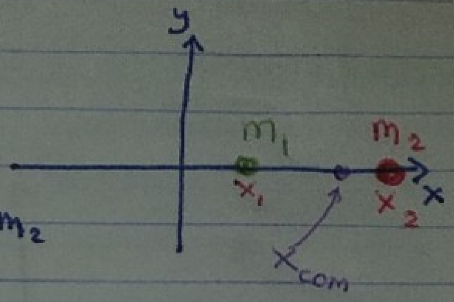
Chapter 9:
Center of Mass and Linear Momentum

Center of Mass

$$x_{com} = \frac{m_1x_1 + m_2x_2}{m_1+m_2}$$

$$= \frac{m_1x_1 + m_2x_2}{M} \quad , M = m_1 + m_2$$



For... Many Particles

$$x_{com} = \frac{m_1x_1 + m_2x_2 + m_3x_3 + \cdots + m_nx_n}{M}$$

$$= \frac{1}{M}\sum_{i=1}^{n} m_ix_i \quad , M = m_1 + m_2 + \cdots + m_n$$

For 3Dimensions $\begin{cases} \to x \\ \to y \\ \to z \end{cases}$

$$x_{com} = \frac{1}{M}\sum_{i=1}^{n} m_ix_i \quad , y_{com} = \frac{1}{M}\sum_{i=1}^{n} m_iy_i \quad , z_{com} = \frac{1}{M}\sum_{i=1}^{n} m_iz_i$$

the Position of each particle is given by

$$\vec{r}_i = x_i\hat{i} + y_i\hat{j} + z_i\hat{k}$$

the Position of the com is given by

$$\vec{r}_{com} = x_{com}\hat{i} + y_{com}\hat{j} + z_{com}\hat{k}$$

$$\vec{r}_{com} = \frac{1}{M}\sum m_i\vec{r}_i$$

For Solid Bodies (infinite number of point masses)

$$\sum \longrightarrow \int$$

$$x_{com} = \frac{1}{M}\int x\,dm \quad , y_{com} = \frac{1}{M}\int dm \quad , z_{com} = \int z\,dm$$

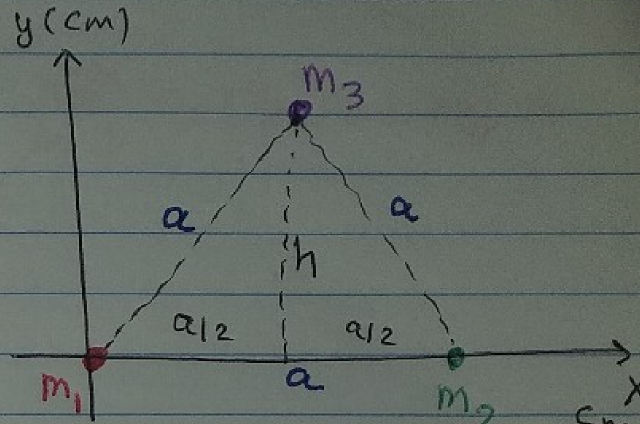
The center of mass of a system of Particles is the point that moves as though

① all of the system's were concentrated there

② all external forces were applied thre.

Sample Problem 9.01:

$m_1 = 1.2kg$, $m_2 = 2.5kg$, $m_3 = 3.4kg$ are three Particles Placed on the verices of equilateral of edge $a = 140 cm$.



$m_1 = 1.2kg\ (0,0)$

$m_2 = 2.5kg\ (140,0)$ cm

$m_3 = 3.4kg\ (70,121)$ cm

$M = m_1 + m_2 + m_3$

$= 7.1kg$

$$a^2 = h^2 + \left(\frac{a}{2}\right)^2$$

$$h = \sqrt{a^2 - \frac{a^2}{4}} = \sqrt{\frac{3a^2}{4}} =$$

$$x_{com} = \frac{1}{M}\sum_{i=1}^{3} m_i x_i = \frac{1.2(0) + 2.5(140) + 3.4(70)}{7.1}$$

$= 83$ cm

$$y_{com} = \frac{1}{M}\sum_{i=1}^{3} m_i y_i = \frac{1.2(0) + 2.5(0) + 3.4(121)}{7.1}$$

$= 58$ cm

$$\vec{r}_{com} = 83\hat{i} + 58\hat{j}$$

Problem (9.01)

$m_1 = 2kg\ (-1.2, 0.5)m \Rightarrow \vec{r}_1 = -1.2\hat{i} + 0.5\hat{j}$ m

$m_2 = 4kg\ (0.6, -0.75)m \Rightarrow \vec{r}_2 = 0.6\hat{i} - 0.75\hat{j}$ m

$m_3 = 3kg\ (x_3, y_3)$?

com $(-0.5, -0.7)$ m $\quad \vec{r}_{com} = -0.5\hat{i} - 0.7\hat{j}$ m

$M = 2 + 4 + 3 = 9kg$

$$x_{com} = \frac{1}{M}\sum m_i x_i$$

$$-0.5 = \frac{2(-1.2)+4(0.6)+3x_3}{9}$$

$$-4.5 = -2.4 + 2.4 + 3x_3$$

$$x_3 = -\frac{4.5}{3} = -1.5\,m$$

$$y_{com} = \frac{1}{M}\sum m_i y_i$$

$$-0.7 = \frac{2(0.5)+4(-0.75)+3y_3}{9}$$

$$-6.3 = 2-3+3y_3$$

$$y_3 = \frac{-5.3}{3} = -1.77\,m$$

$$\vec{r}_3 = -1.5\hat{i} - 1.77\hat{j}\ m$$

Newton's Second Law for a System of Particles:

$$\vec{r}_{com} = \frac{1}{M}\sum m_i \vec{r}_i$$

$$\vec{r}_{com} = \frac{1}{M}(m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + \cdots m_n\vec{r}_n)$$

$$M\vec{r}_{com} = m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + \cdots + m_n\vec{r}_n$$

If the particles are moving

Differentiating with respect to time

$$M\vec{V}_{com} = m_1\vec{V}_1 + m_2\vec{V}_2 + m_3\vec{V}_3 + \cdots + m_n\vec{V}_n$$

Differentiating with respect to time

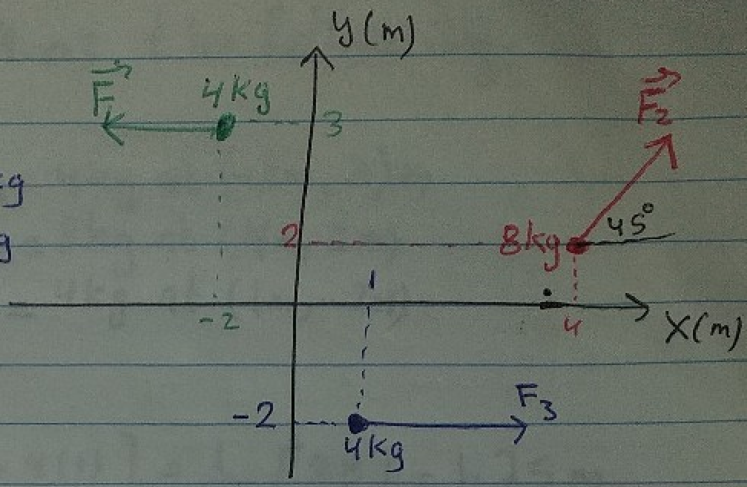
$$M\vec{a}_{com} = m_1\vec{a}_1 + m_2\vec{a}_2 + m_3\vec{a}_3 + \cdots + m_n\vec{a}_n$$

$$M\vec{a}_{com} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \cdots + \vec{F}_n$$

$$M\vec{a}_{com} = \vec{F}_{net}$$

Sample Problem 9.03.

