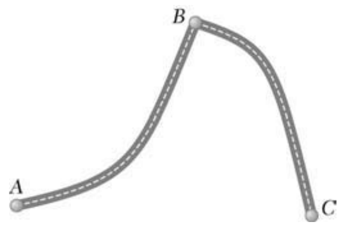
CHAPTER 11



PROBLEM 11.CQ1

A bus travels the 100 miles between A and B at 50 mi/h and then another 100 miles between B and C at 70 mi/h. The average speed of the bus for the entire 200-mile trip is:

(a) more than 60 mi/h

(b) equal to 60 mi/h

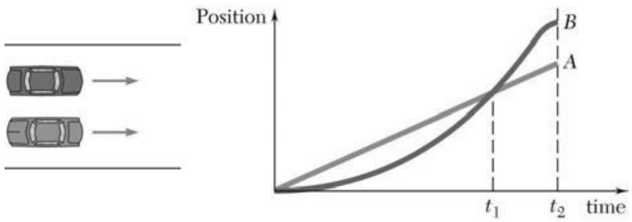
(c) less than 60 mi/h

SOLUTION

The time required for the bus to travel from A to B is 2 h and from B to C is 100/70 = 1.43 h, so the total time is 3.43 h and the average speed is 200/3.43 = 58 mph.

Answer: (c) ◀

PROBLEM 11CQ2



Two cars *A* and *B* race each other down a straight road. The position of each car as a function of time is shown. Which of the following statements are true (more than one answer can be correct)?

(*a*) At time t_2 both cars have traveled the same distance

(*b*) At time t_1 both cars have the same speed

(*c*) Both cars have the same speed at some time $t < t_1$

(*d*) Both cars have the same acceleration at some time $t < t_1$

(*e*) Both cars have the same acceleration at some time $t_1 < t < t_2$

SOLUTION

The speed is the slope of the curve, so answer c) is true.

The acceleration is the second derivative of the position. Since *A*'s position increases linearly the second derivative will always be zero. The second derivative of curve *B* is zero at the pont of inflection which occurs between t_1 and t_2.

Answers: (*c*) and (*e*) ◀

PROBLEM 11.1

The motion of a particle is defined by the relation $x = t^4 - 10t^2 + 8t + 12$, where x and t are expressed in inches and seconds, respectively. Determine the position, the velocity, and the acceleration of the particle when $t = 1$ s.

SOLUTION

$$x = t^4 - 10t^2 + 8t + 12$$

$$v = \frac{dx}{dt} = 4t^3 - 20t + 8$$

$$a = \frac{dv}{dt} = 12t^2 - 20$$

At $t = 1$ s,

$$x = 1 - 10 + 8 + 12 = 11 \qquad x = 11.00 \text{ in.} \blacktriangleleft$$

$$v = 4 - 20 + 8 = -8 \qquad v = -8.00 \text{ in./s} \blacktriangleleft$$

$$a = 12 - 20 = -8 \qquad a = -8.00 \text{ in./s}^2 \blacktriangleleft$$

PROBLEM 11.2

The motion of a particle is defined by the relation $x = 2t^3 - 9t^2 + 12t + 10$, where x and t are expressed in feet and seconds, respectively. Determine the time, the position, and the acceleration of the particle when $v = 0$.

SOLUTION

$$x = 2t^3 - 9t^2 + 12t + 10$$

Differentiating,

$$v = \frac{dx}{dt} = 6t^2 - 18t + 12 = 6(t^2 - 3t + 2)$$

$$= 6(t-2)(t-1)$$

$$a = \frac{dv}{dt} = 12t - 18$$

So $v = 0$ at $t = 1\text{ s}$ and $t = 2\text{ s}$.

At $t = 1\text{ s}$,

$$x_1 = 2 - 9 + 12 + 10 = 15$$

$$a_1 = 12 - 18 = -6$$

$t = 1.000\text{ s}$ ◀

$x_1 = 15.00\text{ ft}$ ◀

$a_1 = -6.00\text{ ft/s}^2$ ◀

At $t = 2\text{ s}$,

$$x_2 = 2(2)^3 - 9(2)^2 + 12(2) + 10 = 14$$

$t = 2.00\text{ s}$ ◀

$x_2 = 14.00\text{ ft}$ ◀

$$a_2 = (12)(2) - 18 = 6$$

$a_2 = 6.00\text{ ft/s}^2$ ◀

PROBLEM 11.3

The vertical motion of mass A is defined by the relation $x = 10\sin 2t + 15\cos 2t + 100$, where x and t are expressed in mm and seconds, respectively. Determine (*a*) the position, velocity and acceleration of A when $t = 1$ s, (*b*) the maximum velocity and acceleration of A.

SOLUTION

$$x = 10\sin 2t + 15\cos 2t + 100$$

$$v = \frac{dx}{dt} = 20\cos 2t - 30\sin 2t$$

$$a = \frac{dv}{dt} = -40\sin 2t - 60\cos 2t$$

For trigonometric functions set calculator to radians:

(*a*) At $t = 1$ s. $x_1 = 10\sin 2 + 15\cos 2 + 100 = 102.9$ $\qquad x_1 = 102.9$ mm ◀

$v_1 = 20\cos 2 - 30\sin 2 = -35.6$ $\qquad v_1 = -35.6$ mm/s ◀

$a_1 = -40\sin 2 - 60\cos 2 = -11.40$ $\qquad a_1 = -11.40$ mm/s^2 ◀

(*b*) Maximum velocity occurs when $a = 0$.

$$-40\sin 2t - 60\cos 2t = 0$$

$$\tan 2t = -\frac{60}{40} = -1.5$$

$$2t = \tan^{-1}(-1.5) = -0.9828 \text{ and } -0.9828 + \pi$$

Reject the negative value. $2t = 2.1588$

$$t = 1.0794 \text{ s}$$

$$t = 1.0794 \text{ s for } v_{max}$$

so $v_{max} = 20\cos(2.1588) - 30\sin(2.1588)$

$= -36.056$ $\qquad v_{max} = -36.1$ mm/s ◀

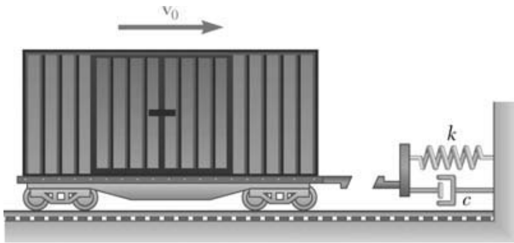
Note that we could have also used

$$v_{max} = \sqrt{20^2 + 30^2} = 36.056$$

by combining the sine and cosine terms.

For a_{max} we can take the derivative and set equal to zero or just combine the sine and cosine terms.

$a_{max} = \sqrt{40^2 + 60^2} = 72.1$ mm/s^2 $\qquad a_{max} = 72.1$ mm/s^2 ◀



PROBLEM 11.4

A loaded railroad car is rolling at a constant velocity when it couples with a spring and dashpot bumper system. After the coupling, the motion of the car is defined by the relation $x = 60e^{-4.8t}\sin 16t$ where x and t are expressed in mm and seconds, respectively. Determine the position, the velocity and the acceleration of the railroad car when (*a*) $t = 0$, (*b*) $t = 0.3$ s.

SOLUTION

$$x = 60e^{-4.8t}\sin 16t$$

$$v = \frac{dx}{dt} = 60(-4.8)e^{-4.8t}\sin 16t + 60(16)e^{-4.8t}\cos 16t$$

$$v = -288e^{-4.8t}\sin 16t + 960e^{-4.8t}\cos 16t$$

$$a = \frac{dv}{dt} = 1382.4e^{-4.8t}\sin 16t - 4608e^{-4.8t}\cos 16t$$
$$- 4608e^{-4.8t}\cos 16t - 15360e^{-4.8t}\sin 16t$$

$$a = -13977.6e^{-4.8t}\sin 16t - 9216e^{-4.8}\cos 16t$$

(*a*) At $t = 0$,

$x_0 = 0$ $\qquad x_0 = 0 \text{ mm}$ ◀

$v_0 = 960 \text{ mm/s}$ $\qquad v_0 = 960 \text{ mm/s} \longrightarrow$ ◀

$a_0 = -9216 \text{ mm/s}^2$ $\qquad a_0 = 9220 \text{ mm/s}^2 \longleftarrow$ ◀

(*b*) At $t = 0.3$ s,

$$e^{-4.8t} = e^{-1.44} = 0.23692$$
$$\sin 16t = \sin 4.8 = -0.99616$$
$$\cos 16t = \cos 4.8 = 0.08750$$

$x_{0.3} = (60)(0.23692)(-0.99616) = -14.16$ $\qquad x_{0.3} = 14.16 \text{ mm} \longleftarrow$ ◀

$v_{0.3} = -(288)(0.23692)(-0.99616)$
$\quad + (960)(0.23692)(0.08750) = 87.9$ $\qquad v_{0.3} = 87.9 \text{ mm/s} \longrightarrow$ ◀

$a_{0.3} = -(13977.6)(0.23692)(-0.99616)$
$\quad - (9216)(0.23692)(0.08750) = 3108$ $\qquad a_{0.3} = 3110 \text{ mm/s}^2$ ◀

or $3.11 \text{ m/s}^2 \longrightarrow$ ◀

PROBLEM 11.5

The motion of a particle is defined by the relation $x = 6t^4 - 2t^3 - 12t^2 + 3t + 3$, where x and t are expressed in meters and seconds, respectively. Determine the time, the position, and the velocity when $a = 0$.

SOLUTION

We have $$x = 6t^4 - 2t^3 - 12t^2 + 3t + 3$$

Then $$v = \frac{dx}{dt} = 24t^3 - 6t^2 - 24t + 3$$

and $$a = \frac{dv}{dt} = 72t^2 - 12t - 24$$

When $a = 0$: $$72t^2 - 12t - 24 = 12(6t^2 - t - 2) = 0$$

or $$(3t - 2)(2t + 1) = 0$$

or $$t = \frac{2}{3}\text{ s} \quad \text{and} \quad t = -\frac{1}{2}\text{ s} \quad \text{(Reject)}$$

$t = 0.667$ s ◀

At $t = \frac{2}{3}$ s: $$x_{2/3} = 6\left(\frac{2}{3}\right)^4 - 2\left(\frac{2}{3}\right)^3 - 12\left(\frac{2}{3}\right)^2 + 3\left(\frac{2}{3}\right) + 3$$

or $x_{2/3} = 0.259$ m ◀

$$v_{2/3} = 24\left(\frac{2}{3}\right)^3 - 6\left(\frac{2}{3}\right)^2 - 24\left(\frac{2}{3}\right) + 3$$

or $v_{2/3} = -8.56$ m/s ◀

PROBLEM 11.6

The motion of a particle is defined by the relation $x = t^3 - 9t^2 + 24t - 8$, where x and t are expressed in inches and seconds, respectively. Determine (*a*) when the velocity is zero, (*b*) the position and the total distance traveled when the acceleration is zero.

SOLUTION

We have $x = t^3 - 9t^2 + 24t - 8$

Then $v = \dfrac{dx}{dt} = 3t^2 - 18t + 24$

and $a = \dfrac{dv}{dt} = 6t - 18$

(*a*) When $v = 0$: $3t^2 - 18t + 24 = 3(t^2 - 6t + 8) = 0$

$$(t-2)(t-4) = 0$$

$t = 2.00$ s and $t = 4.00$ s ◀

(*b*) When $a = 0$: $6t - 18 = 0$ or $t = 3$ s

At $t = 3$ s: $x_3 = (3)^3 - 9(3)^2 + 24(3) - 8$ or $x_3 = 10.00$ in. ◀

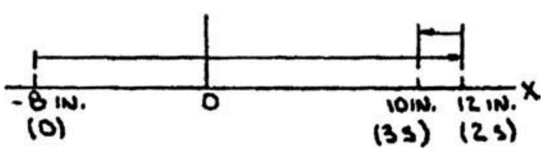
First observe that $0 \le t < 2$ s: $v > 0$

$2 \text{ s} < t \le 3$ s: $v < 0$

Now

At $t = 0$: $x_0 = -8$ in.

At $t = 2$ s: $x_2 = (2)^3 - 9(2)^2 + 24(2) - 8 = 12$ in.



Then $x_2 - x_0 = 12 - (-8) = 20$ in.

$$|x_3 - x_2| = |10 - 12| = 2 \text{ in.}$$

Total distance traveled $= (20 + 2)$ in. Total distance $= 22.0$ in. ◀

PROBLEM 11.7

The motion of a particle is defined by the relation $x = 2t^3 - 15t^2 + 24t + 4$, where x is expressed in meters and t in seconds. Determine (*a*) when the velocity is zero, (*b*) the position and the total distance traveled when the acceleration is zero.

SOLUTION

$$x = 2t^3 - 15t^2 + 24t + 4$$

$$v = \frac{dx}{dt} = 6t^2 - 30t + 24$$

$$a = \frac{dv}{dt} = 12t - 30$$

(*a*) $\underline{v = 0 \text{ when}}$ $\quad 6t^2 - 30t + 24 = 0$

$$6(t-1)(t-4) = 0 \qquad t = 1.000 \text{ s} \quad \text{and} \quad t = 4.00 \text{ s} \blacktriangleleft$$

(*b*) $\underline{a = 0 \text{ when}}$ $\quad 12t - 30 = 0 \qquad t = 2.5 \text{ s}$

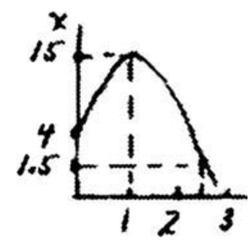
For $t = 2.5$ s: $\quad x_{2.5} = 2(2.5)^3 - 15(2.5)^2 + 24(2.5) + 4$

$$x_{2.5} = +1.500 \text{ m} \blacktriangleleft$$

To find total distance traveled, we note that

$\underline{v = 0 \text{ when } t = 1\text{ s}}$: $\quad x_1 = 2(1)^3 - 15(1)^2 + 24(1) + 4$

$x_1 = +15$ m



For $t = 0$, $\quad x_0 = +4$ m

Distance traveled

From $t = 0$ to $t = 1$ s: $\quad x_1 - x_0 = 15 - 4 = 11$ m $\longrightarrow$

From $t = 1$ s to $t = 2.5$ s: $\quad x_{2.5} - x_1 = 1.5 - 15 = 13.5$ m $\longleftarrow$

Total distance traveled $= 11 \text{ m} + 13.5 \text{ m}$ $\qquad$ Total distance $= 24.5$ m $\blacktriangleleft$

PROBLEM 11.8

The motion of a particle is defined by the relation $x = t^3 - 6t^2 - 36t - 40$, where x and t are expressed in feet and seconds, respectively. Determine (*a*) when the velocity is zero, (*b*) the velocity, the acceleration, and the total distance traveled when $x = 0$.

SOLUTION

We have $$x = t^3 - 6t^2 - 36t - 40$$

Then $$v = \frac{dx}{dt} = 3t^2 - 12t - 36$$

and $$a = \frac{dv}{dt} = 6t - 12$$

(*a*) When $v = 0$: $$3t^2 - 12t - 36 = 3(t^2 - 4t - 12) = 0$$

or $$(t+2)(t-6) = 0$$

or $t = -2$ s (Reject) and $t = 6$ s $\qquad t = 6.00$ s ◀

(*b*) When $x = 0$: $$t^3 - 6t^2 - 36t - 40 = 0$$

Factoring $(t-10)(t+2)(t+2) = 0$ or $t = 10$ s

Now observe that $0 \le t < 6$ s: $\quad v < 0$

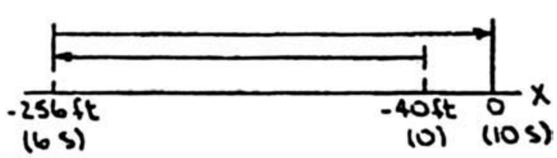
$6\text{ s} < t \le 10$ s: $\quad v > 0$

and at $t = 0$: $\quad x_0 = -40$ ft

$t = 6$ s: $$x_6 = (6)^3 - 6(6)^2 - 36(6) - 40 = -256 \text{ ft}$$

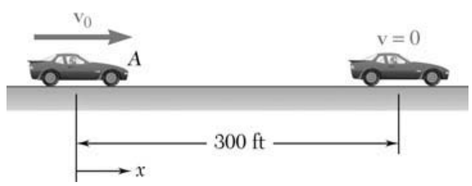
$t = 10$ s: $v_{10} = 3(10)^2 - 12(10) - 36$ $\qquad$ or $v_{10} = 144.0$ ft/s ◀

$a_{10} = 6(10) - 12$ $\qquad$ or $a_{10} = 48.0$ ft/s^2 ◀



Then $$|x_6 - x_0| = |-256 - (-40)| = 216 \text{ ft}$$
$$x_{10} - x_6 = 0 - (-256) = 256 \text{ ft}$$

Total distance traveled $= (216 + 256)$ ft $\qquad$ Total distance $= 472$ ft ◀



PROBLEM 11.9

The brakes of a car are applied, causing it to slow down at a rate of 10 m/s^2. Knowing that the car stops in 100 m, determine (*a*) how fast the car was traveling immediately before the brakes were applied, (*b*) the time required for the car to stop.

SOLUTION

$$a = -10 \text{ ft/s}^2$$

(*a*) Velocity at $x = 0$.

$$v\frac{dv}{dx} = a = -10$$

$$\int_{v_0}^{0} v\,dv = -\int_0^{x_f} (-10)\,dx$$

$$0 - \frac{v_0^2}{2} = -10x_f = -(10)(300)$$

$$v_0^2 = 6000$$

$v_0 = 77.5 \text{ ft/s}^2$ ◀

(*b*) Time to stop.

$$\frac{dv}{dx} = a = -10$$

$$\int_{v_0}^{0} dv = -\int_0^{t_f} -10\,dt$$

$$0 - v_0 = -10t_f$$

$$t_f = \frac{v_0}{10} = \frac{77.5}{10}$$

$t_f = 7.75 \text{ s}$ ◀

PROBLEM 11.10

The acceleration of a particle is directly proportional to the time t. At $t = 0$, the velocity of the particle is $v = 16$ in./s. Knowing that $v = 15$ in./s and that $x = 20$ in. when $t = 1$ s, determine the velocity, the position, and the total distance traveled when $t = 7$ s.

SOLUTION

We have $$a = kt \quad k = \text{constant}$$

Now $$\frac{dv}{dt} = a = kt$$

At $t = 0, v = 16$ in./s: $$\int_{16}^{v} dv = \int_{0}^{t} kt\, dt$$

or $$v - 16 = \frac{1}{2}kt^2$$

or $$v = 16 + \frac{1}{2}kt^2 \text{(in./s)}$$

At $t = 1$ s, $v = 15$ in./s: $$15 \text{ in./s} = 16 \text{ in./s} + \frac{1}{2}k(1 \text{ s})^2$$

or $$k = -2 \text{ in./s}^3 \quad \text{and} \quad v = 16 - t^2$$

Also $$\frac{dx}{dt} = v = 16 - t^2$$

At $t = 1$ s, $x = 20$ in.: $$\int_{20}^{x} dx = \int_{1}^{t} (16 - t^2)dt$$

or $$x - 20 = \left[16t - \frac{1}{3}t^3\right]_1^t$$

or $$x = -\frac{1}{3}t^3 + 16t + \frac{13}{3} \text{(in.)}$$

Then

At $t = 7$ s: $$v_7 = 16 - (7)^2$$ or $v_7 = -33.0$ in./s ◀

$$x_7 = -\frac{1}{3}(7)^3 + 16(7) + \frac{13}{3}$$ or $x_7 = 2.00$ in. ◀

When $v = 0$: $$16 - t^2 = 0 \quad \text{or} \quad t = 4 \text{ s}$$

PROBLEM 11.10 (Continued)

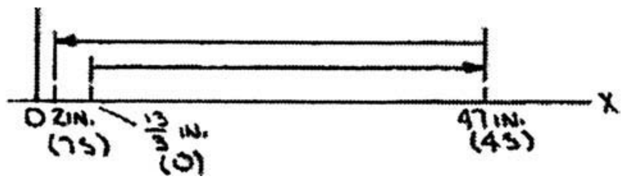
At $t = 0$: $\quad x_0 = \frac{13}{3}$

$t = 4$ s: $\quad x_4 = -\frac{1}{3}(4)^3 + 16(4) + \frac{13}{3} = 47$ in.

Now observe that

$0 \le t < 4$ s: $\quad v > 0$

4 s $< t \le 7$ s: $\quad v < 0$



Then
$$x_4 - x_0 = 47 - \frac{13}{3} = 42.67 \text{ in.}$$
$$|x_7 - x_4| = |2 - 47| = 45 \text{ in.}$$

Total distance traveled $= (42.67 + 45)$ in. $\quad$ Total distance $= 87.7$ in. ◀

PROBLEM 11.11

The acceleration of a particle is directly proportional to the square of the time t. When $t = 0$, the particle is at $x = 24$ m. Knowing that at $t = 6$ s, $x = 96$ m and $v = 18$ m/s, express x and v in terms of t.

SOLUTION

We have $$a = kt^2 \quad k = \text{constant}$$

Now $$\frac{dv}{dt} = a = kt^2$$

At $t = 6$ s, $v = 18$ m/s: $$\int_{18}^{v} dv = \int_{6}^{t} kt^2 dt$$

or $$v - 18 = \frac{1}{3}k(t^3 - 216)$$

or $$v = 18 + \frac{1}{3}k(t^3 - 216)(\text{m/s})$$

Also $$\frac{dx}{dt} = v = 18 + \frac{1}{3}k(t^3 - 216)$$

At $t = 0$, $x = 24$ m: $$\int_{24}^{x} dx = \int_{0}^{t}\left[18 + \frac{1}{3}k(t^3 - 216)\right]dt$$

or $$x - 24 = 18t + \frac{1}{3}k\left(\frac{1}{4}t^4 - 216t\right)$$

Now

At $t = 6$ s, $x = 96$ m: $$96 - 24 = 18(6) + \frac{1}{3}k\left[\frac{1}{4}(6)^4 - 216(6)\right]$$

or $$k = \frac{1}{9} \text{ m/s}^4$$

Then $$x - 24 = 18t + \frac{1}{3}\left(\frac{1}{9}\right)\left(\frac{1}{4}t^4 - 216t\right)$$

or $$x(t) = \frac{1}{108}t^4 + 10t + 24 \quad \blacktriangleleft$$

and $$v = 18 + \frac{1}{3}\left(\frac{1}{9}\right)(t^3 - 216)$$

or $$v(t) = \frac{1}{27}t^3 + 10 \quad \blacktriangleleft$$

PROBLEM 11.12

The acceleration of a particle is defined by the relation $a = kt^2$. (*a*) Knowing that $v = -8$ m/s when $t = 0$ and that $v = +8$ m/s when $t = 2$ s, determine the constant *k*. (*b*) Write the equations of motion, knowing also that $x = 0$ when $t = 2$ s.

SOLUTION

$$a = kt^2 \qquad (1)$$

$$\frac{dv}{dt} = a = kt^2$$

$t = 0,\ v = -8$ m/s and $t = 2$ s, $v = +8$ ft/s

(*a*)
$$\int_{-8}^{8} dv = \int_0^2 kt^2\, dt$$

$$8 - (-8) = \frac{1}{3}k(2)^3 \qquad k = 6.00 \text{ m/s}^4 \blacktriangleleft$$

(*b*) Substituting $k = 6$ m/s^4 into (1)

$$\frac{dv}{dt} = a = 6t^2 \qquad a = 6t^2 \blacktriangleleft$$

$t = 0,\ v = -8$ m/s:
$$\int_{-8}^{v} dv = \int_0^t 6t^2\, dt$$

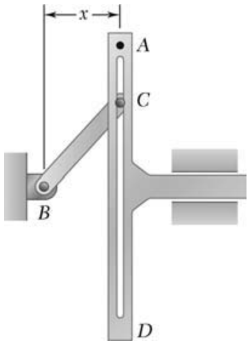
$$v - (-8) = \frac{1}{3}6(t)^3 \qquad v = 2t^3 - 8 \blacktriangleleft$$

$$\frac{dx}{dt} = v = 2t^3 - 8$$

$t = 2$ s, $x = 0$:
$$\int_0^x dx = \int_2^t (2t^3 - 8)dt; \quad x = \left|\frac{1}{2}t^4 - 8t\right|_2^t$$

$$x = \left[\frac{1}{2}t^4 - 8t\right] - \left[\frac{1}{2}(2)^4 - 8(2)\right]$$

$$x = \frac{1}{2}t^4 - 8t - 8 + 16 \qquad x = \frac{1}{2}t^4 - 8t + 8 \blacktriangleleft$$



PROBLEM 11.13

The acceleration of Point A is defined by the relation $a = -1.8 \sin kt$, where a and t are expressed in m/s^2 and seconds, respectively, and $k = 3$ rad/s. Knowing that $x = 0$ and $v = 0.6$ m/s when $t = 0$, determine the velocity and position of Point A when $t = 0.5$ s.

SOLUTION

Given: $a = -1.8 \sin kt \text{ m/s}^2, \quad v_0 = 0.6 \text{ m/s}, \quad x_0 = 0, \quad k = 3 \text{ rad/s}$

$$v - v_0 = \int_0^t a\,dt = -1.8 \int_0^t \sin kt\,dt = \frac{1.8}{k} \cos kt \Big|_0^t$$

$$v - 0.6 = \frac{1.8}{3}(\cos kt - 1) = 0.6 \cos kt - 0.6$$

Velocity: $v = 0.6 \cos kt \text{ m/s}$

$$x - x_0 = \int_0^t v\,dt = 0.6 \int_0^t \cos kt\,dt = \frac{0.6}{k} \sin kt \Big|_0^t$$

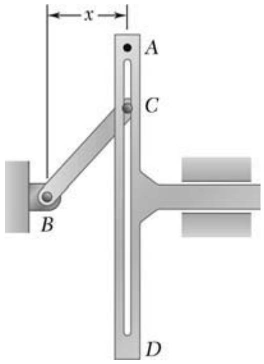
$$x - 0 = \frac{0.6}{3}(\sin kt - 0) = 0.2 \sin kt$$

Position: $x = 0.2 \sin kt \text{ m}$

When $t = 0.5$ s, $kt = (3)(0.5) = 1.5 \text{ rad}$

$v = 0.6 \cos 1.5 = 0.0424 \text{ m/s}$ $\qquad v = 42.4 \text{ mm/s}$ ◀

$x = 0.2 \sin 1.5 = 0.1995 \text{ m}$ $\qquad x = 199.5 \text{ mm}$ ◀



PROBLEM 11.14

The acceleration of Point A is defined by the relation $a = -1.08\sin kt - 1.44\cos kt$, where a and t are expressed in m/s^2 and seconds, respectively, and $k = 3$ rad/s. Knowing that $x = 0.16$ m and $v = 0.36$ m/s when $t = 0$, determine the velocity and position of Point A when $t = 0.5$ s.

SOLUTION

Given: $a = -1.08\sin kt - 1.44\cos kt \text{ m/s}^2, \qquad k = 3 \text{ rad/s}$

$$x_0 = 0.16 \text{ m}, \qquad v_0 = 0.36 \text{ m/s}$$

$$v - v_0 = \int_0^t a\,dt = -1.08\int_0^t \sin kt\,dt - 1.44\int_0^t \cos kt\,dt$$

$$v - 0.36 = \frac{1.08}{k}\cos kt\Big|_0^t - \frac{1.44}{k}\sin kt\Big|_0^t$$

$$= \frac{1.08}{3}(\cos kt - 1) - \frac{1.44}{3}(\sin kt - 0)$$

$$= 0.36\cos kt - 0.36 - 0.48\sin kt$$

Velocity: $v = 0.36\cos kt - 0.48\sin kt \text{ m/s}$

$$x - x_0 = \int_0^t v\,dt = 0.36\int_0^t \cos kt\,dt - 0.48\int_0^t \sin kt\,dt$$

$$x - 0.16 = \frac{0.36}{k}\sin kt\Big|_0^t + \frac{0.48}{k}\cos kt\Big|_0^t$$

$$= \frac{0.36}{3}(\sin kt - 0) + \frac{0.48}{3}(\cos kt - 1)$$

$$= 0.12\sin kt + 0.16\cos kt - 0.16$$

Position: $x = 0.12\sin kt + 0.16\cos kt \text{ m}$

When $t = 0.5$ s, $kt = (3)(0.5) = 1.5$ rad

$v = 0.36\cos 1.5 - 0.48\sin 1.5 = -0.453$ m/s $\qquad v = -453$ mm/s ◀

$x = 0.12\sin 1.5 + 0.16\cos 1.5 = 0.1310$ m $\qquad x = 131.0$ mm ◀

PROBLEM 11.15

A piece of electronic equipment that is surrounded by packing material is dropped so that it hits the ground with a speed of 4 m/s. After contact the equipment experiences an acceleration of $a = -kx$, where k is a constant and x is the compression of the packing material. If the packing material experiences a maximum compression of 20 mm, determine the maximum acceleration of the equipment.

SOLUTION

$$a = \frac{v\,dv}{dx} = -kx$$

Separate and integrate.

$$\int_{v_0}^{v_f} v\,dv = -\int_0^{x_f} kx\,dx$$

$$\frac{1}{2}v_f^2 - \frac{1}{2}v_0^2 = -\frac{1}{2}kx^2\Big|_0^{x_f} = -\frac{1}{2}kx_f^2$$

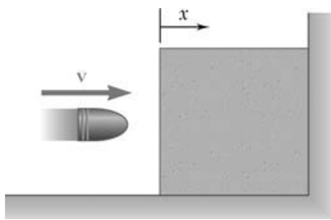
Use $v_0 = 4$ m/s, $x_f = 0.02$ m, and $v_f = 0$. Solve for k.

$$0 - \frac{1}{2}(4)^2 = -\frac{1}{2}k(0.02)^2 \qquad k = 40{,}000\ \text{s}^{-2}$$

Maximum acceleration.

$$a_{\max} = -kx_{\max}: \quad (-40{,}000)(0.02) = -800\ \text{m/s}^2$$

$$a = 800\ \text{m/s}^2 \uparrow \blacktriangleleft$$



PROBLEM 11.16

A projectile enters a resisting medium at $x = 0$ with an initial velocity $v_0 = 900$ ft/s and travels 4 in. before coming to rest. Assuming that the velocity of the projectile is defined by the relation $v = v_0 - kx$, where v is expressed in ft/s and x is in feet, determine (*a*) the initial acceleration of the projectile, (*b*) the time required for the projectile to penetrate 3.9 in. into the resisting medium.

SOLUTION

First note

When $x = \frac{4}{12}$ ft, $v = 0$: $\quad 0 = (900 \text{ ft/s}) - k\left(\frac{4}{12} \text{ ft}\right)$

or $\quad k = 2700 \frac{1}{\text{s}}$

(*a*) We have $\quad v = v_0 - kx$

Then $\quad a = \frac{dv}{dt} = \frac{d}{dt}(v_0 - kx) = -kv$

or $\quad a = -k(v_0 - kx)$

At $t = 0$: $\quad a = 2700 \frac{1}{\text{s}}(900 \text{ ft/s} - 0)$

or $\quad a_0 = -2.43 \times 10^6 \text{ ft/s}^2$ ◀

(*b*) We have $\quad \frac{dx}{dt} = v = v_0 - kx$

At $t = 0, x = 0$: $\quad \int_0^x \frac{dx}{v_0 - kx} = \int_0^t dt$

or $\quad -\frac{1}{k}[\ln(v_0 - kx)]_0^x = t$

or $\quad t = \frac{1}{k}\ln\left(\frac{v_0}{v_0 - kx}\right) = \frac{1}{k}\ln\left(\frac{1}{1 - \frac{k}{v_0}x}\right)$

When $x = 3.9$ in.: $\quad t = \frac{1}{2700 \frac{1}{\text{s}}}\ln\left[\frac{1}{1 - \frac{2700 \text{ 1/s}}{900 \text{ ft/s}}\left(\frac{3.9}{12} \text{ ft}\right)}\right]$

or $\quad t = 1.366 \times 10^{-3}$ s ◀

PROBLEM 11.17

The acceleration of a particle is defined by the relation $a = -k/x$. It has been experimentally determined that $v = 15$ ft/s when $x = 0.6$ ft and that $v = 9$ ft/s when $x = 1.2$ ft. Determine (*a*) the velocity of the particle when $x = 1.5$ ft, (*b*) the position of the particle at which its velocity is zero.

SOLUTION

$$a = \frac{v\,dv}{dx} = \frac{-k}{x}$$

Separate and integrate using $x = 0.6$ ft, $v = 15$ ft/s.

$$\int_{15}^{v} v\,dv = -k\int_{0.6}^{x} \frac{dx}{x}$$

$$\frac{1}{2}v^2\Big|_{15}^{v} = -k \ln x\Big|_{0.6}^{x}$$

$$\frac{1}{2}v^2 - \frac{1}{2}(15)^2 = -k \ln\left(\frac{x}{0.6}\right) \qquad (1)$$

When $v = 9$ ft/s, $x = 1.2$ ft

$$\frac{1}{2}(9)^2 - \frac{1}{2}(15)^2 = -k \ln\left(\frac{1.2}{0.6}\right)$$

Solve for k.

$$k = 103.874 \text{ ft}^2/\text{s}^2$$

(*a*) Velocity when $x = 65$ ft.

Substitute $k = 103.874 \text{ ft}^2/\text{s}^2$ and $x = 1.5$ ft into (1).

$$\frac{1}{2}v^2 - \frac{1}{2}(15)^2 = -103.874 \ln\left(\frac{1.5}{0.6}\right)$$

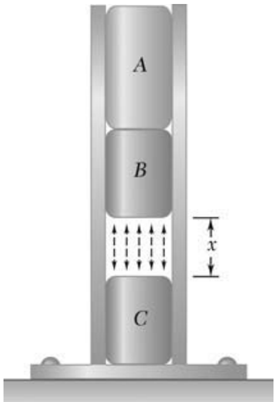
$v = 5.89$ ft/s ◀

(*b*) Position when for $v = 0$,

$$0 - \frac{1}{2}(15)^2 = -103.874 \ln\left(\frac{x}{0.6}\right)$$

$$\ln\left(\frac{x}{0.6}\right) = 1.083$$

$x = 1.772$ ft ◀



PROBLEM 11.18

A brass (nonmagnetic) block A and a steel magnet B are in equilibrium in a brass tube under the magnetic repelling force of another steel magnet C located at a distance $x = 0.004$ m from B. The force is inversely proportional to the square of the distance between B and C. If block A is suddenly removed, the acceleration of block B is $a = -9.81 + k/x^2$, where a and x are expressed in m/s^2 and m, respectively, and $k = 4\times 10^{-4}\ \text{m}^3/\text{s}^2$. Determine the maximum velocity and acceleration of B.

SOLUTION

The maximum veolocity occurs when $a = 0$. $\quad 0 = -9.81 + \dfrac{k}{x_m^2}$

$$x_m^2 = \frac{k}{9.81} = \frac{4\times 10^{-4}}{9.81} = 40.775\times 10^{-6}\ \text{m}^2 \qquad x_m = 0.0063855\ \text{m}$$

The acceleration is given as a function of x.

$$v\frac{dv}{dx} = a = -9.81 + \frac{k}{x^2}$$

Separate variables and integrate:

$$vdv = -9.81dx + \frac{k\,dx}{x^2}$$

$$\int_0^v vdv = -9.81\int_{x_0}^{x} dx + k\int_{x_0}^{x} \frac{dx}{x^2}$$

$$\frac{1}{2}v^2 = -9.81(x - x_0) - k\left(\frac{1}{x} - \frac{1}{x_0}\right)$$

$$\frac{1}{2}v_m^2 = -9.81(x_m - x_0) - k\left(\frac{1}{x_m} - \frac{1}{x_0}\right)$$

$$= -9.81(0.0063855 - 0.004) - (4\times 10^{-4})\left(\frac{1}{0.0063855} - \frac{1}{0.004}\right)$$

$$= -0.023402 + 0.037358 = 0.013956\ \text{m}^2/\text{s}^2$$

Maximum velocity: $\quad v_m = 0.1671\ \text{m/s} \qquad v_m = 167.1\ \text{mm/s}\ \uparrow$ ◀

The maximum acceleration occurs when x is smallest, that is, $x = 0.004$ m.

$$a_m = -9.81 + \frac{4\times 10^{-4}}{(0.004)^2} \qquad a_m = 15.19\ \text{m/s}^2\ \uparrow$$ ◀

PROBLEM 11.19

Based on experimental observations, the acceleration of a particle is defined by the relation $a = -(0.1 + \sin x/b)$, where a and x are expressed in m/s^2 and meters, respectively. Knowing that $b = 0.8$ m and that $v = 1$ m/s when $x = 0$, determine (*a*) the velocity of the particle when $x = -1$ m, (*b*) the position where the velocity is maximum, (*c*) the maximum velocity.

SOLUTION

We have $$v\frac{dv}{dx} = a = -\left(0.1 + \sin\frac{x}{0.8}\right)$$

When $x = 0,\ v = 1$ m/s: $$\int_1^v v\,dv = \int_0^x -\left(0.1 + \sin\frac{x}{0.8}\right)dx$$

or $$\frac{1}{2}(v^2 - 1) = -\left[0.1x - 0.8\cos\frac{x}{0.8}\right]_0^x$$

or $$\frac{1}{2}v^2 = -0.1x + 0.8\cos\frac{x}{0.8} - 0.3$$

(*a*) When $x = -1$ m: $$\frac{1}{2}v^2 = -0.1(-1) + 0.8\cos\frac{-1}{0.8} - 0.3$$

or $v = \pm 0.323$ m/s ◀

(*b*) When $v = v_{\max},\ a = 0$: $$-\left(0.1 + \sin\frac{x}{0.8}\right) = 0$$

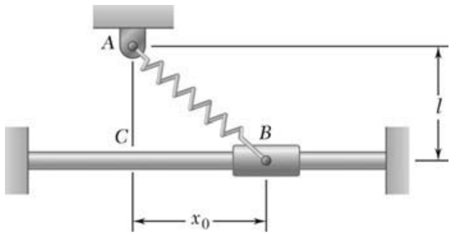
or $x = -0.080\,134$ m $\qquad x = -0.0801$ m ◀

(*c*) When $x = -0.080\,134$ m:

$$\frac{1}{2}v_{\max}^2 = -0.1(-0.080\,134) + 0.8\cos\frac{-0.080\,134}{0.8} - 0.3$$
$$= 0.504\ \text{m}^2/\text{s}^2$$

or $v_{\max} = 1.004$ m/s ◀

PROBLEM 11.20



A spring AB is attached to a support at A and to a collar. The unstretched length of the spring is l. Knowing that the collar is released from rest at $x = x_0$ and has an acceleration defined by the relation $a = -100(x - lx/\sqrt{l^2 + x^2})$, determine the velocity of the collar as it passes through Point C.

SOLUTION

Since a is function of x,

$$a = v\frac{dv}{dx} = -100\left(x - \frac{lx}{\sqrt{l^2 + x^2}}\right)$$

Separate variables and integrate:

$$\int_{v_0}^{v_f} v\,dv = -100\int_{x_0}^{0}\left(x - \frac{lx}{\sqrt{l^2 + x^2}}\right)dx$$

$$\frac{1}{2}v_f^2 - \frac{1}{2}v_0^2 = -100\left(\frac{x^2}{2} - l\sqrt{l^2 + x^2}\right)\Bigg|_{x_0}^{0}$$

$$\frac{1}{2}v_f^2 - 0 = -100\left(-\frac{x_0^2}{2} - l^2 + l\sqrt{l^2 + x_0^2}\right)$$

$$\frac{1}{2}v_f^2 = \frac{100}{2}(-l^2 + x_0^2 - l^2 - 2l\sqrt{l^2 + x_0^2})$$

$$= \frac{100}{2}(\sqrt{l^2 + x_0^2} - l)^2$$

$$v_f = 10(\sqrt{l^2 + x_0^2} - l) \blacktriangleleft$$

PROBLEM 11.21

The acceleration of a particle is defined by the relation $a = -0.8v$ where a is expressed in m/s^2 and v in m/s. Knowing that at $t = 0$ the velocity is 1 m/s, determine (*a*) the distance the particle will travel before coming to rest, (*b*) the time required for the particle's velocity to be reduced by 50 percent of its initial value.

SOLUTION

(*a*) Determine relationship between *x* and *v*.

$$a = \frac{vdv}{dx} = -0.8v \qquad dv = -0.8dx$$

Separate and integrate with $v = 1$ m/s when $x = 0$.

$$\int_1^v dv = -0.8\int_0^x dx$$

$$v - 1 = -0.8x$$

Distance traveled.

For $v = 0$, $x = \frac{-1}{-0.8} \Rightarrow$ $x = 1.25$ m ◀

(*b*) Determine realtionship between *v* and *t*.

$$a = \frac{dv}{dt} = 0.8v$$

$$\int_1^v \frac{dv}{v} = -\int_0^x 0.8dt$$

$$\ln\left(\frac{v}{1}\right) = -0.8t \qquad t = 1.25\ln\left(\frac{1}{v}\right)$$

For $v = 0.5(1 \text{ m/s}) = 0.5$ m/s,

$$t = 1.25\ln\left(\frac{1}{0.5}\right)$$ $t = 0.866$ s ◀

PROBLEM 11.22

Starting from $x = 0$ with no initial velocity, a particle is given an acceleration $a = 0.1\sqrt{v^2 + 16}$, where a and v are expressed in ft/s^2 and ft/s, respectively. Determine (*a*) the position of the particle when $v = 3$ft/s, (*b*) the speed and acceleration of the particle when $x = 4$ ft.

SOLUTION

$$a = \frac{vdv}{dx} = 0.1(v^2 + 16)^{1/2} \qquad (1)$$

Separate and integrate.

$$\int_0^v \frac{vdv}{\sqrt{v^2 + 16}} = \int_0^x 0.1\, dx$$

$$(v^2 + 16)^{1/2}\Big|_0^v = 0.1x$$

$$(v^2 + 16)^{1/2} - 4 = 0.1x$$

$$x = 10[(v^2 + 16)^{1/2} - 4] \qquad (2)$$

(*a*) $v = 3$ ft/s.

$$x = 10[(3^2 + 16)^{1/2} - 4]$$

$x = 10.00$ ft ◀

(*b*) $x = 4$ ft.

From (2), $(v^2 + 16)^{1/2} = 4 + 0.1x = 4 + (0.1)(4) = 4.4$

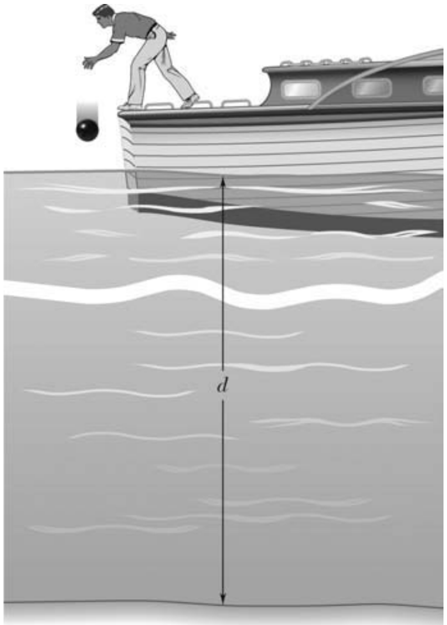
$$v^2 + 16 = 19.36$$

$$v^2 = 3.36\,\text{ft}^2/\text{s}^2$$

$v = 1.833$ ft/s ◀

From (1), $a = 0.1(1.833^2 + 16)^{1/2}$

$a = 0.440\ \text{ft/s}^2$ ◀



PROBLEM 11.23

A ball is dropped from a boat so that it strikes the surface of a lake with a speed of 16.5 ft/s. While in the water the ball experiences an acceleration of $a = 10 - 0.8v,$ where a and v are expressed in ft/s^2 and ft/s, respectively. Knowing the ball takes 3 s to reach the bottom of the lake, determine (*a*) the depth of the lake, (*b*) the speed of the ball when it hits the bottom of the lake.

SOLUTION

$$a = \frac{dv}{dt} = 10 - 0.8v$$

Separate and integrate:

$$\int_{v_0}^{v} \frac{dv}{10 - 0.8v} = \int_0^t dt$$

$$-\frac{1}{0.8} \ln(10 - 0.8v)\Big|_{v_0}^{v} = t$$

$$\ln\left(\frac{10 - 0.8v}{10 - 0.8v_0}\right) = -0.8t$$

$$10 - 0.8v = (10 - 0.8v_0)e^{-0.8t}$$

or $$0.8v = 10 - (10 - 0.8v_0)e^{-0.8t}$$

$$v = 12.5 - (12.5 - v_0)e^{-0.8t}$$

With $v_0 = 16.5\,\text{ft/s}$ $$v = 12.5 + 4e^{-0.8t}$$

PROBLEM 11.23 (Continued)

Integrate to determine x as a function of t.

$$v = \frac{dx}{dt} = 12.5 + 4e^{-0.8t}$$

$$\int_0^x dx = \int_0^t (12.5 + 4e^{-0.8t})dt$$

$$x = 12.5t - 5e^{-0.8t}\Big|_0^t = 12.5t - 5e^{-0.8t} + 5$$

(a) At $t = 35$ s,

$x = 12.5(3) - 5e^{-2.4} + 5 = 42.046$ ft $\qquad x = 42.0$ ft ◀

(b) $v = 12.5 + 4e^{-2.4} = 12.863$ ft/s $\qquad v = 12.86$ ft/s ◀

PROBLEM 11.24

The acceleration of a particle is defined by the relation $a = -k\sqrt{v}$, where k is a constant. Knowing that $x = 0$ and $v = 81$ m/s at $t = 0$ and that $v = 36$ m/s when $x = 18$ m, determine (a) the velocity of the particle when $x = 20$ m, (b) the time required for the particle to come to rest.

SOLUTION

(a) We have $$v\frac{dv}{dx} = a = -k\sqrt{v}$$

so that $$\sqrt{v}\, dv = -k\, dx$$

When $x = 0, v = 81$ m/s: $$\int_{81}^{v}\sqrt{v}\, dv = \int_{0}^{x} -k\, dx$$

or $$\frac{2}{3}[v^{3/2}]_{81}^{v} = -kx$$

or $$\frac{2}{3}[v^{3/2} - 729] = -kx$$

When $x = 18$ m, $v = 36$ m/s: $$\frac{2}{3}(36^{3/2} - 729) = -k(18)$$

or $$k = 19\sqrt{\text{m/s}^2}$$

Finally

When $x = 20$ m: $$\frac{2}{3}(v^{3/2} - 729) = -19(20)$$

or $$v^{3/2} = 159$$

$v = 29.3$ m/s ◀

(b) We have $$\frac{dv}{dt} = a = -19\sqrt{v}$$

At $t = 0, v = 81$ m/s: $$\int_{81}^{v}\frac{dv}{\sqrt{v}} = \int_{0}^{t} -19dt$$

or $$2[\sqrt{v}]_{81}^{v} = -19t$$

or $$2(\sqrt{v} - 9) = -19t$$

When $v = 0$: $$2(-9) = -19t$$

or $t = 0.947$ s ◀

PROBLEM 11.25

A particle is projected to the right from the position $x = 0$ with an initial velocity of 9 m/s. If the acceleration of the particle is defined by the relation $a = -0.6v^{3/2}$, where a and v are expressed in m/s^2 and m/s, respectively, determine (*a*) the distance the particle will have traveled when its velocity is 4 m/s, (*b*) the time when $v = 1$ m/s, (*c*) the time required for the particle to travel 6 m.

SOLUTION

(*a*) We have $$v\frac{dv}{dx} = a = -0.6v^{3/2}$$

When $x = 0,\ v = 9$ m/s: $$\int_9^v v^{-(1/2)}dv = \int_0^x -0.6dx$$

or $$2[v^{1/2}]_9^v = -0.6x$$

or $$x = \frac{1}{0.3}(3 - v^{1/2}) \qquad (1)$$

When $v = 4$ m/s: $$x = \frac{1}{0.3}(3 - 4^{1/2})$$

or $x = 3.33$ m ◀

(*b*) We have $$\frac{dv}{dt} = a = -0.6v^{3/2}$$

When $t = 0,\ v = 9$ m/s: $$\int_9^v v^{-(3/2)}dv = \int_0^t -0.6dt$$

or $$-2[v^{-(1/2)}]_9^v = -0.6t$$

or $$\frac{1}{\sqrt{v}} - \frac{1}{3} = 0.3t$$

When $v = 1$ m/s: $$\frac{1}{\sqrt{1}} - \frac{1}{3} = 0.3t$$

or $t = 2.22$ s ◀

(*c*) We have $$\frac{1}{\sqrt{v}} - \frac{1}{3} = 0.3t$$

or $$v = \left(\frac{3}{1 + 0.9t}\right)^2 = \frac{9}{(1 + 0.9t)^2}$$

Now $$\frac{dx}{dt} = v = \frac{9}{(1 + 0.9t)^2}$$

At $t = 0,\ x = 0$: $$\int_0^x dx = \int_0^t \frac{9}{(1+0.9t)^2}\,dt$$

or $$x = 9\left[-\frac{1}{0.9}\frac{1}{1+0.9t}\right]_0^t$$

$$= 10\left(1 - \frac{1}{1+0.9t}\right)$$

$$= \frac{9t}{1+0.9t}$$

When $x = 6$ m: $$6 = \frac{9t}{1+0.9t}$$

or $t = 1.667$ s ◀

An alternative solution is to begin with Eq. (1).

$$x = \frac{1}{0.3}(3 - v^{1/2})$$

Then $$\frac{dx}{dt} = v = (3 - 0.3x)^2$$

Now

At $t = 0,\ x = 0$: $$\int_0^x \frac{dx}{(3-0.3x)^2} = \int_0^t dt$$

or $$t = \frac{1}{0.3}\left[\frac{1}{3-0.3x}\right]_0^x = \frac{x}{9-0.9x}$$

which leads to the same equation as above.

PROBLEM 11.26

The acceleration of a particle is defined by the relation $a = 0.4(1 - kv)$, where k is a constant. Knowing that at $t = 0$ the particle starts from rest at $x = 4$ m and that when $t = 15$ s, $v = 4$ m/s, determine (a) the constant k, (b) the position of the particle when $v = 6$ m/s, (c) the maximum velocity of the particle.

SOLUTION

(a) We have $$\frac{dv}{dt} = a = 0.4(1 - kv)$$

At $t = 0$, $v = 0$: $$\int_0^v \frac{dv}{1 - kv} = \int_0^t 0.4dt$$

or $$-\frac{1}{k}[\ln(1 - kv)]_0^v = 0.4t$$

or $$\ln(1 - kv) = -0.4kt \qquad (1)$$

At $t = 15$ s, $v = 4$ m/s: $$\ln(1 - 4k) = -0.4k(15)$$
$$= -6k$$

Solving yields $$k = 0.145703 \text{ s/m}$$

or $k = 0.1457$ s/m ◀

(b) We have $$v\frac{dv}{dx} = a = 0.4(1 - kv)$$

When $x = 4$ m, $v = 0$: $$\int_0^v \frac{vdv}{1 - kv} = \int_4^x 0.4dx$$

Now $$\frac{v}{1 - kv} = -\frac{1}{k} + \frac{1/k}{1 - kv}$$

Then $$\int_0^v \left[-\frac{1}{k} + \frac{1}{k(1 - kv)}\right] dv = \int_4^x 0.4dx$$

or $$\left[-\frac{v}{k} - \frac{1}{k^2}\ln(1 - kv)\right]_0^v = 0.4[x]_4^x$$

or $$-\left[\frac{v}{k} + \frac{1}{k^2}\ln(1 - kv)\right] = 0.4(x - 4)$$

When $v = 6$ m/s: $$-\left[\frac{6}{0.145\,703} + \frac{1}{(0.145\,703)^2}\ln(1 - 0.145\,703 \times 6)\right] = 0.4(x - 4)$$

or $$0.4(x - 4) = 56.4778$$

or $x = 145.2$ m ◀

PROBLEM 11.26 (Continued)

(*c*) The maximum velocity occurs when $a = 0$.

$$a = 0: \quad 0.4(1 - k v_{max}) = 0$$

or $$v_{max} = \frac{1}{0.145\,703}$$

or $$v_{max} = 6.86 \text{ m/s} \blacktriangleleft$$

An alternative solution is to begin with Eq. (1).

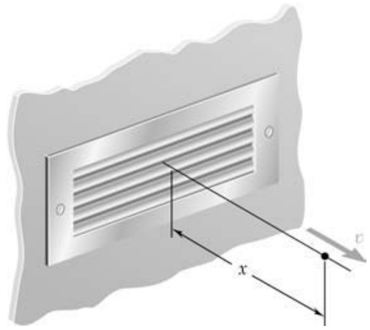
$$\ln(1 - kv) = -0.4kt$$

Then $$v = \frac{1}{k}(1 - k^{-0.4kt})$$

Thus, v_{max} is attained as $t \longrightarrow \infty$

$$v_{max} = \frac{1}{k}$$

as above.



PROBLEM 11.27

Experimental data indicate that in a region downstream of a given louvered supply vent the velocity of the emitted air is defined by $v = 0.18v_0/x$, where v and x are expressed in m/s and meters, respectively, and v_0 is the initial discharge velocity of the air. For $v_0 = 3.6$ m/s, determine (*a*) the acceleration of the air at $x = 2$ m, (*b*) the time required for the air to flow from $x = 1$ to $x = 3$ m.

SOLUTION

(*a*) We have

$$a = v\frac{dv}{dx}$$

$$= \frac{0.18v_0}{x}\frac{d}{dx}\left(\frac{0.18v_0}{x}\right)$$

$$= -\frac{0.0324v_0^2}{x^3}$$

When $x = 2$ m:

$$a = -\frac{0.0324(3.6)^2}{(2)^3}$$

or $a = -0.0525 \text{ m/s}^2$ ◀

(*b*) We have

$$\frac{dx}{dt} = v = \frac{0.18v_0}{x}$$

From $x = 1$ m to $x = 3$ m:

$$\int_1^3 x dx = \int_{t_1}^{t_3} 0.18v_0\, dt$$

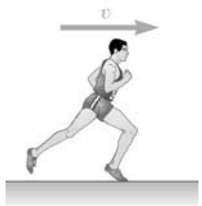
or

$$\left[\frac{1}{2}x^2\right]_1^3 = 0.18v_0(t_3 - t_1)$$

or

$$(t_3 - t_1) = \frac{\frac{1}{2}(9-1)}{0.18(3.6)}$$

or $t_3 - t_1 = 6.17$ s ◀



PROBLEM 11.28

Based on observations, the speed of a jogger can be approximated by the relation $v = 7.5(1-0.04x)^{0.3}$, where v and x are expressed in mi/h and miles, respectively. Knowing that $x = 0$ at $t = 0$, determine (*a*) the distance the jogger has run when $t = 1$ h, (*b*) the jogger's acceleration in ft/s^2 at $t = 0$, (*c*) the time required for the jogger to run 6 mi.

SOLUTION

(*a*) We have
$$\frac{dx}{dt} = v = 7.5(1-0.04x)^{0.3}$$

At $t = 0,\ x = 0$:
$$\int_0^x \frac{dx}{(1-0.04x)^{0.3}} = \int_0^t 7.5dt$$

or
$$\frac{1}{0.7}\left(-\frac{1}{0.04}\right)[(1-0.04x)^{0.7}]_0^x = 7.5t$$

or
$$1-(1-0.04x)^{0.7} = 0.21t \qquad (1)$$

or
$$x = \frac{1}{0.04}[1-(1-0.21t)^{1/0.7}]$$

At $t = 1$ h:
$$x = \frac{1}{0.04}\{1-[1-0.21(1)]^{1/0.7}\}$$

or
$$x = 7.15 \text{ mi} \blacktriangleleft$$

(*b*) We have
$$a = v\frac{dv}{dx}$$
$$= 7.5(1-0.04x)^{0.3}\frac{d}{dx}[7.5(1-0.04x)^{0.3}]$$
$$= 7.5^2(1-0.04x)^{0.3}[(0.3)(-0.04)(1-0.04x)^{-0.7}]$$
$$= -0.675(1-0.04x)^{-0.4}$$

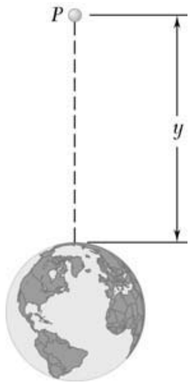
At $t = 0,\ x = 0$:
$$a_0 = -0.675 \text{ mi/h}^2 \times \frac{5280 \text{ ft}}{1 \text{ mi}} \times \left(\frac{1 \text{ h}}{3600 \text{ s}}\right)^2$$

or
$$a_0 = -275\times 10^{-6} \text{ ft/s}^2 \blacktriangleleft$$

(*c*) From Eq. (1)
$$t = \frac{1}{0.21}[1-(1-0.04x)^{0.7}]$$

When $x = 6$ mi:
$$t = \frac{1}{0.21}\{1-[1-0.04(6)]^{0.7}\}$$
$$= 0.83229 \text{ h}$$

or
$$t = 49.9 \text{ min} \blacktriangleleft$$



PROBLEM 11.29

The acceleration due to gravity at an altitude y above the surface of the earth can be expressed as

$$a = \frac{-32.2}{[1 + (y/20.9 \times 10^6)]^2}$$

where a and y are expressed in ft/s^2 and feet, respectively. Using this expression, compute the height reached by a projectile fired vertically upward from the surface of the earth if its initial velocity is (a) 1800 ft/s, (b) 3000 ft/s, (c) 36,700 ft/s.

SOLUTION

We have $$v\frac{dv}{dy} = a = -\frac{32.2}{\left(1 + \frac{y}{20.9 \times 10^6}\right)^2}$$

When $$y = 0, \quad v = v_0$$

provided that v does reduce to zero, $$y = y_{max}, \quad v = 0$$

Then $$\int_{v_0}^{0} v\,dv = \int_{0}^{y_{max}} \frac{-32.2}{\left(1 + \frac{y}{20.9 \times 10^6}\right)^2} dy$$

or $$-\frac{1}{2}v_0^2 = -32.2\left[-20.9 \times 10^6 \frac{1}{1 + \frac{y}{20.9 \times 10^6}}\right]_0^{y_{max}}$$

or $$v_0^2 = 1345.96 \times 10^6 \left(1 - \frac{1}{1 + \frac{y_{max}}{20.9 \times 10^6}}\right)$$

or $$y_{max} = \frac{v_0^2}{64.4 - \frac{v_0^2}{20.9 \times 10^6}}$$

(a) $v_0 = 1800$ ft/s: $$y_{max} = \frac{(1800)^2}{64.4 - \frac{(1800)^2}{20.9 \times 10^6}}$$

or $$y_{max} = 50.4 \times 10^3 \text{ ft} \blacktriangleleft$$

(b) $v_0 = 3000$ ft/s: $$y_{max} = \frac{(3000)^2}{64.4 - \frac{(3000)^2}{20.9 \times 10^6}}$$

or $$y_{max} = 140.7 \times 10^3 \text{ ft} \blacktriangleleft$$

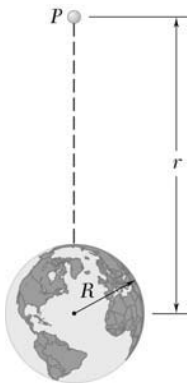
PROBLEM 11.29 (Continued)

(*c*) $v_0 = 36,700$ ft/s: $$y_{\max} = \frac{(36,700)^2}{64.4 - \frac{(36,700)^2}{20.9\times 10^6}} = -3.03\times 10^{10} \text{ ft}$$

This solution is invalid since the velocity does not reduce to zero. The velocity 36,700 ft/s is above the escape velocity v_R from the earth. For v_R and above.

$$y_{\max} \longrightarrow \infty \blacktriangleleft$$

PROBLEM 11.30



The acceleration due to gravity of a particle falling toward the earth is $a = -gR^2/r^2$, where r is the distance from the *center* of the earth to the particle, R is the radius of the earth, and g is the acceleration due to gravity at the surface of the earth. If $R = 3960$ mi, calculate the *escape velocity,* that is, the minimum velocity with which a particle must be projected vertically upward from the surface of the earth if it is not to return to the earth. (*Hint:* $v = 0$ for $r = \infty$.)

SOLUTION

We have $$v\frac{dv}{dr} = a = -\frac{gR^2}{r^2}$$

When $$r = R, \quad v = v_e$$
$$r = \infty, \quad v = 0$$

then $$\int_{v_e}^{0} v\,dv = \int_R^\infty -\frac{gR^2}{r^2}\,dr$$

or $$-\frac{1}{2}v_e^2 = gR^2\left[\frac{1}{r}\right]_R^\infty$$

or $$v_e = \sqrt{2gR}$$
$$= \left(2 \times 32.2 \text{ ft/s}^2 \times 3960 \text{ mi} \times \frac{5280 \text{ ft}}{1 \text{ mi}}\right)^{1/2}$$

or $$v_e = 36.7 \times 10^3 \text{ ft/s} \blacktriangleleft$$

PROBLEM 11.31

The velocity of a particle is $v = v_0[1 - \sin(\pi t/T)]$. Knowing that the particle starts from the origin with an initial velocity v_0, determine (*a*) its position and its acceleration at $t = 3T$, (*b*) its average velocity during the interval $t = 0$ to $t = T$.

