Basic Concepts of Cost:

There are at least three different concepts of costs encountered in economics: opportunity cost, accounting cost, and economic cost. For economists, the most general of these is opportunity cost (sometimes called social cost). Because resources are limited, any decision to produce more of one good means doing without some other good. When an automobile is produced, for example, an implicit decision has been made to do without 15 bicycles, say, that could have been produced using the labor, steel, and glass that goes into the automobile. The opportunity cost of one automobile is 15 bicycles.

هناك ثلاثة مفاهيم مختلفة على الأقل للتكاليف في الاقتصاد: تكلفة الفرصة البديلة ، والتكلفة المحاسبية ، والتكلفة الاقتصادية. بالنسبة للاقتصاديين ، فإن أكثرها عمومية هي تكلفة الفرصة البديلة (تسمى أحيانًا التكلفة الاجتماعية). نظرًا لأن الموارد الاقتصادية محدودة ، فإن أي قرار لإنتاج أكثر من سلعة واحدة يعني الاستغناء عن بعض السلع الآخرى. عندما يتم إنتاج سيارة ، على سبيل المثال ، يتم اتخاذ قرار ضمني بالاستغناء عن 15 دراجة ، على سبيل المثال ، كان من الممكن إنتاجها باستخدام العمالة والفولاذ والزجاج التي تدخل في السيارة. تكلفة الفرصة البديلة لسيارة واحدة هي 15 دراجة.

Opportunity cost :The cost of a good as measured by the alternative uses that are forgone by producing the good.

تكلفة الفرصة البديلة: الكمية التي يتم التضحية بها من سلعة مقابل الحصول على وحدة اضافية من سلعة أخرى.

Economic and Accounting Cost: التكاليف الافتصادية والتكاليف المحاسبية

Costs exist because resources are scare, productive, and have alternative uses. The economic cost of any resource used to produce a good is the value the resource would have in its best alternative use.

التكاليف هي المبالغ المدفوعة صراحة ثمنا للحصول على الاشياء التي تم استخدامها .المفهوم الحقيقي للتكاليف في الاقتصاد هو التضحية. فعندما نقوم بدفع مبلغ معين لشراء قميص مثلاً فقد كان بإمكانك استخدام ذلك المبلغ لشراء شيء أخر ولكنك لم تفعل ,وبالتالي فقد ضحيت بالشيء الاخر من أجل شراء القميصز فبالإضافة الى المبالغ التي يتم دفعها صراحة فإن التكاليف الاقتصادية تتضمن أيضا المبالغ التي يتم التضحية بها حتى لو لم تدفع صراحة.

Explicit and Implicit Costs التكاليف الفعلية والتكاليف الضمنية

Explicit costs are the monetary payment (or cash expenditures) the firm makes to those who supply resources.

التكاليف الصريحة :هي عبارة عن المبالغ التي تدفعها المؤسسة صراحة مقابل خدمات عناصر الانتاج مثل اجور العمال ومصروفات الصيانة وثمن المواد الخام ومصروفات الكهرباء والمياه واجور النقل ومصروفات أخرى مثل التامين والضرائب وغيرها

Firm's implicit costs are the opportunity costs of using its self-owned, self-employed resources. Money payment that self-employed resources could have earned in their best alternative use.

التكاليف الضمنية (تكلفة الفرصة البديلة :)هي عبارة عن التكاليف التي لا تدفعها المؤسسة صراحة مقابل خدمات عناصر الانتاج ولكنها تضحي بها مقابل استخدام عناصر الانتاج المملوكة للمؤسسة.

مثال :منشاة تمتلك مستودع وتقوم بإستخدامه في تخزين البضاعة في هذه الحالة لن تدفع المؤسسة أي ايجار لهذا المستودع لانه ملك لها ولكنها تتحمل تكلفة تتمثل بالتضحية بإيجار الذي كان يمكن الحصول عليع لو لم تستخدمه وقامت بتاجيره وهذا ما يعرف بتكلفة الفرصة البديلة ولو عملت زوجة صاحب المؤسسة ولاولاد في هذه المؤسسة تتمثل تكلفة الفرصة البديلة بالنسبة لهم بمقدار الاجر الذي يمكن الحصول عليه لو عملوا في مؤسسة اخرى.

Accounting cost :The concept that inputs cost what was paid for them.

التكلفة المحاسبية: مفهوم أن المدخلات الانتاج تكلف ما تم دفعه مقابل الحصول عليها . وهي عبارة عن المبالغ التي تدفعها المؤسسة صراحة مقابل خدمات عناصر الانتاج

Economic cost :The amount required to keep an input in its present use; the amount that it would be worth in its next best alternative use.

Accounting cost = Explicit cost

Economic cost = Explicit cost + Implicit cost

To see how the economic definition of cost might be applied in practice and how it differs from accounting ideas, let's look at the economic costs of three inputs: labor, capital, and the services of entrepreneurs (owners).

<u>Labor Costs</u>

Economists and accountants view labor costs in much the same way. To the accountant, firms' spending on wages and salaries is a current expense and therefore is a cost of production. Economists regard wage payments as an explicit cost: labor services (worker-hours) are purchased at some hourly wage rate (which we denote by w), and we presume that this rate is the amount that workers would earn in their next best alternative employment.

ينظر الاقتصاديون والمحاسبون إلى تكاليف العمالة بنفس الطريقة. بالنسبة للمحاسب ، فإن إنفاق الشركات على الأجور والمرتبات هو نفقات جارية ، وبالتالي فهي تكاليف انتاجية. يعتبر الاقتصاديون مدفوعات الأجور تكلفة صريحة: يتم شراء خدمات العمل (ساعات العمل) بمعدل أجر بالساعة (والذي نشير إليه بـ (w)، ونفترض أن هذا المعدل هو المبلغ الذي سيكسبه العمال في عملهم البديل التالي الأفضل.

