7.2 $2+4+10+14+16+22+24+31+34+44+54+64+66+70$ ①
$+71$

7.2 Natural Logarithms

(2.) Express the following in terms of $\ln 5$ and $\ln 7$

a. $\ln 1/125 = \ln 1 - \ln 125$
$= \ln 1 - \ln 5^3$
$= -3\ln 5$

b. $\ln 9.8 = \ln \frac{98}{10} = \ln \frac{49}{5} = \ln 7^2 - \ln 5 = 2\ln 7 - \ln 5$

c. $\ln 7\sqrt{7} = \ln 7^{\frac{3}{2}} = \frac{3}{2}\ln 7$

d. $\ln 1225 = \ln(25 * 49)$
$= \ln 25 + \ln 49$
$= \ln 5^2 + \ln 7^2$
$= 2\ln 5 + 2\ln 7$

e. $\ln 0.056 = \ln \frac{56}{1000} = \ln \frac{28}{500} = \ln \frac{14}{250} = \ln \frac{7}{125} = \ln 7 - \ln 125$
$= \ln 7 - \ln 5^2$
$= \ln 7 - 2\ln 5$.

f. $(\ln 35 + \ln(\frac{1}{7})) / \ln 25 = \frac{\ln(5 \cdot 7) - \ln 7}{2\ln 5}$
$= \frac{\ln 5}{2\ln 5} = \frac{1}{2}$

4. a. $\ln \sec\theta + \ln \cos\theta$

$= \ln (\sec\theta)(\cos\theta)$

$= \boxed{\ln 1 = 0}$

b. $\ln(8x+4) - 2\ln 2$

$= \ln(8x+4) - \ln 4$

$= \ln \dfrac{8x+4}{4} = \boxed{\ln(2x+1)}$

c. $3\ln\sqrt[3]{t^2-1} - \ln t+1$

$= \ln(t^2-1) - \ln(t+1)$

$= \ln \dfrac{t^2-1}{t+1} = \ln\left[\dfrac{(t-1)(t+1)}{t+1}\right]$

$= \boxed{\ln(t-1)}$

10. $y = \ln\left(\dfrac{10}{x}\right)$

$\boxed{y = \ln 10 - \ln x}$

$y' = 0 - \dfrac{1}{x}$

$\boxed{y' = -\dfrac{1}{x}}$

14. $y = (\ln x)^3$

$y' = 3(\ln x)^2 \cdot \dfrac{1}{x} = \boxed{\dfrac{3\ln^2 x}{x}}$

16. $y = t\sqrt{\ln t}$

$y = t(\ln t)^{\frac{1}{2}}$

$y' = t \cdot \frac{1}{2}(\ln t)^{-\frac{1}{2}} \cdot \frac{1}{t} + (\ln t) \cdot 1$

$$\boxed{y' = \frac{1}{2\sqrt{\ln t}} + \ln t}$$

22. $y = \frac{x \ln x}{1 + \ln x}$

$$y' = \frac{(1+\ln x)\left[x \cdot \frac{1}{x} + \ln x \cdot 1\right] - x \ln x \left(0 + \frac{1}{x}\right)}{(1+\ln x)^2}$$

$$= \frac{(1+\ln x)(1+\ln x) - \ln x}{(1+\ln x)^2} = \frac{1 + \ln^2 x + 2\ln x - \ln x}{(1+\ln x)^2}$$

$$\boxed{y' = \frac{\ln^2 x + \ln x + 1}{(1+\ln x)^2}}$$

24. $y = \ln(\ln(\ln x))$

$$y' = \frac{[\ln(\ln x)]'}{\ln[\ln x]} = \frac{\frac{(\ln x)'}{\ln x}}{\ln(\ln x)} = \frac{\frac{\frac{1}{x}}{\ln x}}{\ln(\ln x)}$$

$$\boxed{y' = \frac{1}{x \ln x \ln(\ln x)}}$$

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31. $y = \ln(\sec(\ln\theta))$

$$y' = \frac{[\sec(\ln\theta)]'}{[\sec(\ln\theta)]}$$

$$= \frac{\sec(\ln\theta)\tan(\ln\theta)\cdot\frac{1}{\theta}}{\sec(\ln\theta)}$$

