$$ $\qquad a = 1.864 \text{ m}$ ◀

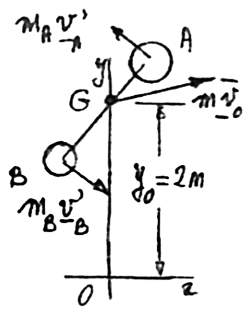
PROBLEM 14.53



Two small disks A and B, of mass 3 kg and 1.5 kg, respectively, may slide on a horizontal, frictionless surface. They are connected by a cord, 600 mm, long, and spin counterclockwise about their mass center G at the rate of 10 rad/s. At $t = 0$, the coordinates of G are $\overline{x}_0 = 0$, $\overline{y}_0 = 2$ m, and its velocity is $\overline{\mathbf{v}}_0 = (1.2 \text{ m/s})\mathbf{i} + (0.96 \text{ m/s})\mathbf{j}$. Shortly thereafter, the cord breaks; disk A is then observed to move along a path parallel to the y axis and disk B along a path which intersects the x axis at a distance $b = 7.5$ m from O. Determine (a) the velocities of A and B after the cord breaks, (b) the distance a from the y-axis to the path of A.

SOLUTION

Initial conditions.



Location of G:

$$\frac{AG}{m_B} = \frac{BG}{m_A} = \frac{AG + GB}{m_B + m_A} = \frac{AB}{m} = \frac{0.6 \text{ m}}{4.5 \text{ kg}}$$

$$AG = 1.5\left(\frac{0.6}{4.5}\right) = 0.2 \text{ m}$$

$$BG = 0.4 \text{ m}$$

Linear momentum:

$$\mathbf{L}_0 = m\overline{\mathbf{v}}_0 = (4.5 \text{ kg})(1.2\mathbf{i} + 0.96\mathbf{j})$$
$$= 5.4\mathbf{i} + 4.32\mathbf{j}$$

Angular momentum:

About G:

$$(\mathbf{H}_G)_0 = \overrightarrow{GA} \times m_A \mathbf{v}'_A + \overrightarrow{GB} \times m_B \mathbf{v}'_B$$
$$= (0.2 \text{ m})(3 \text{ kg})(0.2 \text{ m} \times 10 \text{ rad/s})\mathbf{k}$$
$$+ (0.4 \text{ m})(1.5 \text{ kg})(0.4 \text{ m} \times 10 \text{ rad/s})\mathbf{k}$$
$$(\mathbf{H}_G)_0 = (3.6 \text{ kg} \cdot \text{m}^2/\text{s})\mathbf{k}$$

About O: Using formula derived in Problem 14.27,

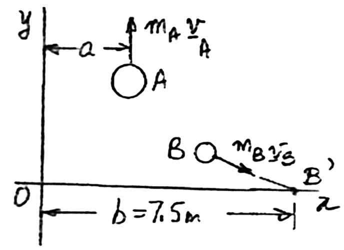
$$(\mathbf{H}_O)_0 = \overline{\mathbf{r}} \times m\overline{\mathbf{v}}_0 + (\mathbf{H}_G)_0$$
$$= 2\mathbf{j} \times (5.4\mathbf{i} + 4.32\mathbf{j}) + 3.6\mathbf{k}$$
$$= -10.8\mathbf{k} + 3.6\mathbf{k} = -(7.2 \text{ kg} \cdot \text{m}^2/\text{s})\mathbf{k}$$

Kinetic energy: Using Eq. (14.29),

$$T_0 = \frac{1}{2}m\bar{v}_0^2 + \frac{1}{2}\sum_i m: \quad v_i'^2 = \frac{1}{2}m\bar{v}_0^2 + \frac{1}{2}m_A v_A'^2 + \frac{1}{2}m_B v_B'^2$$

$$= \frac{1}{2}(4.5)[(1.2)^2 + (0.96)^2] + \frac{1}{2}(3)(0.2\times 10)^2 + \frac{1}{2}(1.5)(0.4\times 10)^2$$

$$= (5.3136 + 6 + 12) = 23.314 \text{ J}$$



(*a*) Conservation of linear momentum:

$$\mathbf{L}_0 = \mathbf{L}$$

$$5.4\mathbf{i} + 4.32\mathbf{j} = m_A \mathbf{v}_A + m_B \mathbf{v}_B$$

$$= 3(v_A\mathbf{j}) + 1.5[(v_B)_z\mathbf{i} + (v_B)_y\mathbf{j}]$$

Equating coefficients of **i**: $5.4 = 1.5(v_B)_x$

$$(v_B)_x = 3.6 \text{ m/s} \qquad (1)$$

Equating coefficients of **j**: $4.32 = 3v_A + 1.5(v_B)_y$

$$(v_B)_y = 2.88 - 2v_A \qquad (2)$$

Conservation of energy:

$$T_0 = T: \quad T_0 = \frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2$$

$$23.314 \text{ J} = \frac{1}{2}(3)\left(v_A^2\right) + \frac{1}{2}(1.5)[(v_B)_x^2 + (v_B)_y^2]$$

Substituting from Eqs. (1) and (2):

$$23.314 = 1.5v_A^2 + 0.75(3.6)^2 + 0.75(2.88 - 2v_A)^2$$

$$4.5v_A^2 - 8.64v_A - 7.373 = 0$$

$$v_A^2 - 1.92v_A - 1.6389 = 0$$

$$v_A = 0.96 + 1.60 = 2.56 \text{ m/s} \qquad \mathbf{v}_A = 2.56 \text{ m/s} \uparrow \blacktriangleleft$$

and $v_A = 0.96 - 1.60 = -0.64$ m/s (rejected, since $\mathbf{v}_A$ is shown directed up)

From Eqs. (1) and (2):

$$(v_B)_x = 3.6 \text{ m/s}$$

$$(v_B)_y = 2.88 - 2(2.56) = -2.24 \text{ m/s}$$

$$\mathbf{v}_B = 3.6\mathbf{i} - 2.24\mathbf{j} \qquad \mathbf{v}_B = 4.24 \text{ m/s} \measuredangle 31.9° \blacktriangleleft$$

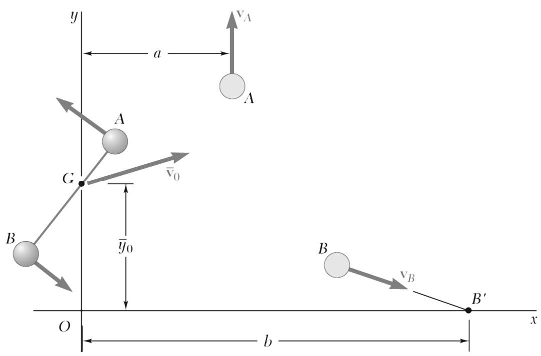
(*b*) Conservation of angular momentum about *O*:

$$(\mathbf{H}_O)_0 = \mathbf{H}_O: \quad -7.2\mathbf{k} = a\mathbf{i}\times m_A\mathbf{v}_A + b\mathbf{i}\times m_B\mathbf{v}_B$$

$$-7.2\mathbf{k} = a\mathbf{i}\times 3(2.56\mathbf{j}) + 7.5\mathbf{i}\times 1.5(3.6\mathbf{i} - 2.24\mathbf{j})$$

$$-7.2\mathbf{k} = 7.68a\mathbf{k} - 25.2\mathbf{k} \qquad a = 2.34 \text{ m} \blacktriangleleft$$

PROBLEM 14.54

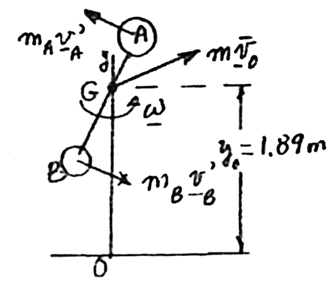


Two small disks A and B, of mass 2 kg and 1 kg, respectively, may slide on a horizontal and frictionless surface. They are connected by a cord of negligible mass and spin about their mass center G. At $t = 0$, G is moving with the velocity $\bar{\mathbf{v}}_0$ and its coordinates are $\bar{x}_0 = 0$, $\bar{y}_0 = 1.89$ m. Shortly thereafter, the cord breaks and disk A is observed to move with a velocity $\mathbf{v}_A = (5 \text{ m/s})\mathbf{j}$ in a straight line and at a distance $a = 2.56$ m from the y-axis, while B moves with a velocity $\mathbf{v}_B = (7.2 \text{ m/s})\mathbf{i} - (4.6 \text{ m/s})\mathbf{j}$ along a path intersecting the x-axis at a distance $b = 7.48$ m from the origin O. Determine (a) the initial velocity $\bar{\mathbf{v}}_0$ of the mass center G of the two disks, (b) the length of the cord initially connecting the two disks, (c) the rate in rad/s at which the disks were spinning about G.

SOLUTION

Initial conditions.

Location of G:



$$\frac{AG}{m_B} = \frac{BG}{m_A} = \frac{AG + GB}{m_B + m_A} = \frac{l}{m}$$

$$AG = \frac{m_B}{m} l = \frac{1}{3} l$$

$$BG = \frac{m_A}{m} l = \frac{2}{3} l$$

Linear momentum:

$$\mathbf{L}_0 = m\bar{\mathbf{v}}_0 = 3\bar{\mathbf{v}}_0$$

Angular momentum about G:

$$(\mathbf{H}_G)_0 = \overrightarrow{GA} \times m_A \mathbf{v}'_A + \overrightarrow{GB} \times m_B \mathbf{v}'_B$$

$$= \left(\frac{1}{3}l\right)(2 \text{ kg})\left(\frac{1}{3}l\omega\right)\mathbf{k} + \left(\frac{2}{3}l\right)(1 \text{ kg})\left(\frac{2}{3}l\omega\right)\mathbf{k}$$

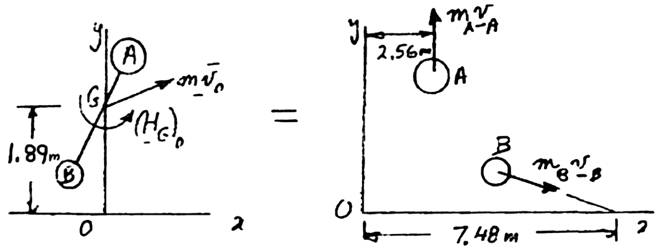
$$= \frac{2}{3}l^2\omega\mathbf{k}^2$$

Kinetic energy: Using Eq. (14.29),

$$T_0 = \frac{1}{2}m\bar{v}_0^2 + \frac{1}{2}\sum_i m_i v_i^2 = \frac{1}{2}m\bar{v}_0^2 + \frac{1}{2}m_A v_A'^2 + \frac{1}{2}m_B v_B'^2$$

$$= \frac{1}{2}(3)\bar{v}_0^2 + \frac{1}{2}(2)\left(\frac{1}{3}l\omega\right)^2 + \frac{1}{2}(1)\left(\frac{2}{3}l\omega\right)^2$$

$$T_0 = \frac{3}{2}\bar{v}_0^2 + \frac{1}{3}l^2\omega^2$$



Conservation of linear momentum:

$$m\bar{\mathbf{v}}_0 = m_A\mathbf{v}_A + m_B\mathbf{v}_B$$
$$3\bar{\mathbf{v}}_0 = (2)(5\mathbf{j}) + (1)(7.2\mathbf{i} - 4.6\mathbf{j}) = 7.2\mathbf{i} + 5.4\mathbf{j}$$

(*a*) $\bar{\mathbf{v}}_0 = (2.4\text{ m/s})\mathbf{i} + (1.8\text{ m/s})\mathbf{j}$ ◀

Conservation of angular momentum about O:

$$+\circlearrowleft: \quad (1.89\mathbf{j})\times m\bar{\mathbf{v}}_0 + (\mathbf{H}_G)_0 = (2.56\mathbf{i})\times m_A\mathbf{v}_A + (7.48\mathbf{i})\times m_B\mathbf{v}_A$$

Substituting for $\bar{\mathbf{v}}_0$, $(\mathbf{H}_G)_0$, $\mathbf{v}_A$, $\mathbf{v}_B$ and masses:

$$(1.89\mathbf{j})\times 3(2.4\mathbf{i} + 1.8\mathbf{j}) + \frac{2}{3}l^2\omega\mathbf{k} = (2.56\mathbf{i})\times 2(5\mathbf{j}) + (7.48\mathbf{i})\times(7.2\mathbf{i} - 4.6\mathbf{j})$$

$$-13.608\mathbf{k} + \frac{2}{3}l^2\omega\mathbf{k} = 25.6\mathbf{k} - 34.408\mathbf{k}$$

$$\frac{2}{3}l^2\omega = 4.80 \qquad l^2\omega = 7.20 \tag{1}$$

Conservation of energy:

$$T_0 = T: \quad \frac{3}{2}\bar{v}_0^2 + \frac{1}{3}l^2\omega^2 = \frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2$$

$$\frac{3}{2}[(2.4)^2 + (1.8)^2] + \frac{1}{3}l^2\omega^2 = \frac{1}{2}(2)(5)^2 + \frac{1}{2}(1)[(7.2)^2 + (4.6)^2]$$

$$13.5 + \frac{1}{3}l^2\omega^2 = 25 + 36.5 \qquad l^2\omega^2 = 144.0 \tag{2}$$

Dividing Eq. (2) by Eq. (1), member by member:

$$\omega = \frac{144.0}{7.20} = 20.0\text{ rad/s}$$

(*c*) Original rate of spin = 20.0 rad/s ◀

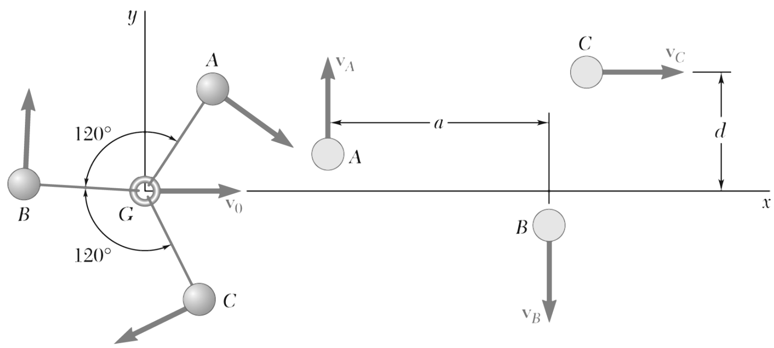
Substituting for ω into Eq. (1):

$$l^2(20.0) = 7.20 \qquad l^2 = 0.360 \qquad l = 0.600\text{ m}$$

(*b*) Length of cord = 600 mm ◀

PROBLEM 14.55

Three small identical spheres A, B, and C, which can slide on a horizontal, frictionless surface, are attached to three 9-in-long strings, which are tied to a ring G. Initially, the spheres rotate clockwise about the ring with a relative velocity of 2.6 ft/s and the ring moves along the x-axis with a velocity $\mathbf{v}_0 = (1.3 \text{ ft/s})\mathbf{i}$. Suddenly, the ring breaks and the three spheres move freely in the xy plane with A and B, following paths parallel to the y-axis at a distance $a = 1.0$ ft from each other and C following a path parallel to the x-axis. Determine (a) the velocity of each sphere, (b) the distance d.



SOLUTION

Conservation of linear momentum:

Before break: $\mathbf{L}_0 = (3m)\overline{\mathbf{v}} = 3m(1.3\mathbf{i}) = m(3.9 \text{ ft/s})\mathbf{i}$

After break: $\mathbf{L} = mv_A\mathbf{j} - mv_B\mathbf{j} + mv_C\mathbf{i}$

$\mathbf{L} = \mathbf{L}_0$: $mv_C\mathbf{i} + m(v_A - v_B)\mathbf{j} = m(3.9 \text{ ft/s})\mathbf{i}$

Therefore, $v_A = v_B$ (1)

$v_C = 3.9000$ ft/s $\quad \mathbf{v}_C = 3.90$ ft/s $\longrightarrow$ (2)

Conservation of angular momentum:

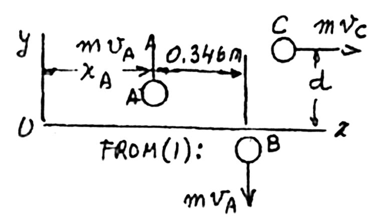
Before break: $+\circlearrowright (H_O)_0 = 3mlv' = 3m(0.75 \text{ ft})(2.6 \text{ ft/s})$

$= 5.85m$

After break: $H_O = -mv_Ax_A$

$+mv_A(x_A + 0.346)$

$+mv_Cd$



$+\circlearrowright H_O = -m_Ax_A + mv_A(x_A + 1.0) + mv_Cd$

$H_O = (H_O)_0$: $1.0mv_A + mv_Cd = 5.85m$

Recalling Eq. (2):

$v_A + 3.9d = 5.85$

$d = 1.5 - 0.25641v_A$ (3)

Conservation of energy.

Before break:

$$T_0 = \frac{1}{2}(3m)\overline{v}^2 + 3\left(\frac{1}{2}mv'^2\right)$$

$$= \frac{3}{2}m\left(v_0^2 + v'^2\right) = \frac{3}{2}[(1.3)^2 + (2.6)^2]m = 12.675m$$

After break:

$$T = \frac{1}{2}mv_A^2 + \frac{1}{2}mv_B^2 + \frac{1}{2}mv_C^2$$

$T = T_0$: Substituting for v_B from Eq. (1) and v_C from Eq. (2),

$$\frac{1}{2}\left[v_A^2 + v_A^2 + (3.900)^2\right] = 12.675$$

$$v_A^2 = 5.0700$$

$$v_A = v_B = 2.2517 \text{ ft/s}$$

(*a*) *Velocities*:

$$\mathbf{v}_A = 2.25 \text{ ft/s} \uparrow; \ \mathbf{v}_B = 2.25 \text{ ft/s} \downarrow; \ \mathbf{v}_C = 3.9 \text{ ft/s} \longrightarrow \blacktriangleleft$$

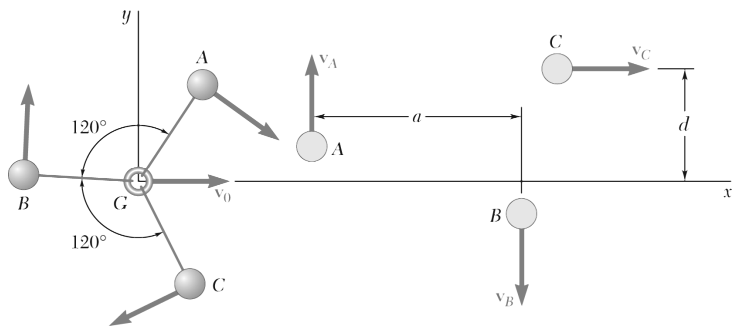
(*b*) *Distance d*:

From Eq. (3): $d = 1.5 - 0.25641(2.2517) = 0.92265$ ft

$$d = 11.1 \text{ in.} \blacktriangleleft$$

PROBLEM 14.56

Three small identical spheres A, B, and C, which can slide on a horizontal, frictionless surface, are attached to three strings of length l which are tied to a ring G. Initially, the spheres rotate clockwise about the ring which moves along the x axis with a velocity $\mathbf{v}_0$. Suddenly the ring breaks and the three spheres move freely in the xy plane. Knowing that $\mathbf{v}_A = (3.5 \text{ ft/s})\mathbf{j}$, $\mathbf{v}_C = (6.0 \text{ ft/s})\mathbf{i}$, $a = 16$ in. and $d = 9$ in., determine (a) the initial velocity of the ring, (b) the length l of the strings, (c) the rate in rad/s at which the spheres were rotating about G.



SOLUTION

Conservation of linear momentum:

$$(3m)\overline{\mathbf{v}} = m\mathbf{v}_A + m\mathbf{v}_B + m\mathbf{v}_C$$
$$3mv_0\mathbf{i} = m(3.5 \text{ ft/s})\mathbf{j} - mv_B\mathbf{j} + m(6.0 \text{ ft/s})\mathbf{i}$$

Equating coefficients of unit vectors:

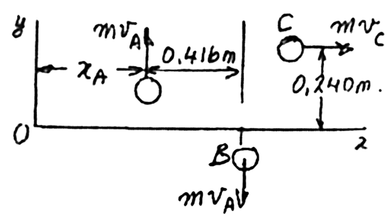
$$3v_0 = 6.00 \text{ ft/s}$$
$$0 = 3.5 \text{ ft/s} - \mathbf{v}_B \qquad v_B = 3.5 \text{ ft/s} \qquad (1)$$

$$v_0 = 2.00 \text{ ft/s} \longrightarrow \blacktriangleleft$$

(a) Conservation of angular momentum:

Before break: $\circlearrowright + \; (H_O)_0 = 3ml^2\dot{\theta}$

After break:
$$H_O = -mv_A x_A + mv_A(x_A + 16/12) + mv_C(9/12)$$
$$= m(3.5)(16/12) + m(6.0)(9/12)$$
$$= m(9.1667)$$



$(H_O)_0 = H_0$: $\qquad 3ml^2\dot{\theta} = m(9.1667) \qquad l^2\dot{\theta} = 3.0556 \qquad (2)$

PROBLEM 14.56 (Continued)

Conservation of energy:

Before break:

$$T_0 = \frac{1}{2}(3m)\bar{v}^2 + 3\left(\frac{1}{2}mv'^2\right) = \frac{3}{2}mv_0^2 + \frac{3}{2}m(l\dot{\theta})^2$$

$$= \frac{3}{2}m(2.0)^2 + \frac{3}{2}ml^2\dot{\theta}^2$$

After break:

$$T = \frac{1}{2}mv_A^2 + \frac{1}{2}mv_B^2 + \frac{1}{2}mv_C^2$$

$$= \frac{1}{2}m[(3.5)^2 + (6.0)^2] = \frac{1}{2}m(60.5)$$

$T = T_0$: $\quad \frac{1}{2}m(60.5) = \frac{3}{2}m(2.0)^2 + \frac{3}{2}ml^2\dot{\theta}^2$

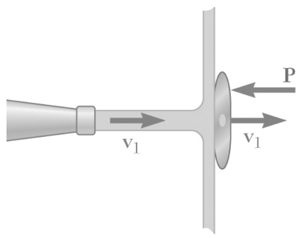
$$l^2\dot{\theta}^2 = 16.167 \qquad (3)$$

Dividing Eq. (3) by Eq. (2): $\quad \dot{\theta} = \frac{16.167}{3.0556} = 5.2909$

(*b*) From Eq. (2): $\quad l^2 = \frac{3.0556}{5.2909} \qquad l = 0.75994 \text{ ft}$

$l = 0.76 \text{ ft}$ ◀

(*c*) Rate of rotation: $\quad \dot{\theta} = 5.29 \text{ rad/s}$ ◀



PROBLEM 14.57

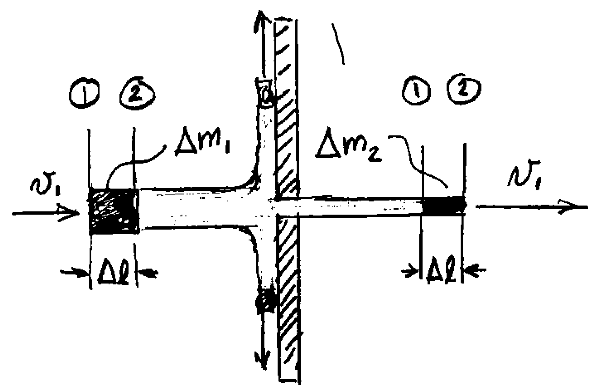
A stream of water of cross-sectional area A_1 and velocity $\mathbf{v}_1$ strikes a circular plate which is held motionless by a force $\mathbf{P}$. A hole in the circular plate of area A_2 results in a discharge jet having a velocity $\mathbf{v}_1$. Determine the magnitude of $\mathbf{P}$.

