Chapter 7.1, Problem 35E

Problem

Let *J*5 = {0, 1, 2, 3, 4}. Then *J*5 − {0} = {1, 2, 3, 4}. Student *A* tries to define a function *R: J*5 − {0} → **Z** as follows: For each $x \in J5 - \{0\}$,

R(*x*) is the number *y* so that (*xy*) *mod* 5 = 1.

Student *B* claims that *R* is not well defined. Who is right: student *A* or student *B*? Justify your answer.

Step-by-step solution

Step 1 of 1

For $J_5 = \{0,1,2,3\}$, the function is defined as,

$R: J_5 - \{0\} \to J_5 - \{0\}$, $R(x)$ is the number *y* so that $(xy) \bmod 5 = 1$

Suppose that *R* is well defined. Then $R(3)$ will have a determined value which is unique for each $x \in J_5\{0\}$.

Since $(3 \cdot 2) mod 5 = 1$, then $R(3) = 2$

Since $(3 \cdot 7) mod 5 = 1$, then $R(3) = 7$

These imply that $R(3)$ has no uniquely determined value.

So *R* is not well defined.

Thus, Student *B* is correct.