$F_1 = 6N$ acts on $m_1 = 4kg$
$F_2 = 12N$ acts on $m_2 = 8kg$
$F_3 = 14N$ acts on $m_3 = 4kg$
the positions are shown.
Find $\vec{a}_{com}$?



$$\vec{F_1} + \vec{F_2} + \vec{F_3} = M\vec{a}_{com}$$

$$\vec{a}_{com} = \frac{\vec{F_1} + \vec{F_2} + \vec{F_3}}{M}, \quad M = 4 + 8 + 4 = 16kg$$

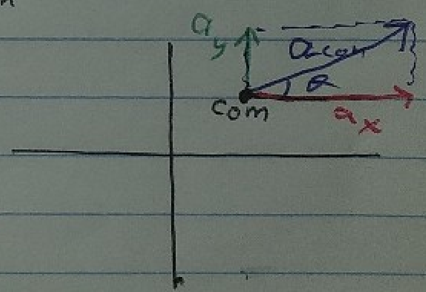
$$a_{com,x} = \frac{F_{1x} + F_{2x} + \vec{F}_{3x}}{16} = \frac{-6 + 12\cos 45 + 14}{16}$$

$$a_{com,x} = 1.03\ m/s^2$$

$$a_{com,y} = \frac{F_{1y} + F_{2y} + F_{3y}}{M} = \frac{0 + 12\sin 45 + 0}{16}$$

$$a_{com,y} = 0.53\ m/s^2$$

$$\vec{a}_{com} = 1.03\hat{i} + 0.53\hat{j}\ m/s^2$$



$$a_{com} = \sqrt{(1.03)^2 + (0.53)^2} = 1.2\ m/s^2 \text{ in the direction}$$

$$\theta = \tan^{-1}\left(\frac{0.53}{1.03}\right) = 27^\circ$$

$$\vec{F}_{net} = M\vec{a}_{com} = 16(1.03\hat{i} + 0.53\hat{j})$$

$$\vec{F}_{net} = 16.5\hat{i} + 8.48\hat{j}\ N$$

or

$$F_{net} = \sqrt{(16.5)^2 + (8.48)^2}$$

$$\vec{F}_{net} = 18.6N, \text{ at } \theta = +27^\circ$$

4

Add to sample Problem 9.03 the following:

① Find $\vec{r}_{com}$ initially?

From the graph $\longrightarrow$ $m_1 = 4kg$ at $(-2m, 3)m$
$m_2 = 8kg$ at $(4m, 2m)$
$m_3 = 4kg$ at $(1m, -2m)$

$$(x_{com})_i = \frac{1}{M}\sum_{i=1}^{3} m_i x_i$$

$$= \frac{1}{16}\left[4(-2) + 8(4) + 4(1)\right] = \frac{1}{16}[28] = 1.75\,m$$

$$(y_{com})_i = \frac{1}{M}\sum m_i y_i$$

$$(y_{com})_i = \frac{1}{16}\left[4(3) + 8(2) + 4(-2)\right] = \frac{20}{16} = 1.25\,m$$

$$(\vec{r}_{com})_i = 1.75\hat{i} + 1.25\hat{j}\ m$$

② After 10s find the position of com?

$$\Delta\vec{r} = \vec{v}_0 t + \frac{1}{2}\vec{a}_{com} t^2$$

$$\Delta\vec{r}_{com} = 0 + \frac{1}{2}(1.03\hat{i} + 0.53\hat{j})(10)^2$$

$$\Delta\vec{r}_{com} = 51.5\hat{i} + 26.5\hat{j}\ m$$

$$\Delta\vec{r}_{com} = \vec{r}_{comf} - \vec{r}_{comi}$$

$$(\vec{r}_{com})_{final} = \Delta\vec{r}_{com} + (\vec{r}_{com})_{initial}$$

$$= 51.5\hat{i} + 26.5\hat{j} + 1.75\hat{i} + 1.25\hat{j}$$

$$(\vec{r}_{com})_{final} = 53.3\hat{i} + 27.8\hat{j}\ m$$

③ ~~find $(\vec{V}_f)_{com}$~~ ? find V_{com} after 10s?

$$\vec{V}_{f\,com} = \vec{V}_{i\,com} + \vec{a}_{com} t \ ,\quad (\vec{V}_f)_{com} = 0 + (1.03\hat{i} + 0.53\hat{j})(10)$$

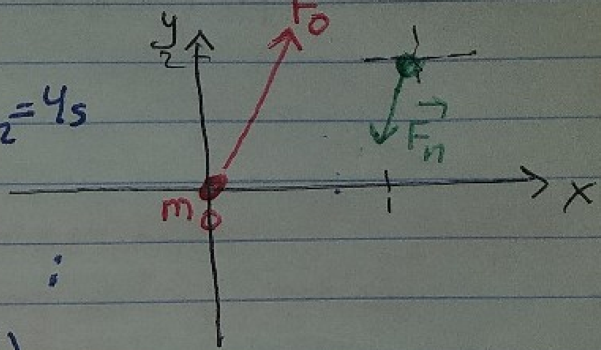
$$= 10.3\hat{i} + 5.3\hat{j}\ m/s$$

Problem 9.11:

A big Olive ($m_o = 0.5$ kg) at $(0,0)$, $\vec{F}_o = 2\hat{i} + 3\hat{j}$ N

A big Brazil nut ($M_n = 1.5$ kg) at $(1.0\text{ m}, 2.0\text{ m})$

$\vec{F}_n = -3\hat{i} - 2\hat{j}$ N

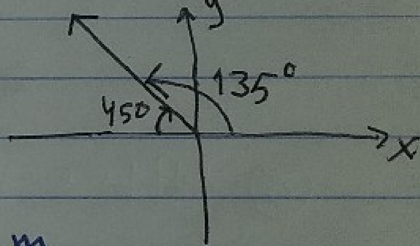


Find $\Delta\vec{r}_{com}$? during $t_1 = 0 \rightarrow t_2 = 4$ s

$$M\vec{a}_{com} = \vec{F}_1 + \vec{F}_2$$

$$(0.5 + 1.5)\vec{a}_{com} = (2\hat{i} + 3\hat{j}) + (-3\hat{i} - 2\hat{j})$$

$$2\vec{a}_{com} = -1\hat{i} + 1\hat{j} \qquad \vec{a}_{com} = -0.5\hat{i} + 0.5\hat{j} \text{ m/s}^2$$



$$(\Delta\vec{r})_{com} = \vec{V}_o t + \frac{1}{2}\vec{a}_{com} t^2$$

$$= 0 + \frac{1}{2}(-0.5\hat{i} + 0.5\hat{j})(4)^2$$

$$\Delta\vec{r}_{com} = -4\hat{i} + 4\hat{j} \text{ m} \longrightarrow \Delta\vec{r} = 4\sqrt{2} \text{ m}$$