SOLUTION

<u>Mass flow rates</u>. As the fluid ahead of the plate moves from section 1 to section 2 Δt, the mass Δm_1 moved is

$$\Delta m_1 = \rho A_1(\Delta t) = \rho A_1 v_1(\Delta t)$$

so that $$\frac{dm_1}{dt} = \frac{\Delta m_1}{\Delta t} = \rho A_1 v_1$$

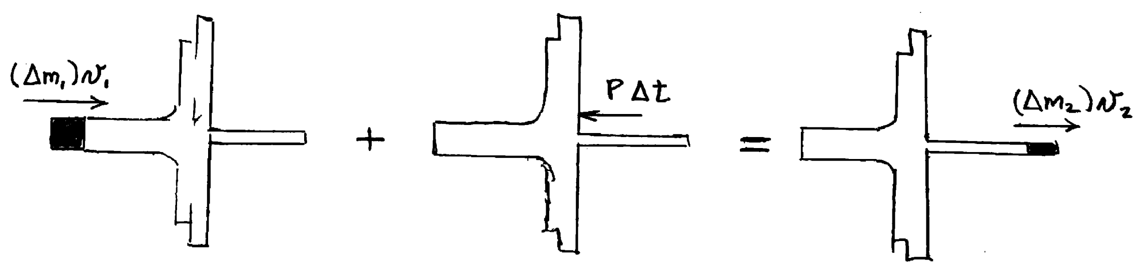


Likewise, for the fluid that has passed through the hole

$$\Delta m_2 = \rho A_2(\Delta l) = \rho A_2 v_1(\Delta t)$$

so that $$\frac{dm_2}{dt} = \rho A_2 v_1$$

Apply the impulse-momentum principle.



$$\Sigma m\mathbf{v}_1 + \int \mathbf{F}\, dt = \Sigma m\mathbf{v}_2$$

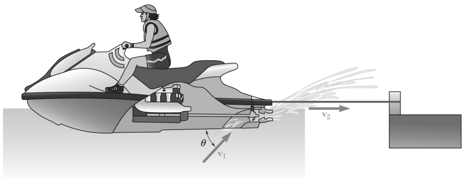
Components in the direction of the flow.

$$(\Delta m_1)v_1 - P\Delta t = (\Delta m_2)v_1$$

$$P = \frac{\Delta m_1}{\Delta t}v_1 - \frac{\Delta m_2}{\Delta t}v_1 = \rho A_1 v_1^2 - \rho A_2 v_1^2$$

$$P = \rho(A_1 - A_2)v_1^2 \blacktriangleleft$$

PROBLEM 14.58



A jet ski is placed in a channel and is tethered so that it is stationary. Water enters the jet ski with velocity $\mathbf{v}_1$ and exits with velocity $\mathbf{v}_2$. Knowing the inlet area is A_1 and the exit area is A_2, determine the tension in the tether.

SOLUTION

Mass flow rates. Consider a cylindrical portion of the fluid lying in a section of pipe of cross sectional area A and length Δl.

The volume and mass are

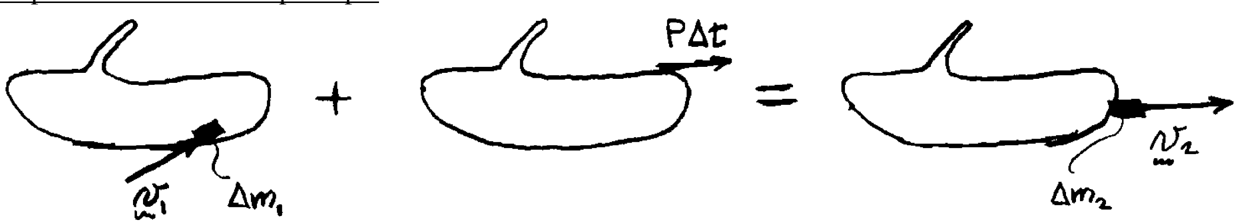
$$\Delta m = \rho A(\Delta l)$$

Then
$$\frac{\Delta m}{\Delta t} = \rho A \frac{\Delta l}{\Delta t} = \rho A v$$

At the pipe inlet and outlet, we get

$$\frac{\Delta m_1}{\Delta t} = \rho A_1 v_1, \qquad \frac{\Delta m_2}{\Delta t} = \rho A_2 v_2$$

Impulse and momentum principle:



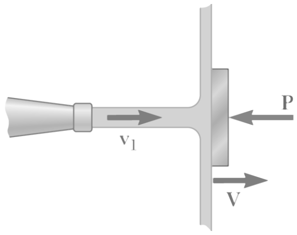
$$\Sigma m\mathbf{v}_1 + \text{Imp}_{1\to 2} = \Sigma m\mathbf{v}_2$$

Using horizontal components $(+\rightarrow)$,

$$(\Delta m_1)v_1 \cos\theta + P(\Delta t) = (\Delta m_2)v_2$$

$$P = \frac{\Delta m_1}{\Delta t}v_2 - \frac{\Delta m_1}{\Delta t}v_1 \cos\theta$$
$$= \rho A_2 v_2^2 - \rho A_1 v_1^2 \cos\theta$$

$$P = \rho A_2 v_2^2 - \rho A_1 v_1^2 \cos\theta \blacktriangleleft$$



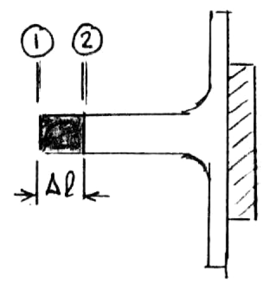
PROBLEM 14.59

A stream of water of cross-sectional area A and velocity $\mathbf{v}_1$ strikes a plate which is held motionless by a force **P**. Determine the magnitude of **P**, knowing that $A = 0.75\ \text{in}^2$, $v_1 = 80\ \text{ft/s}$, and $V = 0$.

SOLUTION

Mass flow rate. As the fluid moves from section 1 to section 2 in time Δt, the mass Δm moved is

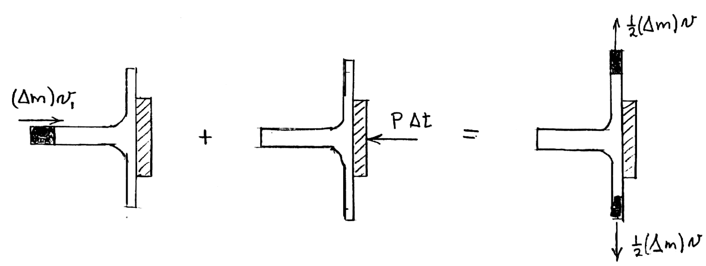
$$\Delta m = \rho A(\Delta l)$$



Then
$$\frac{dm}{dt} = \frac{\Delta m}{\Delta t} = \frac{\rho A(\Delta l)}{\Delta t} = \rho A v_1$$

<u>Data</u>: $\gamma = 62.4\ \text{lb/ft}^3$, $A = 0.75\ \text{in.}^2 = 0.0052083\ \text{ft}^2$, $v_1 = 80\ \text{ft/s}$

$$\frac{dm}{dt} = \frac{(62.4)}{32.2}(0.0052083)(80) = 0.80745\ \text{slug/s}$$



Principle of impulse and momentum:

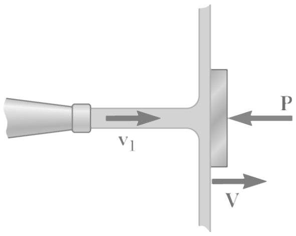
$$\xrightarrow{+}: \quad (\Delta m)v_1 - P\Delta t = 0$$

$$P = \frac{\Delta m}{\Delta t} v = \frac{dm}{dt} v$$

$$P = (0.80745)(80) = 64.596\ \text{lb}$$

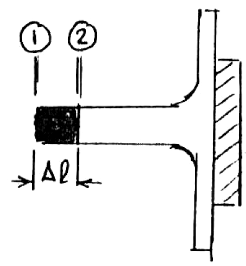
$P = 64.6\ \text{lb}$ ◀

PROBLEM 14.60



A stream of water of cross-sectional area A and velocity $\mathbf{v}_1$ strikes a plate which moves to the right with a velocity $\mathbf{V}$. Determine the magnitude of $\mathbf{V}$, knowing that $A = 1\text{ in}^2$, $v_1 = 100$ ft/s, and $P = 90$ lb.

SOLUTION



Consider velocities measured with respect to the plate, which is moving with velocity V. The velocity of the stream relative to the plate is

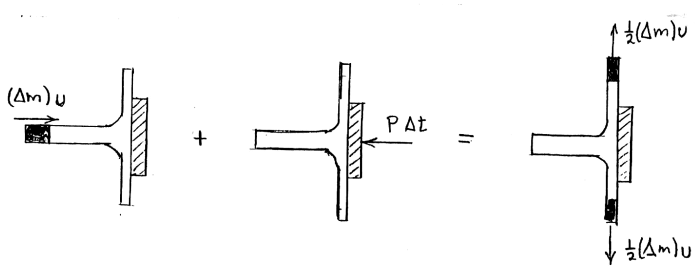
$$\mathbf{u} = \mathbf{v}_1 - \mathbf{V} \qquad (1)$$

Mass flow rate. As the fluid moves from section 1 to section 2 in time Δt, the mass Δm moved is

$$\Delta m = \rho A(\Delta l)$$

Then

$$\frac{dm}{dt} = \frac{\Delta m}{\Delta t} = \frac{\rho A(\Delta l)}{\Delta t} = \rho A u \qquad (2)$$



Principle of impulse and momentum:

$$\xrightarrow{+} \quad (\Delta m)u - P(\Delta t) = 0$$

$$P = \frac{\Delta m}{\Delta t}u = \frac{dm}{dt}u = \rho A u^2$$

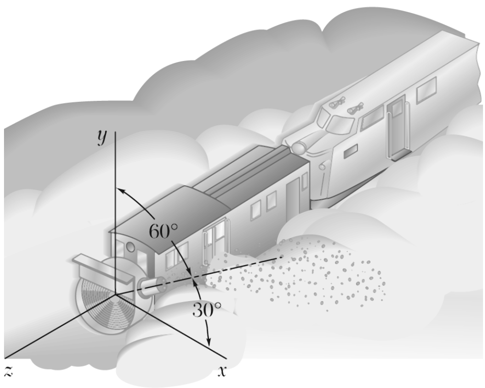
$$u = \sqrt{\frac{P}{\rho A}}$$

PROBLEM 14.60 (Continued)

From Eq. (1),
$$V = v_1 - u = v_1 - \sqrt{\frac{P}{\rho A}}$$

Data:
$$P = 90 \text{ lb}, \qquad A = 1 \text{ in}^2 = 0.0069444 \text{ ft}^2$$
$$V_1 = 100 \text{ ft/s}, \quad \gamma = 62.4 \text{ lb/ft}^3$$
$$v = 100 - \sqrt{\frac{90}{(62.4/32.2)(0.0069444)}}$$

$V = 18.2$ ft/s ◀



PROBLEM 14.61

A rotary power plow is used to remove snow from a level section of railroad track. The plow car is placed ahead of an engine which propels it at a constant speed of 20 km/h. The plow car clears 160 Mg of snow per minute, projecting it in the direction shown with a velocity of 12 m/s relative to the plow car. Neglecting friction, determine (*a*) the force exerted by the engine on the plow car, (*b*) the lateral force exerted by the track on the plow.

SOLUTION

Velocity of the plow: $v_P = 20 \text{ km/h} = 5.5556 \text{ m/s}$

Velocity of thrown snow:

$$\mathbf{v}_s = (12 \text{ m/s})(\cos 30°\mathbf{i} + \sin 30°\mathbf{j}) + (5.5556 \text{ m/s})\mathbf{k}$$

Mass flow rate:

$$\frac{dm}{dt} = \frac{(160000 \text{ kg/min})}{(60 \text{ s/min})} = 2666.7 \text{ kg/s}$$

Let F be the force exerted on the plow and the snow.

Apply impulse-momentum, noting that the snow is initially at rest and that the velocity of the plow is constant. Neglect gravity.

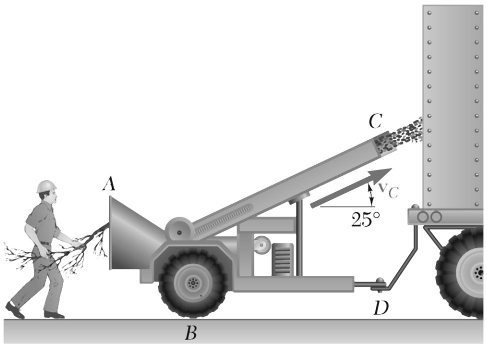
$$\mathbf{F}(\Delta t) = (\Delta m)\mathbf{v}_s$$

$$\mathbf{F} = \left(\frac{dm}{dt}\right)v_s = (2.666.7)(12\cos 30°\mathbf{i} + 12\sin 30°\mathbf{j} + 5.5556\mathbf{k})$$

$$= (27713 \text{ N})\mathbf{i} + (16000 \text{ N})\mathbf{j} + (14815 \text{ N})\mathbf{k}$$

(*a*) Force exerted by engine. $F_z = 14.8 \text{ kN}$ ◀

(*b*) Lateral force exerted by track. $F_x = 27.7 \text{ kN}$ ◀

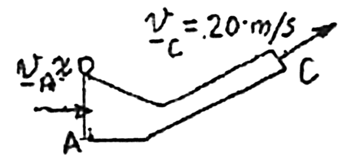


PROBLEM 14.62

Tree limbs and branches are being fed at A at the rate of 5 kg/s into a shredder which spews the resulting wood chips at C with a velocity of 20 m/s. Determine the horizontal component of the force exerted by the shredder on the truck hitch at D.

SOLUTION

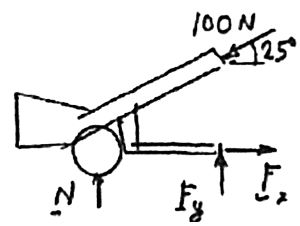
Eq. (14.38):



$$(\Delta m)\,\mathbf{v}_A + \Sigma\mathbf{F}\Delta t = (\Delta m)\mathbf{v}_C$$

$$\Sigma\mathbf{F} = \frac{\Delta m}{\Delta t}\mathbf{v}_C = (5\text{ kg/s})(20\text{ m/s} \measuredangle 25^\circ)$$

Force exerted on chips $= \Sigma\mathbf{F} = 100\text{ N} \measuredangle 25^\circ$

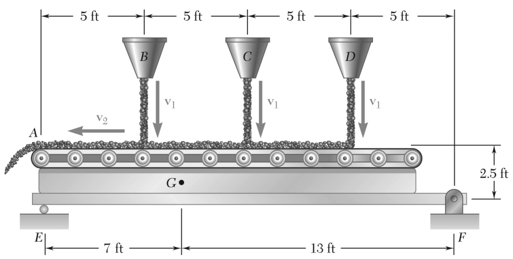


Free body: shredder:

$$\xrightarrow{+} \Sigma F_x = 0: \quad F_x - (100\text{ N})\cos 25^\circ = 0$$

$$\mathbf{F}_x = 90.6\text{ N} \longrightarrow$$

On hitch: $\mathbf{F}_x = 90.6\text{ N} \longleftarrow$ ◀

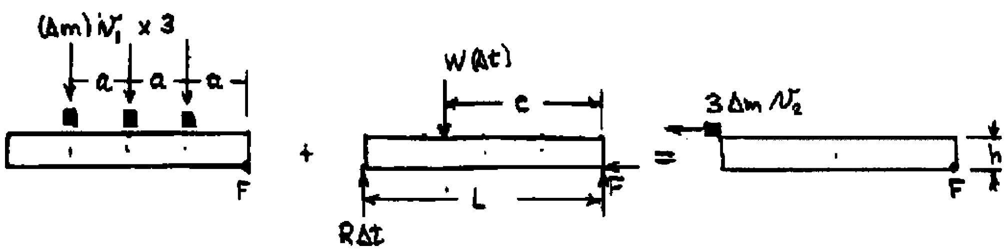


PROBLEM 14.63

Sand falls from three hoppers onto a conveyor belt at a rate of 90 lb/s for each hopper. The sand hits the belt with a vertical velocity $v_1 = 10$ ft/s and is discharged at A with a horizontal velocity $v_2 = 13$ ft/s. Knowing that the combined mass of the beam, belt system, and the sand it supports is 1300 lb with a mass center at G, determine the reaction at E.

SOLUTION

Principle of impulse and momentum:



$+\circlearrowright$ Moments about F:

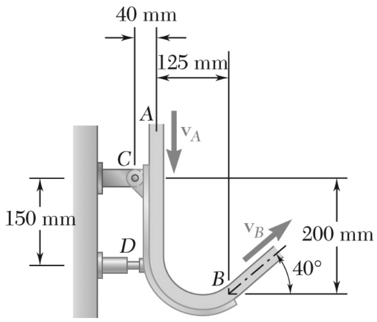
$$(\Delta m)v_1(3a) + \Delta m v_1(2a) + (\Delta m)v_1 a + (W\Delta t)c - (R\Delta t)L = 3(\Delta m)v_2 h$$

$$R = \frac{1}{L}\left[cW + 6av_1\frac{\Delta m}{\Delta t} - 3hv_2\frac{\Delta m}{\Delta t}\right]$$

Data: $L = 20$ ft, $c = 13$ ft, $a = 5$ ft, $h = 2.5$ ft, $\dfrac{\Delta m}{\Delta t} = \dfrac{\Delta W/g}{dt} = \dfrac{1}{g}\dfrac{dW}{dt} = 2.7950$ slug/s

$$R = \frac{1}{20}\left[13(1300) + (6)(5)(10)(2.7950) - (3)(2.5)(13)(2.7950)\right]$$

$\mathbf{R} = 873 \text{ lb}\uparrow$ ◀



PROBLEM 14.64

The stream of water shown flows at a rate of 550 liters/min and moves with a velocity of magnitude 18 m/s at both A and B. The vane is supported by a pin and bracket at C and by a load cell at D which can exert only a horizontal force. Neglecting the weight of the vane, determine the components of the reactions at C and D.

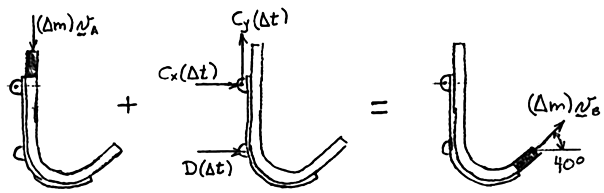
SOLUTION

Mass flow rate: $$\frac{dm}{dt} = \rho Q = \frac{(1000\text{ kg/m}^3)(550\text{ liters/min})(1\text{ min})}{(1000\text{ liters/m}^3)(60\text{ sec})}$$

$$\frac{dm}{dt} = 9.1667\text{ kg/s}$$

Velocity vectors: $\mathbf{v}_A = 18\text{ m/s}\downarrow \qquad \mathbf{v}_B = 18\text{ m/s} \measuredangle 40°$

Apply the impulse-momentum principle.



$+\circlearrowleft$ Moments about C: $-0.040(\Delta m)v_A + 0.150D(\Delta t) = 0.200(\Delta m)v_B\cos 40° + 0.165(\Delta m)v_B\sin 40°$

$$D = \frac{1}{0.150}\left(\frac{\Delta m}{\Delta t}\right)[0.200v_B\cos 40° + 0.165v_B\sin 40° + 0.040v_A]$$

$$= \frac{1}{0.150}(9.1667)[(0.200)(18)\cos 40° + 0.165(18)\sin 40° + 0.040(18)]$$

$$= 329.20\text{ N}$$

$D_x = 329\text{ N}$ ◀

$D_y = 0$ ◀

$\xrightarrow{+}$ x components: $C_x(\Delta t) + D(\Delta t) = (\Delta m)v_B\cos 40°$

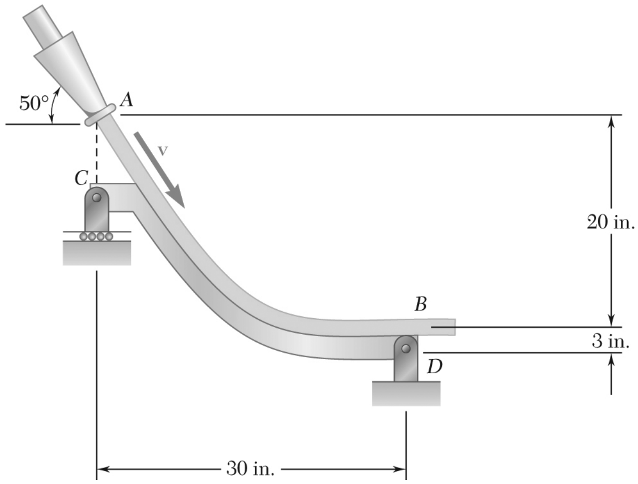
$$C_x = \left(\frac{\Delta m}{\Delta t}\right)v_B\cos 40° - D = (9.1667)(18\cos 40°) - 329.20 = -202.79\text{ N}$$

$C_x = -203\text{ N}$ ◀

$+\uparrow$ y components: $-(\Delta m)v_A + C_y(\Delta t) = (\Delta m)v_B\sin 40°$

$$C_y = \left(\frac{\Delta m}{\Delta t}\right)v_A + \frac{\Delta m}{\Delta t}v_B\sin 40° = (9.1667)(18 + 18\sin 40°) = 271.06\text{ N}$$

$C_y = 271\text{ N}$ ◀



PROBLEM 14.65

The nozzle discharges water at the rate of 340 gal/min. Knowing the velocity of the water at both A and B has a magnitude of 65 ft/s and neglecting the weight of the vane, determine the components of the reactions at C and D. (1 ft^3 = 7.48 gallons)

SOLUTION

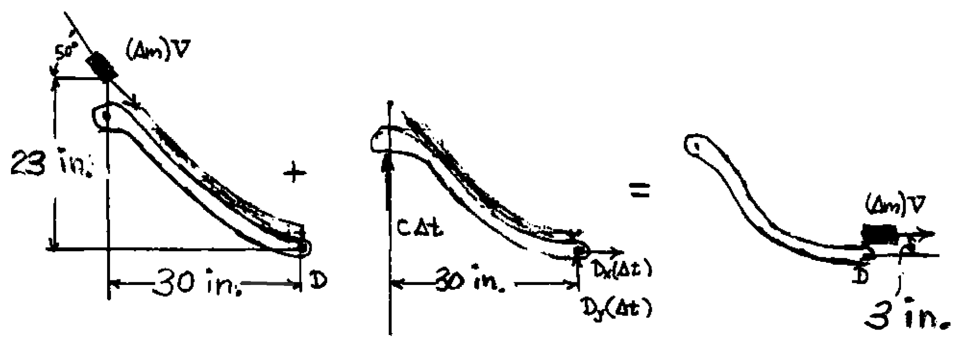
Volumetric flow rate: $Q = 340 \text{ gal/min} \times (1 \text{ ft}^3/7.48 \text{ gal}) \times (1 \text{ min}/60 \text{ sec}) = 0.75758 \text{ ft}^3/\text{s}$

Mass density of water: $\dfrac{\gamma}{g} = \dfrac{62.4 \text{ lb/ft}^3}{32.2 \text{ ft/s}^2}$

Mass flow rate: $\dfrac{dm}{dt} = \dfrac{\gamma}{g} Q = \dfrac{62.4}{32.2}(0.75758) = 1.4681 \text{ lb} \cdot \text{s/ft}$

Assume that the flow speed remains constant.

Principle of impulse and momentum.