*Wage rate (w) :*The cost of hiring one worker for one hour.

معدل الأجر (w): تكلفة تشغيل عامل واحد لمدة ساعة.

<u>Capital Costs</u>

In the case of capital services (machine-hours), accounting and economic definitions of costs differ greatly. Accountants, in calculating capital costs, use the historical price of a particular machine and apply a depreciation rule to determine how much of that machine's original price to charge to current costs. For example, a machine purchased for $1,000 and expected to last 10 years might be said to ''cost'' $100 per year, in the accountant's view.

في حالة خدمات رأس المال (تكلفة المكينة في الساعة) ، تختلف التعريفات المحاسبية والاقتصادية للتكاليف اختلافًا كبيرًا. يستخدم المحاسبون ، في حساب تكاليف رأس المال ، السعر التاريخي لآلة معينة ويطبقون قاعدة الإهلاك لتحديد مقدار السعر الأصلي للجهاز الذي سيتم تحميله على التكاليف الحالية. على سبيل المثال ، آلة تم شراؤها مقابل 1000 دولار ومن المتوقع أن تستمر لمدة 10 سنوات يمكن أن يقال عنها "تكلف" 100 دولار سنويًا ، من وجهة نظر المحاسب.

Economists, on the other hand, regard the amount paid for a machine as a sunk cost. Once such a cost has been incurred, there is no way to get it back. Because sunk costs do not reflect forgone opportunities, economists instead focus on the implicit cost of a machine as being what someone else would be willing to pay to use it. Thus, the cost of one machine-hour is the rental rate for that machines in the best alternative us.

من ناحية أخرى، يعتبر الاقتصاديون أن المبلغ المدفوع مقابل الحصول على الآلة هو تكلفة لا يمكن استردادها. بمجرد تكبد هذه التكلفة ، لا توجد طريقة لاستعادتها. نظرًا لأن التكاليف الغارقة لا تعكس الفرص الضائعة ، يركز الاقتصاديون بدلاً من ذلك على التكلفة الضمنية للآلة باعتبارها ما قد يرغب شخص آخر في دفعه لاستخدامها. وبالتالي ، فإن تكلفة الساعة الآلية الواحدة هي سعر الإيجار لتلك الآلات في أفضل بديل لها.

Sunk cost :Expenditure that once made cannot be recovered.

المصروفات التي تمت مرة واحدة لا يمكن استردادها

Rental rate (v) :The cost of hiring one machine for one hour.

معدل الإيجار (v): تكلفة استئجار آلة واحدة لمدة ساعة

Entrepreneurial Costs

The owner of a firm is entitled to whatever is left from the firm's revenues after all costs have been paid. To an accountant, all of this excess would be called ''profits'' (or ''losses'' if costs exceed revenues). Economists, however, ask whether owners (or entrepreneurs) also encounter opportunity costs by being engaged in a particular business. If so, their entrepreneurial services should be considered an input to the firm, and economic costs should be imputed to that input.

يحق لمالك الشركة الحصول على ما تبقى من إيرادات الشركة بعد دفع جميع التكاليف. بالنسبة للمحاسب ، فإن كل هذا الفائض سيطلق عليه "أرباح" (أو "خسائر" إذا تجاوزت التكاليف الإيرادات). ومع ذلك ، يتساءل الاقتصاديون عما إذا كان المالكون (أو رواد الأعمال) يواجهون أيضًا تكاليف الفرصة البديلة من خلال الانخراط في عمل معين(تم التضحية براتب مقابل ادارة شغل خاص). ذا كان الأمر كذلك ، ينبغي اعتبار خدمات تنظيم المشاريع الخاصة بهم مدخلاً للشركة ، ويجب أن تُحسب التكاليف الاقتصادية إلى هذا المدخل .

The Two-Input Case

We will make two simplifying assumptions about the costs of inputs a firm uses. *First*, we can assume, as before, that there are only two inputs: labor (L, measured in labor-hours) and capital (K, measured in machine-hours). Entrepreneurial services are assumed to be included in capital input. That is, we assume that the primary opportunity costs faced by a firm's owner are those associated with the capital the owner provides.

سنقوم بعمل افتراضين مبسطين حول تكاليف المدخلات التي تستخدمها الشركة. أولاً ، يمكننا أن نفترض ، أن هناك مدخلين فقط: العمالة (L ، تقاس بساعات العمل) ورأس المال (K ، تقاس تكلفة تشغيل مكينة في الساعة). يفترض أن يتم تضمين خدمات ريادة الأعمال في مدخلات رأس المال. بمعنى أننا نفترض أن تكاليف الفرصة البديلة الأساسية التي يواجهها مالك الشركة هي تلك المرتبطة برأس المال الذي يوفره المالك.

A second assumption we make is that inputs are hired in perfectly competitive markets. Firms can buy (or sell) all the labor or capital services they want at the prevailing rental rates (w and v). In graphic terms, the supply curve for these resources that the firm faces is horizontal at the prevailing input prices.

الافتراض الثاني هو أن المدخلات يتم تبادلها في أسواق تنافسية تمامًا. يمكن للشركات شراء (أو بيع) جميع العمالة أو خدمات رأس المال التي تريدها بأسعار الإيجار السائدة (w) و (v). من الناحية الرسومية ، يكون منحنى العرض لهذه الموارد التي تواجهها الشركة أفقيًا عند أسعار المدخلات السائدة.