$= 5.7$ m, at $45°$ with $-x$ clockwise

$\Delta\vec{r} = 5.7$ m; at $135°$ with $+x$ Counterclockwise

Linear Momentum:

$\vec{P} = m\vec{v}$ (Linear Momentum of a particle is a vector quantity

kg.m/s

Newton expressed his second Law in terms of momentum

Newton's second Law in motion:

The time rate of change of the momentum of a particle is equal to the net force acting on the particle and in the direction of that force

$$\vec{F}_{net} = \frac{d\vec{P}}{dt}$$

$\vec{F}_{net} = \frac{d(m\vec{v})}{dt}$ for constant m $\Longrightarrow$

$\vec{F}_{net} = m\frac{d\vec{v}}{dt}$

$\vec{F}_{net} = m\vec{a}$ (Newton's 2^{nd} Law)

The Linear Momentum of a System of particles:

Remember $\Longrightarrow$ $\vec{r}_{com} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + \cdots + m_n\vec{r}_n}{M}$

Differentiate with respect to time $\frac{d}{dt}\vec{r}_{com} \Longrightarrow$

$\vec{V}_{com} = \frac{1}{M}\left[m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + \cdots + m_n\vec{v}_n\right]$

$M\vec{V}_{com} = \vec{P}_1 + \vec{P}_2 + \vec{P}_3 + \cdots \vec{P}_n$

$M\vec{V}_{com} = \vec{P}$

$\vec{P} = \vec{P}_1 + \vec{P}_2 + \cdots + \vec{P}_n$

$$\vec{F}_{net} = \frac{d\vec{P}}{dt} \quad , \quad \vec{P} = \vec{p}_1 + \vec{p}_2 + \cdots + \cdots$$

For a system of Particles

Collision and Impulse:

$$\vec{F}(t) = \frac{d\vec{P}}{dt} \Rightarrow$$

$$d\vec{p} = \vec{F}(t)\,dt$$

$$\int_{t_i}^{t_f} d\vec{p} = \int_{t_i}^{t_f} F(t)\,dt$$

change in momentum

$\overrightarrow{\text{Impulse}} = \vec{J}$

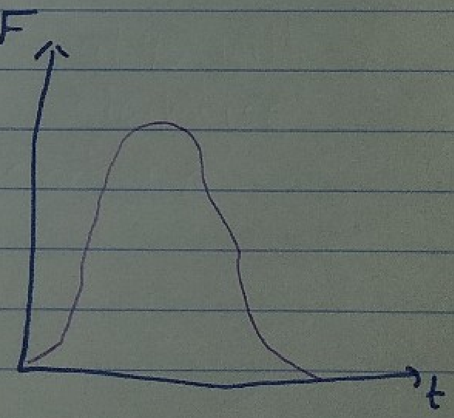
$$\vec{J} = \int_{t_i}^{t_f} \vec{F}(t)\,dt$$

$$\overrightarrow{\Delta P} = \int_{t_i}^{t_f} F(t)\,dt$$

Linear momentum – Impulse theorem

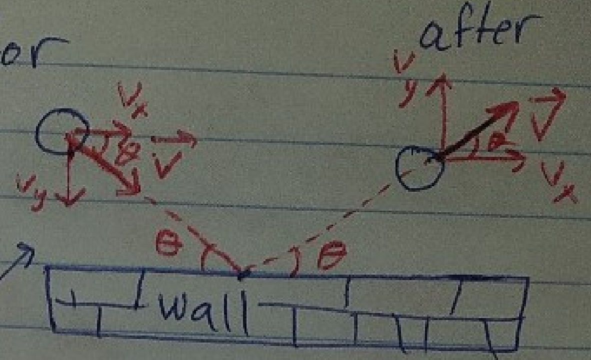
$$\Delta\vec{P} = \vec{J}$$

$$\vec{J} = \int_{t_i}^{t_f} \vec{F}(t)\,dt$$ = Area under the curve of F versus t from $t_i \rightarrow t_f$



$$\vec{J} = \vec{F}_{avg}\,\Delta t$$

(9-38) $m = 300g = 0.3kg$ befor



before collision:
$\vec{V}_b = 6\ m/s$, at $\theta = 30°$ with the Wall

After collision
$\vec{V}_a = 6\ m/s$, at $\theta = 30°$
Collision time, $\Delta t = 10\ ms$.
a) Find Impulse $\vec{J}$?
b) Find $\vec{F}_{avg}$ On the Wall from the ball?

$\vec{V}_a = V\cos\theta\,\hat{i} + V\sin\theta\,\hat{j}$
$\vec{V}_a = 5.2\hat{i} + 3\hat{j}\ m/s$

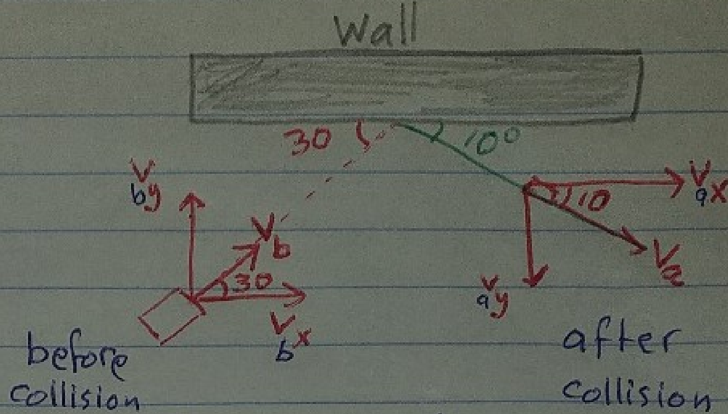
$\vec{V}_b = V\cos\theta\,\hat{i} - V\sin\theta\,\hat{j}$
$\vec{V}_b = 5.2\hat{i} - 3\hat{j}\ m/s$

(a)
$$\underset{\text{on the ball}}{\vec{J}} = \Delta\vec{P} = m\vec{V}_f - m\vec{V}_i$$
$$= m\vec{V}_a - m\vec{V}_b$$
$$= m(\vec{V}_a - \vec{V}_b)$$
$$= 0.3[(5.2\hat{i} + 3\hat{j}) - (5.2\hat{i} - 3\hat{j})]$$
$$= 0.3[2(3\hat{j})]$$
$$= 1.8\hat{j}\ kg\,m/s$$

(b)
$$\vec{J} = \vec{F}_{avg} \cdot \Delta t$$
$$\underset{\text{on the ball}}{\vec{F}_{avg}} = \frac{\vec{J}_{on\,the\,ball}}{\Delta t}$$
$$= \frac{1.8\hat{j}}{10 \times 10^{-3}} = 180\hat{j}\ N$$

$$\vec{F}_{avg,\,on\,the\,Wall} = -180\hat{j}\ N$$

Sample problem 9.04:



$\vec{V}_b = 70\,m/s$, at $30°$
$\vec{V}_a = 50\,m/s$, at $10°$
$m = 80\,kg$

a) What is the impulse $\vec{J}$ on the driver due to collision?