+ Moments about D:

$$C = \frac{((30/12)\sin 50° - (23/12)\cos 50° + (3/12))}{(30/12)} \frac{\Delta m}{\Delta t} V$$

$$= 0.37324 \frac{dm}{dt} V$$

$$= (0.37324)(1.4681 \text{ lb} \cdot \text{s/ft})(65 \text{ ft/s}) = 35.617 \text{ lb}$$

$\mathbf{C} = 35.6 \text{ lb} \uparrow$ ◀

$\xrightarrow{+}$ Horizontal components:

$$(\Delta m)V\cos 50° + D_x(\Delta t) = (\Delta m)V$$

$$D_x = (1-\cos 50°)\frac{\Delta m}{\Delta t}V$$

$$= 0.35721\frac{dm}{dt}V$$

$$= (0.35721)(1.4681\text{ lb}\cdot\text{s/ft})(65\text{ ft/s})$$

$$= 34.087\text{ lb}$$

$\mathbf{D}_x = 34.1\text{ N} \longrightarrow$ ◀

$+\uparrow$ Vertical components: $-(\Delta m)V\sin 50° + C(\Delta t) + D_y(\Delta t) = 0$

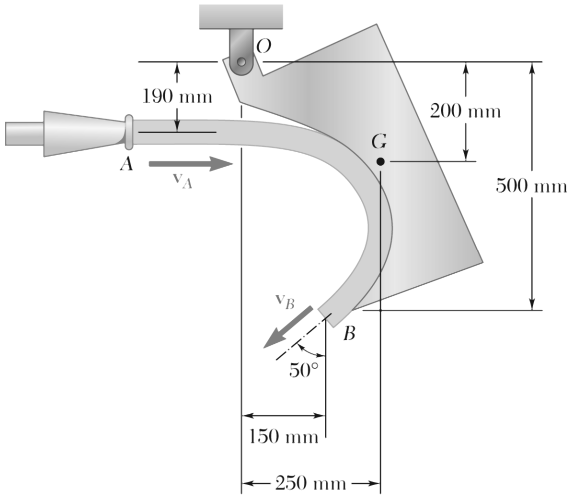
$$D_y = (\sin 50°)\frac{\Delta m}{\Delta t}V - C$$

$$= 0.76604\frac{dm}{dt}V - C$$

$$= (0.76604)(1.4681\text{ lb}\cdot\text{s/ft})(65\text{ ft/s}) - 35.617\text{ N}$$

$$= 37.484\text{ lb}$$

$\mathbf{D}_y = 37.5\text{ lb}\uparrow$ ◀



PROBLEM 14.66

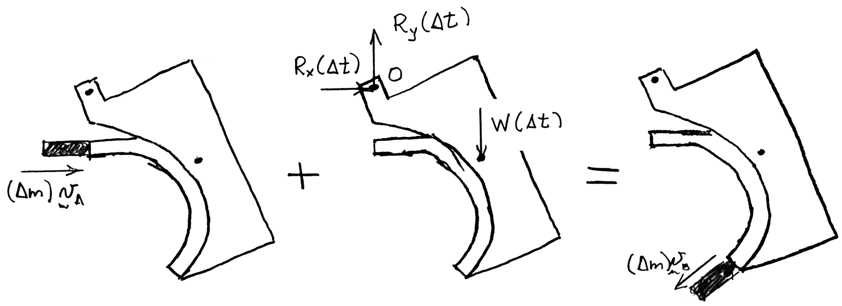
A high speed jet of air issues from the nozzle A with a velocity of $\mathbf{v}_A$ and mass flow rate of 0.36 kg/s. The air impinges on a vane causing it to rotate to the position shown. The vane has a mass of 6-kg. Knowing that the magnitude of the air velocity is equal at A and B determine (*a*) the magnitude of the velocity at A, (*b*) the components of the reactions at O.

SOLUTION

Assume that the speed of the air jet is the same at A and B.

$$v_A = v_B = v$$

Apply the principle of impulse and momentum.



(*a*) $+\circlearrowright$ Moments about O: $(0.190)(\Delta m)v - (0.250)W(\Delta t) = -(0.250)(\Delta m)v\cos 50° - (0.500)(\Delta m)v\sin 50°$

$$v = \frac{\Delta t}{\Delta m} \cdot \frac{0.250W}{0.150\cos 50° + 0.500\sin 50° + 0.190}$$

$$= \frac{0.250W}{0.66944} \frac{dm}{dt}$$

$$= \frac{(0.250)(6)(9.81)}{(0.66944)(0.36)}$$

$$= 61.058 \text{ m/s}$$

$v_A = 61.1$ m/s ◀

PROBLEM 14.66 (Continued)

(*b*) $\xrightarrow{+}$ x components: $(\Delta m)v + R_x(\Delta t) = -(\Delta m)v\sin 50°$

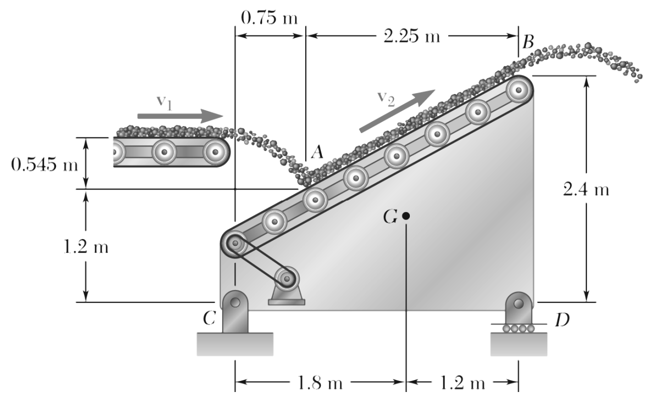
$$R_x = -\frac{\Delta m}{\Delta t}v(1+\sin 50°)$$
$$= -(0.36)(61.058)(1+\sin 50°)$$
$$= -38.82 \text{ N}$$

$+\uparrow$ y components: $0 + R_y(\Delta t) - W(\Delta t) = -(\Delta m)v\cos 50°$

$$R_y = W + \frac{\Delta m}{\Delta t}v\cos 50°$$
$$= (6)(9.81) - (0.36)(61.058)\cos 50°$$
$$= 44.73 \text{ N}$$

$$R = \sqrt{(38.82)^2 + (44.73)^2}$$
$$= 59.2 \text{ N}$$

$\tan\alpha = \dfrac{44.73}{38.82}$ $\alpha = 49.0°$ $\mathbf{R} = 59.2 \text{ N} \measuredangle 49.0°$ ◀



PROBLEM 14.67

Coal is being discharged from a first conveyor belt at the rate of 120 kg/s. It is received at *A* by a second belt which discharges it again at *B*. Knowing that $v_1 = 3$ m/s and $v_2 = 4.25$ m/s and that the second belt assembly and the coal it supports have a total mass of 472 kg, determine the components of the reactions at *C* and *D*.

SOLUTION

Velocity before impact at *A*:

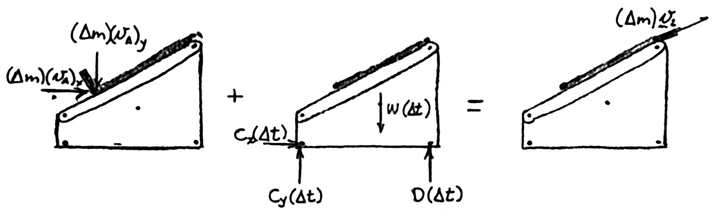
$$(v_A)_x = v_1 = 3 \text{ m/s} \longrightarrow$$

$$(v_A)_y^2 = 2g(\Delta y) = (2)(9.81)(0.545) = 10.693 \text{ m}^2/\text{s}^2 \qquad (v_A)_y = 3.270 \text{ m/s} \downarrow$$

Slope of belt: $\quad \tan\theta = \dfrac{2.4 - 1.2}{2.25}, \qquad \theta = 28.07°$

Velocity of coal leaving at *B*: $\quad \mathbf{v}_2 = 4.25\,(\cos\theta\,\mathbf{i} + \sin\theta\,\mathbf{j})$

Apply the impulse-momentum principle.



$\xrightarrow{+}$ x components: $\quad (\Delta m)(v_A)_x + C_x(\Delta t) = (\Delta m)v_2\cos\theta$

$$C_x = \frac{\Delta m}{\Delta t}[v_2\cos\theta - (v_A)_x] = (120)(4.25\cos 28.07° - 3)$$

$$C_x = 90.0 \text{ N} \longrightarrow$$

$+\circlearrowleft$ moments about C: $\quad (\Delta m)[-1.2(v_A)_x - 0.75(v_A)_y] + 3.00D(\Delta t) - 1.8W(\Delta t) = (\Delta m)[-2.4\;v_2\cos\theta + 3v_2\sin\theta]$

$$\frac{\Delta m}{\Delta t}[-(1.2)(3) - (0.75)(3.270)] + 3D - (1.8)(472)(9.81)$$

$$= \frac{\Delta m}{\Delta t}[-(2.4)(4.25\cos\theta) + (3)(4.25\sin\theta)]$$

$$D = 2775 + 1.0168\frac{dm}{dt} = 2775 + (1.0168)(120) = 2897 \text{ N}\uparrow$$

PROBLEM 14.67 (Continued)

$+\uparrow$ y components: $$(\Delta m)(-v_A)_y + (C_y + D - W)(\Delta t) = (\Delta m)v_2 \sin\theta$$

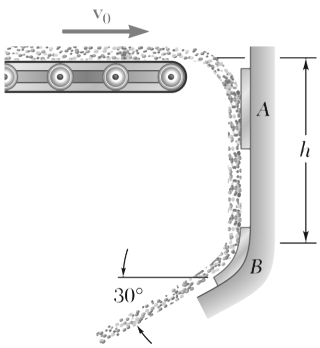
$$C_y + D - W = \frac{\Delta m}{\Delta t}(3.270 + 4.25\sin\theta)$$

$$= (120)(5.268) = 632.2 \text{ N}$$

$$C_y = 4625.6 - 2897 + 632.2 \qquad C_y = 2361 \text{ N}\uparrow$$

$$C_x = 90.0 \text{ N}, \quad C_y = 2360 \text{ N} \blacktriangleleft$$

$$D_x = 0, \quad D_y = 2900 \text{ N} \blacktriangleleft$$

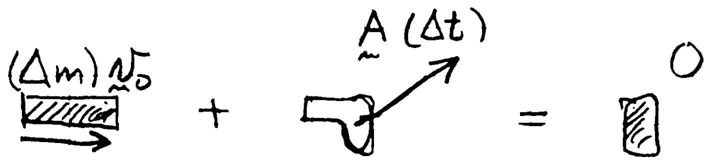


PROBLEM 14.68

A mass q of sand is discharged per unit time from a conveyor belt moving with a velocity $\mathbf{v}_0$. The sand is deflected by a plate at A so that it falls in a vertical stream. After falling a distance h the sand is again deflected by a curved plate at B. Neglecting the friction between the sand and the plates, determine the force required to hold in the position shown (a) plate A, (b) plate B.

SOLUTION

(a) When the sand impacts on plate A, it is momentarily brought to rest. Apply the principle of impulse and momentum to find the force on the sand.



$\xrightarrow{+}$ x component: $\quad (\Delta m)v_0 + A_x(\Delta t) = 0$

$$A_x = -\frac{\Delta m}{\Delta t}v_0 = -qv_0$$

$+\uparrow$ y component: $\quad 0 + A_y(\Delta t) = 0 \qquad A_y = 0$

$$\mathbf{A} = qv_0 \longleftarrow \blacktriangleleft$$

The sand falls vertically. Use conservation of energy for mass element Δm. Let v be the speed at the curved portion of plate B.

$$T_1 + V_1 = T_2 + V_2: \quad 0 + (\Delta m)gh = \frac{1}{2}(\Delta m)v^2 + 0$$

$$v^2 = 2gh$$

$$v = \sqrt{2gh}$$

Over the curved portion of plate B, there is negligible change of elevation. Hence, by conservation of energy, v is both the entrance speed and exit speed of the curved portion of plate B.

(b) Force exerted through plate B:

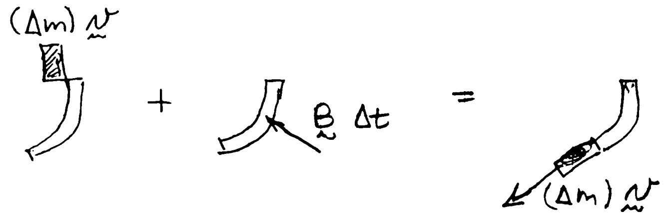
Entrance velocity: $\quad \mathbf{v} = -\sqrt{2gh}\,\mathbf{j}$

Exit velocity: $\quad \mathbf{v}' = \sqrt{2gh}\,(-\cos 30°\mathbf{i} - \sin 30°\mathbf{j})$

Mass flow rate: $\quad \dfrac{dm}{dt} = \dfrac{\Delta m}{\Delta t} = q$

PROBLEM 14.68 (Continued)

Principle of impulse and momentum:



$$(\Delta m)\mathbf{v} + \mathbf{B}(\Delta t) = (\Delta m)\mathbf{v}'$$

$$\mathbf{B} = \left(\frac{\Delta m}{\Delta t}\right)(\mathbf{v}' - \mathbf{v}) = q\sqrt{2gh}(-\cos 30°\mathbf{i} - \sin 30°\mathbf{j} + \mathbf{j})$$

$$B_x = -\sqrt{2gh}\cos 30° = -\frac{\sqrt{3}}{2}\sqrt{2gh}$$

$$B_y = \sqrt{2gh}(1 - \sin 30°) = \frac{1}{2}\sqrt{2gh}$$

$$\mathbf{B} = \sqrt{2gh} \;\searrow\; 30° \blacktriangleleft$$

PROBLEM 14.69

The total drag due to air friction on a jet airplane traveling at 900 km/h is 35 kN. Knowing that the exhaust velocity is 600 m/s relative to the airplane, determine the mass of air which must pass through the engine per second to maintain the speed of 900 km/h in level flight.

SOLUTION

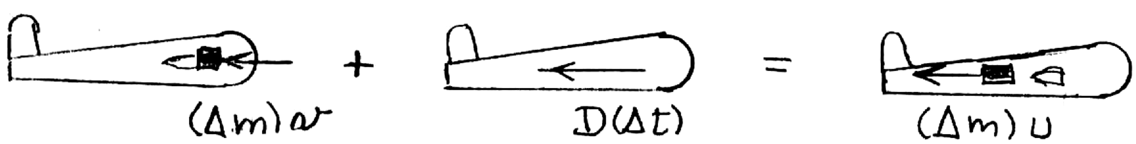
Symbols: $\frac{dm}{dt}$ = mass flow rate

u = exhaust relative to the airplane

v = speed of airplane

D = drag force

Principle of impulse and momentum:



$\xleftarrow{+}$ $(\Delta m)v + D(\Delta t) = (\Delta m)u$

$$\frac{\Delta m}{\Delta t} = \frac{dm}{dt} = \frac{D}{u - v}$$

Data: $v = 900$ km/h $= 250$ m/s

$u = 600$ m/s

$D = 35$ kN $= 35000$ N

$$\frac{dm}{dt} = \frac{35000}{600 - 250}$$

$$\frac{dm}{dt} = 100 \text{ kg/s} \blacktriangleleft$$

PROBLEM 14.70

While cruising in level flight at a speed of 600 mi/h, a jet plane scoops in air at the rate of 200 lb/s and discharges it with a velocity of 2100 ft/s relative to the airplane. Determine the total drag due to air friction on the airplane.

SOLUTION

Flight speed: $v = 600 \text{ mi/h} = 880 \text{ ft/s}$

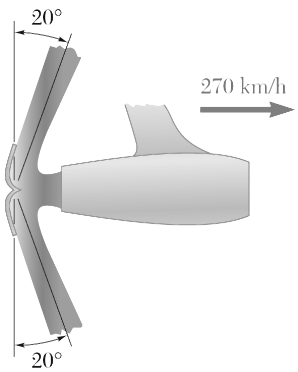
Mass flow rate: $$\frac{dm}{dt} = \frac{200 \text{ lb/s}}{32.2 \text{ ft/s}^2} = 6.2112 \text{ slug/s}$$

$$\Sigma \mathbf{F} = \mathbf{D} = \frac{dm}{dt}(\mathbf{u} - \mathbf{v}) \qquad \text{or} \qquad D = \frac{dm}{dt}(v - u)$$

where, for a frame of reference moving with the plane, v is the free stream velocity (equal to the air speed) and u is the relative exhaust velocity.

$$D = (6.2112)(2100 - 880) = 7577.6 \text{ lb}$$

$D = 7580 \text{ lb}$ ◀



PROBLEM 14.71

In order to shorten the distance required for landing, a jet airplane is equipped with movable vanes, which partially reverse the direction of the air discharged by each of its engines. Each engine scoops in the air at a rate of 120 kg/s and discharges it with a velocity of 600 m/s relative to the engine. At an instant when the speed of the airplane is 270 km/h, determine the reverse thrust provided by each of the engines.

SOLUTION

Apply the impulse-momentum principle to the moving air. Use a frame of reference that is moving with the airplane. Let **F** be the force on the air.

$$v = 270 \text{ km/h} = 75 \text{ m/s}$$

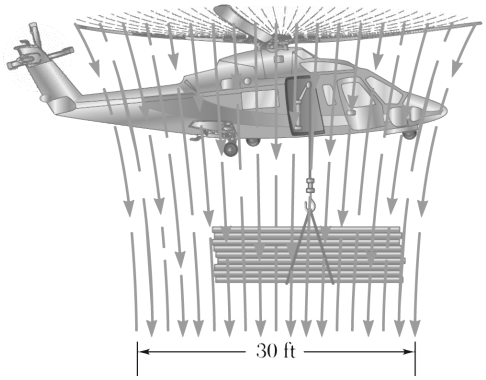
$$u = 600 \text{ m/s}$$

$$-(\Delta m)v + F(\Delta t) = 2\frac{(\Delta m)}{2}u\sin 20°$$

$$F = \frac{\Delta m}{\Delta t}(v + u\sin 20°) = \frac{dm}{dt}(v + u\sin 20°)$$

$$F = (120)(75 + 600\sin 20°) = 33.6\times 10^3 \text{ N}$$

Force on airplane is $-\mathbf{F}$. **F** = 33.6 kN ⟵ ◀



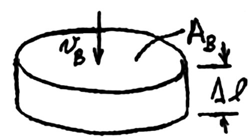
PROBLEM 14.72

The helicopter shown can produce a maximum downward air speed of 80 ft/s in a 30-ft-diameter slipstream. Knowing that the weight of the helicopter and its crew is 3500 lb and assuming $\gamma = 0.076\ \text{lb/ft}^3$ for air, determine the maximum load that the helicopter can lift while hovering in midair.

SOLUTION

The thrust is
$$F = \frac{dm}{dt}(v_B - v_A)$$

Calculation of $\dfrac{dm}{dt}$. mass = density × volume = density × area × length



$$\Delta m = \rho A_B(\Delta l) = \rho A_B v_B(\Delta t)$$
$$\frac{\Delta m}{\Delta t} = \rho A_B v_B = \frac{\gamma}{g} A_B v_B = \frac{dm}{dt}$$

where A_B is the area of the slipstream well below the helicopter and v_B is the corresponding velocity in the slipstream. Well above the blade, $v_A \approx 0$.

Hence,
$$F = \frac{\gamma}{g} A_B v_B^2$$
$$= \left(\frac{0.076\ \text{lb/ft}^3}{32.2\ \text{ft/s}^2}\right)\left(\frac{\pi}{4}\right)(30\ \text{ft})^2(80\ \text{ft/s})^2$$
$$= 10{,}678\ \text{lb}$$
$$F = 10{,}678\ \text{lb}\downarrow$$

The force on the helicopter is 10,678 lb $\uparrow$.

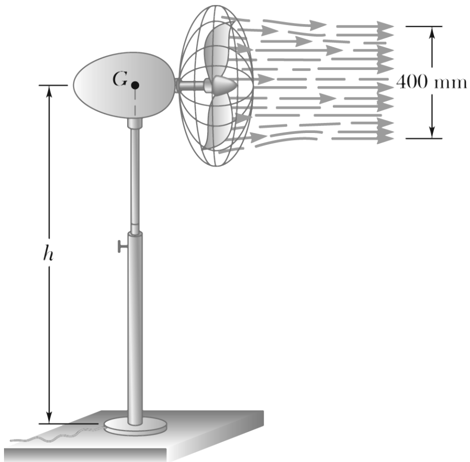
Weight of helicopter: $\mathbf{W}_H = 3500\ \text{lb}\downarrow$

Weight of payload: $\mathbf{W}_P = W_P\downarrow$

Statics: $+\uparrow\ \Sigma F_y = F - W_H - W_P = 0$

$$W_P = F - W_H = 10{,}678 - 3500 = 7178\ \text{lb}$$

$W = 7180\ \text{lb}$ ◀

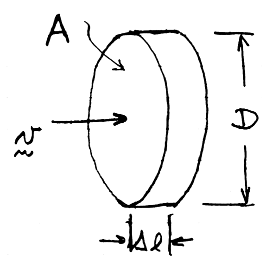


PROBLEM 14.73

A floor fan designed to deliver air at a maximum velocity of 6 m/s in a 400-mm-diameter slipstream is supported by a 200-mm-diameter circular base plate. Knowing that the total weight of the assembly is 60 N and that its center of gravity is located directly above the center of the base plate, determine the maximum height h at which the fan may be operated if it is not to tip over. Assume $\rho = 1.21\ \text{kg/m}^3$ for air and neglect the approach velocity of the air.

SOLUTION

Calculation of $\frac{dm}{dt}$ at a section in the airstream:



$$\text{mass} = \text{density} \times \text{volume} = \text{density} \times \text{area} \times \text{length}$$

$$\Delta m = \rho A(\Delta l) = \rho A v(\Delta t)$$

$$\frac{\Delta m}{\Delta t} = \frac{dm}{dt} = \rho A v$$

Thrust on the airstream:

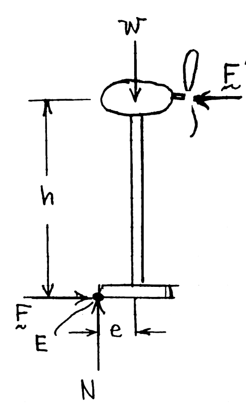
$$\mathbf{F} = \frac{dm}{dt}(\mathbf{v}_B - \mathbf{v}_A)$$

where $\mathbf{v}_B$ is the velocity just downstream of the fan and $\mathbf{v}_A$ is the velocity for upstream. Assume that $\mathbf{v}_A$ is negligible.

$$F = (\rho A v)v = \rho\left(\frac{\pi}{4}D^2\right)v^2$$

$$F = (1.21\ \text{kg/m}^3)\left(\frac{\pi}{4}\right)(0.400\ \text{m})^2(6\ \text{m/s})^2 = 5.474\ \text{N}$$

$$\mathbf{F} = 5.474\ \text{N} \longrightarrow$$



Force on fan: $\mathbf{F}' = -\mathbf{F} = 5.474\ \text{N} \longleftarrow$

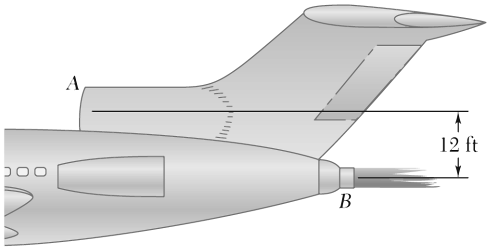
Maximum height h: $\left(e = \frac{1}{2}d = 100\ \text{mm} = 0.1\ \text{m}\right)$

$$+\circlearrowleft\ \Sigma M_E = 0$$

$$F'h - We = 0$$

$$h = \frac{We}{F'} = \frac{(60)(0.1)}{5.474}$$

$h = 1.096\ \text{m}$ ◀

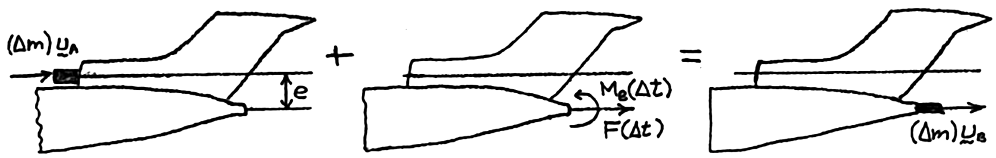


PROBLEM 14.74

The jet engine shown scoops in air at A at a rate of 200 lb/s and discharges it at B with a velocity of 2000 ft/s relative to the airplane. Determine the magnitude and line of action of the propulsive thrust developed by the engine when the speed of the airplane is (*a*) 300 mi/h, (*b*) 600 mi/h.

SOLUTION

Use a frame of reference moving with the plane. Apply the impulse-momentum principle. Let F be the force that the plane exerts on the air.



