النظير الضربي

صباح الخير

اما

- Inverse only for $\underbrace{\text{square matrices}}_{n\times n}$ (non singular) غير منفردة

Def If $A_{2\times 2} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ then

the determinante of A is the number رقم

$$|A| = a_{11} \cdot a_{22} - a_{12}\, a_{21}$$

Exp Find $|A|$ if

① $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \Rightarrow |A| = (1)(4) - (2)(3)$
$= 4 - 6$
$= -2$

A is nonsingular since $|A| \neq 0$

② $A = \begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix} \Rightarrow |A| = (1)(6) - (3)(2)$
$= 6 - 6$
$= 0$

singular matrix since $|A| = 0$

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matrix A with $|A_{n\times n}| = 0$ is

Def • Any matrix $A_{n\times n}$ with $|A_{n\times n}| = 0$ is called singular matrix مصفوفة منفردة

(Any singular matrix has no inverse)

• If $|A_{n\times n}| \neq 0$, then A is nonsingular

it has inverse

Def If $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ is nonsingular ($|A| \neq 0$)

then the inverse of A exists and equal

$$A^{-1} = \frac{1}{|A|}\begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix}$$

Exp $A = \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 2 \\ 1 & -1 \end{bmatrix}$

① Is A singular? nonsingular?

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$|A| = (1)(1) - (2)(1) = 1 - 2 = -1 \neq 0 \Rightarrow A$ nonsingular

$\Downarrow$

A has inverse

② Does B have inverse?

$|B| = (-1)(-1) - (2)(1) = 1 - 2 = -1 \neq 0 \Rightarrow B$ nonsingular

B has inverse

ⓒ Find A^{-1}

$A = \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}, \quad |A| = -1$

$A^{-1} = \frac{1}{-1}\begin{bmatrix} 1 & -2 \\ -1 & 1 \end{bmatrix} = -1\begin{bmatrix} 1 & -2 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 2 \\ 1 & -1 \end{bmatrix}$

$= B$

ⓓ Find B^{-1}

$B = \begin{bmatrix} -1 & 2 \\ 1 & -1 \end{bmatrix}, \quad |B| = -1$

$B^{-1} = \frac{1}{-1}\begin{bmatrix} -1 & -2 \\ -1 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix} = A$

$A^{-1} = B$ and $B^{-1} = A$

check $A^{-1} = B$

ⓔ $\underset{\substack{\downarrow \\ B^{-1}}}{A}\ \underset{\substack{\downarrow \\ A^{-1}}}{B} = \overset{\text{صف}}{\begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}}_{2\times 2} \overset{\text{عمود}}{\begin{bmatrix} -1 & 2 \\ 1 & -1 \end{bmatrix}}_{2\times 2} = \begin{bmatrix} (1)(-1)+(2)(1) & (1)(2)+(2)(-1) \\ (1)(-1)+(1)(1) & (1)(2)+(1)(-1) \end{bmatrix}$

$= \begin{bmatrix} -1+2 & 2+-2 \\ -1+1 & 2+-1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_{2\times 2}$

unit matrix

$A A^{-1} = A^{-1} A = I$

$B^{-1} B = B B^{-1} = I$

ضرب أي مصفوفة في النظير الضربي رح يكون الناتج المصفوفة الوحدة

$A A^{-1} = I$ $\qquad$ $A^{-1} A = I$

Exp $\quad A = \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix} \Rightarrow A^{-1} = \begin{bmatrix} -1 & 2 \\ 1 & -1 \end{bmatrix}$

Find $(A^{-1})^{-1}$

$(A^{-1})^{-1} = A$ في المثال السابق

Exp Given $A = \begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix}$, $B = \begin{bmatrix} 2 & -1 \\ -4 & 2 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix}$

① Does C have Inverse?

No since C is not square matrix

② Does B have inverse?

square ✓ But $|B| = (2)(2) - (-1)(-4) = 4 - 4 = 0$

no inverse for B

③ Does A have inverse?

$A = \begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix} \Rightarrow$ square ✓ $\Rightarrow |A| = (3)(6) - (5)(4)$

$= 18 - 20$

$= -2 \neq 0$

Yes A^{-1} exists

④ Which matrix is nonsingular?