SOLUTION

(*a*) We have

$$\frac{dx}{dt} = v = v_0\left[1 - \sin\left(\frac{\pi t}{T}\right)\right]$$

At $t = 0,\ x = 0$:

$$\int_0^x dx = \int_0^t v_0\left[1 - \sin\left(\frac{\pi t}{T}\right)\right]dt$$

$$x = v_0\left[t + \frac{T}{\pi}\cos\left(\frac{\pi t}{T}\right)\right]_0^t = v_0\left[t + \frac{T}{\pi}\cos\left(\frac{\pi t}{T}\right) - \frac{T}{\pi}\right] \qquad (1)$$

At $t = 3T$:

$$x_{3T} = v_0\left[3T + \frac{T}{\pi}\cos\left(\frac{\pi \times 3T}{T}\right) - \frac{T}{\pi}\right] = v_0\left(3T - \frac{2T}{\pi}\right) \qquad x_{3T} = 2.36\, v_0 T \ ◀$$

$$a = \frac{dv}{dt} = \frac{d}{dt}\left\{v_0\left[1 - \sin\left(\frac{\pi t}{T}\right)\right]\right\} = -v_0\frac{\pi}{T}\cos\frac{\pi t}{T}$$

At $t = 3T$:

$$a_{3T} = -v_0\frac{\pi}{T}\cos\frac{\pi \times 3T}{T} \qquad a_{3T} = \frac{\pi v_0}{T} \ ◀$$

(*b*) Using Eq. (1)

At $t = 0$:

$$x_0 = v_0\left[0 + \frac{T}{\pi}\cos(0) - \frac{T}{\pi}\right] = 0$$

At $t = T$:

$$x_T = v_0\left[T + \frac{T}{\pi}\cos\left(\frac{\pi T}{T}\right) - \frac{T}{\pi}\right] = v_0\left(T - \frac{2T}{\pi}\right) = 0.363 v_0 T$$

Now

$$v_{\text{ave}} = \frac{x_T - x_0}{\Delta t} = \frac{0.363 v_0 T - 0}{T - 0} \qquad v_{\text{ave}} = 0.363 v_0 \ ◀$$

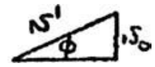
PROBLEM 11.32

The velocity of a slider is defined by the relation $v = v' \sin(\omega_n t + \phi)$. Denoting the velocity and the position of the slider at $t = 0$ by v_0 and x_0, respectively, and knowing that the maximum displacement of the slider is $2x_0$, show that (*a*) $v' = (v_0^2 + x_0^2\omega_n^2)/2x_0\omega_n$, (*b*) the maximum value of the velocity occurs when $x = x_0[3 - (v_0/x_0\omega_n)^2]/2$.

SOLUTION

(*a*) At $t = 0, v = v_0$: $\quad v_0 = v' \sin(0 + \phi) = v' \sin\phi$

Then $\quad \cos\phi = \sqrt{v'^2 - v_0^2}\Big/v'$



Now $\quad \dfrac{dx}{dt} = v = v' \sin(\omega_n t + \phi)$

At $t = 0, x = x_0$: $\quad \displaystyle\int_{x0}^{x} dx = \int_0^t v' \sin(\omega_n t + \phi)dt$

or $$x - x_0 = v'\left[-\frac{1}{\omega_n}\cos(\omega_n t + \phi)\right]_0^t$$

or $$x = x_0 + \frac{v'}{\omega_n}[\cos\phi - \cos(\omega_n t + \phi)]$$

Now observe that x_{max} occurs when $\cos(\omega_n t + \phi) = -1$. Then

$$x_{max} = 2x_0 = x_0 + \frac{v'}{\omega_n}[\cos\phi - (-1)]$$

Substituting for $\cos\phi$ $$x_0 = \frac{v'}{\omega_n}\left(\frac{\sqrt{v'^2 - v_0^2}}{v^1} + 1\right)$$

or $$x_0\omega_n - v' = \sqrt{v'^2 - v_0^2}$$

Squaring both sides of this equation

$$x_0^2\omega_n^2 - 2x_0\omega_n + v'^2 = v'^2 - v_0^2$$

or $$v' = \frac{v_0^2 + x_0^2\omega_n^2}{2x_0\omega_n} \qquad \text{Q. E. D.}$$

PROBLEM 11.32 (Continued)

(*b*) First observe that v_{max} occurs when $\omega_n t + \phi = \frac{\pi}{2}$. The corresponding value of x is

$$x_{v_{max}} = x_0 + \frac{v'}{\omega_n}\left[\cos\phi - \cos\left(\frac{\pi}{2}\right)\right]$$

$$= x_0 + \frac{v'}{\omega_n}\cos\phi$$

Substituting first for $\cos\phi$ and then for v'

$$x_{v_{max}} = x_0 + \frac{v'}{\omega_n}\frac{\sqrt{v'^2 - v_0^2}}{v'}$$

$$= x_0 + \frac{1}{\omega_n}\left[\left(\frac{v_0^2 + x_0^2\omega_n^2}{2x_0\omega_n}\right)^2 - v_0^2\right]^{1/2}$$

$$= x_0 + \frac{1}{2x_0\omega_n^2}\left(v_0^4 + 2v_0^2x_0^2\omega_n^2 + x_0^4\omega_n^4 - 4x_0^2\omega_n^2v_0^2\right)^{1/2}$$

$$= x_0 + \frac{1}{2x_0\omega_n^2}\left[\left(x_0^2\omega_n^2 - v_0^2\right)^2\right]^{1/2}$$

$$= x_0 + \frac{x_0^2\omega_n^2 - v_0^2}{2x_0\omega_n^2}$$

$$= \frac{x_0}{2}\left[3 - \left(\frac{v_0}{x_0\omega_n}\right)^2\right] \qquad \text{Q. E. D.}$$

PROBLEM 11.33

A stone is thrown vertically upward from a point on a bridge located 40 m above the water. Knowing that it strikes the water 4 s after release, determine (*a*) the speed with which the stone was thrown upward, (*b*) the speed with which the stone strikes the water.

SOLUTION

Uniformly accelerated motion. Origin at water. $+\uparrow$

$$y = y_0 + v_0 t + \frac{1}{2} a t^2$$

$$v = v_0 + a t$$

where $y_0 = 40 \text{ m}$ and $a = -9.81 \text{ m/s}^2$.

(*a*) *Initial speed.*

$$y = 0 \text{ when } t = 4 \text{ s.}$$

$$0 = 40 + v_0(4) - \frac{1}{2}(9.81)(4)^2$$

$$v_0 = 9.62 \text{ m/s}$$

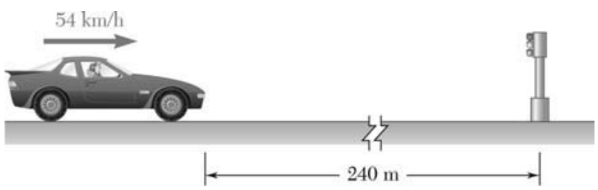
$\mathbf{v}_0 = 9.62 \text{ m/s} \uparrow$ ◀

(*b*) *Speed when striking the water.* (v at $t = 4$ s)

$$v = 9.62 - (9.81)(4) = -29.62 \text{ m/s}$$

$\mathbf{v} = 29.6 \text{ m/s} \downarrow$ ◀

PROBLEM 11.34



A motorist is traveling at 54 km/h when she observes that a traffic light 240 m ahead of her turns red. The traffic light is timed to stay red for 24 s. If the motorist wishes to pass the light without stopping just as it turns green again, determine (*a*) the required uniform deceleration of the car, (*b*) the speed of the car as it passes the light.

SOLUTION

Uniformly accelerated motion:

$$x_0 = 0 \quad v_0 = 54 \text{ km/h} = 15 \text{ m/s}$$

(a) $x = x_0 + v_0 t + \frac{1}{2} a t^2$

when $t = 24 \text{ s}$, $x = 240 \text{ m}$:

$$240 \text{ m} = 0 + (15 \text{ m/s})(24 \text{ s}) + \frac{1}{2} a (24 \text{ s})^2$$

$$a = -0.4167 \text{ m/s}^2 \qquad a = -0.417 \text{ m/s}^2 \blacktriangleleft$$

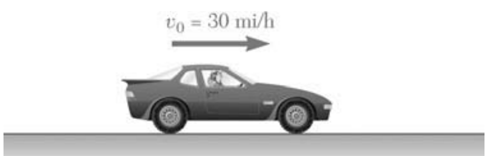
(*b*) $v = v_0 + a t$

when $t = 24 \text{ s}$:

$$v = (15 \text{ m/s}) + (-0.4167 \text{ m/s})(24 \text{ s})$$

$$v = 5.00 \text{ m/s}$$

$$v = 18.00 \text{ km/h} \qquad v = 18.00 \text{ km/h} \blacktriangleleft$$



PROBLEM 11.35

A motorist enters a freeway at 30 mi/h and accelerates uniformly to 60 mi/h. From the odometer in the car, the motorist knows that she traveled 550 ft while accelerating. Determine (*a*) the acceleration of the car, (*b*) the time required to reach 60 mi/h.

SOLUTION

(*a*) Acceleration of the car.

$$v_1^2 = v_0^2 + 2a(x_1 - x_0)$$

$$a = \frac{v_1^2 - v_0^2}{2(x_1 - x_0)}$$

Data:

$$v_0 = 30 \text{ mi/h} = 44 \text{ ft/s}$$

$$v_1 = 60 \text{ mi/h} = 88 \text{ ft/s}$$

$$x_0 = 0$$

$$x_1 = 550 \text{ ft}$$

$$a = \frac{(88)^2 - (44)^2}{(2)(55 - 0)}$$

$a = 5.28 \text{ ft/s}^2$ ◀

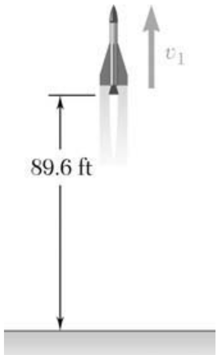
(*b*) Time to reach 60 mi/h.

$$v_1 = v_0 + a(t_1 - t_0)$$

$$t_1 - t_0 = \frac{v_1 - v_0}{a} = \frac{88 - 44}{5.28} = 8.333 \text{ s}$$

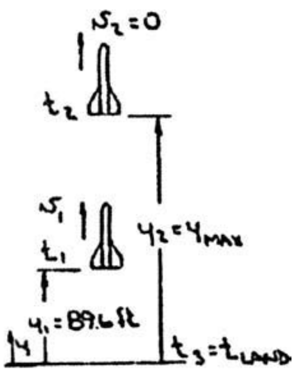
$t_1 - t_0 = 8.33 \text{ s}$ ◀

PROBLEM 11.36



A group of students launches a model rocket in the vertical direction. Based on tracking data, they determine that the altitude of the rocket was 89.6 ft at the end of the powered portion of the flight and that the rocket landed 16 s later. Knowing that the descent parachute failed to deploy so that the rocket fell freely to the ground after reaching its maximum altitude and assuming that $g = 32.2 \text{ ft/s}^2$, determine (*a*) the speed v_1 of the rocket at the end of powered flight, (*b*) the maximum altitude reached by the rocket.

SOLUTION



(*a*) We have $\quad y = y_1 + v_1 t + \frac{1}{2}at^2$

At t_{land}, $\quad y = 0$

Then
$$0 = 89.6 \text{ ft} + v_1(16 \text{ s}) + \frac{1}{2}(-32.2 \text{ ft/s}^2)(16 \text{ s})^2$$

or $\quad v_1 = 252 \text{ ft/s}$ ◀

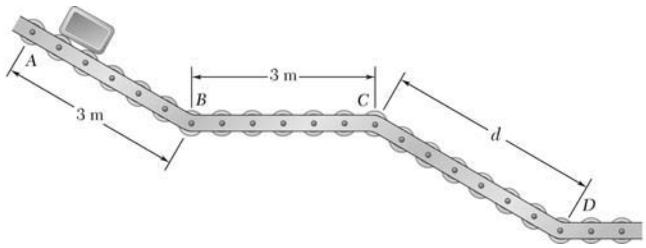
(*b*) We have $\quad v^2 = v_1^2 + 2a(y - y_1)$

At $\quad y = y_{\max}, \quad v = 0$

Then
$$0 = (252 \text{ ft/s})^2 + 2(-32.2 \text{ ft/s}^2)(y_{\max} - 89.6) \text{ ft}$$

or $\quad y_{\max} = 1076 \text{ ft}$ ◀

PROBLEM 11.37



A small package is released from rest at A and moves along the skate wheel conveyor $ABCD$. The package has a uniform acceleration of $4.8\ \text{m/s}^2$ as it moves down sections AB and CD, and its velocity is constant between B and C. If the velocity of the package at D is 7.2 m/s, determine (a) the distance d between C and D, (b) the time required for the package to reach D.

SOLUTION

(a) For $A \longrightarrow B$ and $C \longrightarrow D$ we have

$$v^2 = v_0^2 + 2a(x - x_0)$$

Then, at B $\quad v_{BC}^2 = 0 + 2(4.8\ \text{m/s}^2)(3-0)\ \text{m}$

$$= 28.8\ \text{m}^2/\text{s}^2 \qquad (v_{BC} = 5.3666\ \text{m/s})$$

and at D $\quad v_D^2 = v_{BC}^2 + 2a_{CD}(x_D - x_C) \qquad d = x_D - x_C$

or $\quad (7.2\ \text{m/s})^2 = (28.8\ \text{m}^2/\text{s}^2) + 2(4.8\ \text{m/s}^2)d$

or $\quad d = 2.40\ \text{m}$ ◀

(b) For $A \longrightarrow B$ and $C \longrightarrow D$ we have

$$v = v_0 + at$$

Then $A \longrightarrow B$ $\quad 5.3666\ \text{m/s} = 0 + (4.8\ \text{m/s}^2)t_{AB}$

or $\quad t_{AB} = 1.11804\ \text{s}$

and $C \longrightarrow D$ $\quad 7.2\ \text{m/s} = 5.3666\ \text{m/s} + (4.8\ \text{m/s}^2)t_{CD}$

or $\quad t_{CD} = 0.38196\ \text{s}$

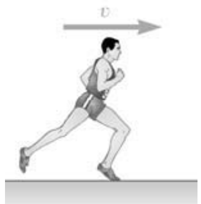
Now, for $B \longrightarrow C$, we have $\quad x_C = x_B + v_{BC}t_{BC}$

or $\quad 3\ \text{m} = (5.3666\ \text{m/s})t_{BC}$

or $\quad t_{BC} = 0.55901\ \text{s}$

Finally, $\quad t_D = t_{AB} + t_{BC} + t_{CD} = (1.11804 + 0.55901 + 0.38196)\ \text{s}$

or $\quad t_D = 2.06\ \text{s}$ ◀



PROBLEM 11.38

A sprinter in a 100-m race accelerates uniformly for the first 35 m and then runs with constant velocity. If the sprinter's time for the first 35 m is 5.4 s, determine (*a*) his acceleration, (*b*) his final velocity, (*c*) his time for the race.

SOLUTION

<u>Given</u>: $0 \le x \le 35$ m, $a = \text{constant}$

$35 \text{ m} < x \le 100 \text{ m}$, $v = \text{constant}$

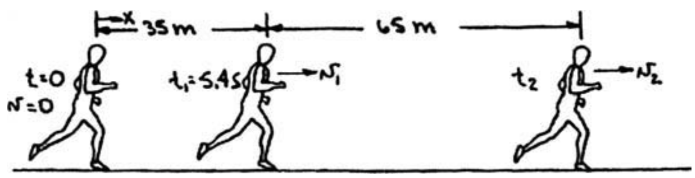
At $t = 0$, $v = 0$ when $x = 35$ m, $t = 5.4$ s

Find:

(*a*) a

(*b*) v when $x = 100$ m

(*c*) t when $x = 100$ m



(*a*) We have $x = 0 + 0t + \frac{1}{2}at^2$ for $0 \le x \le 35$ m

At $t = 5.4$ s: $35 \text{ m} = \frac{1}{2}a(5.4 \text{ s})^2$

or $a = 2.4005 \text{ m/s}^2$

$a = 2.40 \text{ m/s}^2$ ◀

(*b*) First note that $v = v_{max}$ for $35 \text{ m} \le x \le 100$ m.

Now $v^2 = 0 + 2a(x - 0)$ for $0 \le x \le 35$ m

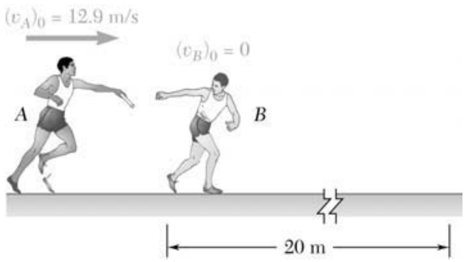
When $x = 35$ m: $v_{max}^2 = 2(2.4005 \text{ m/s}^2)(35 \text{ m})$

or $v_{max} = 12.9628$ m/s $v_{max} = 12.96$ m/s ◀

(*c*) We have $x = x_1 + v_0(t - t_1)$ for $35 \text{ m} < x \le 100$ m

When $x = 100$ m: $100 \text{ m} = 35 \text{ m} + (12.9628 \text{ m/s})(t_2 - 5.4)$ s

or $t_2 = 10.41$ s ◀



PROBLEM 11.39

As relay runner *A* enters the 20-m-long exchange zone with a speed of 12.9 m/s, he begins to slow down. He hands the baton to runner *B* 1.82 s later as they leave the exchange zone with the same velocity. Determine (*a*) the uniform acceleration of each of the runners, (*b*) when runner *B* should begin to run.

SOLUTION

(*a*) For runner *A*: $x_A = 0 + (v_A)_0 t + \frac{1}{2} a_A t^2$

At $t = 1.82$ s: $20 \text{ m} = (12.9 \text{ m/s})(1.82 \text{ s}) + \frac{1}{2} a_A (1.82 \text{ s})^2$

or $a_A = -2.10 \text{ m/s}^2$ ◀

Also $v_A = (v_A)_0 + a_A t$

At $t = 1.82$ s: $(v_A)_{1.82} = (12.9 \text{ m/s}) + (-2.10 \text{ m/s}^2)(1.82 \text{ s})$

$= 9.078 \text{ m/s}$

For runner *B*: $v_B^2 = 0 + 2a_B [x_B - 0]$

When $x_B = 20 \text{ m}, \quad v_B = v_A: \quad (9.078 \text{ m/s})^2 = 2a_B(20 \text{ m})$

or $a_B = 2.0603 \text{ m/s}^2$

$a_B = 2.06 \text{ m/s}^2$ ◀

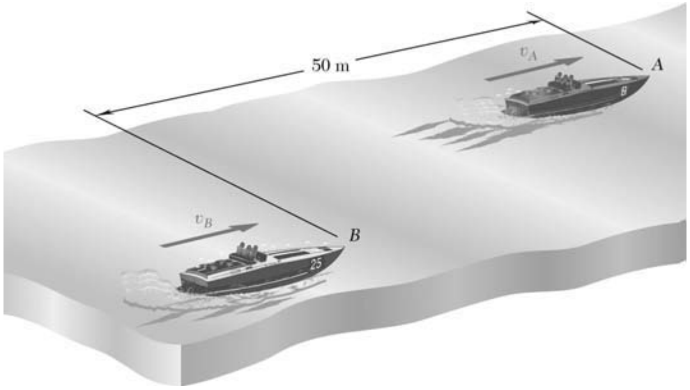
(*b*) For runner *B*: $v_B = 0 + a_B(t - t_B)$

where t_B is the time at which he begins to run.

At $t = 1.82$ s: $9.078 \text{ m/s} = (2.0603 \text{ m/s}^2)(1.82 - t_B)\text{ s}$

or $t_B = -2.59$ s

Runner *B* should start to run 2.59 s before *A* reaches the exchange zone. ◀



PROBLEM 11.40

In a boat race, boat *A* is leading boat *B* by 50 m and both boats are traveling at a constant speed of 180 km/h. At $t = 0$, the boats accelerate at constant rates. Knowing that when *B* passes *A*, $t = 8$ s and $v_A = 225$ km/h, determine (*a*) the acceleration of *A*, (*b*) the acceleration of *B*.

SOLUTION

(*a*) We have

$$v_A = (v_A)_0 + a_A t$$

$$(v_A)_0 = 180 \text{ km/h} = 50 \text{ m/s}$$

At $t = 8$ s:

$$v_A = 225 \text{ km/h} = 62.5 \text{ m/s}$$

Then

$$62.5 \text{ m/s} = 50 \text{ m/s} + a_A (8 \text{ s})$$

or

$$a_A = 1.563 \text{ m/s}^2 \blacktriangleleft$$

(*b*) We have

$$x_A = (x_A)_0 + (v_A)_0 t + \frac{1}{2} a_A t^2 = 50 \text{ m} + (50 \text{ m/s})(8 \text{ s}) + \frac{1}{2}(1.5625 \text{ m/s}^2)(8 \text{ s})^2 = 500 \text{ m}$$

and

$$x_B = 0 + (v_B)_0 t + \frac{1}{2} a_B t^2 \qquad (v_B)_0 = 50 \text{ m/s}$$

At $t = 8$ s:

$$x_A = x_B$$

$$500 \text{ m} = (50 \text{ m/s})(8 \text{ s}) + \frac{1}{2} a_B (8 \text{ s})^2$$

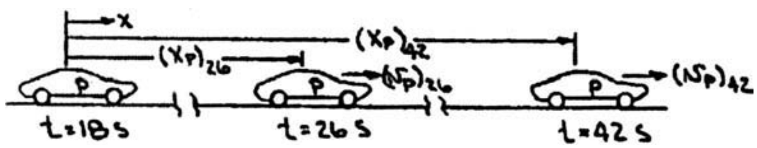
or

$$a_B = 3.13 \text{ m/s}^2 \blacktriangleleft$$

PROBLEM 11.41

A police officer in a patrol car parked in a 45 mi/h speed zone observes a passing automobile traveling at a slow, constant speed. Believing that the driver of the automobile might be intoxicated, the officer starts his car, accelerates uniformly to 60 mi/h in 8 s, and, maintaining a constant velocity of 60 mi/h, overtakes the motorist 42 s after the automobile passed him. Knowing that 18 s elapsed before the officer began pursuing the motorist, determine (*a*) the distance the officer traveled before overtaking the motorist, (*b*) the motorist's speed.

SOLUTION



$(v_P)_{18} = 0 \qquad (v_P)_{26} = 60 \text{ mi/h} = 88 \text{ ft/s} \qquad (v_P)_{42} = 90 \text{ mi/h} = 88 \text{ ft/s}$

(*a*) Patrol car:

For $18 \text{ s} < t \le 26 \text{ s}$: $\quad v_P = 0 + a_P(t - 18)$

At $t = 26$ s: $\quad 88 \text{ ft/s} = a_P(26 - 18) \text{ s}$

or $\quad a_P = 11 \text{ ft/s}^2$

Also, $\quad x_P = 0 + 0(t - 18) - \dfrac{1}{2}a_P(t - 18)^2$

At $t = 26$ s: $\quad (x_P)_{26} = \dfrac{1}{2}(11 \text{ ft/s}^2)(26 - 18)^2 = 352 \text{ ft}$

For $26 \text{ s} < t \le 42 \text{ s}$: $\quad x_P = (x_P)_{26} + (v_P)_{26}(t - 26)$

At $t = 42$ s: $\quad (x_P)_{42} = 352 \text{ m} + (88 \text{ ft/s})(42 - 26) \text{ s}$
$\quad = 1760 \text{ ft}$

$(x_P)_{42} = 1760 \text{ ft}$ ◀

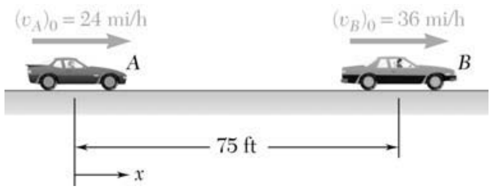
(*b*) For the motorist's car: $\quad x_M = 0 + v_M t$

At $t = 42$ s, $x_M = x_P$: $\quad 1760 \text{ ft} = v_M(42 \text{ s})$

or $\quad v_M = 41.9048 \text{ ft/s}$

or $\quad v_M = 28.6 \text{ mi/h}$ ◀

PROBLEM 11.42



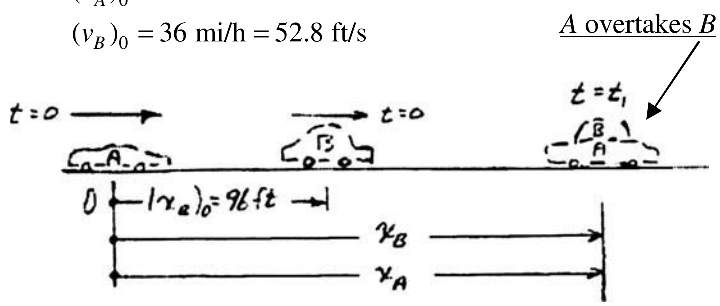
Automobiles A and B are traveling in adjacent highway lanes and at $t = 0$ have the positions and speeds shown. Knowing that automobile A has a constant acceleration of 1.8 ft/s^2 and that B has a constant deceleration of 1.2 ft/s^2, determine (*a*) when and where A will overtake B, (*b*) the speed of each automobile at that time.

SOLUTION

$$a_A = +1.8 \text{ ft/s}^2 \quad a_B = -1.2 \text{ ft/s}^2$$

$$(v_A)_0 = 24 \text{ mi/h} = 35.2 \text{ ft/s}$$

$$(v_B)_0 = 36 \text{ mi/h} = 52.8 \text{ ft/s}$$



Motion of auto A:

$$v_A = (v_A)_0 + a_A t = 35.2 + 1.8t \quad (1)$$

$$x_A = (x_A)_0 + (v_A)_0 t + \frac{1}{2}a_A t^2 = 0 + 35.2t + \frac{1}{2}(1.8)t^2 \quad (2)$$

Motion of auto B:

$$v_B = (v_B)_0 + a_B t = 52.8 - 1.2t \quad (3)$$

$$x_B = (x_B)_0 + (v_B)_0 t + \frac{1}{2}a_B t^2 = 75 + 52.8t + \frac{1}{2}(-1.2)t^2 \quad (4)$$

(*a*) A overtakes B at $t = t_1$.

$$x_A = x_B: \quad 35.2t + 0.9t_1^2 = 75 + 52.8t_1 - 0.6t_1^2$$

$$1.5t_1^2 - 17.6t_1 - 75 = 0$$

$$t_1 = -3.22 \text{ s} \quad \text{and} \quad t_1 = 15.0546$$

$t_1 = 15.05$ s ◀

Eq. (2): $x_A = 35.2(15.05) + 0.9(15.05)^2$

$x_A = 734$ ft ◀

PROBLEM 11.42 (Continued)

(*b*) Velocities when $t_1 = 15.05$ s

Eq. (1): $v_A = 35.2 + 1.8(15.05)$

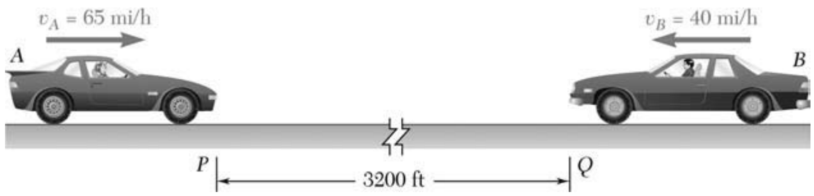
$v_A = 62.29$ ft/s $\qquad v_A = 42.5$ mi/h $\longrightarrow$ ◀

Eq. (3): $v_B = 52.8 - 1.2(15.05)$

$v_B = 34.74$ ft/s $\qquad v_B = 23.7$ mi/h $\longrightarrow$ ◀

PROBLEM 11.43

Two automobiles A and B are approaching each other in adjacent highway lanes. At $t = 0$, A and B are 3200 ft apart, their speeds are $v_A = 65$ mi/h and $v_B = 40$ mi/h, and they are at Points P and Q, respectively. Knowing that A passes Point Q 40 s after B was there and that B passes Point P 42 s after A was there, determine (a) the uniform accelerations of A and B, (b) when the vehicles pass each other, (c) the speed of B at that time.



SOLUTION

(a) We have $$x_A = 0 + (v_A)_0 t + \frac{1}{2}a_A t^2 \quad (v_A)_0 = 65 \text{ mi/h} = 95.33 \text{ ft/s}$$

(x is positive ⟶; origin at P.)

At $t = 40$ s: $$3200 \text{ m} = (95.333 \text{ m/s})(40 \text{ s}) + \frac{1}{2}a_A(40 \text{ s})^2$$ $a_A = -0.767 \text{ ft/s}^2$ ◀

Also, $x_B = 0 + (v_B)_0 t + \frac{1}{2}a_B t^2 \quad (v_B)_0 = 40 \text{ mi/h} = 58.667 \text{ ft/s}$

(x_B is positive ⟵; origin at Q.)

At $t = 42$ s: $$3200 \text{ ft} = (58.667 \text{ ft/s})(42 \text{ s}) + \frac{1}{2}a_B(42 \text{ s})^2$$

or $a_B = 0.83447 \text{ ft/s}^2$ $a_B = 0.834 \text{ ft/s}^2$ ◀

(b) When the cars pass each other $x_A + x_B = 3200$ ft

Then $$(95.333 \text{ ft/s})t_{AB} + \frac{1}{2}(-0.76667 \text{ ft/s})t_{AB}^2 + (58.667 \text{ ft/s})t_{AB} + \frac{1}{2}(0.83447 \text{ ft/s}^2)t_{AB}^2 = 3200 \text{ ft}$$

or $$0.03390 t_{AB}^2 + 154 t_{AB} - 3200 = 0$$

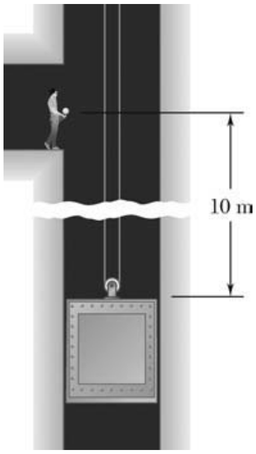
Solving $t = 20.685$ s and $t = -4563$ s $t > 0 \Rightarrow$ $t_{AB} = 20.7$ s ◀

(c) We have $v_B = (v_B)_0 + a_B t$

At $t = t_{AB}$: $v_B = 58.667 \text{ ft/s} + (0.83447 \text{ ft/s}^2)(20.685 \text{ s})$

$= 75.927$ ft/s $v_B = 51.8$ mi/h ◀

PROBLEM 11.44



An elevator is moving upward at a constant speed of 4 m/s. A man standing 10 m above the top of the elevator throws a ball upward with a speed of 3 m/s. Determine (*a*) when the ball will hit the elevator, (*b*) where the ball will hit the elevator with respect to the location of the man.

SOLUTION

Place the origin of the position coordinate at the level of the standing man, the positive direction being up. The ball undergoes uniformly accelerated motion.

$$y_B = (y_B)_0 + (v_B)_0 t - \frac{1}{2} g t^2$$

with $(y_B)_0 = 0$, $(v_B)_0 = 3$ m/s, and $g = 9.81 \text{ m/s}^2$.

$$y_B = 3t - 4.905t^2$$

The elevator undergoes uniform motion.

$$y_E = (y_E)_0 + v_E t$$

with $(y_E)_0 = -10$ m and $v_E = 4$ m/s.

(*a*) Time of impact. Set $y_B = y_E$

$$3t - 4.905t^2 = -10 + 4t$$

$$4.905t^2 + t - 10 = 0$$

$t = 1.3295$ and -1.5334 $\qquad t = 1.330 \text{ s}$ ◀

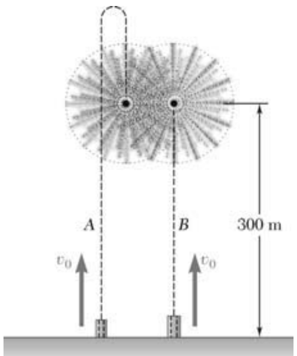
(*b*) Location of impact.

$$y_B = (3)(1.3295) - (4.905)(1.3295)^2 = -4.68 \text{ m}$$

$$y_E = -10 + (4)(1.3295) = -4.68 \text{ m} \qquad \text{(checks)}$$

4.68 m below the man ◀

PROBLEM 11.45



Two rockets are launched at a fireworks display. Rocket A is launched with an initial velocity $v_0 = 100$ m/s and rocket B is launched t_1 seconds later with the same initial velocity. The two rockets are timed to explode simultaneously at a height of 300 m as A is falling and B is rising. Assuming a constant acceleration $g = 9.81\ \text{m/s}^2$, determine (a) the time t_1, (b) the velocity of B relative to A at the time of the explosion.

SOLUTION

Place origin at ground level. The motion of rockets A and B is

Rocket A: $$v_A = (v_A)_0 - gt = 100 - 9.81t \quad (1)$$

$$y_A = (y_A)_0 + (v_A)_0 t - \frac{1}{2}gt^2 = 100t - 4.905t^2 \quad (2)$$

Rocket B: $$v_B = (v_B)_0 - g(t - t_1) = 100 - 9.81(t - t_1) \quad (3)$$

$$y_B = (y_B)_0 + (v_B)_0(t - t_1) - \frac{1}{2}g(t - t_1)^2$$
$$= 100(t - t_1) - 4.905(t - t_1)^2 \quad (4)$$

Time of explosion of rockets A and B. $y_A = y_B = 300$ ft

From (2), $$300 = 100t - 4.905t^2$$

$$4.905t^2 - 100t + 300 = 0$$

$$t = 16.732 \text{ s} \quad \text{and} \quad 3.655 \text{ s}$$

From (4), $$300 = 100(t - t_1) - 4.905(t - t_1^2)$$

$$t - t_1 = 16.732 \text{ s} \quad \text{and} \quad 3.655 \text{ s}$$

Since rocket A is falling, $t = 16.732$ s

Since rocket B is rising, $t - t_1 = 3.655$ s

(a) Time t_1: $t_1 = t - (t - t_1)$ $\qquad t_1 = 13.08$ s ◀

(b) Relative velocity at explosion.

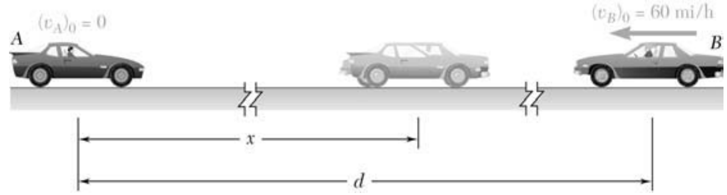
From (1), $v_A = 100 - (9.81)(16.732) = -64.15$ m/s

From (3), $v_B = 100 - (9.81)(16.732 - 13.08) = 64.15$ m/s

Relative velocity: $v_{B/A} = v_B - v_A$ $\qquad v_{B/A} = 128.3$ m/s ↑ ◀

PROBLEM 11.46

Car A is parked along the northbound lane of a highway, and car B is traveling in the southbound lane at a constant speed of 60 mi/h. At $t = 0$, A starts and accelerates at a constant rate a_A, while at $t = 5$ s, B begins to slow down with a constant deceleration of magnitude $a_A/6$. Knowing that when the cars pass each other $x = 294$ ft and $v_A = v_B$, determine (*a*) the acceleration a_A, (*b*) when the vehicles pass each other, (*c*) the distance d between the vehicles at $t = 0$.



SOLUTION

For $t \geq 0$: $v_A = 0 + a_A t$

$$x_A = 0 + 0 + \frac{1}{2} a_A t^2$$

$0 \leq t < 5$ s: $x_B = 0 + (v_B)_0 t \quad (v_B)_0 = 60 \text{ mi/h} = 88 \text{ ft/s}$

At $t = 5$ s: $x_B = (88 \text{ ft/s})(5 \text{ s}) = 440 \text{ ft}$

For $t \geq 5$ s: $v_B = (v_B)_0 + a_B(t-5) \qquad a_B = -\frac{1}{6} a_A$

$$x_B = (x_B)_S + (v_B)_0(t-5) + \frac{1}{2} a_B (t-5)^2$$

Assume $t > 5$ s when the cars pass each other.

At that time (t_{AB}),

$v_A = v_B$: $\quad a_A t_{AB} = (88 \text{ ft/s}) - \frac{a_A}{6}(t_{AB} - 5)$

$x_A = 294$ ft: $\quad 294 \text{ ft} = \frac{1}{2} a_A t_{AB}^2$

Then $$\frac{a_A\left(\frac{7}{6} t_{AB} - \frac{5}{6}\right)}{\frac{1}{2} a_A t_{AB}^2} = \frac{88}{294}$$

or $\quad 44 t_{AB}^2 - 343 t_{AB} + 245 = 0$

PROBLEM 11.46 (Continued)

Solving $t_{AB} = 0.795\text{ s}$ and $t_{AB} = 7.00\text{ s}$

(*a*) With $t_{AB} > 5\text{ s}$, $294\text{ ft} = \frac{1}{2}a_A(7.00\text{ s})^2$

or $a_A = 12.00\text{ ft/s}^2$ ◀

(*b*) From above $t_{AB} = 7.00\text{ s}$ ◀

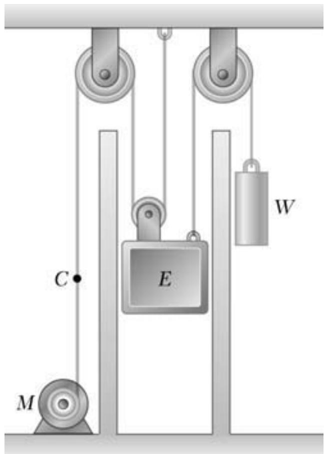
Note: An acceptable solution cannot be found if it is assumed that $t_{AB} \le 5\text{ s}$.

(*c*) We have

$$d = x + (x_B)_{t_{AB}}$$
$$= 294\text{ ft} + 440\text{ ft} + (88\text{ ft/s})(2.00\text{ s})$$
$$+\frac{1}{2}\left(-\frac{1}{6}\times 12.00\text{ ft/s}^2\right)(2.00\text{ s})^2$$

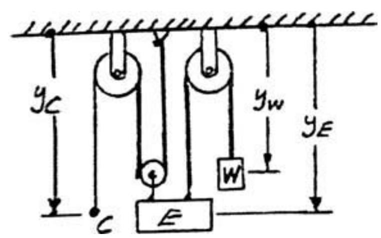
or $d = 906\text{ ft}$ ◀

PROBLEM 11.47



The elevator shown in the figure moves downward with a constant velocity of 4 m/s. Determine (*a*) the velocity of the cable *C*, (*b*) the velocity of the counterweight *W*, (*c*) the relative velocity of the cable *C* with respect to the elevator, (*d*) the relative velocity of the counterweight *W* with respect to the elevator.

SOLUTION



Choose the positive direction downward.

(*a*) Velocity of cable *C*.

$$y_C + 2y_E = \text{constant}$$

$$v_C + 2v_E = 0$$

But, $v_E = 4 \text{ m/s}$

or $v_C = -2v_E = -8 \text{ m/s}$ $\quad \mathbf{v}_C = 8.00 \text{ m/s} \uparrow$ ◀

(*b*) Velocity of counterweight *W*.

$$y_W + y_E = \text{constant}$$

$$v_W + v_E = 0 \quad v_W = -v_E = -4 \text{ m/s}$$ $\quad \mathbf{v}_W = 4.00 \text{ m/s} \uparrow$ ◀

(*c*) Relative velocity of *C* with respect to *E*.

$$v_{C/E} = v_C - v_E = (-8 \text{ m/s}) - (+4 \text{ m/s}) = -12 \text{ m/s}$$

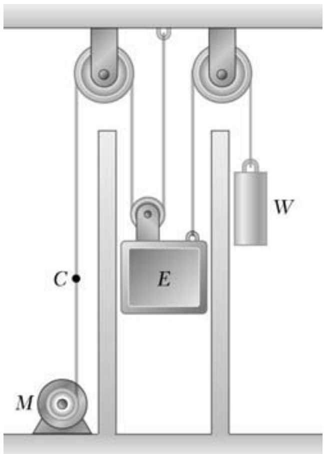
$\mathbf{v}_{C/E} = 12.00 \text{ m/s} \uparrow$ ◀

(*d*) Relative velocity of *W* with respect to *E*.

$$v_{W/E} = v_W - v_E = (-4 \text{ m/s}) - (4 \text{ m/s}) = -8 \text{ m/s}$$

$\mathbf{v}_{W/E} = 8.00 \text{ m/s} \uparrow$ ◀

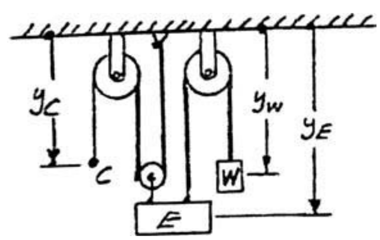
PROBLEM 11.48



The elevator shown starts from rest and moves upward with a constant acceleration. If the counterweight W moves through 30 ft in 5 s, determine (*a*) the acceleration of the elevator and the cable C, (*b*) the velocity of the elevator after 5 s.

SOLUTION

We choose positive direction downward for motion of counterweight.



$$y_W = \frac{1}{2}a_W t^2$$

At $t = 5$ s,

$$y_W = 30 \text{ ft}$$

$$30 \text{ ft} = \frac{1}{2}a_W (5 \text{ s})^2$$

$$a_W = 2.4 \text{ ft/s}^2 \qquad \mathbf{a}_W = 2.4 \text{ ft/s}^2 \downarrow$$

(*a*) Accelerations of E and C.

Since $\quad y_W + y_E = \text{constant} \quad v_W + v_E = 0, \quad \text{and} \quad a_W + a_E = 0$

Thus: $\quad a_E = -a_W = -(2.4 \text{ ft/s}^2), \qquad \mathbf{a}_E = 2.40 \text{ ft/s}^2 \uparrow$ ◀

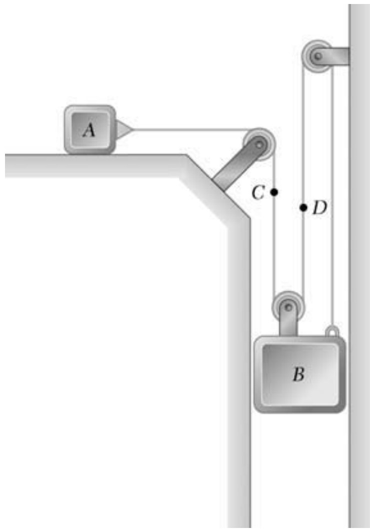
Also, $\quad y_C + 2y_E = \text{ constant}, \quad v_C + 2v_E = 0, \quad \text{and} \quad a_C + 2a_E = 0$

Thus: $\quad a_C = -2a_E = -2(-2.4 \text{ ft/s}^2) = +4.8 \text{ ft/s}^2, \qquad \mathbf{a}_C = 4.80 \text{ ft/s}^2 \downarrow$ ◀

(*b*) Velocity of elevator after 5 s.

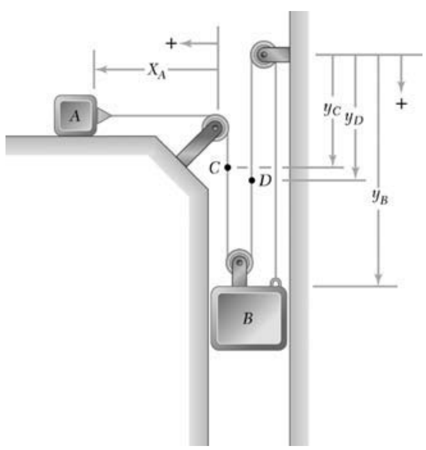
$$v_E = (v_E)_0 + a_E t = 0 + (-2.4 \text{ ft/s}^2)(5 \text{ s}) = -12 \text{ ft/s} \qquad (\mathbf{v}_E)_5 = 12.00 \text{ ft/s} \uparrow$$ ◀

PROBLEM 11.49



Slider block A moves to the left with a constant velocity of 6 m/s. Determine (*a*) the velocity of block B, (*b*) the velocity of portion D of the cable, (*c*) the relative velocity of portion C of the cable with respect to portion D.

SOLUTION



From the diagram, we have

$$x_A + 3y_B = \text{constant}$$

Then $\quad v_A + 3v_B = 0 \quad (1)$

and $\quad a_A + 3a_B = 0 \quad (2)$

(*a*) Substituting into Eq. (1) $\quad 6 \text{ m/s} + 3v_B = 0$

or $\quad \mathbf{v}_B = 2.00 \text{ m/s} \uparrow$ ◀

(*b*) From the diagram $\quad y_B + y_D = \text{constant}$

Then $\quad v_B + v_D = 0$

$\mathbf{v}_D = 2.00 \text{ m/s} \downarrow$ ◀

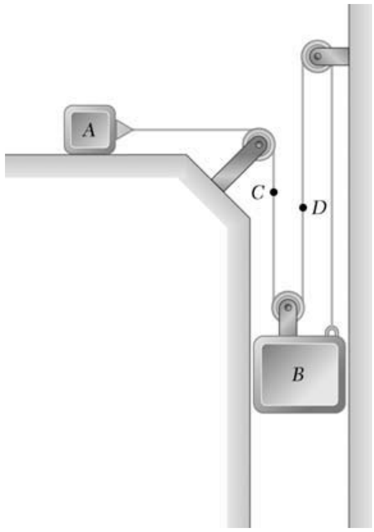
(*c*) From the diagram $\quad x_A + y_C = \text{constant}$

Then $\quad v_A + v_C = 0 \qquad v_C = -6 \text{ m/s}$

Now $\quad v_{C/D} = v_C - v_D = (-6 \text{ m/s}) - (2 \text{ m/s}) = -8 \text{ m/s}$

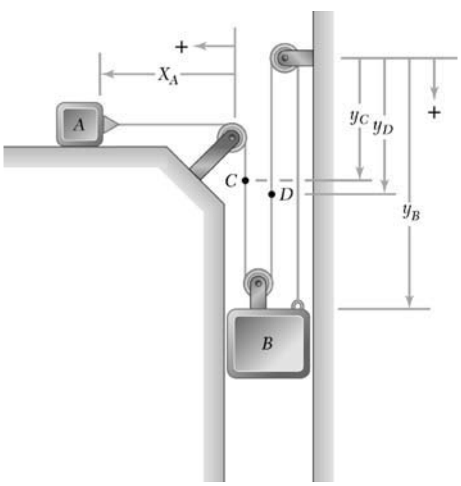
$\mathbf{v}_{C/D} = 8.00 \text{ m/s} \uparrow$ ◀

PROBLEM 11.50



Block B starts from rest and moves downward with a constant acceleration. Knowing that after slider block A has moved 9 in. its velocity is 6 ft/s, determine (*a*) the accelerations of A and B, (*b*) the velocity and the change in position of B after 2 s.

SOLUTION



From the diagram, we have

$$x_A + 3y_B = \text{constant}$$

Then $\quad v_A + 3v_B = 0 \quad (1)$

and $\quad a_A + 3a_B = 0 \quad (2)$

(*a*) Eq. (2): $a_A + 3a_B = 0$ and $\mathbf{a}_B$ is constant and positive $\Rightarrow \mathbf{a}_A$ is constant and negative

Also, Eq. (1) and $(v_B)_0 = 0 \Rightarrow (v_A)_0 = 0$

Then $\quad v_A^2 = 0 + 2a_A[x_A - (x_A)_0]$

When $|\Delta x_A| = 0.4$ m: $\quad (6\text{ ft/s})^2 = 2a_A(9/12\text{ ft})$

or $\quad \mathbf{a}_A = 24.0\text{ ft/s}^2 \longrightarrow$ ◀

Then, substituting into Eq. (2):

$$-24\text{ ft/s}^2 + 3a_B = 0$$

or $\quad a_B = \dfrac{24}{3}\text{ ft/s}^2 \qquad \mathbf{a}_B = 8.00\text{ ft/s}^2 \downarrow$ ◀

PROBLEM 11.50 (Continued)

(*b*) We have $v_B = 0 + a_B t$

At $t = 2$ s: $v_B = \left(\frac{24}{3}\text{ ft/s}^2\right)(2\text{ s})$

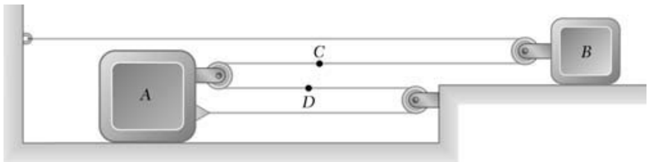
or $\mathbf{v}_B = 16.00\text{ ft/s}\downarrow$ ◀

Also $y_B = (y_B)_0 + 0 + \frac{1}{2}a_B t^2$

At $t = 2$ s: $y_B - (y_B)_0 = \frac{1}{2}\left(\frac{24}{3}\text{ ft/s}^2\right)(2\text{ s})^2$

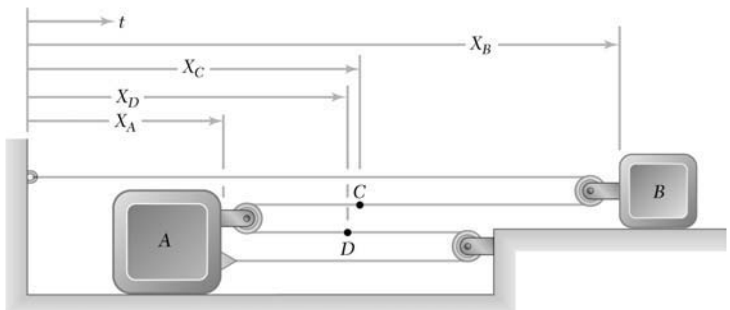
or $\mathbf{y}_B - (\mathbf{y}_B)_0 = 16.00\text{ ft}\downarrow$ ◀

PROBLEM 11.51



Slider block B moves to the right with a constant velocity of 300 mm/s. Determine (a) the velocity of slider block A, (b) the velocity of portion C of the cable, (c) the velocity of portion D of the cable, (d) the relative velocity of portion C of the cable with respect to slider block A.

SOLUTION



From the diagram $\qquad x_B + (x_B - x_A) - 2x_A = \text{constant}$

Then $\qquad 2v_B - 3v_A = 0 \qquad (1)$

and $\qquad 2a_B - 3a_A = 0 \qquad (2)$

Also, we have $\qquad -x_D - x_A = \text{constant}$

Then $\qquad v_D + v_A = 0 \qquad (3)$

(a) Substituting into Eq. (1) $\qquad 2(300 \text{ mm/s}) - 3v_A = 0$

or $\qquad \mathbf{v}_A = 200 \text{ mm/s} \longrightarrow$ ◀

(b) From the diagram $\qquad x_B + (x_B - x_C) = \text{constant}$

Then $\qquad 2v_B - v_C = 0$

Substituting $\qquad 2(300 \text{ mm/s}) - v_C = 0$

or $\qquad \mathbf{v}_C = 600 \text{ mm/s} \longrightarrow$ ◀

PROBLEM 11.51 (Continued)

(*c*) From the diagram $(x_C - x_A) + (x_D - x_A) = \text{constant}$

Then $v_C - 2v_A + v_D = 0$

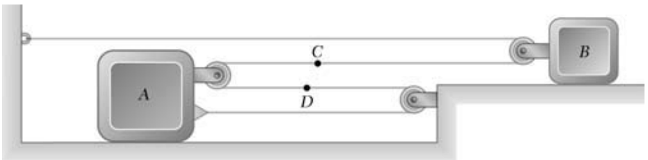
Substituting $600 \text{ mm/s} - 2(200 \text{ mm/s}) + v_D = 0$

or $\mathbf{v}_D = 200 \text{ mm/s} \longleftarrow$ ◀

(*d*) We have $v_{C/A} = v_C - v_A$

$= 600 \text{ mm/s} - 200 \text{ mm/s}$

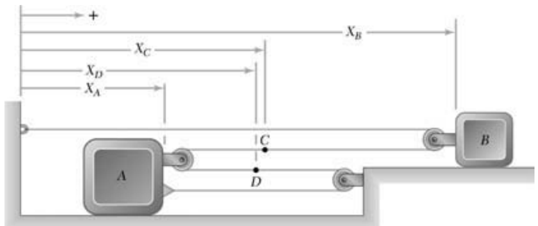
or $\mathbf{v}_{C/A} = 400 \text{ mm/s} \longrightarrow$ ◀



PROBLEM 11.52

At the instant shown, slider block B is moving with a constant acceleration, and its speed is 150 mm/s. Knowing that after slider block A has moved 240 mm to the right its velocity is 60 mm/s, determine (*a*) the accelerations of A and B, (*b*) the acceleration of portion D of the cable, (*c*) the velocity and change in position of slider block B after 4 s.

SOLUTION



From the diagram $\quad x_B + (x_B - x_A) - 2x_A = \text{constant}$

Then $\quad 2v_B - 3v_A = 0 \quad (1)$

and $\quad 2a_B - 3a_A = 0 \quad (2)$

(*a*) First observe that if block A moves to the right, $\mathbf{v}_A \rightarrow$ and Eq. (1) $\Rightarrow \mathbf{v}_B \rightarrow$. Then, using Eq. (1) at $t = 0$

$$2(150 \text{ mm/s}) - 3(v_A)_0 = 0$$

or $\quad (v_A)_0 = 100 \text{ mm/s}$

Also, Eq. (2) and $a_B =$ constant $\Rightarrow a_A =$ constant

Then $\quad v_A^2 = (v_A)_0^2 + 2a_A[x_A - (x_A)_0]$

When $x_A - (x_A)_0 = 240$ mm:

$$(60 \text{ mm/s})^2 = (100 \text{ mm/s})^2 + 2a_A(240 \text{ mm})$$

or $\quad a_A = -\dfrac{40}{3} \text{ mm/s}^2$

or $\quad \mathbf{a}_A = 13.33 \text{ mm/s}^2 \leftarrow$ ◀

PROBLEM 11.52 (Continued)

Then, substituting into Eq. (2)

$$2a_B - 3\left(-\frac{40}{3}\ \text{mm/s}^2\right) = 0$$

or $a_B = -20\ \text{mm/s}^2$ $\qquad \mathbf{a}_B = 20.0\ \text{mm/s}^2 \longleftarrow$ ◀

(*b*) From the diagram, $-x_D - x_A = \text{constant}$

$$v_D + v_A = 0$$

Then $a_D + a_A = 0$

Substituting $a_D + \left(-\frac{40}{3}\ \text{mm/s}^2\right) = 0$

or $\qquad \mathbf{a}_D = 13.33\ \text{mm/s}^2 \longrightarrow$ ◀

(*c*) We have $v_B = (v_B)_0 + a_B t$

At $t = 4$ s: $v_B = 150\ \text{mm/s} + (-20.0\ \text{mm/s}^2)(4\ \text{s})$

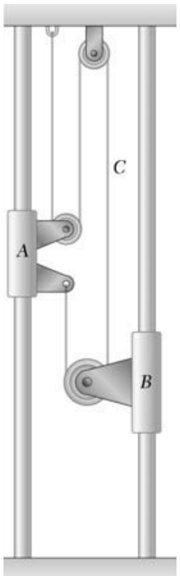
or $\qquad \mathbf{v}_B = 70.0\ \text{mm/s} \longrightarrow$ ◀

Also $x_B = (x_B)_0 + (v_B)_0 t + \frac{1}{2} a_B t^2$

At $t = 4$ s: $x_B - (x_B)_0 = (150\ \text{mm/s})(4\ \text{s}) + \frac{1}{2}(-20.0\ \text{mm/s}^2)(4\ \text{s})^2$

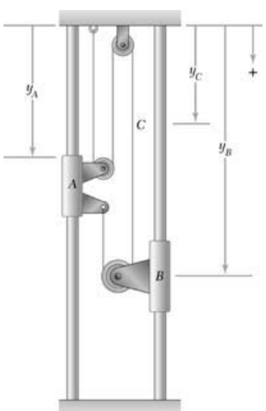
or $\qquad \mathbf{x}_B - (\mathbf{x}_B)_0 = 440\ \text{mm} \longrightarrow$ ◀

PROBLEM 11.53



Collar A starts from rest and moves upward with a constant acceleration. Knowing that after 8 s the relative velocity of collar B with respect to collar A is 24 in./s, determine (*a*) the accelerations of A and B, (*b*) the velocity and the change in position of B after 6 s.

SOLUTION



From the diagram

$$2y_A + y_B + (y_B - y_A) = \text{constant}$$

Then $$v_A + 2v_B = 0 \qquad (1)$$

and $$a_A + 2a_B = 0 \qquad (2)$$

(*a*) Eq. (1) and $(v_A)_0 = 0 \Rightarrow (v_B)_0 = 0$

Also, Eq. (2) and $\mathbf{a}_A$ is constant and negative $\Rightarrow \mathbf{a}_B$ is constant and positive.

Then $$v_A = 0 + a_A t \qquad v_B = 0 + a_B t$$

Now $$v_{B/A} = v_B - v_A = (a_B - a_A)t$$

From Eq. (2) $$a_B = -\frac{1}{2}a_A$$

So that $$v_{B/A} = -\frac{3}{2}a_A t$$

PROBLEM 11.53 (Continued)

At $t = 8$ s: $$24 \text{ in./s} = -\frac{3}{2}a_A(8 \text{ s})$$

or $$\mathbf{a}_A = 2.00 \text{ in./s}^2 \uparrow \blacktriangleleft$$

and then $$a_B = -\frac{1}{2}(-2 \text{ in./s}^2)$$

or $$\mathbf{a}_B = 1.000 \text{ in./s}^2 \downarrow \blacktriangleleft$$

(*b*) At $t = 6$ s: $$v_B = (1 \text{ in./s}^2)(6 \text{ s})$$

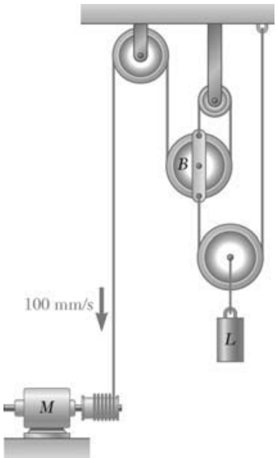
or $$\mathbf{v}_B = 6.00 \text{ in./s} \downarrow \blacktriangleleft$$

Now $$y_B = (y_B)_0 + 0 + \frac{1}{2}a_B t^2$$

At $t = 6$ s: $$y_B - (y_B)_0 = \frac{1}{2}(1 \text{ in./s}^2)(6 \text{ s})^2$$

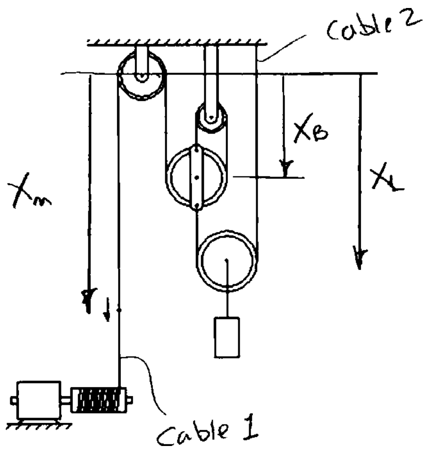
or $$\mathbf{y}_B - (\mathbf{y}_B)_0 = 18.00 \text{ in.} \downarrow \blacktriangleleft$$

PROBLEM 11.54



The motor *M* reels in the cable at a constant rate of 100 mm/s. Determine (*a*) the velocity of load *L*, (*b*) the velocity of pulley *B* with respect to load *L*.

SOLUTION



Let x_B and x_L be the positions, respectively, of pulley *B* and load *L* measured downward from a fixed elevation above both. Let x_M be the position of a point on the cable about to enter the reel driven by the motor. Then, considering the lengths of the two cables,

$$x_M + 3x_B = \text{constant} \qquad v_M + 3v_B = 0$$

$$x_L + (x_L - x_B) = \text{constant} \qquad 2v_L + v_B = 0$$

with

$$v_M = 100 \text{ mm/s}$$

$$v_B = -\frac{v_M}{3} = -33.333 \text{ m/s}$$

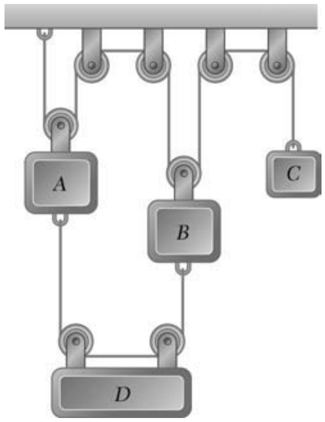
$$v_L = \frac{v_B}{2} = -16.667 \text{ mm/s}$$

(*a*) Velocity of load *L*. $\mathbf{v}_L = 16.67 \text{ mm/s} \uparrow$ ◀

(*b*) Velocity of pulley *B* with respect to load *L*. $v_{B/L} = v_B - v_L = -33.333 - (-16.667) = -16.667$

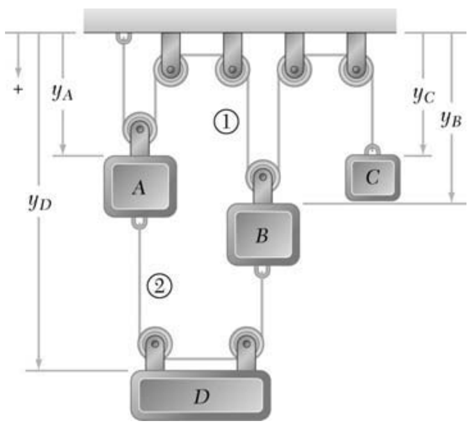
$\mathbf{v}_{B/L} = 16.67 \text{ mm/s} \uparrow$ ◀

PROBLEM 11.55



Block C starts from rest at $t = 0$ and moves downward with a constant acceleration of 4 in./s^2. Knowing that block B has a constant velocity of 3 in./s upward, determine (*a*) the time when the velocity of block A is zero, (*b*) the time when the velocity of block A is equal to the velocity of block D, (*c*) the change in position of block A after 5 s.