$\xrightarrow{+}$ x components: $(\Delta m)u_A + F(\Delta t) = (\Delta m)u_B$

$$F = \frac{\Delta m}{\Delta t}(u_B - u_A) = \frac{dm}{dt}(u_B - u_A) \quad (1)$$

$+\circlearrowleft$ moments about B: $-e(\Delta m)u_A + M_B(\Delta t) = 0$

$$M_B = e\frac{dm}{dt}u_A \quad (2)$$

Let d be the distance that the line of action is below B.

$$Fd = M_B \qquad d = \frac{M_B}{F} = \frac{eu_A}{u_B - u_A} \quad (3)$$

Data: $\frac{dm}{dt} = 200 \text{ lb/s} = \frac{200}{32.2} = 6.2112 \text{ slugs/s}, \quad u_B = 2000 \text{ ft/s}, \quad e = 12 \text{ ft}$

(*a*) $u_A = 300 \text{ mi/h} = 440 \text{ ft/s}$

From Eq. (1), $F = (6.2112)(2000 - 440)$ $F = 9690 \text{ lb}$ ◀

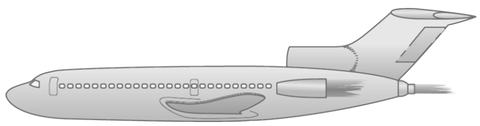
From Eq. (3), $d = \frac{(12)(440)}{2000 - 440}$ $d = 3.38 \text{ ft}$ ◀

(*b*) $u_A = 600 \text{ mi/h} = 880 \text{ ft/s}$

From Eq. (1), $F = (6.2112)(2000 - 880)$ $F = 6960 \text{ lb}$ ◀

From Eq. (3), $d = \frac{(12)(880)}{2000 - 880}$ $d = 9.43 \text{ ft}$ ◀

PROBLEM 14.75



A jet airliner is cruising at a speed of 900 km/h with each of its three engines discharging air with a velocity of 800 m/s relative to the plane. Determine the speed of the airliner after it has lost the use of (*a*) one of its engines, (*b*) two of its engines. Assume that the drag due to air friction is proportional to the square of the speed and that the remaining engines keep operating at the same rate.

SOLUTION

Let v be the airliner speed and u be the discharge relative velocity.

$$u = 800 \text{ m/s.}$$

Thrust formula for one engine: $$F = \frac{dm}{dt}(u - v)$$

Drag formula: $$D = kv^2$$

Three engines working. Cruising speed $= v_0 = 900$ km/h $= 250$ m/s

$$3F - D = 3\frac{dm}{dt}(u - v_0) - kv_0^2 = 0$$

$$\frac{dm}{dt} = \frac{kv_0^2}{3(u - v_0)} = \frac{k(250)^2}{3(800 - 250)} = 37.879k$$

(*a*) *One engine fails*. Two engines working. Cruising speed $= v_1$

$$2F - D = 2\frac{dm}{dt}(u - v_1) - kv_1^2 = 0$$

$$(2)(37.879k)(800 - v_1) - kv_1^2 = 0$$

$$v_1^2 + 75.758v_1 - 60.606 \times 10^3 = 0$$

$$v_1 = 211.20 \text{ m/s} \qquad v_1 = 760 \text{ km/h} \blacktriangleleft$$

(*b*) *Two engines fail*. One engine working. Cruising speed $= v_2$

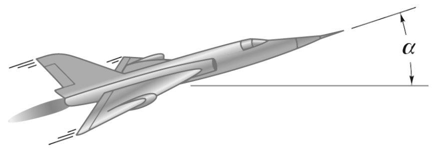
$$F - D = \frac{dm}{dt}(u - v_2) - kv_2^2 = 0$$

$$(37.879k)(800 - v_2) - kv_2^2 = 0$$

$$v_2^2 + 37.879v_2 - 30.303 \times 10^3 = 0$$

$$v_2 = 156.17 \text{ m/s} \qquad v_2 = 562 \text{ km/h} \blacktriangleleft$$

PROBLEM 14.76



A 16-Mg jet airplane maintains a constant speed of 774 km/h while climbing at an angle $\alpha = 18°$. The airplane scoops in air at a rate of 300 kg/s and discharges it with a velocity of 665 m/s relative to the airplane. If the pilot changes to a horizontal flight while maintaining the same engine setting, determine (*a*) the initial acceleration of the plane, (*b*) the maximum horizontal speed that will be attained. Assume that the drag due to air friction is proportional to the square of the speed.

SOLUTION

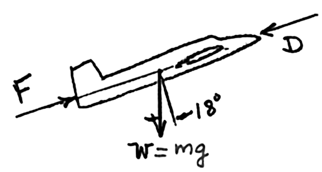
Calculate the propulsive force using velocities relative to the airplane.

$$F = \frac{dm}{dt}(v_B - v_A)$$

Data:

$$\frac{dm}{dt} = 300 \text{ kg/s}$$

$$v_A = 774 \text{ km/h}$$
$$= 215 \text{ m/s}$$
$$v_B = 665 \text{ m/s}$$
$$F = (300)(665 - 215)$$
$$= 135,000 \text{ N}$$



Since there is no acceleration while the airplane is climbing, the forces are in equilibrium.

$$+\measuredangle 18° \Sigma F = 0: \quad F - D - mg\sin\alpha = 0$$

$$F - D = mg\sin\alpha = (16,000)(9.8)\sin 18° = 48,454 \text{ N}$$

(*a*) *Initial acceleration of airplane in horizontal flight*:

$$ma = F - D: \quad 16,000a = 48.454 \times 10^3$$

$\mathbf{a} = 3.03 \text{ m/s}^2 \measuredangle 18°$ ◀

Corresponding drag force:

$$D = 135,000 - 48,454$$
$$= 86,546 \text{ N}$$

Drag force factor:

$$D = kv_A^2$$

or

$$k = \frac{D}{v_A^2}$$
$$= \frac{86,546}{(215)^2}$$
$$= 1.87228 \text{ N}\cdot\text{s}^2/\text{m}^2$$

PROBLEM 14.76 (Continued)

(*b*) *Maximum speed in horizontal flight*:

Since the acceleration is zero, the forces are in equilibrium.

$$F - D = 0$$

$$\frac{dm}{dt}(v_B - v_A) - kv_A^2 = 0 \quad kv_A^2 + \frac{dm}{dt}v_A - \frac{dm}{dt}v_B = 0$$

$$1.87228v_A^2 + 300v_A - (300)(665) = 0$$

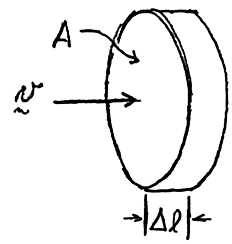
$$v_A = 256.0 \text{ m/s} \qquad v_A = 922 \text{ km/h} \blacktriangleleft$$

PROBLEM 14.77

The propeller of a small airplane has a 2-m-diameter slipstream and produces a thrust of 3600 N when the airplane is at rest on the ground. Assuming $\rho = 1.225 \text{ kg/m}^3$ for air, determine (*a*) the speed of the air in the slipstream, (*b*) the volume of air passing through the propeller per second, (*c*) the kinetic energy imparted per second to the air in the slipstream.

SOLUTION

Calculation of $\frac{dm}{dt}$ at a section in the airstream:



$$\text{mass} = \text{density} \times \text{volume}$$
$$= \text{density} \times \text{area} \times \text{length}$$
$$\Delta m = \rho A(\Delta l) = \rho A v \, \Delta t$$
$$\frac{\Delta m}{\Delta t} = \frac{dm}{dt} = \rho A v$$

(*a*) Thrust $= \frac{dm}{dt}(\mathbf{v}_B - \mathbf{v}_A)$ where $\mathbf{v}_B$ is the velocity just downstream of propeller and $\mathbf{v}_A$ is the velocity far upstream. Assume $\mathbf{v}_A$ is negligible.

$$\text{Thrust} = (\rho A v)v = \rho\left(\frac{\pi}{4}D^2\right)v^2$$
$$3600 = 1.225\left(\frac{\pi}{4}\right)(2)^2 v^2$$
$$v^2 = 935.44$$
$$v = 30.585 \text{ m/s}$$

$v = 30.6 \text{ m/s}$ ◀

(*b*)
$$Q = \frac{1}{\rho}\frac{dm}{dt} = Av = \left(\frac{\pi}{4}D^2\right)v = \frac{\pi}{4}(2)^2(30.585) = 96.086$$

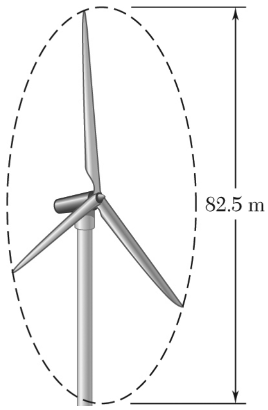
$Q = 96.1 \text{ m}^3/\text{s}$ ◀

(*c*) Kinetic energy of mass Δm:

$$\Delta T = \frac{1}{2}(\Delta m)v^2 = \frac{1}{2}\rho A(\Delta l)v^2 = \frac{1}{2}\rho A v(\Delta t)v^3$$
$$\frac{\Delta T}{\Delta t} = \frac{dT}{dt}$$
$$= \frac{1}{2}\rho A v^3$$
$$= \frac{1}{2}\rho\left(\frac{\pi}{4}D^2\right)v^3$$
$$= \frac{1}{2}(1.225)\left(\frac{\pi}{4}\right)(2)^2(30.585)^3$$
$$= 55{,}053 \text{ N}\cdot\text{m/s}$$

$\frac{dT}{dt} = 55{,}100 \text{ N}\cdot\text{m/s}$ ◀

PROBLEM 14.78



The wind turbine-generator shown has an output-power rating of 1.5 MW for a wind speed of 36 km/h. For the given wind speed, determine (*a*) the kinetic energy of the air particles entering the 82.5-m-diameter circle per second, (*b*) the efficiency of this energy conversion system. Assume $\rho = 1.21$ kg/m^3 for air.

SOLUTION

(*a*) Rate of kinetic energy in the slipstream.

Let Δm be the mass moving through the slipstream of area A in the time Δt. Then,

$$\Delta m = \rho A(\Delta l) = \rho A v(\Delta t)$$

The kinetic energy carried by this mass is

$$\Delta t = \frac{1}{2}(\Delta m)v^2 = \frac{1}{2}\rho A v^3(\Delta t)$$

$$\frac{dT}{dt} = \frac{\Delta T}{\Delta t} = \frac{1}{2}\rho A v^3$$

Data:

$$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(82.5 \text{ m})^2 = 5345.6 \text{ m}^2$$

$$v = 36 \text{ km/h} = 10 \text{ m/s}$$

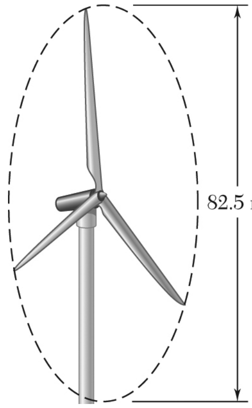
$$\frac{dT}{dt} = \frac{1}{2}(1.21 \text{ kg/m}^3)(5345.6 \text{ m}^2)(10 \text{ m/s})^3$$

$$= 3.234 \times 10^6 \text{ kg} \cdot \text{m}^2/\text{s}^3$$

$$\frac{dT}{dt} = 3.234 \text{ MW} \blacktriangleleft$$

(*b*) Efficiency *n*:

$$\eta = \frac{\text{output power}}{\text{available input power}} = \frac{1.5 \text{ MW}}{3.234 \text{ MW}} \qquad \eta = 0.464 \blacktriangleleft$$



PROBLEM 14.79

A wind turbine-generator system having a diameter of 82.5 m produces 1.5 MW at a wind speed of 12 m/s. Determine the diameter of blade necessary to produce 10 MW of power assuming the efficiency is the same for both designs and $\rho = 1.21 \text{ kg/m}^3$ for air.

SOLUTION

Rate of kinetic energy in the slipstream.

Let Δm be the mass moving through the slipstream of area A in time Δt. Then

$$\Delta m = \rho A(\Delta l) = \rho A v(\Delta t)$$

The kinetic energy carried by this mass is

$$\Delta T = \frac{1}{2}(\Delta m)v^2 = \frac{1}{2}\rho A v^3(\Delta t)$$

$$\frac{dT}{dt} = \frac{\Delta T}{\Delta t} = \frac{1}{2}\rho A v^3$$

This is the available input power for the wind turbine. For a wind turbine of efficiency η, the output power P is

$$P = \eta\frac{dT}{dt} = \frac{\eta}{2}\rho A v^3$$

We want to compare two turbines having $P_1 = 1.5 \text{ MW}$ and $P_2 = 10 \text{ MW}$, respectively. Then

$$\frac{P_2}{P_1} = \frac{\eta_2 \rho_2 A_2 v_2^3}{\eta_1 \rho_1 A_1 v_1^3}$$

Since $\eta_2 = \eta_1$, $\rho_2 = \rho_1$, and $v_2 = v_1$, we get

$$\frac{P_2}{P_1} = \frac{A_2}{A_1} = \frac{d_2^2}{d_1^2} = \frac{10}{1.5} = 6.6667$$

$$d_2^2 = 6.6667 d_1^2 = (6.6667)(82.5 \text{ in})^2$$

$$d_2 = 213 \text{ m} \blacktriangleleft$$

PROBLEM 14.80

While cruising in level flight at a speed of 570 mi/h, a jet airplane scoops in air at a rate of 240 lb/s and discharges it with a velocity of 2200 ft/s relative to the airplane. Determine (*a*) the power actually used to propel the airplane, (*b*) the total power developed by the engine, (*c*) the mechanical efficiency of the airplane.

SOLUTION

Data:
$$\frac{dm}{dt} = \frac{240}{32.2} = 7.4534 \text{ slugs/s}$$
$$u = 2200 \text{ ft/s}$$
$$v = 570 \text{ mi/h} = 836 \text{ ft/s}$$
$$F = \frac{dm}{dt}(u - v)$$
$$= (7.4534)(2200 - 836)$$
$$= 10{,}166 \text{ lb}$$

(*a*) Power used to propel airplane:

$$P_1 = Fv$$
$$= (10{,}166)(836)$$
$$= 8.499 \times 10^6 \text{ ft} \cdot \text{lb/s}$$

Propulsion power $= 15{,}450$ hp ◀

Power of kinetic energy of exhaust:

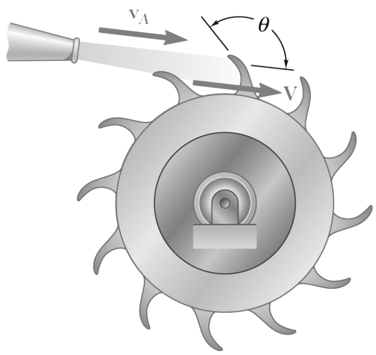
$$P_2(\Delta t) = \frac{1}{2}(\Delta m)(u - v)^2$$
$$P_2 = \frac{1}{2}\frac{dm}{dt}(u - v)^2$$
$$= \frac{1}{2}(7.4534)(2200 - 836)^2$$
$$= 6.934 \times 10^6 \text{ ft} \cdot \text{lb/s}$$

(*b*) Total power:
$$P = P_1 + P_2$$
$$= 15.433 \times 10^6 \text{ ft} \cdot \text{lb/s}$$

Total power $= 28{,}060$ hp ◀

(*c*) Mechanical efficiency:
$$\frac{P_1}{P} = \frac{8.499 \times 10^6}{15.433 \times 10^6}$$
$$= 0.551$$

Mechanical efficiency $= 0.551$ ◀



PROBLEM 14.81

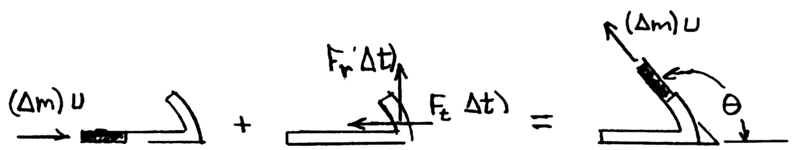
In a Pelton-wheel turbine, a stream of water is deflected by a series of blades so that the rate at which water is deflected by the blades is equal to the rate at which water issues from the nozzle ($\Delta m/\Delta t = A\rho v_A$). Using the same notation as in Sample Problem 14.7, (*a*) determine the velocity **V** of the blades for which maximum power is developed, (*b*) derive an expression for the maximum power, (*c*) derive an expression for the mechanical efficiency.

SOLUTION

Let **u** be the velocity of the stream relative to the velocity of the blade.

$$u = (v - V)$$

Mass flow rate: $$\frac{\Delta m}{\Delta t} = \rho A v_A$$



Principle of impulse and momentum:

$$\xrightarrow{+} (\Delta m)u - F_t(\Delta t) = (\Delta m)u\cos\theta$$

$$F_t = \frac{\Delta m}{\Delta t}u(1-\cos\theta)$$

$$= \rho A v_A(v_A - V)(1-\cos\theta)$$

where F_t is the tangential force on the fluid.

The force F_t on the fluid is directed to the left as shown. By Newton's law of action and reaction, the tangential force on the blade is F_t to the right.

Output power: $$P_{\text{out}} = F_t V$$

$$= \rho A v_A(v_A - V)V(1-\cos\theta)$$

(*a*) V for maximum power output:

$$\frac{dP_{\text{out}}}{dV} = \rho A(v_A - 2V)(1-\cos\theta) = 0 \qquad v_A = \frac{1}{2}V \blacktriangleleft$$

(*b*) Maximum power:

$$(P_{\text{out}})_{\max} = \rho A v_A\left(v_A - \frac{1}{2}v_A\right)\left(\frac{1}{2}v_A\right)(1-\cos\theta)$$

$$(P_{\text{out}})_{\max} = \frac{1}{4}\rho A v_A^3(1-\cos\theta) \blacktriangleleft$$

PROBLEM 14.81 (Continued)

Input power = rate of supply of kinetic energy of the stream

$$P_{\text{in}} = \frac{1}{\Delta t}\left[\frac{1}{2}(\Delta m)v_A^2\right]$$

$$= \frac{1}{2}\frac{\Delta m}{\Delta t}v_A^2$$

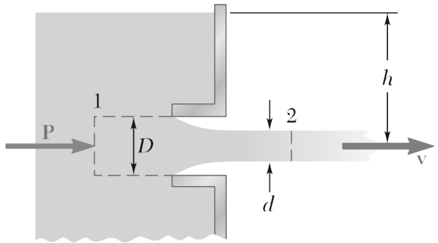
$$= \frac{1}{2}\rho A v_A^3$$

(*c*) Efficiency:

$$\eta = \frac{P_{\text{out}}}{P_{\text{in}}}$$

$$\eta = \frac{\rho A v_A (v_A - V)V(1-\cos\theta)}{\frac{1}{2}\rho A v_A^3}$$

$$\eta = 2\left(1 - \frac{V}{v_A}\right)\frac{V}{v_A}(1-\cos\theta) \blacktriangleleft$$



PROBLEM 14.82

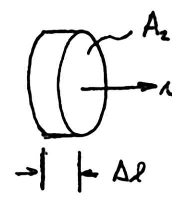
A circular reentrant orifice (also called Borda's mouthpiece) of diameter D is placed at a depth h below the surface of a tank. Knowing that the speed of the issuing stream is $v = \sqrt{2gh}$ and assuming that the speed of approach v_1 is zero, show that the diameter of the stream is $d = D/\sqrt{2}$. (*Hint*: Consider the section of water indicated, and note that P is equal to the pressure at a depth h multiplied by the area of the orifice).

SOLUTION

From hydrostatics, the pressure at section 1 is $p_1 = \gamma h = \rho g h.$

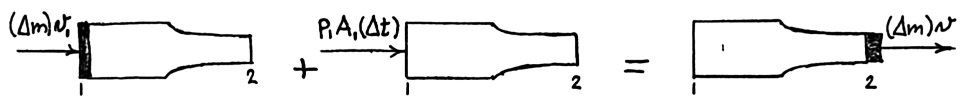
The pressure at section 2 is $p_2 = 0.$

Calculate the mass flow rate using section 2.



$$\begin{aligned} \text{mass} &= \text{density} \times \text{volume} \\ &= \text{density} \times \text{area} \times \text{length} \\ \Delta m &= \rho A_2 (\Delta l) = \rho A_2 v (\Delta t) \\ \frac{dm}{dt} &= \frac{\Delta m}{\Delta t} = \rho A_2 v \end{aligned}$$

Apply the impulse-momentum principle to fluid between sections 1 and 2.



$$(\Delta m)v_1 + p_1 A_1 (\Delta t) = (\Delta m) v$$

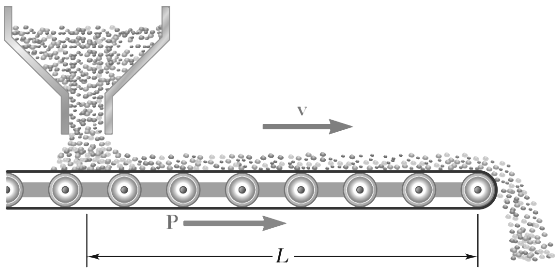
$$\frac{dm}{dt} v_1 + p_1 A_1 = \frac{dm}{dt} v$$

$$\begin{aligned} p_1 A_1 &= \frac{dm}{dt}(v - v_1) \\ &= \rho A_2 v (v - v_1) \end{aligned}$$

But v_1 is negligible, $\quad p_1 = \rho g h \quad \text{and} \quad v = \sqrt{2gh}$

$$\rho g h A_1 = \rho A_2 (2gh) \quad \text{or} \quad A_1 = 2A_2$$

$$\frac{\pi}{4} D^2 = 2\left(\frac{\pi}{4} d^2\right) \qquad d = \frac{D}{\sqrt{2}} \blacktriangleleft$$

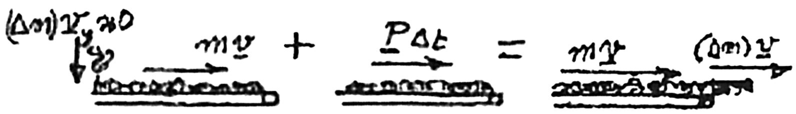


PROBLEM 14.83

Gravel falls with practically zero velocity onto a conveyor belt at the constant rate $q = dm/dt$. (*a*) Determine the magnitude of the force **P** required to maintain a constant belt speed v. (*b*) Show that the kinetic energy acquired by the gravel in a given time interval is equal to half the work done in that interval by the force **P**. Explain what happens to the other half of the work done by **P**.

SOLUTION

(*a*) We apply the impulse-momentum principle to the gravel on the belt and to the mass Δm of gravel hitting and leaving belt in interval Δt.



$\xrightarrow{+}$ x comp.: $mv + P\Delta t = mv + (\Delta m)v$

$$P = \frac{\Delta m}{\Delta t} v = qv \qquad\qquad P = qv \blacktriangleleft$$

(*b*) Kinetic energy acquired for unit time:

$$\Delta T = \frac{1}{2}(\Delta m)v^2$$

$$\frac{\Delta T}{\Delta t} = \frac{1}{2}\frac{\Delta m}{\Delta t}v^2 = \frac{1}{2}qv^2 \qquad (1)$$

Work done per unit time:

$$\frac{\Delta U}{\Delta t} = \frac{P\Delta x}{\Delta t} = Pv$$

Recalling the result of part *a*:

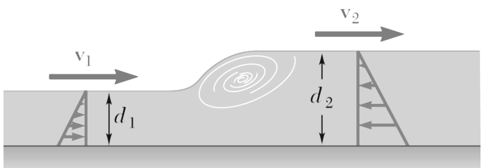
$$\Delta U = P(\Delta x)$$

$$\frac{\Delta U}{\Delta t} = (qv)v = qv^2 \qquad (2)$$

Comparing Eqs. (1) and (2), we conclude that

$$\frac{\Delta T}{\Delta t} = \frac{1}{2}\frac{\Delta U}{\Delta t} \qquad \text{Q.E.D.} \blacktriangleleft$$

The other half of the work of **P** is dissipated into heat by friction as the gravel slips on the belt before reaching the speed v. ◀

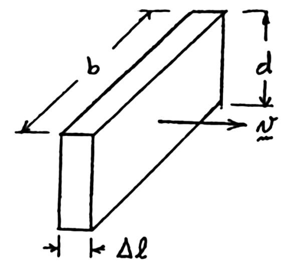


PROBLEM 14.84*

The depth of water flowing in a rectangular channel of width b at a speed v_1 and a depth d_1 increases to a depth d_2 at a *hydraulic jump*. Express the rate of flow Q in terms of b, d_1, and d_2.