Economic Profits and Cost Minimization

Given these simplifying assumptions, total costs for the firm during a period are:

Total costs = $TC = wL + vK$

Where, L and K represent input usage during the period.

If the firm produces only one output, its total revenues are given by the price of its product (P) times its total output [$q = f(K, L)$, where $f(K, L)$ is the firm's production function].

Economic profits (π) are then the difference between total revenues and total economic costs:

$$Economic\ profits\ (\pi) = total\ revenues - total\ costs$$

$$\pi = Pq - wL - vK$$

Cost-Minimizing Input Choice

To minimize the cost of producing q_1, a firm should choose that point on the q_1 isoquant that has the lowest cost. That is, it should explore all feasible input combinations to find the cheapest one. This will require the firm to choose that input combination for which the marginal rate of technical substitution (RTS) of L for K is equal to the ratio of the inputs' costs, w/v.

The Isocost Line

isocost line: Graph showing all possible combinations of labor and capital that can be purchased for a given total cost.

To see what an isocost line looks like, recall that the total cost TC of producing any particular output is given by the sum of the firm's labor cost wL and its capital cost vK:

$$TC = wL + vK$$

If we rewrite the total cost equation as an equation for a straight line, we get

$$K = \frac{\mathrm{TC}}{\mathrm{v}} - \frac{\mathrm{w}}{\mathrm{v}} L$$

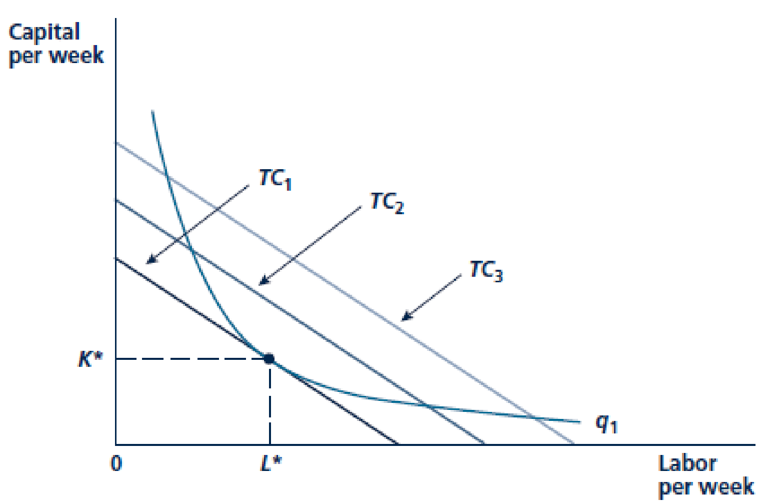
It follows that the isocost line has a slope of $\frac{\Delta K}{\Delta L} = -(w/v)$, which is the ratio of the wage rate to the rental cost of capital.

The slope of isocost tells us that if the firm gave up a unit of labor to buy w/r units of capital at a cost or r dollar per unit, its total cost of production would remain the same. For example, if the wage rate were $10 and the rental cost of capital $5, the firm could replace one unit of labor with two units of capital with no change in total cost.

Choosing Inputs

Suppose we wish to produce at an output level q_1. How can we do so at minimum cost?

Output q_1 can be achieved with the expenditure of TC_1, either by using K* units of capital and L* units of labor or by using K_3 units of capital and L_3 units of labor. But TC_2 is not the minimum cost. The same output q_1 can be produced more cheaply, at a cost of TC_1, by using K* units of capital and L* units of labor. In fact, isocost line TC_1 is the lowest isocost line that allows output q_1 to be produced. The point of tangency of the isoquant q_1 and the isocost line TC_1 at point A gives us the cost-minimizing choice of inputs, L* and K*, which can be read directly from the diagram.



At this point, the slopes of the isoquant and the isocost line are just equal.

The slopes of the isoquant = $RTS = \frac{MPL}{MPK}$

The slopes of the isocost line = $\frac{W}{V}$

When a firm minimizes the cost of producing a particular output, the following condition holds:

$$\frac{MPL}{MPK} = \frac{w}{v} \quad \text{or} \quad \frac{MPL}{w} = \frac{MPK}{v}$$

Example

A widget manufacturer has a production function of the form $q = 4KL$. If the wage rate (w) is $8 and the rental rate on capital (v) is $20, what cost minimization combination of K and L will the manufacturer employ to produce 160 units of output? What is the total cost at that output level?

When a firm minimizes the cost of producing a particular output, the following condition holds:

$$\frac{MPL}{MPK} = \frac{w}{v} \Rightarrow \frac{4K}{4L} = \frac{8}{20} \Rightarrow \frac{K}{L} = \frac{2}{5} \Rightarrow 5K = 2L \text{ .... (1)}$$

$$q = 4KL \Rightarrow 160 = 4KL \rightarrow 40 = KL \rightarrow K = \frac{40}{L} \text{ .... (2)}$$

Solve (1) and (2):

$$\frac{2}{5}L = \frac{40}{L} \rightarrow 2L^2 = 200 \rightarrow L^2 = 100 \rightarrow L = 10$$

$$K = \frac{40}{L} = \frac{40}{10} = 4$$

$$Tc = wL + vK = (8x10) + (20x4) = 160$$

or:

$$q = 4KL \Rightarrow 160 = 4KL \rightarrow 40 = KL \Rightarrow K = \frac{40}{L}$$

L	K	TC = 8 L + 20 K
1	40	808
2	20	416
3	13.4	292
4	10	232
5	8	200
6	6.67	181.4
7	5.71	170.2
8	5	164
9	4.45	161
10	4	160 (min)
11	3.63	160.6
12	3.34	162.8

Example:
A firm producing good A has a production function of the form $q = LK + 2L$. If the wage rate (w) is \$5 and the rental rate on capital (v) is \$3, what cost minimization combination of K and L will the manufacturer employ to produce 60 units of output?