$\vec{V}_b = 70\cos 30\,\hat{i} + 70\sin 30\,\hat{j}$
$= 60.6\,\hat{i} + 35\,\hat{j}\ m/s$
$\vec{V}_a = 50\cos 10\,\hat{i} - 50\sin 10\,\hat{j}$
$= 49\,\hat{i} - 8.7\,\hat{j}\ m/s$

$\vec{J} = \Delta\vec{P}$
$= \vec{P}_a - \vec{P}_b$
$= m(\vec{V}_a - \vec{V}_b)$
$= 80[(49\hat{i} - 8.7\hat{j}) - (60.6\hat{i} + 35\hat{j})]$
$= 80[-11.6\hat{i} - 43.7\hat{j}]$
$\vec{J} = -928\hat{i} - 3496\hat{j}\ kg\cdot m/s$
$\vec{J}_{\text{on driver}} \approx -910\hat{i} - 3500\hat{j}\ kg\cdot m/s$

b) the collision lasts for 14ms.
Find $\vec{F}_{avg}$? + F_{avg}?

$\vec{J} = \vec{F}_{avg}\cdot\Delta t$

$\vec{F}_{avg} = \dfrac{\vec{J}}{\Delta t} = \dfrac{-910\hat{i} - 3500\hat{j}}{14\times10^{-3}}$

$\vec{F}_{avg} = -6.5\times10^{4}\,\hat{i} - 2.5\times10^{5}\,\hat{j}$

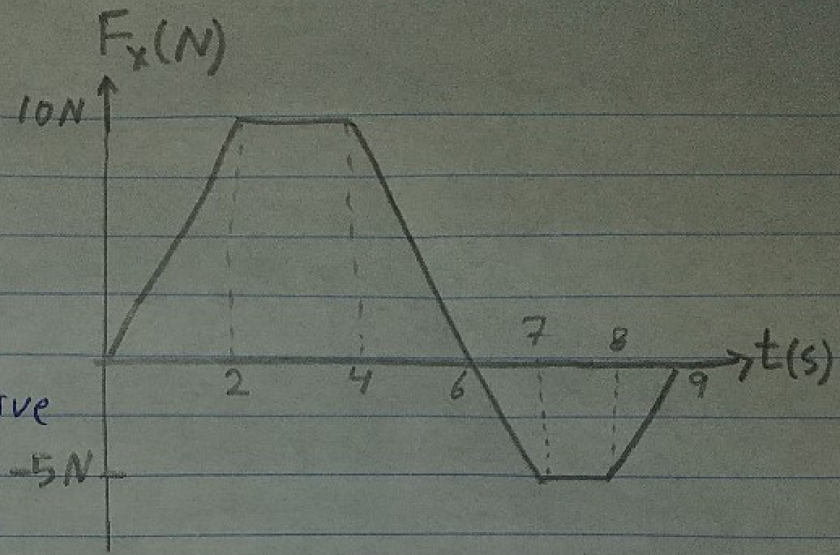
$\vec{F}_{avg} = -6.5\times10^{4}\,\hat{i} - 25\times10^{4}\,\hat{j}\ N$

$F_{avg} = \sqrt{(-6.5\times10^4)^2 + (-25\times10^4)^2}$
$= 26\times10^4\,N = 2.6\times10^5\,N$

(9-32) $m = 5kg$

at $t=0$, $v=0$

find $\vec{P}$?



a) Find $\vec{P}$ at $t = 4s$?

$\Delta\vec{P} = \vec{J}$

$\vec{P}_4 - \vec{P}_0$ = Area Under the curve of (F Vs. t) from $t_1 = 0 \to t_2 = 4s$

$P_4 - 0 = \triangle_{0\ 2} + \square_{2\ 4}$

$= \frac{1}{2}(2)(10) + 2(10) = 30$

$\boxed{P_4 = +30 \text{ kg.ms} \quad +x}$

b) find $\vec{P}$ at $t = 7s$

$P_7 - P_0 = \triangle_{0\ 2} + \square_{2\ 4} + \triangle_{4\ 6} + \triangledown_{6\ 7}$

$P_7 = 10 + 20 + \frac{1}{2}(2)(10) + \frac{1}{2}(1)(-5) = 10 + 20 + 10 - 2.5$

$P_7 = 37.5 \text{ kg.m/s} \quad +x$

c) Find P at $t = 9s$

$P_9 - P_0 = \triangle_{0\ 2} + \square_{2\ 4} + \triangle_{4\ 6} + \triangledown_{6\ 7} + \square_{7\ 8} + \triangledown_{8\ 9}$

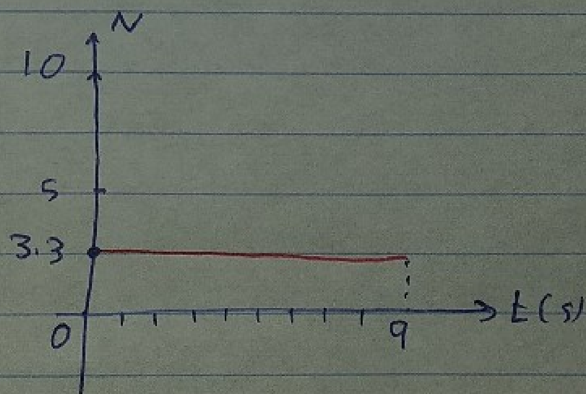
$P_9 = (10 + 20 + 10) + -2.5 + 1(-5) + \frac{1}{2}(1)(-5)$

$= (40) - 2.5 - 5 - 2.5$

$= (40) - (10)$

$= +30 \text{ kg.m/s} \quad +x$

Add to the Problem:



Find $\vec{F}_{avg}$?

$\vec{J} = \vec{F}_{avg} \cdot \Delta t$

$\vec{F}_{avg} = \frac{\vec{J}}{\Delta t} = \frac{+30}{9} = 3.3N$

Find $\vec{v}$ at, $t = 4s$? $7s$? $9s$? $\vec{P} = m\vec{v}$, $\vec{v} = \frac{\vec{P}}{m}$

$V_4 = \frac{+30}{5} = +6 \text{ m/s}$, $V_7 = \frac{37.5}{5} = 7.5 \text{ m/s}$ $\quad P_9 = \frac{30}{5} = +6 \text{ m/s}$

Problem (9-36)