SOLUTION



From the diagram:

Cord 1: $2y_A + 2y_B + y_C = \text{constant}$

Then $2v_A + 2v_B + v_C = 0$

and $2a_A + 2a_B + a_C = 0$ (1)

Cord 2: $(y_D - y_A) + (y_D - y_B) = \text{constant}$

Then $2v_D - v_A - v_B = 0$

and $2a_D - a_A - a_B = 0$ (2)

Use units of inches and seconds.

Motion of block C:

$$v_C = v_{C0} + a_C t$$
$$= 0 + 4t \quad \text{where} \quad a_C = -4 \text{ in./s}^2$$

Motion of block B: $v_B = -3 \text{ in./s}; \quad a_B = 0$

Motion of block A: From (1) and (2),

$$v_A = -v_B - \frac{1}{2}v_C = 3 - \frac{1}{2}(4t) = 3 - 2t \text{ in./s}$$

$$a_A = -a_B - \frac{1}{2}a_C = 0 - \frac{1}{2}(4) = -2 \text{ in./s}^2$$

PROBLEM 11.55 (Continued)

(*a*) Time when v_B is zero.

$$3 - 2t = 0 \qquad t = 1.500 \text{ s} \blacktriangleleft$$

Motion of block *D*: From (3),

$$v_D = \frac{1}{2}v_A + \frac{1}{2}v_B = \frac{1}{2}(3 - 2t) - \frac{1}{2}(3) = -1t$$

(*b*) Time when v_A is equal to v_0.

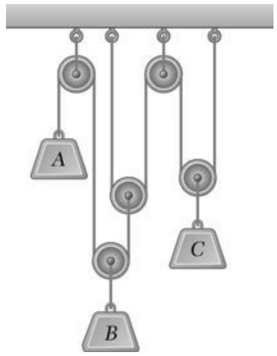
$$3 - 2t = -t \qquad t = 3.00 \text{ s} \blacktriangleleft$$

(*c*) Change in position of block *A* ($t = 5$ s).

$$\Delta y_A = (v_A)_0 t + \frac{1}{2}a_A t^2$$

$$= (3)(5) + \frac{1}{2}(-2)(5)^2 = -10 \text{ in.}$$

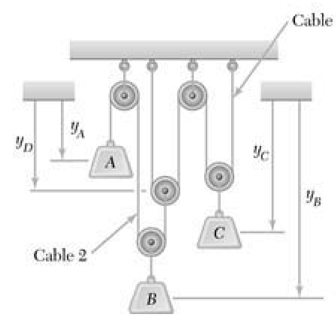
Change in position $= 10.00 \text{ in.}\uparrow \blacktriangleleft$



PROBLEM 11.56

Block *A* starts from rest at $t = 0$ and moves downward with a constant acceleration of 6 in./s^2. Knowing that block *B* moves up with a constant velocity of 3 in./s, determine (*a*) the time when the velocity of block *C* is zero, (*b*) the corresponding position of block *C*.

SOLUTION



The cable lengths are constant.

$$L_1 = 2y_C + 2y_D + \text{constant}$$

$$L_2 = y_A + y_B + (y_B - y_D) + \text{constant}$$

Eliminate y_D.

$$L_1 + 2L_2 = 2y_C + 2y_D + 2y_A + 2y_B + 2(y_B - y_D) + \text{constant}$$

$$2(y_C + y_A + 2y_B) = \text{constant}$$

Differentiate to obtain relationships for velocities and accelerations, positive downward.

$$v_C + v_A + 2v_B = 0 \quad (1)$$

$$a_C + a_A + 2a_B = 0 \quad (2)$$

Use units of inches and seconds.

Motion of block *A*: $v_A = a_A t + 6t$

$$\Delta y_A = \frac{1}{2}a_A t^2 = \frac{1}{2}(6)t^2 = 3t^2$$

Motion of block *B*: $\mathbf{v}_B = 3 \text{ in./s}\uparrow \qquad v_B = -3 \text{ in./s}$

$$\Delta y_B = v_B t = -3t$$

Motion of block *C*: From (1),

$$v_C = -v_A - 2v_B = -6t - 2(-3) = 6 - 6t$$

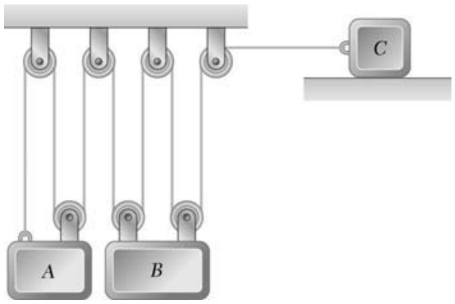
$$\Delta y_C = \int_0^t v_C\, dt = 6t - 3t^2$$

(*a*) Time when v_C is zero. $6 - 6t = 0$ $\qquad t = 1.000 \text{ s}$ ◀

(*b*) Corresponding position.

$$\Delta y_C = (6)(1) - (3)(1)^2 = 3 \text{ in.}$$

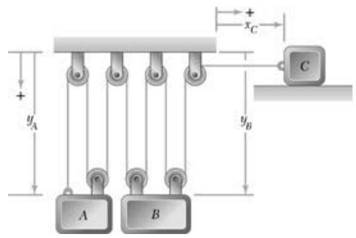
$\Delta y_C = 3.00 \text{ in.}\downarrow$ ◀



PROBLEM 11.57

Block B starts from rest, block A moves with a constant acceleration, and slider block C moves to the right with a constant acceleration of 75 mm/s^2. Knowing that at $t = 2$ s the velocities of B and C are 480 mm/s downward and 280 mm/s to the right, respectively, determine (*a*) the accelerations of A and B, (*b*) the initial velocities of A and C, (*c*) the change in position of slider block C after 3 s.

SOLUTION



From the diagram

$$3y_A + 4y_B + x_C = \text{constant}$$

Then $$3v_A + 4v_B + v_C = 0 \quad (1)$$

and $$3a_A + 4a_B + a_C = 0 \quad (2)$$

Given: $$(v_B) = 0,$$

$$a_A = \text{constent}$$

$$(\mathbf{a}_C) = 75 \text{ mm/s}^2 \longrightarrow$$

At $t = 2$ s, $$\mathbf{v}_B = 480 \text{ mm/s}\downarrow$$

$$\mathbf{v}_C = 280 \text{ mm/s} \longrightarrow$$

(*a*) Eq. (2) and $a_A = \text{constant}$ and $a_C = \text{constant} \Rightarrow a_B = \text{constant}$

Then $$v_B = 0 + a_B t$$

At $t = 2$ s: $$480 \text{ mm/s} = a_B(2 \text{ s})$$

$$a_B = 240 \text{ mm/s}^2 \qquad \text{or} \qquad \mathbf{a}_B = 240 \text{ mm/s}^2\downarrow \blacktriangleleft$$

Substituting into Eq. (2)

$$3a_A + 4(240 \text{ mm/s}^2) + (75 \text{ mm/s}^2) = 0$$

$$a_A = -345 \text{ mm/s} \qquad \text{or} \qquad \mathbf{a}_A = 345 \text{ mm/s}^2\uparrow \blacktriangleleft$$

PROBLEM 11.57 (Continued)

(*b*) We have $v_C = (v_C)_0 + a_C t$

At $t = 2$ s: $280 \text{ mm/s} = (v_C)_0 + (75 \text{ mm/s})(2 \text{ s})$

$v_C = 130 \text{ mm/s}$ or $(\mathbf{v}_C)_0 = 130.0 \text{ mm/s} \longrightarrow$ ◀

Then, substituting into Eq. (1) at $t = 0$

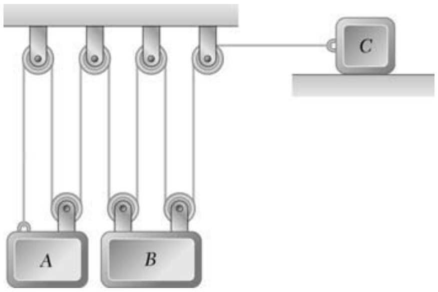
$$3(v_A)_0 + 4(0) + (130 \text{ mm/s}) = 0$$

$v_A = -43.3 \text{ mm/s}$ or $(\mathbf{v}_A)_0 = 43.3 \text{ mm/s} \uparrow$ ◀

(*c*) We have $x_C = (x_C)_0 + (v_C)_0 t + \frac{1}{2} a_C t^2$

At $t = 3$ s: $x_C - (x_C)_0 = (130 \text{ mm/s})(3 \text{ s}) + \frac{1}{2}(75 \text{ mm/s}^2)(3 \text{ s})^2$

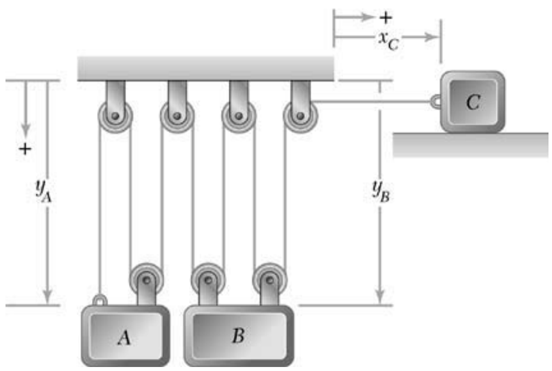
$= 728 \text{ mm}$ or $\mathbf{x}_C - (\mathbf{x}_C)_0 = 728 \text{ mm} \longrightarrow$ ◀



PROBLEM 11.58

Block B moves downward with a constant velocity of 20 mm/s. At $t = 0$, block A is moving upward with a constant acceleration, and its velocity is 30 mm/s. Knowing that at $t = 3$ s slider block C has moved 57 mm to the right, determine (*a*) the velocity of slider block C at $t = 0$, (*b*) the accelerations of A and C, (*c*) the change in position of block A after 5 s.

SOLUTION



From the diagram

$$3y_A + 4y_B + x_C = \text{constant}$$

Then $$3v_A + 4v_B + v_C = 0 \qquad (1)$$

and $$3a_A + 4a_B + a_C = 0 \qquad (2)$$

<u>Given</u>: $\mathbf{v}_B = 20 \text{ mm/s} \downarrow;$

$$(\mathbf{v}_A)_0 = 30 \text{ mm/s} \uparrow$$

(*a*) Substituting into Eq. (1) at $t = 0$

$$3(-30 \text{ mm/s}) + 4(20 \text{ mm/s}) + (v_C)_0 = 0$$

$$(v_C)_0 = 10 \text{ mm/s} \qquad \text{or} \qquad (\mathbf{v}_C)_0 = 10.00 \text{ mm/s} \longrightarrow \blacktriangleleft$$

(*b*) We have $$x_C = (x_C)_0 + (v_C)_0 t + \frac{1}{2} a_C t^2$$

At $t = 3$ s: $$57 \text{ mm} = (10 \text{ mm/s})(3 \text{ s}) + \frac{1}{2} a_C (3 \text{ s})^2$$

$$a_C = 6 \text{ mm/s}^2 \qquad \text{or} \qquad \mathbf{a}_C = 6.00 \text{ mm/s}^2 \longrightarrow \blacktriangleleft$$

Now $$\mathbf{v}_B = \text{constant} \rightarrow a_B = 0$$

PROBLEM 11.58 (Continued)

Then, substituting into Eq. (2)

$$3a_A + 4(0) + (6 \text{ mm/s}^2) = 0$$

$$a_A = -2 \text{ mm/s}^2 \qquad \text{or} \qquad \mathbf{a}_A = 2.00 \text{ mm/s}^2 \uparrow \blacktriangleleft$$

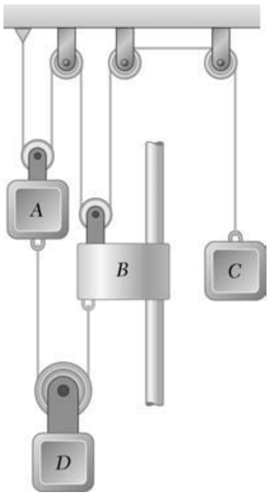
(*c*) We have $\quad y_A = (y_A)_0 + (v_A)_0 t + \frac{1}{2} a_A t^2$

At $t = 5$ s: $\quad y_A - (y_A)_0 = (-30 \text{ mm/s})(5 \text{ s}) + \frac{1}{2}(-2 \text{ mm/s}^2)(5 \text{ s})^2$

$$= -175 \text{ mm}$$

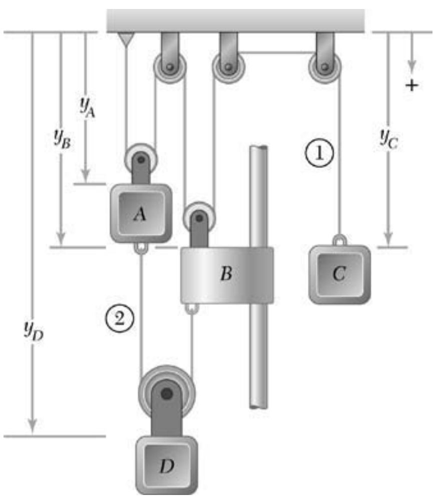
or $\quad \mathbf{y}_A - (\mathbf{y}_A)_0 = 175.0 \text{ mm} \uparrow \blacktriangleleft$

PROBLEM 11.59



The system shown starts from rest, and each component moves with a constant acceleration. If the relative acceleration of block C with respect to collar B is 60 mm/s^2 upward and the relative acceleration of block D with respect to block A is 110 mm/s^2 downward, determine (*a*) the velocity of block C after 3 s, (*b*) the change in position of block D after 5 s.

SOLUTION



From the diagram

Cable 1: $2y_A + 2y_B + y_C = \text{constant}$

Then $2v_A + 2v_B + v_C = 0$ (1)

and $2a_A + 2a_B + a_C = 0$ (2)

Cable 2: $(y_D - y_A) + (y_D - y_B) = \text{constant}$

Then $-v_A - v_B + 2v_D = 0$ (3)

and $-a_A - a_B + 2a_D = 0$ (4)

Given: At $t = 0,\ v = 0;$ all accelerations constant;
$a_{C/B} = 60 \text{ mm/s}^2\uparrow,\ a_{D/A} = 110 \text{ mm/s}^2\downarrow$

(a) We have $a_{C/B} = a_C - a_B = -60$ or $a_B = a_C + 60$

and $a_{D/A} = a_D - a_A = 110$ or $a_A = a_D - 110$

Substituting into Eqs. (2) and (4)

Eq. (2): $2(a_D - 110) + 2(a_C + 60) + a_C = 0$

or $3a_C + 2a_D = 100$ (5)

Eq. (4): $-(a_D - 110) - (a_C + 60) + 2a_D = 0$

or $-a_C + a_D = -50$ (6)

PROBLEM 11.59 (Continued)

Solving Eqs. (5) and (6) for a_C and a_D

$$a_C = 40 \text{ mm/s}^2$$

$$a_D = -10 \text{ mm/s}^2$$

Now $\quad v_C = 0 + a_C t$

At $t = 3$ s: $\quad v_C = (40 \text{ mm/s}^2)(3 \text{ s})$

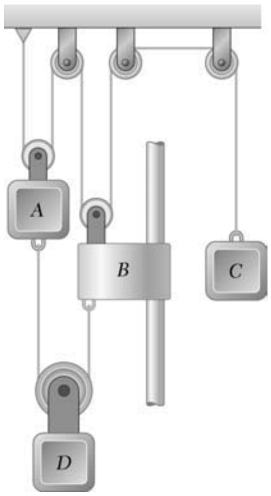
or $\quad \mathbf{v}_C = 120.0 \text{ mm/s} \downarrow$ ◀

(*b*) We have $\quad y_D = (y_D)_0 + (0)t + \frac{1}{2}a_D t^2$

At $t = 5$ s: $\quad y_D - (y_D)_0 = \frac{1}{2}(-10 \text{ mm/s}^2)(5 \text{ s})^2$

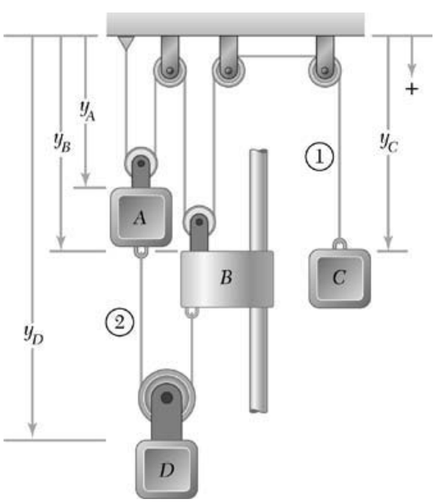
or $\quad \mathbf{y}_D - (\mathbf{y}_D)_0 = 125.0 \text{ mm} \uparrow$ ◀

PROBLEM 11.60*



The system shown starts from rest, and the length of the upper cord is adjusted so that A, B, and C are initially at the same level. Each component moves with a constant acceleration, and after 2 s the relative change in position of block C with respect to block A is 280 mm upward. Knowing that when the relative velocity of collar B with respect to block A is 80 mm/s downward, the displacements of A and B are 160 mm downward and 320 mm downward, respectively, determine (*a*) the accelerations of A and B if $a_B > 10\ \text{mm/s}^2$, (*b*) the change in position of block D when the velocity of block C is 600 mm/s upward.

SOLUTION



From the diagram

Cable 1: $\quad 2y_A + 2y_B + y_C = \text{constant}$

Then $$2v_A + 2v_B + v_C = 0 \qquad (1)$$

and $$2a_A + 2a_B + a_C = 0 \qquad (2)$$

Cable 2: $\quad (y_D - y_A) + (y_D - y_B) = \text{constant}$

Then $$-v_A - v_B - 2v_D = 0 \qquad (3)$$

and $$-a_A - a_B + 2a_D = 0 \qquad (4)$$

Given: At $\quad t = 0$

$$v = 0$$

$$(y_A)_0 = (y_B)_0 = (y_C)_0$$

All accelerations constant.

At $t = 2$ s

$$y_{C/A} = 280 \text{ mm} \uparrow$$

When $\quad v_{B/A} = 80 \text{ mm/s} \downarrow$

$$y_A - (y_A)_0 = 160 \text{ mm} \uparrow$$

$$y_B - (y_B)_0 = 320 \text{ mm} \downarrow$$

$$a_B > 10 \text{ mm/s}^2$$

(*a*) We have $y_A = (y_A)_0 + (0)t + \frac{1}{2}a_A t^2$

and $y_C = (y_C)_0 + (0)t + \frac{1}{2}a_C t^2$

Then $y_{C/A} = y_C - y_A = \frac{1}{2}(a_C - a_A)t^2$

At $t = 2$ s, $y_{C/A} = -280$ mm:

$$-280 \text{ mm} = \frac{1}{2}(a_C - a_A)(2 \text{ s})^2$$

or $a_C = a_A - 140$ (5)

Substituting into Eq. (2)

$$2a_A + 2a_B + (a_A - 140) = 0$$

or $a_A = \frac{1}{3}(140 - 2a_B)$ (6)

Now $v_B = 0 + a_B t$

$$v_A = 0 + a_A t$$

$$v_{B/A} = v_B - v_A = (a_B - a_A)t$$

Also $y_B = (y_B)_0 + (0)t + \frac{1}{2}a_B t^2$

When $\mathbf{v}_{B/A} = 80 \text{ mm/s}\downarrow$: $80 = (a_B - a_A)t$ (7)

$\Delta y_A = 160 \text{ mm}\downarrow$: $160 = \frac{1}{2}a_A t^2$

$\Delta y_B = 320 \text{ mm}\downarrow$: $320 = \frac{1}{2}a_B t^2$

Then $160 = \frac{1}{2}(a_B - a_A)t^2$

Using Eq. (7) $320 = (80)t$ or $t = 4$ s

Then $160 = \frac{1}{2}a_A(4)^2$ or $\mathbf{a}_A = 20.0 \text{ mm/s}^2\downarrow$ ◀

and $320 = \frac{1}{2}a_B(4)^2$ or $\mathbf{a}_B = 40.0 \text{ mm/s}^2\downarrow$ ◀

Note that Eq. (6) is not used; thus, the problem is over-determined.

PROBLEM 11.60* (Continued)

(*b*) Substituting into Eq. (5)

$$a_C = 20 - 140 = -120 \text{ mm/s}^2$$

and into Eq. (4)

$$-(20 \text{ mm/s}^2) - (40 \text{ mm/s}^2) + 2a_D = 0$$

or $$a_D = 30 \text{ mm/s}^2$$

Now $$v_C = 0 + a_C t$$

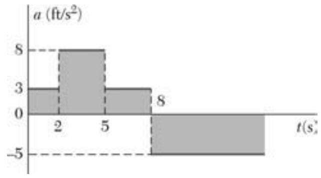
When $v_C = -600$ mm/s: $$-600 \text{ mm/s} = (-120 \text{ mm/s}^2)t$$

or $$t = 5 \text{ s}$$

Also $$y_D = (y_D)_0 + (0)t + \frac{1}{2}a_D t^2$$

At $t = 5$ s: $$y_D - (y_D)_0 = \frac{1}{2}(30 \text{ mm/s}^2)(5 \text{ s})^2$$

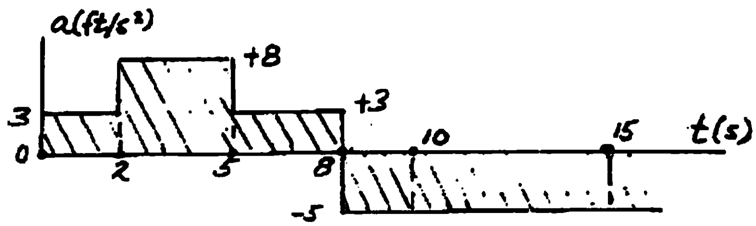
or $$\mathbf{y}_D - (\mathbf{y}_D)_0 = 375 \text{ mm}\downarrow \blacktriangleleft$$



PROBLEM 11.61

A particle moves in a straight line with the acceleration shown in the figure. Knowing that it starts from the origin with $v_0 = -14$ ft/s, plot the v–t and x–t curves for $0 < t < 15$ s and determine (*a*) the maximum value of the velocity of the particle, (*b*) the maximum value of its position coordinate.

SOLUTION



Change in v = area under a–t curve. $v_0 = -14$ ft/s

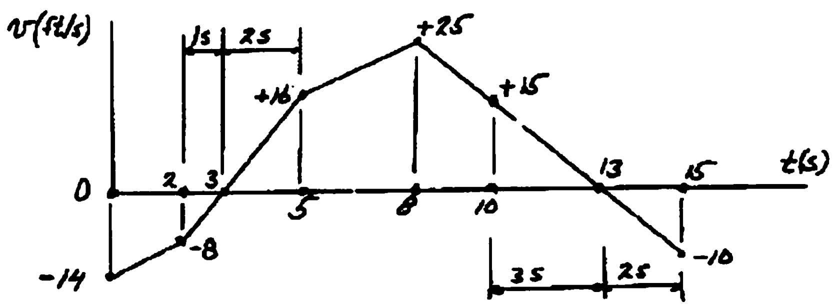
$t = 0$ to $t = 2$ s: $v_2 - v_0 = (3 \text{ ft/s}^2)(2 \text{ s}) = +6$ ft/s $v_2 = -8$ ft/s

$t = 2$ s to $t = 5$ s: $v_5 - v_2 = (8 \text{ ft/s}^2)(3 \text{ s}) = +24$ ft/s $v_5 = +16$ ft/s

$t = 5$ s to $t = 8$ s: $v_8 - v_5 = (3 \text{ ft/s}^2)(3 \text{ s}) = +9$ ft/s $v_8 = +25$ ft/s

$t = 8$ s to $t = 10$ s: $v_{10} - v_8 = (-5 \text{ ft/s}^2)(2 \text{ s}) = -10$ ft/s $v_{10} = +15$ ft/s

$t = 10$ s to $t = 15$ s: $v_{15} - v_{10} = (-5 \text{ ft/s}^2)(5 \text{ s}) = -25$ ft/s $v_{15} = -10$ ft/s



PROBLEM 11.61 (Continued)

Plot v–t curve. Then by similar triangles Δ's find t for $v = 0$.

Change in x = area under v–t curve $\qquad x_0 = 0$

$t = 0$ to $t = 2$ s: $\quad x_2 - x_0 = \frac{1}{2}(-14-8)(2) = -22$ ft $\qquad x_2 = -22$ ft

$t = 2$ s to $t = 3$ s: $\quad x_3 - x_2 = \frac{1}{2}(-8)(1) = -4$ ft $\qquad x_8 = -26$ ft

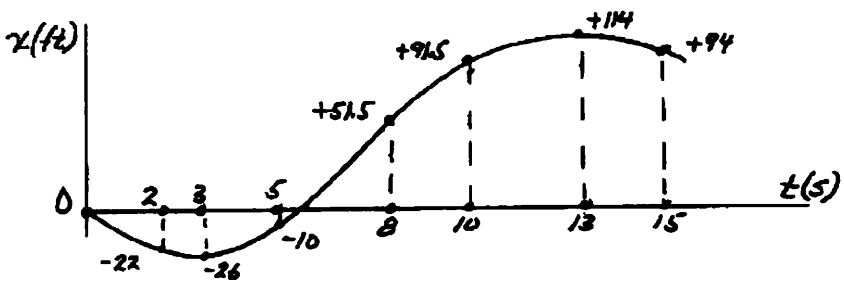
$t = 3$ s to $t = 5$ s: $\quad x_5 - x_3 = \frac{1}{2}(+16)(2) = +16$ ft $\qquad x_5 = -10$ ft

$t = 5$ s to $t = 8$ s: $\quad x_8 - x_5 = \frac{1}{2}(+16+25)(3) = +61.5$ ft $\qquad x_8 = +51.6$ ft

$t = 8$ s to $t = 10$ s: $\quad x_{10} - x_8 = \frac{1}{2}(+25+15)(2) = +40$ ft $\qquad x_{10} = +91.6$ ft

$t = 10$ s to $t = 13$ s: $\quad x_{13} - x_{10} = \frac{1}{2}(+15)(3) = +22.5$ ft $\qquad x_{13} = +114$ ft

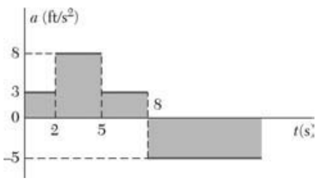
$t = 13$ s to $t = 15$ s: $\quad x_{15} - x_{13} = \frac{1}{2}(-10)(2) = -10$ ft $\qquad x_{15} = +94$ ft



(a) Maximum velocity: When $t = 8$ s, $\qquad v_m = 25.0$ ft/s ◀

(b) Maximum x: When $t = 13$ s, $\qquad x_m = 114.0$ ft ◀

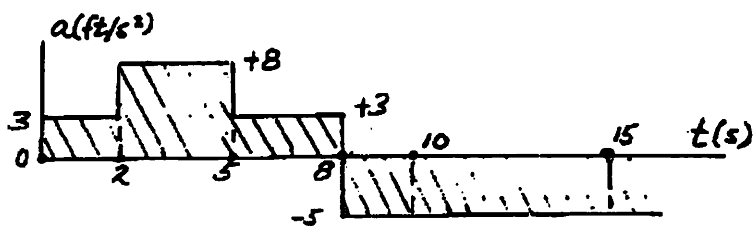
PROBLEM 11.62



For the particle and motion of Problem 11.61, plot the v–t and x–t curves for $0 < t < 15$ s and determine the velocity of the particle, its position, and the total distance traveled after 10 s.

PROBLEM 11.61 A particle moves in a straight line with the acceleration shown in the figure. Knowing that it starts from the origin with $v_0 = -14$ ft/s, plot the v–t and x–t curves for $0 < t < 15$ s and determine (*a*) the maximum value of the velocity of the particle, (*b*) the maximum value of its position coordinate.

SOLUTION



Change in v = area under a–t curve. $\qquad v_0 = -14$ ft/s

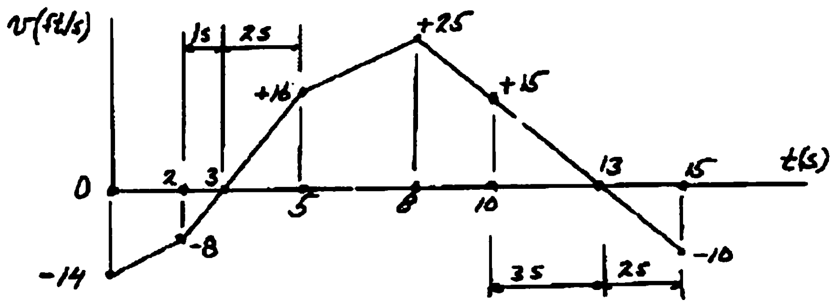
$t = 0$ to $t = 2$ s: $\quad v_2 - v_0 = (3\text{ ft/s}^2)(2\text{ s}) = +6$ ft/s $\qquad v_2 = -8$ ft/s

$t = 2$ s to $t = 5$ s: $\quad v_5 - v_2 = (8\text{ ft/s}^2)(3\text{ s}) = +24$ ft/s $\qquad v_5 = +16$ ft/s

$t = 5$ s to $t = 8$ s: $\quad v_8 - v_5 = (3\text{ ft/s}^2)(3\text{ s}) = +9$ ft/s $\qquad v_8 = +25$ ft/s

$t = 8$ s to $t = 10$ s: $\quad v_{10} - v_8 = (-5\text{ ft/s}^2)(2\text{ s}) = -10$ ft/s $\qquad v_{10} = +15$ ft/s

$t = 10$ s to $t = 15$ s: $\quad v_{15} - v_{10} = (-5\text{ ft/s}^2)(5\text{ s}) = -25$ ft/s $\qquad v_{15} = -10$ ft/s



PROBLEM 11.62 (Continued)

Plot v–t curve. Then by similar triangles Δ's find t for $v = 0$.

Change in x = area under v–t curve $\qquad x_0 = 0$

$t = 0$ to $t = 2$ s: $\quad x_2 - x_0 = \frac{1}{2}(-14-8)(2) = -22$ ft $\qquad x_2 = -22$ ft

$t = 2$ s to $t = 3$ s: $\quad x_3 - x_2 = \frac{1}{2}(-8)(1) = -4$ ft $\qquad x_8 = -26$ ft

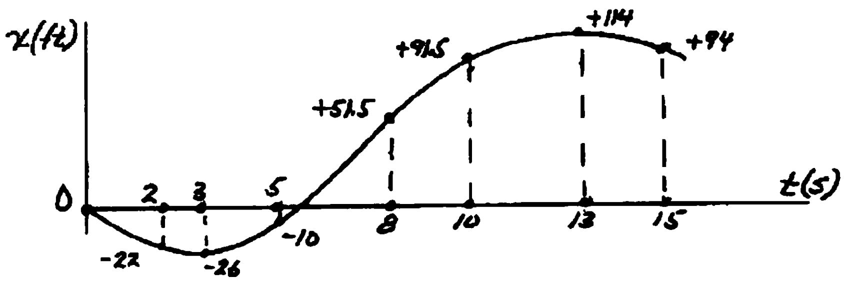
$t = 3$ s to $t = 5$ s: $\quad x_5 - x_3 = \frac{1}{2}(+16)(2) = +16$ ft $\qquad x_5 = -10$ ft

$t = 5$ s to $t = 8$ s: $\quad x_8 - x_5 = \frac{1}{2}(+16+25)(3) = +61.5$ ft $\qquad x_8 = +51.6$ ft

$t = 8$ s to $t = 10$ s: $\quad x_{10} - x_8 = \frac{1}{2}(+25+15)(2) = +40$ ft $\qquad x_{10} = +91.6$ ft

$t = 10$ s to $t = 13$ s: $\quad x_{13} - x_{10} = \frac{1}{2}(+15)(3) = +22.5$ ft $\qquad x_{13} = +114$ ft

$t = 13$ s to $t = 15$ s: $\quad x_{15} - x_{13} = \frac{1}{2}(-10)(2) = -10$ ft $\qquad x_{15} = +94$ ft



when $t = 10$ s: $\qquad v_{10} = +15$ ft/s ◀

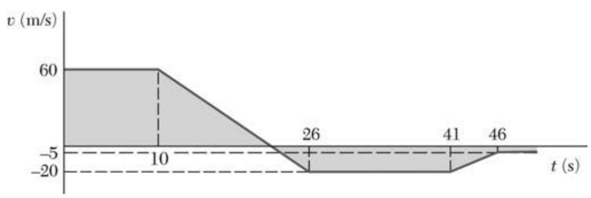
$x_{10} = +91.5$ ft/s ◀

Distance traveled: $t = 0$ to $t = 105$

$t = 0$ to $t = 3$ s: $\qquad$ Distance traveled = 26 ft

$t = 3$ s to $t = 10$ s $\qquad$ Distance traveled = 26 ft + 91.5 ft = 117.5 ft

Total distance traveled = 26 + 117.5 = 143.5 ft ◀



PROBLEM 11.63

A particle moves in a straight line with the velocity shown in the figure. Knowing that $x = -540$ m at $t = 0$, (a) construct the a–t and x–t curves for $0 < t < 50$ s, and determine (b) the total distance traveled by the particle when $t = 50$ s, (c) the two times at which $x = 0$.

SOLUTION

(a) $a_t =$ slope of v–t curve at time t

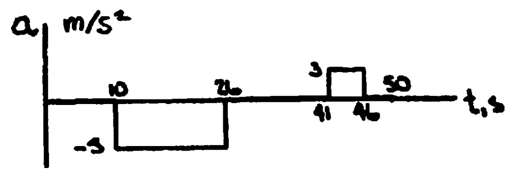
From $t = 0$ to $t = 10$ s: $v = \text{constant} \Rightarrow a = 0$

$t = 10$ s to $t = 26$ s: $a = \dfrac{-20 - 60}{26 - 10} = -5 \text{ m/s}^2$

$t = 26$ s to $t = 41$ s: $v = \text{constant} \Rightarrow a = 0$

$t = 41$ s to $t = 46$ s: $a = \dfrac{-5 - (-20)}{46 - 41} = 3 \text{ m/s}^2$

$t = 46$ s: $v = \text{constant} \Rightarrow a = 0$



$x_2 = x_1 +$ (area under v–t curve from t_1 to t_2)

At $t = 10$ s: $x_{10} = -540 + 10(60) + 60$ m

Next, find time at which $v = 0$. Using similar triangles

$$\frac{t_{v=0} - 10}{60} = \frac{26 - 10}{80} \quad \text{or} \quad t_{v=0} = 22 \text{ s}$$

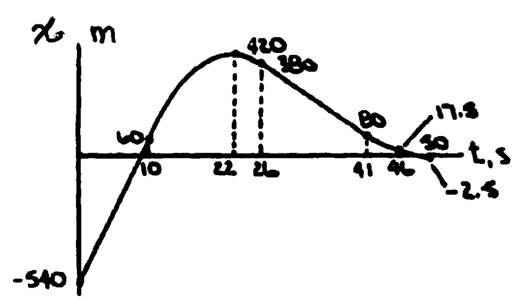
At $t = 22$ s: $x_{22} = 60 + \dfrac{1}{2}(12)(60) = 420$ m

$t = 26$ s: $x_{26} = 420 - \dfrac{1}{2}(4)(20) = 380$ m

$t = 41$ s: $x_{41} = 380 - 15(20) = 80$ m

$t = 46$ s: $x_{46} = 80 - 5\left(\dfrac{20 + 5}{2}\right) = 17.5$ m

$t = 50$ s: $x_{50} = 17.5 - 4(5) = -2.5$ m



PROBLEM 11.63 (Continued)

(*b*) From $t = 0$ to $t = 22$ s: Distance traveled $= 420 - (-540)$

$$= 960 \text{ m}$$

$t = 22$ s to $t = 50$ s: Distance traveled $= |-2.5 - 420|$

$$= 422.5 \text{ m}$$

Total distance traveled $= (960 + 422.5) \text{ ft} = 1382.5 \text{ m}$

Total distance traveled $= 1383 \text{ m}$ ◀

(*c*) Using similar triangles

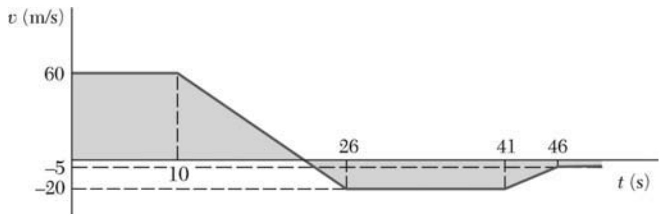
Between 0 and 10 s: $$\frac{(t_{x=0})_1 - 0}{540} = \frac{10}{600}$$

$(t_{x=0})_1 = 9.00 \text{ s}$ ◀

Between 46 s and 50 s: $$\frac{(t_{x=0})_2 - 46}{17.5} = \frac{4}{20}$$

$(t_{x=0})_2 = 49.5 \text{ s}$ ◀

PROBLEM 11.64



A particle moves in a straight line with the velocity shown in the figure. Knowing that $x = -540$ m at $t = 0$, (*a*) construct the $a-t$ and $x-t$ curves for $0 < t < 50$ s, and determine (*b*) the maximum value of the position coordinate of the particle, (*c*) the values of t for which the particle is at $x = 100$ m.

SOLUTION

(*a*) a_t = slope of $v-t$ curve at time t

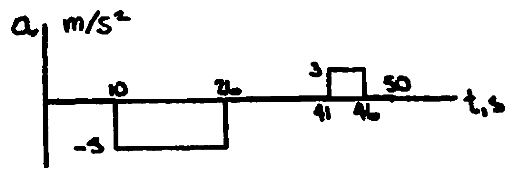
From $t = 0$ to $t = 10$ s: $v = \text{constant} \Rightarrow a = 0$

$t = 10$ s to $t = 26$ s: $a = \dfrac{-20 - 60}{26 - 10} = -5 \text{ m/s}^2$

$t = 26$ s to $t = 41$ s: $v = \text{constant} \Rightarrow a = 0$

$t = 41$ s to $t = 46$ s: $a = \dfrac{-5 - (-20)}{46 - 41} = 3 \text{ m/s}^2$

$t = 46$ s: $v = \text{constant} \Rightarrow a = 0$



$x_2 = x_1 +$ (area under $v-t$ curve from t_1 to t_2)

At $t = 10$ s: $x_{10} = -540 + 10(60) = 60$ m

Next, find time at which $v = 0$. Using similar triangles

$$\frac{t_{v=0} - 10}{60} = \frac{26 - 10}{80} \quad \text{or} \quad t_{v=0} = 22 \text{ s}$$

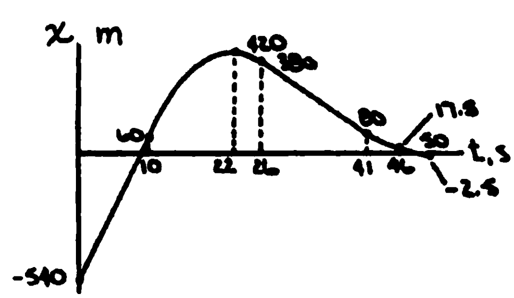
At $t = 22$ s: $x_{22} = 60 + \frac{1}{2}(12)(60) = 420$ m

$t = 26$ s: $x_{26} = 420 - \frac{1}{2}(4)(20) = 380$ m

$t = 41$ s: $x_{41} = 380 - 15(20) = 80$ m

$t = 46$ s: $x_{46} = 80 - 5\left(\dfrac{20 + 5}{2}\right) = 17.5$ m

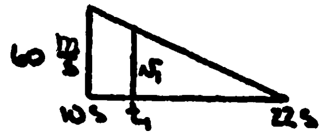
$t = 50$ s: $x_{50} = 17.5 - 4(5) = -2.5$ m



(*b*) Reading from the x–t curve $\qquad x_{max} = 420 \text{ m}$ ◀

(*c*) Between 10 s and 22 s

$100 \text{ m} = 420 \text{ m} - (\text{area under } v\text{–}t \text{ curve from } t_1 \text{ to } 22 \text{ s}) \text{ m}$



$$100 = 420 - \frac{1}{2}(22 - t_1)(v_1)$$

$$(22 - t_1)(v_1) = 640$$

Using similar triangles

$$\frac{v_1}{22 - t_1} = \frac{60}{12} \qquad \text{or} \qquad v_1 = 5(22 - t_1)$$

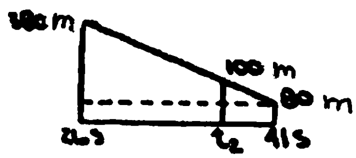
Then $\qquad (22 - t_1)[5(22 - t_1)] = 640$

$$t_1 = 10.69 \text{ s} \quad \text{and} \quad t_1 = 33.3 \text{ s}$$

We have $\qquad 10 \text{ s} < t_1 < 22 \text{ s} \Rightarrow \qquad t_1 = 10.69 \text{ s}$ ◀

Between 26 s and 41 s:

Using similar triangles



$$\frac{41 - t_2}{20} = \frac{15}{300}$$

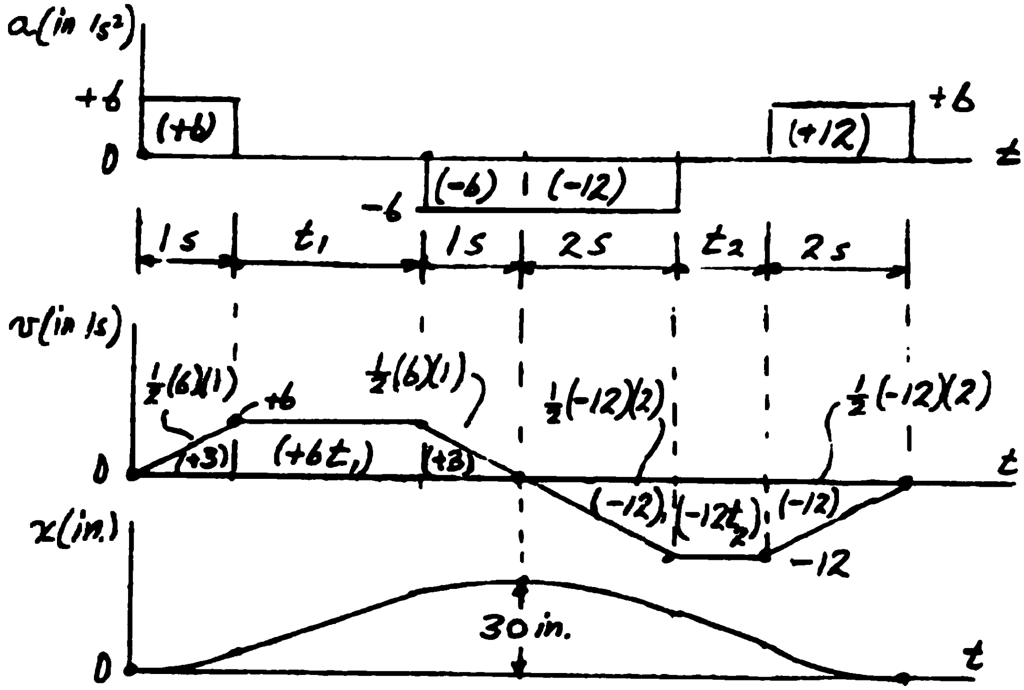
$t_2 = 40.0 \text{ s}$ ◀

PROBLEM 11.65

During a finishing operation the bed of an industrial planer moves alternately 30 in. to the right and 30 in. to the left. The velocity of the bed is limited to a maximum value of 6 in./s to the right and 12 in./s to the left; the acceleration is successively equal to 6 in./s^2 to the right, zero 6 in./s^2 to the left, zero, etc. Determine the time required for the bed to complete a full cycle, and draw the v–t and x–t curves.

SOLUTION

We choose positive to the right, thus the range of permissible velocities is $-12 \text{ in./s} < v < 6 \text{ in./s}$ since acceleration is $-6 \text{ in./s}^2, 0, \text{ or } +6 \text{ in./s}^2$. The slope the v–t curve must also be -6 in./s^2, 0, or $+6$ in./s^2.

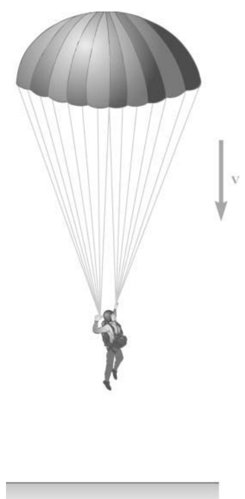


Planer moves = 30 in. to right: $+30 \text{ in.} = 3 + 6t_1 + 3$ $\qquad t_1 = 4.00 \text{ s}$

Planer moves = 30 in. to left: $-30 \text{ in.} = -12 - 12t_2 + 12$ $\qquad t_2 = 0.50 \text{ s}$

Total time $= 1 \text{ s} + 4 \text{ s} + 1 \text{ s} + 2 \text{ s} + 0.5 \text{ s} + 2 \text{ s} = 10.5 \text{ s}$ $\qquad t_{\text{total}} = 10.50 \text{ s}$ ◀

PROBLEM 11.66



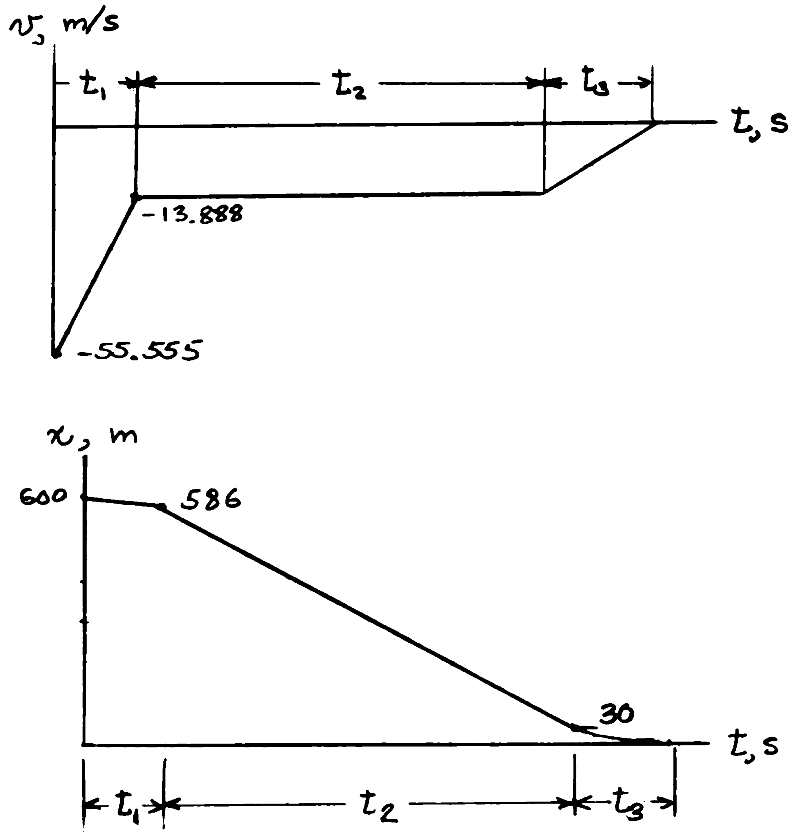
A parachutist is in free fall at a rate of 200 km/h when he opens his parachute at an altitude of 600 m. Following a rapid and constant deceleration, he then descends at a constant rate of 50 km/h from 586 m to 30 m, where he maneuvers the parachute into the wind to further slow his descent. Knowing that the parachutist lands with a negligible downward velocity, determine (*a*) the time required for the parachutist to land after opening his parachute, (*b*) the initial deceleration.

SOLUTION

Assume second deceleration is constant. Also, note that

$200 \text{ km/h} = 55.555 \text{ m/s},$

$50 \text{ km/h} = 13.888 \text{ m/s}$



PROBLEM 11.66 (Continued)

(*a*) Now $\Delta x =$ area under v–t curve for given time interval

Then

$$(586 - 600)\text{ m} = -t_1\left(\frac{55.555 + 13.888}{2}\right)\text{ m/s}$$

$$t_1 = 0.4032\text{ s}$$

$$(30 - 586)\text{ m} = -t_2(13.888\text{ m/s})$$

$$t_2 = 40.0346\text{ s}$$

$$(0 - 30)\text{ m} = -\frac{1}{2}(t_3)(13.888\text{ m/s})$$

$$t_3 = 4.3203\text{ s}$$

$$t_{\text{total}} = (0.4032 + 40.0346 + 4.3203)\text{ s}$$

$t_{\text{total}} = 44.8\text{ s}$ ◀

(*b*) We have

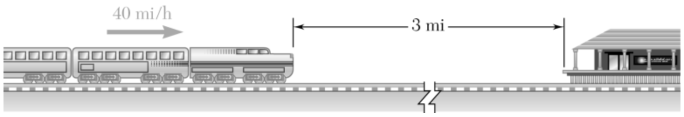
$$a_{\text{initial}} = \frac{\Delta v_{\text{initial}}}{t_1}$$

$$= \frac{[-13.888 - (-55.555)]\text{ m/s}}{0.4032\text{ s}}$$

$$= 103.3\text{ m/s}^2$$

$\mathbf{a}_{\text{initial}} = 103.3\text{ m/s}^2 \uparrow$ ◀

PROBLEM 11.67

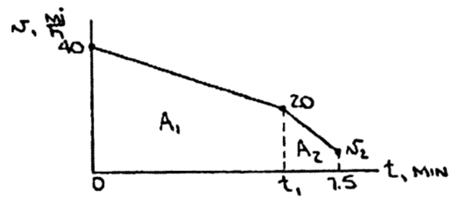


A commuter train traveling at 40 mi/h is 3 mi from a station. The train then decelerates so that its speed is 20 mi/h when it is 0.5 mi from the station. Knowing that the train arrives at the station 7.5 min after beginning to decelerate and assuming constant decelerations, determine (*a*) the time required for the train to travel the first 2.5 mi, (*b*) the speed of the train as it arrives at the station, (*c*) the final constant deceleration of the train.

SOLUTION

Given: At $t = 0$, $v = 40$ mi/h, $x = 0$; when $x = 2.5$ mi, $v = 20$ mi/h;
at $t = 7.5$ min, $x = 3$ mi; constant decelerations.

The $v-t$ curve is first drawn as shown.



(*a*) We have $A_1 = 2.5$ mi

$$(t_1 \text{ min})\left(\frac{40+20}{2}\right)\text{mi/h} \times \frac{1\text{ h}}{60\text{ min}} = 2.5\text{ mi}$$

$$t_1 = 5.00\text{ min} \blacktriangleleft$$

(*b*) We have $A_2 = 0.5$ mi

$$(7.5-5)\text{ min} \times \left(\frac{20+v_2}{2}\right)\text{mi/h} \times \frac{1\text{ h}}{60\text{ min}} = 0.5\text{ mi}$$

$$v_2 = 4.00\text{ mi/h} \blacktriangleleft$$

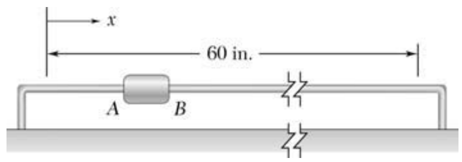
(*c*) We have

$$a_{\text{final}} = a_{12}$$

$$= \frac{(4-20)\text{ mi/h}}{(7.5-5)\text{ min}} \times \frac{5280\text{ ft}}{\text{mi}} \times \frac{1\text{ min}}{60\text{ s}} \times \frac{1\text{ h}}{3600\text{ s}}$$

$$a_{\text{final}} = -0.1564\text{ ft/s}^2 \blacktriangleleft$$

PROBLEM 11.68



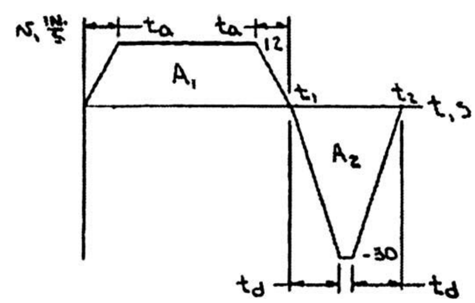
A temperature sensor is attached to slider AB which moves back and forth through 60 in. The maximum velocities of the slider are 12 in./s to the right and 30 in./s to the left. When the slider is moving to the right, it accelerates and decelerates at a constant rate of 6 in./s^2; when moving to the left, the slider accelerates and decelerates at a constant rate of 20 in./s^2. Determine the time required for the slider to complete a full cycle, and construct the $v-t$ and $x-t$ curves of its motion.

SOLUTION

The $v-t$ curve is first drawn as shown. Then

$$t_a = \frac{v_{\text{right}}}{a_{\text{right}}} = \frac{12 \text{ in./s}}{6 \text{ in./s}^2} = 2 \text{ s}$$

$$t_d = \frac{v_{\text{left}}}{a_{\text{left}}} = \frac{30 \text{ in./s}}{20 \text{ in./s}}$$
$$= 1.5 \text{ s}$$



Now $A_1 = 60$ in.

or $[(t_1 - 2) \text{ s}](12 \text{ in./s}) = 60 \text{ in.}$

or $t_1 = 7$ s

and $A_2 = 60$ in.

or $\{[(t_2 - 7) - 1.5] \text{ s}\}(30 \text{ in./s}) = 60 \text{ in.}$

or $t_2 = 10.5$ s

Now $t_{\text{cycle}} = t_2$ $\qquad t_{\text{cycle}} = 10.5$ s ◀

We have $x_{ii} = x_i + (\text{area under } v\text{–}t \text{ curve from } t_i \text{ to } t_{ii})$

At $t = 2$ s: $\quad x_2 = \frac{1}{2}(2)(12) = 12$ in.

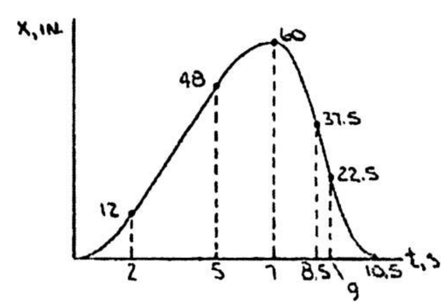
$t = 5$ s: $\quad x_5 = 12 + (5-2)(12)$
$= 48$ in.

$t = 7$ s: $\quad x_7 = 60$ in.

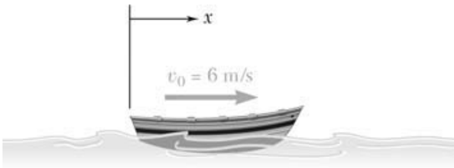
$t = 8.5$ s: $\quad x_{8.5} = 60 - \frac{1}{2}(1.5)(30)$
$= 37.5$ in.

$t = 9$ s: $\quad x_9 = 37.5 - (0.5)(30)$
$= 22.5$ in.

$t = 10.5$ s: $\quad x_{10.5} = 0$



PROBLEM 11.69



In a water-tank test involving the launching of a small model boat, the model's initial horizontal velocity is 6 m/s, and its horizontal acceleration varies linearly from -12 m/s^2 at $t = 0$ to -2 m/s^2 at $t = t_1$ and then remains equal to -2 m/s^2 until $t = 1.4$ s. Knowing that $v = 1.8$ m/s when $t = t_1$, determine (*a*) the value of t_1, (*b*) the velocity and the position of the model at $t = 1.4$ s.

SOLUTION

<u>Given</u>: $v_0 = 6\text{ m/s; for } 0 < t < t_1,$

for $t_1 < t < 1.4\text{ s} \quad a = -2\text{ m/s}^2;$

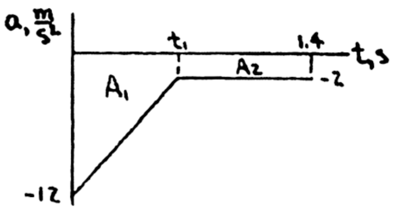
at $t = 0 \quad a = -12\text{ m/s}^2;$

at $t = t_1$ $\quad a = -2\text{ m/s}^2,\ v = 1.8\text{ m/s}^2$

The $a\!-\!t$ and $v\!-\!t$ curves are first drawn as shown. The time axis is not drawn to scale.

(*a*) We have $v_{t_1} = v_0 + A_1$

$$1.8\text{ m/s} = 6\text{ m/s} - (t_1\text{ s})\left(\frac{12+2}{2}\right)\text{m/s}^2$$

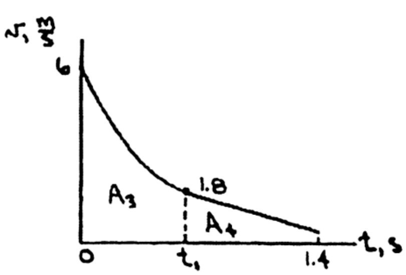


$t_1 = 0.6\text{ s}$ ◀

(*b*) We have $v_{1.4} = v_{t_1} + A_2$

$$v_{1.4} = 1.8\text{ m/s} - (1.4 - 0.6)\text{ s} \times 2\text{ m/s}^2$$

$v_{1.4} = 0.20\text{ m/s}$ ◀



Now $x_{1.4} = A_3 + A_4$, where A_3 is most easily determined using integration. Thus,

for $0 < t < t_1$: $\quad a = \dfrac{-2-(-12)}{0.6}t - 12 = \dfrac{50}{3}t - 12$

Now $\quad \dfrac{dv}{dt} = a = \dfrac{50}{3}t - 12$

PROBLEM 11.69 (Continued)

At $t = 0$, $v = 6$ m/s: $\int_6^v dv = \int_0^t \left(\frac{50}{3}t - 12\right)dt$

or $v = 6 + \frac{25}{3}t^2 - 12t$

We have $\frac{dx}{dt} = v = 6 - 12t + \frac{25}{3}t^2$

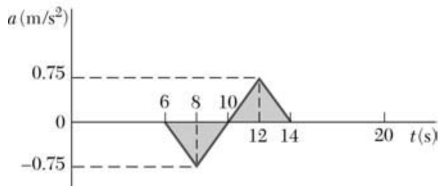
Then $A_3 = \int_0^{x_{t_1}} dx = \int_0^{0.6} (6 - 12t + \frac{25}{3}t^2)dt$

$$= \left[6t - 6t^2 + \frac{25}{9}t^3\right]_0^{0.6} = 2.04 \text{ m}$$

Also $A_4 = (1.4 - 0.6)\left(\frac{1.8 + 0.2}{2}\right) = 0.8$ m

Then $x_{1.4} = (2.04 + 0.8)$ m

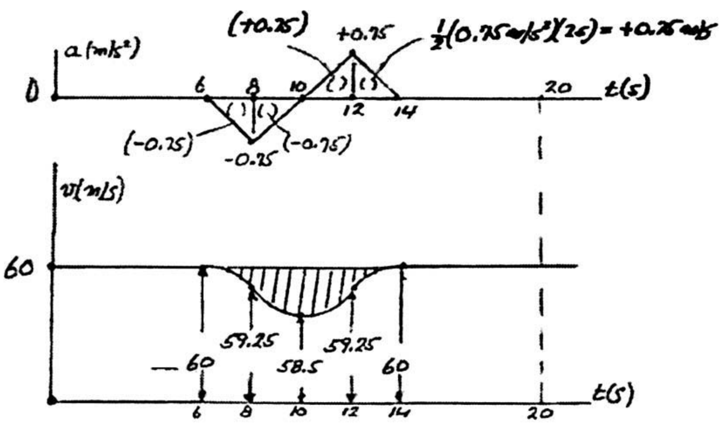
or $x_{1.4} = 2.84$ m ◀



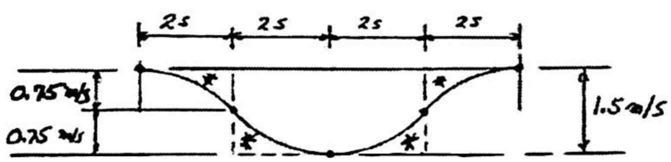
PROBLEM 11.70

The acceleration record shown was obtained for a small airplane traveling along a straight course. Knowing that $x = 0$ and $v = 60$ m/s when $t = 0$, determine (*a*) the velocity and position of the plane at $t = 20$ s, (*b*) its average velocity during the interval $6 \text{ s} < t < 14$ s.