$\frac{MPL}{MPK} = \frac{w}{v} \rightarrow \frac{K+2}{L} = \frac{5}{3} \rightarrow 5L = 3K + 6 \rightarrow K = \frac{5L-6}{3} \rightarrow K = \frac{5L}{3} - 2$ ………….. (1)

$q = LK + 2L \Rightarrow 60 = LK + 2L \rightarrow K = \frac{60-2L}{L} = \frac{60}{L} - 2$ ………. (2)

Solve (1) and (2):

$\frac{5L}{3} - 2 = \frac{60}{L} - 2 \rightarrow \frac{5L}{3} = \frac{60}{L} \rightarrow 5L^2 = 180 \rightarrow L^2 = \frac{180}{5} \rightarrow L^2 = 36 \rightarrow L = \sqrt{36} = 6$

$K = \frac{5L}{3} - 2 = \frac{5(6)}{3} - 2 = 10 - 2 = 8$

$Tc = wL + vK = (5 * 6) + (3 * 8) = 30 + 24 = \$54$

Example
A firm producing good A has a production function of the form $q = 2K + L$. If the wage rate (w) is \$1 and the rental rate on capital (r) is \$1, what cost minimization combination of K and L will the manufacturer employ to produce 40 units of output?

$\frac{MPL}{MPK} = \frac{w}{v}$

$MPL = 1 \quad MPK = 2$

$\frac{1}{2} = \frac{1}{1}$

½ ≠ 1 ⇨ condition fail

$q = 2K + L \rightarrow 40 = 2K + L \rightarrow 2K = 40 - L \rightarrow K = 20 - ½ L$

L	K	TC = L + K
0	20	20 (min)
40	0	40

To produce 40 units of output: L = 0 and K = 20 with total cost 20.

- If $\frac{\text{MPL}}{\text{w}} > \frac{\text{MPK}}{\text{r}}$ the firm does not minimize the cost. *To reduce the cost of producing its current output level it should employing more labor and less capital*

- If $\frac{\text{MPL}}{\text{w}} < \frac{\text{MPK}}{\text{r}}$ the firm does not minimize the cost. *To reduce the cost of producing its current output level it should employing more capital and less labor*

Example:

A firm employs 100 workers, each at $8 per hour, and 50 units of capital, each at $10 per hour. The marginal product of labor is 3 and the marginal product of capital is 5. Is the firm minimizing the cost? If not, what it do to reduce the cost of producing its current output level?

To minimize cost: $\frac{\text{MPL}}{\text{w}} = \frac{\text{MPK}}{v}$

$\frac{\text{MPL}}{\text{w}} = \frac{3}{8}$

$\frac{\text{MPK}}{v} = \frac{5}{10}$

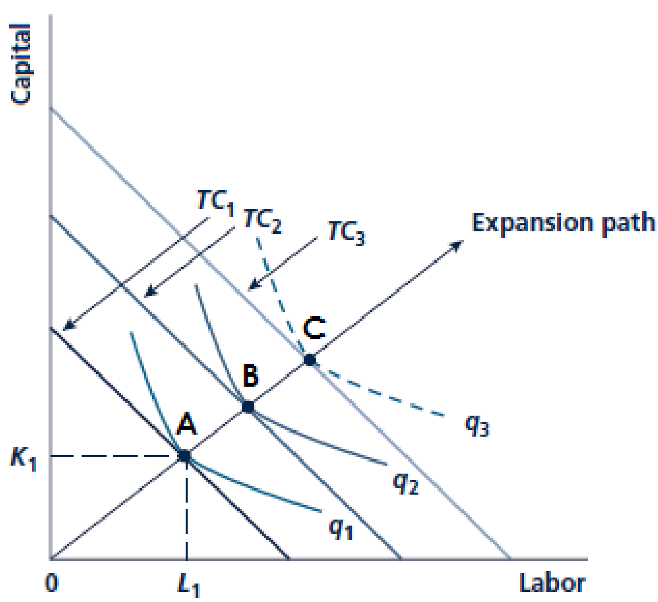
$\frac{\text{MPL}}{\text{w}} < \frac{\text{MPK}}{v}$

The firm does not minimize the cost. *To reduce the cost of producing its current output level it should employing more capital and less labor*

The Firm's Expansion Path

The firm's *expansion path* is the set of cost-minimizing input combinations a firm will choose to produce various levels of output (when the prices of inputs are held constant).

The expansion path (from the origin through points *A*, *B*, and *C*) illustrates the lowest-cost combinations of labor and capital that can be used to produce each level of output



Example

A widget manufacturer has a production function of the form $q = 4KL$. If the wage rate (w) is $8 and the rental rate on capital (r) is $20.

1. What cost minimization combination of K and L will the manufacturer employs to produce 160 units of output? What is the total cost at that output level?

   When a firm minimizes the cost of producing a particular output, the following condition holds:

   $\frac{MPL}{MPK} = \frac{w}{r} \rightarrow \frac{4K}{4L} = \frac{8}{20} \rightarrow \frac{K}{L} = \frac{2}{5} \rightarrow 5K = 2L$ .... *(1)*

   $q = 4KL \rightarrow 160 = 4KL \rightarrow 40 = KL \rightarrow K = \frac{40}{L}$ .... *(2)*

   Solve (1) and (2):

   $\frac{2}{5}L = \frac{40}{L} \rightarrow 2L^2 = 200 \rightarrow L^2 = 100 \rightarrow L = 10$

   $K = \frac{40}{L} = \frac{40}{10} = 4$

   $Tc = wL + vK = (8x10) + (20x4) = 160$

2. If the firm wants to increase output level to 360. What cost minimization combination of K and L will the manufacturer employs? What is the total cost at that output level?