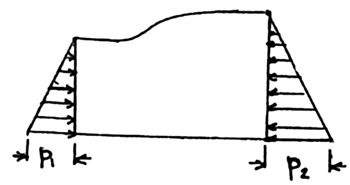
SOLUTION

Mass flow rate:



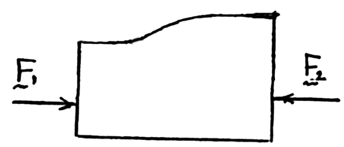
$$\text{mass} = \text{density} \times \text{volume}$$
$$= \text{density} \times \text{area} \times \text{length}$$
$$\Delta m = \rho b d(\Delta l) = \rho b d v(\Delta t)$$
$$\frac{dm}{dt} = \frac{\Delta m}{\Delta t} = \rho b d v$$
$$Q = \frac{1}{\rho}\frac{dm}{dt} = bdv$$

Continuity of flow:



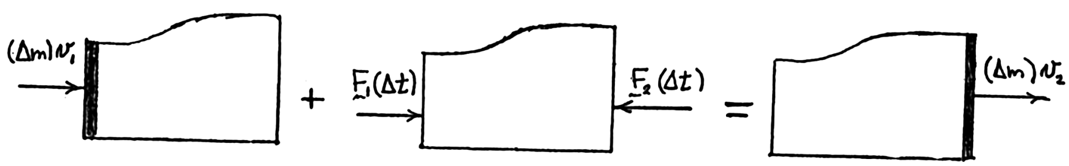
$$Q_1 = Q_2 = Q$$
$$v_1 = \frac{Q}{bd_1} \qquad v_2 \frac{Q}{bd_2}$$

Resultant pressure forces:



$$p_1 = \gamma d_1 \qquad p_2 = \gamma d_2$$
$$F_1 = \frac{1}{2}p_1 b d_1 = \frac{1}{2}\gamma b d_1^2$$
$$F_2 = \frac{1}{2}p_2 b d_2 = \frac{1}{2}\gamma b d_2^2$$

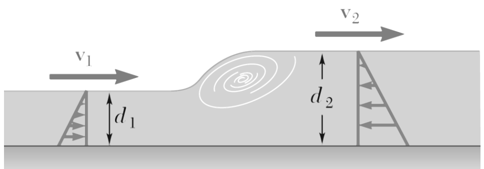
Apply impulse-momentum principle to water between sections 1 and 2.



$$(\Delta m)v_1 + F_1(\Delta t) - F_2(\Delta t) = (\Delta m)v_2$$
$$\frac{\Delta m}{\Delta t}(v_1 - v_2) = F_2 - F_1 \qquad \rho Q\cdot\left(\frac{Q}{bd_1} - \frac{Q}{bd_2}\right) = \frac{1}{2}\gamma b\left(d_2^2 - d_1^2\right)$$
$$\frac{\rho Q^2(d_2 - d_1)}{bd_1d_2} = \frac{1}{2}\gamma b(d_1 + d_2)(d_2 - d_1)$$

Noting that $\gamma = \rho g$,
$$Q = b\sqrt{\frac{1}{2}g d_1 d_2 (d_1 + d_2)} \blacktriangleleft$$

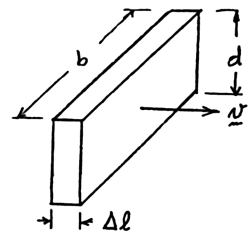
PROBLEM 14.85*



Determine the rate of flow in the channel of Problem 14.84, knowing that $b = 12$ ft, $d_1 = 4$ ft, and $d_2 = 5$ ft.

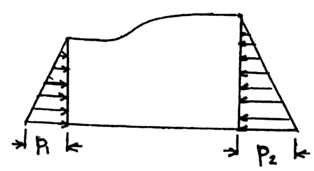
PROBLEM 14.84 The depth of water flowing in a rectangular channel of width b at a speed v_1 and a depth d_1 increases to a depth d_2 at a *hydraulic jump*. Express the rate of flow Q in terms of b, d_1, and d_2.

SOLUTION



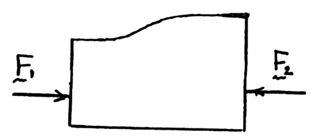
Mass flow rate:

$$\text{mass} = \text{density} \times \text{volume}$$
$$= \text{density} \times \text{area} \times \text{length}$$
$$\Delta m = \rho bd(\Delta l) = \rho bdv(\Delta t)$$
$$\frac{dm}{dt} = \frac{\Delta m}{\Delta t} = \rho bdv$$
$$Q = \frac{1}{\rho}\frac{dm}{dt} = bdv$$



Continuity of flow:

$$Q_1 = Q_2 = Q$$
$$v_1 = \frac{Q}{bd_1} \qquad v_2 \frac{Q}{bd_2}$$



Resultant pressure forces:

$$p_1 = \gamma d_1 \qquad p_2 = \gamma d_2$$
$$F_1 = \frac{1}{2}p_1 bd_1 = \frac{1}{2}\gamma bd_1^2$$
$$F_2 = \frac{1}{2}p_2 bd_2 = \frac{1}{2}\gamma bd_2^2$$

Apply impulse-momentum principle to water between sections 1 and 2.

$$(\Delta m)v_1 + F_1(\Delta t) - F_2(\Delta t) = (\Delta m)v_2$$
$$\frac{\Delta m}{\Delta t}(v_1 - v_2) = F_2 - F_1 \quad \rho Q \cdot \left(\frac{Q}{bd_1} - \frac{Q}{bd_2}\right) = \frac{1}{2}\gamma b\left(d_2^2 - d_1^2\right)$$
$$\frac{\rho Q^2(d_2 - d_1)}{bd_1 d_2} = \frac{1}{2}\gamma b(d_1 + d_2)(d_2 - d_1)$$

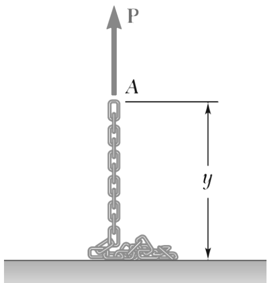
PROBLEM 14.85* (Continued)

Noting that $\gamma = \rho g$, $$Q = b\sqrt{\frac{1}{2} g d_1 d_2 (d_1 + d_2)}$$

Data: $g = 32.2\ \text{ft/s},^2 \quad b = 12\ \text{ft}, \quad d_1 = 4\ \text{ft}, \quad d_2 = 5\ \text{ft}$

$$Q = 12\sqrt{\frac{1}{2}(32.2)(4)(5)(9)}$$

$Q = 646\ \text{ft}^3/\text{s}$ ◀



PROBLEM 14.86

A chain of length l and mass m lies in a pile on the floor. If its end A is raised vertically at a constant speed v, express in terms of the length y of chain which is off the floor at any given instant (*a*) the magnitude of the force **P** applied at A, (*b*) the reaction of the floor.

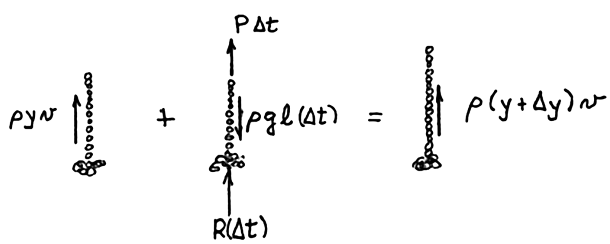
SOLUTION

Let ρ be the mass per unit length of chain. Apply the impulse-momentum to the entire chain. Assume that the reaction from the floor is equal to the weight of chain still in contact with the floor.

Calculate the floor reaction.

$$R = \rho g(l - y)$$

$$R = mg\left(1 - \frac{y}{l}\right)$$



Apply the impulse-momentum principle.

$$\rho yv + P(\Delta t) + R(\Delta t) - \rho gL(\Delta t) = \rho(y + \Delta y)v$$

$$P\Delta t = \rho(\Delta y)v + \rho gL(\Delta t) - R(\Delta t)$$

(*a*)

$$P = \rho\frac{\Delta y}{\Delta t}v + \rho gL - \rho(L - y)g$$

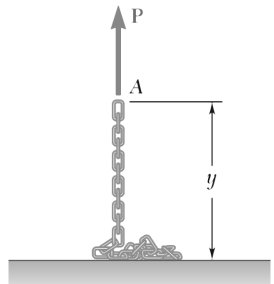
$$= \rho v^2 + \rho gy \qquad P = \frac{m}{l}(v^2 + gy) \blacktriangleleft$$

Let

$$\frac{\Delta y}{\Delta t} = \frac{dy}{dt} = v$$

(*b*) From above,

$$\mathbf{R} = mg\left(1 - \frac{y}{l}\right)\uparrow \blacktriangleleft$$



PROBLEM 14.87

Solve Problem 14.86, assuming that the chain is being *lowered* to the floor at a constant speed v.

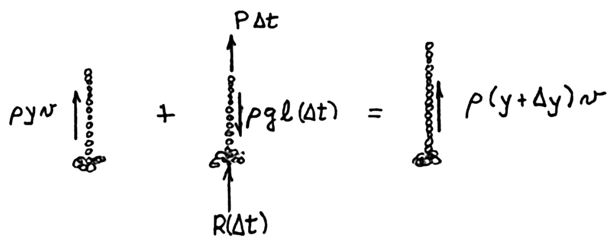
PROBLEM 14.86 A chain of length l and mass m lies in a pile on the floor. If its end A is raised vertically at a constant speed v, express in terms of the length y of chain which is off the floor at any given instant (a) the magnitude of the force **P** applied at A, (b) the reaction of the floor.

SOLUTION

(a) Let ρ be the mass per unit length of chain. The force P supports the weight of chain still off the floor.

$$P = \rho g y \qquad\qquad P = \frac{mgy}{l} \blacktriangleleft$$

(b) Apply the impulse-momentum principle to the entire chain.



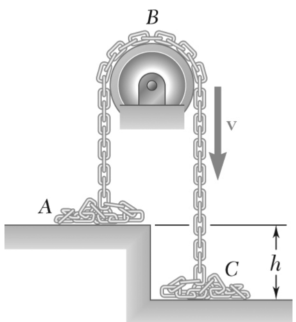
$$-\rho y v + P(\Delta t) + R(\Delta t) - \rho g L(\Delta t) = -\rho g(y + \Delta y)v$$

$$R(\Delta t) = \rho g L(\Delta t) - P(\Delta t) - \rho g(\Delta y)v$$

$$R = \rho g L - \rho g y - \rho \frac{\Delta y}{\Delta t} v$$

Let $\Delta t \to 0$. Then $\dfrac{\Delta y}{\Delta t} = \dfrac{dy}{dt} = -v$

$$R = \rho g(L - y) + \rho v^2 \qquad\qquad \mathbf{R} = \frac{m}{l}[g(L - y) + v^2] \uparrow \blacktriangleleft$$



PROBLEM 14.88

The ends of a chain lie in piles at A and C. When released from rest at time $t = 0$, the chain moves over the pulley at B, which has a negligible mass. Denoting by L the length of chain connecting the two piles and neglecting friction, determine the speed v of the chain at time t.

SOLUTION

Let m be the mass of the portion of the chain between the two piles. This is the portion of the chain that is moving with speed v. The remainder of the chain lies in either of the two piles. Consider the time period between t and $t + \Delta t$ and apply the principle impulse and momentum. Let Δm be the amount of chain that is picket up at A and deposited at C during the time period Δt.

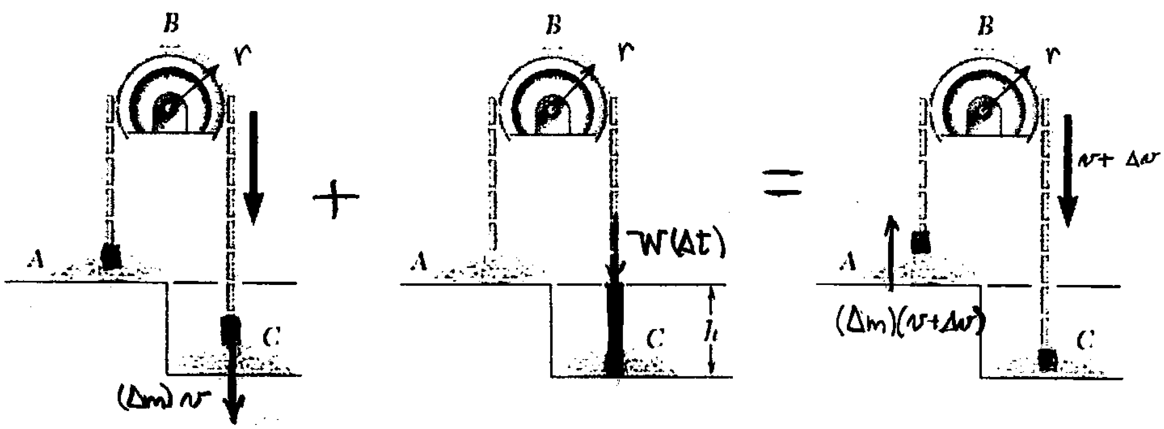
At time t, Δm is still in pile A while Δm has a downward at speed v just above pile C. The remaining mass $(m - \Delta m)$ is moving with speed v.

At time $t + \Delta t$, Δm is moving with speed $v + \Delta v$ just above pile A and Δm is at rest in pile C.

Over the time period an unbalanced weight of chain acts on the system. The weight is

$$W = \frac{mgh}{L}$$

Apply the impulse-momentum principle to the system.



Consider moments about the pulley axle.

$$\begin{aligned} &r[(\Delta m)v + (m - \Delta m)v] + rW(\Delta t) \\ &= r[(\Delta m)(v + \Delta v) + (m - \Delta m)(v + \Delta v)] \end{aligned}$$

PROBLEM 14.88 (Continued)

Dividing by r and canceling the terms $(\Delta m)v$ and $(m-\Delta m)v$

$$0+\frac{mgh}{L}(\Delta t)=(\Delta m)(v+\Delta v)+(m-\Delta m)(\Delta v)$$
$$=v(\Delta m)+m(\Delta v)$$

But $$\Delta m=\frac{m}{L}v(\Delta t)$$

Hence, $$\frac{mgh}{L}(\Delta t)=\frac{mv^2}{L}(\Delta t)+m(\Delta v)$$

Solving for Δt, $$\Delta t=\frac{L(\Delta v)}{gh-v^2}$$

Letting $c^2=gh,$ and considering the limit as Δt and Δv become infinitesimal, gives

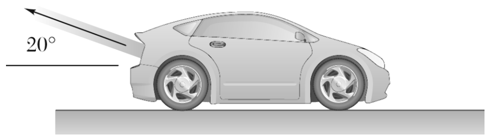
$$dt=\frac{L\,dv}{c^2-v^2}$$

Integrate, noting that $v=0$ when $t=0$

$$t=L\int_0^v\frac{dv}{c^2-v^2}=\frac{L}{c}\tanh^{-1}\frac{v}{c}\Big|_0^v$$

$$\tanh\frac{ct}{L}=\frac{v}{c}$$

$$v=\sqrt{gh}\tanh\left(\frac{\sqrt{gh}}{L}t\right)\blacktriangleleft$$

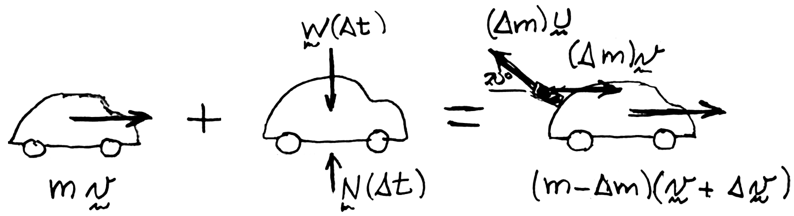


PROBLEM 14.89

A toy car is propelled by water that squirts from an internal tank at a constant 6 ft/s relative to the car. The weight of the empty car is 0.4 lb and it holds 2 lb of water. Neglecting other tangential forces, determine the top speed of the car.

SOLUTION

Consider a time interval Δt. Let m be the mass of the car plus the water in the tank at the beginning of the interval and $(m - \Delta m)$ the corresponding mass at the end of the interval. m_0 is the initial value of m. Let **v** be the velocity of the car. Apply the impulse and momentum principle over the time interval.



Horizontal components $\xrightarrow{+}$:

$$mv + 0 = (\Delta m)(\cancel{v} - u\cos 20°) + (\cancel{m} - \Delta m)(\cancel{v} + \Delta v)$$

$$\Delta v = u \cos 20° \frac{\Delta m}{m - \Delta m}$$

Let Δv be replaced by differential dv and Δm be replaced by the small differential $-dm$, the minus sign meaning that dm is the infinitesimal increase in m.

$$dv = -u \cos 20° \frac{dm}{m}$$

Integrating, $$v = v_0 - u \cos 20° \ln \frac{m}{m_0}$$

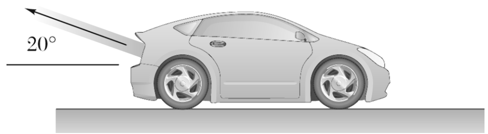
Since $v_0 = 0$, $$v = u \cos 20° \ln \frac{m_0}{m}$$

The velocity is maximum when $m = m_f$, the value of m when all of the water is expelled.

$$v_{\max} = u \cos 20° \ln \frac{m_0}{m_f}$$

$$v_{\max} = (6 \text{ ft/s}) \cos 20° \ln \frac{0.4 + 2}{0.4}$$

$v_{\max} = 10.10$ ft/s ◀

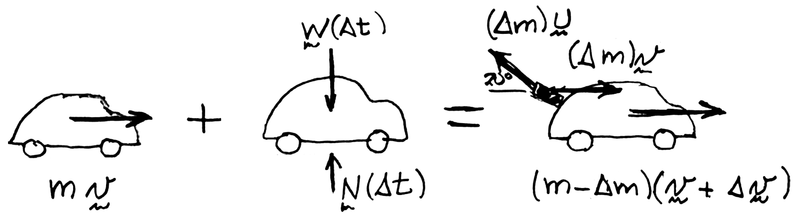


PROBLEM 14.90

A toy car is propelled by water that squirts from an internal tank. The weight of the empty car is 0.4 lb and it holds 2 lb of water. Knowing the top speed of the car is 8 ft/s, determine the relative velocity of the water that is being ejected.

SOLUTION

Consider a time interval Δt. Let m be the mass of the car plus the water in the tank at the beginning of the interval and $(m - \Delta m)$ the corresponding mass at the end of the interval. m_0 is the initial value of m. Let $\mathbf{v}$ be the velocity of the car. Apply the impulse and momentum principle over the time interval.



Horizontal components $\xrightarrow{+}$:

$$mv + 0 = (\Delta m)(\cancel{v} - u\cos 20°) + (\cancel{m} - \Delta m)(\cancel{v} + \Delta v)$$

$$\Delta v = u\cos 20° \frac{\Delta m}{m - \Delta m}$$

Let Δv be replaced by differential dv and Δm be replaced by the small differential $-dm$, the minus sign meaning that dm is the infinitesimal increase in m.

$$dv = -u\cos 20° \frac{dm}{m}$$

Integrating, $$v = v_0 - u\cos 20° \ln\frac{m}{m_0}$$

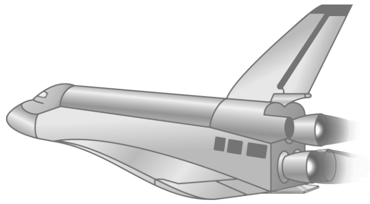
Since $v_0 = 0$, $$v = u\cos 20° \ln\frac{m_0}{m}$$

The velocity is maximum when $m = m_f$, the value of m when all of the water is expelled.

$$v_{\max} = u\cos 20° \ln\frac{m_0}{m_f}$$

$$8 \text{ ft/s} = u\cos 20° \ln\frac{0.4+2}{0.4}$$

$u = 4.75$ ft/s ◀



PROBLEM 14.91

The main propulsion system of a space shuttle consists of three identical rocket engines which provide a total thrust of 6 MN. Determine the rate at which the hydrogen-oxygen propellant is burned by each of the three engines, knowing that it is ejected with a relative velocity of 3750 m/s.

SOLUTION

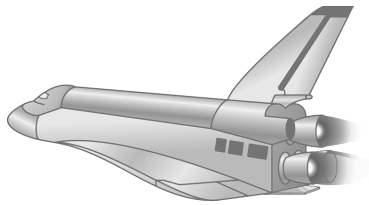
Thrust of each engine: $P = \frac{1}{3}(6 \text{ MN}) = 2\times 10^6 \text{ N}$

Eq. (14.44):

$$P = \frac{dm}{dt}u$$

$$2\times 10^6 \text{ N} = \frac{dm}{dt}(3750 \text{ m/s})$$

$$\frac{dm}{dt} = \frac{2\times 10^6 \text{ N}}{3750 \text{ m/s}} \qquad \frac{dm}{dt} = 533 \text{ kg/s} \blacktriangleleft$$



PROBLEM 14.92

The main propulsion system of a space shuttle consists of three identical rocket engines, each of which burns the hydrogen-oxygen propellant at the rate of 750 lb/s and ejects it with a relative velocity of 12000 ft/s. Determine the total thrust provided by the three engines.

SOLUTION

From Eq. (14.44) for each engine:

$$\begin{aligned} P &= \frac{dm}{dt}u \\ &= \frac{(750 \text{ lb/s})}{32.2 \text{ ft/s}^2}(12000 \text{ ft/s}) \\ &= 279.50\times 10^3 \text{ lb} \end{aligned}$$

For the 3 engines:

$$\text{Total thrust} = 3(279.50\times 10^3 \text{ lb})$$

$\text{Total thrust} = 839{,}000 \text{ lb}$ ◀

PROBLEM 14.93

A rocket weighs 2600 lb, including 2200 lb of fuel, which is consumed at the rate of 25 lb/s and ejected with a relative velocity of 13000 ft/s. Knowing that the rocket is fired vertically from the ground, determine its acceleration (*a*) as it is fired, (*b*) as the last particle of fuel is being consumed.

SOLUTION

From Eq. (14.44) of the textbook, the thrust is

$$P = \frac{dm}{dt}u$$

$$= \frac{(25 \text{ lb/s})}{32.2 \text{ ft/s}^2}(13000 \text{ ft/s})$$

$$= 10.093 \times 10^3 \text{ lb}$$

$$\Sigma F = ma$$

$$P - mg = ma \qquad a = \frac{P}{m} - g \qquad (1)$$

(*a*) *At the start of firing,* $W = W_0 = 2600 \text{ lb}$

$$g = 32.2 \text{ ft/s}^2 \qquad m = \frac{2600}{32.2} = 80.745 \text{ slug}$$

From Eq. (1), $a = \dfrac{10.093 \times 10^3 \text{ lb}}{80.745 \text{ slug}} - 32.2 = 92.80 \text{ ft/s}^2$ $\qquad \mathbf{a} = 92.8 \text{ ft/s}^2 \uparrow$ ◀

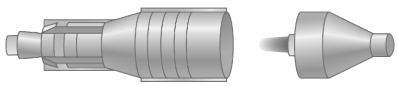
(*b*) *As the last particle of fuel is consumed,*

$$W = 2600 - 2200 = 400 \text{ lb}$$

$$g = 32.2 \text{ ft/s}^2 \text{ (assumed)} \qquad m = \frac{400}{32.2} = 12.422 \text{ slug}$$

From Eq. (1), $a = \dfrac{10.093 \times 10^3 \text{ lb}}{12.422} - 32.2 = 780.30 \text{ ft/s}^2$ $\qquad \mathbf{a} = 780 \text{ ft/s}^2 \uparrow$ ◀

PROBLEM 14.94



A space vehicle describing a circular orbit at a speed of 24×10^3 km/h releases its front end, a capsule which has a gross mass of 600 kg, including 400 kg of fuel. If the fuel is consumed at the rate of 18 kg/s and ejected with a relative velocity of 3000 m/s, determine (*a*) the tangential acceleration of the capsule as the engine is fired, (*b*) the maximum speed attained by the capsule.