محدد ≠ صفر

A

⑤ Find A^{-1}

$A = \begin{pmatrix} 3 & 5 \\ 4 & 6 \end{pmatrix}$, $|A| = -2$

$$A^{-1} = \frac{1}{-2} \begin{bmatrix} 6 & -5 \\ -4 & 3 \end{bmatrix} = \begin{bmatrix} -3 & \frac{5}{2} \\ 2 & -\frac{3}{2} \end{bmatrix}$$

⑥ Check your solution in ⑤

صف عمود

$$A^{-1}A = \underset{2\times 2}{\begin{bmatrix} -3 & \frac{5}{2} \\ 2 & -\frac{3}{2} \end{bmatrix}} \underset{2\times 2}{\begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix}}$$

$$= \begin{bmatrix} (-3)(3) + (\frac{5}{\cancel{2}})(\overset{2}{\cancel{4}}) & (-3)(5) + (\frac{5}{\cancel{2}})(\overset{3}{\cancel{6}}) \\ (2)(3) + (\frac{-3}{\cancel{2}})(\overset{2}{\cancel{4}}) & (2)(5) + (\frac{-3}{\cancel{2}})(\overset{3}{\cancel{6}}) \end{bmatrix}$$

$$= \begin{bmatrix} -9+10 & -15+15 \\ 6-6 & 10-9 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{2\times 2} = I$$

$$AA^{-1} = \text{- - - - -} = I_{2\times 2} \checkmark$$

Exp Let $A = \begin{bmatrix} 2 & 5 & 4 \\ 1 & 4 & 3 \\ 1 & -3 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 2 & 1 \\ -5 & 8 & 2 \\ 7 & -11 & -3 \end{bmatrix}$

Does $B = A^{-1}$?

صف عمود

$$AB = \begin{bmatrix} 2 & 5 & 4 \\ 1 & 4 & 3 \\ & & -2 \end{bmatrix} \begin{bmatrix} -1 & 2 & 1 \\ -5 & 8 & 2 \\ 7 & -11 & -3 \end{bmatrix} \overset{\text{Check}}{=} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

$$AB = \begin{bmatrix} \cdots & \cdots & \cdots \\ 1 & 4 & 3 \\ 1 & -3 & -2 \end{bmatrix}_{3\times 3} \begin{bmatrix} -5 & 0 & 2 \\ 7 & -11 & -3 \end{bmatrix}_{3\times 3} = \begin{pmatrix} \cdots & \cdots & \cdots \\ 0 & 0 & 1 \end{pmatrix}_{3\times 3} = I$$

$$= \begin{bmatrix} -2-25+28 & 4+40-44 & 2+10-12 \\ -1-20+21 & 2+32-33 & 1+8-9 \\ -1+15-14 & 2-24+22 & 1-6+6 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I_{3\times 3}$$

Yes it does $\Rightarrow$ $A = B^{-1}$

$A^{-1} = B$

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linear system

$$Ax = b \Rightarrow \underbrace{\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}}_{A} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

A: coefficient matrix مصفوفة المعاملات

x: مصفوفة المتغيرات

b: مصفوفة الثوابت

To solve for x_1, x_2 $\Downarrow$

$A^{-1}Ax = A^{-1}b$

$Ix = A^{-1}b$

A Nonsingular

$\downarrow$

A^{-1} exists

$Ix = A^{-1}b$

$\boxed{x = A^{-1}b}$ $\rightarrow$ القانون

Exp Given $A^{-1} = \begin{bmatrix} 1 & -2 \\ 1 & 3 \end{bmatrix}$. Solve this linear system

$$Ax = \begin{bmatrix} 5 \\ -1 \end{bmatrix}$$

$$x = A^{-1}b = \begin{array}{c} \text{صف} \\ \begin{bmatrix} 1 & -2 \\ 1 & 3 \end{bmatrix} \\ 2\times 2 \end{array} \begin{array}{c} \text{عمود} \\ \begin{bmatrix} 5 \\ -1 \end{bmatrix} \\ 2\times 1 \end{array}$$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{bmatrix} (1)(5) + (-2)(-1) \\ (1)(5) + (3)(-1) \end{bmatrix}$$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 7 \\ 2 \end{pmatrix}$$

$x_1 = 7$
$x_2 = 2$

Exp If $B^{-1} = \begin{bmatrix} 3 & 0 & -3 \\ 1 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix}$ then solve $BX = \begin{bmatrix} -1 \\ 2 \\ 3 \end{bmatrix}$

صف ___ عمود

$\begin{bmatrix} 1 & 2 & 1 \end{bmatrix}$

صف   عمود

$$x = B^{-1} b = \underset{3\times 3}{\begin{bmatrix} 3 & 0 & -3 \\ 1 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix}} \underset{3\times 1}{\begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix}}$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{bmatrix} -3+0-9 \\ -1+2+6 \\ -1+4+3 \end{bmatrix} = \begin{pmatrix} -12 \\ 7 \\ 6 \end{pmatrix}$$

$x_1 = -12$

$x_2 = 7$

$x_3 = 6$