   $\frac{MPL}{MPK} = \frac{w}{r} \rightarrow \frac{4K}{4L} = \frac{8}{20} \rightarrow \frac{K}{L} = \frac{2}{5} \rightarrow 5K = 2L$ .... *(1)*

   $q = 4KL \rightarrow 360 = 4KL \rightarrow 90 = KL \rightarrow K = \frac{90}{L}$ .... *(2)*

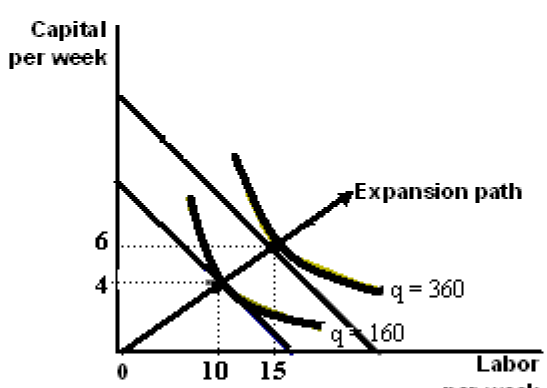
   Solve (1) and (2):

   $\frac{2}{5}L = \frac{90}{L} \rightarrow 2L^2 = 450 \rightarrow L^2 = 225 \rightarrow L = 15$

   $K = \frac{90}{L} = \frac{90}{15} = 6$

   $Tc = wL + vK = (8x15) + (20x6) = 240$

3. Illustrate your results graphically with a representative isoquant and isocost line consistent with the answers you derived in parts (1) and (2) above and then draw the firm expansion path

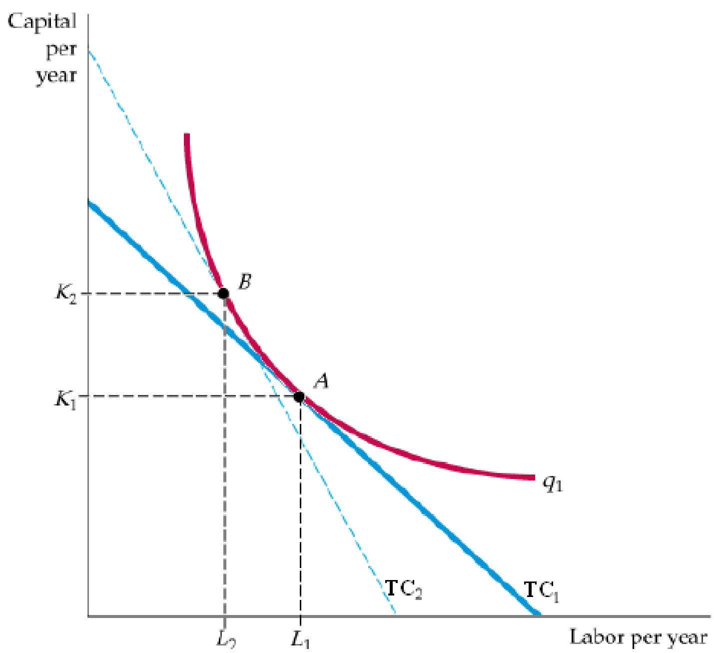


Input substitution with an input price changes:

When the expenditure on all inputs increase, the slope of the isocost line does not change because the prices of the inputs have not changed, but the intercept, will increase (isocost line shift to the right).

Suppose that the price of one of the input, such as labor were to increase. In that case the slope of the isocost line (w/r) would increase and the isocost line would become steeper.

Facing an isocost curve TC_1, the firm produces output q_1 at point A using L_1 units of labor and K_1 units of capital.
When the price of labor increases, the isocost curves become steeper. Output q_1 is now produced at point B on isocost curve TC_2 by using L_2 units of labor and K_2 units of capital. The firm has responded to the higher price of labor by substituting capital for labor in the production process.



Example:
Nike's total cost of producing sport shoes is given by $TC = wL + vK$, where w is the price of labor (= \$1) per unit and r is the rental price of capital (= \$2) per unit. Nike just won an order to supply q pairs of sport shoes and Nike's production function is of the form $q = 2L^2 K$.

1. What is Nike's cost minimization choice of inputs (capital and labor) in order to produce 32 pairs of sport shoes?

   When a firm minimizes the cost of producing a particular output, the following condition holds:

   $\frac{MPL}{MPK} = \frac{w}{r} \rightarrow \frac{4LK}{2L^2} = \frac{1}{2} \rightarrow \frac{2K}{L} = \frac{1}{2} \rightarrow K = ¼L$.......... (1)

   $q = 2L^2 K \rightarrow 32 = 2L^2 K \rightarrow K = \frac{32}{2L^2}$ .... (2)

   Solve (1) and (2):

   $¼L = \frac{32}{2L^2} \rightarrow ½L3 = 32 \rightarrow L^3 = 64 \rightarrow L = 4$

   $K = ½L = ½ x 4 = 2$

2. Suppose that the price of capital increases to \$3 per unit. If Nike continues to produce 32 pairs of sport shoes. What cost minimization choice of inputs capital and labor should the firm used.