SOLUTION

Thrust:
$$P = \left|\frac{dm}{dt}\right| u$$
$$= (18 \text{ kg/s})(3000 \text{ m/s})$$
$$= 54 \times 10^3 \text{ N}$$

(*a*)
$$(a_t)_0 = \frac{P}{m_0} = \frac{54 \times 10^3}{600} = 90 \text{ m/s}^2$$

$(a_t)_0 = 90.0 \text{ m/s}^2$ ◀

(*b*) Maximum speed is attained when all the fuel is used up:

$$v_1 = v_0 + \int_0^{t_1} a_t dt = v_0 + \int_0^{t_1} \frac{P}{m} dt$$
$$= v_0 + \int_0^{t_1} \frac{u\left(\frac{dm}{dt}\right)}{m} dt = v_0 + u\int_{m_0}^{m_1}\left(-\frac{dm}{m}\right)$$
$$v_1 = v_0 + u\left(-\ln\frac{m_1}{m_0}\right) = v_0 + u\ln\frac{m_0}{m_1}$$

Data:
$$v_0 = 24 \times 10^3 \text{ km/h}$$
$$= 6.6667 \times 10^3 \text{ m/s}$$
$$u = 3000 \text{ m/s}$$
$$m_0 = 600 \text{ kg}$$
$$m_1 = 600 - 400 = 200 \text{ kg}$$
$$v_1 = 6.6667 \times 10^3 + 3000 \ln\frac{600}{200}$$
$$= 9.9625 \times 10^3 \text{ m/s}$$

$v_1 = 35.9 \times 10^3$ km/h ◀

PROBLEM 14.95

A 540-kg spacecraft is mounted on top of a rocket with a mass of 19 Mg, including 17.8 Mg of fuel. Knowing that the fuel is consumed at a rate of 225 kg/s and ejected with a relative velocity of 3600 m/s, determine the maximum speed imparted to the spacecraft if the rocket is fired vertically from the ground.

SOLUTION

See sample Problem 14.8 for derivation of

$$v = u \ln \frac{m_0}{m_0 - qt} - gt \qquad (1)$$

Data: $u = 3600 \text{ m/s} \quad q = 225 \text{ kg/s}, \quad m_{\text{fuel}} = 17{,}800 \text{ kg}$

$$m_0 = 19{,}000 \text{ kg} + 540 \text{ kg} = 19{,}540 \text{ kg}$$

We have $m_{\text{fuel}} = qt, \quad 17{,}800 \text{ kg} = (225 \text{ kg/s})t$

$$t = \frac{17{,}800 \text{ kg}}{225 \text{ kg/s}} = 79.111 \text{ s}$$

Maximum velocity is reached when all fuel has been consumed, that is, when $qt = m_{\text{fuel}}$. Eq. (1) yields

$$v_m = u \ln \frac{m_0}{m_0 - m_{\text{fuel}}} - gt$$

$$= (3600 \text{ m/s}) \ln \frac{19{,}540}{19{,}540 - 17{,}800} - (9.81 \text{ m/s}^2)(79.111 \text{ s})$$

$$= (3600 \text{ m/s}) \ln 11.230 - 776.1 \text{ m/s}$$

$$= 7930.8 \text{ m/s}$$

$v_m = 7930 \text{ m/s}$ ◀

PROBLEM 14.96

The rocket used to launch the 540-kg spacecraft of Problem 14.95 is redesigned to include two stages A and B, each of mass 9.5 Mg, including 8.9 Mg of fuel. The fuel is again consumed at a rate of 225 kg/s and ejected with a relative velocity of 3600 m/s. Knowing that when stage A expels its last particle of fuel its casing is released and jettisoned, determine (*a*) the speed of the rocket at that instant, (*b*) the maximum speed imparted to the spacecraft.

SOLUTION

Thrust force: $P = u\dfrac{dm}{dt} = uq$

Mass of rocket + unspent fuel: $m = m_0 - qt$

Corresponding weight force: $W = mg$

Acceleration: $a = \dfrac{F}{m} = \dfrac{P-W}{m} = \dfrac{P}{m} - g = \dfrac{uq}{m_0 - qt} - g$

Integrating with respect to time to obtain the velocity,

$$v = v_0 + \int_0^t a\,dt = v_0 + u\int_0^t \frac{q\,dt}{m_0 - qt} - gt$$

$$= v_0 - u\ln\frac{m_0 - qt}{m_0} - gt \qquad (1)$$

For each stage, $m_{\text{fuel}} = 8900$ kg $\quad u = 3600$ m/s

$q = 225$ kg/s $\quad t = \dfrac{m_{\text{fuel}}}{q} = \dfrac{8900}{225} = 39.556$ s

For the first stage, $v_0 = 0 \quad m_0 = 540 + (2)(9500) = 19{,}540$ kg

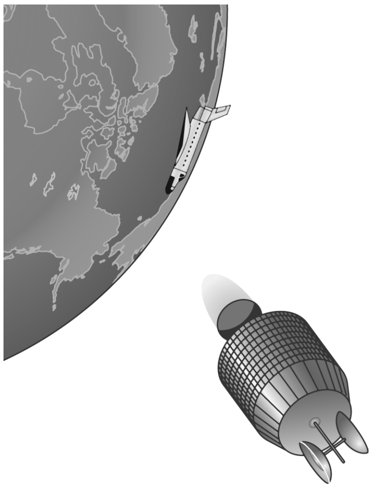
(*a*) $$v_1 = 0 - 3600\ln\frac{19{,}540 - 8900}{19{,}540} - (9.81)(39.556) = 1800.1 \text{ m/s}$$

$v_1 = 1800$ m/s ◀

For the second stage, $v_0 = 1800.1$ m/s, $\quad m_0 = 540 + 9500 = 10{,}040$ kg

(*b*) $$v_2 = 1800.1 - 3600\ln\frac{10{,}040 - 8900}{10{,}040} - (9.81)(39.556) = 9244 \text{ m/s}$$

$v_2 = 9240$ m/s ◀

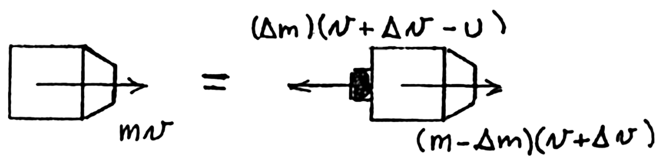


PROBLEM 14.97

A communication satellite weighing 10,000 lb, including fuel, has been ejected from a space shuttle describing a low circular orbit around the earth. After the satellite has slowly drifted to a safe distance from the shuttle, its engine is fired to increase its velocity by 8000 ft/s as a first step to its transfer to a geosynchronous orbit. Knowing that the fuel is ejected with a relative velocity of 13,750 ft/s, determine the weight of fuel consumed in this maneuver.

SOLUTION

Apply the *principle of impulse and momentum* to the satellite plus the fuel expelled in time Δt.



$$\xrightarrow{+}: \quad mv = (m - \Delta m)(v + \Delta v) + (\Delta m)(v + \Delta v - v)$$

$$= mv + m(\Delta v) - (\Delta m)v - (\Delta m)(\Delta v) + (\Delta m)v + (\Delta m)(\Delta v) - (\Delta m)v$$

$$m(\Delta v) - u(\Delta m) = 0$$

$$\Delta m = -\frac{dm}{dt}(\Delta t)$$

$$\frac{\Delta v}{\Delta t} = \frac{dv}{dt} = -\frac{u}{m}\frac{dm}{dt}$$

$$\int_{v_0}^{v_1} dv = -\int_0^{t_1} \frac{u}{m}\frac{dm}{dt}dt = -\int_{m_0}^{m_1} u\frac{dm}{m}$$

$$v_1 - v_0 = -u\ln\frac{m_1}{m_0} = u\ln\frac{m_0}{m_1}$$

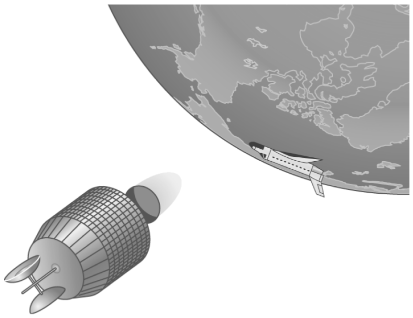
$$\frac{m_0}{m_1} = \exp\left(\frac{v_1 - v_0}{u}\right)$$

PROBLEM 14.97 (Continued)

Data:

$$v_1 - v_0 = 8000 \text{ ft/s}$$
$$u = 13,750 \text{ ft/s}$$
$$m_0 = 10,000 \text{ lb}$$
$$\frac{10,000}{m_1} = \exp\frac{8000}{13,750}$$
$$= 1.7893$$
$$m_1 = 5589 \text{ kg}$$
$$m_{\text{fuel}} = m_0 - m_1 = 10,000 - 5589 \qquad m_{\text{fuel}} = 4410 \text{ lb} \blacktriangleleft$$

PROBLEM 14.98



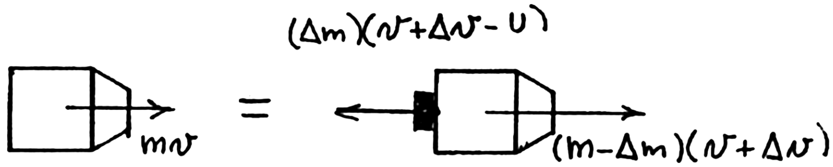
Determine the increase in velocity of the communication satellite of Problem 14.97 after 2500 lb of fuel has been consumed.

SOLUTION

Data from Problem 14.95: $m_0 = 10,000 \text{ lb} \quad u = 13,750 \text{ ft/s}$

$$m_1 = m_0 - m_{\text{fuel}} = 10,000 - 2500 = 7500 \text{ lb.}$$

Apply the *principle of impulse and momentum* to the satellite plus the fuel expelled in time Δt.



$$\xrightarrow{+}: \quad mv = (m - \Delta m)(v + \Delta v) + (\Delta m)(v + \Delta v - v)$$

$$= mv + m(\Delta v) - (\Delta m)v - (\Delta m)(\Delta v)$$

$$+ (\Delta m)v + (\Delta m)(\Delta v) - (\Delta m)v$$

$$m(\Delta v) - u(\Delta m) = 0$$

$$\Delta m = -\frac{dm}{dt}(\Delta t)$$

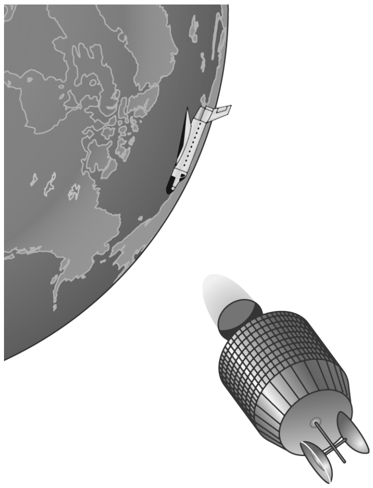
$$\frac{\Delta v}{\Delta t} = \frac{dv}{dt} = -\frac{u}{m}\frac{dm}{dt}$$

$$\int_{v_0}^{v_1} dv = -\int_0^{t_1} \frac{u}{m}\frac{dm}{dt}\,dt = -\int_{m_0}^{m_1} u\frac{dm}{m}$$

$$v_1 - v_0 = -u \ln\frac{m_1}{m_0} = u \ln\frac{m_0}{m_1}$$

$$\Delta v = v_1 - v_0 = 13,750 \ln\frac{10,000}{7500}$$

$\Delta v = 3960 \text{ ft/s}$ ◀



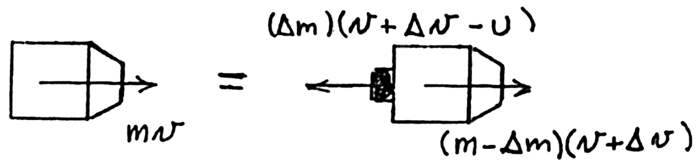
PROBLEM 14.99

Determine the distance separating the communication satellite of Problem 14.97 from the space shuttle 60 s after its engine has been fired, knowing that the fuel is consumed at a rate of 37.5 lb/s.

PROBLEM 14.97 A communication satellite weighing 10,000 lb, including fuel, has been ejected from a space shuttle describing a low circular orbit around the earth. After the satellite has slowly drifted to a safe distance from the shuttle, its engine is fired to increase its velocity by 8000 ft/s as a first step to its transfer to a geosynchronous orbit. Knowing that the fuel is ejected with a relative velocity of 13,750 ft/s, determine the weight of fuel consumed in this maneuver.

SOLUTION

Apply the *principle of impulse and momentum* to the satellite plus the fuel expelled in time Δt.



$$\xrightarrow{+}: \quad mv = (m - \Delta m)(v + \Delta v) + (\Delta m)(v + \Delta v - v)$$

$$= mv + m(\Delta v) - (\Delta m)v - (\Delta m)(\Delta v)$$

$$+ (\Delta m)v + (\Delta m)(\Delta v) - (\Delta m)v$$

$$m(\Delta v) - u(\Delta m) = 0$$

$$\Delta m = -\frac{dm}{dt}(\Delta t)$$

$$\frac{\Delta v}{\Delta t} = \frac{dv}{dt} = -\frac{u}{m}\frac{dm}{dt} = -\frac{uq}{m} = -\frac{uq}{m_0 - qt}$$

$$v = v_0 + \int_0^t \frac{uq}{m_0 - qt}\,dt = v_0 - u\ln(m_0 - qt)\Big|_0^t$$

$$= v_0 + u\ln(m_0 - qt) + u\ln m_0$$

$$= v_0 - u\ln\left(\frac{m_0 - qt}{m_0}\right) \qquad (1)$$

Set $\frac{dx}{dt} = v$ in Eq. (1) and integrate with respect to time.

$$x = x_0 + v_0 t + u\int_0^t \ln\left(\frac{m_0 - qt}{m_0}\right)dt$$

PROBLEM 14.99 (Continued)

Call the last term x' and let

$$z = \frac{m_0 - qt}{m_0} \qquad dz = -\frac{q}{m_0}dt \qquad \text{or} \qquad dt = -\frac{m_0}{q}dz$$

$$x' = \frac{m_0 u}{q}\int_{z_0}^{z} \ln z\, dz = \frac{m_0 u}{q}[(z\ln z + z)]_{z_0}^{z}$$

$$= \frac{m_0 u}{q}\left[\frac{m_0 - qt}{m_0}\left(\ln\frac{m_0 - qt}{m_0} - 1\right) - \frac{m_0}{m_0}\left(\ln\frac{m_0}{m_0} - 1\right)\right]$$

$$= \frac{m_0 u}{q}\left[\left(1 - \frac{qt}{m_0}\right)\left(\ln\frac{m_0 - qt}{m_0} - 1\right) + 1\right]$$

$$= \frac{m_0 u}{q}\left[\ln\frac{m_0 - qt}{m_0} - 1 + 1\right] - ut\left[\ln\frac{m_0 - qt}{m_0} - 1\right]$$

$$= ut + \left(\frac{m_0 u}{q} - ut\right)\ln\frac{m_0 - qt}{m_0}$$

$$= u\left[t - \left(\frac{m_0}{q} - t\right)\ln\frac{m_0}{m_0 - qt}\right]$$

$$x = x_0 + v_0 t + u\left[t - \left(\frac{m_0}{q} - t\right)\ln\frac{m_0}{m_0 - qt}\right] \qquad (2)$$

Data: $x_0 = 0 \qquad v_0 = 0 \qquad q = 37.5\ \text{lb/s.}$

$$m_0 = 10{,}000\ \text{lb}, \quad t = 60\ \text{sec} \quad u = 13{,}750\ \text{ft/s}$$

$$x = 0 + 0 + (13{,}750)\left[60 - \left(\frac{10{,}000}{37.5} - 60\right)\ln\frac{10{,}000}{10{,}000 - (37.5)(60)}\right]$$

$$= 100{,}681\ \text{ft} \qquad\qquad x = 19.07\ \text{mi} \blacktriangleleft$$

PROBLEM 14.100

For the rocket of Problem 14.93, determine (*a*) the altitude at which all the fuel has been consumed, (*b*) the velocity of the rocket at that time.

PROBLEM 14.93 A rocket weighs 2600 lb, including 2200 lb of fuel, which is consumed at the rate of 25 lb/s and ejected with a relative velocity of 13000 ft/s. Knowing that the rocket is fired vertically from the ground, determine its acceleration (*a*) as it is fired, (*b*) as the last particle of fuel is being consumed.

SOLUTION

See Sample Problem 14.8 for derivation of

$$v = u\ln\frac{m_0}{m_0 - qt} - gt = -u\ln\frac{m_0 - qt}{m_0} - gt \qquad (1)$$

Note that g is assumed to be constant.

Set $\frac{dy}{dt} = v$ in Eq. (1) and integrate with respect to time.

$$h = \int_0^h dy = \int_0^t v\,dt = \int_0^t \left(u\ln\frac{m_0}{m_0 - qt} - gt\right)dt$$

$$= -u\int_0^t \ln\frac{m_0 - qt}{m_0}\,dt - \frac{1}{2}gt^2$$

Let $\quad z = \dfrac{m_0 - qt}{m_0} \qquad dz = -\dfrac{q}{m_0}dt \qquad \text{or} \qquad dt = -\dfrac{m_0}{q}dz$

$$h = \frac{m_0 u}{q}\int_{z_0}^{z} \ln z\,dz - \frac{1}{2}gt^2 = \frac{m_0 u}{q}[(z\ln z + z)]_{z_0}^{z} - \frac{1}{2}gt^2$$

$$= \frac{m_0 u}{q}\left[\frac{m_0 - qt}{m_0}\left(\ln\frac{m_0 - qt}{m_0} - 1\right) - \frac{m_0}{m_0}\left(\ln\frac{m_0}{m_0} - 1\right)\right] - \frac{1}{2}gt^2$$

$$= \frac{m_0 u}{q}\left[\left(1 - \frac{qt}{m_0}\right)\left(\ln\frac{m_0 - qt}{m_0} - 1\right) + 1\right] - \frac{1}{2}gt^2$$

$$= \frac{m_0 u}{q}\left[\ln\frac{m_0 - qt}{m_0} - 1 + 1\right] - ut\left[\ln\frac{m_0 - qt}{m_0} - 1\right] - \frac{1}{2}gt^2$$

$$= ut + \left(\frac{m_0 u}{q} - ut\right)\ln\frac{m_0 - qt}{m_0} - \frac{1}{2}gt^2$$

$$h = u\left[t - \left(\frac{m_0}{q} - t\right)\ln\frac{m_0}{m_0 - qt}\right] - \frac{1}{2}gt^2 \qquad (2)$$

PROBLEM 14.100 (Continued)

Data:

$$W_0 = 2600 \text{ lb} \qquad qt = W_{\text{fuel}} = 2200 \text{ lb} \qquad q = 25 \text{ lb/s}$$

$$u = 13000 \text{ ft/s} \qquad g = 32.2 \text{ ft/s}^2$$

$$t = \frac{W_{\text{fuel}}}{q} = \frac{2200}{25} = 88 \text{ s}$$

(*a*) From Eq. (2),

$$h = (13000)\left[88 - \left(\frac{2600}{25} - 88\right)\ln\frac{2600}{2600-2200}\right] - \frac{1}{2}(32.2)(88)^2$$

$$= (13000)(88 - 16 \ln 6.5) - 124680$$

$$= 630,000 \text{ ft}$$

$h = 119.3$ mi ◀

(*b*) From Eq. (1),

$$v = -13000 \ln\frac{2600-2200}{2600} - (32.2)(88)$$

$$= 13000 \ln 6.5 - 2834$$

$$= 21500 \text{ ft/s}$$

$v = 14,660$ mi/h ◀

PROBLEM 14.101

Determine the altitude reached by the spacecraft of Problem 14.95 when all the fuel of its launching rocket has been consumed.

SOLUTION

See Sample Problem 14.8 for derivation of

$$v = u \ln \frac{m_0}{m_0 - qt} - gt = -u \ln \frac{m_0 - qt}{m_0} - gt \qquad (1)$$

Note that g is assumed to be constant.

Set $\frac{dy}{dt} = v$ in Eq. (1) and integrate with respect to time.

$$h = \int_0^h dy = \int_0^t v dt = \int_0^t \left(u \ln \frac{m_0}{m_0 - qt} - gt \right) dt$$

$$= -u \int_0^t \ln \frac{m_0 - qt}{m_0} dt - \frac{1}{2} gt^2$$

Let $$z = \frac{m_0 - qt}{m_0} \qquad dz = -\frac{q}{m_0} dt \qquad \text{or} \qquad dt = -\frac{m_0}{q} dz$$

$$h = \frac{m_0 u}{q} \int_{z_0}^{z} \ln z \, dz - \frac{1}{2} gt^2 = \frac{m_0 u}{q} [(z \ln z + z)]_{z_0}^{z} - \frac{1}{2} gt^2$$

$$= \frac{m_0 u}{q} \left[\frac{m_0 - qt}{m_0} \left(\ln \frac{m_0 - qt}{m_0} - 1 \right) - \frac{m_0}{m_0} \left(\ln \frac{m_0}{m_0} - 1 \right) \right] - \frac{1}{2} gt^2$$

$$= \frac{m_0 u}{q} \left[\left(1 - \frac{qt}{m_0} \right) \left(\ln \frac{m_0 - qt}{m_0} - 1 \right) + 1 \right] - \frac{1}{2} gt^2$$

$$= \frac{m_0 u}{q} \left[\ln \frac{m_0 - qt}{m_0} - 1 + 1 \right] - ut \left[\ln \frac{m_0 - qt}{m_0} - 1 \right] - \frac{1}{2} gt^2$$

$$= ut + \left(\frac{m_0 u}{q} - ut \right) \ln \frac{m_0 - qt}{m_0} - \frac{1}{2} gt^2$$

$$h = u \left[t - \left(\frac{m_0}{q} - t \right) \ln \frac{m_0}{m_0 - qt} \right] - \frac{1}{2} gt^2 \qquad (2)$$

PROBLEM 14.101 (Continued)

Data:

$$u = 3600 \text{ m/s} \qquad m_0 = 19{,}000 + 540 = 19{,}540 \text{ kg}$$

$$q = 225 \text{ kg/s} \qquad m_{\text{fuel}} = 17{,}800 \text{ kg}$$

$$t = \frac{m_{\text{fuel}}}{q} = \frac{17{,}800}{225} = 79.111 \text{ s} \qquad g = 9.81 \text{ m/s}^2$$

$$m_0 - qt = 1740 \text{ kg}$$

From Eq. (2),

$$h = (3600)\left[79.111 - \left(\frac{19{,}540}{225} - 79.111\right)\ln\frac{19{,}540}{1740}\right] - \frac{1}{2}(9.81)(79.111)^2$$

$$= 186{,}766 \text{ m}$$

$h = 186.8 \text{ km}$ ◀

PROBLEM 14.102

For the spacecraft and the two-stage launching rocket of Problem 14.96, determine the altitude at which (*a*) stage *A* of the rocket is released, (*b*) the fuel of both stages has been consumed.