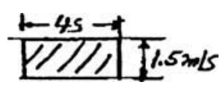
SOLUTION



Geometry of "bell-shaped" portion of $v-t$ curve



The parabolic spandrels marked by * are of equal area. Thus, total area of shaded portion of $v-t$ diagram is:



$= \Delta x = 6$ m

(*a*) When $t = 20$ s: $v_{20} = 60$ m/s ◀

$$x_{20} = (60 \text{ m/s})(20 \text{ s}) - (\text{shaded area})$$

$$= 1200 \text{ m} - 6 \text{ m}$$

$x_{20} = 1194$ m ◀

(*b*) From $t = 6$ s to $t = 14$ s: $\Delta t = 8$ s

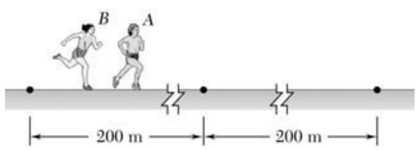
$$\Delta x = (60 \text{ m/s})(14 \text{ s} - 6 \text{ s}) - (\text{shaded area})$$

$$= (60 \text{ m/s})(8 \text{ s}) - 6 \text{ m} = 480 \text{ m} - 6 \text{ m} = 474 \text{ m}$$

$$v_{\text{average}} = \frac{\Delta x}{\Delta t} = \frac{474 \text{ m}}{8 \text{ s}}$$

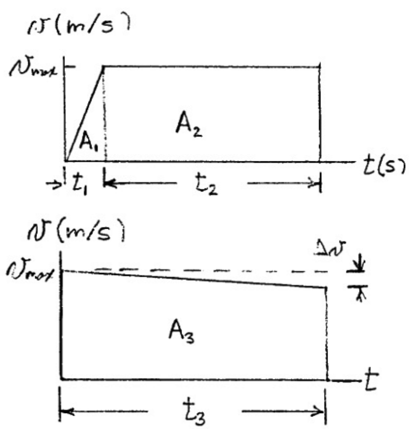
$v_{\text{average}} = 59.25$ m/s ◀

PROBLEM 11.71



In a 400-m race, runner A reaches her maximum velocity v_A in 4 s with constant acceleration and maintains that velocity until she reaches the half-way point with a split time of 25 s. Runner B reaches her maximum velocity v_B in 5 s with constant acceleration and maintains that velocity until she reaches the half-way point with a split time of 25.2 s. Both runners then run the second half of the race with the same constant deceleration of 0.1 m/s^2. Determine (*a*) the race times for both runners, (*b*) the position of the winner relative to the loser when the winner reaches the finish line.

SOLUTION



Sketch v–t curves for first 200 m.

Runner A: $t_1 = 4$ s, $t_2 = 25 - 4 = 21$ s

$$A_1 = \frac{1}{2}(4)(v_A)_{max} = 2(v_A)_{max}$$

$$A_2 = 21(v_A)_{max}$$

$$A_1 + A_2 = \Delta x = 200 \text{ m}$$

$$23(v_A)_{max} = 200 \quad \text{or} \quad (v_A)_{max} = 8.6957 \text{ m/s}$$

Runner B: $t_1 = 5$ s, $t_2 = 25.2 - 5 = 20.2$ s

$$A_1 = \frac{1}{2}(5)(v_B)_{max} = 2.5(v_B)_{max}$$

$$A_2 = 20.2(v_B)_{max}$$

$$A_1 + A_2 = \Delta x = 200 \text{ m}$$

$$22.7(v_B)_{max} = 200 \quad \text{or} \quad (v_B)_{max} = 8.8106 \text{ m/s}$$

Sketch v–t curve for second 200 m. $\Delta v = |a|t_3 = 0.1t_3$

$$A_3 = v_{max}t_3 - \frac{1}{2}\Delta v t_3 = 200 \quad \text{or} \quad 0.05t_3^2 - v_{max}t_3 + 200 = 0$$

$$t_3 = \frac{v_{max} \pm \sqrt{(v_{max})^2 - (4)(0.05)(200)}}{(2)(0.05)} = 10\left(v_{max} \pm \sqrt{(v_{max})^2 - 40}\right)$$

Runner A: $(v_{max})_A = 8.6957$, $(t_3)_A = 146.64$ s and 27.279 s

Reject the larger root. Then total time $t_A = 25 + 27.279 = 52.279$ s $\qquad t_A = 52.2 \text{ s}$ ◀

PROBLEM 11.71 (Continued)

Runner B: $(v_{\text{max}})_B = 8.8106$, $(t_3)_B = 149.45$ s and 26.765 s

Reject the larger root. Then total time $t_B = 25.2 + 26.765 = 51.965$ s

$t_B = 52.0$ s ◀

Velocity of A at $t = 51.965$ s:

$$v_1 = 8.6957 - (0.1)(51.965 - 25) = 5.999 \text{ m/s}$$

Velocity of A at $t = 51.279$ s:

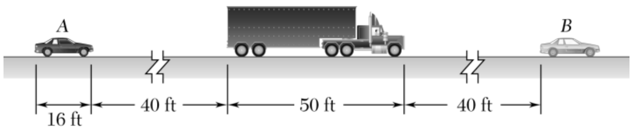
$$v_2 = 8.6957 - (0.1)(52.279 - 25) = 5.968 \text{ m/s}$$

Over $51.965 \text{ s} \leq t \leq 52.965$ s, runner A covers a distance Δx

$$\Delta x = v_{\text{ave}}(\Delta t) = \frac{1}{2}(5.999 + 5.968)(52.279 - 51.965)$$

$\Delta x = 1.879$ m ◀

PROBLEM 11.72

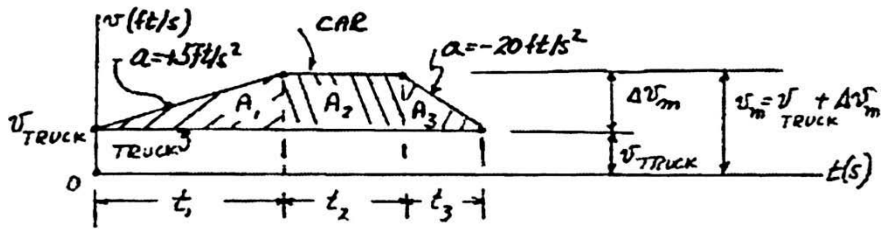


A car and a truck are both traveling at the constant speed of 35 mi/h; the car is 40 ft behind the truck. The driver of the car wants to pass the truck, i.e., he wishes to place his car at B, 40 ft in front of the truck, and then resume the speed of 35 mi/h. The maximum acceleration of the car is 5 ft/s^2 and the maximum deceleration obtained by applying the brakes is 20 ft/s^2. What is the shortest time in which the driver of the car can complete the passing operation if he does not at any time exceed a speed of 50 mi/h? Draw the $v-t$ curve.

SOLUTION

Relative to truck, car must move a distance: $\Delta x = 16 + 40 + 50 + 40 = 146\text{ ft}$

Allowable increase in speed: $\Delta v_m = 50 - 35 = 15\text{ mi/h} = 22\text{ ft/s}$



Acceleration Phase: $t_1 = 22/5 = 4.4\text{ s}$ $\quad A_1 = \frac{1}{2}(22)(4.4) = 48.4\text{ ft}$

Deceleration Phase: $t_3 = 22/20 = 1.1\text{ s}$ $\quad A_3 = \frac{1}{2}(22)(1.1) = 12.1\text{ ft}$

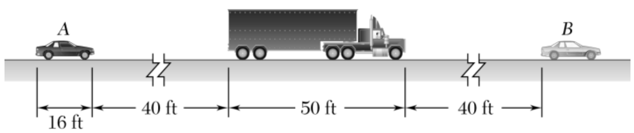
But: $\Delta x = A_1 + A_2 + A_3$: $\quad 146\text{ ft} = 48.4 + (22)t_2 + 12.1$ $\quad t_2 = 3.89\text{ s}$

$t_{\text{total}} = t_1 + t_2 + t_3 = 4.4\text{ s} + 3.89\text{ s} + 1.1\text{ s} = 9.39\text{ s}$ $\quad t_B = 9.39\text{ s}$ ◀

PROBLEM 11.73

Solve Problem 11.72, assuming that the driver of the car does not pay any attention to the speed limit while passing and concentrates on reaching position *B* and resuming a speed of 35 mi/h in the shortest possible time. What is the maximum speed reached? Draw the $v\!-\!t$ curve.

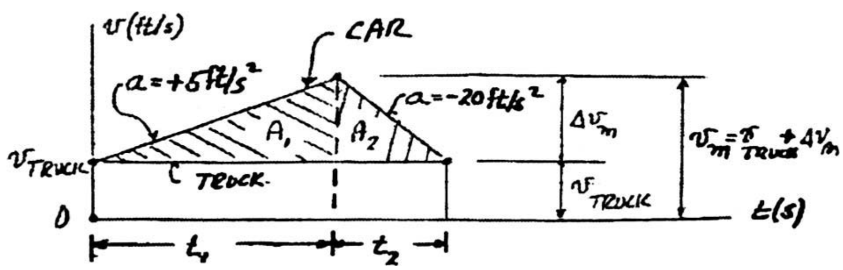
PROBLEM 11.72 A car and a truck are both traveling at the constant speed of 35 mi/h; the car is 40 ft behind the truck. The driver of the car wants to pass the truck, i.e., he wishes to place his car at *B*, 40 ft in front of the truck, and then resume the speed of 35 mi/h. The maximum acceleration of the car is $5\ \text{ft/s}^2$ and the maximum deceleration obtained by applying the brakes is $20\ \text{ft/s}^2$. What is the shortest time in which the driver of the car can complete the passing operation if he does not at any time exceed a speed of 50 mi/h? Draw the $v\!-\!t$ curve.



SOLUTION

Relative to truck, car must move a distance:

$$\Delta x = 16 + 40 + 50 + 40 = 146\ \text{ft}$$



$$\Delta v_m = 5t_1 = 20t_2; \quad t_2 = \frac{1}{4}t_1$$

$\Delta x = A_1 + A_2$:

$$146\ \text{ft} = \frac{1}{2}(\Delta v_m)(t_1 + t_2)$$

$$146\ \text{ft} = \frac{1}{2}(5t_1)\left(t_1 + \frac{1}{4}t_1\right)$$

$$t_1^2 = 46.72 \qquad t_1 = 6.835\ \text{s} \qquad t_2 = \frac{1}{4}t_1 = 1.709$$

$$t_{\text{total}} = t_1 + t_2 = 6.835 + 1.709$$

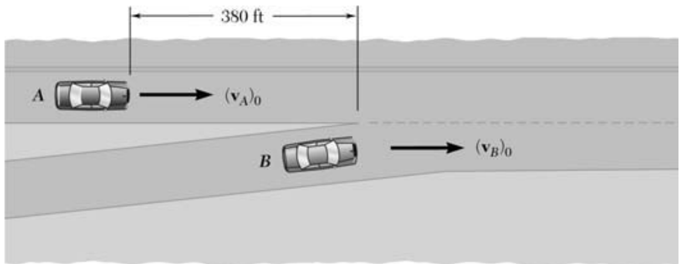
$t_B = 8.54\ \text{s}$ ◀

$$\Delta v_m = 5t_1 = 5(6.835) = 34.18\ \text{ft/s} = 23.3\ \text{mi/h}$$

Speed $v_{\text{total}} = 35\ \text{mi/h}$, $v_m = 35\ \text{mi/h} + 23.3\ \text{mi/h}$

$v_m = 58.3\ \text{mi/h}$ ◀

PROBLEM 11.74



Car A is traveling on a highway at a constant speed $(v_A)_0 = 60$ mi/h, and is 380 ft from the entrance of an access ramp when car B enters the acceleration lane at that point at a speed $(v_B)_0 = 15$ mi/h. Car B accelerates uniformly and enters the main traffic lane after traveling 200 ft in 5 s. It then continues to accelerate at the same rate until it reaches a speed of 60 mi/h, which it then maintains. Determine the final distance between the two cars.

SOLUTION

Given: $(v_A)_0 = 60$ mi/h, $(v_B)_0 = 1.5$ mi/h; at $t = 0$,

$(x_A)_0 = -380$ ft, $(x_B)_0 = 0$; at $t = 5$ s,

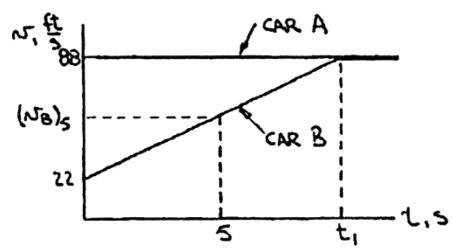
$x_B = 200$ ft; for 15 mi/h $< v_B \leq 60$ mi/h,

$a_B =$ constant; for $v_B = 60$ mi/h,

$a_B = 0$

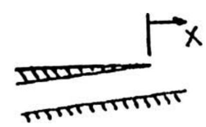
First note 60 mi/h = 88 ft/s

15 mi/h = 22 ft/s



The v–t curves of the two cars are then drawn as shown.

Using the coordinate system shown, we have



at $t = 5$ s, $x_B = 200$ ft: $$(5\text{ s})\left[\frac{22 + (v_B)_5}{2}\right]\text{ft/s} = 200\text{ ft}$$

or $$(v_B)_5 = 58\text{ ft/s}$$

Then, using similar triangles, we have

$$\frac{(88-22)\text{ ft/s}}{t_1} = \frac{(58-22)\text{ ft/s}}{5\text{ s}} (= a_B)$$

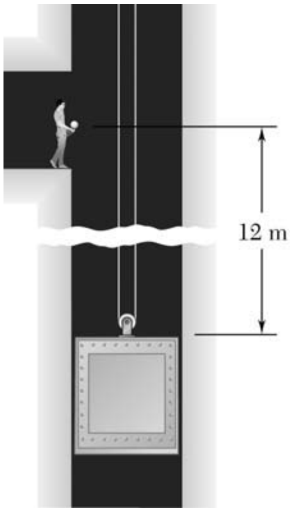
or $$t_1 = 9.1667\text{ s}$$

Finally, at $t = t_1$

$$x_{B/A} = x_B - x_A = \left[(9.1667\text{ s})\left(\frac{22+88}{2}\right)\text{ft/s}\right] - [-380\text{ ft} + (9.1667\text{ s})(88\text{ ft/s})]$$

or $$x_{B/A} = 77.5\text{ ft} \blacktriangleleft$$

PROBLEM 11.75



An elevator starts from rest and moves upward, accelerating at a rate of 1.2 m/s^2 until it reaches a speed of 7.8 m/s, which it then maintains. Two seconds after the elevator begins to move, a man standing 12 m above the initial position of the top of the elevator throws a ball upward with an initial velocity of 20 m/s. Determine when the ball will hit the elevator.

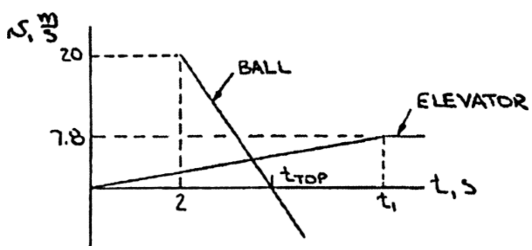
SOLUTION

Given: At $t = 0$ $v_E = 0$; For $0 < v_E \leq 7.8$ m/s, $a_E = 1.2$ m/s^2 $\uparrow$;

For $v_E = 7.8$ m/s, $a_E = 0$;

At $t = 2$ s, $v_B = 20$m/s $\uparrow$

The $v-t$ curves of the ball and the elevator are first drawn as shown. Note that the initial slope of the curve for the elevator is 1.2 m/s^2, while the slope of the curve for the ball is $-g$ (-9.81 m/s^2).



The time t_1 is the time when v_E reaches 7.8 m/s.

Thus, $$v_E = (0) + a_E t$$

or $$7.8 \text{ m/s} = (1.2 \text{ m/s}^2)t_1$$

or $$t_1 = 6.5 \text{ s}$$

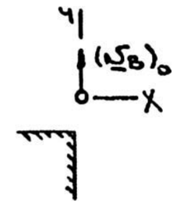
The time t_{top} is the time at which the ball reaches the top of its trajectory.

Thus, $$v_B = (v_B)_0 - g(t-2)$$

or $$0 = 20 \text{ m/s} - (9.81 \text{ m/s}^2)(t_{top} - 2) \text{ s}$$

or $$t_{top} = 4.0387 \text{ s}$$

Using the coordinate system shown, we have



$0 < t < t_1$: $\quad y_E = -12 \text{ m} + \left(\frac{1}{2} a_E t^2\right) \text{ m}$

At $t = t_{\text{top}}$: $\quad y_B = \frac{1}{2}(4.0387 - 2) \text{ s} \times (20 \text{ m/s})$

$$= 20.387 \text{ m}$$

and $\quad y_E = -12 \text{ m} + \frac{1}{2}(1.2 \text{ m/s}^2)(4.0387 \text{ s})^2$

$$= -2.213 \text{ m}$$

At $\quad t = [2 + 2(4.0387 - 2)] \text{ s} = 6.0774 \text{ s}, \;\; y_B = 0$

and at $t = t_1$, $\quad y_E = -12 \text{ m} + \frac{1}{2}(6.5 \text{ s})(7.8 \text{ m/s}) = 13.35 \text{ m}$

The ball hits the elevator $(y_B = y_E)$ when $t_{\text{top}} \leq t \leq t_1$.

For $t \geq t_{\text{top}}$: $\quad y_B = 20.387 \text{ m} - \left[\frac{1}{2} \text{ g } (t - t_{\text{top}})^2\right] \text{ m}$

Then,

when $\quad y_B = y_E$

$$20.387 \text{ m} - \frac{1}{2}(9.81 \text{ m/s}^2)(t - 4.0387)^2$$

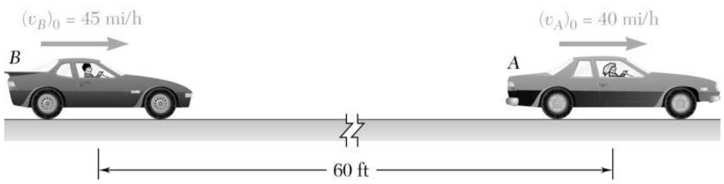
$$= -12 \text{ m} + \frac{1}{2}(1.2 \text{ m/s}^2)(t \text{ s})^2$$

or $\quad 5.505t^2 - 39.6196t + 47.619 = 0$

Solving $\quad t = 1.525 \text{ s} \quad \text{and} \quad t = 5.67 \text{ s}$

Since 1.525 s is less than 2 s, $\quad t = 5.67 \text{ s}$ ◀

PROBLEM 11.76



Car A is traveling at 40 mi/h when it enters a 30 mi/h speed zone. The driver of car A decelerates at a rate of 16 ft/s^2 until reaching a speed of 30 mi/h, which she then maintains. When car B, which was initially 60 ft behind car A and traveling at a constant speed of 45 mi/h, enters the speed zone, its driver decelerates at a rate of 20 ft/s^2 until reaching a speed of 28 mi/h. Knowing that the driver of car B maintains a speed of 28 mi/h, determine (*a*) the closest that car B comes to car A, (*b*) the time at which car A is 70 ft in front of car B.

SOLUTION

<u>Given</u>: $(v_A)_0 = 40$ mi/h; For 30 mi/h $< v_A \le 40$ mi/h, $a_A = -16$ ft/s^2; For $v_A = 30$ mi/h, $a_A = 0$;

$(x_{A/B})_0 = 60$ ft; $(v_B)_0 = 45$ mi/h;

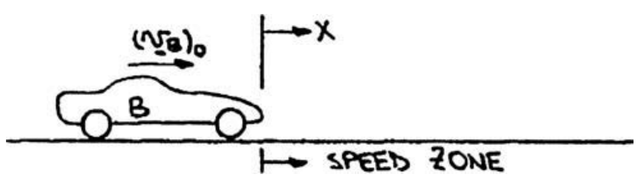
When $x_B = 0$, $a_B = -20$ ft/s^2;

For $v_B = 28$ mi/h, $a_B = 0$

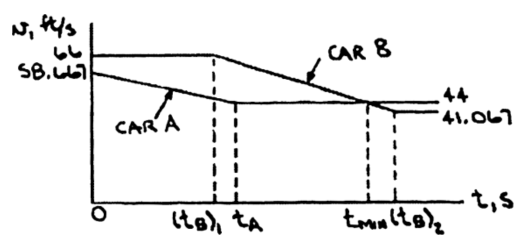
First note 40 mi/h = 58.667 ft/s 30 mi/h = 44 ft/s

45 mi/h = 66 ft/s 28 mi/h = 41.067 ft/s

At $t = 0$



The $v-t$ curves of the two cars are as shown.



At $t = 0$: Car A enters the speed zone.

$t = (t_B)_1$: Car B enters the speed zone.

$t = t_A$: Car A reaches its final speed.

$t = t_{\min}$: $v_A = v_B$

$t = (t_B)_2$: Car B reaches its final speed.

(*a*) We have $$a_A = \frac{(v_A)_{\text{final}} - (v_A)_0}{t_A}$$

or $$-16\text{ ft/s}^2 = \frac{(44 - 58.667)\text{ ft/s}}{t_A}$$

or $$t_A = 0.91669\text{ s}$$

Also $$60\text{ ft} = (t_B)_1(v_B)_0$$

or $$60\text{ ft} = (t_B)_1(66\text{ ft/s}) \qquad \text{or} \qquad (t_B)_1 = 0.90909\text{ s}$$

and $$a_B = \frac{(v_B)_{\text{final}} - (v_B)_0}{(t_B)_2 - (t_B)_1}$$

or $$-20\text{ ft/s}^2 = \frac{(41.067 - 66)\text{ ft/s}}{[(t_B)_2 - 0.90909]\text{ s}}$$

Car *B* will continue to overtake car *A* while $v_B > v_A$. Therefore, $(x_{A/B})_{\min}$ will occur when $v_A = v_B$, which occurs for

$$(t_B)_1 < t_{\min} < (t_B)_2$$

For this time interval

$$v_A = 44\text{ ft/s}$$
$$v_B = (v_B)_0 + a_B[t - (t_B)_1]$$

Then at $t = t_{\min}$: $$44\text{ ft/s} = 66\text{ ft/s} + (-20\text{ ft/s}^2)(t_{\min} - 0.90909)\text{ s}$$

or $$t_{\min} = 2.00909\text{ s}$$

Finally $$(x_{A/B})_{\min} = (x_A)_{t_{\min}} - (x_B)_{t_{\min}}$$

$$= \left\{t_A\left[\frac{(v_A)_0 + (v_A)_{\text{final}}}{2}\right] + (t_{\min} - t_A)(v_A)_{\text{final}}\right\}$$
$$- \left\{(x_B)_0 + (t_B)_1(v_B)_0 + [t_{\min} - (t_B)_1]\left[\frac{(v_B)_0 + (v_A)_{\text{final}}}{2}\right]\right\}$$

$$= \left[(0.91669\text{ s})\left(\frac{58.667 + 44}{2}\right)\text{ft/s} + (2.00909 - 0.91669)\text{ s} \times (44\text{ ft/s})\right]$$
$$- \left[-60\text{ ft} + (0.90909\text{ s})(66\text{ ft/s}) + (2.00909 - 0.90909)\text{ s} \times \left(\frac{66 + 44}{2}\right)\text{ft/s}\right]$$
$$= (47.057 + 48.066)\text{ ft} - (-60 + 60.000 + 60.500)\text{ ft}$$
$$= 34.623\text{ ft}$$

or $(x_{A/B})_{\min} = 34.6\text{ ft}$ ◀

PROBLEM 11.76 (Continued)

(*b*) Since $(x_{A/B}) \le 60$ ft for $t \le t_{\min}$, it follows that $x_{A/B} = 70$ ft for $t > (t_B)_2$

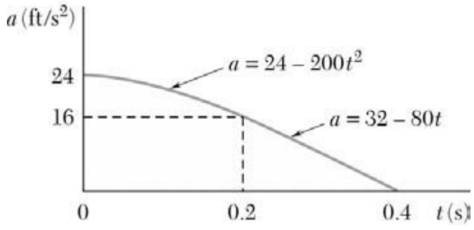
[Note $(t_B)_2 \simeq t_{\min}$]. Then, for $t > (t_B)_2$

$$x_{A/B} = (x_{A/B})_{\min} + [(t - t_{\min})(v_A)_{\text{final}}] - \left\{[(t_B)_2 - (t_{\min})]\left[\frac{(v_A)_{\text{final}} + (v_B)_{\text{final}}}{2}\right] + [t - (t_B)_2](v_B)_{\text{final}}\right\}$$

or

$$70 \text{ ft} = 34.623 \text{ ft} + [(t - 2.00909) \text{ s} \times (44 \text{ ft/s})] - \left[(2.15574 - 2.00909) \text{ s} \times \left(\frac{44 + 41.06}{2}\right) \text{ft/s} + (t - 2.15574) \text{ s} \times (41.067) \text{ ft/s}\right]$$

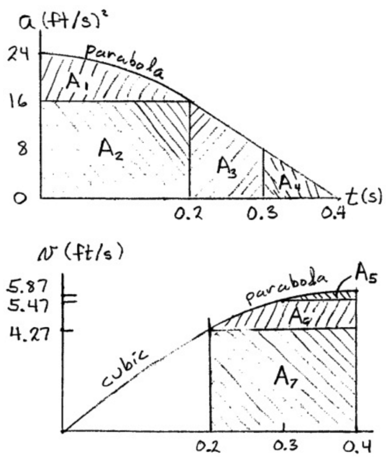
or $t = 14.14$ s ◀



PROBLEM 11.77

An accelerometer record for the motion of a given part of a mechanism is approximated by an arc of a parabola for 0.2 s and a straight line for the next 0.2 s as shown in the figure. Knowing that $v = 0$ when $t = 0$ and $x = 0.8$ ft when $t = 0.4$ s, (*a*) construct the $v{-}t$ curve for $0 \leq t \leq 0.4$ s, (*b*) determine the position of the part at $t = 0.3$ s and $t = 0.2$ s.

SOLUTION



Divide the area of the $a{-}t$ curve into the four areas A_1, A_2, A_3 and A_4.

$$A_1 = \frac{2}{3}(8)(0.2) = 1.0667 \text{ ft/s}$$

$$A_2 = (16)(0.2) = 3.2 \text{ ft/s}$$

$$A_3 = \frac{1}{2}(16+8)(0.1) = 1.2 \text{ ft/s}$$

$$A_4 = \frac{1}{2}(8)(0.1) = 0.4 \text{ ft/s}$$

Velocities: $v_0 = 0$

$v_{0.2} = v_0 + A_1 + A_2$ $\qquad v_{0.2} = 4.27$ ft/s ◀

$v_{0.3} = v_{0.2} + A_3$ $\qquad v_{0.3} = 5.47$ ft/s ◀

$v_{0.4} = v_{0.3} + A_4$ $\qquad v_{0.4} = 5.87$ ft/s ◀

Sketch the $v{-}t$ curve and divide its area into A_5, A_6, and A_7 as shown.

$$\int_x^{0.8} dx = 0.8 - x = \int_t^{0.4} v\,dt \quad \text{or} \quad x = 0.8 - \int_t^{0.4} v\,dt$$

At $t = 0.3$ s, $\qquad x_{0.3} = 0.8 - A_5 - (5.47)(0.1)$

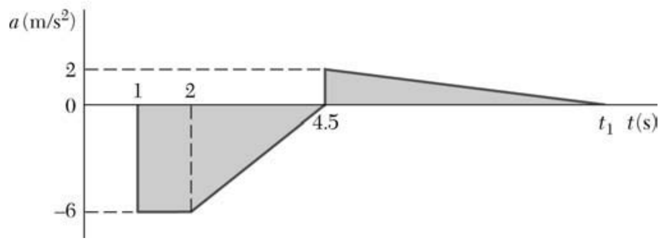
With $A_5 = \frac{2}{3}(0.4)(0.1) = 0.0267$ ft, $\qquad x_{0.3} = 0.227$ ft ◀

At $t = 0.2$ s, $\qquad x_{0.2} = 0.8 - (A_5 + A_6) - A_7$

With $A_5 + A_6 = \frac{2}{3}(1.6)(0.2) = 0.2133$ ft,

and $A_7 = (4.27)(0.2) = 0.8533$ ft

$x_{0.2} = 0.8 - 0.2133 - 0.8533$ $\qquad x_{0.2} = -0.267$ ft ◀



PROBLEM 11.78

A car is traveling at a constant speed of 54 km/h when its driver sees a child run into the road. The driver applies her brakes until the child returns to the sidewalk and then accelerates to resume her original speed of 54 km/h; the acceleration record of the car is shown in the figure. Assuming $x = 0$ when $t = 0$, determine (*a*) the time t_1 at which the velocity is again 54 km/h, (*b*) the position of the car at that time, (*c*) the average velocity of the car during the interval $1\text{ s} \le t \le t_1$.

SOLUTION

Given: At $t = 0, \quad x = 0, \quad v = 54$ km/h;

For $t = t_1$, $\quad v = 54$ km/h

First note $\quad 54 \text{ km/h} = 15 \text{ m/s}$

(*a*) We have $\quad v_b = v_a + (\text{area under } a\text{–}t \text{ curve from } t_a \text{ to } t_b)$

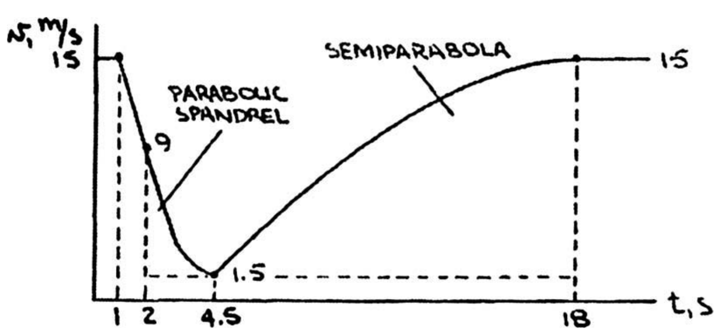
Then at $t = 2\text{ s}$: $\quad v = 15 - (1)(6) = 9 \text{ m/s}$

$t = 4.5\text{ s}$: $\quad v = 9 - \frac{1}{2}(2.5)(6) = 1.5 \text{ m/s}$

$t = t_1$: $\quad 15 = 1.5 + \frac{1}{2}(t_1 - 4.5)(2)$

or $\quad t_1 = 18.00 \text{ s}$ ◀

(*b*) Using the above values of the velocities, the v–t curve is drawn as shown.



Now x at $t = 18$ s

$$x_{18} = 0 + \Sigma\,(\text{area under the } v\text{–}t \text{ curve from } t = 0 \text{ to } t = 18\text{ s})$$

$$= (1\text{ s})(15\text{ m/s}) + (1\text{ s})\left(\frac{15+9}{2}\right)\text{m/s}$$

$$+\left[(2.5\text{ s})(1.5\text{ m/s}) + \frac{1}{3}(2.5\text{ s})(7.5\text{ m/s})\right]$$

$$+\left[(13.5\text{ s})(1.5\text{ m/s}) + \frac{2}{3}(13.5\text{ s})(13.5\text{ m/s})\right]$$

$$= [15 + 12 + (3.75 + 6.25) + (20.25 + 121.50)]\text{ m}$$

$$= 178.75\text{ m}$$

or $x_{18} = 178.8\text{ m}$ ◀

(*c*) First note

$$x_1 = 15\text{ m}$$
$$x_{18} = 178.75\text{ m}$$

Now

$$v_{\text{ave}} = \frac{\Delta x}{\Delta t} = \frac{(178.75 - 15)\text{ m}}{(18 - 1)\text{ s}} = 9.6324\text{ m/s}$$

or $v_{\text{ave}} = 34.7\text{ km/h}$ ◀

PROBLEM 11.79

An airport shuttle train travels between two terminals that are 1.6 mi apart. To maintain passenger comfort, the acceleration of the train is limited to ±4 ft/s^2, and the jerk, or rate of change of acceleration, is limited to ±0.8 ft/s^2 per second. If the shuttle has a maximum speed of 20 mi/h, determine (*a*) the shortest time for the shuttle to travel between the two terminals, (*b*) the corresponding average velocity of the shuttle.

SOLUTION

Given: $x_{\max} = 1.6 \text{ mi}; \quad |a_{\max}| = 4 \text{ ft/s}^2$

$$\left|\left(\frac{da}{dt}\right)_{\max}\right| = 0.8 \text{ ft/s}^2\text{/s}; \quad v_{\max} = 20 \text{ mi/h}$$

First note $\quad 20 \text{ mi/h} = 29.333 \text{ ft/s}$

$$1.6 \text{ mi} = 8448 \text{ ft}$$

(*a*) To obtain $t_{\min}$, the train must accelerate and decelerate at the maximum rate to maximize the time for which $v = v_{\max}$. The time Δt required for the train to have an acceleration of 4 ft/s^2 is found from

$$\left(\frac{da}{dt}\right)_{\max} = \frac{a_{\max}}{\Delta t}$$

or
$$\Delta t = \frac{4 \text{ ft/s}^2}{0.8 \text{ ft/s}^2\text{/s}}$$

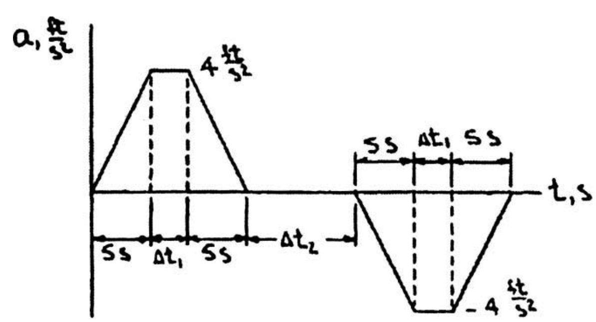
or
$$\Delta t = 5 \text{ s}$$

Now,

after 5 s, the speed of the train is $\quad v_5 = \frac{1}{2}(\Delta t)(a_{\max}) \quad \left(\text{since } \frac{da}{dt} = \text{constant}\right)$

or
$$v_5 = \frac{1}{2}(5 \text{ s})(4 \text{ ft/s}^2) = 10 \text{ ft/s}$$

Then, since $v_5 < v_{\max}$, the train will continue to accelerate at 4 ft/s^2 until $v = v_{\max}$. The a–t curve must then have the shape shown. Note that the magnitude of the slope of each inclined portion of the curve is 0.8 ft/s^2/s.



Now at $t = (10 + \Delta t_1)$ s, $v = v_{max}$:

$$2\left[\frac{1}{2}(5 \text{ s})(4 \text{ ft/s}^2)\right] + (\Delta t_1)(4 \text{ ft/s}^2) = 29.333 \text{ ft/s}$$

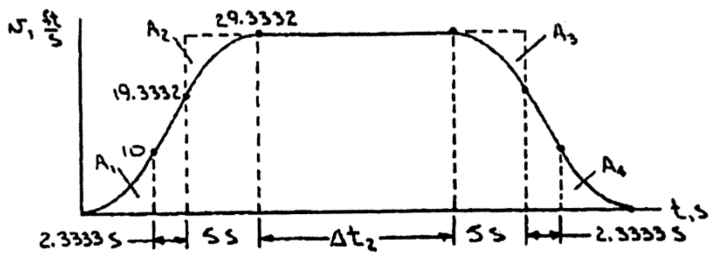
or $\Delta t_1 = 2.3333$ s

Then at $t = 5$ s: $v = 0 + \frac{1}{2}(5)(4) = 10$ ft/s

$t = 7.3333$ s: $v = 10 + (2.3333)(4) = 19.3332$ ft/s

$t = 12.3333$ s: $v = 19.3332 + \frac{1}{2}(5)(4) = 29.3332$ ft/s

Using symmetry, the v–t curve is then drawn as shown.



Noting that $A_1 = A_2 = A_3 = A_4$ and that the area under the v–t curve is equal to x_{max}, we have

$$2\left[(2.3333 \text{ s})\left(\frac{10 + 19.3332}{2}\right) \text{ft/s}\right]$$
$$+ (10 + \Delta t_2) \text{ s} \times (29.3332 \text{ ft/s}) = 8448 \text{ ft}$$

or $\Delta t_2 = 275.67$ s

Then $t_{min} = 4(5 \text{ s}) + 2(2.3333 \text{ s}) + 275.67 \text{ s}$

$= 300.34$ s

or $t_{min} = 5.01$ min ◀

(*b*) We have $v_{ave} = \frac{\Delta x}{\Delta t} = \frac{1.6 \text{ mi}}{300.34 \text{ s}} \times \frac{3600 \text{ s}}{1 \text{ h}}$

or $v_{ave} = 19.18$ mi/h ◀

PROBLEM 11.80

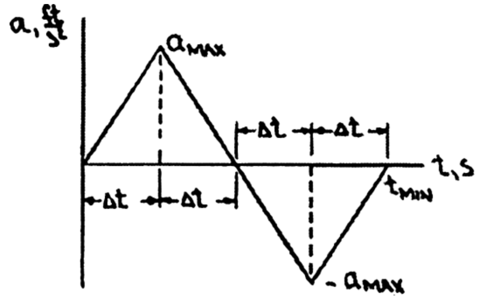
During a manufacturing process, a conveyor belt starts from rest and travels a total of 1.2 ft before temporarily coming to rest. Knowing that the jerk, or rate of change of acceleration, is limited to ±4.8 ft/s^2 per second, determine (*a*) the shortest time required for the belt to move 1.2 ft, (*b*) the maximum and average values of the velocity of the belt during that time.

SOLUTION

Given: At $t = 0, \quad x = 0, \quad v = 0; \quad x_{max} = 1.2$ ft;

when $x = x_{max}, \quad v = 0; \quad \left|\left(\frac{da}{dt}\right)_{max}\right| = 4.8$ ft/s^2

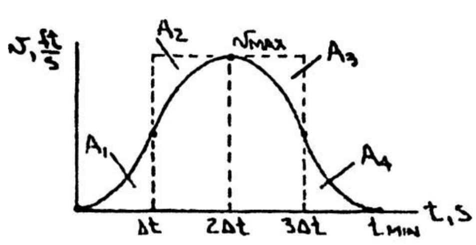
(*a*) Observing that v_{max} must occur at $t = \frac{1}{2}t_{min}$, the $a-t$ curve must have the shape shown. Note that the magnitude of the slope of each portion of the curve is 4.8 ft/s^2/s.



We have at $t = \Delta t$: $v = 0 + \frac{1}{2}(\Delta t)(a_{max}) = \frac{1}{2}a_{max}\Delta t$

$t = 2\Delta t$: $v_{max} = \frac{1}{2}a_{max}\Delta t + \frac{1}{2}(\Delta t)(a_{max}) = a_{max}\Delta t$

Using symmetry, the $v-t$ is then drawn as shown.



Noting that $A_1 = A_2 = A_3 = A_4$ and that the area under the $v-t$ curve is equal to x_{max}, we have

$$(2\Delta t)(v_{max}) = x_{max}$$

$$v_{max} = a_{max}\Delta t \Rightarrow 2a_{max}\Delta t^2 = x_{max}$$

PROBLEM 11.80 (Continued)

Now $\frac{a_{max}}{\Delta t} = 4.8 \text{ ft/s}^2\text{/s}$ so that

$$2(4.8\Delta t \text{ ft/s}^3)\Delta t^2 = 1.2 \text{ ft}$$

or $\Delta t = 0.5 \text{ s}$

Then $t_{min} = 4\Delta t$

or $t_{min} = 2.00 \text{ s}$ ◀

(*b*) We have

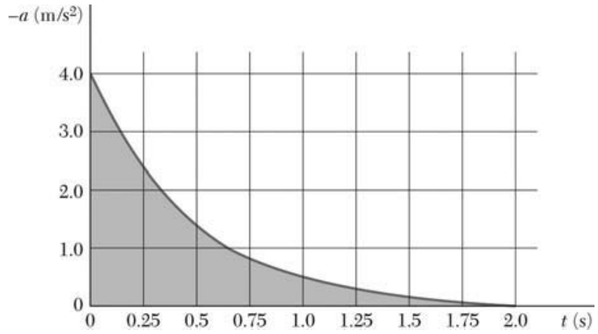
$$v_{max} = a_{max}\Delta t$$
$$= (4.8 \text{ ft/s}^2\text{/s} \times \Delta \text{t})\Delta t$$
$$= 4.8 \text{ ft/s}^2\text{/s} \times (0.5 \text{ s})^2$$

or $v_{max} = 1.2 \text{ ft/s}$ ◀

Also $v_{ave} = \frac{\Delta x}{\Delta t_{total}} = \frac{1.2 \text{ ft}}{2.00 \text{ s}}$

or $v_{ave} = 0.6 \text{ ft/s}$ ◀

PROBLEM 11.81



Two seconds are required to bring the piston rod of an air cylinder to rest; the acceleration record of the piston rod during the 2 s is as shown. Determine by approximate means (*a*) the initial velocity of the piston rod, (*b*) the distance traveled by the piston rod as it is brought to rest.

SOLUTION

Given: $a\text{–}t$ curve; at $t = 2\text{ s}$, $v = 0$

1. The $a\text{–}t$ curve is first approximated with a series of rectangles, each of width $\Delta t = 0.25$ s. The area $(\Delta t)(a_{ave})$ of each rectangle is approximately equal to the change in velocity Δv for the specified interval of time. Thus,

$$\Delta v \cong a_{ave}\ \Delta t$$

where the values of a_{ave} and Δv are given in columns 1 and 2, respectively, of the following table.

2. Now

$$v(2) = v_0 + \int_0^2 a\, dt = 0$$

and approximating the area $\int_0^2 a\, dt$ under the $a\text{–}t$ curve by $\Sigma a_{ave}\Delta t \approx \Sigma \Delta v$, the initial velocity is then equal to

$$v_0 = -\Sigma\Delta v$$

Finally, using

$$v_2 = v_1 + \Delta v_{12}$$

where Δv_{12} is the change in velocity between times t_1 and t_2, the velocity at the end of each 0.25 interval can be computed; see column 3 of the table and the $v\text{–}t$ curve.

3. The $v\text{–}t$ curve is then approximated with a series of rectangles, each of width 0.25 s. The area $(\Delta t)(v_{ave})$ of each rectangle is approximately equal to the change in position Δx for the specified interval of time. Thus

$$\Delta x \approx v_{ave}\Delta t$$

where v_{ave} and Δx are given in columns 4 and 5, respectively, of the table.

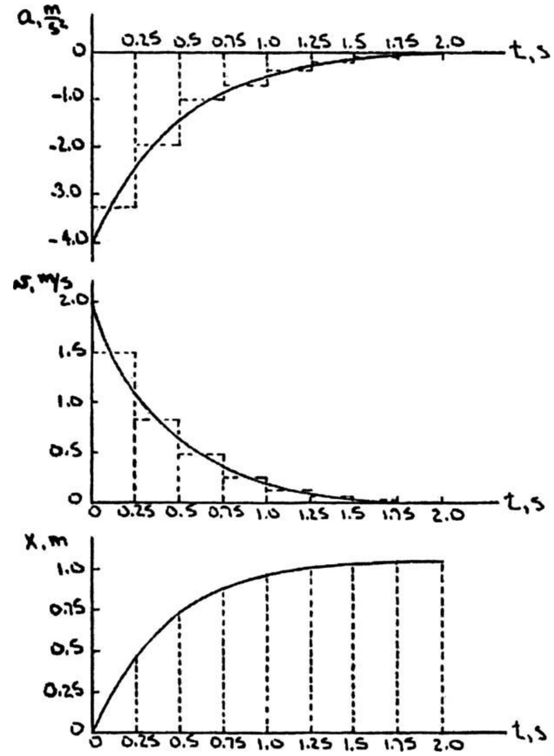
PROBLEM 11.81 (Continued)

4. With $x_0 = 0$ and noting that

$$x_2 = x_1 + \Delta x_{12}$$

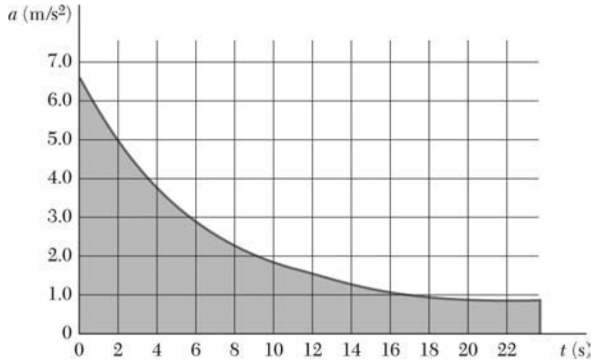
where Δx_{12} is the change in position between times t_1 and t_2, the position at the end of each 0.25 s interval can be computed; see column 6 of the table and the x–t curve.

		1	2	3	4	5	6
t, s	a, m/s²	a_AVE, m/s²	Δv, m/s	v, m/s	v_AVE, m/s	Δx, m	x, m
0	−4.00			1.914			0
		−3.215	−0.804		1.512	0.378	
0.25	−2.43			1.110			0.378
		−1.915	−0.479		0.871	0.218	
0.50	−1.40			0.631			0.596
		−1.125	−0.281		0.491	0.123	
0.75	−0.85			0.350			0.719
		−0.675	−0.169		0.266	0.067	
1.00	−0.50			0.181			0.786
		−0.390	−0.098		0.132	0.033	
1.25	−0.28			0.083			0.819
		−0.205	−0.051		0.058	0.015	
1.50	−0.13			0.032			0.834
		−0.095	−0.024		0.020	0.005	
1.75	−0.06			0.008			0.839
		−0.030	−0.008		0.004	0.001	
2.00	0			0			0.840



(*a*) We had found $v_0 = 1.914$ m/s ◀

(*b*) At $t = 2$ s $x = 0.840$ m ◀



PROBLEM 11.82

The acceleration record shown was obtained during the speed trials of a sports car. Knowing that the car starts from rest, determine by approximate means (*a*) the velocity of the car at $t = 8$ s, (*b*) the distance the car has traveled at $t = 20$ s.

SOLUTION

Given: $a{-}t$ curve; at $t = 0, \; x = 0, \; v = 0$

1. The $a{-}t$ curve is first approximated with a series of rectangles, each of width $\Delta t = 2$ s. The area $(\Delta t)(a_{\text{ave}})$ of each rectangle is approximately equal to the change in velocity Δv for the specified interval of time. Thus,

$$\Delta v \cong a_{\text{ave}} \; \Delta t$$

where the values of a_{ave} and Δv are given in columns 1 and 2, respectively, of the following table.

2. Noting that $v_0 = 0$ and that

$$v_2 = v_1 + \Delta v_{12}$$

where Δv_{12} is the change in velocity between times t_1 and t_2, the velocity at the end of each 2 s interval can be computed; see column 3 of the table and the $v{-}t$ curve.

3. The $v{-}t$ curve is next approximated with a series of rectangles, each of width $\Delta t = 2$ s. The area $(\Delta t)(v_{\text{ave}})$ of each rectangle is approximately equal to the change in position Δx for the specified interval of time.

Thus, $$\Delta x \cong v_{\text{ave}} \; \Delta t$$

where v_{ave} and Δx are given in columns 4 and 5, respectively, of the table.

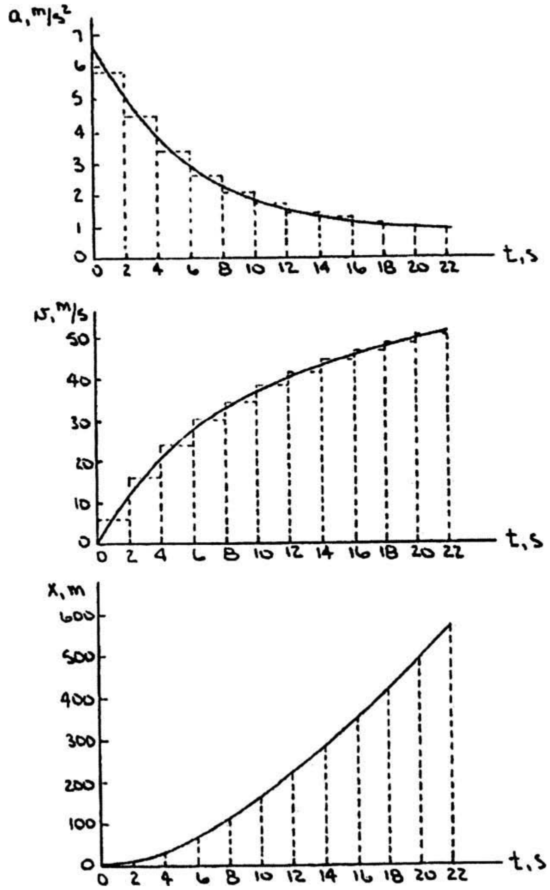
4. With $x_0 = 0$ and noting that

$$x_2 = x_1 + \Delta x_{12}$$

where Δx_{12} is the change in position between times t_1 and t_2, the position at the end of each 2 s interval can be computed; see column 6 of the table and the $x{-}t$ curve.

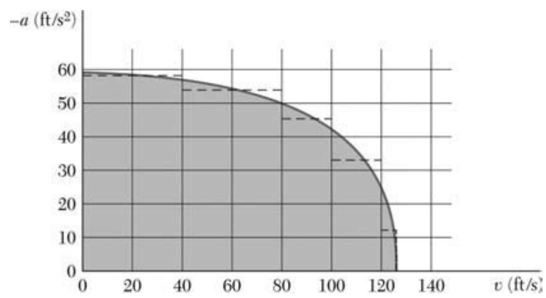
PROBLEM 11.82 (Continued)

		1	2	3	4	5	6
t, s	a, m/s²	a_{AVE}, m/s²	Δv, m/s	v, m/s	v_{AVE}, m/s	Δx, m	x, m
0	6.63			0			0
		5.86	11.72		5.86	11.72	
2	5.08			11.72			11.72
		4.47	8.94		16.19	32.38	
4	3.86			20.66			44.10
		3.38	6.76		24.04	48.08	
6	2.90			27.42			92.18
		2.58	5.16		30.00	60.00	
8	2.25			32.58			152.18
		2.06	4.12		34.64	69.28	
10	1.87			36.70			221.46
		1.71	3.42		38.41	76.82	
12	1.54			40.12			298.28
		1.42	2.84		41.54	83.08	
14	1.29			42.96			381.36
		1.23	2.46		44.19	88.38	
16	1.16			45.42			469.74
		1.10	2.20		46.52	93.04	
18	1.03			47.62			562.78
		1.00	2.00		48.62	97.24	
20	0.97			49.62			660.02
		0.94	1.88		50.56	101.12	
22	0.90			51.50			761.14



(*a*) At $t = 8$ s, $v = 32.58$ m/s or $v = 117.3$ km/h ◀

(*b*) At $t = 20$ s $x = 660$ m ◀



PROBLEM 11.83

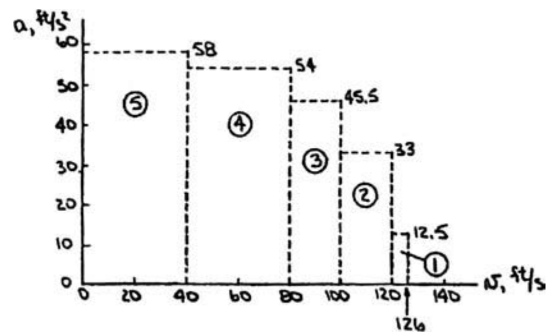
A training airplane has a velocity of 126 ft/s when it lands on an aircraft carrier. As the arresting gear of the carrier brings the airplane to rest, the velocity and the acceleration of the airplane are recorded; the results are shown (solid curve) in the figure. Determine by approximate means (*a*) the time required for the airplane to come to rest, (*b*) the distance traveled in that time.

SOLUTION

<u>Given</u>: $a-v$ curve:

$$v_0 = 126 \text{ ft/s}$$

The given curve is approximated by a series of uniformly accelerated motions (the horizontal dashed lines on the figure).



For uniformly accelerated motion

$$v_2^2 = v_1^2 + 2a(x_2 - x_1)$$

$$v_2 = v_1 + a(t_2 - t_1)$$

or

$$\Delta x = \frac{v_2^2 - v_1^2}{2a}$$

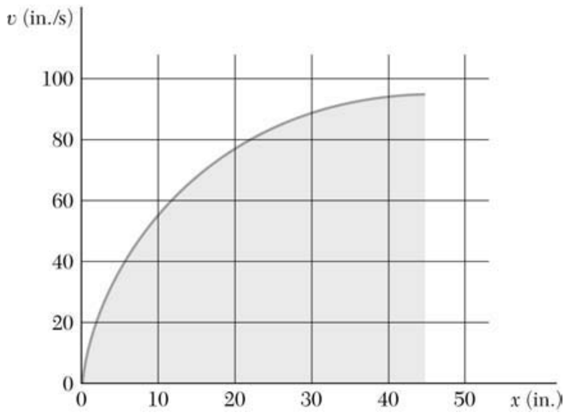
$$\Delta t = \frac{v_2 - v_1}{a}$$

For the five regions shown above, we have

Region	v_1, ft/s	v_2, ft/s	a, ft/s^2	Δx, ft	Δt, s
1	126	120	−12.5	59.0	0.480
2	120	100	−33	66.7	0.606
3	100	80	−45.5	39.6	0.440
4	80	40	−54	44.4	0.741
5	40	0	−58	13.8	0.690
Σ				223.5	2.957

(*a*) From the table, when $v = 0$ $t = 2.96$ s ◀

(*b*) From the table and assuming $x_0 = 0$, when $v = 0$ $x = 224$ ft ◀

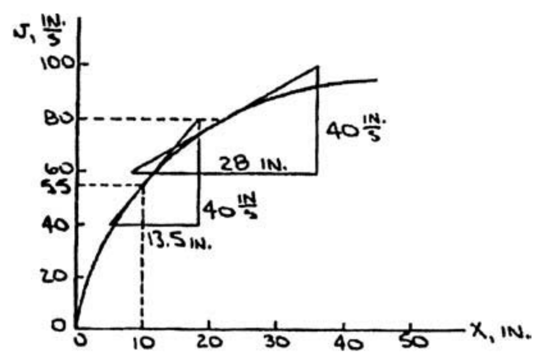


PROBLEM 11.84

Shown in the figure is a portion of the experimentally determined $v-x$ curve for a shuttle cart. Determine by approximate means the acceleration of the cart (*a*) when $x = 10$ in., (*b*) when $v = 80$ in./s.

SOLUTION

Given: $v-x$ curve



First note that the slope of the above curve is $\frac{dv}{dx}$. Now

$$a = v\frac{dv}{dx}$$

(*a*) When $x = 10$ in., $v = 55$ in./s

Then $$a = 55 \text{ in./s}\left(\frac{40 \text{ in./s}}{13.5 \text{ in.}}\right)$$

or $$a = 163.0 \text{ in./s}^2 \blacktriangleleft$$

(*b*) When $v = 80$ in./s, we have

$$a = 80 \text{ in./s}\left(\frac{40 \text{ in./s}}{28 \text{ in.}}\right)$$

or $$a = 114.3 \text{ in./s}^2 \blacktriangleleft$$

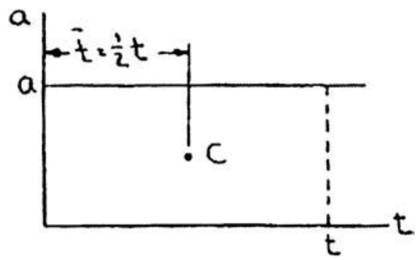
Note: To use the method of measuring the subnormal outlined at the end of Section 11.8, it is necessary that the same scale be used for the x and v axes (e.g., 1 in. = 50 in., 1 in. = 50 in./s). In the above solution, Δv and Δx were measured directly, so different scales could be used.

PROBLEM 11.85

Using the method of Section 11.8, derive the formula $x = x_0 + v_0 t + \frac{1}{2}at^2$ for the position coordinate of a particle in uniformly accelerated rectilinear motion.

SOLUTION

The $a\!-\!t$ curve for uniformly accelerated motion is as shown.



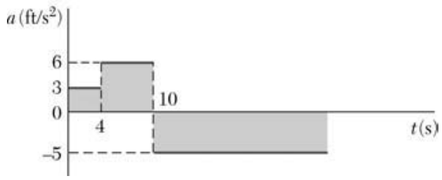
Using Eq. (11.13), we have

$$x = x_0 + v_0 t + (\text{area under } a\!-\!t \text{ curve})\,(t - \bar{t})$$

$$= x_0 + v_0 t + (t \times a)\left(t - \frac{1}{2}t\right)$$

$$= x_0 + v_0 t + \frac{1}{2}at^2 \qquad \text{Q.E.D.}$$ ◀

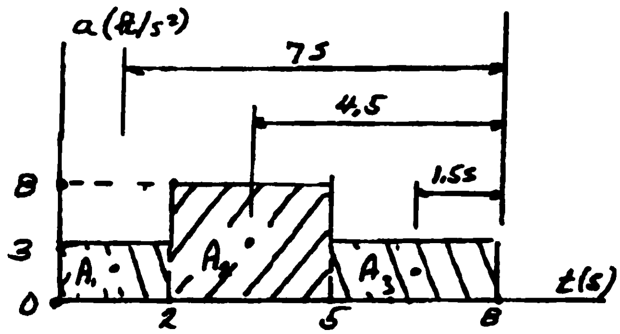
PROBLEM 11.86



Using the method of Section 11.8 determine the position of the particle of Problem 11.61 when $t = 8$ s.

PROBLEM 11.61 A particle moves in a straight line with the acceleration shown in the figure. Knowing that it starts from the origin with $v_0 = -14$ ft/s, plot the v–t and x–t curves for $0 < t < 15$ s and determine (*a*) the maximum value of the velocity of the particle, (*b*) the maximum value of its position coordinate.

SOLUTION



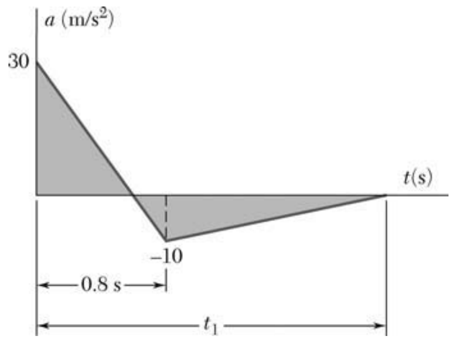
$$x_0 = 0$$
$$v_0 = -14 \text{ ft/s}$$

when $t = 8$ s:

$$x = x_0 + v_0 t + \Sigma A(t_1 - t)$$
$$= 0 - (14 \text{ ft/s})(8 \text{ s}) + [(3 \text{ ft/s}^2)(2 \text{ s})](7 \text{ s}) + [(8 \text{ ft/s}^2)(3 \text{ s})](4.5 \text{ s}) + [(3 \text{ ft/s})(3 \text{ s})](1.5 \text{ s})$$

$$x_8 = -112 \text{ ft} + 42 \text{ ft} + 108 \text{ ft} + 13.5 \text{ ft}$$

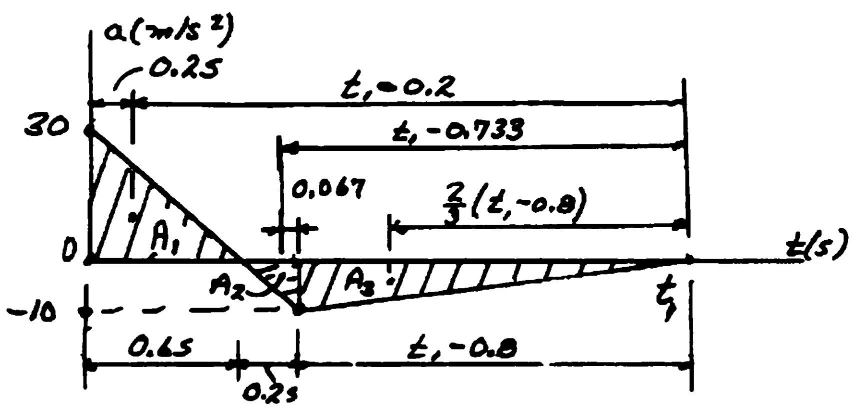
$$x_8 = 51.5 \text{ ft} \blacktriangleleft$$



PROBLEM 11.87

The acceleration of an object subjected to the pressure wave of a large explosion is defined approximately by the curve shown. The object is initially at rest and is again at rest at time t_1. Using the method of section 11.8, determine (*a*) the time t_1, (b) the distance through which the object is moved by the pressure wave.

SOLUTION



(*a*) Since $v = 0$ when $t = 0$ and when $t = t_1$ the change in v between $t = 0$ and $t = t_1$ is zero.

Thus, area under *a*–*t* curve is zero

$$A_1 + A_2 + A_3 = 0$$

$$\frac{1}{2}(30)(0.6) + \frac{1}{2}(-10)(0.2) + \frac{1}{2}(-10)(t_1 - 0.8) = 0$$

$$9 - 1 - 5t_1 + 4 = 0$$

$t_1 = 2.40$ s ◀

(*b*) <u>Position when $t = t_1 = 2.4$ s</u>

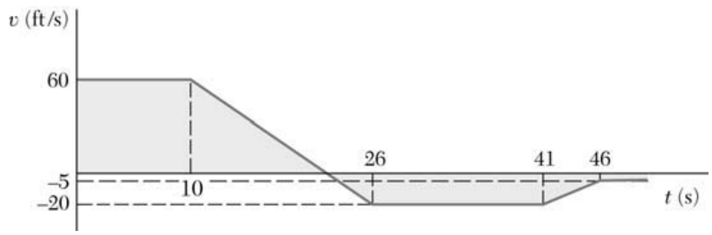
$$x = x_0 + v_0 t_1 + A_1(t_1 - 0.2) + A_2(t_1 - 0.733) + A_3\left(\frac{2}{3}\right)(t_1 - 0.8)$$

$$= 0 + 0 + (9)(2.4 - 0.2) + (-1)(2.4 - 0.733) + \left[\frac{1}{2}(-10)(2.4 - 0.8)\right]\frac{2}{3}(2.4 - 0.8)$$

$$= 19.8 \text{ m} - 1.667 \text{ m} - 8.533 \text{ m}$$

$x = 9.60$ m ◀

PROBLEM 11.88



For the particle of Problem 11.63, draw the $a\!-\!t$ curve and determine, using the method of Section 11.8, (*a*) the position of the particle when $t = 52$ s, (*b*) the maximum value of its position coordinate.