$$\frac{MPL}{MPK} = \frac{w}{r} \rightarrow \frac{4LK}{2L^2} = \frac{1}{3} \rightarrow \frac{2K}{L} = \frac{1}{3} \rightarrow K = \frac{L}{6} \text{.......... (1)}$$

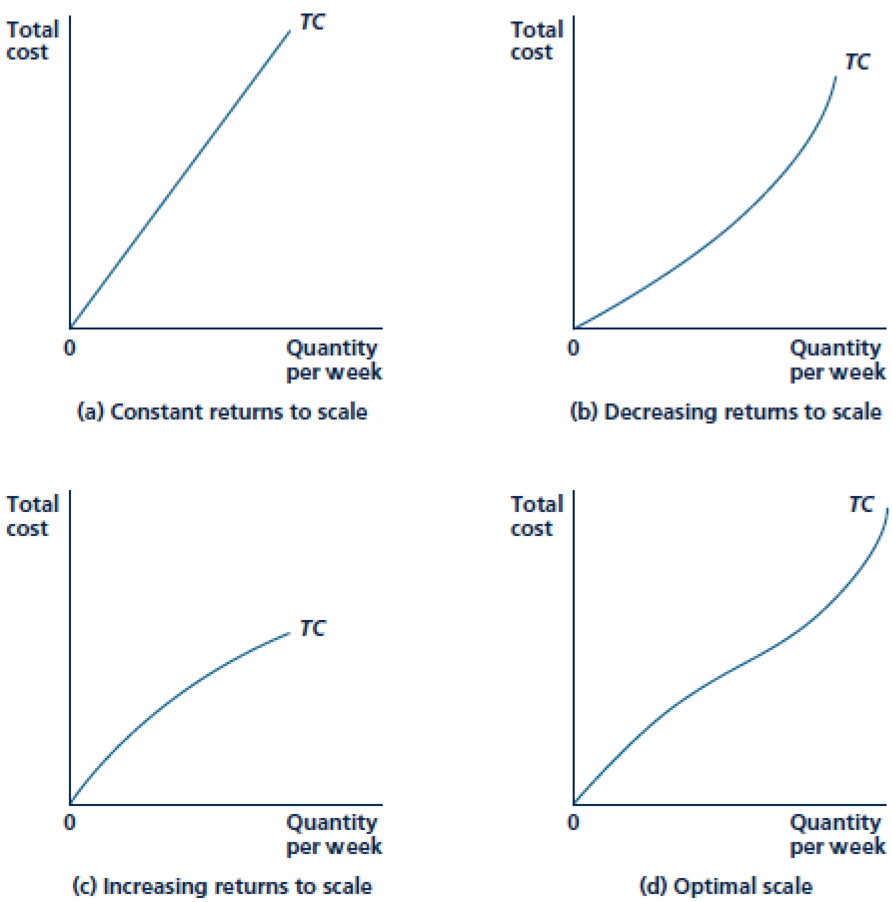
$$q = 2L^2K \Rightarrow 32 = 2L^2K \Rightarrow K = \frac{32}{2L^2} \text{ .... (2)}$$

Solve (1) and (2):

$$\frac{L}{6} = \frac{32}{2L^2} \Rightarrow 2L^3 = 192 \Rightarrow L^3 = 96 \Rightarrow L = 4.5 \quad ; \; K = \frac{L}{6} = \frac{4.5}{6} = 0.75$$

Cost Curves

The shape of the total cost curve depends on the nature of the production function. Panel a represents constant returns to scale: As output expands, input costs expand proportionately. Panel b and panel c show decreasing returns to scale and increasing returns to scale, respectively. Panel d represents costs where the firm has an ‘‘optimal scale’’ of operations.



Average Costs:

Average cost is total cost divided by output; a common measure of cost per unit.

Average cost = AC = $\frac{TC}{q}$

Marginal Cost:

The additional cost of producing one more unit of output

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If the cost of producing 24 units is $98 and the cost of producing 25 units is $100, the marginal cost of the 25th unit is $2.

MC = $\frac{\Delta TC}{\Delta q}$ = $\frac{dTC}{dq}$

Marginal costs are reflected by the slope of the total cost curve.

Example

The total cost curve of a firm is $\mathrm{TC} = 0.2\,\mathrm{q}^2 + 5\mathrm{q} + 200$. What is the average total cost and marginal cost to produce 100 units of output?

Average cost = AC = $\frac{TC}{q}$ = $\frac{0.2q^2 + 5q + 200}{q}$ = $0.2q + 5 + \frac{200}{q}$

AC = $0.2(100) + 5 + \frac{200}{100}$ = 27

MC = $\frac{dTC}{dq}$ = $0.4q + 5$

MC = $0.4(100) + 5$ = 45

Marginal and Average Cost Curves:

The constant returns to scale total cost curve has a constant slope, so the marginal cost is constant as shown by the horizontal marginal cost curve in Panel a of Figure.

With decreasing returns to scale, the total cost curve is convex. This means that marginal costs are increasing which is shown by the positively sloped marginal cost curve in Panel b of Figure.

Increasing returns to scale results in a concave total cost curve. This causes the marginal costs to decrease as output increases as shown in the negatively sloped marginal cost curve in Panel c of Figure.

When the total cost curve is first concave followed by convex, marginal costs initially decrease but eventually increase. Thus, the marginal cost curve is first negatively sloped followed by a positively sloped curve as shown in Panel d of Figure.