$$\boxed{y' = \frac{\tan(\ln\theta)}{\theta}}$$

34. $y = \ln\sqrt{\frac{(x+1)^5}{(x+2)^{20}}}$

$$y = \frac{1}{2}\ln\frac{(x+1)^5}{(x+2)^{20}}$$

$$= \frac{1}{2}\left[5\ln(x+1) - 20\ln(x+2)\right]$$

$$y = \frac{5}{2}\ln(x+1) - 10\ln(x+2)$$

$$y' = \frac{5}{2}\cdot\frac{1}{x+1} - \frac{10}{x+2}$$

$$\boxed{y' = \frac{5}{2(x+1)} - \frac{10}{(x+2)}}$$

44. $\displaystyle\int_2^4 \frac{dx}{x\ln x}$

$u = \ln x$

$du = \dfrac{dx}{x}$

$\displaystyle\int \frac{x\,d\cancel{u}}{x\cdot u} = \int \frac{du}{u}$

$= \ln|u| = \ln|\ln x|\Big|_2^4$

$= \ln(\ln 4) - \ln(\ln 2)$

$= \ln(2\ln 2) - \ln(\ln 2)$

$= \ln 2 + \ln(\ln 2) - \ln(\ln 2)$

$= \ln 2$

54. $\displaystyle\int \frac{\sec x\, dx}{\sqrt{\ln(\sec x + \tan x)}}$

$u = \ln(\sec x + \tan x)$

$\dfrac{du}{dx} = \dfrac{\sec x \cdot \tan x + \sec^2 x}{\sec x + \tan x} = \sec x\left[\dfrac{\cancel{\sec}\cdot\tan x + \sec x}{\sec x + \tan x}\right]$

$= \sec x$

$\displaystyle\int \frac{du}{\sqrt{u}} = \int u^{-\frac{1}{2}}\,du = \frac{u^{\frac{1}{2}}}{\frac{1}{2}} + c = 2\sqrt{u} + c$

So $\boxed{I = 2\sqrt{\ln(\sec x + \tan x)} + C}$

64. $y = \dfrac{\theta \sin\theta}{\sqrt{\sec\theta}}$

$$\ln y = \ln\left(\frac{\theta \sin\theta}{\sqrt{\sec\theta}}\right)$$

$$\ln y = \ln\theta + \ln\sin\theta - \frac{1}{2}\ln\sec\theta$$

$$\frac{y'}{y} = \frac{1}{\theta} + \frac{\cos\theta}{\sin\theta} - \frac{1}{2}\,\frac{\sec\theta\tan\theta}{\sec\theta}$$

$$\frac{y'}{y} = \frac{1}{\theta} + \cot\theta - \frac{1}{2}\tan\theta$$

$$\boxed{y' = \frac{\theta\sin\theta}{\sqrt{\sec\theta}}\left[\frac{1}{\theta} + \cot\theta - \frac{1}{2}\tan\theta\right]}$$

66. $y = \sqrt{\dfrac{(x+1)^{10}}{(2x+1)^5}}$

$$\ln y = \frac{1}{2}\ln(x+1)^{10} - \frac{1}{2}\ln(2x+1)^5$$

$$\ln y = 5\ln(x+1) - \frac{5}{2}\ln(2x+1)$$

$$\frac{y'}{y} = \frac{5}{x+1} - \frac{5}{2}\cdot\frac{2}{2x+1}$$

$$\boxed{y' = \sqrt{\frac{(x+1)^{10}}{(2x+1)^5}}\left[\frac{5}{x+1} - \frac{5}{2x+1}\right]}$$

70 (a) $f(x) = x - \ln x$ increasing for $x > 1$

$$f' = 1 - \frac{1}{x}$$

$$1 - \frac{1}{x} = 0$$

$$1 = \frac{1}{x}$$

$\rightarrow x = 1$

$+ + + +$ f'

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(b). Use part (a) to show that $\ln x < x$ if $x > 1$

If $x > 1$

$$f(x) > f(1)$$

$$x - \ln x > 1 - \ln 1$$

$$x - \ln x > 1$$

$$x - \ln x > 0$$

$$\boxed{x > \ln x}$$

71. $A = \int_1^5 \ln 2x - \ln x \, dx = \int_1^5 \ln 2 \, dx = (\ln 2) x \Big|_1^5 = 4 \ln 2 = \ln 8$

s area between $y = \ln x$ and $y = \ln 2x$ is $\ln 8$