PROBLEM 11.63 A particle moves in a straight line with the velocity shown in the figure. Knowing that $x = -540$ m at $t = 0$, (*a*) construct the $a\!-\!t$ and $x\!-\!t$ curves for $0 < t < 50$ s, and determine (*b*) the total distance traveled by the particle when $t = 50$ s, (*c*) the two times at which $x = 0$.

SOLUTION

We have $$a = \frac{dv}{dt}$$

where $\frac{dv}{dt}$ is the slope of the $v\!-\!t$ curve. Then

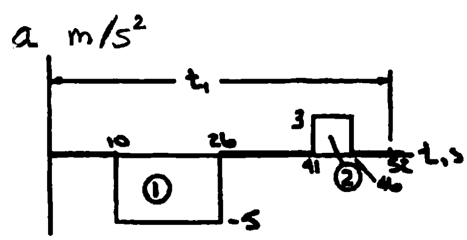
from $t = 0$ to $t = 10$ s: $v = \text{constant} \Rightarrow a = 0$

$t = 10$ s to $t = 26$ s: $a = \dfrac{-20-60}{26-10} = -5 \text{ m/s}^2$

$t = 26$ s to $t = 41$ s: $v = \text{constant} \Rightarrow a = 0$

$t = 41$ s to $t = 46$ s: $a = \dfrac{-5-(-20)}{46-41} = 3 \text{ m/s}^2$

$t > 46$ s: $v = \text{constant} \Rightarrow a = 0$



The $a\!-\!t$ curve is then drawn as shown.

(*a*) From the discussion following Eq. (11.13),

we have $$x = x_0 + v_0 t_1 + \Sigma A(t_1 - \bar{t})$$

where A is the area of a region and $\bar{t}$ is the distance to its centroid. Then, for $t_1 = 52$ s

$$x = -540 \text{ m} + (60 \text{ m/s})(52 \text{ s}) + \{-[(16 \text{ s})(5 \text{ m/s}^2)](52-18) \text{ s}$$
$$+ [(5 \text{ s})(3 \text{ m/s}^2)](52-43.5)\text{ s}\}$$
$$= [-540 + (3120) + (-2720 + 127.5)] \text{ m}$$

or $x = -12.50$ m ◀

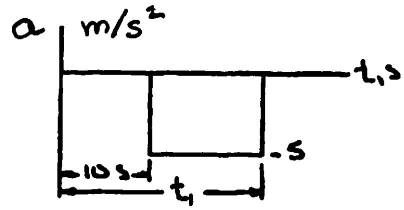
PROBLEM 11.88 (Continued)

(*b*) Noting that x_{max} occurs when $v = 0\left(\frac{dx}{dt} = 0\right)$, it is seen from the v–t curve that x_{max} occurs for $10\text{ s} < t < 26\text{ s}$. Although similar triangles could be used to determine the time at which $x = x_{max}$ (see the solution to Problem 11.63), the following method will be used.

For $\quad 10\text{ s} < t_1 < 26\text{ s}$, we have

$$x = -540 + 60t_1 - [(t_1 - 10)(5)]\left[\frac{1}{2}(t_1 - 10)\right]\text{ m}$$

$$= -540 + 60t_1 - \frac{5}{2}(t_1 - 10)^2$$

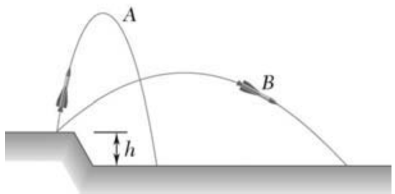


When $x = x_{max}$: $\quad \dfrac{dx}{dt} = 60 - 5(t_1 - 10) = 0$

or $\quad (t_1)_{x_{max}} = 22\text{ s}$

Then $\quad x_{max} = -540 + 60(22) - \frac{5}{2}(22 - 10)^2$

or $\quad x_{max} = 420\text{ m}$ ◀



PROBLEM 11.CQ3

Two model rockets are fired simultaneously from a ledge and follow the trajectories shown. Neglecting air resistance, which of the rockets will hit the ground first?

(*a*) *A*

(*b*) *B*

(*c*) They hit at the same time.

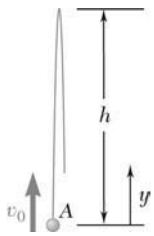
(*d*) The answer depends on *h*.

SOLUTION

The motion in the vertical direction depends on the initial velocity in the *y*-direction. Since *A* has a larger initial velocity in this direction it will take longer to hit the ground.

Answer: (*b*) ◀

PROBLEM 11.CQ4



Ball A is thrown straight up. Which of the following statements about the ball are true at the highest point in its path?

(a) The velocity and acceleration are both zero.

(b) The velocity is zero, but the acceleration is not zero.

(c) The velocity is not zero, but the acceleration is zero.

(d) Neither the velocity nor the acceleration are zero.

SOLUTION

At the highest point the velocity is zero. The acceleration is never zero.

Answer: (b) ◀

PROBLEM 11.CQ5

Ball *A* is thrown straight up with an initial speed v_0 and reaches a maximum elevation h before falling back down. When *A* reaches its maximum elevation, a second ball is thrown straight upward with the same initial speed v_0. At what height, y, will the balls cross paths?

(*a*) $y = h$

(*b*) $y > h/2$

(*c*) $y = h/2$

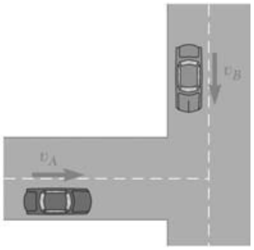
(*d*) $y < h/2$

(*e*) $y = 0$

SOLUTION

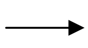
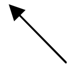
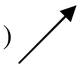
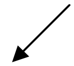
When the ball is thrown up in the air it will be constantly slowing down until it reaches its apex, at which point it will have a speed of zero. So, the time it will take to travel the last half of the distance to the apex will be longer than the time it takes for the first half. This same argument can be made for the ball falling from the maximum elevation. It will be speeding up, so the first half of the distance will take longer than the second half. Therefore, the balls should cross above the half-way point.

Answer: (*b*) ◀



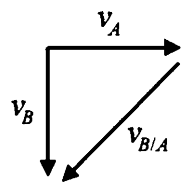
PROBLEM 11.CQ6

Two cars are approaching an intersection at constant speeds as shown. What velocity will car *B* appear to have to an observer in car *A*?

(*a*)  (*b*)  (*c*)  (*d*)  (*e*) 

SOLUTION

Since $\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$ we can draw the vector triangle and see

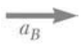


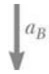
$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$

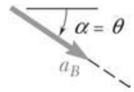
Answer: (*e*) ◀

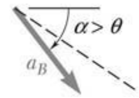
PROBLEM 11.CQ7

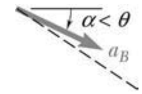
Blocks A and B are released from rest in the positions shown. Neglecting friction between all surfaces, which figure below best indicates the direction α of the acceleration of block B?

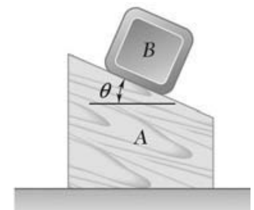
(a) 

(b) 

(c) 

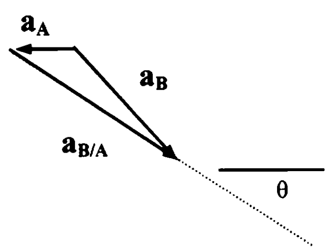
(d) 

(e) 

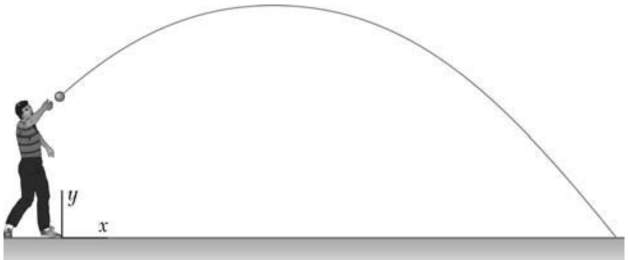


SOLUTION

Since $\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$ we get



Answer: (d) ◀



PROBLEM 11.89

A ball is thrown so that the motion is defined by the equations $x = 5t$ and $y = 2 + 6t - 4.9t^2$, where x and y are expressed in meters and t is expressed in seconds. Determine (*a*) the velocity at $t = 1$ s, (*b*) the horizontal distance the ball travels before hitting the ground.

SOLUTION

Units are meters and seconds.

Horizontal motion: $v_x = \dfrac{dx}{dt} = 5$

Vertical motion: $v_y = \dfrac{dy}{dt} = 6 - 9.8t$

(*a*) Velocity at $t = 1$ s. $v_x = 5$

$v_y = 6 - 9.8 = -3.8$

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{5^2 + 3.8^2} = 6.28 \text{ m/s}$$

$$\tan\theta = \frac{v_y}{v_x} = \frac{-3.8}{5} \qquad \theta = -37.2°$$

$v = 6.28$ m/s ◥ 37.2° ◀

(*b*) Horizontal distance: $(y = 0)$

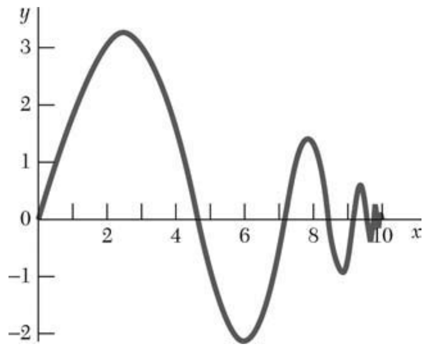
$y = 2 + 6t - 4.9t^2$

$t = 1.4971$ s

$x = (5)(1.4971) = 7.4856$ m

$x = 7.49$ m ◀

PROBLEM 11.90



The motion of a vibrating particle is defined by the position vector $\mathbf{r} = 10(1 - e^{-3t})\mathbf{i} + (4e^{-2t}\sin 15t)\mathbf{j}$, where $\mathbf{r}$ and t are expressed in millimeters and seconds, respectively. Determine the velocity and acceleration when (*a*) $t = 0$, (*b*) $t = 0.5$ s.

SOLUTION

$$\mathbf{r} = 10(1 - e^{-3t})\mathbf{i} + (4e^{-2t}\sin 15t)\mathbf{j}$$

Then $$\mathbf{v} = \frac{d\mathbf{r}}{dt} = 30e^{-3t}\mathbf{i} + [60e^{-2t}\cos 15t - 8e^{-2t}\sin 15t]\mathbf{j}$$

and $$\mathbf{a} = \frac{d\mathbf{v}}{dt} = -90e^{-3t}\mathbf{i} + [-120e^{-2t}\cos 15t - 900e^{-2t}\sin 15t - 120e^{-2t}\cos 15t + 16e^{-2t}\sin 15t]\mathbf{j}$$

$$= -90e^{-3t}\mathbf{i} + [-240e^{-2t}\cos 15t - 884e^{-2t}\sin 15t]\mathbf{j}$$

(*a*) When $t = 0$:

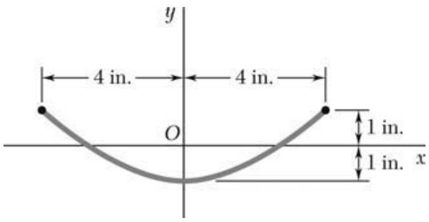
$\mathbf{v} = 30\mathbf{i} + 60\mathbf{j}$ mm/s $\qquad \mathbf{v} = 67.1$ mm/s ⦣ 63.4° ◀

$\mathbf{a} = -90\mathbf{i} - 240\mathbf{j}$ mm/s^2 $\qquad \mathbf{a} = 256$ mm/s^2 ⦣ 69.4° ◀

When $t = 0.5$ s:

$$\mathbf{v} = 30e^{-1.5}\mathbf{i} + [60e^{-1}\cos 7.5 - 8e^{-1}\sin 7.5]$$
$= 6.694\mathbf{i} + 4.8906\mathbf{j}$ mm/s $\qquad \mathbf{v} = 8.29$ mm/s ⦣ 36.2° ◀

$$\mathbf{a} = 90e^{-1.5}\mathbf{i} + [-240e^{-1}\cos 7.5 - 884e^{-1}\sin 7.5\mathbf{j}]$$
$= -20.08\mathbf{i} - 335.65\mathbf{j}$ mm/s^2 $\qquad \mathbf{a} = 336$ mm/s^2 ⦣ 86.6° ◀



PROBLEM 11.91

The motion of a vibrating particle is defined by the position vector $\mathbf{r} = (4\sin \pi t)\mathbf{i} - (\cos 2\pi t)\mathbf{j}$, where r is expressed in inches and t in seconds. (*a*) Determine the velocity and acceleration when $t = 1$ s. (*b*) Show that the path of the particle is parabolic.

SOLUTION

$$\mathbf{r} = (4\sin \pi t)\mathbf{i} - (\cos 2\pi t)\mathbf{j}$$
$$\mathbf{v} = (4\pi \cos \pi t)\mathbf{i} + (2\pi \sin 2\pi t)\mathbf{j}$$
$$\mathbf{a} = -(4\pi^2 \sin \pi t)\mathbf{i} + (4\pi^2 \cos 2\pi t)\mathbf{j}$$

(*a*) When $t = 1$ s:

$$\mathbf{v} = (4\pi \cos \pi)\mathbf{i} + (2\pi \sin 2\pi)\mathbf{j} \qquad \mathbf{v} = -(4\pi \text{ in/s})\mathbf{i} \blacktriangleleft$$

$$\mathbf{a} = -(4\pi^2 \sin \pi)\mathbf{i} - (4\pi^2 \cos \pi)\mathbf{j} \qquad \mathbf{a} = -(4\pi^2 \text{in/s}^2)\mathbf{j} \blacktriangleleft$$

(*b*) Path of particle:

Since $\mathbf{r} = x\mathbf{i} + y\,\mathbf{j}$; $\quad x = 4\sin \pi t,\quad y = -\cos 2\pi t$

Recall that $\cos 2\theta = 1 - 2\sin^2 \theta$ and write

$$y = -\cos 2\pi t = -(1 - 2\sin^2 \pi t) \qquad (1)$$

But since $x = 4\sin \pi t$ or $\sin \pi t = \frac{1}{4}x$, Eq.(1) yields

$$y = -\left[1 - 2\left(\frac{1}{4}x\right)^2\right] \qquad y = \frac{1}{8}x^2 - 1 \text{ (Parabola)} \blacktriangleleft$$

PROBLEM 11.92

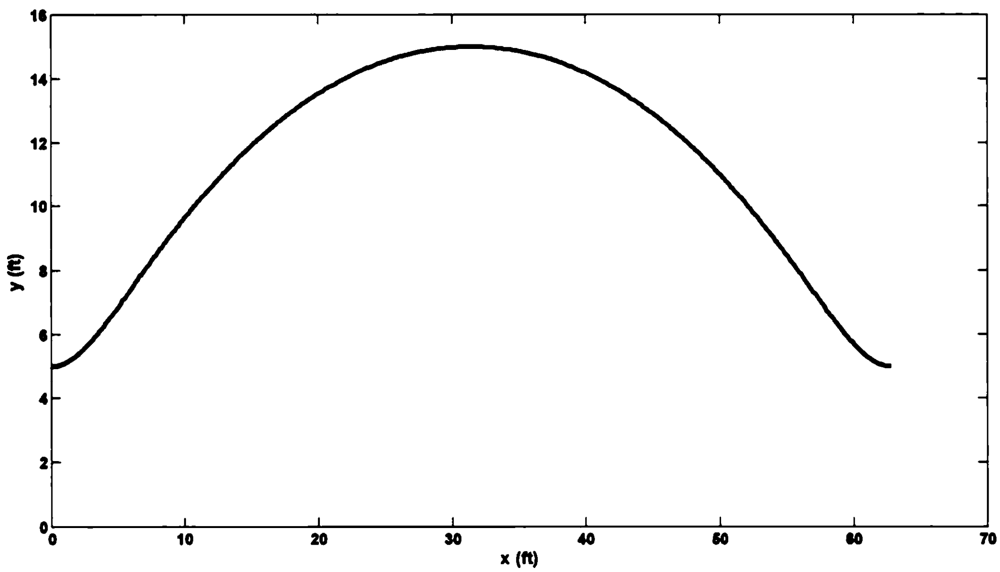
The motion of a particle is defined by the equations $x = 10t - 5\sin t$ and $y = 10 - 5\cos t$, where x and y are expresed in feet and t is expressed in seconds. Sketch the path of the particle for the time interval $0 \le t \le 2\pi$, and determine (*a*) the magnitudes of the smallest and largest velocities reached by the particle, (*b*) the corresponding times, positions, and directions of the velocities.

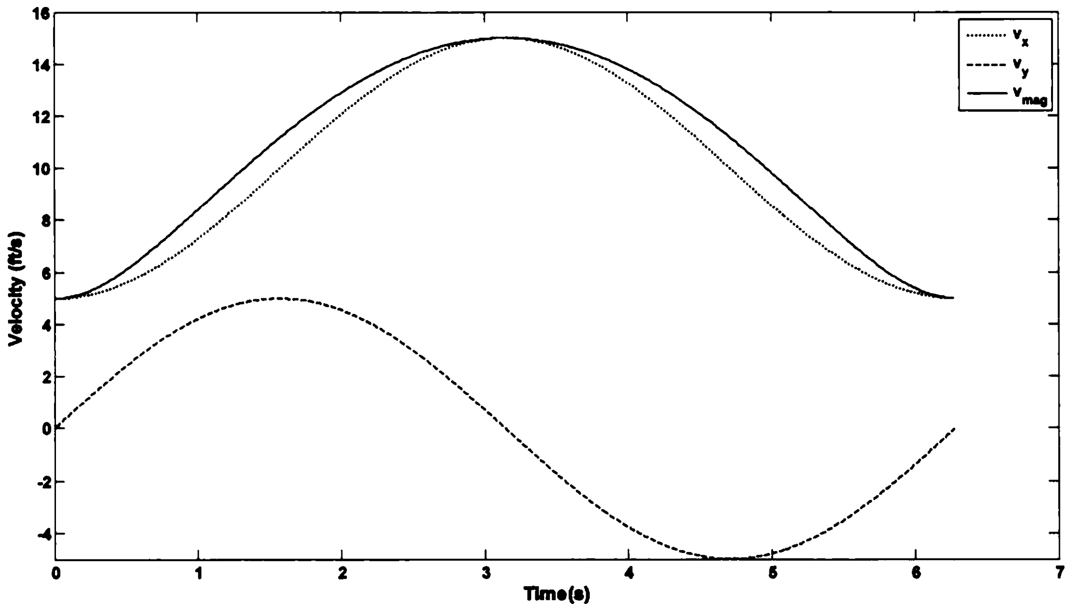
SOLUTION

Sketch the path of the particle, i.e., plot of y versus x.

Using $x = 10t - 5\sin t$, and $y = 10 - 5\cos t$ obtain the values in the table below. Plot as shown.

$t(s)$	x(ft)	y(ft)
0	0.00	5
$\frac{\pi}{2}$	10.71	10
π	31.41	15
$3\frac{\pi}{2}$	52.12	10
2π	62.83	5





(*a*) Differentiate with respect to *t* to obtain velocity components.

$$v_x = \frac{dx}{dt} = 10 - 5\cos t \quad \text{and} \quad v_y = 5\sin t$$

$$v^2 = v_x^2 + v_y^2 = (10 - 5\cos t)^2 + 25\sin^2 t = 125 - 100\cos t$$

$$\frac{d(v)^2}{dt} = 100\sin t = 0 \quad t = 0.\ \pm\pi.\ \pm 2\pi \ldots \pm N\pi$$

When $t = 2N\pi.$ $\cos t = 1.$ and v^2 is minimum.

When $t = (2N+1)\pi.$ $\cos t = -1.$ and v^2 is maximum.

$$(v^2)_{\min} = 125 - 100 = 25(\text{ft/s})^2 \qquad v_{\min} = 5 \text{ ft/s} \blacktriangleleft$$

$$(v^2)_{\max} = 125 + 100 = 225(\text{ft/s})^2 \qquad v_{\max} = 15 \text{ ft/s} \blacktriangleleft$$

(*b*) When $v = v_{\min}$.

When $N = 0, 1, 2, \ldots$ $x = 10(2\pi N) - 5\sin(2\pi N)$ $\qquad x = 20\pi N$ ft ◀

$y = 10 - 5\cos(2\pi N)$ $\qquad y = 5$ ft ◀

$v_x = 10 - 5\cos(2\pi N)$ $\qquad v_x = 5$ ft/s ◀

$v_y = 5\sin(2\pi N)$ $\qquad v_y = 0$ ◀

$$\tan\theta = \frac{v_y}{v_x} = 0, \qquad \theta = 0 \blacktriangleleft$$

PROBLEM 11.92 (Continued)

When $v = v_{\max}$. $t = (2N+1)\pi \text{ s}$ ◀

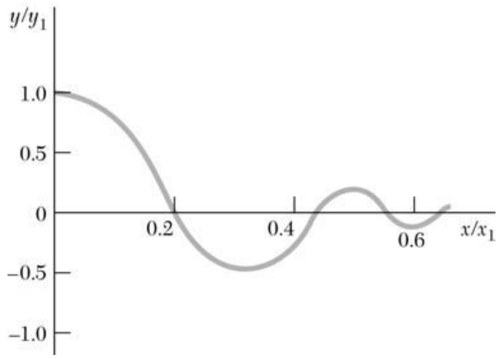
$$x = 10[2\pi(N-1)] - 5\sin[2\pi(N+1)] \qquad x = 20\pi(N+1) \text{ ft} \blacktriangleleft$$

$$y = 10 - 5\cos[2\pi(N+1)] \qquad y = 15 \text{ ft} \blacktriangleleft$$

$$v_x = 10 - 5\cos[2\pi(N+1)] \qquad v_x = 15 \text{ ft/s} \blacktriangleleft$$

$$v_y = 5\sin[2\pi(N+1)] \qquad v_y = 0 \blacktriangleleft$$

$$\tan\theta = \frac{v_y}{v_x} = 0, \qquad \theta = 0 \blacktriangleleft$$



PROBLEM 11.93

The damped motion of a vibrating particle is defined by the position vector $\mathbf{r} = x_1[1-1/(t+1)]\mathbf{i} + (y_1 e^{-\pi t/2}\cos 2\pi t)\mathbf{j}$, where t is expressed in seconds. For $x_1 = 30$ mm and $y_1 = 20$ mm, determine the position, the velocity, and the acceleration of the particle when (*a*) $t = 0$, (*b*) $t = 1.5$ s.

SOLUTION

We have $\mathbf{r} = 30\left(1 - \frac{1}{t+1}\right)\mathbf{i} + 20(e^{-\pi t/2}\cos 2\pi t)\mathbf{j}$

Then $\mathbf{v} = \frac{d\mathbf{r}}{dt}$

$$= 30\frac{1}{(t+1)^2}\mathbf{i} + 20\left(-\frac{\pi}{2}e^{-\pi t/2}\cos 2\pi t - 2\pi e^{-\pi t/2}\sin 2\pi t\right)\mathbf{j}$$

$$= 30\frac{1}{(t+1)^2}\mathbf{i} - 20\pi\left[e^{-\pi t/2}\left(\frac{1}{2}\cos 2\pi t + 2\sin 2\pi t\right)\right]\mathbf{j}$$

and $\mathbf{a} = \frac{d\mathbf{v}}{dt}$

$$= -30\frac{2}{(t+1)^3}\mathbf{i} - 20\pi\left[-\frac{\pi}{2}e^{-\pi t/2}\left(\frac{1}{2}\cos 2\pi t + 2\sin 2\pi t\right) + e^{-\pi t/2}(-\pi\sin 2\pi t + 4\cos 2\pi t)\right]\mathbf{j}$$

$$= -\frac{60}{(t+1)^3}\mathbf{i} + 10\pi^2 e^{-\pi t/2}(4\sin 2\pi t - 7.5\cos 2\pi t)\mathbf{j}$$

(*a*) At $t = 0$: $\mathbf{r} = 30\left(1 - \frac{1}{1}\right)\mathbf{i} + 20(1)\mathbf{j}$

or $\mathbf{r} = 20 \text{ mm}\uparrow$ ◀

$$\mathbf{v} = 30\left(\frac{1}{1}\right)\mathbf{i} - 20\pi\left[(1)\left(\frac{1}{2} + 0\right)\right]\mathbf{j}$$

or $\mathbf{v} = 43.4 \text{ mm/s} \measuredangle 46.3°$ ◀

$$\mathbf{a} = -\frac{60}{(1)}\mathbf{i} + 10\pi^2(1)(0 - 7.5)\mathbf{j}$$

or $\mathbf{a} = 743 \text{ mm/s}^2 \measuredangle 85.4°$ ◀

PROBLEM 11.93 (Continued)

(*b*) At $t = 1.5$ s: $$\mathbf{r} = 30\left(1 - \frac{1}{2.5}\right)\mathbf{i} + 20e^{-0.75\pi}(\cos 3\pi)\mathbf{j}$$

$$= (18 \text{ mm})\mathbf{i} + (-1.8956 \text{ mm})\mathbf{j}$$

or $\mathbf{r} = 18.10$ mm ⦪ $6.01°$ ◀

$$\mathbf{v} = \frac{30}{(2.5)^2}\mathbf{i} - 20\pi e^{-0.75\pi}\left(\frac{1}{2}\cos 3\pi + 0\right)\mathbf{j}$$

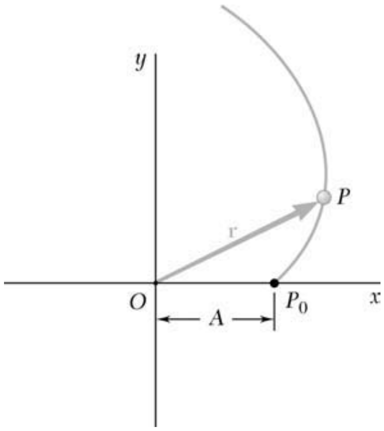
$$= (4.80 \text{ mm/s})\mathbf{i} + (2.9778 \text{ mm/s})\mathbf{j}$$

or $\mathbf{v} = 5.65$ mm/s ⦫ $31.8°$ ◀

$$\mathbf{a} = -\frac{60}{(2.5)^3}\mathbf{i} + 10\pi^2 e^{-0.75\pi}(0 - 7.5\cos 3\pi)\mathbf{j}$$

$$= (-3.84 \text{ mm/s}^2)\mathbf{i} + (70.1582 \text{ mm/s}^2)\mathbf{j}$$

or $\mathbf{a} = 70.3 \text{ mm/s}^2$ ⦩ $86.9°$ ◀



PROBLEM 11.94

The motion of a particle is defined by the position vector $\mathbf{r} = A(\cos t + t\sin t)\mathbf{i} + A(\sin t - t\cos t)\mathbf{j}$, where t is expressed in seconds. Determine the values of t for which the position vector and the acceleration are (*a*) perpendicular, (*b*) parallel.

SOLUTION

We have $$\mathbf{r} = A(\cos t + t\sin t)\mathbf{i} + A(\sin t - t\cos t)\mathbf{j}$$

Then $$\mathbf{v} = \frac{d\mathbf{r}}{dt} = A(-\sin t + \sin t + t\cos t)\mathbf{i} + A(\cos t - \cos t + t\sin t)\mathbf{j}$$

$$= A(t\cos t)\mathbf{i} + A(t\sin t)\mathbf{j}$$

and $$\mathbf{a} = \frac{d\mathbf{v}}{dt} = A(\cos t - t\sin t)\mathbf{i} + A(\sin t + t\cos t)\mathbf{j}$$

(*a*) When **r** and **a** are perpendicular, $\mathbf{r}\cdot\mathbf{a} = 0$

$$A[(\cos t + t\sin t)\mathbf{i} + (\sin t - t\cos t)\mathbf{j}]\cdot A[(\cos t - t\sin t)\mathbf{i} + (\sin t + t\cos t)\mathbf{j}] = 0$$

or $$(\cos t + t\sin t)(\cos t - t\sin t) + (\sin t - t\cos t)(\sin t + t\cos t) = 0$$

or $$(\cos^2 t - t^2\sin^2 t) + (\sin^2 t - t^2\cos^2 t) = 0$$

or $$1 - t^2 = 0 \quad \text{or} \quad t = 1\text{ s} \blacktriangleleft$$

(*b*) When **r** and **a** are parallel, $\mathbf{r}\times\mathbf{a} = 0$

$$A[(\cos t + t\sin t)\mathbf{i} + (\sin t - t\cos t)\mathbf{j}] \times A[(\cos t - t\sin t)\mathbf{i} + (\sin t + t\cos t)\mathbf{j}] = 0$$

or $$[(\cos t + t\sin t)(\sin t + t\cos t) - (\sin t - t\cos t)(\cos t - t\sin t)]\mathbf{k} = 0$$

Expanding $$(\sin t\cos t + t + t^2\sin t\cos t) - (\sin t\cos t - t + t^2\sin t\cos t) = 0$$

or $$2t = 0 \quad \text{or} \quad t = 0 \blacktriangleleft$$

PROBLEM 11.95

The three-dimensional motion of a particle is defined by the position vector $\mathbf{r} = (Rt\cos\omega_n t)\mathbf{i} + ct\mathbf{j} + (Rt\sin\omega_n t)\mathbf{k}$. Determine the magnitudes of the velocity and acceleration of the particle. (The space curve described by the particle is a conic helix.)

SOLUTION

We have $\mathbf{r} = (Rt\cos\omega_n t)\mathbf{i} + ct\mathbf{j} + (Rt\sin\omega_n t)\mathbf{k}$

Then $\mathbf{v} = \frac{d\mathbf{r}}{dt} = R(\cos\omega_n t - \omega_n t\sin\omega_n t)\mathbf{i} + c\mathbf{j} + R(\sin\omega_n t + \omega_n t\cos\omega_n t)\mathbf{k}$

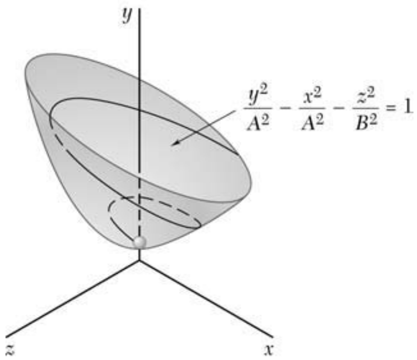
and
$$\mathbf{a} = \frac{d\mathbf{v}}{dt}$$
$$= R(-\omega_n\sin\omega_n t - \omega_n\sin\omega_n t - \omega_n^2 t\cos\omega_n t)\mathbf{i}$$
$$+ R(\omega_n\cos\omega_n t + \omega_n\cos\omega_n t - \omega_n^2 t\sin\omega_n t)\mathbf{k}$$
$$= R(-2\omega_n\sin\omega_n t - \omega_n^2 t\cos\omega_n t)\mathbf{i} + R(2\omega_n\cos\omega_n t - \omega_n^2 t\sin\omega_n t)\mathbf{k}$$

Now
$$v^2 = v_x^2 + v_y^2 + v_z^2$$
$$= [R(\cos\omega_n t - \omega_n t\sin\omega_n t)]^2 + (c)^2 + [R(\sin\omega_n t + \omega_n t\cos\omega_n t)]^2$$
$$= R^2\left[\left(\cos^2\omega_n t - 2\omega_n t\sin\omega_n t\cos\omega_n t + \omega_n^2 t^2\sin^2\omega_n t\right)\right.$$
$$\left. + \left(\sin^2\omega_n t + 2\omega_n t\sin\omega_n t\cos\omega_n t + \omega_n^2 t^2\cos^2\omega_n t\right)\right] + c^2$$
$$= R^2\left(1 + \omega_n^2 t^2\right) + c^2$$

or $v = \sqrt{R^2\left(1 + \omega_n^2 t^2\right) + c^2}$ ◀

Also,
$$a^2 = a_x^2 + a_y^2 + a_z^2$$
$$= \left[R\left(-2\omega_n\sin\omega_n t - \omega_n^2 t\cos\omega_n t\right)\right]^2 + (0)^2$$
$$+ \left[R\left(2\omega_n\cos\omega_n t - \omega_n^2 t\sin\omega_n t\right)\right]^2$$
$$= R^2\left[\left(4\omega_n^2\sin^2\omega_n t + 4\omega_n^3 t\sin\omega_n t\cos\omega_n t + \omega_n^4 t^2\cos^2\omega_n t\right)\right.$$
$$\left. + \left(4\omega_n^2\cos^2\omega_n t - 4\omega_n^3 t\sin\omega_n t\cos\omega_n t + \omega_n^4 t^2\sin^2\omega_n t\right)\right]$$
$$= R^2\left(4\omega_n^2 + \omega_n^4 t^2\right)$$

or $a = R\omega_n\sqrt{4 + \omega_n^2 t^2}$ ◀



PROBLEM 11.96

The three-dimensional motion of a particle is defined by the position vector $\mathbf{r} = (At \cos t)\mathbf{i} + (A\sqrt{t^2+1})\mathbf{j} + (Bt \sin t)\mathbf{k}$, where r and t are expressed in feet and seconds, respectively. Show that the curve described by the particle lies on the hyperboloid $(y/A)^2 - (x/A)^2 - (z/B)^2 = 1$. For $A = 3$ and $B = 1$, determine (*a*) the magnitudes of the velocity and acceleration when $t = 0$, (*b*) the smallest nonzero value of t for which the position vector and the velocity are perpendicular to each other.

SOLUTION

We have $$\mathbf{r} = (At \cos t)\mathbf{i} + (A\sqrt{t^2+1})\mathbf{j} + (Bt \sin t)\mathbf{k}$$

or $$x = At \cos t \qquad y = A\sqrt{t^2+1} \qquad z = Bt \sin t$$

Then $$\cos t = \frac{x}{At} \qquad \sin t = \frac{z}{Bt} \qquad t^2 = \left(\frac{y}{A}\right)^2 - 1$$

Now $$\cos^2 t + \sin^2 t = 1 \Rightarrow \left(\frac{x}{At}\right)^2 + \left(\frac{z}{Bt}\right)^2 = 1$$

or $$t^2 = \left(\frac{x}{A}\right)^2 + \left(\frac{z}{B}\right)^2$$

Then $$\left(\frac{y}{A}\right)^2 - 1 = \left(\frac{x}{A}\right)^2 + \left(\frac{z}{B}\right)^2$$

or $$\left(\frac{y}{A}\right)^2 - \left(\frac{x}{A}\right)^2 - \left(\frac{z}{B}\right)^2 = 1 \qquad \text{Q.E.D.}$$ ◀

(*a*) With $A = 3$ and $B = 1$, we have

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = 3(\cos t - t \sin t)\mathbf{i} + 3\frac{t}{\sqrt{t^2+1}}\mathbf{j} + (\sin t + t \cos t)\mathbf{k}$$

and $$\mathbf{a} = \frac{d\mathbf{v}}{dt} = 3(-\sin t - \sin t - t \cos t)\mathbf{i} + 3\frac{\sqrt{t^2+1} - t\left(\frac{t}{\sqrt{t^2+1}}\right)}{(t^2+1)}\mathbf{j}$$

$$+ (\cos t + \cos t - t \sin t)\mathbf{k}$$

$$= -3(2 \sin t + t \cos t)\mathbf{i} + 3\frac{1}{(t^2+1)^{3/2}}\mathbf{j} + (2 \cos t - t \sin t)\mathbf{k}$$

PROBLEM 11.96 (Continued)

At $t = 0$: $\mathbf{v} = 3(1-0)\mathbf{i} + (0)\mathbf{j} + (0)\mathbf{k}$

$$v = \sqrt{v_x^2 + v_y^2 + v_z^2}$$

or $v = 3 \text{ ft/s}$ ◀

and $\mathbf{a} = -3(0)\mathbf{i} + 3(1)\mathbf{j} + (2-0)\mathbf{k}$

Then $a^2 = (0)^2 + (3)^2 + (2)^2 = 13$

or $a = 3.61 \text{ ft/s}^2$ ◀

(*b*) If **r** and **v** are perpendicular, $\mathbf{r} \cdot \mathbf{v} = 0$

$$[(3t \cos t)\mathbf{i} + (3\sqrt{t^2+1})\mathbf{j} + (t \sin t)\mathbf{k}] \cdot [3(\cos t - t \sin t)\mathbf{i} + \left(3\frac{t}{\sqrt{t^2+1}}\right)\mathbf{j} + (\sin t + t \cos t)\mathbf{k}] = 0$$

or $$(3t \cos t)[3(\cos t - t \sin t)] + (3\sqrt{t^2+1})\left(3\frac{t}{\sqrt{t^2+1}}\right) + (t \sin t)(\sin t + t \cos t) = 0$$

Expanding $(9t \cos^2 t - 9t^2 \sin t \cos t) + (9t) + (t \sin^2 t + t^2 \sin t \cos t) = 0$

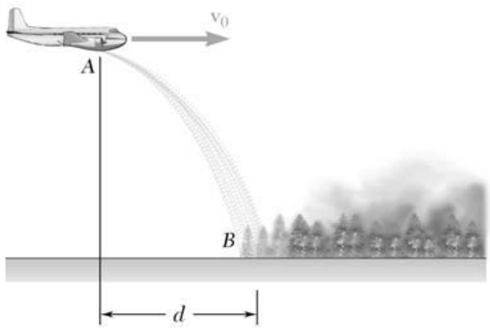
or (with $t \neq 0$) $10 + 8 \cos^2 t - 8t \sin t \cos t = 0$

or $7 + 2 \cos 2t - 2t \sin 2t = 0$

Using "trial and error" or numerical methods, the smallest root is $t = 3.82 \text{ s}$ ◀

Note: The next root is $t = 4.38$ s.

PROBLEM 11.97



An airplane used to drop water on brushfires is flying horizontally in a straight line at 180 mi/h at an altitude of 300 ft. Determine the distance d at which the pilot should release the water so that it will hit the fire at B.

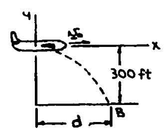
SOLUTION

First note $\quad v_0 = 180 \text{ km/h} = 264 \text{ ft/s}$

Place origin of coordinates at Point A.

Vertical motion. (Uniformly accelerated motion)

$$y = 0 + (0)t - \frac{1}{2}gt^2$$



At B: $$-300 \text{ ft} = -\frac{1}{2}(32.2 \text{ ft/s}^2)t^2$$

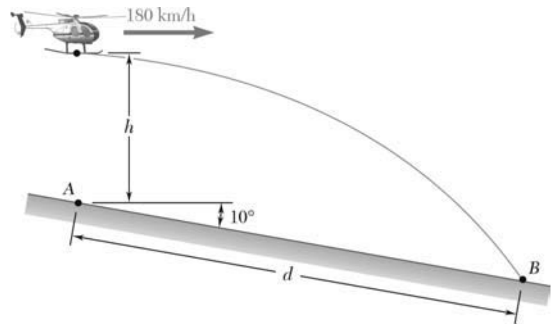
or $$t_B = 4.31666 \text{ s}$$

Horizontal motion. (Uniform)

$$x = 0 + (v_x)_0 t$$

At B: $$d = (264 \text{ ft/s})(4.31666 \text{ s})$$

or $$d = 1140 \text{ ft} \blacktriangleleft$$



PROBLEM 11.98

A helicopter is flying with a constant horizontal velocity of 180 km/h and is directly above Point *A* when a loose part begins to fall. The part lands 6.5 s later at Point *B* on an inclined surface. Determine (*a*) the distance *d* between Points *A* and *B*, (*b*) the initial height *h*.

SOLUTION

Place origin of coordinates at Point *A*.

Horizontal motion: $(v_x)_0 = 180 \text{ km/h} = 50 \text{ m/s}$

$$x = x_0 + (v_x)_0 t = 0 + 50t \text{ m}$$

At Point *B* where $t_B = 6.5$ s, $x_B = (50)(6.5) = 325 \text{ m}$

(*a*) Distance *AB*.

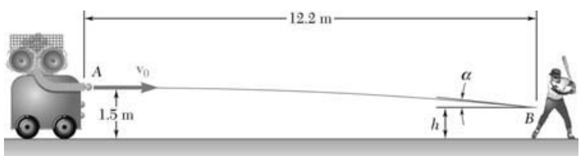
From geometry $d = \dfrac{325}{\cos 10°}$ $d = 330 \text{ m}$ ◀

Vertical motion: $y = y_0 + (v_y)_0 t - \frac{1}{2} g t^2$

At Point *B* $-x_B \tan 10° = h + 0 - \frac{1}{2}(9.81)(6.5)^2$

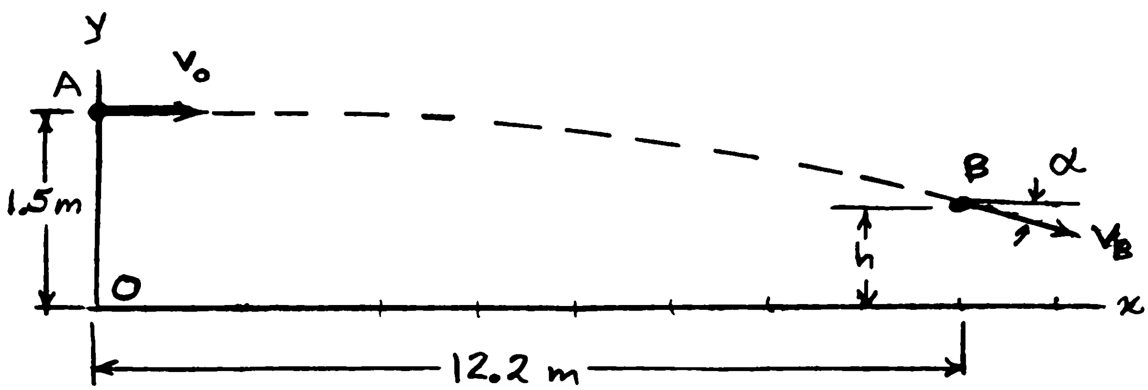
(*b*) Initial height. $h = 149.9 \text{ m}$ ◀

PROBLEM 11.99



A baseball pitching machine "throws" baseballs with a horizontal velocity $\mathbf{v}_0$. Knowing that height h varies between 788 mm and 1068 mm, determine (*a*) the range of values of v_0, (*b*) the values of α corresponding to $h = 788$ mm and $h = 1068$ mm.

SOLUTION



(*a*) Vertical motion: $y_0 = 1.5 \text{ m}, \ (v_y)_0 = 0$

$$y = y_0 + (v_y)_0 t - \frac{1}{2}gt^2 \quad \text{or} \quad t = \sqrt{\frac{2(y_0 - y)}{g}}$$

At Point B, $\quad y = h \quad$ or $\quad t_B = \sqrt{\frac{2(y_0 - h)}{g}}$

When $h = 788 \text{ mm} = 0.788 \text{ m}$, $\quad t_B = \sqrt{\frac{(2)(1.5 - 0.788)}{9.81}} = 0.3810 \text{ s}$

When $h = 1068 \text{ mm} = 1.068 \text{ m}$, $\quad t_B = \sqrt{\frac{(2)(1.5 - 1.068)}{9.81}} = 0.2968 \text{ s}$

Horizontal motion: $x_0 = 0, \ (v_x)_0 = v_0,$

$$x = v_0 t \quad \text{or} \quad v_0 = \frac{x}{t} = \frac{x_B}{t_B}$$

PROBLEM 11.99 (Continued)

With $x_B = 12.2$ m, we get $v_0 = \dfrac{12.2}{0.3810} = 32.02$ m/s

and $v_0 = \dfrac{12.2}{0.2968} = 41.11$ m/s

$32.02 \text{ m/s} \le v_0 \le 41.11 \text{ m/s}$ or $115.3 \text{ km/h} \le v_0 \le 148.0 \text{ km/h}$ ◀

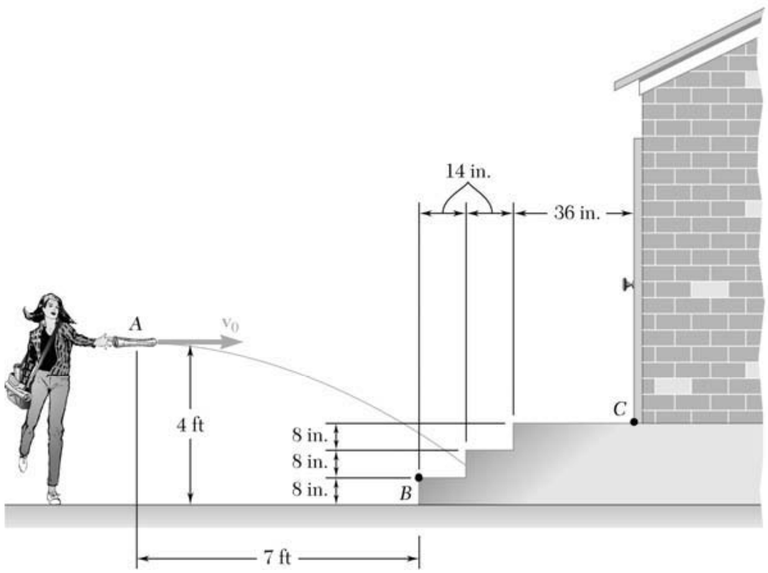
(*b*) Vertical motion: $v_y = (v_y)_0 - gt = -gt$

Horizontal motion: $v_x = v_0$

$$\tan\alpha = -\frac{dy}{dx} = -\frac{(v_y)_B}{(v_x)_B} = \frac{gt_B}{v_0}$$

For $h = 0.788$ m, $\tan\alpha = \dfrac{(9.81)(0.3810)}{32.02} = 0.11673,$ $\alpha = 6.66°$ ◀

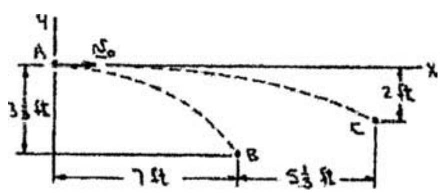
For $h = 1.068$ m, $\tan\alpha = \dfrac{(9.81)(0.2968)}{41.11} = 0.07082,$ $\alpha = 4.05°$ ◀



PROBLEM 11.100

While delivering newspapers, a girl throws a newspaper with a horizontal velocity $\mathbf{v}_0$. Determine the range of values of v_0 if the newspaper is to land between Points B and C.

SOLUTION



Vertical motion. (Uniformly accelerated motion)

$$y = 0 + (0)t - \frac{1}{2}gt^2$$

Horizontal motion. (Uniform)

$$x = 0 + (v_x)_0 t = v_0 t$$

At B: y: $-3\frac{1}{3}\text{ ft} = -\frac{1}{2}(32.2\text{ ft/s}^2)t^2$

or $t_B = 0.455016\text{ s}$

Then x: $7\text{ ft} = (v_0)_B(0.455016\text{ s})$

or $(v_0)_B = 15.38\text{ ft/s}$

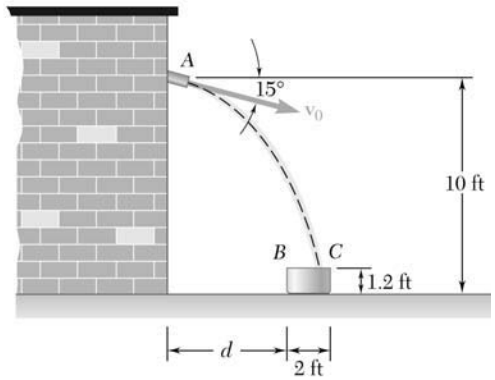
At C: y: $-2\text{ ft} = -\frac{1}{2}(32.2\text{ ft/s}^2)t^2$

or $t_C = 0.352454\text{ s}$

Then x: $12\frac{1}{3}\text{ ft} = (v_0)_C(0.352454\text{ s})$

or $(v_0)_C = 35.0\text{ ft/s}$

$$15.38\text{ ft/s} < v_0 < 35.0\text{ ft/s} \blacktriangleleft$$



PROBLEM 11.101

Water flows from a drain spout with an initial velocity of 2.5 ft/s at an angle of 15° with the horizontal. Determine the range of values of the distance d for which the water will enter the trough BC.

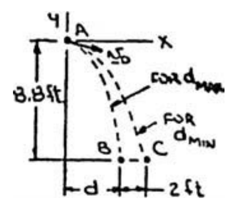
SOLUTION

First note $\quad (v_x)_0 = (2.5\text{ ft/s})\cos 15° = 2.4148\text{ ft/s}$

$$(v_y)_0 = -(2.5\text{ ft/s})\sin 15° = -0.64705\text{ ft/s}$$

Vertical motion. (Uniformly accelerated motion)

$$y = 0 + (v_y)_0 t - \frac{1}{2}gt^2$$



At the top of the trough

$$-8.8\text{ ft} = (-0.64705\text{ ft/s})t - \frac{1}{2}(32.2\text{ ft/s}^2)t^2$$

or $\quad t_{BC} = 0.719491\text{ s} \quad$ (the other root is negative)

Horizontal motion. (Uniform)

$$x = 0 + (v_x)_0 t$$

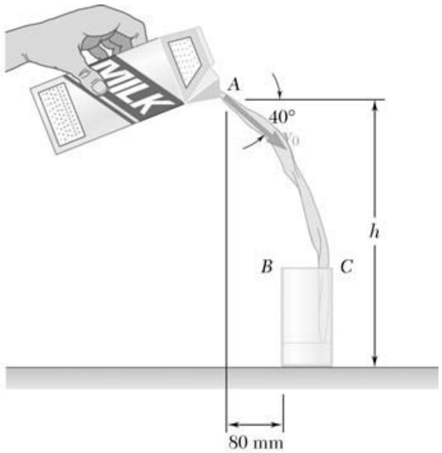
In time t_{BC} $\quad x_{BC} = (2.4148\text{ ft/s})(0.719491\text{ s}) = 1.737\text{ ft}$

Thus, the trough must be placed so that

$$x_B < 1.737\text{ ft or } x_C \geq 1.737\text{ ft}$$

Since the trough is 2 ft wide, it then follows that $\quad 0 < d < 1.737\text{ ft}$ ◀

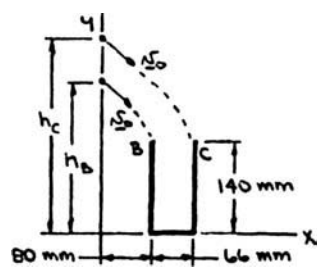
PROBLEM 11.102



Milk is poured into a glass of height 140 mm and inside diameter 66 mm. If the initial velocity of the milk is 1.2 m/s at an angle of 40° with the horizontal, determine the range of values of the height h for which the milk will enter the glass.

SOLUTION

First note



$$(v_x)_0 = (1.2 \text{ m/s}) \cos 40° = 0.91925 \text{ m/s}$$
$$(v_y)_0 = -(1.2 \text{ m/s}) \sin 40° = -0.77135 \text{ m/s}$$

Horizontal motion. (Uniform)

$$x = 0 + (v_x)_0 t$$

Vertical motion. (Uniformly accelerated motion)

$$y = y_0 + (v_y)_0 t - \frac{1}{2} g t^2$$

Milk enters glass at B.

$$x: \quad 0.08 \text{ m} = (0.91925 \text{ m/s})t \quad \text{or} \quad t_B = 0.087028 \text{ s}$$
$$y: \quad 0.140 \text{ m} = h_B + (-0.77135 \text{ m/s})(0.087028 \text{ s}) - \frac{1}{2}(9.81 \text{ m/s}^2)(0.087028 \text{ s})^2$$

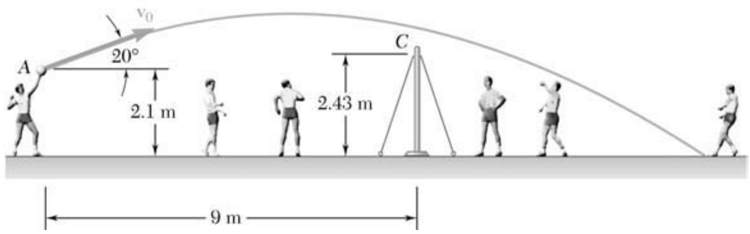
or $$h_B = 0.244 \text{ m}$$

Milk enters glass at C.

$$x: \quad 0.146 \text{ m} = (0.91925 \text{ m/s})t \quad \text{or} \quad t_C = 0.158825 \text{ s}$$
$$y: \quad 0.140 \text{ m} = h_C + (-0.77135 \text{ m/s})(0.158825 \text{ s}) - \frac{1}{2}(9.81 \text{ m/s}^2)(0.158825 \text{ s})^2$$

or $$h_C = 0.386 \text{ m}$$

$$0.244 \text{ m} < h < 0.386 \text{ m} \blacktriangleleft$$



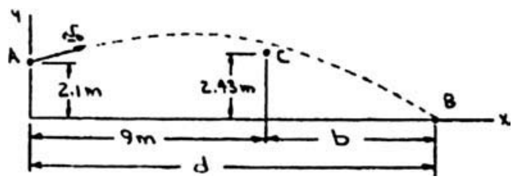
PROBLEM 11.103

A volleyball player serves the ball with an initial velocity $\mathbf{v}_0$ of magnitude 13.40 m/s at an angle of 20° with the horizontal. Determine (*a*) if the ball will clear the top of the net, (*b*) how far from the net the ball will land.

SOLUTION

First note $\quad (v_x)_0 = (13.40 \text{ m/s}) \cos 20° = 12.5919 \text{ m/s}$

$$(v_y)_0 = (13.40 \text{ m/s}) \sin 20° = 4.5831 \text{ m/s}$$



(*a*) Horizontal motion. (Uniform)

$$x = 0 + (v_x)_0 t$$

At *C* $\quad 9 \text{ m} = (12.5919 \text{ m/s})t \quad \text{or} \quad t_C = 0.71475 \text{ s}$

Vertical motion. (Uniformly accelerated motion)

$$y = y_0 + (v_y)_0 t - \frac{1}{2}gt^2$$

At *C*:

$$y_C = 2.1 \text{ m} + (4.5831 \text{ m/s})(0.71475 \text{ s}) - \frac{1}{2}(9.81 \text{ m/s}^2)(0.71475 \text{ s})^2$$
$$= 2.87 \text{ m}$$

$y_C > 2.43$ m (height of net) $\Rightarrow$ ball clears net ◀

(*b*) At *B*, $y = 0$: $\quad 0 = 2.1 \text{ m} + (4.5831 \text{ m/s})t - \frac{1}{2}(9.81 \text{ m/s}^2)t^2$

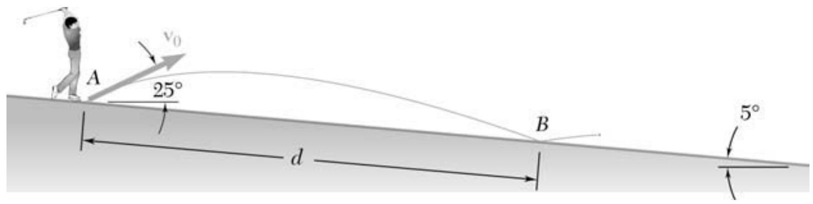
Solving $\quad t_B = 1.271175 \text{ s} \quad$ (the other root is negative)

Then $\quad d = (v_x)_0 t_B = (12.5919 \text{ m/s})(1.271175 \text{ s})$
$$= 16.01 \text{ m}$$

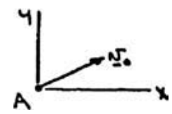
The ball lands $\quad b = (16.01 - 9.00) \text{ m} = 7.01 \text{ m}$ from the net ◀

PROBLEM 11.104

A golfer hits a golf ball with an initial velocity of 160 ft/s at an angle of 25° with the horizontal. Knowing that the fairway slopes downward at an average angle of 5°, determine the distance *d* between the golfer and Point *B* where the ball first lands.



SOLUTION



First note

$$(v_x)_0 = (160 \text{ ft/s})\cos 25°$$
$$(v_y)_0 = (160 \text{ ft/s})\sin 25°$$

and at *B*

$$x_B = d\cos 5° \qquad y_B = -d\sin 5°$$

Now **Horizontal motion**. (Uniform)

$$x = 0 + (v_x)_0 t$$

At *B*

$$d\cos 5° = (160\cos 25°)t \quad \text{or} \quad t_B = \frac{\cos 5°}{160\cos 25°}d$$

Vertical motion. (Uniformly accelerated motion)

$$y = 0 + (v_y)_0 t - \frac{1}{2}gt^2 \quad (g = 32.2 \text{ ft/s}^2)$$

At *B*:

$$-d\sin 5° = (160\sin 25°)t_B - \frac{1}{2}gt_B^2$$

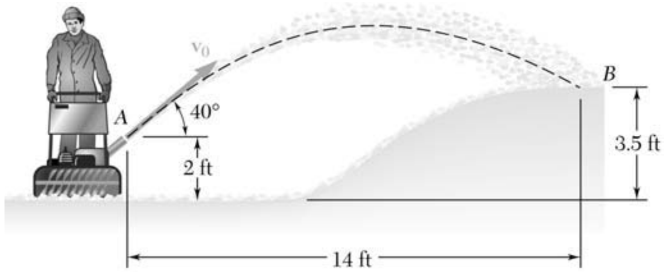
Substituting for t_B

$$-d\sin 5° = (160\sin 25°)\left(\frac{\cos 5°}{160\cos 25°}\right)d - \frac{1}{2}g\left(\frac{\cos 5°}{160\cos 25°}\right)^2 d^2$$

or

$$d = \frac{2}{32.2\cos 5°}(160\cos 25°)^2(\tan 5° + \tan 25°)$$
$$= 726.06 \text{ ft}$$

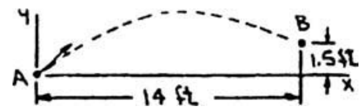
or $d = 242$ yd ◀



PROBLEM 11.105

A homeowner uses a snowblower to clear his driveway. Knowing that the snow is discharged at an average angle of 40° with the horizontal, determine the initial velocity v_0 of the snow.

SOLUTION



First note $\quad (v_x)_0 = v_0 \cos 40°$

$$(v_y)_0 = v_0 \sin 40°$$

<u>Horizontal motion</u>. (Uniform)

$$x = 0 + (v_x)_0 t$$

At B: $\quad 14 = (v_0 \cos 40°)t \quad \text{or} \quad t_B = \frac{14}{v_0 \cos 40°}$

<u>Vertical motion</u>. (Uniformly accelerated motion)

$$y = 0 + (v_y)_0 t - \frac{1}{2}gt^2 \qquad (g = 32.2 \text{ ft/s}^2)$$

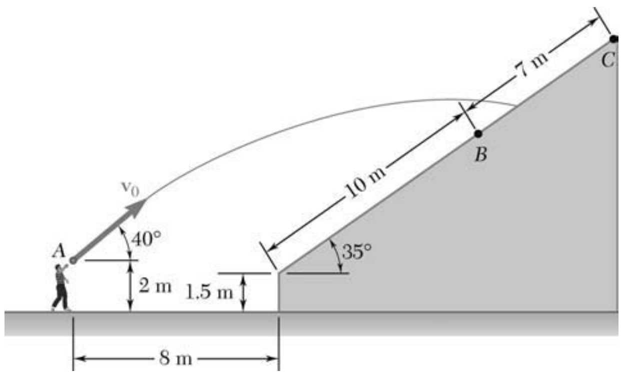
At B: $\quad 1.5 = (v_0 \sin 40°)t_B - \frac{1}{2}gt_B^2$

Substituting for t_B

$$1.5 = (v_0 \sin 40°)\left(\frac{14}{v_0 \cos 40°}\right) - \frac{1}{2}g\left(\frac{14}{v_0 \cos 40°}\right)^2$$

or $\quad v_0^2 = \dfrac{\frac{1}{2}(32.2)(196)/\cos^2 40°}{-1.5 + 14 \tan 40°}$

or $\quad v_0 = 22.9 \text{ ft/s}$ ◀



PROBLEM 11.106

At halftime of a football game souvenir balls are thrown to the spectators with a velocity $\mathbf{v}_0$. Determine the range of values of v_0 if the balls are to land between Points B and C.