$m = 0.25\,kg$

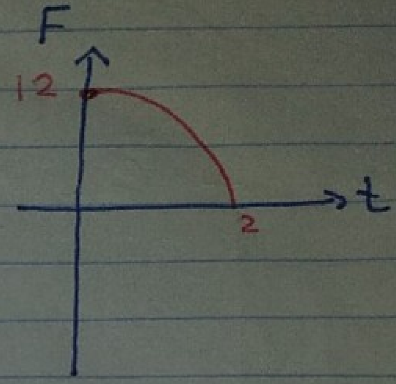
at $t = 0$, $v_0 = 0$

$\vec{F} = (12 - 3t^2)\,\hat{i}$ N the

force acts from $t_1 = 0 \rightarrow t_f$, when $F = 0$

$0 = 12 - 3t_f^2$

$t_f = 2s.$



a) Find Impulse from $t_1 = 0 \rightarrow t_2 = 1.25s$

$$\vec{J} = \int_{t_1}^{t_2} \vec{F}\,dt = \int_0^{1.25} (12 - 3t^2)\,dt$$

$$J_x = \left[12t - t^3\right]_0^{1.25} = \left[12t - (1.25)^3\right] - \left[\ -0\right]$$

$$= (15 - 1.953) -$$

$$= 13.05\,N.m$$

b) Find Δp from $t_1 = 0 \longrightarrow t_2 = 2s$ were $F = 0$

$$\Delta\vec{P} = \int \vec{F}\,dt = \int (12 - 3t)\,dt$$

$$(\Delta P)_x = \int_0^2 (12 - 3t)\,dt = \left[12t - t^3\right]_0^2$$

$$= \left[12(2) - (2)^3\right] - \left[\ 0\ \right]$$

$$= 24 - 8$$

$$(\Delta P)_x = 16\,N.m$$

Conservation Of Linear Momentum:

$$\vec{F}_{net} = \frac{d\vec{P}}{dt}$$

if the net $\vec{F}_{ext} = 0 \Rightarrow$

$$\frac{d\vec{P}}{dt} = 0 \Rightarrow d\vec{P} = 0 \Rightarrow \vec{P} = \text{Constant}$$

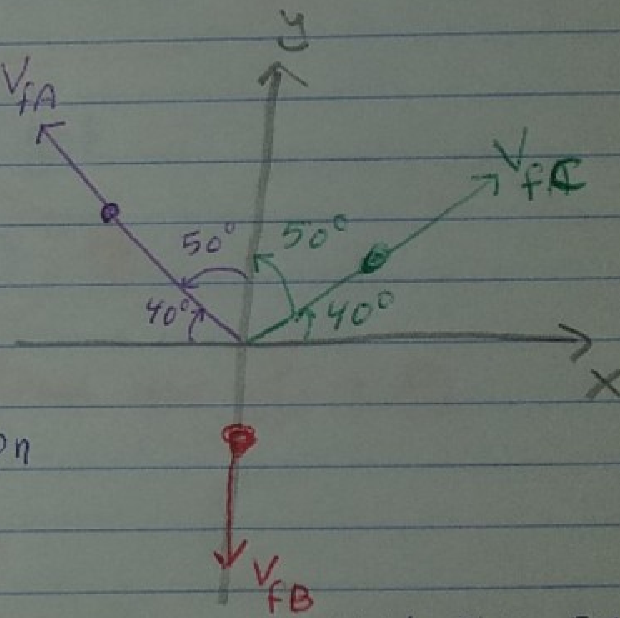
$\Delta\vec{P} = 0$
$\vec{P} = \text{Constant}$
$\vec{P}_i = \vec{P}_f$

Conservation Law of Linear Momentum

If no net external force acts on a system of Particles, the total linear momentum $\vec{P}$ of the system Can not change

- Collisions
- Explusions
- Rocket

Sample Problem 9.06:



M is initially at rest $V_i = 0$
M explodes to three Pieces

$M_A = 0.5M$, V_{fA} ?
$M_B = 0.2M$, V_{fB} ?
$M_C = 0.3M$, $V_{fC} = 5\,m/s$

as shown $\Rightarrow$

Let V_{fB} is $-y$

$\vec{P}_i = \vec{P}_f$, $\vec{P}_i = 0$

$P_{ix} = P_{fx}$

$P_{iy} = P_{fy}$

$0 = M_C V_{fC} \sin 40 + M_A V_{fA} \sin 140 + {}^{-}M_B V_{fB}$

$0 = 0.3M(5\sin 40) + 0.5M(3\sin 140) - 0.2MV_{fB}$

$0 = 0.964 + 0.964 - 0.2V_{fB}$

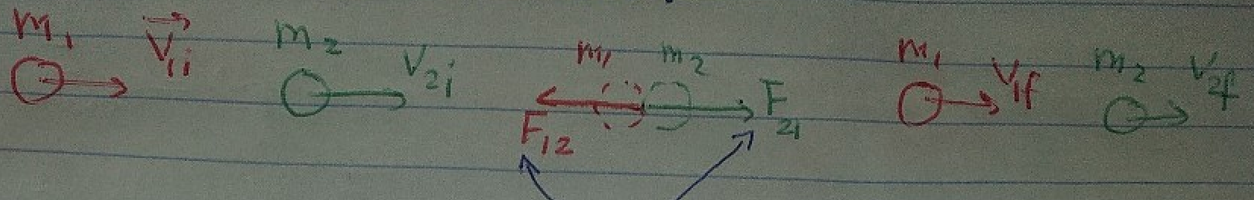
$V_{fB} = 9.64\,m/s$

$0 = {}^{+}M_C V_{fC} \cos 40 + M_A V_{fA} \cos 140°$

$0 = 0.3M(5\cos 40) + 0.5MV_{fA} \cos 140$

$0 = 1.15 + {}^{-}0.383V_{fA}$, $V_{fA} = 3\,m/s$

Collisions in one-dimension:



internal forces

$\vec{F}_{21} = -\vec{F}_{12}$

$\vec{F}_{21} \cdot \Delta t = -\vec{F}_{12} \cdot \Delta t$

$\vec{J}_{21} = -\vec{J}_{12}$

$\Rightarrow$

$\vec{P}_i = \vec{P}_f$

$$m_1 \vec{V}_{1i} + m_2 \vec{V}_{2i} = m_1 \vec{V}_{1f} + m_2 \vec{V}_{2f} \quad (1) \Rightarrow (\vec{V}_{com})_i = (\vec{V}_{com})_f$$

Momentum $(\vec{P})$ + Kinetic Energy (K) in Collisions

1) Elastic Collision

$K_i = K_f$

$$\frac{1}{2} m_1 V_{1i}^2 + \frac{1}{2} m_2 V_{2i}^2 = \frac{1}{2} m_1 V_{1f}^2 + \frac{1}{2} m_2 V_{2f}^2 \quad (2)$$

Solve (1) + (2) $\Rightarrow$ you will get

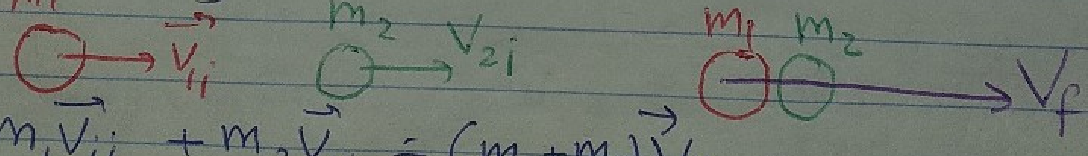
$$\vec{V}_{1i} - \vec{V}_{2i} = -(\vec{V}_{1f} - \vec{V}_{2f}) \quad (3)$$

During solving Elastic Problems use (1) + (3)