SOLUTION

Thrust force: $$P = u\frac{dm}{dt} = uq$$

Mass of rocket + unspent fuel: $$m = m_0 - qt$$

Corresponding weight force: $$W = mg$$

Acceleration: $$a = \frac{F}{m} = \frac{P - W}{m} = \frac{P}{m} - g = \frac{uq}{m_0 - qt} - g$$

Integrating with respect to time to obtain velocity,

$$v = v_0 + \int_0^t a dt = v_0 + u\int_0^t \frac{q dt}{m_0 - qt} - gt$$

$$= v_0 - u\ln\frac{m_0 - qt}{m_0} - gt \qquad (1)$$

Integrating again to obtain the displacement,

$$s = s_0 + v_0 t - u\int_0^t \ln\frac{m_0 - qt}{m_0} dt - \frac{1}{2}gt^2$$

Let $$z = \frac{m_0 - qt}{m_0} \qquad dz = -\frac{q}{m_0}dt \qquad dt = -\frac{m_0}{q}dz$$

Then $$s = s_0 + v_0 t + \frac{m_0 u}{q}\int_{z_0}^{z} \ln z\, dz - \frac{1}{2}gt^2$$

$$= s_0 + v_0 t + \frac{m_0 u}{q}(z\ln z + z)\Big|_{z_0}^{z} - \frac{1}{2}gt^2$$

$$= s_0 + v_0 t + \frac{m_0 u}{q}\left[\frac{m_0 - qt}{m_0}\ln\frac{m_0 - qt}{m_0} + \frac{m_0 - qt}{m_0} - \frac{m_0}{m_0}\ln\frac{m_0}{m_0} + \frac{m_0}{m_0}\right] - \frac{1}{2}gt^2$$

$$= s_0 + v_0 t + u\left[t + \left(\frac{m_0}{q} - t\right)\ln\frac{m_0 - qt}{m_0}\right] - \frac{1}{2}gt^2 \qquad (2)$$

For each stage, $m_{\text{fuel}} = 8900 \text{ kg} \qquad u = 3600 \text{ m/s}$

$$q = 225 \text{ kg/s} \qquad t = \frac{m_{\text{fuel}}}{q} = \frac{8900}{225} = 39.556 \text{ s}$$

For the first stage, $v_0 = 0 \qquad s_0 = 0$

$$m_0 = 540 + (2)(9500) = 19,540 \text{ kg}$$

From Eq. (1),

$$v_1 = 0 - 3600 \ln \frac{19,540 - 8900}{19,540} - (9.81)(39.556)$$

$$= 1800.1 \text{ m/s}$$

From Eq. (2),

(*a*) $$s_1 = 0 + 0 + 3600\left[39.556 + \left(\frac{19,540}{225} - 39.556\right)\ln\frac{19,540 - 8900}{19,540}\right] - \frac{1}{2}(9.81)(39.556)^2$$

$$= 31,249 \text{ m}$$

$h_1 = 31.2 \text{ km}$ ◀

For the second stage, $v_0 = 1800.1 \text{ m/s} \qquad s_0 = 31,249 \text{ m}$

$$m_0 = 540 + 9500 = 10,040 \text{ kg}$$

From Eq. (2),

(*b*) $$s_2 = 31,249 + (1800.1)(39.556) + 3600\left[39.556 + \left(\frac{10,040}{225} - 39.556\right)\ln\frac{10,040 - 8900}{10,040}\right]$$

$$- \frac{1}{2}(9.81)(39.556)^2$$

$$= 197,502 \text{ m}$$

$h_2 = 197.5 \text{ km}$ ◀

PROBLEM 14.103

In a jet airplane, the kinetic energy imparted to the exhaust gases is wasted as far as propelling the airplane is concerned. The useful power is equal to the product of the force available to propel the airplane and the speed of the airplane. If v is the speed of the airplane and u is the relative speed of the expelled gases, show that the mechanical efficiency of the airplane is $\eta = 2v/(u+v)$. Explain why $\eta = 1$ when $u = v$.

SOLUTION

Let F be the thrust force, and $\frac{dm}{dt}$ be the mass flow rate.

Absolute velocity of exhaust: $v_e = u - v$

Thrust force: $F = \dfrac{dm}{dt}(u - v)$

Power of thrust force: $P_1 = Fv = \dfrac{dm}{dt}(u - v)v$

Power associated with exhaust: $P_2(\Delta t) = \dfrac{1}{2}(\Delta m)v_e^2 = \dfrac{1}{2}(\Delta m)(u - v)^2$

$$P_2 = \frac{1}{2}\frac{dm}{dt}(u - v)^2$$

Total power supplied by engine: $P = P_1 + P_2$

$$P = \frac{dm}{dt}\left[(u - v)v - \frac{1}{2}(u - v)^2\right]$$

$$= \frac{1}{2}\frac{dm}{dt}(u^2 - v^2)$$

Mechanical efficiency: $\eta = \dfrac{\text{useful power}}{\text{total power}} = \dfrac{P_1}{P}$

$$\eta = \frac{2(u - v)v}{u^2 - v^2}$$

$$\eta = \frac{2v}{(u + v)} \quad ◀$$

$\eta = 1$ when $u = v$. The exhaust, having zero velocity, carries no power away.

PROBLEM 14.104

In a rocket, the kinetic energy imparted to the consumed and ejected fuel is wasted as far as propelling the rocket is concerned. The useful power is equal to the product of the force available to propel the rocket and the speed of the rocket. If v is the speed of the rocket and u is the relative speed of the expelled fuel, show that the mechanical efficiency of the rocket is $\eta = 2uv/(u^2 + v^2)$. Explain why $\eta = 1$ when $u = v$.

SOLUTION

Let F be the thrust force and $\frac{dm}{dt}$ be the mass flow rate.

Absolute velocity of exhaust: $v_e = u - v$

Thrust force: $F = \dfrac{dm}{dt}u$

Power of thrust force: $P_1 = Fv = \dfrac{dm}{dt}uv$

Power associated with exhaust: $P_2(\Delta t) = \dfrac{1}{2}(\Delta m)v_e^2 = \dfrac{1}{2}(\Delta m)(u-v)^2$

$$P_2 = \frac{1}{2}\frac{dm}{dt}(u-v)^2$$

Total power supplied by engine: $P = P_1 + P_2$

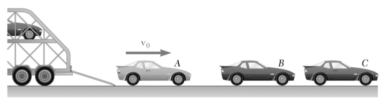
$$P = \frac{dm}{dt}\left[uv - \frac{1}{2}(u-v)^2\right] = \frac{1}{2}\frac{dm}{dt}(u^2 - v^2)$$

Mechanical efficiency: $\eta = \dfrac{\text{useful power}}{\text{total power}} = \dfrac{P_1}{P}$

$$\eta = \frac{2uv}{(u^2 + v^2)} \quad \blacktriangleleft$$

$\eta = 1$ when $u = v$. The exhaust, having zero velocity, carries no power away.

PROBLEM 14.105



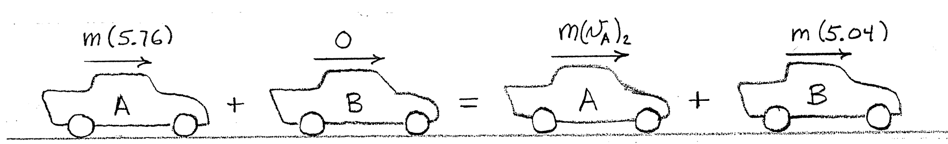
Three identical cars are being unloaded from an automobile carrier. Cars *B* and *C* have just been unloaded and are at rest with their brakes off when car *A* leaves the unloading ramp with a velocity of 5.76 ft/s and hits car *B*, which hits car *C*. Car *A* then again hits car *B*. Knowing that the velocity of car *B* is 5.04 ft/s after the first collision, 0.630 ft/s after the second collision, and 0.709 ft/s after the third collision, determine (*a*) the final velocities of cars *A* and *C*, (*b*) the coefficient of restitution for each of the collisions.

SOLUTION

There are no horizontal forces acting. Horizontal momentum is conserved.

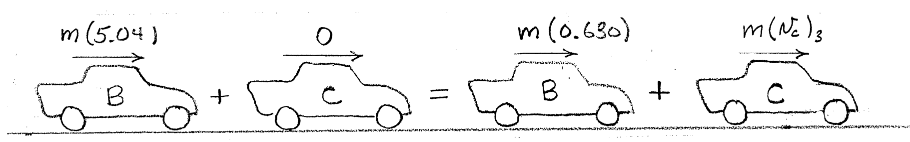
(a) *Velocities*:

Event $1 \longrightarrow 2$: Car *A* hits car *B*.



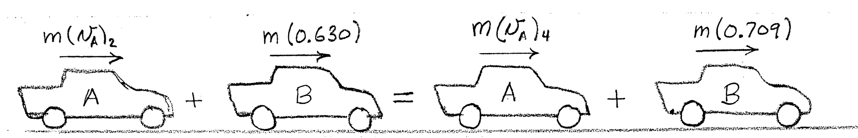
$$m(5.76) + 0 = m(v_A)_2 + m(5.04) \qquad (\mathbf{v}_A)_2 = 0.720 \text{ ft/s} \longrightarrow$$

Event $2 \longrightarrow 3$: Car *B* hits car *C*.



$$m(5.04) + 0 = m(0.630) + m(v_C)_3 \qquad (\mathbf{v}_C)_3 = 4.41 \text{ ft/s} \longrightarrow \blacktriangleleft$$

Event $3 \longrightarrow 4$: Car *A* hits car *B* again.



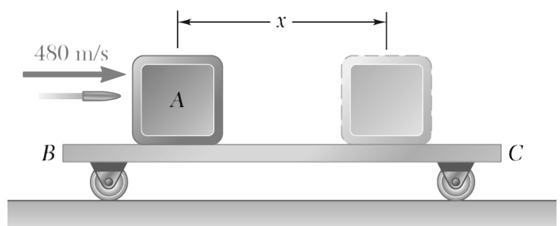
$$m(0.720) + m(0.630) = m(v_A)_4 + m(0.709) \qquad (\mathbf{v}_A)_4 = 0.641 \text{ ft/s} \longrightarrow \blacktriangleleft$$

(*b*) *Coefficients of restitution*:

Event $1 \longrightarrow 2$: $$e_{1\to 2} = \left|\frac{(v_A)_2 - (v_B)_2}{(v_A)_1 - (v_B)_1}\right| = \frac{5.04 - 0.720}{5.76 - 0} \qquad e_{1\to 2} = 0.750 \blacktriangleleft$$

Event $2 \longrightarrow 3$: $$e_{2\to 3} = \left|\frac{(v_B)_3 - (v_C)_3}{(v_B)_2 - (v_C)_2}\right| = \frac{4.41 - 0.630}{5.04 - 0} \qquad e_{2\to 3} = 0.750 \blacktriangleleft$$

Event $3 \longrightarrow 4$: $$e_{3\to 4} = \left|\frac{(v_A)_4 - (v_B)_4}{(v_A)_3 - (v_B)_3}\right| = \frac{0.709 - 0.641}{0.720 - 0.630} \qquad e_{3\to 4} = 0.756 \blacktriangleleft$$

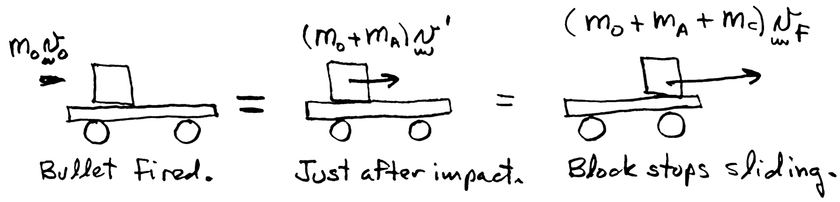


PROBLEM 14.106

A 30-g bullet is fired with a velocity of 480 m/s into block A, which has a mass of 5 kg. The coefficient of kinetic friction between block A and cart BC is 0.50. Knowing that the cart has a mass of 4 kg and can roll freely, determine (a) the final velocity of the cart and block, (b) the final position of the block on the cart.

SOLUTION

(a) Conservation of linear momentum:



$$m_0 v_0 = (m_0 + m_A)v' = (m_0 + m_A + m_C)v_f$$

$$(0.030 \text{ kg})(480 \text{ m/s}) = (5.030 \text{ kg})v' = (9.030 \text{ kg})v_f$$

$$v' = \frac{0.030}{5.030}(480 \text{ m/s}) = 2.863 \text{ m/s}$$

$$v_f = \frac{0.030}{9.030}(480 \text{ m/s}) = 1.5947 \text{ m/s} \qquad v_f = 1.595 \text{ m/s} \blacktriangleleft$$

(b) Work-energy principle:

Just after impact:
$$T' = \frac{1}{2}(m_0 + m_A)v'^2$$
$$= \frac{1}{2}(5.030 \text{ kg})(2.863 \text{ m/s})^2$$
$$= 20.615 \text{ J}$$

Final kinetic energy:
$$T_f = \frac{1}{2}(m_0 + m_A + m_C)\frac{1}{2}v_f^2$$
$$= \frac{1}{2}(9.030 \text{ kg})(1.5947 \text{ m/s})^2$$
$$= 11.482 \text{ J}$$

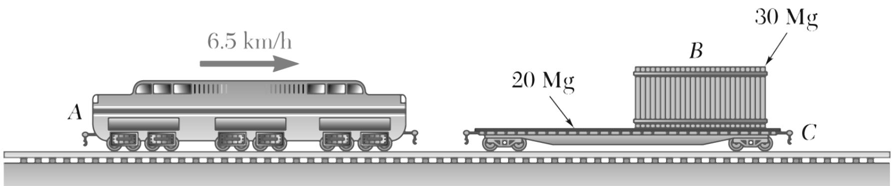
Work of friction force:
$$F = \mu_k N$$
$$= \mu_k (m_0 + m_A)g$$
$$= 0.50(5.030)(9.81)$$
$$= 24.672 \text{ N}$$

$$\text{Work} = U = -Fx = -24.672x$$

$$T' + U = T_f: \quad 20.615 - 24.672x = 11.482 \qquad x = 0.370 \text{ m} \blacktriangleleft$$

PROBLEM 14.107

An 80-Mg railroad engine A coasting at 6.5 km/h strikes a 20-Mg flatcar C carrying a 30-Mg load B which can slide along the floor of the car $(\mu_k = 0.25)$. Knowing that the car was at rest with its brakes released and that it automatically coupled with the engine upon impact, determine the velocity of the car (*a*) immediately after impact, (*b*) after the load has slid to a stop relative to the car.



SOLUTION

The masses are the engine $(m_A = 80\times 10^3 \text{ kg})$, the load $(m_B = 30\times 10^3 \text{ kg})$, and the flat car $(m_C = 20\times 10^3 \text{ kg})$.

Initial velocities: $(v_A)_0 = 6.5 \text{ km/h}$

$$= 1.80556 \text{ m/s}$$

$$(v_B)_0 = (v_C)_0 = 0.$$

No horizontal external forces act on the system during the impact and while the load is sliding relative to the flat car. Momentum is conserved.

Initial momentum: $$m_A(v_A)_0 + m_B(0) + m_C(0) = m_A(v_A)_0 \qquad (1)$$

(*a*) Let v' be the common velocity of the engine and flat car immediately after impact. Assume that the impact takes place before the load has time to acquire velocity.

Momentum immediately after impact:

$$m_A v' + m_B(0) + m_C v' = (m_A + m_C)v' \qquad (2)$$

Equating (1) and (2) and solving for v',

$$v' = \frac{m_A(v_A)_0}{m_A + m_C}$$

$$= \frac{(80\times 10^3)(1.80556)}{(100\times 10^3)}$$

$$= 1.44444 \text{ m/s}$$

$$\mathbf{v}' = 5.20 \text{ km/h} \longrightarrow \blacktriangleleft$$

(*b*) Let v_f be the common velocity of all three masses after the load has slid to a stop relative to the car.

Corresponding momentum:

$$m_A v_f + m_B v_f + m_C v_f = (m_A + m_B + m_C)v_f \qquad (3)$$

PROBLEM 14.107 (Continued)

Equating (1) and (3) and solving for v_f,

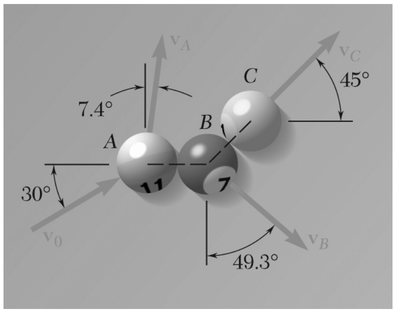
$$v_f = \frac{m_A (v_A)_0}{m_A + m_B + m_C}$$

$$= \frac{(80\times 10^3)(1.80556)}{(130\times 10^3)}$$

$$= 1.11111 \text{ m/s}$$

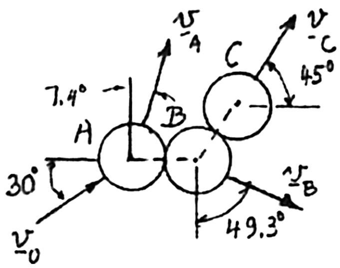
$\mathbf{v}_f = 4.00 \text{ km/h} \longrightarrow$ ◀

PROBLEM 14.108



In a game of pool, ball A is moving with a velocity $\mathbf{v}_0$ when it strikes balls B and C which are at rest and aligned as shown. Knowing that after the collision the three balls move in the directions indicated and that $v_0 = 12$ ft/s and $v_C = 6.29$ ft/s, determine the magnitude of the velocity of (*a*) ball A, (*b*) ball B.

SOLUTION



Conservation of linear momentum. In x direction:

$$m(12 \text{ ft/s})\cos 30° = mv_A \sin 7.4° + mv_B \sin 49.3° + m(6.29)\cos 45°$$

$$0.12880v_A + 0.75813v_B = 5.9446 \qquad (1)$$

In y direction:

$$m(12 \text{ ft/s})\sin 30° = mv_A \cos 7.4° - mv_B \cos 49.3° + m(6.29)\sin 45°$$

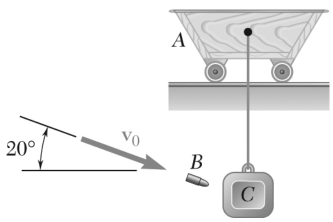
$$0.99167v_A - 0.65210v_B = 1.5523 \qquad (2)$$

(*a*) Multiply (1) by 0.65210, (2) by 0.75813, and add:

$$0.83581\, v_A = 5.0533 \qquad v_A = 6.05 \text{ ft/s} \blacktriangleleft$$

(*b*) Multiply (1) by 0.99167, (2) by –0.12880, and add:

$$0.83581\, v_B = 5.6951 \qquad v_B = 6.81 \text{ ft/s} \blacktriangleleft$$

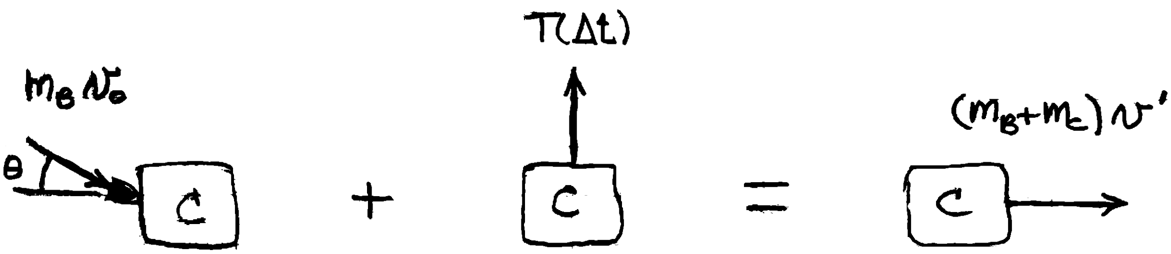


PROBLEM 14.109

Mass C, which has a mass of 4 kg, is suspended from a cord attached to cart A, which has a mass of 5 kg and can roll freely on a frictionless horizontal track. A 60-g bullet is fired with a speed v_0 = 500 m/s and gets lodged in block C. Determine (a) the velocity of C as it reaches its maximum elevation, (b) the maximum vertical distance h through which C will rise.

SOLUTION

Consider the impact as bullet B hits mass C. Apply the principle of impulse-momentum to the two particle system.



$$\Sigma m\mathbf{v}_1 + \Sigma \mathbf{Imp}_{1\to 2} = \Sigma m\mathbf{v}_2$$

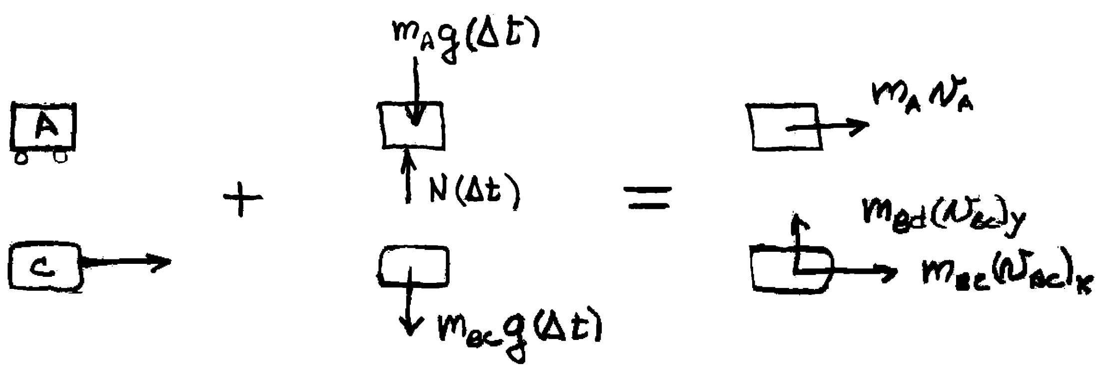
Using both B and C and taking horizontal components gives

$$m_B v_0 \cos\theta + O = (m_B + m_C)v' = m_{BC}v'$$

$$v' = \frac{m_B v_0 \cos\theta}{m_{BC}}$$

$$= \frac{(0.060\text{ kg})(500\text{ m/s})\cos 20°}{(4.06\text{ kg})} = 6.9435\text{ m/s}$$

Now consider the system of m_A and m_{BC} after the impact, and apply to impulse momentum principle.



$$\Sigma m\mathbf{v}_2 + \Sigma \mathbf{Imp}_{2\to 3} = \Sigma m\mathbf{v}_3$$

Horizontal components: $+\longrightarrow$

$$m_{BC}v' + 0 = m_A v_A + m_{BC}v_{cx}$$

$$v_A = \frac{m_{BC}}{m_A}(v' - v_{cx})$$

$$= \frac{4.06}{5}(6.9435 - v_{cx})$$

$$v_A = 5.6381 - 0.812v_{cx} \quad \text{in m/s} \qquad (1)$$

(a) At maximum elevation.

Both particles have the same velocity, thus

$$v_{cx} = v_A$$
$$v_A = 5.6381 - 0.812v_A$$

$$v_A = 3.1115 \text{ m/s} \qquad v_A = 3.11 \text{ m/s} \blacktriangleleft$$

(b) Conservation of energy: $T_2 + V_2 = T_3 + V_3$

$$T_2 = \frac{1}{2}m_A(0) + \frac{1}{2}m_{BC}(v')^2$$

$$= \frac{1}{2}(4.06)(6.9435)^2 = 97.871 \text{ J}$$

$$V_2 = 0 \qquad \text{(datum)}$$

$$T_3 = \frac{1}{2}m_A v_A^2 + \frac{1}{2}m_{BC}(v_{Bx}^2 + v_{By}^2)^2$$

$$= \frac{1}{2}(5)(3.1115)^2 + \frac{1}{2}(4.06)[(3.1115)^2 + 0] = 43.857 \text{ J}$$

$$V_3 = m_{BC}gh = (4.06)(9.81)h = 39.829 \text{ h}$$

$$97.871 + 0 = 43.857 + 39.829 \text{ h}$$

$$h = 1.356 \text{ m} \blacktriangleleft$$

<u>Another method</u>: We observe that no external horizontal forces are exerted on the system consisting of A, B, and C. Thus the horizontal component of the velocity of the mass center remains constant.

$$m = m_A + m_B + m_C = 5 + 0.06 + 4 = 9.06 \text{ kg}$$

$$\bar{v}_x = \frac{m_B v_0 \cos\theta}{m_A + m_B + m_C} = \frac{(0.060 \text{ kg})(500 \text{ m/s})\cos 20^\circ}{9.06 \text{ kg}} = 3.1115 \text{ m/s}$$

(a) At maximum elevation, v_A and v_{BC} are equal.

$$v_A = 3.1115 \text{ m/s} \qquad \mathbf{v}_A = 3.11 \text{ m/s} \longrightarrow \blacktriangleleft$$

Immediately after the impact of B on C, the velocity v_A is zero.

$$(m_B + m_C)v' = (m_A + m_B + m_C)\overline{v}_x$$

$$v' = \frac{m_A + m_B + m_C}{m_B + m_C}\overline{v}_x = \frac{9.06}{4.06}(3.1115 \text{ m/s}) = 6.9435 \text{ m/s}$$

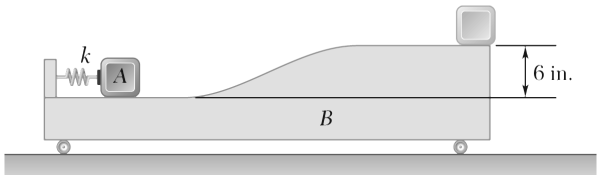
(b) Principle of work and energy: $T_2 + V_2 = T_3 + V_3$

$T_2, V_2,$ and V_3 are calculated as before.