SOLUTION

The motion is projectile motion. Place the origin of the xy-coordinate system at ground level just below Point A. The coordinates of Point A are $x_0 = 0,\ y_0 = 2\text{m}.$ The components of initial velocity are $(v_x)_0 = v_0 \cos 40°$ m/s and $(v_y)_0 = v_0 \sin 40°.$

Horizontal motion: $$x = x_0 + (v_x)_0 t = (v_0 \cos 40°)t \qquad (1)$$

Vertical motion: $$y = y_0 + (v_y)_0 t = \frac{1}{2} g t^2$$
$$= 2 + (v_0 \sin 40°) = -\frac{1}{2}(9.81)t^2 \qquad (2)$$

From (1), $$v_0 t = \frac{x}{\cos 40°} \qquad (3)$$

Then $$y = 2 + x \tan 40° - 4.905 t^2$$
$$t^2 = \frac{2 + x \tan 40° - y}{4.905} \qquad (4)$$

Point B: $$x = 8 + 10\cos 35° = 16.1915 \text{ m}$$
$$y = 1.5 + 10 \sin 35° = 7.2358 \text{ m}$$
$$v_0 t = \frac{16.1915}{\cos 40°} = 21.1365 \text{ m}$$
$$t^2 = \frac{2 + 16.1915 \tan 40° - 7.2358}{4.905} \qquad t = 1.3048 \text{ s}$$
$$v_0 = \frac{21.1365}{1.3048} \qquad v_0 = 16.199 \text{ m/s}$$

PROBLEM 11.106 (Continued)

Point C:

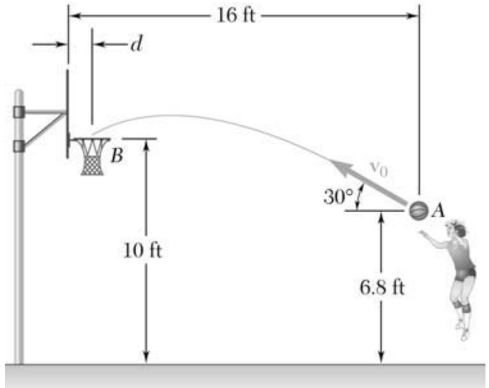
$$x = 8 + (10+7)\cos 35° = 21.9256 \text{ m}$$
$$y = 1.5 + (10+7)\sin 35° = 11.2508 \text{ m}$$

$$v_0 t = \frac{21.9256}{\cos 40°} = 28.622 \text{ m}$$

$$t^2 = \frac{2 + 21.9256\tan 40° - 11.2508}{4.905} \qquad t = 1.3656 \text{ s}$$

$$v_0 = \frac{28.622}{1.3656} \qquad v_0 = 20.96 \text{ m/s}$$

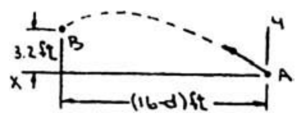
Range of values of v_0. $16.20 \text{ m/s} < v_0 < 21.0 \text{ m/s}$ ◀



PROBLEM 11.107

A basketball player shoots when she is 16 ft from the backboard. Knowing that the ball has an initial velocity $\mathbf{v}_0$ at an angle of 30° with the horizontal, determine the value of v_0 when d is equal to (*a*) 9 in., (*b*) 17 in.

SOLUTION



First note $\quad (v_x)_0 = v_0 \cos 30° \qquad (v_y)_0 = v_0 \sin 30°$

Horizontal motion. (Uniform) $\quad x = 0 + (v_x)_0 t$

At B: $\quad (16-d) = (v_0 \cos 30°)t \quad$ or $\quad t_B = \dfrac{16-d}{v_0 \cos 30°}$

Vertical motion. (Uniformly accelerated motion) $\quad y = 0 + (v_y)_0 t - \frac{1}{2}gt^2 \quad (g = 32.2 \text{ ft/s}^2)$

At B: $\quad 3.2 = (v_0 \sin 30°)t_B - \frac{1}{2}gt_B^2$

Substituting for t_B $\quad 3.2 = (v_0 \sin 30°)\left(\dfrac{16-d}{v_0 \cos 30°}\right) - \dfrac{1}{2}g\left(\dfrac{16-d}{v_0 \cos 30°}\right)^2$

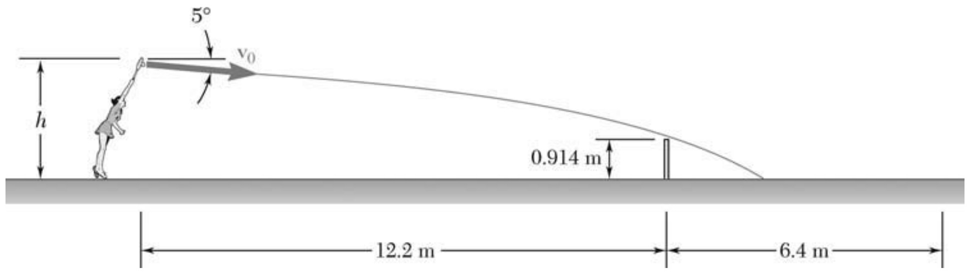
or $\quad v_0^2 = \dfrac{2g(16-d)^2}{3\left[\frac{1}{\sqrt{3}}(16-d) - 3.2\right]}$

(*a*) $\quad d = 9$ in.: $\quad v_0^2 = \dfrac{2(32.2)\left(16-\frac{9}{12}\right)^2}{3\left[\frac{1}{\sqrt{3}}\left(16-\frac{9}{12}\right) - 3.2\right]}$ $\quad v_0 = 29.8$ ft/s ◀

(*b*) $\quad d = 17$ in.: $\quad v_0^2 = \dfrac{2(32.2)\left(16-\frac{17}{12}\right)^2}{3\left[\frac{1}{\sqrt{3}}\left(16-\frac{17}{12}\right) - 3.2\right]}$ $\quad v_0 = 29.6$ ft/s ◀

PROBLEM 11.108

A tennis player serves the ball at a height h = 2.5 m with an initial velocity of $\mathbf{v}_0$ at an angle of 5° with the horizontal. Determine the range for which of v_0 for which the ball will land in the service area which extends to 6.4 m beyond the net.



SOLUTION

The motion is projectile motion. Place the origin of the xy-coordinate system at ground level just below the point where the racket impacts the ball. The coordinates of this impact point are $x_0 = 0,\ y_0 = h = 2.5$ m. The components of initial velocity are $(v_x)_0 = v_0 \cos 5°$ and $(v_y)_0 = v_0 \sin 5°$.

Horizontal motion: $$x = x_0 + (v_x)_0 t = (v_0 \cos 5°)t \qquad (1)$$

Vertical motion: $$y = y_0 + (v_y)_0 t = \frac{1}{2} gt^2$$

$$= 2.5 - (v_0 \sin 5°)t = -\frac{1}{2}(9.81)t^2 \qquad (2)$$

From (1), $$v_0 t = \frac{x}{\cos 5°} \qquad (3)$$

Then $$y = 2.5 - x \tan 5° - 4.905t^2$$

$$t^2 = \frac{2.5 - x \tan 5° - y}{4.905} \qquad (4)$$

At the minimum speed the ball just clears the net.

$$x = 12.2 \text{ m}, \quad y = 0.914 \text{ m}$$

$$v_0 t = \frac{12.2}{\cos 5°} = 12.2466 \text{ m}$$

$$t^2 = \frac{2.5 - 12.2 \tan 5° - 0.914}{4.905} \qquad t = 0.32517 \text{ s}$$

$$v_0 = \frac{12.2466}{0.32517} \qquad v_0 = 37.66 \text{ m/s}$$

PROBLEM 11.108 (Continued)

At the maximum speed the ball lands 6.4 m beyond the net.

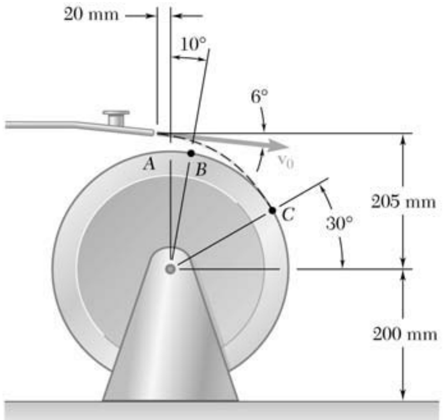
$$x = 12.2 + 6.4 = 18.6 \text{ m} \qquad y = 0$$

$$v_0 t = \frac{18.6}{\cos 5°} = 18.6710 \text{ m}$$

$$t^2 = \frac{2.5 - 18.6 \tan 5° - 0}{4.905} \qquad t = 0.42181 \text{ s}$$

$$v_0 = \frac{18.6710}{0.42181} \qquad v_0 = 44.26 \text{ m/s}$$

Range for v_0. $37.7 \text{ m/s} < v_0 < 44.3 \text{ m/s}$ ◀



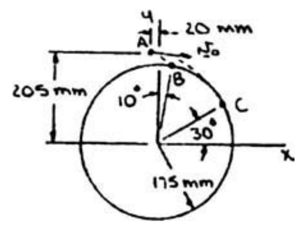
PROBLEM 11.109

The nozzle at A discharges cooling water with an initial velocity $\mathbf{v}_0$ at an angle of 6° with the horizontal onto a grinding wheel 350 mm in diameter. Determine the range of values of the initial velocity for which the water will land on the grinding wheel between Points B and C.

SOLUTION

First note

$$(v_x)_0 = v_0 \cos 6°$$
$$(v_y)_0 = -v_0 \sin 6°$$



Horizontal motion. (Uniform)

$$x = x_0 + (v_x)_0 t$$

Vertical motion. (Uniformly accelerated motion)

$$y = y_0 + (v_y)_0 t - \frac{1}{2}gt^2 \quad (g = 9.81 \text{ m/s}^2)$$

At Point B: $x = (0.175 \text{ m}) \sin 10°$

$y = (0.175 \text{ m}) \cos 10°$

$$x: \quad 0.175 \sin 10° = -0.020 + (v_0 \cos 6°)t$$

or

$$t_B = \frac{0.050388}{v_0 \cos 6°}$$

$$y: \quad 0.175 \cos 10° = 0.205 + (-v_0 \sin 6°)t_B - \frac{1}{2}gt_B^2$$

Substituting for t_B

$$-0.032659 = (-v_0 \sin 6°)\left(\frac{0.050388}{v_0 \cos 6°}\right) - \frac{1}{2}(9.81)\left(\frac{0.050388}{v_0 \cos 6°}\right)^2$$

or
$$v_0^2 = \frac{\frac{1}{2}(9.81)(0.050388)^2}{\cos^2 6°(0.032659 - 0.050388 \tan 6°)}$$

or
$$(v_0)_B = 0.678 \text{ m/s}$$

At Point C:
$$x = (0.175 \text{ m}) \cos 30°$$
$$y = (0.175 \text{ m}) \sin 30°$$

$$x: \quad 0.175 \cos 30° = -0.020 + (v_0 \cos 6°)t$$

or
$$t_C = \frac{0.171554}{v_0 \cos 6°}$$

$$y: \quad 0.175 \sin 30° = 0.205 + (-v_0 \sin 6°)t_C - \frac{1}{2}gt_C^2$$

Substituting for t_C

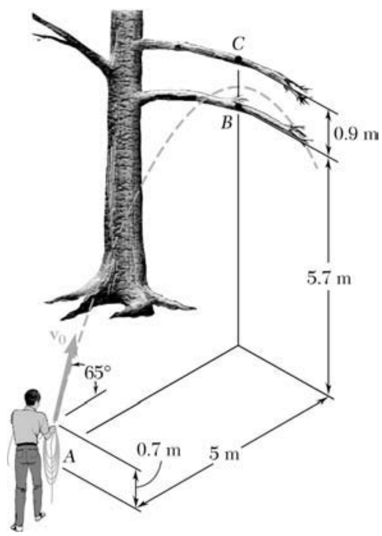
$$-0.117500 = (-v_0 \sin 6°)\left(\frac{0.171554}{v_0 \cos 6°}\right) - \frac{1}{2}(9.81)\left(\frac{0.171554}{v_0 \cos 6°}\right)^2$$

or
$$v_0^2 = \frac{\frac{1}{2}(9.81)(0.171554)^2}{\cos^2 6°(0.117500 - 0.171554 \tan 6°)}$$

or
$$(v_0)_C = 1.211 \text{ m/s}$$

$$0.678 \text{ m/s} < v_0 < 1.211 \text{ m/s} \blacktriangleleft$$

PROBLEM 11.110

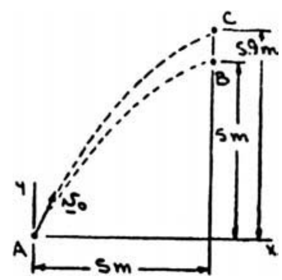


While holding one of its ends, a worker lobs a coil of rope over the lowest limb of a tree. If he throws the rope with an initial velocity $\mathbf{v}_0$ at an angle of 65° with the horizontal, determine the range of values of v_0 for which the rope will go over only the lowest limb.

SOLUTION

First note

$$(v_x)_0 = v_0 \cos 65°$$
$$(v_y)_0 = v_0 \sin 65°$$



Horizontal motion. (Uniform)

$$x = 0 + (v_x)_0 t$$

At either B or C, $x = 5$ m

$$s = (v_0 \cos 65°)t_{B,C}$$

or

$$t_{B,C} = \frac{5}{(v_0 \cos 65°)}$$

Vertical motion. (Uniformly accelerated motion)

$$y = 0 + (v_y)_0 t - \frac{1}{2}gt^2 \qquad (g = 9.81 \text{ m/s}^2)$$

At the tree limbs, $t = t_{B,C}$

$$y_{B,C} = (v_0 \sin 65°)\left(\frac{5}{v_0 \cos 65°}\right) - \frac{1}{2}g\left(\frac{5}{v_0 \cos 65°}\right)^2$$

PROBLEM 11.110 (Continued)

or $$v_0^2 = \frac{\frac{1}{2}(9.81)(25)}{\cos^2 65°(5 \tan 65° - y_{B,C})}$$

$$= \frac{686.566}{5 \tan 65° - y_{B,C}}$$

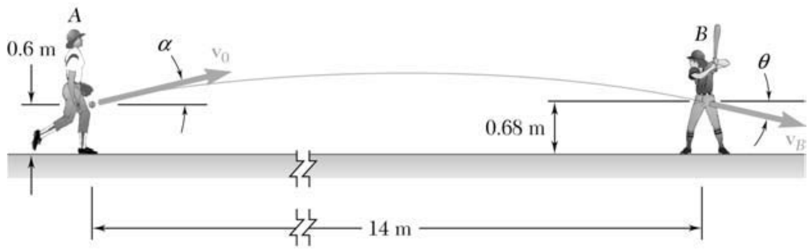
At Point B: $$v_0^2 = \frac{686.566}{5 \tan 65° - 5} \quad \text{or} \quad (v_0)_B = 10.95 \text{ m/s}$$

At Point C: $$v_0^2 = \frac{686.566}{5 \tan 65° - 5.9} \quad \text{or} \quad (v_0)_C = 11.93 \text{ m/s}$$

$$10.95 \text{ m/s} < v_0 < 11.93 \text{ m/s} \blacktriangleleft$$

PROBLEM 11.111

The pitcher in a softball game throws a ball with an initial velocity $\mathbf{v}_0$ of 72 km/h at an angle α with the horizontal. If the height of the ball at Point B is 0.68 m, determine (a) the angle α, (b) the angle θ that the velocity of the ball at Point B forms with the horizontal.



SOLUTION

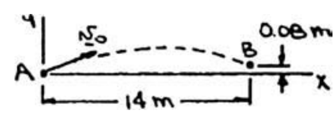
First note

$$v_0 = 72 \text{ km/h} = 20 \text{ m/s}$$

and

$$(v_x)_0 = v_0 \cos\alpha = (20 \text{ m/s}) \cos\alpha$$

$$(v_y)_0 = v_0 \sin\alpha = (20 \text{ m/s}) \sin\alpha$$



(a) Horizontal motion. (Uniform)

$$x = 0 + (v_x)_0 t = (20\cos\alpha)t$$

At Point B:

$$14 = (20\cos\alpha)t \quad \text{or} \quad t_B = \frac{7}{10\cos\alpha}$$

Vertical motion. (Uniformly accelerated motion)

$$y = 0 + (v_y)_0 t - \frac{1}{2}gt^2 = (20\sin\alpha)t - \frac{1}{2}gt^2 \quad (g = 9.81 \text{ m/s}^2)$$

At Point B:

$$0.08 = (20\sin\alpha)t_B - \frac{1}{2}gt_B^2$$

Substituting for t_B

$$0.08 = (20\sin\alpha)\left(\frac{7}{10\cos\alpha}\right) - \frac{1}{2}g\left(\frac{7}{10\cos\alpha}\right)^2$$

or

$$8 = 1400\tan\alpha - \frac{1}{2}g\frac{49}{\cos^2\alpha}$$

Now

$$\frac{1}{\cos^2\alpha} = \sec^2\alpha = 1 + \tan^2\alpha$$

PROBLEM 11.111 (Continued)

Then $$8 = 1400 \tan\alpha - 24.5g(1+\tan^2\alpha)$$

or $$240.345\tan^2\alpha - 1400\tan\alpha + 248.345 = 0$$

Solving $$\alpha = 10.3786° \quad \text{and} \quad \alpha = 79.949°$$

Rejecting the second root because it is not physically reasonable, we have

$$\alpha = 10.38° \blacktriangleleft$$

(*b*) We have $$v_x = (v_x)_0 = 20\cos\alpha$$

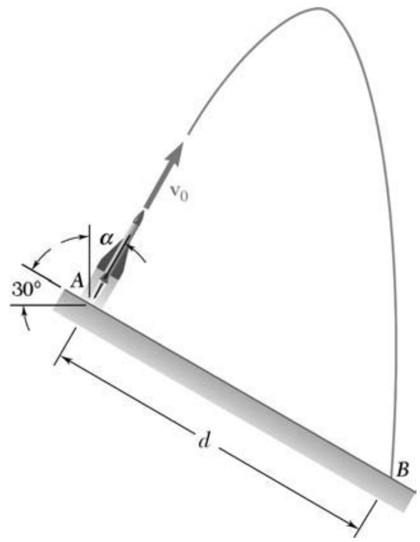
and $$v_y = (v_y)_0 - gt = 20\sin\alpha - gt$$

At Point *B*: $$(v_y)_B = 20\sin\alpha - gt_B$$
$$= 20\sin\alpha - \frac{7g}{10\cos\alpha}$$

Noting that at Point *B*, $v_y < 0$, we have

$$\tan\theta = \frac{|(v_y)_B|}{v_x}$$
$$= \frac{\frac{7g}{10\cos\alpha} - 20\sin\alpha}{20\cos\alpha}$$
$$= \frac{\frac{7}{200}\frac{9.81}{\cos 10.3786°} - \sin 10.3786°}{\cos 10.3786°}$$

or $$\theta = 9.74° \blacktriangleleft$$



PROBLEM 11.112

A model rocket is launched from Point A with an initial velocity $\mathbf{v}_0$ of 75 m/s. If the rocket's descent parachute does not deploy and the rocket lands a distance $d = 100$ m from A, determine (a) the angle α that $\mathbf{v}_0$ forms with the vertical, (b) the maximum height above Point A reached by the rocket, and (c) the duration of the flight.

SOLUTION

Set the origin at Point A. $\quad x_0 = 0, \quad y_0 = 0$

Horizontal motion: $\quad x = v_0 t \sin\alpha \qquad \sin\alpha = \dfrac{x}{v_0 t} \qquad (1)$

Vertical motion: $\quad y = v_0 t \cos\alpha - \dfrac{1}{2} g t^2$

$$\cos\alpha = \frac{1}{v_0 t}\left(y + \frac{1}{2} g t^2\right) \qquad (2)$$

$$\sin^2\alpha + \cos^2\alpha = \frac{1}{(v_0 t)^2}\left[x^2 + \left(y + \frac{1}{2} g t^2\right)^2\right] = 1$$

$$x^2 + y^2 + gyt^2 + \frac{1}{4} g^2 t^4 = v_0^2 t^2$$

$$\frac{1}{4} g^2 t^4 - \left(v_0^2 - gy\right) t^2 + (x^2 + y^2) = 0 \qquad (3)$$

At Point B, $\quad \sqrt{x^2 + y^2} = 100 \text{ m}, \quad x = 100 \cos 30° \text{ m}$

$$y = -100 \sin 30° = -50 \text{ m}$$

$$\frac{1}{4}(9.81)^2 t^4 - [75^2 - (9.81)(-50)] t^2 + 100^2 = 0$$

$$24.0590\, t^4 - 6115.5\, t^2 + 10000 = 0$$

$$t^2 = 252.54 \text{ s}^2 \quad \text{and} \quad 1.6458 \text{ s}^2$$

$$t = 15.8916 \text{ s} \quad \text{and} \quad 1.2829 \text{ s}$$

Restrictions on α: $0 < \alpha < 120°$

$$\tan\alpha = \frac{x}{y + \frac{1}{2}gt^2} = \frac{100\cos 30°}{-50 + (4.905)(15.8916)^2} = 0.0729$$

$$\alpha = 4.1669°$$

and
$$\frac{100\cos 30°}{-50 + (4.905)(1.2829)^2} = -2.0655$$

$$\alpha = 115.8331°$$

Use $\alpha = 4.1669°$ corresponding to the steeper possible trajectory.

(*a*) Angle α. $\alpha = 4.17°$ ◀

(*b*) Maximum height. $v_y = 0$ at $y = y_{max}$

$$v_y = v_0\cos\alpha - gt = 0$$

$$t = \frac{v_0\cos\alpha}{g}$$

$$y_{max} = v_0 t\cos\alpha - \frac{1}{2}gt = \frac{v_0^2\cos^2\alpha}{2g}$$

$$= \frac{(75)^2\cos^2 4.1669°}{(2)(9.81)}$$

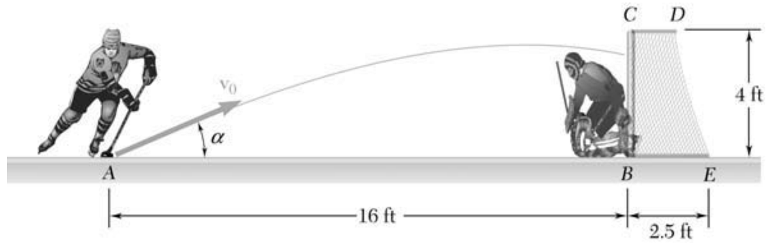
$y_{max} = 285$ m ◀

(*c*) Duration of the flight. (time to reach B)

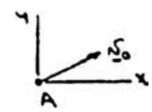
$t = 15.89$ s ◀

PROBLEM 11.113

The initial velocity $\mathbf{v}_0$ of a hockey puck is 105 mi/h. Determine (*a*) the largest value (less than 45°) of the angle α for which the puck will enter the net, (*b*) the corresponding time required for the puck to reach the net.



SOLUTION



First note $\quad v_0 = 105 \text{ mi/h} = 154 \text{ ft/s}$

and
$$(v_x)_0 = v_0 \cos\alpha = (154 \text{ ft/s}) \cos\alpha$$
$$(v_y)_0 = v_0 \sin\alpha = (154 \text{ ft/s}) \sin\alpha$$

(*a*) Horizontal motion. (Uniform)

$$x = 0 + (v_x)_0 t = (154 \cos\alpha)t$$

At the front of the net, $\quad x = 16$ ft

Then $\quad 16 = (154 \cos\alpha)t$

or
$$t_{\text{enter}} = \frac{8}{77 \cos\alpha}$$

Vertical motion. (Uniformly accelerated motion)

$$y = 0 + (v_y)_0 t - \frac{1}{2}gt^2$$
$$= (154 \sin\alpha)t - \frac{1}{2}gt^2 \quad (g = 32.2 \text{ ft/s}^2)$$

At the front of the net,

$$y_{\text{front}} = (154 \sin\alpha)t_{\text{enter}} - \frac{1}{2}gt_{\text{enter}}^2$$
$$= (154 \sin\alpha)\left(\frac{8}{77 \cos\alpha}\right) - \frac{1}{2}g\left(\frac{8}{77 \cos\alpha}\right)^2$$
$$= 16 \tan\alpha - \frac{32g}{5929 \cos^2\alpha}$$

Now $$\frac{1}{\cos^2\alpha} = \sec^2\alpha = 1 + \tan^2\alpha$$

Then $$y_{\text{front}} = 16\tan\alpha - \frac{32g}{5929}(1 + \tan^2\alpha)$$

or $$\tan^2\alpha - \frac{5929}{2g}\tan\alpha + \left(1 + \frac{5929}{32g}y_{\text{front}}\right) = 0$$

Then $$\tan\alpha = \frac{\frac{5929}{2g} \pm \left[\left(-\frac{5929}{2g}\right)^2 - 4\left(1 + \frac{5929}{32g}y_{\text{front}}\right)\right]^{1/2}}{2}$$

or $$\tan\alpha = \frac{5929}{4\times 32.2} \pm \left[\left(-\frac{5929}{4\times 32.2}\right)^2 - \left(1 + \frac{5929}{32\times 32.2}y_{\text{front}}\right)\right]^{1/2}$$

or $$\tan\alpha = 46.0326 \pm [(46.0326)^2 - (1 + 5.7541\, y_{\text{front}})]^{1/2}$$

Now $0 < y_{\text{front}} < 4$ ft so that the positive root will yield values of $\alpha > 45°$ for all values of y_{front}. When the negative root is selected, α increases as y_{front} is increased. Therefore, for $\alpha_{\max}$, set

$$y_{\text{front}} = y_C = 4 \text{ ft}$$

Then $$\tan\alpha = 46.0326 - [(46.0326)^2 - (1 + 5.7541 + 4)]^{1/2}$$

or $$\alpha_{\max} = 14.6604°$$

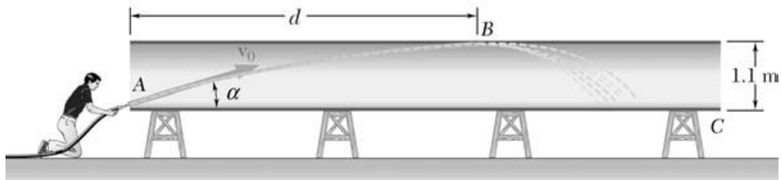
$\alpha_{\max} = 14.66°$ ◀

(*b*) We had found $$t_{\text{enter}} = \frac{8}{77\cos\alpha} = \frac{8}{77\cos 14.6604°}$$

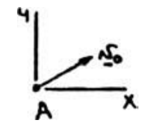
or $t_{\text{enter}} = 0.1074$ s ◀

PROBLEM 11.114

A worker uses high-pressure water to clean the inside of a long drainpipe. If the water is discharged with an initial velocity $\mathbf{v}_0$ of 11.5 m/s, determine (*a*) the distance *d* to the farthest Point *B* on the top of the pipe that the worker can wash from his position at *A*, (*b*) the corresponding angle α.



SOLUTION



First note $(v_x)_0 = v_0 \cos\alpha = (11.5 \text{ m/s})\cos\alpha$

$(v_y)_0 = v_0 \sin\alpha = (11.5 \text{ m/s})\sin\alpha$

By observation, d_{max} occurs when $y_{max} = 1.1$ m.

<u>Vertical motion</u>. (Uniformly accelerated motion)

$$v_y = (v_y)_0 - gt \qquad y = 0 + (v_y)_0 t - \frac{1}{2}gt^2$$

$$= (11.5 \sin\alpha) - gt \qquad = (11.5\sin\alpha)t - \frac{1}{2}gt^2$$

When $y = y_{max}$ at B, $(v_y)_B = 0$

Then $(v_y)_B = 0 = (11.5 \sin\alpha) - gt$

or $t_B = \dfrac{11.5\sin\alpha}{g} \quad (g = 9.81 \text{ m/s}^2)$

and $y_B = (11.5\sin\alpha)t_B - \dfrac{1}{2}gt_B^2$

Substituting for t_B and noting $y_B = 1.1$ m

$$1.1 = (11.5\sin\alpha)\left(\frac{11.5\sin\alpha}{g}\right) - \frac{1}{2}g\left(\frac{11.5\sin\alpha}{g}\right)^2$$

$$= \frac{1}{2g}(11.5)^2\sin^2\alpha$$

or $\sin^2\alpha = \dfrac{2.2\times 9.81}{11.5^2} \qquad \alpha = 23.8265°$

PROBLEM 11.114 (Continued)

(*a*) Horizontal motion. (Uniform)

$$x = 0 + (v_x)_0 t = (11.5 \cos \alpha) t$$

At Point *B*: $x = d_{max}$ and $t = t_B$

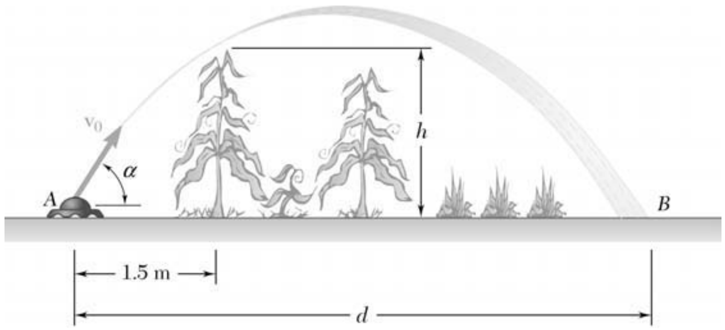
where $t_B = \dfrac{11.5}{9.81} \sin 23.8265° = 0.47356 \text{ s}$

Then $d_{max} = (11.5)(\cos 23.8265°)(0.47356)$

or $d_{max} = 4.98 \text{ m}$ ◀

(*b*) From above $\alpha = 23.8°$ ◀

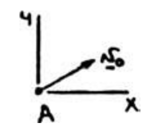
PROBLEM 11.115



An oscillating garden sprinkler which discharges water with an initial velocity $\mathbf{v}_0$ of 8 m/s is used to water a vegetable garden. Determine the distance d to the farthest Point B that will be watered and the corresponding angle α when (a) the vegetables are just beginning to grow, (b) the height h of the corn is 1.8 m.

SOLUTION

First note $$(v_x)_0 = v_0 \cos\alpha = (8 \text{ m/s}) \cos\alpha$$
$$(v_y)_0 = v_0 \sin\alpha = (8 \text{ m/s}) \sin\alpha$$



Horizontal motion. (Uniform)

$$x = 0 + (v_x)_0 t = (8\cos\alpha)t$$

At Point B: $$x = d: \quad d = (8\cos\alpha)t$$

or $$t_B = \frac{d}{8\cos\alpha}$$

Vertical motion. (Uniformly accelerated motion)

$$y = 0 + (v_y)_0 t - \frac{1}{2}gt^2$$
$$= (8\sin\alpha)t - \frac{1}{2}gt^2 \quad (g = 9.81 \text{ m/s}^2)$$

At Point B: $$0 = (8\sin\alpha)t_B - \frac{1}{2}gt_B^2$$

Simplifying and substituting for t_B

$$0 = 8\sin\alpha - \frac{1}{2}g\left(\frac{d}{8\cos\alpha}\right)$$

or $$d = \frac{64}{g}\sin 2\alpha \qquad (1)$$

(a) When $h = 0$, the water can follow any physically possible trajectory. It then follows from Eq. (1) that d is maximum when $2\alpha = 90°$

or $$\alpha = 45° \blacktriangleleft$$

Then $$d = \frac{64}{9.81}\sin(2\times 45°)$$

or $$d_{max} = 6.52 \text{ m} \blacktriangleleft$$

(*b*) Based on Eq. (1) and the results of Part *a*, it can be concluded that *d* increases in value as α increases in value from 0 to 45° and then *d* decreases as α is further increased. Thus, d_{max} occurs for the value of α closest to 45° and for which the water just passes over the first row of corn plants. At this row, $x_{com} = 1.5$ m

so that $$t_{corn} = \frac{1.5}{8\cos\alpha}$$

Also, with $y_{corn} = h$, we have

$$h = (8\sin\alpha)t_{corn} - \frac{1}{2}gt_{corn}^2$$

Substituting for t_{corn} and noting $h = 1.8$ m,

$$1.8 = (8\sin\alpha)\left(\frac{1.5}{8\cos\alpha}\right) - \frac{1}{2}g\left(\frac{1.5}{8\cos\alpha}\right)^2$$

or $$1.8 = 1.5\tan\alpha - \frac{2.25g}{128\cos^2\alpha}$$

Now $$\frac{1}{\cos^2\alpha} = \sec^2\alpha = 1 + \tan^2\alpha$$

Then $$1.8 = 1.5\tan\alpha - \frac{2.25(9.81)}{128}(1 + \tan^2\alpha)$$

or $$0.172441\tan^2\alpha - 1.5\tan\alpha + 1.972441 = 0$$

Solving $$\alpha = 58.229° \quad \text{and} \quad \alpha = 81.965°$$

From the above discussion, it follows that $d = d_{max}$ when

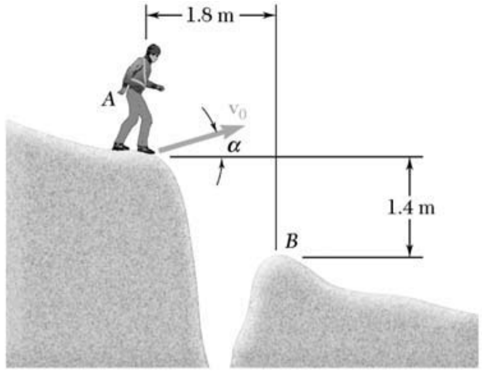
$$\alpha = 58.2° \blacktriangleleft$$

Finally, using Eq. (1)

$$d = \frac{64}{9.81}\sin(2\times 58.229°)$$

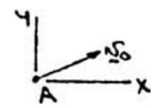
or $$d_{max} = 5.84 \text{ m} \blacktriangleleft$$

PROBLEM 11.116*



A mountain climber plans to jump from A to B over a crevasse. Determine the smallest value of the climber's initial velocity $\mathbf{v}_0$ and the corresponding value of angle α so that he lands at B.

SOLUTION



First note

$$(v_x)_0 = v_0 \cos \alpha$$
$$(v_y)_0 = v_0 \sin \alpha$$

Horizontal motion. (Uniform)

$$x = 0 + (v_x)_0 t = (v_0 \cos \alpha) t$$

At Point B:

$$1.8 = (v_0 \cos \alpha) t$$

or

$$t_B = \frac{1.8}{v_0 \cos \alpha}$$

Vertical motion. (Uniformly accelerated motion)

$$y = 0 + (v_y)_0 t - \frac{1}{2} g t^2$$
$$= (v_0 \sin \alpha) t - \frac{1}{2} g t^2 \quad (g = 9.81 \text{ m/s}^2)$$

At Point B:

$$-1.4 = (v_0 \sin \alpha) t_B - \frac{1}{2} g t_B^2$$

Substituting for t_B

$$-1.4 = (v_0 \sin \alpha)\left(\frac{1.8}{v_0 \cos \alpha}\right) - \frac{1}{2} g \left(\frac{1.8}{v_0 \cos \alpha}\right)^2$$

or

$$v_0^2 = \frac{1.62 g}{\cos^2 \alpha (1.8 \tan \alpha + 1.4)}$$
$$= \frac{1.62 g}{0.9 \sin 2\alpha + 1.4 \cos^2 \alpha}$$

PROBLEM 11.116* (Continued)

Now minimize v_0^2 with respect to α.

We have
$$\frac{dv_0^2}{d\alpha} = 1.62g \frac{-(1.8 \cos 2\alpha - 2.8 \cos \alpha \sin \alpha)}{(0.9 \sin 2\alpha + 1.4 \cos^2 \alpha)^2} = 0$$

or
$$1.8 \cos 2\alpha - 1.4 \sin 2\alpha = 0$$

or
$$\tan 2\alpha = \frac{18}{14}$$

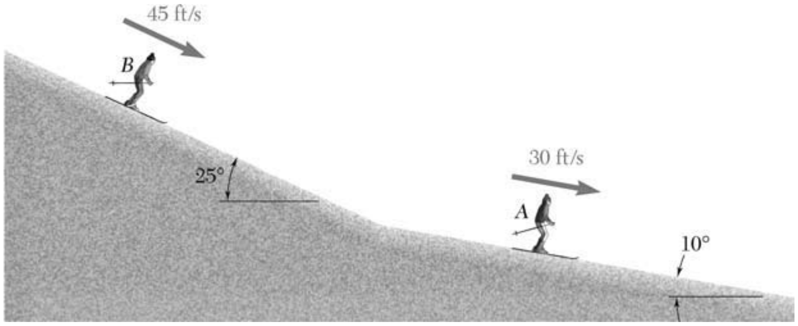
or
$$\alpha = 26.0625° \quad \text{and} \quad \alpha = 206.06°$$

Rejecting the second value because it is not physically possible, we have

$$\alpha = 26.1° \blacktriangleleft$$

Finally,
$$v_0^2 = \frac{1.62 \times 9.81}{\cos^2 26.0625°(1.8 \tan 26.0625° + 1.4)}$$

or
$$(v_0)_{min} = 2.94 \text{ m/s} \blacktriangleleft$$

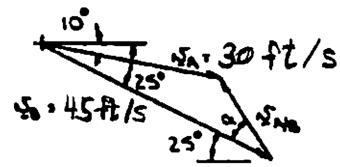


PROBLEM 11.117

The velocities of skiers *A* and *B* are as shown. Determine the velocity of *A* with respect to *B*.

SOLUTION

We have $\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$



The graphical representation of this equation is then as shown.

Then $$\mathbf{v}_{A/B}^2 = 30^2 + 45^2 - 2(30)(45)\cos 15°$$

or $$v_{A/B} = 17.80450 \text{ ft/s}$$

and $$\frac{30}{\sin\alpha} = \frac{17.80450}{\sin 15°}$$

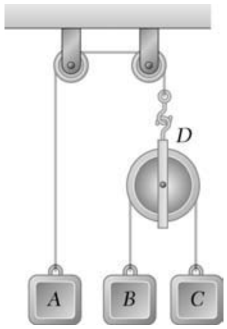
or $$\alpha = 25.8554°$$

$$\alpha + 25° = 50.8554°$$

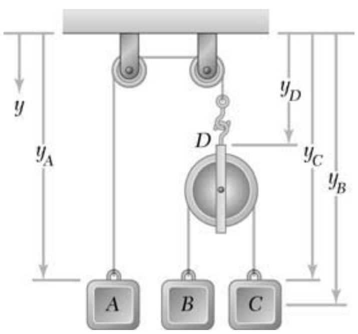
$\mathbf{v}_{A/B} = 17.8$ ft/s ⦩ 50.9° ◀

<u>Alternative solution</u>.

$$\begin{aligned}\mathbf{v}_{A/B} &= \mathbf{v}_A - \mathbf{v}_B \\ &= 30\cos 10°\,\mathbf{i} - 30\sin 10°\mathbf{j} - (45\cos 25°\mathbf{i} - 45°\sin 25°\mathbf{j}) \\ &= 11.2396\mathbf{i} + 13.8084\,\mathbf{j} \\ &= 5.05 \text{ m/s} = 17.8 \text{ ft/s} ⦩ 50.9°\end{aligned}$$

	**PROBLEM 11.118** The three blocks shown move with constant velocities. Find the velocity of each block, knowing that the relative velocity of *A* with respect to *C* is 300 mm/s upward and that the relative velocity of *B* with respect to *A* is 200 mm/s downward.

SOLUTION



From the diagram

Cable 1: $y_A + y_D = \text{constant}$

Then $v_A + v_D = 0$ (1)

Cable 2: $(y_B - y_D) + (y_C - y_D) = \text{constant}$

Then $v_B + v_C - 2v_D = 0$ (2)

Combining Eqs. (1) and (2) to eliminate v_D,

$$2v_A + v_B + v_C = 0 \quad (3)$$

Now $v_{A/C} = v_A - v_C = -300 \text{ mm/s}$ (4)

and $v_{B/A} = v_B - v_A = 200 \text{ mm/s}$ (5)

Then $(3) + (4) - (5) \Rightarrow$

$$(2v_A + v_B + v_C) + (v_A - v_C) - (v_B - v_A) = (-300) - (200)$$

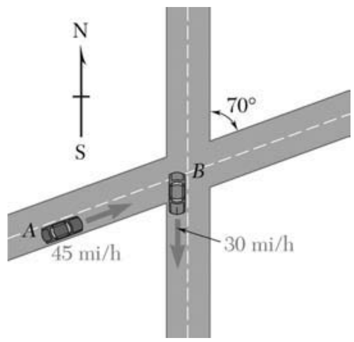
or $\mathbf{v}_A = 125 \text{ mm/s}\uparrow$ ◀

and using Eq. (5) $v_B - (-125) = 200$

or $\mathbf{v}_B = 75 \text{ mm/s}\downarrow$ ◀

Eq. (4) $-125 - v_C = -300$

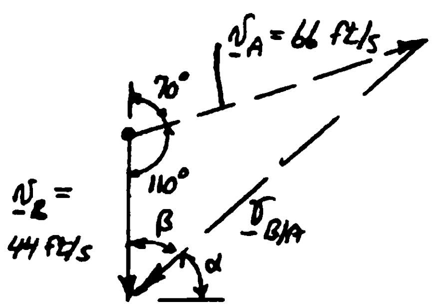
or $\mathbf{v}_C = 175 \text{ mm/s}\downarrow$ ◀



PROBLEM 11.119

Three seconds after automobile B passes through the intersection shown, automobile A passes through the same intersection. Knowing that the speed of each automobile is constant, determine (*a*) the relative velocity of B with respect to A, (*b*) the change in position of B with respect to A during a 4-s interval, (*c*) the distance between the two automobiles 2 s after A has passed through the intersection.

SOLUTION



$$v_A = 45 \text{ mi/h} = 66 \text{ ft/s}$$
$$v_B = 30 \text{ mi/h} = 44 \text{ ft/s}$$

Law of cosines

$$v_{B/A}^2 = 66^2 + 44^2 - 2(66)(44)\cos 110°$$
$$v_{B/A} = 90.99 \text{ ft/s}$$

$$v_B = v_A + v_{B/A}$$

Law of sines

$$\frac{\sin\beta}{66} = \frac{\sin 110°}{90.99} \qquad \beta = 42.97°$$

$$\alpha = 90° - \beta = 90° - 42.97° = 47.03°$$

(*a*) Relative velocity: $\mathbf{v}_{B/A} = 91.0 \text{ ft/s} \measuredangle\ 47.0°$ ◀

(*b*) Change in position for $\Delta t = 4$ s.

$$\Delta r_{B/A} = v_{B/A}\Delta t = (91.0 \text{ ft/s})(4 \text{ s})$$

$\mathbf{r}_{B/A} = 364 \text{ ft} \measuredangle\ 47.0°$ ◀

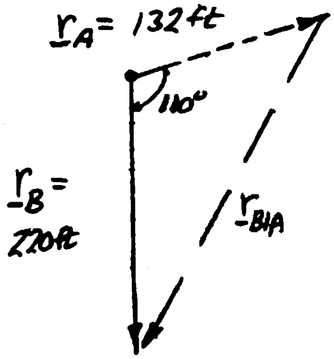
(*c*) Distance between autos 2 seconds after auto A has passed intersection.

Auto A travels for 2 s.

$$v_A = 66 \text{ ft/s} \measuredangle\ 20°$$

$$r_A = v_A t = (66 \text{ ft/s})(2 \text{ s}) = 132 \text{ ft}$$

$$\mathbf{r}_A = 132 \text{ ft} \measuredangle\ 20°$$



<u>Auto *B*</u>

$$\mathbf{v}_B = 44 \text{ ft/s} \downarrow$$

$$\mathbf{r}_B = \mathbf{v}_B t = (44 \text{ ft/s})(5 \text{ s}) = 220 \text{ ft} \downarrow$$

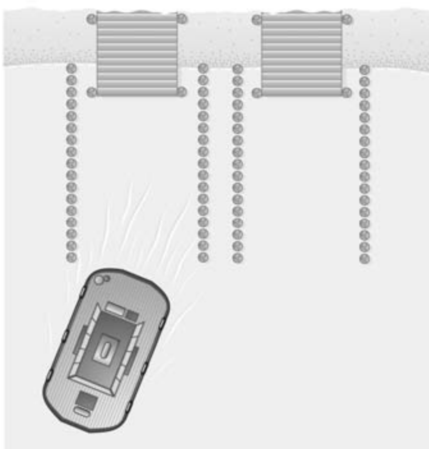
$$\mathbf{r}_B = \mathbf{r}_A + \mathbf{r}_{B/A}$$

<u>Law of cosines</u>

$$r_{B/A}^2 = (132)^2 + (220)^2 - 2(132)(220)\cos 110°$$

$$\mathbf{r}_{B/A} = 292.7 \text{ ft}$$

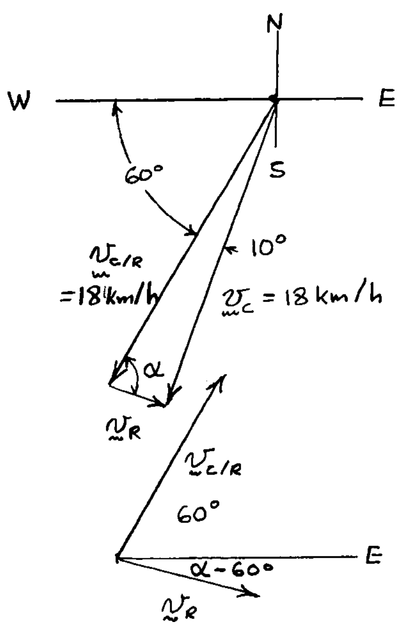
Distance between autos = 293 ft ◀



PROBLEM 11.120

Shore-based radar indicates that a ferry leaves its slip with a velocity $\mathbf{v} = 18$ km/h ↙ 70°, while instruments aboard the ferry indicate a speed of 18.4 km/h and a heading of 30° west of south relative to the river. Determine the velocity of the river.

SOLUTION



We have $\mathbf{v}_F = \mathbf{v}_R + \mathbf{v}_{F/R}$ or $\mathbf{v}_F = \mathbf{v}_{F/R} + \mathbf{v}_R$

The graphical representation of the second equation is then as shown.

We have $v_R^2 = 18^2 + 18.4^2 - 2(18)(18.4)\cos 10°$

or $v_R = 3.1974$ km/h

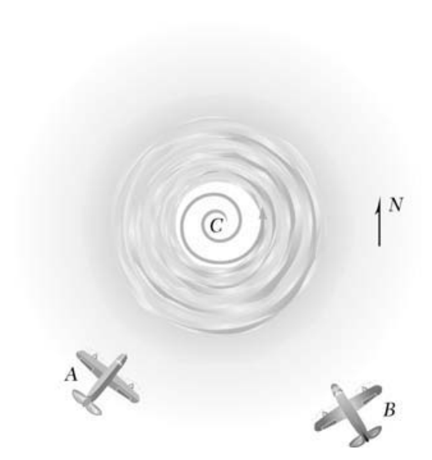
and $\dfrac{18}{\sin\alpha} = \dfrac{3.1974}{\sin 10°}$

or $\alpha = 77.84°$

Noting that

$\mathbf{v}_R = 3.20$ km/h ↘ 17.8° ◀

Alternatively one could use vector algebra.



PROBLEM 11.121

Airplanes A and B are flying at the same altitude and are tracking the eye of hurricane C. The relative velocity of C with respect to A is $\mathbf{v}_{C/A} = 350$ km/h ⟋ 75°, and the relative velocity of C with respect to B is $\mathbf{v}_{C/B} = 400$ km/h ⟍ 40°. Determine (*a*) the relative velocity of B with respect to A, (*b*) the velocity of A if ground-based radar indicates that the hurricane is moving at a speed of 30 km/h due north, (*c*) the change in position of C with respect to B during a 15-min interval.

SOLUTION

(*a*) We have $\mathbf{v}_C = \mathbf{v}_A + \mathbf{v}_{C/A}$

and $\mathbf{v}_C = \mathbf{v}_B + \mathbf{v}_{C/B}$

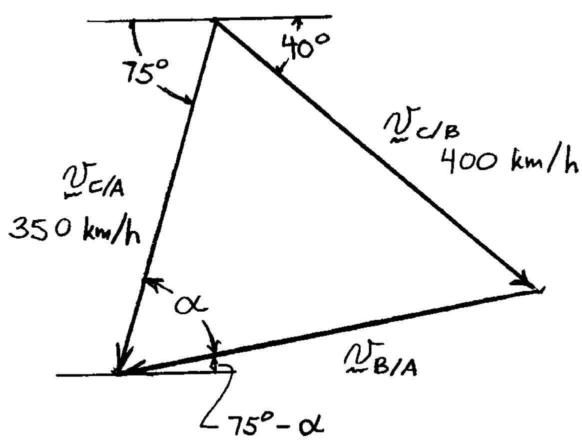
Then $\mathbf{v}_A + \mathbf{v}_{C/A} = \mathbf{v}_B + \mathbf{v}_{C/B}$

or $\mathbf{v}_B - \mathbf{v}_A = \mathbf{v}_{C/A} - \mathbf{v}_{C/B}$

Now $\mathbf{v}_B - \mathbf{v}_A = \mathbf{v}_{B/A}$

so that $\mathbf{v}_{B/A} = \mathbf{v}_{C/A} - \mathbf{v}_{C/B}$

or $\mathbf{v}_{C/A} = \mathbf{v}_{C/B} + \mathbf{v}_{B/A}$



The graphical representation of the last equation is then as shown.

We have $v_{B/A}^2 = 350^2 + 400^2 - 2(350)(400)\cos 65°$

or $v_{B/A} = 405.175$ km/h

and $\dfrac{400}{\sin\alpha} = \dfrac{405.175}{\sin 65°}$

or $\alpha = 63.474°$

$75° - \alpha = 11.526°$

$\mathbf{v}_{B/A} = 405$ km/h ⟋ 11.53° ◀

PROBLEM 11.121 (Continued)

(*b*) We have $\mathbf{v}_C = \mathbf{v}_A + \mathbf{v}_{C/A}$

or $\mathbf{v}_A = (30 \text{ km/h})\mathbf{j} - (350 \text{ km/h})(-\cos 75°\mathbf{i} - \sin 75°\mathbf{j})$

$$\mathbf{v}_A = (90.587 \text{ km/h})\mathbf{i} + (368.07 \text{ km/h})\mathbf{j}$$

or $\mathbf{v}_A = 379 \text{ km/h} \measuredangle 76.17°$ ◀

(*c*) Noting that the velocities of *B* and *C* are constant, we have

$$\mathbf{r}_B = (\mathbf{r}_B)_0 + \mathbf{v}_B t \qquad \mathbf{r}_C = (\mathbf{r}_C)_0 + \mathbf{v}_C t$$

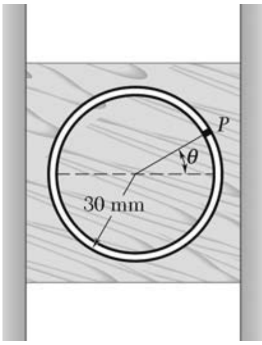
Now $\mathbf{r}_{C/B} = \mathbf{r}_C - \mathbf{r}_B = [(\mathbf{r}_C)_0 - (\mathbf{r}_B)_0] + (\mathbf{v}_C - \mathbf{v}_B)t$

$$= [(\mathbf{r}_C)_0 - (\mathbf{r}_B)_0] + \mathbf{v}_{C/B} t$$

Then $\Delta\mathbf{r}_{C/B} = (\mathbf{r}_{C/B})_{t_2} - (\mathbf{r}_{C/B})_{t_1} = \mathbf{v}_{C/B}(t_2 - t_1) = \mathbf{v}_{C/B}\Delta t$

For $\Delta t = 15$ min: $\Delta r_{C/B} = (400 \text{ km/h})\left(\frac{1}{4}\text{h}\right) = 100 \text{ km}$

$\Delta\mathbf{r}_{C/B} = 100 \text{ km} \searrow 40°$ ◀



PROBLEM 11.122

Pin P moves at a constant speed of 150 mm/s in a counterclockwise sense along a circular slot which has been milled in the slider block A shown. Knowing that the block moves downward at a constant speed 100 mm/s determine the velocity of pin P when (*a*) $\theta = 30°$, (*b*) $\theta = 120°$.

SOLUTION

$$\mathbf{v}_P = \mathbf{v}_A + \mathbf{v}_{P/A}$$

$$\mathbf{v}_P = 100 \text{ ms } (-\mathbf{j}) + 150(\cos\theta\mathbf{i} + \sin\theta\mathbf{j})\text{mm/s}$$

(*a*) For $\theta = 30°$

$$\mathbf{v}_P = -100 \text{ mm/s } (\mathbf{j}) + 150(-\cos(30°)\mathbf{i} + \sin(30°)\mathbf{j})\text{mm/s}$$

$$\mathbf{v}_P = (-75\mathbf{i} + 29.9038\mathbf{j})\text{mm/s}$$

$$\mathbf{v}_P = 80.7 \text{ mm/s} \measuredangle 21.7° \blacktriangleleft$$

(*b*) For $\theta = 120°$

$$\mathbf{v}_P = -100 \text{ mm/s } (\mathbf{j}) + 150(-\cos(120°)\mathbf{i} + \sin(120°)\mathbf{j})\text{mm/s}$$

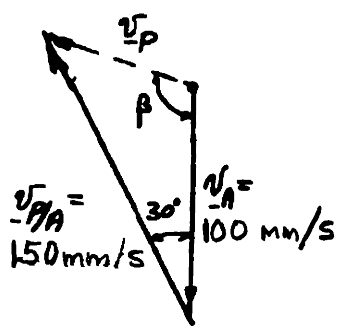
$$\mathbf{v}_P = (-129.9038\mathbf{i} + -175\mathbf{j}) \text{ mm/s}$$

$$\mathbf{v}_P = 218 \text{ mm/s} \measuredangle 53.4° \blacktriangleleft$$

Alternative Solution

(*a*) For $\theta = 30°$, $v_{P/A} = 7.5 \text{ in./s} \measuredangle 30°$

$$v_P = v_A + v_{P/A}$$



Law of cosines

$$v_P^2 = (150)^2 + (100)^2 - 2(100)(150)\cos 30°$$

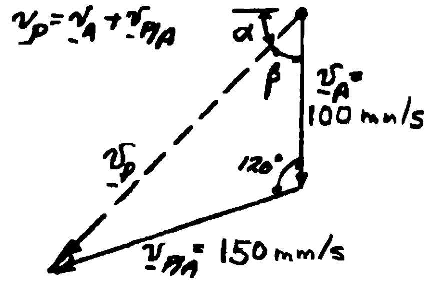
$$v_P = 80.7418 \text{ mm/s}$$

Law of sines

$$\frac{\sin\beta}{150} = \frac{\sin 30°}{80.7418} \qquad \beta = 111.7°$$

$$v_P = 80.7 \text{ mm/s} \measuredangle 21.7° \blacktriangleleft$$

(*b*) For $\theta = 120°$, $v_{P/A} = 150 \text{ mm/s} \measuredangle 30°$



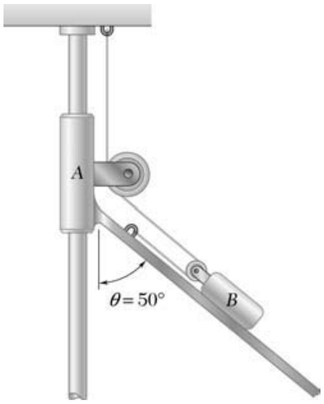
Law of cosines

$$v_P^2 = (150)^2 + (100)^2 - 2(100)(150)\cos 120°$$
$$v_P = 217.9449 \text{ mm/s}$$

Law of sines

$$\frac{\sin\beta}{150} = \frac{\sin 120°}{217.9449} \quad \beta = 36.6°$$
$$\alpha = 90 - \beta = 90° - 36.6 = 53.4°$$

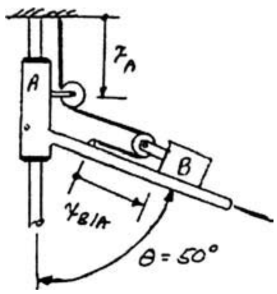
$v_P = 218 \text{ mm/s} \measuredangle 53.4°$ ◀



PROBLEM 11.123

Knowing that at the instant shown assembly A has a velocity of 9 in./s and an acceleration of 15 in./s^2 both directed downward, determine (a) the velocity of block B, (b) the acceleration of block B.

SOLUTION



Length of cable = constant

$$L = x_A + 2x_{B/A} = \text{constant}$$

$$v_A + 2v_{B/A} = 0 \qquad (1)$$

$$a_A + 2a_{B/A} = 0 \qquad (2)$$

Data:

$$\mathbf{a}_A = 15 \text{ in./s}^2 \downarrow$$

$$\mathbf{v}_A = 9 \text{ in./s} \downarrow$$

Eqs. (1) and (2)

$$a_A = -2a_{B/A} \qquad v_A = -2v_{B/A}$$

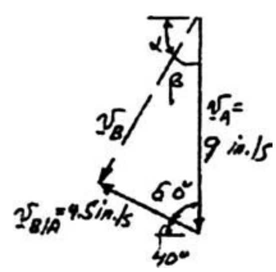
$$15 = -2a_{B/A} \qquad 9 = -2v_{B/A}$$

$$a_{B/A} = -7.5 \text{ in./s}^2 \qquad v_{B/A} = -4.5 \text{ in./s}$$

$$\mathbf{a}_{B/A} = 7.5 \text{ in./s}^2 \searrow 40° \qquad \mathbf{v}_{B/A} = -4.5 \text{ in./s} \searrow 40°$$

(a) Velocity of B.

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$$



Law of cosines:

$$v_B^2 = (9)^2 + (4.5)^2 - 2(9)(4.5)\cos 50°$$

$$v_B = 7.013 \text{ in./s}$$

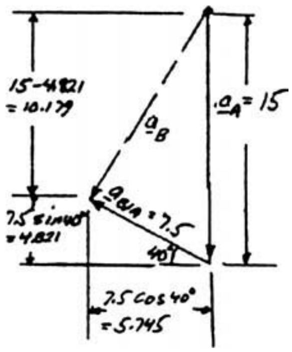
Law of sines:

$$\frac{\sin\beta}{4.5} = \frac{\sin 50°}{7.013} \qquad \beta = 29.44°$$

$$\alpha = 90° - \beta = 90° - 29.44° = 60.56°$$

$$\mathbf{v}_B = 7.01 \text{ in./s} \swarrow 60.6° \blacktriangleleft$$

(*b*) Acceleration of *B*. $\mathbf{a}_B$ may be found by using analysis similar to that used above for v_B. An alternate method is



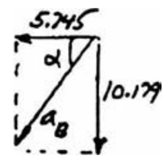
$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$$

$$\mathbf{a}_B = 15 \text{ in./s}^2 \downarrow + 7.5 \text{ in./s}^2 \measuredangle 40^\circ$$

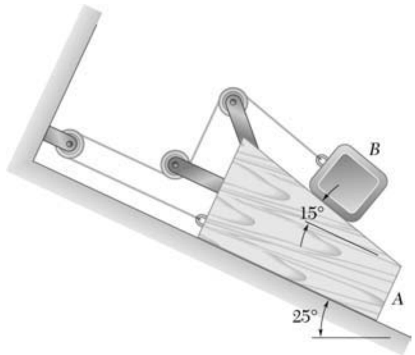
$$= -15\mathbf{j} - (7.5 \cos 40^\circ)\mathbf{i} + (7.5 \sin 40^\circ)\mathbf{j}$$

$$= -15\mathbf{j} - 5.745\mathbf{i} + 4.821\mathbf{j}$$

$$\mathbf{a}_B = -5.745\mathbf{i} - 10.179\mathbf{j}$$



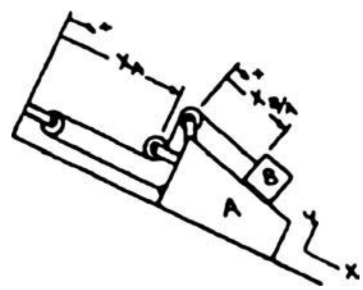
$\mathbf{a}_B = 11.69 \text{ in./s}^2 \measuredangle 60.6^\circ$ ◀



PROBLEM 11.124

Knowing that at the instant shown block A has a velocity of 8 in./s and an acceleration of 6 in./s^2 both directed down the incline, determine (*a*) the velocity of block B, (*b*) the acceleration of block B.

SOLUTION



From the diagram $\quad 2x_A + x_{B/A} = \text{constant}$

Then $\quad 2v_A + v_{B/A} = 0$

or $\quad |v_{B/A}| = 16 \text{ in./s}$

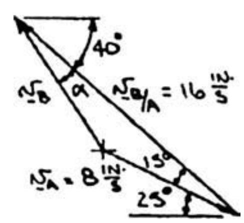
and $\quad 2a_A + a_{B/A} = 0$

or $\quad |a_{B/A}| = 12 \text{ in./s}^2$

Note that $\mathbf{v}_{B/A}$ and $\mathbf{a}_{B/A}$ must be parallel to the top surface of block A.

(*a*) We have $\quad \mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$

The graphical representation of this equation is then as shown. Note that because A is moving downward, B must be moving upward relative to A.



We have $\quad v_B^2 = 8^2 + 16^2 - 2(8)(16)\cos 15°$

or $\quad v_B = 8.5278 \text{ in./s}$

and $\quad \dfrac{8}{\sin\alpha} = \dfrac{8.5278}{\sin 15°}$

or $\quad \alpha = 14.05°$

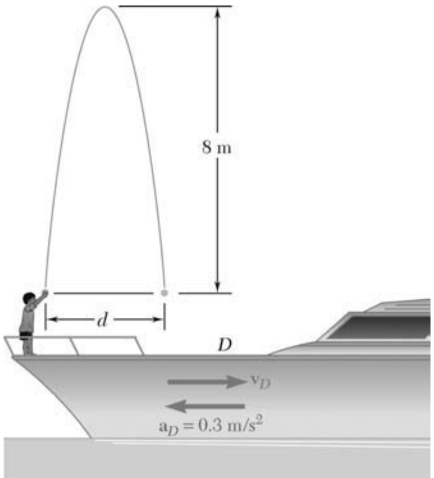
$\mathbf{v}_B = 8.53 \text{ in./s} \;\searrow 54.1°$ ◀

(*b*) The same technique that was used to determine $\mathbf{v}_B$ can be used to determine $\mathbf{a}_B$. An alternative method is as follows.

We have
$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$$
$$= (6\mathbf{i}) + 12(-\cos 15°\mathbf{i} + \sin 15°\mathbf{j})^*$$
$$= -(5.5911 \text{ in./s}^2)\mathbf{i} + (3.1058 \text{ in./s}^2)\mathbf{j}$$

or $\quad \mathbf{a}_B = 6.40 \text{ in./s}^2 \;\searrow 54.1°$ ◀

* Note the orientation of the coordinate axes on the sketch of the system.