2) Inelastic Collision

$K_i \neq K_f$

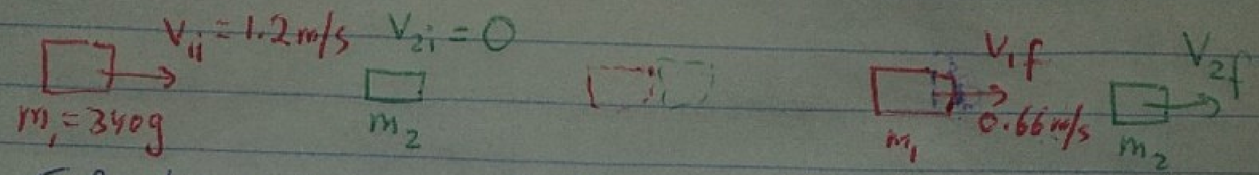
3) Completly Inelastic collision



$$m_1 \vec{V}_{1i} + m_2 \vec{V}_{2i} = (m_1 + m_2) \vec{V}_f$$

$K_i \neq K_f$

Problem 9.61:



Elastic collision. Find m_2? $\vec{v}_{2f}$ $\vec{v}_{com}$

In Any Collision $\vec{P}_i = \vec{P}_f$

$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$$

$$(0.34)(\overset{+}{1.2}) + 0 = 0.34(\overset{+}{0.66}) + m_2 v_{2f}$$

$$0.408 = 0.2244 + m_2 v_{2f}$$

$$m_2 v_{2f} = 0.1836 \quad (1)$$

For Elastic Collision $\rightarrow$ $\vec{v}_{1i} - \vec{v}_{2i} = -(\vec{v}_{1f} - \vec{v}_{2f})$

$$\overset{+}{1.2} - 0 = -(\overset{+}{0.66} - v_{2f})$$

$$1.2 = -0.66 + v_{2f}$$

$$v_{2f} = 1.86 \, m/s \longrightarrow \text{in } (1) \quad m_2 = \frac{0.1836}{v_{2f}}$$

$$m_2 = \frac{0.1836}{1.86} = 0.0987 \, kg$$

$$m = 98.7g$$

$$(\vec{v}_{com})_i = \frac{m_1\vec{v}_{1i} + m_2\vec{v}_{2i}}{m_1 + m_2} = \frac{0.34(1.2) + 0}{0.34 + 98.7 \times 10^{-3}}$$

$$= \frac{0.408}{0.4387} = 0.93 \, m/s$$

$(\vec{v}_{com})$ finally must be the same.

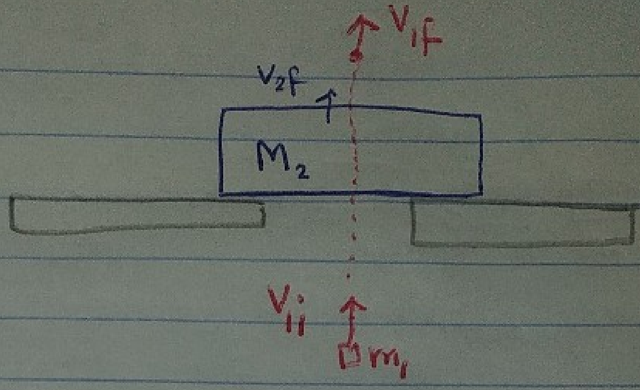
$$(v_{com})_f = \frac{m_1\vec{v}_{1f} + m_2\vec{v}_{2f}}{m_1 + m_2} = \frac{0.34(0.66) + 0.0987(1.86)}{0.34 + 0.0987}$$

$$= \frac{0.2244 + 0.183582}{0.4387} = \frac{0.408}{0.4387}$$

$$= 0.93 \, m/s$$

Remember: in any Collision $(\vec{v}_{com})_i = (\vec{v}_{com})_f$

Problem 9.52:



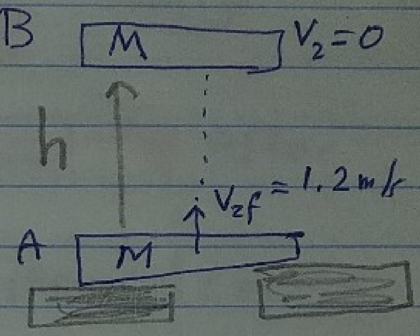
$m_1 = 10g$

$v_{1i} = 1000 m/s \longrightarrow v_{1f} = 400 m/s$

$M_2 = 5kg$, $v_{2i} = 0 \longrightarrow v_{2f}$

Collision $\Longrightarrow$

$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$$



$$(0.01)(1000) + 0 = 0.01(400) + 5V_{2f}$$

$$V_{2f} = \frac{10-4}{5} = \frac{6}{5} = 1.2 m/s$$

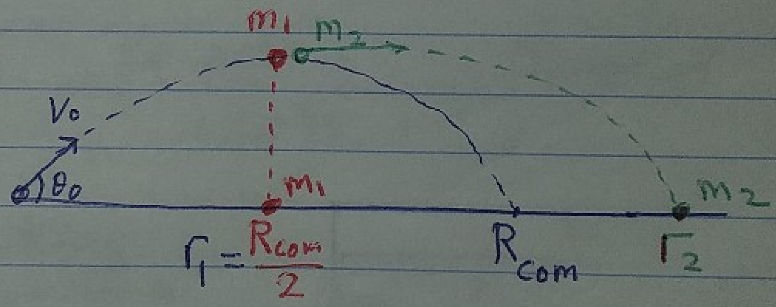
$$V_B^2 = V_A^2 + 2a_y \Delta y$$

$$0 = (1.2)^2 + 2(-9.8)(h)$$

$$h = \frac{(1.2)^2}{19.6} = 0.087 m = 8.7 cm$$

$\boxed{\text{Problem 9.13}}$ $V_0 = 20 m/s$, $\theta_0 = 60°$

$m_1 = m_2 = \frac{M}{2}$



$$R_{com} = \frac{V_0^2 \sin 2\theta_0}{g} = \frac{(20)^2 \sin(120)}{9.8}$$

$(R_{com})_b = 35.35 m$ (without explusion) (before explusion)

$(R_{com})_{before} = (R_{com})_{after\ explusion}$ No external forces in x-axis, $(F_x)_{ext} = 0$

$$(R_{com})_b = \frac{m_1 r_1 + m_2 r_2}{m_1 + m_2} = \frac{(M/2)(\frac{35.35}{2}) + (M/2) r_2}{M}$$

$$35.35 = \frac{35.35}{4} + \frac{r_2}{2} \quad , \quad r_2 = 2\left(35.35 - \frac{35.35}{4}\right)$$

$$r_2 = 53 m$$

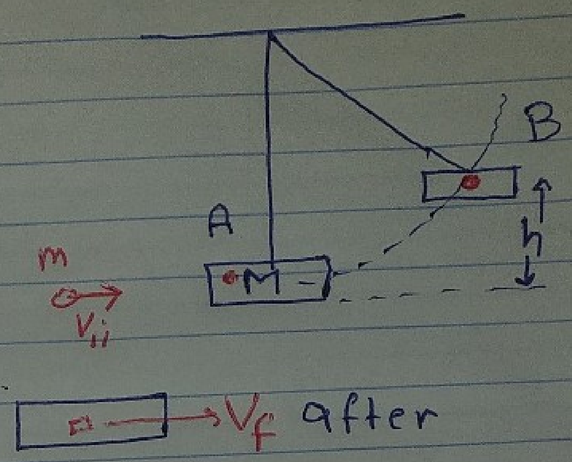
Sample Problem 9.07: ballistic Pendulum:

$M = 5.4\,kg$, $V_i = 0$

$m = 9.5\,g$ at $V_{1i} = ?$

$h = 6.3\,cm$

Completly Inelastic Collision



$$mV_{1i} + M\vec{V}_{2i} = (m+M)V_f$$

$$mV_{1i} + 0 = (m+M)V_f$$

$$V_{1i} = \left(\frac{m+M}{m}\right)V_f \quad (1)$$

To find V_f , use Conservation of Mechanical energy from $A \rightarrow B$

$$(K+U)_A = (K+U)_B$$

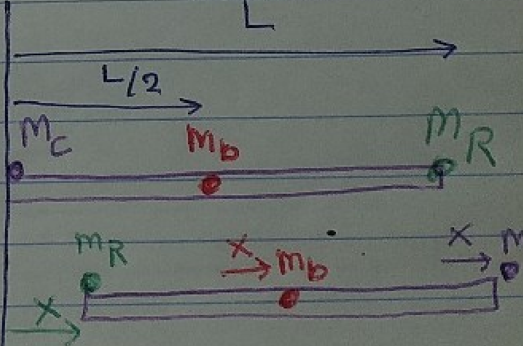
$$\frac{1}{2}(m+M)V_f^2 + 0 = 0 + (m+M)gh$$

$$V_f = \sqrt{2gh} \quad (2) \text{ in } (1)$$

$$V_{1i} = \left(\frac{m+M}{m}\right)\sqrt{2gh} = \left(\frac{0.0095+5.4}{0.0095}\right)\sqrt{(2)(9.8)(0.063)}$$

$$= 630\,m/s$$

Problem 9.16 $L = 3\,m$, $m_b = 30\,kg$, $M_R = 80\,kg$, $x = 40\,cm$



$$(R_{com})_{initial} = \frac{M_c(0) + M_b\frac{L}{2} + M_R L}{M}$$

$$(R_{com})_{final} = \frac{m_R x + m_b(\frac{L}{2}+x) + M_c(L+x)}{M}$$

$$(R_{com})_i = (R_{com})_{final} \qquad \sum \vec{F}_{ext} = 0$$

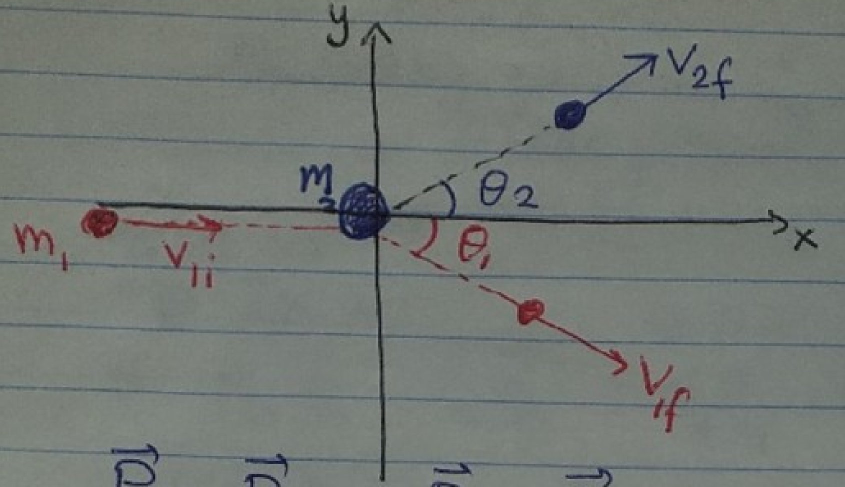
$$m_b\frac{L}{2} + M_R L = M_R x + m_b(\frac{L}{2}+x) + m_c(L+x)$$

$$m_c(L+x) = m_b\frac{L}{2} + M_R L - M_R x - m_b(\frac{L}{2}+x)$$

$$m_c = \frac{M_R(L-x) - m_b x}{L+x} = \frac{80(3-0.4) - 30(0.4)}{3+0.4} = \frac{208-12}{3.4} = 57.6\,kg$$

$$M_c = 57.6\,kg = 58\,kg$$

Collisions in 2 Dimensions:



$$\vec{P}_{1i} + \vec{P}_{2i} = \vec{P}_{1f} + \vec{P}_{2f}$$

$$m_1\vec{V}_{1i} + m_2\vec{V}_{2i} = m_1\vec{V}_{1f} + m_2\vec{V}_{2f}$$

In this Problem $V_{2i} = 0$

$$m\vec{V}_{1i} = m_1\vec{V}_{1f} + m_2\vec{V}_{2f}$$

$$(mV_{1i} + m_1V_{1f} + m_2V_{2f})_x$$

$$mV_{1i} + m_1V_{1f}\cos\theta_1 + m_2V_{2f}\cos\theta_2 \quad ①$$

$$(mV_{1i} = m_1V_{1f} + m_2V_{2f})_y$$

$$0 = -m_1V_{1f}\sin\theta + m_2V_{2f}\sin\theta_2 \quad ②$$

If the Collision is Elastic $\Rightarrow$

$$K_i = K_f$$

$$\frac{1}{2}m_1V_{1i}^2 + \frac{1}{2}m_2V_{2i}^2 = \frac{1}{2}m_1V_{1f}^2 + \frac{1}{2}m_2V_{2f}^2$$

In this Problem

$$\frac{1}{2}m_1V_{1i}^2 = \frac{1}{2}m_1V_{1f}^2 + \frac{1}{2}m_2V_{2f}^2 \quad ③$$

(9-74) $m_A = 2kg$ moves at $\vec{V}_{Ai} = (15\hat{i} + 30\hat{j})\ m/s$

collide with

$m_B = 2kg$ moves at $\vec{V}_{Bi} = (-10\hat{i} + 5\hat{j})\ m/s$

After collision

$\vec{V}_{Af} = (-5\hat{i} + 20\hat{j})\ m/s$

What are:

ⓐ $\vec{V}_{Bf}$? b) ΔK ?

a) $m_A \vec{V}_{Ai} + m_B \vec{V}_{Bi} = m_A \vec{V}_{Af} + m_B \vec{V}_{Bf}$ $m_A = m_B = 2kg$

$(15\hat{i} + 30\hat{j}) + (-10\hat{i} + 5\hat{j}) = (-5\hat{i} + 20\hat{j}) + \vec{V}_{Bf}$

$(+5\hat{i} + 35\hat{j}) + 5\hat{i} - 20\hat{j} = \vec{V}_{Bf}$

$\vec{V}_{Bf} = 10\hat{i} + 15\hat{j}\ m/s$

b) $K_i = \frac{1}{2} m_A V_{Ai}^2 + \frac{1}{2} m_B V_{Bi}^2$

$= \frac{1}{2}(2)\left[(15)^2 + (30)^2\right] + \frac{1}{2}(2)\left[(-10)^2 + (5)^2\right]$