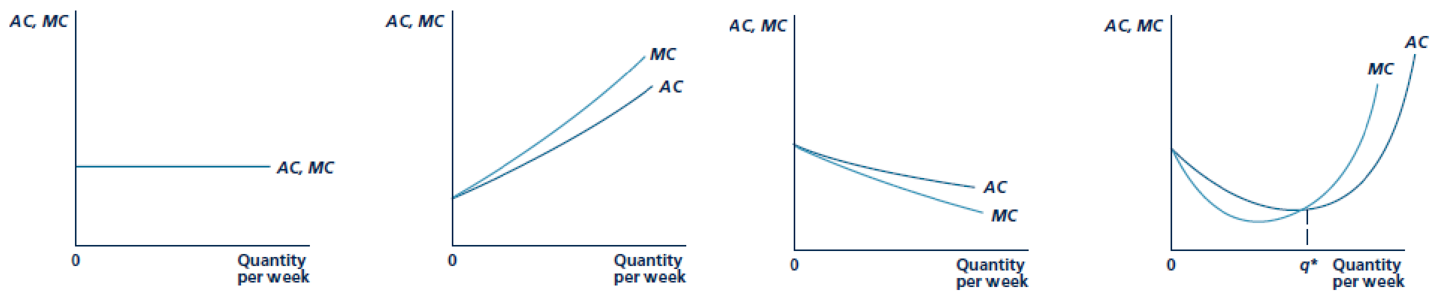
Average Cost Curves:

- If a firm produces only one unit of output, marginal cost would be the same as average cost. Thus, the graph of the average cost curve begins at the point where the marginal cost curve intersects the vertical axis.

- For the constant returns to scale case, marginal cost never varies from its initial level, so average cost must stay the same as well. Thus, the average cost curve are the same horizontal line as shown in Panel a of Figure.

- With convex total costs and increasing marginal costs, average costs also rise, because the first few units are produced at low marginal costs, average costs will always b less than marginal cost, so the average cost curve lies below the marginal cost curve.

- With concave total cost and decreasing marginal costs, average costs will also decrease, Because the first few units are produced at relatively high marginal costs, average is less than marginal cost, so the average cost curve lies below the marginal cost curve.



(a) Constant returns to scale (b) Decreasing returns to scale (c) Increasing returns to scale (d) Optimal scale

The Relationship between the Short Run and the Long Run Cost:

The *short run* is the period of time in which a firm must consider some inputs to be absolutely fixed in making its decisions.

The *long run* is the period of time in which a firm may consider all of its inputs to be variable in making its decisions.

Holding Capital Input Constant

For the following, the capital input is assumed to be held constant at a level of K1, so that, with only two inputs, labor is the only input the firm can vary.

Short Run Total Cost $(STC) = \mathrm{w\,L} + \mathrm{v}\,K_1$

Fixed Cost and Variable cost

Some costs vary with output, while others remain unchanged as long as the firm is producing any output at all.

Total cost (TC): Total economic cost of production, consisting of fixed and variable costs.

Fixed cost (FC): Cost that does not vary with the level of output and that can be eliminated only by shutting down.

Variable cost (VC): Cost that varies as output varies.

Fixed cost does not vary with the level of output—it must be paid even if there is no output. The only way that a firm can eliminate its fixed costs is by shutting down.

$Total\ cost\ (TC) = Fixed\ cost\ (FC) + Variable\ cost\ (VC)$

Example:
Suppose a firm's short run cost curves were found to be: $TC = Q^2 + 4Q + 5$, where Q is output.
What are the firm's FC, VC and TC when the firm producing 10 units of output?

$FC = 5$

$VC = Q^2 + 4Q = 102 + 4(10) = 100 + 40 = 140$

$TC = FC + VC = 5 + 140 = 145$

Average Total Cost:

Average total cost (ATC): Firms total cost dividing by its level of output

ATC = $\frac{TC}{q}$

Average fixed cost (AFC): Firms fixed cost dividing by its level of output

AFC = $\frac{FC}{q}$

Average variable cost (AVC): Firms variable cost dividing by its level of output

AVC = $\frac{VC}{q}$

$TC = VC + FC \;\; ⇨ ATC = AVC + AFC$

Example
The total cost curve of a firm is $TC = 0.2\,q^2 + 5q + 200$. What are the AFC, AVC, and ATC to produce 100 units of output?

$AFC = \frac{FC}{q} = \frac{200}{100} = 2 \qquad AVC = \frac{VC}{q} = \frac{0.2\,q^2 + 5q}{q} = 0.2\,q + 5 = 0.2(100) + 5 = 20 + 5 = 25$

$ATC = AFC + AVC = 2 + 25 = 27$

Example
If average total cost rises from \$10 to \$30 as total production rises from 100 to 300 units. What is marginal cost?

ATC = $\frac{TC}{q}$ = 10 when q = 100 ⇨ 10 = $\frac{TC}{100}$ ⇨ TC = 1000

ATC = $\frac{TC}{q}$ = 30 when q = 300 ⇨ 30 = $\frac{TC}{300}$ ⇨ TC = 9000

MC = $\frac{\Delta TC}{\Delta q} = \frac{(9000-1000)}{(300-100)} = \frac{8000}{200}$ = 40

Example

A firm producing hockey sticks has a production function given by: $q = 2\sqrt{KL}$. In the short run, the firm's amount of capital is fixed at K = 100. The rental rate for K is r = $1, and the wage rate for L is w = $4.

a. Calculate the firm's short run total cost function.

$$STC = w\,L + v\,K1 = 4L + 100$$

$$q = 2\sqrt{KL} \quad \rightarrow \quad q = 2\sqrt{100L} \quad \rightarrow \quad q = 20\sqrt{L}$$

$$q2 = 400L \quad \rightarrow \quad L = \frac{q2}{400}$$

$$STC = 4L + 100 = 4(\frac{q2}{400}) + 100 \quad \rightarrow \quad STC = \frac{q2}{100} + 100$$

b. What are the STC, SAC, and SMC for the firm if it produces 25 hockey sticks?