PROBLEM 11.125

A boat is moving to the right with a constant deceleration of 0.3 m/s^2 when a boy standing on the deck D throws a ball with an initial velocity relative to the deck which is vertical. The ball rises to a maximum height of 8 m above the release point and the boy must step forward a distance d to catch it at the same height as the release point. Determine (*a*) the distance d, (*b*) the relative velocity of the ball with respect to the deck when the ball is caught.

SOLUTION

Horizontal motion of the ball: $v_x = (v_x)_0, \quad x_{ball} = (v_x)_0 t$

Vertical motion of the ball: $v_y = (v_y)_0 - gt$

$$y_B = (v_y)_0 t - \frac{1}{2}gt^2, \quad (v_y)^2 - (v_y)_0^2 = -2gy$$

At maximum height, $v_y = 0$ and $y = y_{max}$

$$(v_y)^2 = 2gy_{max} = (2)(9.81)(8) = 156.96 \text{ m}^2/\text{s}^2$$

$$(v_y)_0 = 12.528 \text{ m/s}$$

At time of catch, $y = 0 = 12.528 - \frac{1}{2}(9.81)t^2$

or $t_{catch} = 2.554$ s and $v_y = 12.528$ m/s $\downarrow$

Motion of the deck: $v_x = (v_x)_0 + a_D t, \quad x_{deck} = (v_x)_0 t + \frac{1}{2}a_D t^2$

Motion of the ball relative to the deck:

$$(v_{B/D})_x = (v_x)_0 - [(v_x)_0 + a_D t] = -a_D t$$

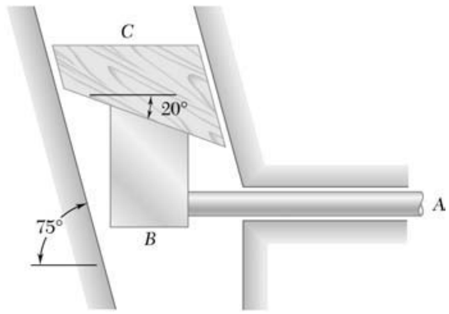
$$x_{B/D} = (v_x)_0 t - \left[(v_x)_0 t + \frac{1}{2}a_D t^2\right] = -\frac{1}{2}a_D t^2$$

$$(v_{B/D})_y = (v_y)_0 - gt, \quad y_{B/D} = y_B$$

(*a*) At time of catch, $d = x_{D/B} = -\frac{1}{2}(-0.3)(2.554)^2$ $d = 0.979$ m ◀

(*b*) $(v_{B/D})_x = -(-0.3)(2.554) = +0.766$ m/s or 0.766 m/s $\rightarrow$

$(v_{B/D})_y = 12.528$ m/s $\downarrow$ $\mathbf{v}_{B/D} = 12.55$ m/s ↘ 86.5° ◀



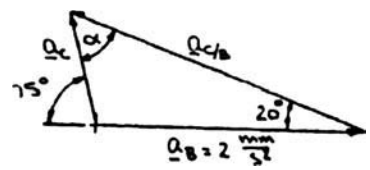
PROBLEM 11.126

The assembly of rod A and wedge B starts from rest and moves to the right with a constant acceleration of 2 mm/s^2. Determine (a) the acceleration of wedge C, (b) the velocity of wedge C when $t = 10$ s.

SOLUTION

(a) We have $\mathbf{a}_C = \mathbf{a}_B + \mathbf{a}_{C/B}$

The graphical representation of this equation is then as shown.



First note $\alpha = 180° - (20° + 105°)$

$= 55°$

Then
$$\frac{a_C}{\sin 20°} = \frac{2}{\sin 55°}$$

$$a_C = 0.83506 \text{ mm/s}^2$$

$\mathbf{a}_C = 0.835$ mm/s^2 ⦪ 75° ◀

(b) For uniformly accelerated motion

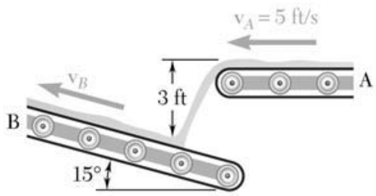
$$v_C = 0 + a_C t$$

At $t = 10$ s: $v_C = (0.83506 \text{ mm/s}^2)(10 \text{ s})$

$= 8.3506$ mm/s

or $\mathbf{v}_C = 8.35$ mm/s ⦪ 75° ◀

PROBLEM 11.127



Determine the required velocity of the belt B if the relative velocity with which the sand hits belt B is to be (*a*) vertical, (*b*) as small as possible.

SOLUTION

A grain of sand will undergo projectile motion.

$$v_{s_x} = v_{s_{x0}} = \text{constant} = -5 \text{ ft/s}$$

<u>*y*-direction</u>. $$v_{s_y} = \sqrt{2gh} = \sqrt{(2)(32.2 \text{ ft/s}^2)(3 \text{ ft})} = 13.90 \text{ ft/s} \downarrow$$

<u>Relative velocity</u>. $$\mathbf{v}_{S/B} = \mathbf{v}_S - \mathbf{v}_B \qquad (1)$$

(*a*) If $v_{S/B}$ is vertical,

$$-v_{S/B}\mathbf{j} = -5\mathbf{i} - 13.9\mathbf{j} - (-v_B \cos 15°\mathbf{i} + v_B \sin 15°\mathbf{j})$$
$$= -5\mathbf{i} - 13.9\mathbf{j} + v_B \cos 15°\mathbf{i} - v_B \sin 15°\mathbf{j}$$

<u>Equate components</u>. $\mathbf{i}$: $0 = -5 + v_B \cos 15°$ $\qquad v_B = \dfrac{5}{\cos 15°} = 5.176 \text{ ft/s}$

$$\mathbf{v}_B = 5.18 \text{ ft/s} \measuredangle 15° \blacktriangleleft$$

(*b*) $v_{S/C}$ is as small as possible, so make $v_{S/B} \perp$ to v_B into (1).

$$-v_{S/B} \sin 15°\mathbf{i} - v_{S/B} \cos 15°\mathbf{j} = -5\mathbf{i} - 13.9\mathbf{j} + v_B \cos 15°\mathbf{i} - v_B \sin 15°\mathbf{j}$$

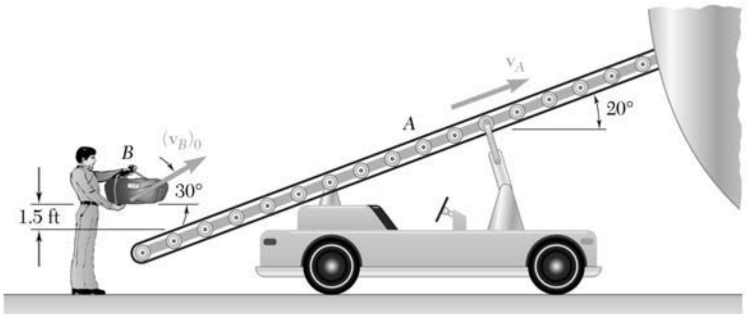
Equate components and transpose terms.

$$(\sin 15°)\, v_{S/B} + (\cos 15°)\, v_B = 5$$
$$(\cos 15°)\, v_{S/B} - (\sin 15°)\, v_B = 13.90$$

Solving, $\qquad v_{S/B} = 14.72 \text{ ft/s} \qquad v_B = 1.232 \text{ ft/s}$

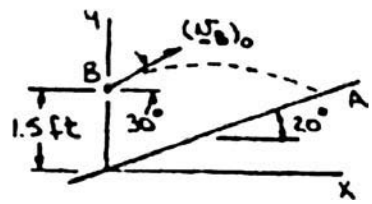
$$\mathbf{v}_B = 1.232 \text{ ft/s} \measuredangle 15° \blacktriangleleft$$

PROBLEM 11.128



Conveyor belt A, which forms a 20° angle with the horizontal, moves at a constant speed of 4 ft/s and is used to load an airplane. Knowing that a worker tosses duffel bag B with an initial velocity of 2.5 ft/s at an angle of 30° with the horizontal, determine the velocity of the bag relative to the belt as it lands on the belt.

SOLUTION



First determine the velocity of the bag as it lands on the belt. Now

$$[(v_B)_x]_0 = (v_B)_0 \cos 30° = (2.5 \text{ ft/s})\cos 30°$$

$$[(v_B)_y]_0 = (v_B)_0 \sin 30° = (2.5 \text{ ft/s})\sin 30°$$

Horizontal motion. (Uniform)

$$x = 0 + [(v_B)_x]_0 t = (2.5 \cos 30°)t \qquad (v_B)_x = [(v_B)_x]_0 = 2.5 \cos 30°$$

Vertical motion. (Uniformly accelerated motion)

$$y = y_0 + [(v_B)_y]_0 t - \frac{1}{2}gt^2 = 1.5 + (2.5 \sin 30°)t - \frac{1}{2}gt^2 \qquad (v_B)_y = [(v_B)_y]_0 - gt = 2.5 \sin 30° - gt$$

The equation of the line collinear with the top surface of the belt is

$$y = x \tan 20°$$

Thus, when the bag reaches the belt

$$1.5 + (2.5 \sin 30°)t - \frac{1}{2}gt^2 = [(2.5 \cos 30°)t]\tan 20°$$

or

$$\frac{1}{2}(32.2)t^2 + 2.5(\cos 30° \tan 20° - \sin 30°)t - 1.5 = 0$$

or

$$16.1t^2 - 0.46198t - 1.5 = 0$$

Solving $t = 0.31992$ s and $t = -0.29122$ s (Reject)

PROBLEM 11.128 (Continued)

The velocity $\mathbf{v}_B$ of the bag as it lands on the belt is then

$$\mathbf{v}_B = (2.5\cos 30°)\mathbf{i} + [2.5\sin 30° - 32.2(0.319\,92)]\mathbf{j}$$
$$= (2.1651\text{ ft/s})\mathbf{i} - (9.0514\text{ ft/s})\mathbf{j}$$

Finally $\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$

or $\mathbf{v}_{B/A} = (2.1651\mathbf{i} - 9.0514\mathbf{j}) - 4(\cos 20°\mathbf{i} + \sin 20°\mathbf{j})$
$$= -(1.59367\text{ ft/s})\mathbf{i} - (10.4195\text{ ft/s})\mathbf{j}$$

or $\mathbf{v}_{B/A} = 10.54\text{ ft/s} \measuredangle 81.3°$ ◀

PROBLEM 11.129

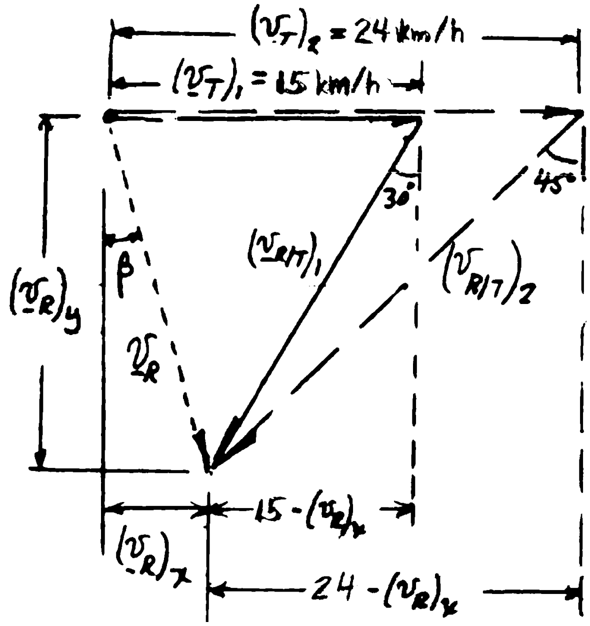
During a rainstorm the paths of the raindrops appear to form an angle of 30° with the vertical and to be directed to the left when observed from a side window of a train moving at a speed of 15 km/h. A short time later, after the speed of the train has increased to 24 km/h, the angle between the vertical and the paths of the drops appears to be 45°. If the train were stopped, at what angle and with what velocity would the drops be observed to fall?

SOLUTION

$$v_{\text{rain}} = v_{\text{train}} + v_{\text{rain/train}}$$

Case ①: $v_T = 15 \text{ km/h} \longrightarrow; \quad v_{R/T} \swarrow 30°$

Case ②: $v_T = 24 \text{ km/h} \longrightarrow; \quad v_{R/T} \swarrow 45°$



Case ①: $(v_R)_y \tan 30 = 15 - (v_R)_x$ (1)

Case ②: $(v_R)_y \tan 45° = 24 - (v_R)_x$ (2)

Substract (1) from (2) $(v_R)_y (\tan 45° - \tan 30°) = 9$

$$(v_R)_y = 21.294 \text{ km/h}$$

Eq. (2): $21.294 \tan 45° = 25 - (v_R)_x$

$$(v_R)_x = 2.706 \text{ km/h}$$

PROBLEM 11.129 (Continued)

$$\tan\beta = \frac{3.706}{21.294}$$
$$\beta = 7.24°$$
$$v_R = \frac{21.294}{\cos 7.24°} = 21.47 \text{ km/h} = 5.96 \text{ m/s}$$

$v_R = 5.96$ m/s ⦪ 82.8° ◀

Alternate solution

Alternate, vector equation $\mathbf{v}_R = \mathbf{v}_T + \mathbf{v}_{R/T}$

For first case, $\mathbf{v}_R = 15\mathbf{i} + v_{R/T-1}(-\sin 30°\mathbf{i} - \cos 30°\mathbf{j})$

For second case, $\mathbf{v}_R = 24\mathbf{i} + v_{R/T-2}(-\sin 45°\mathbf{i} - \cos 45°\mathbf{j})$

Set equal

$$15\mathbf{i} + v_{R/T-1}(-\sin 30°\mathbf{i} - \cos 30°\mathbf{j}) = 24\mathbf{i} + v_{R/T-2}(-\sin 45°\mathbf{i} - \cos 45°\mathbf{j})$$

Separate into components:

i: $$15 - v_{R/T-1}\sin 30° = 24 - v_{R/T-2}\sin 45°$$

$$-v_{R/T-1}\sin 30° + v_{R/T-2}\sin 45° = 9 \quad (3)$$

j: $$-v_{R/T-1}\cos 30° = -v_{R/T-2}\cos 45°$$

$$v_{R/T-1}\cos 30° + v_{R/T-2}\cos 45° = 0 \quad (4)$$

Solving Eqs. (3) and (4) simultaneously,

$v_{R/T-1} = 24.5885$ km/h $\qquad v_{R/T-2} = 30.1146$ km/h

Substitute $\mathbf{v}_{R/T-2}$ back into equation for $\mathbf{v}_R$.

$$\mathbf{v}_R = 24\mathbf{i} + 30.1146(-\sin 45°\mathbf{i} - \cos 45°\mathbf{j})$$

$\mathbf{v}_R = 2.71\mathbf{i} - 21.29\mathbf{j} \qquad \mathbf{v}_R = 21.4654$ km/hr $= 5.96$ m/s

$\theta = \tan^{-1}\left(\frac{-21.29}{2.71}\right) = -82.7585° \qquad \mathbf{v}_R = 5.96$ m/s ⦪ 82.8° ◀

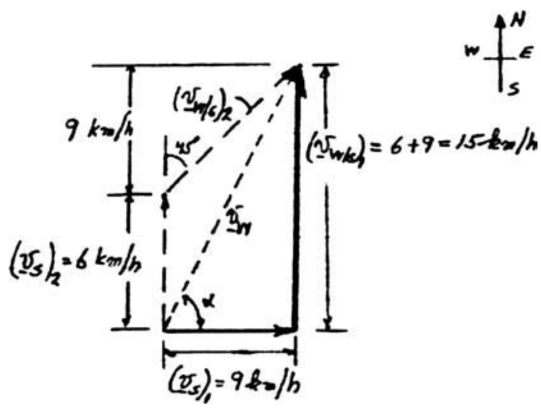
PROBLEM 11.130

As observed from a ship moving due east at 9 km/h, the wind appears to blow from the south. After the ship has changed course and speed, and as it is moving north at 6 km/h, the wind appears to blow from the southwest. Assuming that the wind velocity is constant during the period of observation, determine the magnitude and direction of the true wind velocity.

SOLUTION

$$\mathbf{v}_{\text{wind}} = \mathbf{v}_{\text{ship}} + \mathbf{v}_{\text{wind/ship}}$$

$$\mathbf{v}_w = \mathbf{v}_s + \mathbf{v}_{w/s}$$



Case ①

$$\mathbf{v}_s = 9 \text{ km/h} \rightarrow; \quad \mathbf{v}_{w/s} \uparrow$$

Case ②

$$\mathbf{v}_s = 6 \text{ km/h} \uparrow; \quad \mathbf{v}_{w/s} \nearrow$$

$$\tan\alpha = \frac{15}{9} = 1.6667$$

$$\alpha = 59.0°$$

$$v_w = \sqrt{9^2 + 15^2} = 17.49 \text{ km/h}$$

$$\mathbf{v}_w = 17.49 \text{ km/h} \measuredangle 59.0° \blacktriangleleft$$

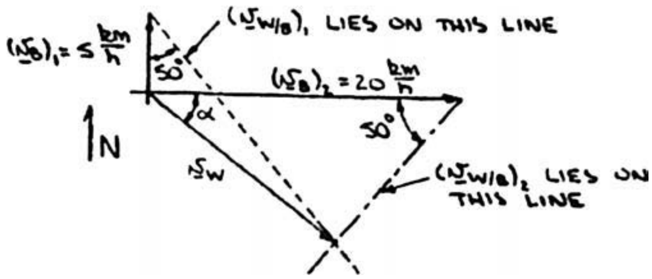
PROBLEM 11.131

When a small boat travels north at 5 km/h, a flag mounted on its stern forms an angle $\theta = 50°$ with the centerline of the boat as shown. A short time later, when the boat travels east at 20 km/h, angle θ is again 50°. Determine the speed and the direction of the wind.

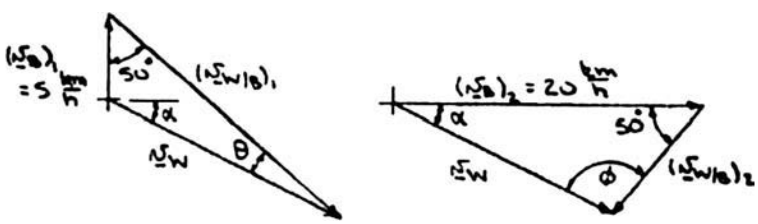
SOLUTION

We have $$\mathbf{v}_W = \mathbf{v}_B + \mathbf{v}_{W/B}$$

Using this equation, the two cases are then graphically represented as shown.



With $\mathbf{v}_W$ now defined, the above diagram is redrawn for the two cases for clarity.



Noting that

$$\theta = 180° - (50° + 90° + \alpha) \qquad \phi = 180° - (50° + \alpha)$$
$$= 40° - \alpha \qquad\qquad = 130° - \alpha$$

We have $$\frac{v_W}{\sin 50°} = \frac{5}{\sin (40° - \alpha)} \qquad \frac{v_W}{\sin 50°} = \frac{20}{\sin (130° - \alpha)}$$

PROBLEM 11.131 (Continued)

Therefore $$\frac{5}{\sin(40^\circ - \alpha)} = \frac{20}{\sin(130^\circ - \alpha)}$$

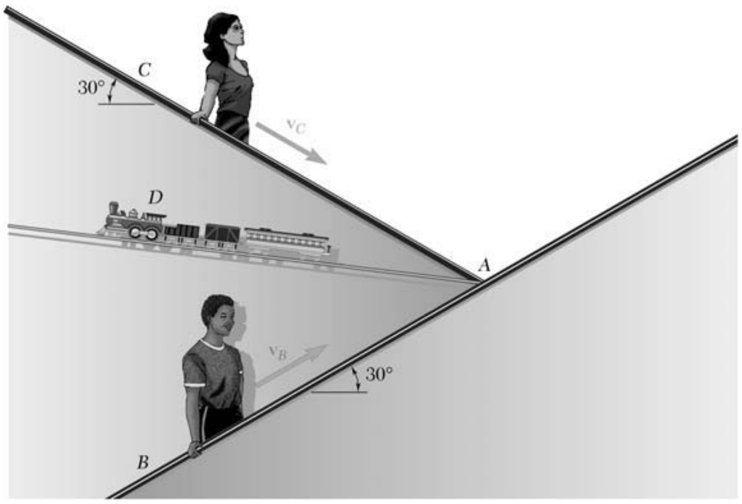
or $$\sin 130^\circ \cos\alpha - \cos 130^\circ \sin\alpha = 4(\sin 40^\circ \cos\alpha - \cos 40^\circ \sin\alpha)$$

or $$\tan\alpha = \frac{\sin 130^\circ - 4\sin 40^\circ}{\cos 130^\circ - 4\cos 40^\circ}$$

or $$\alpha = 25.964^\circ$$

Then $$v_W = \frac{5\sin 50^\circ}{\sin(40^\circ - 25.964^\circ)} = 15.79 \text{ km/h}$$

$\mathbf{v}_W = 15.79$ km/h ⦪ 26.0° ◀



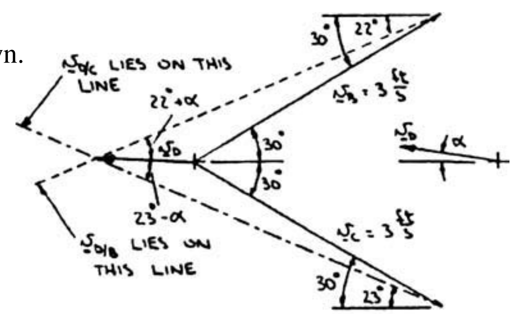
PROBLEM 11.132

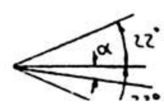
As part of a department store display, a model train D runs on a slight incline between the store's up and down escalators. When the train and shoppers pass Point A, the train appears to a shopper on the up escalator B to move downward at an angle of 22° with the horizontal, and to a shopper on the down escalator C to move upward at an angle of 23° with the horizontal and to travel to the left. Knowing that the speed of the escalators is 3 ft/s, determine the speed and the direction of the train.

SOLUTION

We have
$$\mathbf{v}_D = \mathbf{v}_B + \mathbf{v}_{D/B}$$
$$\mathbf{v}_D = \mathbf{v}_C + \mathbf{v}_{D/C}$$

The graphical representations of these equations are then as shown.





Then $\dfrac{v_D}{\sin 8°} = \dfrac{3}{\sin(22° + \alpha)}$ $\quad \dfrac{v_D}{\sin 7°} = \dfrac{3}{\sin(23° - \alpha)}$

Equating the expressions for $\dfrac{v_D}{3}$

$$\frac{\sin 8°}{\sin(22° + \alpha)} = \frac{\sin 7°}{\sin(23° - \alpha)}$$

or $\sin 8°(\sin 23° \cos\alpha - \cos 23° \sin\alpha)$
$$= \sin 7°(\sin 22° \cos\alpha + \cos 22° \sin\alpha)$$

or
$$\tan\alpha = \frac{\sin 8° \sin 23° - \sin 7° \sin 22°}{\sin 8° \cos 23° + \sin 7° \cos 22°}$$

or
$$\alpha = 2.0728°$$

Then
$$v_D = \frac{3 \sin 8°}{\sin(22° + 2.0728°)} = 1.024 \text{ ft/s}$$

$$\mathbf{v}_D = 1.024 \text{ ft/s} \;\measuredangle\; 2.07° \blacktriangleleft$$

PROBLEM 11.132 (Continued)

Alternate solution using components.

$$\mathbf{v}_B = (3\text{ ft/s}) \measuredangle\, 30^\circ = (2.5981\text{ ft/s})\mathbf{i} + (1.5\text{ ft/s})\mathbf{j}$$

$$\mathbf{v}_C = (3\text{ ft/s}) \searrow 30^\circ = (2.5981\text{ ft/s})\mathbf{i} - (1.5\text{ ft/s})\mathbf{j}$$

$$\mathbf{v}_{D/B} = u_1 \swarrow 22^\circ = -(u_1 \cos 22^\circ)\mathbf{i} - (u_1 \sin 22^\circ)\mathbf{j}$$

$$\mathbf{v}_{D/C} = u_2 \nwarrow 23^\circ = -(u_2 \cos 23^\circ)\mathbf{i} + (u_2 \sin 23^\circ)\mathbf{j}$$

$$\mathbf{v}_D = v_D \nwarrow \alpha = -(v_D \cos\alpha)\mathbf{i} + (v_D \sin\alpha)\mathbf{j}$$

$$\mathbf{v}_D = \mathbf{v}_B + \mathbf{v}_{D/B} = \mathbf{v}_C + \mathbf{v}_{D/C}$$

$$2.5981\mathbf{i} + 1.5\mathbf{j} - (u_1 \cos 22^\circ)\mathbf{i} - (u_1 \sin 22^\circ)\mathbf{j} = 2.5981\mathbf{i} - 1.5\mathbf{j} - (u_2 \cos 23^\circ)\mathbf{i} + (u_2 \sin 23^\circ)\mathbf{j}$$

Separate into components, transpose, and change signs.

$$u_1 \cos 22^\circ - u_2 \cos 23^\circ = 0$$

$$u_1 \sin 22^\circ + u_1 \sin 23^\circ = 3$$

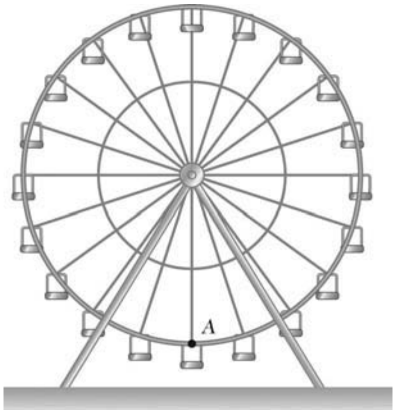
Solving for u_1 and u_2, $u_1 = 3.9054$ ft/s $\quad u_2 = 3.9337$ ft/s

$$\mathbf{v}_D = 2.5981\mathbf{i} + 1.5\mathbf{j} - (3.9054 \cos 22^\circ)\mathbf{i} - (3.9054 \sin 22^\circ)\mathbf{j}$$
$$= -(1.0229\text{ ft/s})\mathbf{i} + (0.0370\text{ ft/s})\mathbf{j}$$

or

$$\mathbf{v}_D = 2.5981\mathbf{i} - 1.5\mathbf{j} - (3.9337 \cos 23^\circ)\mathbf{i} + (3.9337 \sin 23^\circ)\mathbf{j}$$
$$= -(1.0229\text{ ft/s})\mathbf{i} + (0.0370\text{ ft/s})\mathbf{j}$$

$$\mathbf{v}_D = 1.024\text{ ft/s} \nwarrow 2.07^\circ \blacktriangleleft$$



PROBLEM 11.CQ8

The Ferris wheel is rotating with a constant angular velocity ω. What is the direction of the acceleration of Point A?

(a) →

(b) ↑

(c) ↓

(d) ←

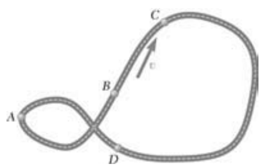
(e) The acceleration is zero.

SOLUTION

The tangential acceleration is zero since the speed is constant, so there will only be normal acceleration pointed upwards.

Answer: (b) ◀

PROBLEM 11.CQ9



A racecar travels around the track shown at a constant speed. At which point will the racecar have the largest acceleration?

(*a*) A

(*b*) B

(*c*) C

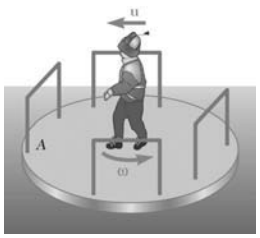
(*d*) The acceleration will be zero at all the points.

SOLUTION

The tangential acceleration is zero since the speed is constant, so there will only be normal acceleration. The normal acceleration will be maximum where the radius of curvature is a minimum, that is at Point A.

Answer: (*a*) ◀

PROBLEM 11.CQ10



A child walks across merry-go-round A with a constant speed u relative to A. The merry-go-round undergoes fixed axis rotation about its center with a constant angular velocity ω counterclockwise. When the child is at the center of A, as shown, what is the direction of his acceleration when viewed from above.

(*a*) $\rightarrow$

(*b*) $\leftarrow$

(*c*) $\uparrow$

(*d*) $\downarrow$

(*e*) The acceleration is zero.

SOLUTION

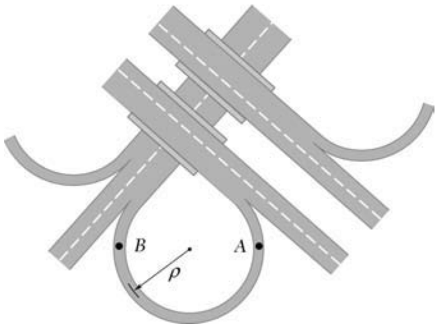
Polar coordinates are most natural for this problem, that is,

$$\vec{a} = (\ddot{r} - r\dot{\theta}^2)\mathbf{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\mathbf{e}_\theta \qquad (1)$$

From the information given, we know $\ddot{r} = 0,\ \ddot{\theta} = 0,\ r = 0,\ \dot{\theta} = \omega,\ \dot{r} = -u.$ When we substitute these values into (1), we will only have a term in the $-\theta$ direction.

Answer: (*d*) ◀

PROBLEM 11.133



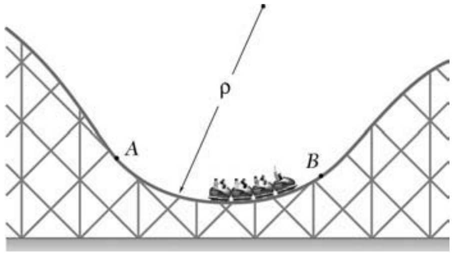
Determine the smallest radius that should be used for a highway if the normal component of the acceleration of a car traveling at 72 km/h is not to exceed $0.8\ \text{m/s}^2$.

SOLUTION

$$a_n = \frac{v^2}{\rho} \qquad a_n = 0.8\ \text{m/s}^2$$

$$v = 72\ \text{km/h} = 20\ \text{m/s}$$

$$0.8\ \text{m/s}^2 = \frac{(20\ \text{m/s})^2}{\rho} \qquad \rho = 500\ \text{m} \blacktriangleleft$$



PROBLEM 11.134

Determine the maximum speed that the cars of the roller-coaster can reach along the circular portion AB of the track if ρ is 25 m and the normal component of their acceleration cannot exceed 3 g.

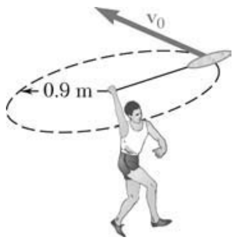
SOLUTION

We have $$a_n = \frac{v^2}{\rho}$$

Then $$(v_{\max})^2_{AB} = (3 \times 9.81 \text{ m/s}^2)(25 \text{ m})$$

or $$(v_{\max})_{AB} = 27.124 \text{ m/s}$$

or $$(v_{\max})_{AB} = 97.6 \text{ km/h} \blacktriangleleft$$



PROBLEM 11.135

A bull-roarer is a piece of wood that produces a roaring sound when attached to the end of a string and whirled around in a circle. Determine the magnitude of the normal acceleration of a bull-roarer when it is spun in a circle of radius 0.9 m at a speed of 20 m/s.

SOLUTION

$$a_n = \frac{v^2}{\rho} = \frac{(20 \text{ m/s})^2}{0.9 \text{ m}} = 444.4 \text{ m/s}^2$$

$$a_n = 444 \text{ m/s}^2 \blacktriangleleft$$

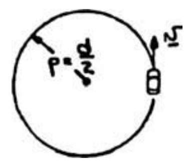
PROBLEM 11.136

To test its performance, an automobile is driven around a circular test track of diameter d. Determine (a) the value of d if when the speed of the automobile is 45 mi/h, the normal component of the acceleration is $11\ \text{ft/s}^2$, (b) the speed of the automobile if $d = 600\ \text{ft}$ and the normal component of the acceleration is measured to be 0.6 g.

SOLUTION

(a) First note $v = 45\ \text{mi/h} = 66\ \text{ft/s}$

Now $$a_n = \frac{v^2}{\rho}$$



$$\rho = \frac{(66\ \text{ft/s})^2}{11\ \text{ft/s}^2} = 396\ \text{ft}$$

$$d = 2\rho$$

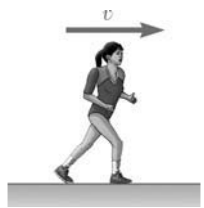
$d = 792\ \text{ft}$ ◀

(b) We have $$a_n = \frac{v^2}{\rho}$$

Then $$v^2 = (0.6 \times 32.2\ \text{ft/s}^2)\left(\frac{1}{2} \times 600\ \text{ft}\right)$$

$$v = 76.131\ \text{ft/s}$$

$v = 51.9\ \text{mi/h}$ ◀



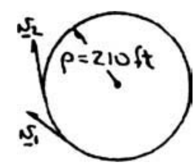
PROBLEM 11.137

An outdoor track is 420 ft in diameter. A runner increases her speed at a constant rate from 14 to 24 ft/s over a distance of 95 ft. Determine the magnitude of the total acceleration of the runner 2 s after she begins to increase her speed.

SOLUTION

We have uniformly accelerated motion

$$v_2^2 = v_1^2 + 2a_t \Delta s_{12}$$



Substituting $\quad (24 \text{ ft/s})^2 = (14 \text{ ft/s})^2 + 2a_t(95 \text{ ft})$

or $\quad a_t = 2 \text{ ft/s}^2$

Also $\quad v = v_1 + a_t t$

At $t = 2$ s: $\quad v = 14 \text{ ft/s} + (2 \text{ ft/s}^2)(2 \text{ s}) = 18 \text{ ft/s}$

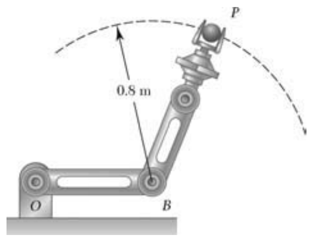
Now $\quad a_n = \dfrac{v^2}{\rho}$

At $t = 2$ s: $\quad a_n = \dfrac{(18 \text{ ft/s})^2}{210 \text{ ft}} = 1.54286 \text{ ft/s}^2$

Finally $\quad a^2 = a_t^2 + a_n^2$

At $t = 2$ s: $\quad a^2 = 2^2 + 1.54286^2$

or $\quad a = 2.53 \text{ ft/s}^2$ ◀



PROBLEM 11.138

A robot arm moves so that *P* travels in a circle about Point *B*, which is not moving. Knowing that *P* starts from rest, and its speed increases at a constant rate of 10 mm/s^2, determine (*a*) the magnitude of the acceleration when $t = 4$ s, (*b*) the time for the magnitude of the acceleration to be 80 mm/s^2.

SOLUTION

Tangential acceleration: $a_t = 10 \text{ mm/s}^2$

Speed: $v = a_t t$

Normal acceleration: $a_n = \dfrac{v^2}{\rho} = \dfrac{a_t^2 t^2}{\rho}$

where $\rho = 0.8 \text{ m} = 800 \text{ mm}$

(*a*) When $t = 4$ s $\quad v = (10)(4) = 40 \text{ mm/s}$

$$a_n = \frac{(40)^2}{800} = 2 \text{ mm/s}^2$$

Acceleration: $a = \sqrt{a_t^2 + a_n^2} = \sqrt{(10)^2 + (2)^2}$

$$a = 10.20 \text{ mm/s}^2 \blacktriangleleft$$

(*b*) Time when $a = 80 \text{ mm/s}^2$

$$a^2 = a_n^2 + a_t^2$$

$$(80)^2 = \left[\frac{(10)^2 t^2}{800}\right]^2 + 10^2 \qquad t^4 = 403200 \text{ s}^4$$

$$t = 25.2 \text{ s} \blacktriangleleft$$

PROBLEM 11.139

A monorail train starts from rest on a curve of radius 400 m and accelerates at the constant rate a_t. If the maximum total acceleration of the train must not exceed 1.5 m/s^2, determine (*a*) the shortest distance in which the train can reach a speed of 72 km/h, (*b*) the corresponding constant rate of acceleration a_t.

SOLUTION

When $v = 72$ km/h $= 20$ m/s and $\rho = 400$ m,

$$a_n = \frac{v^2}{\rho} = \frac{(20)^2}{400} = 1.000 \text{ m/s}^2$$

But $$a = \sqrt{a_n^2 + a_t^2}$$

$$a_t = \sqrt{a^2 - a_n^2} = \sqrt{(1.5)^2 - (1.000)^2} = \pm 1.11803 \text{ m/s}^2$$

Since the train is accelerating, reject the negative value.

(*a*) Distance to reach the speed.

$$v_0 = 0$$

Let $$x_0 = 0$$

$$v_1^2 = v_0^2 + 2a_t(x_1 - x_0) = 2a_t x_1$$

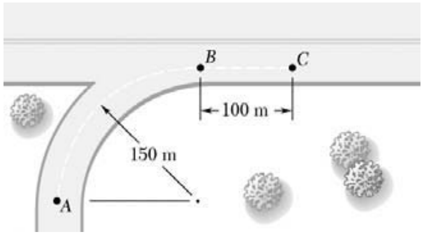
$$x_1 = \frac{v_1^2}{2a_t} = \frac{(20)^2}{(2)(1.11803)}$$

$x_1 = 178.9$ m ◀

(*b*) Corresponding tangential acceleration.

$a_t = 1.118$ m/s^2 ◀

PROBLEM 11.140



A motorist starts from rest at Point A on a circular entrance ramp when $t = 0$, increases the speed of her automobile at a constant rate and enters the highway at Point B. Knowing that her speed continues to increase at the same rate until it reaches 100 km/h at Point C, determine (a) the speed at Point B, (b) the magnitude of the total acceleration when $t = 20$ s.

SOLUTION

Speeds: $v_0 = 0 \qquad v_1 = 100 \text{ km/h} = 27.78 \text{ m/s}$

Distance: $s = \frac{\pi}{2}(150) + 100 = 335.6 \text{ m}$

Tangential component of acceleration: $v_1^2 = v_0^2 + 2a_t s$

$$a_t = \frac{v_1^2 - v_0^2}{2s} = \frac{(27.78)^2 - 0}{(2)(335.6)} = 1.1495 \text{ m/s}^2$$

At Point B, $v_B^2 = v_0^2 + 2a_t s_B$ where $s_B = \frac{\pi}{2}(150) = 235.6 \text{ m}$

$$v_B^2 = 0 + (2)(1.1495)(235.6) = 541.69 \text{ m}^2/\text{s}^2$$

$$v_B = 23.27 \text{ m/s} \qquad v_B = 83.8 \text{ km/h} \blacktriangleleft$$

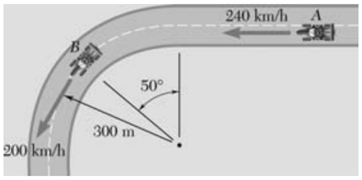
(a) At $t = 20$ s, $v = v_0 + a_t t = 0 + (1.1495)(20) = 22.99 \text{ m/s}$

Since $v < v_B$, the car is still on the curve. $\rho = 150 \text{ m}$

Normal component of acceleration: $a_n = \frac{v^2}{\rho} = \frac{(22.99)^2}{150} = 3.524 \text{ m/s}^2$

(b) Magnitude of total acceleration: $|a| = \sqrt{a_t^2 + a_n^2} = \sqrt{(1.1495)^2 + (3.524)^2} \qquad |a| = 3.71 \text{ m/s}^2 \blacktriangleleft$

PROBLEM 11.141



Racecar A is traveling on a straight portion of the track while racecar B is traveling on a circular portion of the track. At the instant shown, the speed of A is increasing at the rate of 10 m/s^2, and the speed of B is decreasing at the rate of 6 m/s^2. For the position shown, determine (a) the velocity of B relative to A, (b) the acceleration of B relative to A.

SOLUTION

Speeds:

$$v_A = 240 \text{ km/h} = 66.67 \text{ m/s}$$
$$v_B = 200 \text{ km/h} = 55.56 \text{ m/s}$$

Velocities:

$$\mathbf{v}_A = 66.67 \text{ m/s} \longleftarrow$$
$$\mathbf{v}_B = 55.56 \text{ m/s} \angle 50°$$

(a) Relative velocity:

$$\mathbf{v}_{B/A} = \mathbf{v}_B - \mathbf{v}_A$$
$$\mathbf{v}_{B/A} = (55.56\cos 50°) \leftarrow + 55.56\sin 50° \downarrow + 66.67 \longrightarrow$$
$$= 30.96 \rightarrow + 42.56 \downarrow$$
$$= 52.63 \text{ m/s} \searrow 53.96°$$

$$\mathbf{v}_{B/A} = 189.5 \text{ km/h} \searrow 54.0° \blacktriangleleft$$

Tangential accelerations:

$$(\mathbf{a}_A)_t = 10 \text{ m/s}^2 \longleftarrow$$
$$(\mathbf{a}_B)_t = 6 \text{ m/s}^2 \angle 50°$$

Normal accelerations:

$$a_n = \frac{v^2}{\rho}$$

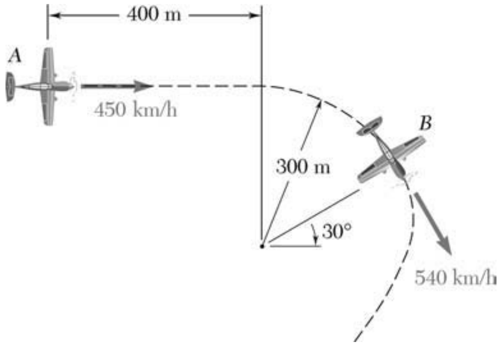
Car A: $(\rho = \infty)$ $\qquad (\mathbf{a}_A)_n = 0$

Car B: $(\rho = 300 \text{ m})$

$$(\mathbf{a}_B)_n = \frac{(55.56)^2}{300} = 10.288 \qquad (\mathbf{a}_B)_n = 10.288 \text{ m/s}^2 \searrow 40°$$

(b) Acceleration of B relative to A:

$$\mathbf{a}_{B/A} = \mathbf{a}_B - \mathbf{a}_A$$
$$\mathbf{a}_{B/A} = (\mathbf{a}_B)_t + (\mathbf{a}_B)_n - (\mathbf{a}_A)_t - (\mathbf{a}_A)_n$$
$$= 6 \angle 50° + 10.288 \searrow 40° + 10 \rightarrow + 0$$
$$= (6\cos 50° + 10.288\cos 40° + 10) \longrightarrow$$
$$+ (6\sin 50° - 10.288\sin 40°) \uparrow$$
$$= 21.738 \rightarrow + 2.017 \downarrow \qquad \mathbf{a}_{B/A} = 21.8 \text{ m/s}^2 \searrow 5.3° \blacktriangleleft$$



PROBLEM 11.142

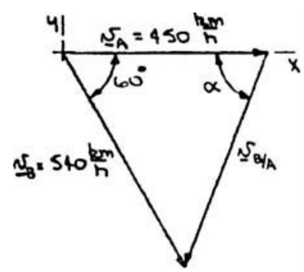
At a given instant in an airplane race, airplane A is flying horizontally in a straight line, and its speed is being increased at the rate of $8\ \text{m/s}^2$. Airplane B is flying at the same altitude as airplane A and, as it rounds a pylon, is following a circular path of 300-m radius. Knowing that at the given instant the speed of B is being decreased at the rate of $3\ \text{m/s}^2$, determine, for the positions shown, (*a*) the velocity of B relative to A, (*b*) the acceleration of B relative to A.

SOLUTION

First note $\qquad v_A = 450\ \text{km/h} \quad v_B = 540\ \text{km/h} = 150\ \text{m/s}$

(*a*) We have $\qquad \mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$

The graphical representation of this equation is then as shown.



We have $\qquad v_{B/A}^2 = 450^2 + 540^2 - 2(450)(540)\cos 60°$

$$v_{B/A} = 501.10\ \text{km/h}$$

and

$$\frac{540}{\sin\alpha} = \frac{501.10}{\sin 60°}$$

$$\alpha = 68.9°$$

$$\mathbf{v}_{B/A} = 501\ \text{km/h} \nearrow 68.9° \blacktriangleleft$$

(*b*) First note $\qquad \mathbf{a}_A = 8\ \text{m/s}^2 \longrightarrow \qquad (\mathbf{a}_B)_t = 3\ \text{m/s}^2 \searrow 60°$

Now

$$(a_B)_n = \frac{v_B^2}{\rho_B} = \frac{(150\ \text{m/s})^2}{300\ \text{m}}$$

$$(\mathbf{a}_B)_n = 75\ \text{m/s}^2 \swarrow 30°$$

Then

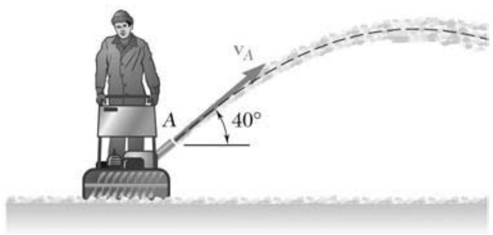
$$\begin{aligned}\mathbf{a}_B &= (\mathbf{a}_B)_t + (\mathbf{a}_B)_n \\ &= 3(-\cos 60°\,\mathbf{i} + \sin 60°\,\mathbf{j}) + 75(-\cos 30°\,\mathbf{i} - \sin 30°\,\mathbf{j}) \\ &= -(66.452\ \text{m/s}^2)\mathbf{i} - (34.902\ \text{m/s}^2)\mathbf{j}\end{aligned}$$

Finally

$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$$

$$\begin{aligned}\mathbf{a}_{B/A} &= (-66.452\mathbf{i} - 34.902\mathbf{j}) - (8\mathbf{i}) \\ &= -(74.452\ \text{m/s}^2)\mathbf{i} - (34.902\ \text{m/s}^2)\mathbf{j}\end{aligned}$$

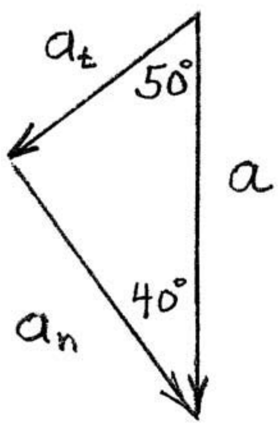
$$\mathbf{a}_{B/A} = 82.2\ \text{m/s}^2 \nearrow 25.1° \blacktriangleleft$$



PROBLEM 11.143

From a photograph of a homeowner using a snowblower, it is determined that the radius of curvature of the trajectory of the snow was 30 ft as the snow left the discharge chute at A. Determine (a) the discharge velocity $\mathbf{v}_A$ of the snow, (b) the radius of curvature of the trajectory at its maximum height.

SOLUTION



(a) The acceleration vector is $32.2 \text{ ft/s} \downarrow$.

At Point A, tangential and normal components of **a** are as shown in the sketch.

$$a_n = a\cos 40° = 32.2\cos 40° = 24.67 \text{ ft/s}^2$$

$$v_A^2 = \rho_A (a_A)_n = (30)(24.67) = 740.0 \text{ ft}^2/\text{s}^2$$

$$\mathbf{v}_A = 27.2 \text{ ft/s} \measuredangle 40° \blacktriangleleft$$

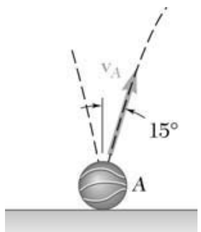
$$v_x = 27.20\cos 40° = 20.84 \text{ ft/s}$$

(b) At maximum height, $v = v_x = 20.84$ ft/s

$$a_n = g = 32.2 \text{ ft/s}^2,$$

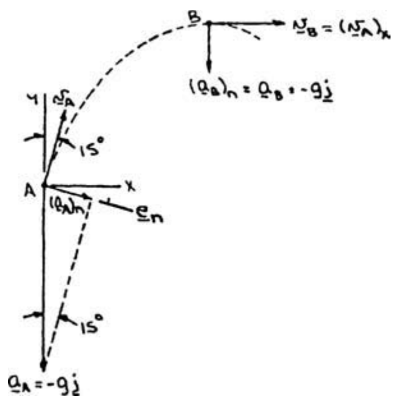
$$\rho = \frac{v^2}{a_n} = \frac{(20.84)^2}{32.2} \qquad \rho = 13.48 \text{ ft} \blacktriangleleft$$

PROBLEM 11.144



A basketball is bounced on the ground at Point A and rebounds with a velocity $\mathbf{v}_A$ of magnitude 2.5 m/s as shown. Determine the radius of curvature of the trajectory described by the ball (*a*) at Point A, (*b*) at the highest point of the trajectory.

SOLUTION



(*a*) We have $$(a_A)_n = \frac{v_A^2}{\rho_A}$$

or $$\rho_A = \frac{(2.5 \text{ m/s})^2}{(9.81 \text{ m/s}^2)\sin 15°}$$

or $$\rho_A = 2.46 \text{ m} \blacktriangleleft$$

(*b*) We have $$(a_B)_n = \frac{v_B^2}{\rho_B}$$

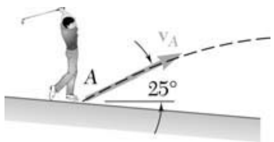
where Point B is the highest point of the trajectory, so that

$$v_B = (v_A)_x = v_A \sin 15°$$

Then $$\rho_B = \frac{[(2.5 \text{ m/s})\sin 15°]^2}{9.81 \text{ m/s}^2} = 0.0427 \text{ m}$$

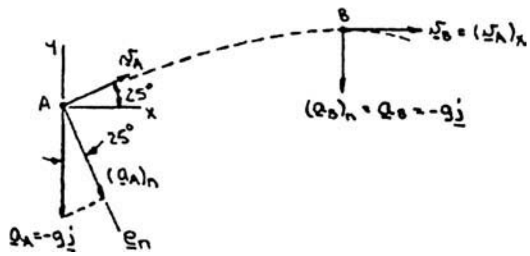
or $$\rho_B = 42.7 \text{ mm} \blacktriangleleft$$

PROBLEM 11.145



A golfer hits a golf ball from Point A with an initial velocity of 50 m/s at an angle of 25° with the horizontal. Determine the radius of curvature of the trajectory described by the ball (a) at Point A, (b) at the highest point of the trajectory.

SOLUTION



(a) We have $$(a_A)_n = \frac{v_A^2}{\rho_A}$$

or $$\rho_A = \frac{(50 \text{ m/s})^2}{(9.81 \text{ m/s}^2)\cos 25°}$$

or $$\rho_A = 281 \text{ m} \blacktriangleleft$$

(b) We have $$(a_B)_n = \frac{v_B^2}{\rho_B}$$

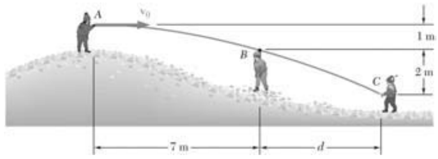
where Point B is the highest point of the trajectory, so that

$$v_B = (v_A)_x = v_A \cos 25°$$

Then $$\rho_B = \frac{[(50 \text{ m/s}) \cos 25°]^2}{9.81 \text{ m/s}^2}$$

or $$\rho_B = 209 \text{ m} \blacktriangleleft$$

PROBLEM 11.146



Three children are throwing snowballs at each other. Child *A* throws a snowball with a horizontal velocity $\mathbf{v}_0$. If the snowball just passes over the head of child *B* and hits child C, determine the radius of curvature of the trajectory described by the snowball (*a*) at Point *B*, (*b*) at Point *C*.

SOLUTION

The motion is projectile motion. Place the origin at Point *A*.

Horizontal motion: $\mathbf{v}_x = v_0 \qquad x = v_0 t$

Vertical motion: $y_0 = 0, \qquad (v_y) = 0$

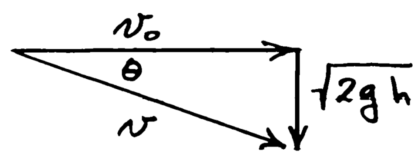
$$v_y = -gt \qquad y = -\frac{1}{2}gt^2$$

$$t = \sqrt{\frac{2h}{g}}, \qquad \text{where } h \text{ is the vertical distance fallen.}$$

$$|v_y| = \sqrt{2gh}$$

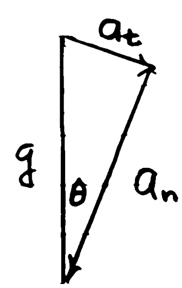
Speed: $v^2 = v_x^2 + v_y^2 = v_0^2 + 2gh$

Direction of velocity.



$$\cos\theta = \frac{v_0}{v}$$

Direction of normal acceleration.



$$a_n = g\cos\theta = \frac{gv_0}{v} = \frac{v^2}{\rho}$$

Radius of curvature: $\rho = \dfrac{v^3}{gv_0}$

At Point *B*, $h_B = 1 \text{ m}; \quad x_B = 7 \text{ m}$

$$t_B = \sqrt{\frac{(2)(1 \text{ m})}{9.81 \text{ m/s}^2}} = 0.45152 \text{ s}$$

$$x_B = v_0 t_B \qquad v_0 = \frac{x_B}{t_B} = \frac{7 \text{ m}}{0.45152 \text{ s}} = 15.504 \text{ m/s}$$

$$v_B^2 = (15.504)^2 + (2)(9.81)(1) = 259.97 \text{ m}^2/\text{s}^2$$

PROBLEM 11.146 (Continued)

(*a*) Radius of curvature at Point *B*.

$$\rho_B = \frac{(259.97\ \text{m}^2/\text{s}^2)^{3/2}}{(9.81\ \text{m/s}^2)(15.504\ \text{m/s})}$$

$\rho_B = 27.6$ m ◀

At Point *C* $\quad h_C = 1\ \text{m} + 2\ \text{m} = 3\ \text{m}$

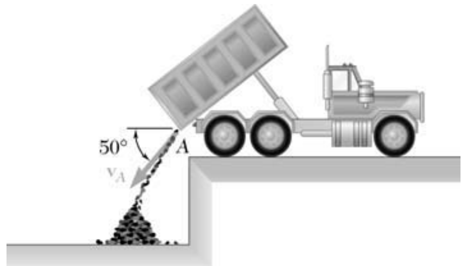
$$v_C^2 = (15.504)^2 + (2)(9.81)(3) = 299.23\ \text{m}^2/\text{s}^2$$

(*b*) Radius of curvature at Point *C*.

$$\rho_C = \frac{(299.23\ \text{m}^2/\text{s}^2)^{3/2}}{(9.81\ \text{m/s}^2)(15.504\ \text{m/s})}$$

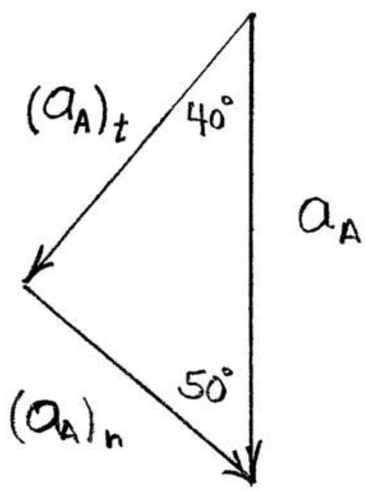
$\rho_C = 34.0$ m ◀

PROBLEM 11.147



Coal is discharged from the tailgate A of a dump truck with an initial velocity $\mathbf{v}_A = 2$ m/s $\measuredangle$ 50°. Determine the radius of curvature of the trajectory described by the coal (*a*) at Point A, (*b*) at the point of the trajectory 1 m below Point A.

SOLUTION



(*a*) At Point A. $a_A = g \downarrow = 9.81 \text{ m/s}^2 \downarrow$

Sketch tangential and normal components of acceleration at A.

$$(a_A)_n = g\cos 50°$$

$$\rho_A = \frac{v_A^2}{(a_A)_n} = \frac{(2)^2}{9.81\cos 50°} \qquad \rho_A = 0.634 \text{ m} ◀$$

(*b*) At Point B, 1 meter below Point A.

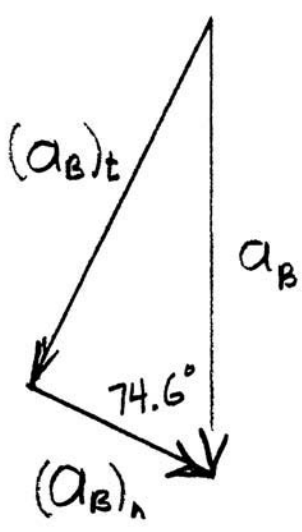
Horizontal motion: $(v_B)_x = (v_A)_x = 2\cos 50° = 1.286 \text{ m/s} \leftarrow$

Vertical motion: $(v_B)_y^2 = (v_A)_y^2 + 2a_y(y_B - y_A)$

$$= (2\cos 40°)^2 + (2)(-9.81)(-1)$$

$$= 21.97 \text{ m}^2/\text{s}^2$$

$$(v_B)_y = 4.687 \text{ m/s} \downarrow$$

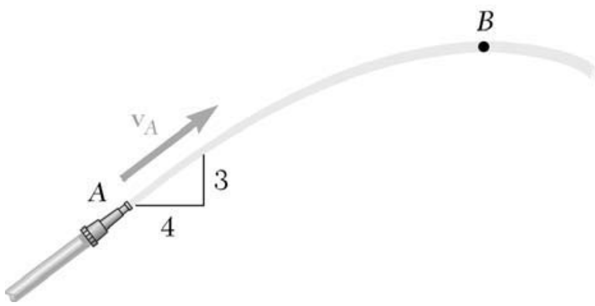


$$\tan\theta = \frac{(v_B)_y}{(v_B)_x} = \frac{4.687}{1.286}, \qquad \text{or} \qquad \theta = 74.6°$$

$$a_B = g\cos 74.6°$$

$$\rho_B = \frac{v_B^2}{(a_B)_n} = \frac{(v_B)_x^2 + (v_B)_y^2}{g\cos 74.6°}$$

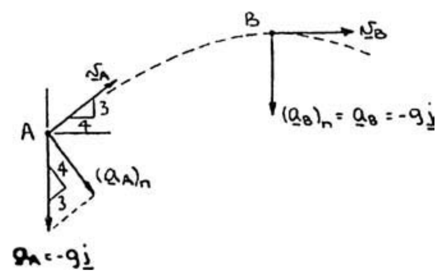
$$= \frac{(1.286)^2 + 21.97}{9.81\cos 74.6°} \qquad \rho_B = 9.07 \text{ m} ◀$$



PROBLEM 11.148

From measurements of a photograph, it has been found that as the stream of water shown left the nozzle at A, it had a radius of curvature of 25 m. Determine (*a*) the initial velocity $\mathbf{v}_A$ of the stream, (*b*) the radius of curvature of the stream as it reaches its maximum height at B.

SOLUTION



(*a*) We have $$(a_A)_n = \frac{v_A^2}{\rho_A}$$

or $$v_A^2 = \left[\frac{4}{5}(9.81 \text{ m/s}^2)\right](25 \text{ m})$$

or $$v_A = 14.0071 \text{ m/s}$$

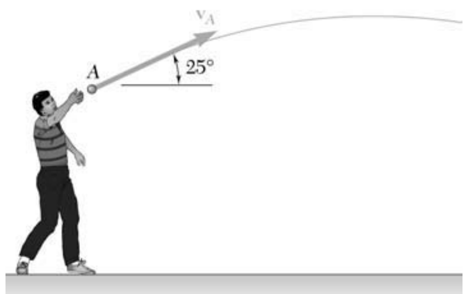
$\mathbf{v}_A = 14.01$ m/s ∠ 36.9° ◀

(*b*) We have $$(a_B)_n = \frac{v_B^2}{\rho_B}$$

Where $$v_B = (v_A)_x = \frac{4}{5} v_A$$

Then $$\rho_B = \frac{\left(\frac{4}{5} \times 14.0071 \text{ m/s}\right)^2}{9.81 \text{ m/s}^2}$$

or $\rho_B = 12.80$ m ◀

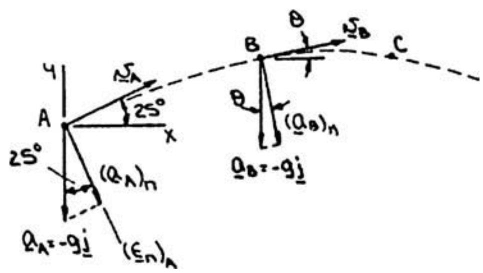


PROBLEM 11.149

A child throws a ball from Point A with an initial velocity $\mathbf{v}_A$ of 20 m/s at an angle of 25° with the horizontal. Determine the velocity of the ball at the points of the trajectory described by the ball where the radius of curvature is equal to three-quarters of its value at A.

SOLUTION

Assume that Points B and C are the points of interest, where $y_B = y_C$ and $v_B = v_C$.