$= [1125] + [125)$

$= 1250\ J$

$K_f = \frac{1}{2} m_A V_{Af}^2 + \frac{1}{2} m_B V_{Bf}^2$

$= \frac{1}{2}(2)\left[(-5)^2 + (20)^2\right] + \frac{1}{2}(2)\left[(10)^2 + (15)^2\right]$

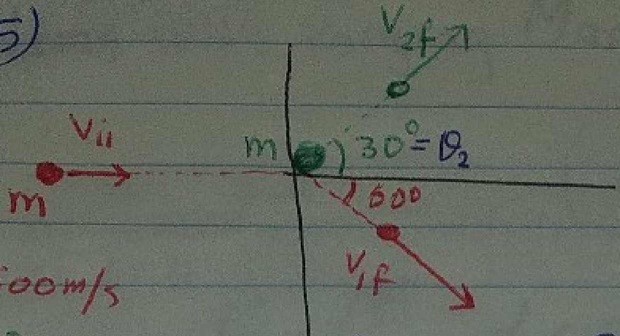
$= [425] + [325]$

$= 750\ J$

$\Delta K = K_f - K_i = 750 - 1250$

$= -500\ J$ lost

(9-75)



$V_{1i} = 500 m/s$

$V_{2i} = 0$

$\theta_1 = 60^\circ$

Elastic Collision.

find V_{1f} ? V_{2f} ? θ_2 ?

Collision in 2D

$m_1\vec{V}_{1i} + m_2\vec{V}_{2i} = m_1\vec{V}_{1f} + m_2\vec{V}_{2f}$ $\quad m_1 = m_2 = m$

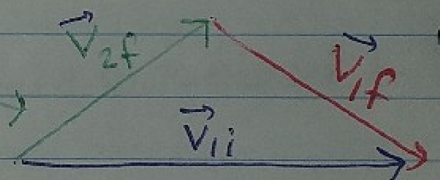
$\vec{V}_{1i} + \vec{V}_{2i} = \vec{V}_{1f} + \vec{V}_{2f} \Rightarrow \vec{V}_{1i} = \vec{V}_{1f} + \vec{V}_{2f}$

x-components $\longrightarrow 500 = V_{1f}\cos 60 + V_{2f}\cos\theta_2$ ①

y-components $\longrightarrow 0 = V_{2f}\sin\theta_2 + -V_{1f}\sin 60$ ②

$K_i = K_f$ (for Elastic Collision)

$\frac{1}{2}mV_{1i}^2 = \frac{1}{2}mV_{1f}^2 + \frac{1}{2}mV_{2f}^2$



$(500)^2 = V_{1f}^2 + V_{2f}^2$ ③

The angle between $\vec{V}_{1f}$ & $\vec{V}_{2f}$ is $90^\circ \Rightarrow$

$\theta_2 = 30^\circ$ in ① & ②

$500 = V_{1f}\cos 60 + V_{2f}\cos 30 \Rightarrow 500 = 0.5V_{1f} + 0.87V_{2f}$ ①'

$1000 = V_{1f} + 1.74V_{2f}$

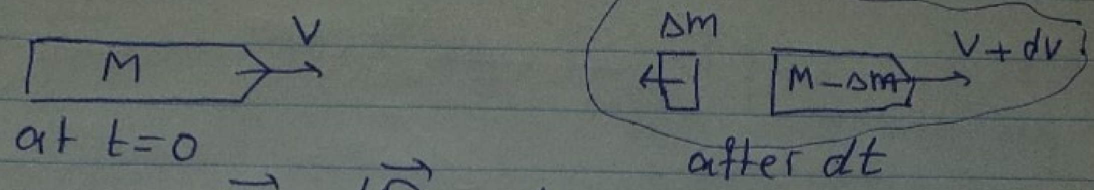
in ② $\quad 0 = V_{2f}\sin 30 - V_{1f}\sin 60$

$0 = 0.5V_{2f} - 0.87V_{1f}$ ②' $\longrightarrow$ $0 = -V_{1f} + 0.575V_{2f}$

$1000 = 2.315V_{2f} \longrightarrow V_{2f} = 432 m/s$

$V_{1f} = 248 m/s$

Systems with varying Mass: A ROCKET



$$\vec{F}_{net} = \frac{d\vec{P}}{dt} = \frac{d}{dt}(M\vec{V})$$

$$\vec{F}_{ext} = M\frac{d\vec{V}}{dt} + \vec{V}_{rel}\frac{dM}{dt}$$

In Free space $\vec{F}_{ext} = 0$

$$\Rightarrow M\frac{d\vec{V}}{dt} = -\vec{V}_{rel}\frac{dM}{dt}$$

$$\underbrace{V_{rel}\frac{dM}{dt}}_{\text{Thrust}} = Ma \quad \text{(First Rocket equation)}$$

Finding the velocity at any time...

$$M\frac{dv}{dt} = -V_{rel}\frac{dM}{dt} \quad \text{integrate with respect to } t$$

$$M\,dv = -V_{rel}\,dM$$

$$dV = -V_{rel}\frac{dM}{M}$$

$$\int_{V_i}^{V_f} dv = -V_{rel}\int_{M_i}^{M_f}\frac{dM}{M} = -V_{rel}\left[\ln M\right]_{M_i}^{M_f}$$

$$V_f - V_i = V_{re}\ln\left(\frac{M_i}{M_f}\right) \quad \text{(2}^{nd}\text{ Rocket equation)}$$

Problem 9.79:

A rocket in deep space

$V_i = 0$

$M_i = 2.55 \times 10^5$ kg, Part of this Mass is fuel

$m_{fuel} = 1.81 \times 10^5$ kg

time of firing the rocket = 250 sec.

$\frac{dM}{dt} = 480$ kg/s (the rate of Consuming fuel)

$V_{rel} = 3.27$ km/s (the speed of exhaust product relative to the rocket)

a) What is the Rocket Thrust?

$$\text{Thrust} = R V_{rel} = \frac{dM}{dt} V_{rel}$$

$$= (480)(3.27 \times 10^3)$$

$$= 1.57 \times 10^6 \text{ N}$$

After 250s of firing

b) find the mass of the rocke?

$$M_f = M_i - \Delta M \quad , \quad \Delta M = \frac{dM}{dt}(\Delta t) = (480 \tfrac{kg}{s})(250s)$$

$$= 2.55 \times 10^5 - 1.2 \times 10^5 \qquad = 1.2 \times 10^5 \text{ kg}$$

$$= 1.35 \times 10^5 \text{ kg}$$

c) find the final speed?

$$V_f - V_i = V_{rel} \ln\left(\frac{M_i}{M_f}\right)$$

$$V_f - 0 = (3.27 \times 10^3) \ln\left(\frac{2.55 \times 10^5}{1.35 \times 10^5}\right) = (3.27 \times 10^3) \ln(1.88$$

$$V_f = 3.27 \times 10^3 (0.636)$$

$$= 2.1 \times 10^3 \text{ m/s}$$