$$STC = \frac{q2}{100} + 100$$

$$SATC = \frac{STC}{q} = \frac{q}{100} + \frac{100}{q}$$

$$SMC = \frac{\partial STC}{\partial q} = \frac{q}{50}$$

$$When\,q = 25$$

$$STC = \frac{q2}{100} + 100 = \frac{(25)2}{100} + 100 = 106.25$$

$$SATC = \frac{STC}{q} = \frac{q}{100} + \frac{100}{q} = \frac{25}{100} + \frac{100}{25} = 4.5$$

$$SMC = \frac{\partial STC}{\partial q} = \frac{q}{50} = \frac{25}{50} = ½$$

Example

Output for a simple production function is given by: $q = LK + 3L$. The price of capital is $20 per unit and capital is fixed at 5 units in the short run. The price of labor is $4 per unit. How many labors would the firm employ in order to producing 80 units of output?

$$STC = w\,L + v\,K1 = 4L + 100$$

$$q = LK + 3L \Rightarrow 80 = 5L + 3L \Rightarrow 80 = 8L \Rightarrow L = \frac{80}{8} = 10$$

$$STC = 4L + 100 = 4x10 + 100 = 140$$

Shifts in Cost Curves:

Any change in *input prices* and *technological innovations*, will affect the shape and position of the firm's cost curves.

Changes in Input Prices

A change in the price of an input will tilt the firm's total cost lines. For example, a rise in wage rates will cause firms to use more capital (to the extent allowed by the technology).

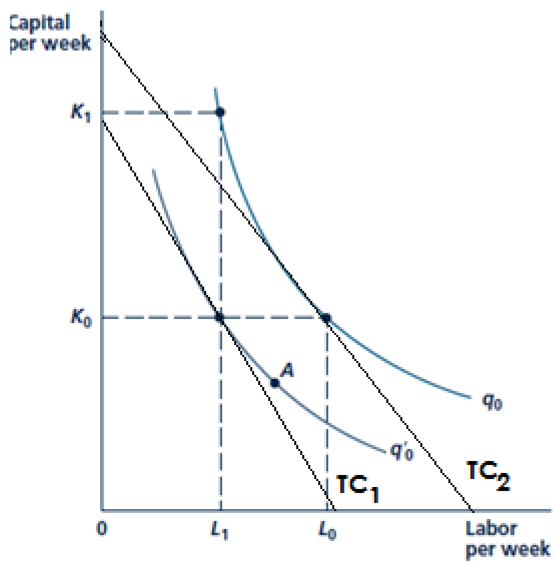
Generally, all cost curves will shift upward with the extent of the shift depending upon how important labor is in production and how successful the firm is in substituting other inputs for labor.

Increase in input price → increase total cost → shift cost curves to the right.

Decrease in input price → decrease total cost → shift cost curves to the left.

Technological Innovation

Technological improvements would shift isoquants toward the origin enabling firms to produce the same level of output with less of all inputs → shift the costs curve to the left.



PROBLEMS

1. A firm production function is $q = KL + L$, where q is output, K is hours of capital services and L is hours of labor.
   a. Does this production function exhibit increasing, constant, or decreasing return to scale?
   b. The firm operates so as to minimize its costs and faces a wage of w =$1 per hour of work and a rental rate of capital of v =$2. If it uses 10 units of L, how many units of K does it use?
   c. How many units of output does it produce?

2. A firm producing Rice has a production function given by: $q = KL^{1/3}$. The rental rate (v) is $8, and the wage rate (w) is $16.
   a. What is the marginal rate of technical substitution between labor and capital if K = 9 and L= 1.
   b. How many labors would minimize cost to produce 96 units of output?

c. If output *(q)* were to increase to 144 units, would this firm exhibit constant, increasing, or decreasing returns to scale? Explain
d. In the short run, the firm amount of capital is fixed at K= 4.Write the short run total cost curve in term of q.

3. Output for a simple production process is given by $q = 4KL$. The price of capital is $20 per unit and capital is fixed at 5 units in the short run. The price of labor is $8 per unit. What is the total cost of producing 100 units of output?

4. Let the firm's production function be given by $q = 2\,min\,\{½K, L\}$. suppose that v = 2 and w = 8.
   a. How much K and L are employed in the efficient production of 20 units of output?
   b. What is the minimum cost of producing 20 units of output?
   c. Determine whether this production function reflects increasing, decreasing, or constant returns to scale?

5. A firm producing Hamburger has a production function given by: $q = 2\sqrt{KL}$. In the short run, the firm amount of capital is fixed at K= 100. The rental rate (v) is $1, and the wage rate (w) is $4.

   a. Determine whether Hamburger production function reflects increasing, decreasing, or constant returns to scale?
   b. Calculate the firm short run total cost curve
   c. What are the STC, SAC, and SMC for the firm if it produce 200 units of hamburger

6. Assume the AXZ Company has the following production function: $q = 25KL$. And they are currently using 100 units of K in the short run. If the price of labor is 50 and the price of capital is 200. If they are using 20 units of labor, what is their (MRTS)

7. Suppose a production function is given by $q = K\sqrt{L}$. the price of capital is $10 and the price of labor is $16. The capital is fixed at the level K = 8.
   a. What is the Marginal rate of technical substitution (MRTS), if they are using 16 units of labor?
   b. Calculate the total cost function (function of output (q))
   c. What is the quantity of labor that minimizes the cost of producing any level of output?
   d. What is the minimum cost of producing q units of output?