For T_3 we note that the velocities $\mathbf{v}'_A$ and $\mathbf{v}'_{BC}$ relative to the mass center are zero. Thus, T_3 is given by

$$T_3 = \frac{1}{2}m\overline{\mathbf{v}}^2 = \frac{1}{2}(9.06)(3.1115)^2 = 43.857 \text{ J}$$

As before, h is found to be $h = 1.356 \text{ m}$ ◀



PROBLEM 14.110

A 15-lb block B is at rest and a spring of constant $k = 72$ lb/in. is held compressed 3 in. by a cord. After 5-lb block A is placed against the end of the spring, the cord is cut causing A and B to move. Neglecting friction, determine the velocities of blocks A and B immediately after A leaves B.

SOLUTION

$$m_A = \frac{5}{32.2} = 0.15528 \text{ lb}\cdot\text{s}^2/\text{ft}$$

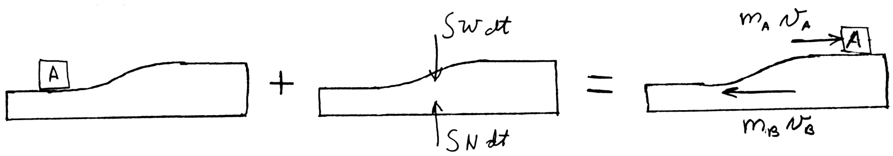
$$m_B = \frac{15}{32.2} = 0.46584 \text{ lb}\cdot\text{s}^2/\text{ft}$$

$$k = 72 \text{ lb/in} = 864 \text{ lb/ft}$$

$$e = 3 \text{ in.} = 0.25 \text{ ft}$$

$$h = 6 \text{ in.} = 0.5 \text{ ft}$$

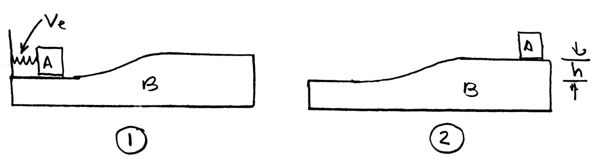
Conservation of linear momentum:



Horizontal components $\xrightarrow{+}$: $\qquad 0 + 0 = m_A v_A - m_B v_B$

$$v_B = \frac{m_A}{m_B} v_A = \frac{1}{3} v_A$$

Conservation of energy:



State 1: $\qquad V_{1e} = \frac{1}{2}ke^2 = \frac{1}{2}(864)(0.25)^2 = 27 \text{ ft}\cdot\text{lb}$

$$V_{1g} = 0$$

$$T_1 = 0$$

State 2: $\qquad V_{2e} = 0$

$$V_{2g} = W_A h = (5)(0.5) = 2.5 \text{ ft}\cdot\text{lb}$$

$$T_2 = \frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2$$

$$= \frac{1}{2}(0.15528)v_A^2 + \frac{1}{2}(0.46584)\left(\frac{v_A}{3}\right)^2 = 0.10352 v_A^2$$

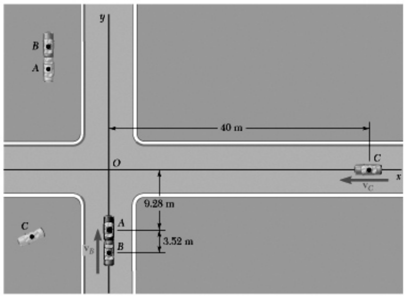
PROBLEM 14.110 (Continued)

$T_1 + V_1 = T_2 + V_2$: $\qquad 0 + 27 = 0.10352 v_A^2 + 2.5$

$$v_A^2 = 236.67 \text{ ft}^2$$

$\mathbf{v}_A = 15.38 \text{ ft/s} \longrightarrow$ ◀

$\mathbf{v}_B = 5.13 \text{ ft/s} \longleftarrow$ ◀



PROBLEM 14.111

Car A was at rest 9.28 m south of the Point O when it was struck in the rear by car B, which was traveling north at a speed v_B. Car C, which was traveling west at a speed v_C, was 40 m east of Point O at the time of the collision. Cars A and B stuck together and, because the pavement was covered with ice, they slid into the intersection and were struck by car C which had not changed its speed. Measurements based on a photograph taken from a traffic helicopter shortly after the second collision indicated that the positions of the cars, expressed in meters, were $\mathbf{r}_A = -10.1\mathbf{i} + 16.9\mathbf{j}$, $\mathbf{r}_B = -10.1\mathbf{i} + 20.4\mathbf{j}$, and $\mathbf{r}_C = -19.8\mathbf{i} - 15.2\mathbf{j}$. Knowing that the masses of cars A, B, and C are, respectively, 1400 kg, 1800 kg, and 1600 kg, and that the time elapsed between the first collision and the time the photograph was taken was 3.4 s, determine the initial speeds of cars B and C.

SOLUTION

Mass center at time of first collision.

$$(m_A + m_B + m_C)\overline{\mathbf{r}}_1 = m_A(\mathbf{r}_A)_1 + m_B(\mathbf{r}_B)_1 + m_C(\mathbf{r}_C)_1$$

$$4800\,\overline{\mathbf{r}}_1 = (1400)(-9.28\mathbf{j}) + (1800)(-12.8\mathbf{j}) + (1600)(40\mathbf{i})$$

$$\overline{\mathbf{r}}_1 = (13.3333\text{ m})\mathbf{i} - (7.5067\text{ m})\mathbf{j}$$

Mass center at time of photo.

$$(m_A + m_B + m_C)\overline{\mathbf{r}}_2 = m_A(\mathbf{r}_A)_2 + m_B(\mathbf{r}_B)_2 + m_C(\mathbf{r}_C)_2$$

$$4800\,\overline{\mathbf{r}}_2 = (1400)(-10.1\mathbf{i} + 16.9\mathbf{j}) + (1800)(-10.1\mathbf{i} + 20.4\mathbf{j}) + (1600)(-19.8\mathbf{i} - 15.2\mathbf{j})$$

$$\overline{\mathbf{r}}_2 = -(13.3333\text{ m})\mathbf{i} + (7.5125\text{ m})\mathbf{j}$$

Since no external horizontal forces act, momentum is conserved and the mass center moves at constant velocity.

$$(m_A + m_B + m_C)\overline{\mathbf{v}} = m_A(\mathbf{v}_A)_1 + m_B(\mathbf{v}_B)_1 + m_C(\mathbf{v}_C)_1 \tag{1}$$

$$\overline{\mathbf{r}}_2 - \overline{\mathbf{r}}_1 = \overline{\mathbf{v}}t \tag{2}$$

Combining (1) and (2),

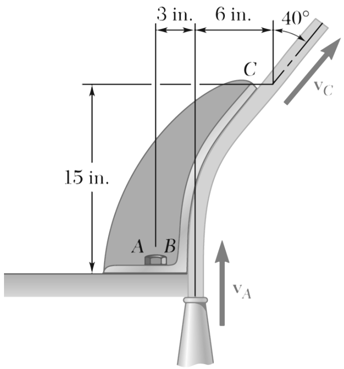
$$(m_A + m_B + m_C)(\overline{\mathbf{r}}_2 - \overline{\mathbf{r}}_1) = [m_A(\mathbf{v}_A)_1 + m_B(\mathbf{v}_B)_1 + m_C(\mathbf{v}_C)_1]\,t$$

$$(4800)(-26.6667\mathbf{i} + 15.0192\mathbf{j}) = [0 + (1800)(v_B)_1\mathbf{j} - (1600)(v_C)_1\mathbf{i}](3.4)$$

Components. $\mathbf{j}$: $72092 = 6120(v_B)_1$, $(v_B)_1 = 11.78$ m/s, $v_B = 42.4$ km/h ◀

$\mathbf{i}$: $-128000 = -5440\,(v_C)_1$, $(v_C)_1 = 23.53$ m/s, $v_C = 84.7$ km/h ◀

PROBLEM 14.112

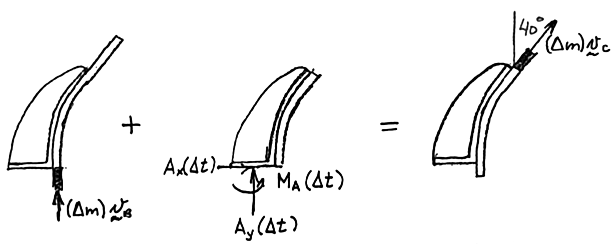


The nozzle shown discharges water at the rate of 200 gal/min. Knowing that at both B and C the stream of water moves with a velocity of magnitude 100 ft/s, and neglecting the weight of the vane, determine the force-couple system which must be applied at A to hold the vane in place $(1\ \text{ft}^3 = 7.48\ \text{gal})$.

SOLUTION

$$Q = \frac{200\ \text{gal/min}}{(7.48\ \text{gal/ft}^3)(60\ \text{s/min})}$$
$$= 0.44563\ \text{ft}^3/\text{s}$$
$$\frac{dm}{dt} = \frac{\gamma Q}{g}$$
$$= \frac{(62.4\ \text{lb/ft}^3)(0.44563\ \text{ft}^3/\text{s})}{32.2\ \text{ft/s}^2}$$
$$= 0.8636\ \text{lb}\cdot\text{s/ft}$$
$$\mathbf{v}_B = (100\ \text{ft/s})\mathbf{j}$$
$$\mathbf{v}_C = (100\ \text{ft/s})(\sin 40°\mathbf{i} + \cos 40°\mathbf{j})$$

Apply the impulse-momentum principle.



$\xrightarrow{+}$ x components:
$$0 + A_x(\Delta t) = (\Delta m)(100 \sin 40°)$$
$$A_x = \frac{\Delta m}{\Delta t}(100 \sin 40°)$$
$$= (0.8636)(100 \sin 40°) \qquad A_x = 55.5\ \text{lb} \longrightarrow$$

$+\uparrow$ y components:
$$(\Delta m)(100) + A_y(\Delta t) = (\Delta m)(100 \cos 40°)$$
$$A_y = \frac{\Delta m}{\Delta t}(100)(\cos 40° - 1)$$
$$= (0.8636)(100)(\cos 40° - 1)$$
$$= -20.2\ \text{lb} \qquad A_y = 20.2\ \text{lb} \downarrow$$

PROBLEM 14.112 (Continued)

$+\circlearrowright$ Moments about A:

$$\left(\frac{3}{12}\right)(\Delta m)(100)+M_A(\Delta t)=\left(\frac{9}{12}\right)(\Delta m)(100\cos 40°)-\left(\frac{15}{12}\right)(\Delta m)(100\sin 40°)$$

$$M_A=\left(\frac{\Delta m}{\Delta t}\right)(75\cos 40°-125\sin 40°-25)$$

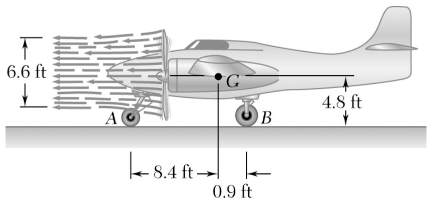
$$=(0.8636)(-47.895)$$

$$=-41.36\ \text{lb}\cdot\text{ft}$$

$$\mathbf{M}_A=41.4\ \text{lb}\cdot\text{ft} \circlearrowright \blacktriangleleft$$

$$\mathbf{A}=59.1\ \text{lb} \searrow 20.0° \blacktriangleleft$$

PROBLEM 14.113



Prior to takeoff the pilot of a 6000-lb twin-engine airplane tests the reversible-pitch propellers with the brakes at Point B locked. Knowing that the velocity of the air in the two 6.6-ft-diameter slipstreams is 60 ft/s and that Point G is the center of gravity of the airplane, determine the reactions at Points A and B. Assume $\gamma = 0.075\ \text{lb/ft}^3$ and neglect the approach velocity of the air.

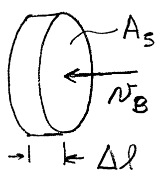
SOLUTION

Let F be the force exerted on the slipstream of one engine.

$$F = \frac{dm}{dt}(v_B - v_A)$$

Calculation of $\dfrac{dm}{dt}$.

mass = density × volume = density × area × length



$$\Delta m = \rho A_B(\Delta l) = \rho A_B v_B(\Delta t) = \frac{\gamma A_B v_B(\Delta t)}{g}$$

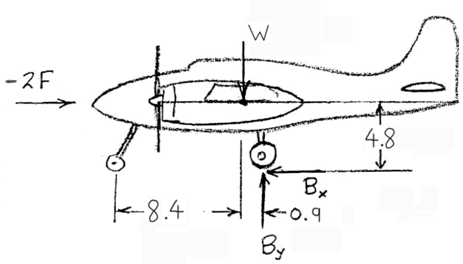
$$\frac{\Delta m}{\Delta t} = \frac{\gamma A_B v_B}{g} \quad \text{or} \quad \frac{dm}{dt} = \frac{\gamma}{g}\left(\frac{\pi}{4}D^2\right)v_B$$

Assume that v_A, the velocity far upstream, is negligible.

$$F = \frac{\gamma}{g}\left(\frac{\pi}{4}D^2\right)v_B(v_B - 0) = \left(\frac{0.075}{32.2}\right)\left(\frac{\pi}{4}\right)(6.6)^2(60)^2 = 286.87\ \text{lb} \longleftarrow$$

The force exerted by two slipstreams on the airplane is $-2F$. $\quad -2F = 573.74\ \text{lb} \longrightarrow$

Statics.



$$+\circlearrowleft \Sigma M_B = 0:$$

$$A = \frac{1}{9.3}[(0.9)(6000) - (4.8)(573.74)]$$

$$= 284.5\ \text{lb} \qquad \mathbf{A} = 285\ \text{lb}\uparrow \blacktriangleleft$$

$$\xrightarrow{+} F_x = 0: \quad -2F - B_x = 0$$

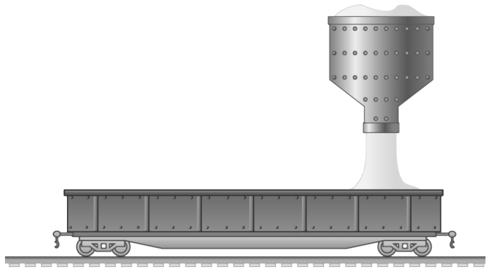
$$B_x = -2F = 573.74\ \text{lb} \longleftarrow$$

$$+\uparrow \Sigma F_y = 0: \quad A + B_y - W = 284.5 + B_y - (6000) = 0$$

$$B_y = 5715.5\ \text{lb}\uparrow$$

$$\mathbf{B} = [573.74\ \text{lb} \longleftarrow] + [5715.5\ \text{lb}\uparrow]$$

$$\mathbf{B} = 5740\ \text{lb} \measuredangle\ 84.3° \blacktriangleleft$$

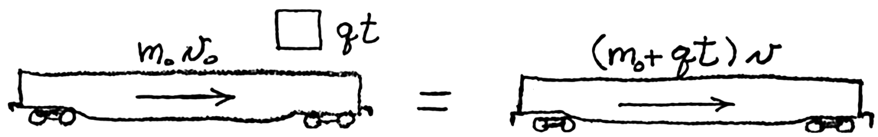


PROBLEM 14.114

A railroad car of length L and mass m_0 when empty is moving freely on a horizontal track while being loaded with sand from a stationary chute at a rate $dm/dt = q$. Knowing that the car was approaching the chute at a speed v_0, determine (*a*) the mass of the car and its load after the car has cleared the chute, (*b*) the speed of the car at that time.

SOLUTION

Consider the conservation of the horizontal component of momentum of the railroad car of mass m_0 and the sand mass qt.



$$m_0 v_0 = (m_0 + qt)v \qquad v = \frac{m_0 v_0}{m_0 + qt} \tag{1}$$

$$\frac{dx}{dt} = v = \frac{m_0 v_0}{m_0 + qt}$$

Integrating, using $\quad x_0 = 0 \quad$ and $\quad x = L$ when $t = t_L$,

$$L = \int_0^{t_L} v\,dt = \int_0^{t_L} \frac{m_0 v_0}{m_0 + qt}\,dt = \frac{m_0 v_0}{q}[\ln(m_0 + qt_L) - \ln m_0]$$

$$= \frac{m_0 v_0}{q} \ln \frac{m_0 + qt_L}{m_0}$$

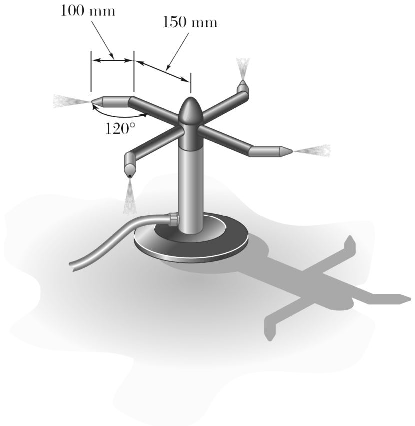
$$\ln \frac{m_0 + qt_L}{m_0} = \frac{qL}{m_0 v_0} \qquad \frac{m_0 + qt_L}{m_0} = e^{qL/m_0 v_0}$$

(*a*) Final mass of railroad car and sand $\qquad m_0 + qt_L = m_0 e^{qL/m_0 v_0}$ ◀

(*b*) Using Eq. (1),

$$v_L = \frac{m_0 v_0}{m_0 + qt_L} = \frac{m_0 v_0}{m_0} e^{-qL/m_0 v_0}$$

$$v_L = v_0 e^{-qL/m_0 v_0}$$ ◀



PROBLEM 14.115

A garden sprinkler has four rotating arms, each of which consists of two horizontal straight sections of pipe forming an angle of 120° with each other. Each arm discharges water at a rate of 20 L/min with a velocity of 18 m/s relative to the arm. Knowing that the friction between the moving and stationary parts of the sprinkler is equivalent to a couple of magnitude $M = 0.375\ \text{N}\cdot\text{m}$, determine the constant rate at which the sprinkler rotates.

SOLUTION

The flow through each arm is 20 L/min.

$$Q = \frac{20\ \text{L/min}}{1000\ \text{L/m}^3} \times \frac{1\,\text{min}}{60\ \text{s}} = 333.33\times 10^{-6}\ \text{m}^3/\text{s}$$

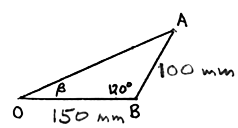
$$\frac{dm}{dt} = \rho Q = (1000\ \text{kg/m}^3)(333.33\times 10^{-6})$$
$$= 0.33333\ \text{kg/s}$$

Consider the moment about O exerted on the fluid stream of one arm. Apply the impulse-momentum principle. Compute moments about O. First, consider the geometry of triangle OAB. Using first the law of cosines,

$$(OA)^2 = 150^2 + 100^2 - (2)(150)(100)\cos 120°$$
$$OA = 217.95\ \text{mm} = 0.21795\ \text{m}$$

Law of sines: $$\frac{\sin\beta}{100} = \frac{\sin 120°}{217.95}$$

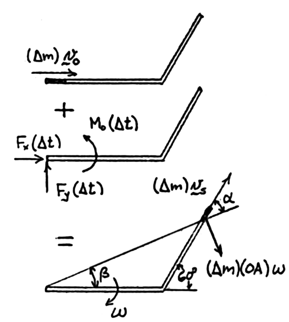
$$\beta = 23.413°, \quad \alpha = 60° - \beta = 36.587°$$



+↻ Moments about O:

$$(\Delta m)(v_O)(0) + M_O(\Delta t) = (OA)(\Delta m)v_s \sin\alpha - (OA)(\Delta m)(OA)\omega$$

$$M_O = \frac{\Delta m}{\Delta t}[(OA)v_s \sin\alpha - (OA)^2\omega]$$
$$= (0.33333)[(0.21795)(18)\sin 36.587° - (0.21795)^2\omega]$$
$$= 0.77945 - 0.015834\omega$$



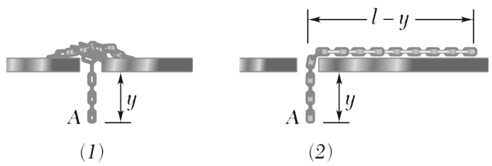
Moment that the stream exerts on the arm is $-M_O$.

Balance of the friction couple and the four streams

$$M_F - 4M_O = 0$$
$$0.375 - 4(0.77945 - 0.015834\omega) = 0$$
$$\omega = 43.305\ \text{rad/s}$$

$\omega = 414\ \text{rpm}$ ◀

PROBLEM 14.116

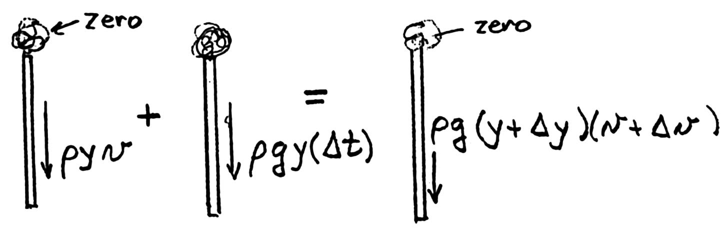


A chain of length l and mass m falls through a small hole in a plate. Initially, when y is very small, the chain is at rest. In each case shown, determine (a) the acceleration of the first link A as a function of y, (b) the velocity of the chain as the last link passes through the hole. In case 1, assume that the individual links are at rest until they fall through the hole; in case 2, assume that at any instant all links have the same speed. Ignore the effect of friction.

SOLUTION

Let ρ be the mass per unit length of chain. Assume that the weight of any chain above the hole is supported by the floor. It and the corresponding upward reaction of the floor are not shown in the diagrams.

Case 1: Apply the impulse-momentum principle to the entire chain.



$$\rho yv + \rho gy\Delta t = \rho(y+\Delta y)(v+\Delta v)$$

$$= \rho yv + \rho(\Delta y)v + \rho y(\Delta v) + \rho(\Delta y)(\Delta v)$$

$$\rho gy = \rho\frac{\Delta y}{\Delta t}v + \rho y\frac{\Delta v}{\Delta t} + \rho\frac{(\Delta y)(\Delta v)}{\Delta t}$$

Let $\Delta t \to 0$.

$$\rho gy = \rho\frac{dy}{dt}v + \rho y\frac{dv}{dt}$$

$$= \rho\frac{d}{dt}(yv)$$

Multiply both sides by yv.

$$\rho gy^2 v = \rho yv\frac{d}{dt}(yv)$$

Let $v = \dfrac{dy}{dt}$ on left hand side.

$$\rho gy^2\frac{dy}{dt} = \rho yv\frac{d}{dt}(yv)$$

Integrate with respect to time.

$$\rho g\int y^2 dy = \rho\int (yv)\,d(yv)$$

$$\frac{1}{3}\rho gy^3 = \frac{1}{2}\rho(yv)^2 \quad \text{or} \quad v^2 = \frac{2}{3}gy \qquad (1)$$

Differentiate with respect to time.

$$2v\frac{dv}{dt} = \frac{2}{3}g\frac{dy}{dt} = \frac{2}{3}gv$$

(*a*) $$a = \frac{dv}{dt} = \frac{1}{3}g$$ $\mathbf{a} = 0.333g \downarrow$ ◀

(*b*) Set $y = l$ in Eq. (1). $$v^2 = \frac{2}{3}gl$$ $\mathbf{v} = 0.817\sqrt{gl} \downarrow$ ◀

Case 2: Apply conservation of energy using the floor as the level from which the potential energy is measured.

$$T_1 = 0 \qquad V_1 = 0$$

$$T_2 = \frac{1}{2}mv^2 \quad V_2 = -\rho g y \frac{y}{2}$$

$$T_1 + V_1 = T_2 + V_2$$

$$0 = \frac{1}{2}mv^2 - \frac{1}{2}\rho g y^2 \qquad v^2 = \frac{\rho g y^2}{m} = \frac{g y^2}{l} \tag{2}$$

Differentiating with respect to *y*, $$2v\frac{dv}{dy} = \frac{2gy}{l}$$

(*a*) Acceleration: $$a = v\frac{dv}{dy} = \frac{gy}{l}$$ $\mathbf{a} = \frac{gy}{l} \downarrow$ ◀

(*b*) Setting $y = l$ in Eq. (2), $$v^2 = gl$$ $\mathbf{v} = \sqrt{gl} \downarrow$ ◀

Note: The impulse-momentum principle may be used to obtain the force that the edge of the hole exerts on the chain.