Now $$(a_A)_n = \frac{v_A^2}{\rho_A}$$

or $$\rho_A = \frac{v_A^2}{g \cos 25°}$$

Then $$\rho_B = \frac{3}{4}\rho_A = \frac{3}{4}\frac{v_A^2}{g \cos 25°}$$

We have $$(a_B)_n = \frac{v_B^2}{\rho_B}$$

where $$(a_B)_n = g\cos\theta$$

so that $$\frac{3}{4}\frac{v_A^2}{g\cos 25°} = \frac{v_B^2}{g\cos\theta}$$

or $$v_B^2 = \frac{3}{4}\frac{\cos\theta}{\cos 25°}v_A^2 \qquad (1)$$

Noting that the horizontal motion is uniform, we have

$$(v_A)_x = (v_B)_x$$

where $$(v_A)_x = v_A\cos 25° \qquad (v_B)_x = v_B\cos\theta$$

Then $$v_A\cos 25° = v_B\cos\theta$$

or $$\cos\theta = \frac{v_A}{v_B}\cos 25°$$

PROBLEM 11.149 (Continued)

Substituting for $\cos\theta$ in Eq. (1), we have

$$v_B^2 = \frac{3}{4}\left(\frac{v_A}{v_B}\cos 25°\right)\frac{v_A^2}{\cos 25°}$$

or
$$v_B^3 = \frac{3}{4}v_A^3$$

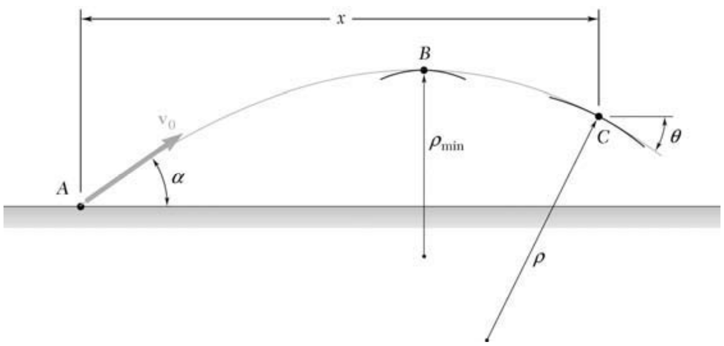
$$v_B = \sqrt[3]{\frac{3}{4}}v_A = 18.17 \text{ m/s}$$

$$\cos\theta = \sqrt[3]{\frac{4}{3}}\cos 25°$$

$$\theta = \pm 4.04°$$

$\mathbf{v}_B = 18.17$ m/s ⦤ 4.04° ◀

and $\mathbf{v}_B = 18.17$ m/s ⦨ 4.04° ◀

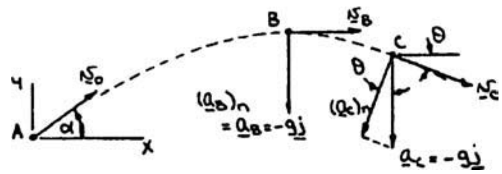


PROBLEM 11.150

A projectile is fired from Point A with an initial velocity $\mathbf{v}_0$. (a) Show that the radius of curvature of the trajectory of the projectile reaches its minimum value at the highest Point B of the trajectory. (b) Denoting by θ the angle formed by the trajectory and the horizontal at a given Point C, show that the radius of curvature of the trajectory at C is $\rho = \rho_{min}/\cos^3\theta$.

SOLUTION

For the arbitrary Point C, we have



$$(a_C)_n = \frac{v_C^2}{\rho_C}$$

or

$$\rho_C = \frac{v_C^2}{g\cos\theta}$$

Noting that the horizontal motion is uniform, we have

$$(v_A)_x = (v_C)_x$$

where

$$(v_A)_x = v_0\cos\alpha \qquad (v_C)_x = v_C\cos\theta$$

Then

$$v_0\cos\alpha = v_C\cos\theta$$

or

$$v_C = \frac{\cos\alpha}{\cos\theta}v_0$$

so that

$$\rho_C = \frac{1}{g\cos\theta}\left(\frac{\cos\alpha}{\cos\theta}v_0\right)^2 = \frac{v_0^2\cos^2\alpha}{g\cos^3\theta}$$

(a) In the expression for ρ_C, v_0, α, and g are constants, so that ρ_C is minimum where $\cos\theta$ is maximum. By observation, this occurs at Point B where $\theta = 0$.

$$\rho_{min} = \rho_B = \frac{v_0^2\cos^2\alpha}{g} \qquad \text{Q.E.D.} \quad ◀$$

(b)

$$\rho_C = \frac{1}{\cos^3\theta}\left(\frac{v_0^2\cos^2\alpha}{g}\right)$$

$$\rho_C = \frac{\rho_{min}}{\cos^3\theta} \qquad \text{Q.E.D.} \quad ◀$$

PROBLEM 11.151*

Determine the radius of curvature of the path described by the particle of Problem 11.95 when $t = 0$.

PROBLEM 11.95 The three-dimensional motion of a particle is defined by the position vector $\mathbf{r} = (Rt\cos\omega_n t)\mathbf{i} + ct\mathbf{j} + (Rt\sin\omega_n t)\mathbf{k}$. Determine the magnitudes of the velocity and acceleration of the particle. (The space curve described by the particle is a conic helix.)

SOLUTION

We have
$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = R(\cos\omega_n t - \omega_n t\sin\omega_n t)\mathbf{i} + c\mathbf{j} + R(\sin\omega_n t + \omega_n t\cos\omega_n t)\mathbf{k}$$

and
$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = R\left(-\omega_n \sin\omega_n t - \omega_n \sin\omega_n t - \omega_n^2 t\cos\omega_n t\right)\mathbf{i}$$
$$+ R\left(\omega_n \cos\omega_n t + \omega_n \cos\omega_n t - \omega_n^2 t\sin\omega_n t\right)\mathbf{k}$$

or
$$\mathbf{a} = \omega_n R[-(2\sin\omega_n t + \omega_n t\cos\omega_n t)\mathbf{i} + (2\cos\omega_n t - \omega_n t\sin\omega_n t)\mathbf{k}]$$

Now
$$v^2 = R^2(\cos\omega_n t - \omega_n t\sin\omega_n t)^2 + c^2 + R^2(\sin\omega_n t + \omega_n t\cos\omega_n t)^2$$
$$= R^2\left(1 + \omega_n^2 t^2\right) + c^2$$

Then
$$v = \left[R^2\left(1 + \omega_n^2 t^2\right) + c^2\right]^{1/2}$$

and
$$\frac{dv}{dt} = \frac{R^2\omega_n^2 t}{\left[R^2\left(1 + \omega_n^2 t^2\right) + c^2\right]^{1/2}}$$

Now
$$a^2 = a_t^2 + a_n^2 = \left(\frac{dv}{dt}\right)^2 + \left(\frac{v^2}{\rho}\right)^2$$

At $t = 0$:
$$\frac{dv}{dt} = 0$$
$$\mathbf{a} = \omega_n R(2\mathbf{k}) \quad \text{or} \quad a = 2\omega_n R$$
$$v^2 = R^2 + c^2$$

Then, with
$$\frac{dv}{dt} = 0,$$

we have
$$a = \frac{v^2}{\rho}$$

or
$$2\omega_n R = \frac{R^2 + c^2}{\rho} \qquad \rho = \frac{R^2 + c^2}{2\omega_n R} \blacktriangleleft$$

PROBLEM 11.152*

Determine the radius of curvature of the path described by the particle of Problem 11.96 when $t = 0$, $A = 3$, and $B = 1$.

SOLUTION

With $A = 3, \quad B = 1$

we have $\mathbf{r} = (3t\cos t)\mathbf{i} + \left(3\sqrt{t^2+1}\right)\mathbf{j} + (t\sin t)\mathbf{k}$

Now $$\mathbf{v} = \frac{d\mathbf{r}}{dt} = 3(\cos t - t\sin t)\mathbf{i} + \left(\frac{3t}{\sqrt{t^2+1}}\right)\mathbf{j} + (\sin t + t\cos t)\mathbf{k}$$

and $$\mathbf{a} = \frac{d\mathbf{v}}{dt} = 3(-\sin t - \sin t - t\cos t)\mathbf{i} + 3\left[\frac{\sqrt{t^2+1} - t\left(\frac{t}{\sqrt{t^2+1}}\right)}{t^2+1}\right]\mathbf{j}$$
$$+ (\cos t + \cos t - t\sin t)\mathbf{k}$$
$$= -3(2\sin t + t\cos t)\mathbf{i} + 3\frac{1}{(t^2+1)^{1/2}}\mathbf{j}$$
$$+ (2\cos t - t\sin t)\mathbf{k}$$

Then $$v^2 = 9(\cos t - t\sin t)^2 + 9\frac{t^2}{t^2+1} + (\sin t + t\cos t)^2$$

Expanding and simplifying yields

$$v^2 = t^4 + 19t^2 + 1 + 8(\cos^2 t + t^4\sin^2 t) - 8(t^3 + t)\sin 2t$$

Then $$v = [t^4 + 19t^2 + 1 + 8(\cos^2 t + t^4\sin^2 t) - 8(t^3 + t)\sin 2t]^{1/2}$$

and $$\frac{dv}{dt} = \frac{4t^3 + 38t + 8(-2\cos t\sin t + 4t^3\sin^2 t + 2t^4\sin t\cos t) - 8[(3t^2+1)\sin 2t + 2(t^3+t)\cos 2t]}{2[t^4 + 19t^2 + 1 + 8(\cos^2 t + t^4\sin^2 t) - 8(t^3+t)\sin 2t]^{1/2}}$$

Now $$a^2 = a_t^2 + a_n^2 = \left(\frac{dv}{dt}\right)^2 + \left(\frac{v^2}{\rho}\right)^2$$

PROBLEM 11.152* (Continued)

At $t = 0$: $\mathbf{a} = 3\mathbf{j} + 2\mathbf{k}$

or $a = \sqrt{13} \text{ ft/s}^2$

$$\frac{dv}{dt} = 0$$

$$v^2 = 9 \text{ (ft/s)}^2$$

Then, with $\frac{dv}{dt} = 0,$

we have $a = \frac{v^2}{\rho}$

or $\rho = \frac{9 \text{ ft}^2/\text{s}^2}{\sqrt{13} \text{ ft/s}^2}$ $\rho = 2.50 \text{ ft}$ ◀

PROBLEM 11.153

A satellite will travel indefinitely in a circular orbit around a planet if the normal component of the acceleration of the satellite is equal to $g(R/r)^2$, where g is the acceleration of gravity at the surface of the planet, R is the radius of the planet, and r is the distance from the center of the planet to the satellite. Knowing that the diameter of the sun is 1.39 Gm and that the acceleration of gravity at its surface is 274 m/s^2, determine the radius of the orbit of the indicated planet around the sun assuming that the orbit is circular.

Earth: $(v_{\text{mean}})_{\text{orbit}} = 107 \text{ Mm/h}$.

SOLUTION

For the sun, $$g = 274 \text{ m/s}^2,$$

and $$R = \frac{1}{2}D = \left(\frac{1}{2}\right)(1.39 \times 10^9) = 0.695 \times 10^9 \text{ m}$$

Given that $a_n = \dfrac{gR^2}{r^2}$ and that for a circular orbit $a_n = \dfrac{v^2}{r}$

Eliminating a_n and solving for r, $$r = \frac{gR^2}{v^2}$$

For the planet Earth, $$v = 107 \times 10^6 \text{ m/h} = 29.72 \times 10^3 \text{ m/s}$$

Then $$r = \frac{(274)(0.695 \times 10^9)^2}{(29.72 \times 10^3)^2} = 149.8 \times 10^9 \text{ m}$$

$r = 149.8 \text{ Gm}$ ◀

PROBLEM 11.154

A satellite will travel indefinitely in a circular orbit around a planet if the normal component of the acceleration of the satellite is equal to $g(R/r)^2$, where g is the acceleration of gravity at the surface of the planet, R is the radius of the planet, and r is the distance from the center of the planet to the satellite. Knowing that the diameter of the sun is 1.39 Gm and that the acceleration of gravity at its surface is 274 m/s^2, determine the radius of the orbit of the indicated planet around the sun assuming that the orbit is circular.

Saturn: $(v_{\text{mean}})_{\text{orbit}} = 34.7 \text{ Mm/h}$.

SOLUTION

For the sun, $g = 274 \text{ m/s}^2$

and $R = \frac{1}{2}D = \left(\frac{1}{2}\right)(1.39 \times 10^9) = 0.695 \times 10^9 \text{ m}$

Given that $a_n = \dfrac{gR^2}{r^2}$ .and that for a circular orbit: $a_n = \dfrac{v^2}{r}$

Eliminating a_n and solving for r, $r = \dfrac{gR^2}{v^2}$

For the planet Saturn, $v = 34.7 \times 10^6 \text{m/h} = 9.639 \times 10^3 \text{m/s}$

Then, $r = \dfrac{(274)(0.695 \times 10^9)^2}{(9.639 \times 10^3)^2} = 1.425 \times 10^{12} \text{ m}$ $r = 1425 \text{ Gm}$ ◀

PROBLEM 11.155

Determine the speed of a satellite relative to the indicated planet if the satellite is to travel indefinitely in a circular orbit 100 mi above the surface of the planet. (See information given in Problems 11.153–11.154).

Venus: $g = 29.20 \text{ ft/s}^2$, $R = 3761$ mi.

SOLUTION

From Problems 11.153 and 11.154, $$a_n = \frac{gR^2}{r^2}$$

For a circular orbit, $$a_n = \frac{v^2}{r}$$

Eliminating a_n and solving for v, $$v = R\sqrt{\frac{g}{r}}$$

For Venus, $$g = 29.20 \text{ ft/s}^2$$

$$R = 3761 \text{ mi} = 19.858 \times 10^6 \text{ ft.}$$

$$r = 3761 + 100 = 3861 \text{ mi} = 20.386 \times 10^6 \text{ ft}$$

Then, $$v = 19.858 \times 10^6 \sqrt{\frac{29.20}{20.386 \times 10^6}} = 23.766 \times 10^3 \text{ ft/s}$$

$v = 16200$ mi/h ◀

PROBLEM 11.156

Determine the speed of a satellite relative to the indicated planet if the satellite is to travel indefinitely in a circular orbit 100 mi above the surface of the planet. (See information given in Problems 11.153–11.154).

Mars: $g = 12.17 \text{ ft/s}^2, \ R = 2102 \text{ mi.}$

SOLUTION

From Problems 11.153 and 11.154, $$a_n = \frac{gR^2}{r^2}$$

For a circular orbit, $$a_n = \frac{v^2}{r}$$

Eliminating a_n and solving for v, $$v = R\sqrt{\frac{g}{r}}$$

For Mars, $g = 12.17 \text{ ft/s}^2$

$$R = 2102 \text{ mi} = 11.0986 \times 10^6 \text{ ft}$$

$$r = 2102 + 100 = 2202 \text{ mi} = 11.6266 \times 10^6 \text{ ft}$$

Then, $$v = 11.0986 \times 10^6 \sqrt{\frac{12.17}{11.6266 \times 10^6}} = 11.35 \times 10^3 \text{ ft/s}$$

$$v = 7740 \text{ mi/h} \blacktriangleleft$$

PROBLEM 11.157

Determine the speed of a satellite relative to the indicated planet if the satellite is to travel indefinitely in a circular orbit 100 mi above the surface of the planet. (See information given in Problems 11.153–11.154).

Jupiter: $g = 75.35 \text{ ft/s}^2$, $R = 44,432$ mi.

SOLUTION

From Problems 11.153 and 11.154, $$a_n = \frac{gR^2}{r^2}$$

For a circular orbit, $$a_n = \frac{v^2}{r}$$

Eliminating a_n and solving for v, $$v = R\sqrt{\frac{g}{r}}$$

For Jupiter, $$g = 75.35 \text{ ft/s}^2$$

$$R = 44432 \text{ mi} = 234.60 \times 10^6 \text{ ft}$$

$$r = 44432 + 100 = 44532 \text{ mi} = 235.13 \times 10^6 \text{ ft}$$

Then, $$v = (234.60 \times 10^6)\sqrt{\frac{75.35}{235.13 \times 10^6}} = 132.8 \times 10^3 \text{ ft/s}$$

$v = 90600$ mi/h ◀

PROBLEM 11.158

A satellite is traveling in a circular orbit around Mars at an altitude of 300 km. After the altitude of the satellite is adjusted, it is found that the time of one orbit has increased by 10 percent. Knowing that the radius of Mars is 3382 km, determine the new altitude of the satellite. (See information given in Problems 11.153–11.155.)

SOLUTION

We have $$a_n = g\frac{R^2}{r^2} \quad \text{and} \quad a_n = \frac{v^2}{r}$$

Then $$g\frac{R^2}{r^2} = \frac{v^2}{r}$$

$$v = R\sqrt{\frac{g}{r}} \quad \text{where} \quad r = R + h$$

The circumference s of a circular orbit is equal to

$$s = 2\pi r$$

Assuming that the speed of the satellite in each orbit is constant, we have

$$s = vt_{\text{orbit}}$$

Substituting for s and v

$$2\pi r = R\sqrt{\frac{g}{r}}\, t_{\text{orbit}}$$

$$t_{\text{orbit}} = \frac{2\pi}{R}\frac{r^{3/2}}{\sqrt{g}}$$

$$= \frac{2\pi}{R}\frac{(R+h)^{3/2}}{\sqrt{g}}$$

Now $$(t_{\text{orbit}})_2 = 1.1(t_{\text{orbit}})_1$$

$$\frac{2\pi}{R}\frac{(R+h_2)^{3/2}}{\sqrt{g}} = 1.1\frac{2\pi}{R}\frac{(R+h_1)^{3/2}}{\sqrt{g}}$$

$$h_2 = (1.1)^{2/3}(R+h_1) - R$$

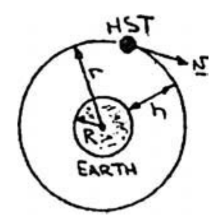
$$= (1.1)^{2/3}(3382+300)\text{ km} - (3382\text{ km})$$

$h_2 = 542$ km ◀

PROBLEM 11.159

Knowing that the radius of the earth is 6370 km, determine the time of one orbit of the Hubble Space Telescope, knowing that the telescope travels in a circular orbit 590 km above the surface of the earth. (See information given in Problems 11.153–11.155.)

SOLUTION



We have $$a_n = g\frac{R^2}{r^2} \quad \text{and} \quad a_n = \frac{v^2}{r}$$

Then $$g\frac{R^2}{r^2} = \frac{v^2}{r}$$

or $$v = R\sqrt{\frac{g}{r}} \quad \text{where} \quad r = R + h$$

The circumference s of the circular orbit is equal to

$$s = 2\pi r$$

Assuming that the speed of the telescope is constant, we have

$$s = vt_{\text{orbit}}$$

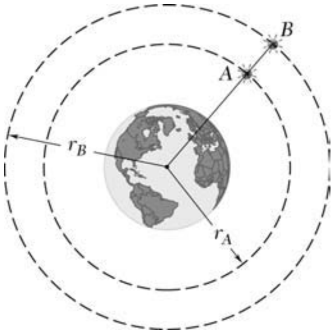
Substituting for s and v

$$2\pi r = R\sqrt{\frac{g}{r}}\, t_{\text{orbit}}$$

or $$t_{\text{orbit}} = \frac{2\pi}{R}\frac{r^{3/2}}{\sqrt{g}}$$

$$= \frac{2\pi}{6370 \text{ km}}\frac{[(6370+590)\text{ km}]^{3/2}}{[9.81\times10^{-3}\text{ km/s}^2]^{1/2}}\times\frac{1 \text{ h}}{3600 \text{ s}}$$

or $$t_{\text{orbit}} = 1.606 \text{ h} \blacktriangleleft$$



PROBLEM 11.160

Satellites A and B are traveling in the same plane in circular orbits around the earth at altitudes of 120 and 200 mi, respectively. If at $t = 0$ the satellites are aligned as shown and knowing that the radius of the earth is $R = 3960$ mi, determine when the satellites will next be radially aligned. (See information given in Problems 11.153–11.155.)

SOLUTION

We have $$a_n = g\frac{R^2}{r^2} \quad \text{and} \quad a_n = \frac{v^2}{r}$$

Then $$g\frac{R^2}{r^2} = \frac{v^2}{r} \quad \text{or} \quad v = R\sqrt{\frac{g}{r}}$$

where $$r = R + h$$

The circumference s of a circular orbit is

equal to $$s = 2\pi r$$

Assuming that the speeds of the satellites are constant, we have

$$s = vT$$

Substituting for s and v

$$2\pi r = R\sqrt{\frac{g}{r}}T$$

or $$T = \frac{2\pi}{R}\frac{r^{3/2}}{\sqrt{g}} = \frac{2\pi}{R}\frac{(R+h)^{3/2}}{\sqrt{g}}$$

Now $$h_B > h_A \Rightarrow (T)_B > (T)_A$$

Next let time T_C be the time at which the satellites are next radially aligned. Then, if in time T_C satellite B completes N orbits, satellite A must complete $(N+1)$ orbits.

Thus,

$$T_C = N(T)_B = (N+1)(T)_A$$

or $$N\left[\frac{2\pi}{R}\frac{(R+h_B)^{3/2}}{\sqrt{g}}\right] = (N+1)\left[\frac{2\pi}{R}\frac{(R+h_A)^{3/2}}{\sqrt{g}}\right]$$

or
$$N = \frac{(R+h_A)^{3/2}}{(R+h_B)^{3/2}-(R+h_A)^{3/2}} = \frac{1}{\left(\frac{R+h_B}{R+h_A}\right)^{3/2}-1}$$
$$= \frac{1}{\left(\frac{3960+200}{3960+120}\right)^{3/2}-1} = 33.835 \text{ orbits}$$

Then
$$T_C = N(T)_B = N\frac{2\pi}{R}\frac{(R+h_B)^{3/2}}{\sqrt{g}}$$
$$= 33.835\frac{2\pi}{3960 \text{ mi}}\frac{[(3960+200)\text{ mi}]^{3/2}}{\left(32.2 \text{ ft/s}^2 \times \frac{1 \text{ mi}}{5280 \text{ ft}}\right)^{1/2}} \times \frac{1 \text{ h}}{3600 \text{ s}}$$

or $T_C = 51.2 \text{ h}$ ◀

<u>Alternative solution</u>

From above, we have $(T)_B > (T)_A$. Thus, when the satellites are next radially aligned, the angles θ_A and θ_B swept out by radial lines drawn to the satellites must differ by 2π. That is,
$$\theta_A = \theta_B + 2\pi$$

For a circular orbit
$$s = r\theta$$

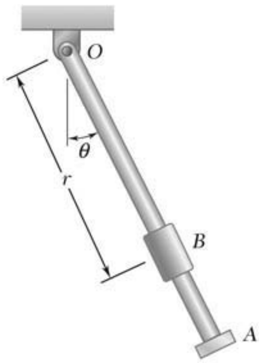
From above
$$s = vt \quad \text{and} \quad v = R\sqrt{\frac{g}{r}}$$

Then
$$\theta = \frac{s}{r} = \frac{vt}{r} = \frac{1}{r}\left(R\sqrt{\frac{g}{r}}\right)t = \frac{R\sqrt{g}}{r^{3/2}}t = \frac{R\sqrt{g}}{(R+h)^{3/2}}t$$

At time T_C:
$$\frac{R\sqrt{g}}{(R+h_A)^{3/2}}T_C = \frac{R\sqrt{g}}{(R+h_B)^{3/2}}T_C + 2\pi$$

or
$$T_C = \frac{2\pi}{R\sqrt{g}\left[\frac{1}{(R+h_A)^{3/2}} - \frac{1}{(R+h_B)^{3/2}}\right]}$$
$$= \frac{2\pi}{(3960 \text{ mi})\left(32.2 \text{ ft/s}^2 \times \frac{1 \text{ mi}}{5280 \text{ ft}}\right)^{1/2}}$$
$$\times \frac{1}{\frac{1}{[(3960+120)\text{ mi}]^{3/2}} - \frac{1}{[(3960+200)\text{ mi}]^{3/2}}}$$
$$\times \frac{1 \text{ h}}{3600 \text{ s}}$$

or $T_C = 51.2 \text{ h}$ ◀



PROBLEM 11.161

The oscillation of rod OA about O is defined by the relation $\theta = (3/\pi)(\sin \pi t)$, where θ and t are expressed in radians and seconds, respectively. Collar B slides along the rod so that its distance from O is $r = 6(1 - e^{-2t})$ where r and t are expressed in inches and seconds, respectively. When $t = 1$ s, determine (a) the velocity of the collar, (b) the acceleration of the collar, (c) the acceleration of the collar relative to the rod.

SOLUTION

Calculate the derivatives with respect to time.

$$r = 6 - 6e^{-2t} \text{ in.} \qquad \theta = \frac{3}{\pi}\sin \pi t \text{ rad}$$

$$\dot{r} = 12e^{-2t} \text{ in/s} \qquad \dot{\theta} = 3\cos \pi t \text{ rad/s}$$

$$\ddot{r} = -24e^{-2t} \text{ in/s}^2 \qquad \ddot{\theta} = -3\pi \sin \pi t \text{ rad/s}^2$$

At $t = 1$ s,

$$r = 6 - 6e^{-2} = 5.1880 \text{ in.} \qquad \theta = \frac{3}{\pi}\sin \pi = 0$$

$$\dot{r} = 12e^{-2} = 1.6240 \text{ in/s} \qquad \dot{\theta} = 3\cos \pi = -3 \text{ rad/s}$$

$$\ddot{r} = -24e^{-2} = -3.2480 \text{ in/s}^2 \qquad \ddot{\theta} = -3\pi \sin \pi = 0$$

(a) *Velocity of the collar*.

$$\mathbf{v} = \dot{r}\mathbf{e}_r + r\dot{\theta}\mathbf{e}_\theta = 1.6240\,\mathbf{e}_r + (5.1880)(-3)\mathbf{e}_\theta$$

$$\mathbf{v} = (1.624 \text{ in/s})\mathbf{e}_r + (15.56 \text{ in/s})\mathbf{e}_\theta \blacktriangleleft$$

(b) *Acceleration of the collar*.

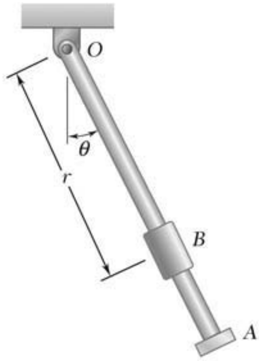
$$\mathbf{a} = (\ddot{r} - r\dot{\theta}^2)\mathbf{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\mathbf{e}_\theta$$

$$= [-3.2480 - (5.1880)(-3)^2]\mathbf{e}_r + (5.1880)(0) + (2)(1.6240)(-3)]\mathbf{e}_\theta$$

$$(-49.9 \text{ in/s}^2)\mathbf{e}_r + (-9.74 \text{ in/s}^2)\mathbf{e}_\theta \blacktriangleleft$$

(c) *Acceleration of the collar relative to the rod.*

$$\mathbf{a}_{B/OA} = \ddot{r}\mathbf{e}_r \qquad \mathbf{a}_{B/OA} = (-3.25 \text{ in/s}^2)\mathbf{e}_r \blacktriangleleft$$



PROBLEM 11.162

The rotation of rod OA about O is defined by the relation $\theta = t^3 - 4t$, where θ and t are expressed in radians and seconds, respectively. Collar B slides along the rod so that its distance from O is $r = 2.5t^3 - 5t^2$, where r and t are expressed in inches and seconds, respectively. When $t = 1$ s, determine (a) the velocity of the collar, (b) the acceleration of the collar, (c) the radius of curvature of the path of the collar.

SOLUTION

Calculate the derivatives with respect to time.

$$r = 2.5t^3 - 5t^2 \qquad \theta = t^3 - 4t$$
$$\dot{r} = 7.5t^2 - 10t \qquad \dot{\theta} = 3t^2 - 4$$
$$\ddot{r} = 15t - 10 \qquad \ddot{\theta} = 6t$$

At $t = 1$ s,

$$r = 2.5 - 5 = -2.5 \text{ in.} \qquad \theta = 1 - 4 = -3 \text{ rad}$$
$$\dot{r} = 7.5 - 10 = -2.5 \text{ in./s} \qquad \dot{\theta} = 3 - 4 = -1 \text{ rad/s}$$
$$\ddot{r} = 15 - 10 = 5 \text{ in./s}^2 \qquad \ddot{\theta} = 6 \text{ rad/s}^2$$

(a) *Velocity of the collar.*

$$\mathbf{v} = \dot{r}\mathbf{e}_r + r\dot{\theta}\mathbf{e}_\theta = -2.5\mathbf{e}_r + (-2.5)(-1)\mathbf{e}_\theta$$

$$\mathbf{v} = (-2.50 \text{ in./s})\mathbf{e}_r + (2.50 \text{ in./s})\mathbf{e}_\theta \blacktriangleleft$$

$$v = \sqrt{(2.50)^2 + (2.50)^2} = 3.5355 \text{ in./s}$$

Unit vector tangent to the path.

$$\mathbf{e}_t = \frac{\mathbf{v}}{v} = -0.70711\mathbf{e}_r + 0.70711\mathbf{e}_\theta$$

(b) *Acceleration of the collar.*

$$\mathbf{a} = (\ddot{r} - r\dot{\theta}^2)\mathbf{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\mathbf{e}_\theta$$
$$= [5 - (-2.5)(-1)^2]\mathbf{e}_r + [(-2.5)(6) + (2)(-2.5)(-1)]\mathbf{e}_\theta$$

$$\mathbf{a} = (7.50 \text{ in/s}^2)\mathbf{e}_r + (-10.00 \text{ in/s}^2)\mathbf{e}_\theta \blacktriangleleft$$

PROBLEM 11.162 (Continued)

Magnitude: $a = \sqrt{(7.50)^2 + (10.00)^2} = 12.50 \text{ in./s}^2$

Tangential component: $a_t = \mathbf{a}\mathbf{e}_t$

$$a_t = (7.50)(-0.70711) + (-10.00)(0.70711) = -12.374 \text{ in./s}^2$$

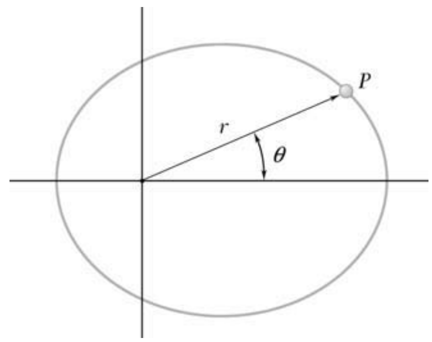
Normal component: $a_n = \sqrt{a^2 - a_t^2} = 1.7674 \text{ in./s}^2$

(*c*) <u>*Radius of curvature of path*</u>.

$$a_n = \frac{v^2}{\rho}$$

$$\rho = \frac{v^2}{a_n} = \frac{(3.5355 \text{ in./s})^2}{1.7674 \text{ in./s}^2}$$

$\rho = 7.07$ in. ◀



PROBLEM 11.163

The path of particle P is the ellipse defined by the relations $r = 2/(2-\cos \pi t)$ and $\theta = \pi t$, where r is expressed in meters, t is in seconds, and θ is in radians. Determine the velocity and the acceleration of the particle when (*a*) $t = 0$, (*b*) $t = 0.5$ s.

SOLUTION

We have $\qquad r = \dfrac{2}{2-\cos \pi t} \qquad \theta = \pi t$

Then $\qquad \dot{r} = \dfrac{-2\pi \sin \pi t}{(2-\cos \pi t)^2} \qquad \dot{\theta} = \pi$

and
$$\ddot{r} = -2\pi \frac{\pi \cos \pi t(2-\cos \pi t) - \sin \pi t(2\pi \sin \pi t)}{(2-\cos \pi t)^3} \qquad \ddot{\theta} = 0$$
$$= -2\pi^2 \frac{2\cos \pi t - 1 - \sin^2 \pi t}{(2-\cos \pi t)^3}$$

(*a*) At $t = 0$: $\qquad r = 2$ m $\qquad \theta = 0$

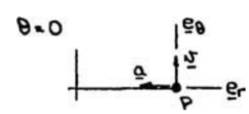
$\dot{r} = 0 \qquad \dot{\theta} = \pi$ rad/s

$\ddot{r} = -2\pi^2 \text{m/s}^2 \qquad \ddot{\theta} = 0$

Now $\qquad \mathbf{v} = \dot{r}\mathbf{e}_r + r\dot{\theta}\mathbf{e}_\theta = (2)(\pi)\mathbf{e}_\theta$

or $\qquad \mathbf{v} = (2\pi \text{ m/s})\mathbf{e}_\theta$ ◀

and
$$\mathbf{a} = (\ddot{r} - r\dot{\theta}^2)\mathbf{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\mathbf{e}_\theta$$
$$= [-2\pi^2 - (2)(\pi)^2]\mathbf{e}_r$$



or $\qquad \mathbf{a} = -(4\pi^2 \text{m/s}^2)\mathbf{e}_r$ ◀

(*b*) At $t = 0.5$ s: $\qquad r = 1$ m $\qquad \theta = \dfrac{\pi}{2}$ rad

$\dot{r} = \dfrac{-2\pi}{(2)^2} = -\dfrac{\pi}{2}$ m/s $\qquad \dot{\theta} = \pi$ rad/s

$\ddot{r} = -2\pi^2 \dfrac{-1-1}{(2)^3} = \dfrac{\pi^2}{2}$ m/s$^2 \qquad \ddot{\theta} = 0$

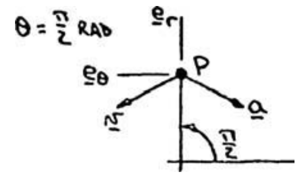
PROBLEM 11.163 (Continued)

Now $$\mathbf{v} = \dot{r}\mathbf{e}_r + r\dot{\theta}\mathbf{e}_\theta = \left(-\frac{\pi}{2}\right)\mathbf{e}_r + (1)(\pi)\mathbf{e}_\theta$$

or $$\mathbf{v} = -\left(\frac{\pi}{2}\text{ m/s}\right)\mathbf{e}_r + (\pi\text{ m/s})\mathbf{e}_\theta \blacktriangleleft$$

and $$\mathbf{a} = (\ddot{r} - r\dot{\theta}^2)\mathbf{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\mathbf{e}_\theta$$

$$= \left[\frac{\pi^2}{2} - (1)(\pi)^2\right]\mathbf{e}_r + \left[2\left(-\frac{\pi}{2}\right)(\pi)\right]\mathbf{e}_\theta$$



or $$\mathbf{a} = -\left(\frac{\pi^2}{2}\text{ m/s}^2\right)\mathbf{e}_r - (\pi^2\text{ m/s}^2)\mathbf{e}_\theta \blacktriangleleft$$

PROBLEM 11.164

The two-dimensional motion of a particle is defined by the relations $r = 2a\cos\theta$ and $\theta = bt^2/2$, where a and b are constants. Determine (*a*) the magnitudes of the velocity and acceleration at any instant, (*b*) the radius of curvature of the path. What conclusion can you draw regarding the path of the particle?

SOLUTION

(*a*) We have $r = 2a\cos\theta \qquad \theta = \frac{1}{2}bt^2$

Then $\dot{r} = -2a\dot{\theta}\sin\theta \qquad \dot{\theta} = bt$

and $\ddot{r} = -2a(\ddot{\theta}\sin\theta + \dot{\theta}^2\cos\theta) \qquad \ddot{\theta} = b$

Substituting for $\dot{\theta}$ and $\ddot{\theta}$

$$\dot{r} = -2abt\sin\theta$$

$$\ddot{r} = -2ab(\sin\theta + bt^2\cos\theta)$$

Now $v_r = \dot{r} = -2abt\sin\theta \qquad v_\theta = r\dot{\theta} = 2abt\cos\theta$

Then $v = \sqrt{v_r^2 + v_\theta^2} = 2abt[(-\sin\theta)^2 + (\cos\theta)^2]^{1/2}$

or $v = 2abt$ ◀

Also $a_r = \ddot{r} - r\dot{\theta}^2 = -2ab(\sin\theta + bt^2\cos\theta) - 2ab^2t^2\cos\theta$

$$= -2ab(\sin\theta + 2bt^2\cos\theta)$$

and $a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 2ab\cos\theta - 4ab^2t^2\sin\theta$

$$= -2ab(\cos\theta - 2bt^2\sin\theta)$$

Then $a = \sqrt{a_r^2 + a_B^2} = 2ab[(\sin\theta + 2bt^2\cos\theta)^2 + (\cos\theta - 2bt^2\sin\theta)^2]^{1/2}$

or $a = 2ab\sqrt{1 + 4b^2t^4}$ ◀

(*b*) Now $a^2 = a_t^2 + a_n^2 = \left(\frac{dv}{dt}\right)^2 + \left(\frac{v^2}{\rho}\right)^2$

Then $\frac{dv}{dt} = \frac{d}{dt}(2abt) = 2ab$

PROBLEM 11.164 (Continued)

so that $$\left(2ab\sqrt{1+4b^2t^4}\right)^2 = (2ab)^2 + a_n^2$$

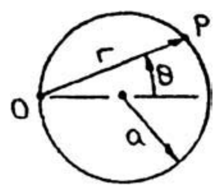
or $$4a^2b^2(1+4b^2t^4) = 4a^2b^2 + a_n^2$$

or $$a_n = 4ab^2t^2$$

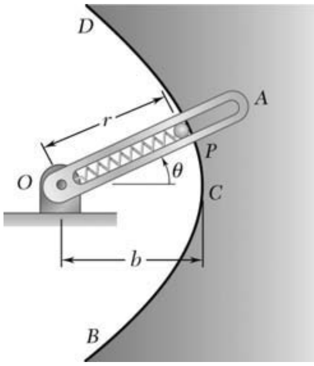
Finally $$a_n = \frac{v^2}{\rho} \Rightarrow \rho = \frac{(2abt)^2}{4ab^2t^2}$$

or $\rho = a$ ◀

Since the radius of curvature is a constant, the path is a circle of radius a. ◀

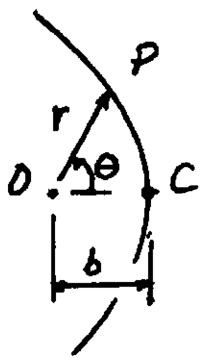


PROBLEM 11.165



As rod OA rotates, pin P moves along the parabola BCD. Knowing that the equation of this parabola is $r = 2b/(1+\cos\theta)$ and that $\theta = kt$, determine the velocity and acceleration of P when (*a*) $\theta = 0$, (*b*) $\theta = 90°$.

SOLUTION



$$r = \frac{2b}{1+\cos kt} \qquad \theta = kt$$

$$\dot{r} = \frac{2bk\sin kt}{(1+\cos kt)^2} \qquad \dot{\theta} = k \qquad \ddot{\theta} = 0$$

$$\ddot{r} = \frac{2bk}{(1+\cos kt)^4}[(1+\cos kt)^2 k\cos kt + (\sin kt)2(1+\cos kt)(k\sin kt)]$$

(*a*) When $\theta = kt = 0$:

$$r = b \qquad \dot{r} = 0 \qquad \ddot{r} = \frac{2bk}{(2)^4}[(2)^2 k(1) + 0] = \frac{1}{2}bk^2$$

$$\theta = 0 \qquad \dot{\theta} = k \qquad \ddot{\theta} = 0$$

$$v_r = \dot{r} = 0 \qquad v_\theta = r\dot{\theta} = bk$$

$$\mathbf{v} = bk\,\mathbf{e}_\theta \blacktriangleleft$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = \frac{1}{2}bk^2 - bk^2 = -\frac{1}{2}bk^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = b(0) + 2(0) = 0$$

$$\mathbf{a} = -\frac{1}{2}bk^2\mathbf{e}_r \blacktriangleleft$$

(*b*) When $\theta = kt = 90°$:

$$r = 2b \qquad \dot{r} = 2bk \qquad \ddot{r} = \frac{2bk}{19}[0 + 2k] = 4bk^2$$

$$\theta = 90° \qquad \dot{\theta} = k \qquad \ddot{\theta} = 0$$

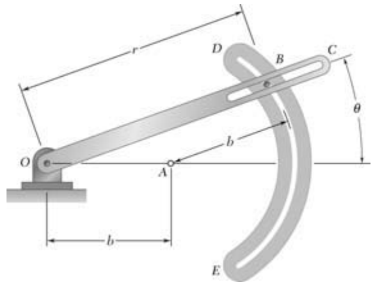
$$v_r = \dot{r} = 2bk \qquad v_\theta = r\dot{\theta} = 2bk$$

$$\mathbf{v} = 2bk\,\mathbf{e}_r + 2bk\,\mathbf{e}_\theta \blacktriangleleft$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = 4bk^2 - 2bk^2 = 2bk^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 2b(0) + 2(2bk)k = 4bk^2$$

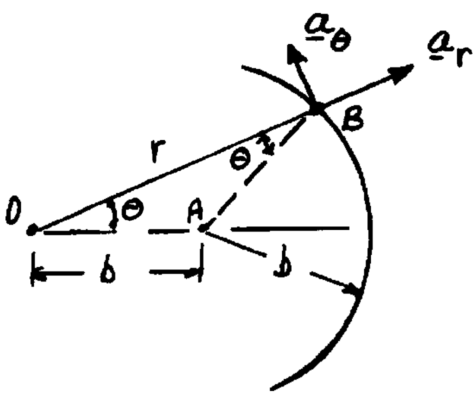
$$\mathbf{a} = 2bk^2\mathbf{e}_r + 4bk^2\mathbf{e}_\theta \blacktriangleleft$$



PROBLEM 11.166

The pin at B is free to slide along the circular slot DE and along the rotating rod OC. Assuming that the rod OC rotates at a constant rate $\dot{\theta}$, (a) show that the acceleration of pin B is of constant magnitude, (b) determine the direction of the acceleration of pin B.

SOLUTION



From the sketch:

$$r = 2b\cos\theta$$
$$\dot{r} = -2b\sin\theta\,\dot{\theta}$$

Since $\dot{\theta} = \text{constant}, \quad \ddot{\theta} = 0$

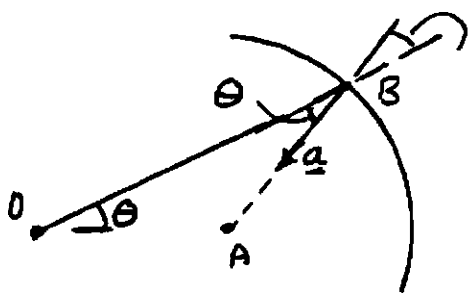
$$\ddot{r} = -2b\cos\theta\,\dot{\theta}^2$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = -2b\cos\theta\,\dot{\theta}^2 - (2b\cos\theta)\dot{\theta}^2$$
$$a_r = -4b\cos\theta\,\dot{\theta}^2$$
$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = (2b\cos\theta)(0) + 2(-2b\sin\theta)\dot{\theta}^2$$
$$a_\theta = -4b\sin\theta\,\dot{\theta}^2$$

$$a = \sqrt{a_r^2 + a_\theta^2} = 4b\dot{\theta}^2\sqrt{(-\cos\theta)^2 + (-\sin\theta)^2}$$
$$a = 4b\dot{\theta}^2$$

Since both b and $\dot{\theta}$ are constant, we find that

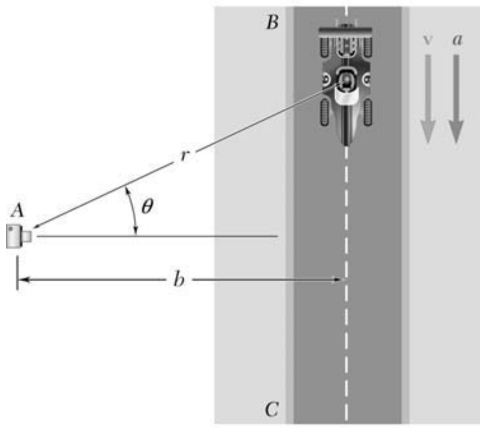
$a = \text{constant}$ ◀



$$\gamma = \tan^{-1}\frac{a_\theta}{a_r} = \tan^{-1}\left(\frac{-4b\sin\theta\,\dot{\theta}^2}{-4b\cos\theta\,\dot{\theta}^2}\right)$$
$$\gamma = \tan^{-1}(\tan\theta)$$
$$\gamma = \theta$$

Thus, **a** is directed toward A ◀

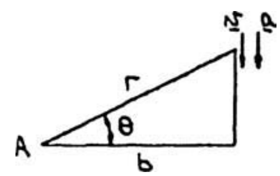
PROBLEM 11.167



To study the performance of a racecar, a high-speed camera is positioned at Point *A*. The camera is mounted on a mechanism which permits it to record the motion of the car as the car travels on straightway *BC*. Determine (*a*) the speed of the car in terms of b, θ, and $\dot{\theta}$, (*b*) the magnitude of the acceleration in terms of b, θ, $\dot{\theta}$, and $\ddot{\theta}$.

SOLUTION

(*a*) We have $$r = \frac{b}{\cos\theta}$$



Then $$\dot{r} = \frac{b\dot{\theta}\sin\theta}{\cos^2\theta}$$

We have $$v^2 = v_r^2 + v_\theta^2 = (\dot{r})^2 + (r\dot{\theta})^2$$

$$= \left(\frac{b\theta\sin\theta}{\cos^2\theta}\right)^2 + \left(\frac{b\theta}{\cos\theta}\right)^2$$

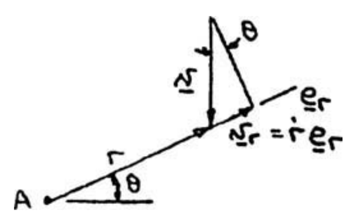
$$= \frac{b^2\dot{\theta}^2}{\cos^2\theta}\left(\frac{\sin^2\theta}{\cos^2\theta} + 1\right) = \frac{b^2\dot{\theta}^2}{\cos^4\theta}$$

or $$v = \pm\frac{b\dot{\theta}}{\cos^2\theta}$$

For the position of the car shown, θ is decreasing; thus, the negative root is chosen. $$v = -\frac{b\dot{\theta}}{\cos^2\theta} \blacktriangleleft$$

<u>Alternative solution</u>.

From the diagram $$\dot{r} = -v\sin\theta$$



or $$\frac{b\dot{\theta}\sin\theta}{\cos^2\theta} = -v\sin\theta$$

or $$v = -\frac{b\dot{\theta}}{\cos^2\theta} \blacktriangleleft$$

(*b*) For rectilinear motion $$a = \frac{dv}{dt}$$

Using the answer from Part *a*

$$v = -\frac{b\dot{\theta}}{\cos^2\theta}$$

Then $$a = \frac{d}{dt}\left(-\frac{b\dot{\theta}}{\cos^2\theta}\right)$$

$$= -b\frac{\ddot{\theta}\cos^2\theta - \dot{\theta}(-2\dot{\theta}\cos\theta\sin\theta)}{\cos^4\theta}$$

or $$a = -\frac{b}{\cos^2\theta}(\ddot{\theta} + 2\dot{\theta}^2\tan\theta) \blacktriangleleft$$

<u>Alternative solution</u>

From above $$r = \frac{b}{\cos\theta} \qquad \dot{r} = \frac{b\dot{\theta}\sin\theta}{\cos^2\theta}$$

Then $$\ddot{r} = b\frac{(\ddot{\theta}\sin\theta + \dot{\theta}^2\cos\theta)(\cos^2\theta) - (\dot{\theta}\sin\theta)(-2\dot{\theta}\cos\theta\sin\theta)}{\cos^4\theta}$$

$$= b\left[\frac{\ddot{\theta}\sin\theta}{\cos^2\theta} + \frac{\dot{\theta}^2(1+\sin^2\theta)}{\cos^3\theta}\right]$$

Now $$a^2 = a_r^2 + a_\theta^2$$

where $$a_r = \ddot{r} - r\dot{\theta}^2 = b\left[\frac{\ddot{\theta}\sin\theta}{\cos^2\theta} + \frac{\dot{\theta}^2(1+\sin^2\theta)}{\cos^2\theta}\right] - \frac{b\dot{\theta}^2}{\cos\theta}$$

$$= \frac{b}{\cos^2\theta}\left(\ddot{\theta}\sin\theta + \frac{2\dot{\theta}^2\sin^2\theta}{\cos\theta}\right)$$

$$a_r = \frac{b\sin\theta}{\cos^2\theta}(\ddot{\theta} + 2\dot{\theta}^2\tan\theta)$$

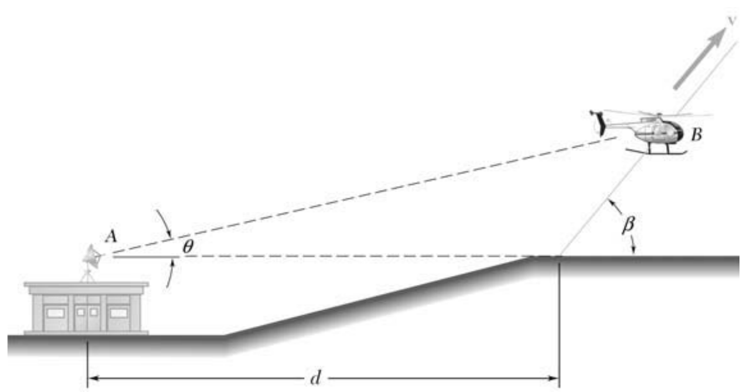
and $$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = \frac{b\ddot{\theta}}{\cos\theta} + 2\frac{b\dot{\theta}^2\sin\theta}{\cos^2\theta}$$

$$= \frac{b\cos\theta}{\cos^2\theta}(\ddot{\theta} + 2\dot{\theta}\tan\theta)$$

Then $$a = \pm\frac{b}{\cos^2\theta}(\ddot{\theta} + 2\dot{\theta}^2\tan\theta)[(\sin\theta)^2 + (\cos\theta)^2]^{1/2}$$

For the position of the car shown, $\ddot{\theta}$ is negative; for *a* to be positive, the negative root is chosen.

$$a = -\frac{b}{\cos^2\theta}(\ddot{\theta} + 2\dot{\theta}^2\tan\theta) \blacktriangleleft$$

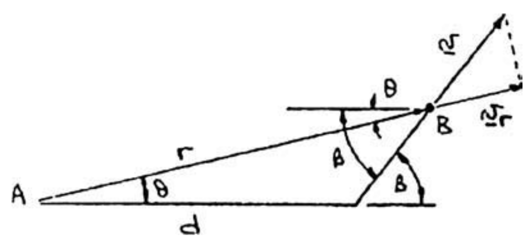


PROBLEM 11.168

After taking off, a helicopter climbs in a straight line at a constant angle β. Its flight is tracked by radar from Point A. Determine the speed of the helicopter in terms of d, β, θ, and $\dot{\theta}$.

SOLUTION

From the diagram



$$\frac{r}{\sin(180° - \beta)} = \frac{d}{\sin(\beta - \theta)}$$

or $$d\sin\beta = r(\sin\beta\cos\theta - \cos\beta\sin\theta)$$

or $$r = d\frac{\tan\beta}{\tan\beta\cos\theta - \sin\theta}$$

Then $$\dot{r} = d\tan\beta\frac{-(-\tan\beta\sin\theta - \cos\theta)}{(\tan\beta\cos\theta - \sin\theta)^2}\dot{\theta}$$

$$= d\dot{\theta}\tan\beta\frac{\tan\beta\sin\theta + \cos\theta}{(\tan\beta\cos\theta - \sin\theta)^2}$$

From the diagram

$$v_r = v\cos(\beta - \theta) \qquad \text{where} \qquad v_r = \dot{r}$$

Then

$$d\dot{\theta}\tan\beta\frac{\tan\beta\sin\theta + \cos\theta}{(\tan\beta\cos\theta - \sin\theta)^2} = v(\cos\beta\cos\theta + \sin\beta\sin\theta)$$

$$= v\cos\beta(\tan\beta\sin\theta + \cos\theta)$$

or $$v = \frac{d\dot{\theta}\tan\beta\sec\beta}{(\tan\beta\cos\theta - \sin\theta)^2}$$ ◀

Alternative solution.

We have $$v^2 = v_r^2 + v_\theta^2 = (\dot{r})^2 + (r\dot{\theta})^2$$

PROBLEM 11.168 (Continued)

Using the expressions for r and $\dot{r}$ from above

$$v = \left[d\dot{\theta} \tan\beta \frac{\tan\beta \sin\theta + \cos\theta}{(\tan\beta \cos\theta - \sin\theta)^2} \right]^2$$

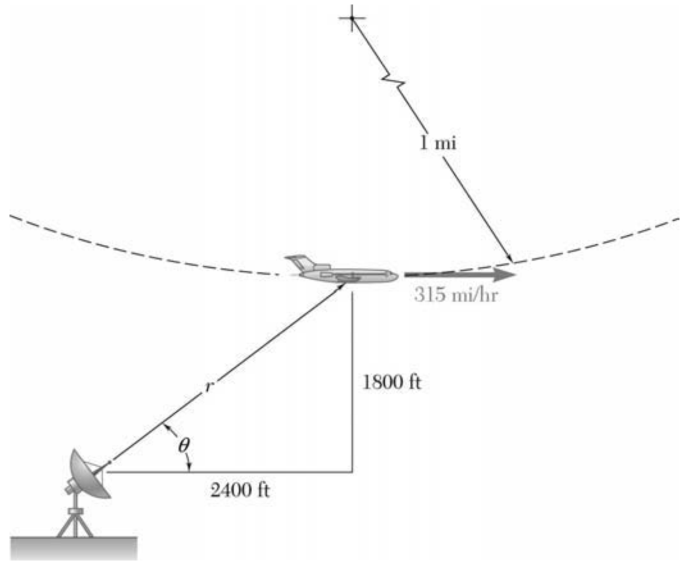
or

$$v = \pm \frac{d\dot{\theta} \tan\beta}{(\tan\beta \cos\theta - \sin\theta)} \left[\frac{(\tan\beta \sin\theta + \cos\theta)^2}{(\tan\beta \cos\theta - \sin\theta)^2} + 1 \right]^{1/2}$$

$$= \pm \frac{d\dot{\theta} \tan\beta}{(\tan\beta \cos\theta - \sin\theta)} \left[\frac{\tan^2\beta + 1}{(\tan\beta \cos\theta - \sin\theta)^2} \right]^{1/2}$$

Note that as θ increases, the helicopter moves in the indicated direction. Thus, the positive root is chosen.

$$v = \frac{d\dot{\theta} \tan\beta \sec\beta}{(\tan\beta \cos\theta - \sin\theta)^2} \blacktriangleleft$$



PROBLEM 11.169

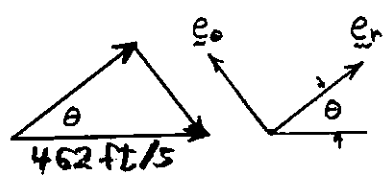
At the bottom of a loop in the vertical plane, an airplane has a horizontal velocity of 315 mi/h and is speeding up at a rate of 10 ft/s^2. The radius of curvature of the loop is 1 mi. The plane is being tracked by radar at O. What are the recorded values of $\dot{r}$, $\ddot{r}$, $\dot{\theta}$ and $\ddot{\theta}$ for this instant?

SOLUTION

Geometry. The polar coordinates are

$$r = \sqrt{(2400)^2 + (1800)^2} = 3000 \text{ ft} \qquad \theta = \tan^{-1}\left(\frac{1800}{2400}\right) = 36.87°$$

Velocity Analysis. $\mathbf{v} = 315 \text{ mi/h} = 462 \text{ ft/s} \longrightarrow$



$$v_r = 462\cos\theta = 369.6 \text{ ft/s}$$

$$v_\theta = -462\sin\theta = -277.2 \text{ ft/s}$$

$$v_r = \dot{r} \qquad \dot{r} = 370 \text{ ft/s} \blacktriangleleft$$

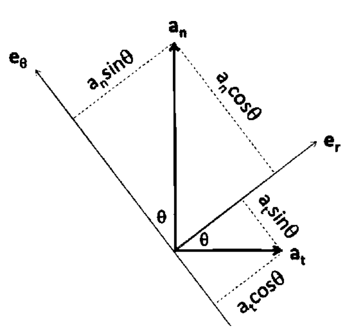
$$v_\theta = r\dot{\theta} \qquad \dot{\theta} = \frac{v_\theta}{r} = -\frac{277.2}{3000}$$

$$\dot{\theta} = -0.0924 \text{ rad/s} \blacktriangleleft$$

Acceleration analysis.

$$a_t = 10 \text{ ft/s}^2$$

$$a_n = \frac{v^2}{\rho} = \frac{(462)^2}{5280} = 40.425 \text{ ft/s}^2$$



PROBLEM 11.169 (Continued)

$$a_r = a_t \cos\theta + a_n \sin\theta = 10\cos 36.87° + 40.425\sin 36.87° = 32.255 \text{ ft/s}^2$$

$$a_\theta = -a_t \sin\theta + a_n \cos\theta = -10\sin 36.87° + 40.425\cos 36.87° = 26.34 \text{ ft/s}^2$$

$$a_r = \ddot{r} - r\dot{\theta}^2 \quad \ddot{r} = a_r + r\dot{\theta}^2$$

$$\ddot{r} = 32.255 + (3000)(-0.0924)^2$$

$\ddot{r} = 57.9 \text{ ft/s}^2$ ◀

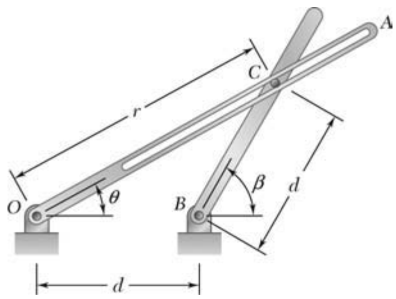
$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$$

$$\ddot{\theta} = \frac{a_\theta}{r} - \frac{2\dot{r}\dot{\theta}}{r}$$

$$= \frac{26.34}{3000} - \frac{(2)(369.6)(-0.0924)}{3000}$$

$\ddot{\theta} = 0.0315 \text{ rad/s}^2$ ◀

PROBLEM 11.170



Pin C is attached to rod BC and slides freely in the slot of rod OA which rotates at the constant rate ω. At the instant when $\beta = 60°$, determine (a) $\dot{r}$ and $\dot{\theta}$, (b) $\ddot{r}$ and $\ddot{\theta}$. Express your answers in terms of d and ω.

SOLUTION

Looking at d and β as polar coordinates with $\dot{d} = 0$,

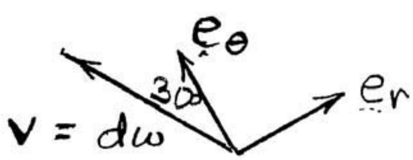
$$v_\beta = d\dot{\beta} = d\omega, \qquad v_d = \dot{d} = 0$$

$$a_\beta = d\ddot{\beta} + 2\dot{d}\dot{\beta} = 0, \qquad a_d = \ddot{d} - d\dot{\beta}^2 = -d\omega^2$$

Geometry analysis: $\quad r = d\sqrt{3}$ for angles shown.

(a) Velocity analysis:

Sketch the directions of $\mathbf{v}$, $\mathbf{e}_r$ and $\mathbf{e}_\theta$.



$$v_r = \dot{r} = \mathbf{v}\cdot\mathbf{e}_r = d\omega\cos 120°$$

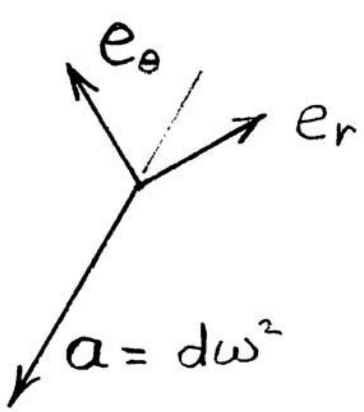
$$\dot{r} = -\frac{1}{2}d\omega \blacktriangleleft$$

$$v_\theta = r\dot{\theta} = \mathbf{v}\cdot\mathbf{e}_\theta = d\omega\cos 30°$$

$$\dot{\theta} = \frac{d\omega\cos 30°}{r} = \frac{d\omega\frac{\sqrt{3}}{2}}{d\sqrt{3}} \qquad \dot{\theta} = \frac{1}{2}\omega \blacktriangleleft$$

(b) Acceleration analysis:

Sketch the directions of $\mathbf{a}$, $\mathbf{e}_r$ and $\mathbf{e}_\theta$.



$$a_r = \mathbf{a}\cdot\mathbf{e}_r = a\cos 150° = -\frac{\sqrt{3}}{2}d\omega^2$$

$$\ddot{r} - r\dot{\theta}^2 = -\frac{\sqrt{3}}{2}d\omega^2$$

$$\ddot{r} = -\frac{\sqrt{3}}{2}d\omega^2 + r\dot{\theta}^2 = -\frac{\sqrt{3}}{2}d\omega^2 + d\sqrt{3}\left(\frac{1}{2}\omega\right)^2$$

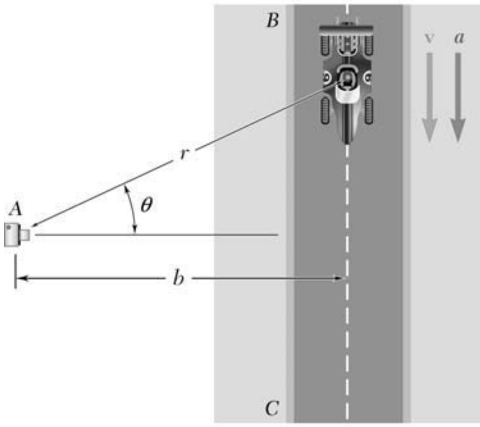
$$\ddot{r} = -\frac{\sqrt{3}}{4}d\omega^2 \blacktriangleleft$$

$$a_\theta = \mathbf{a}\cdot\mathbf{e}_\theta = d\omega^2\cos 120° = -\frac{1}{2}d\omega^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$$

$$\ddot{\theta} = \frac{1}{r}(a_\theta - 2\dot{r}\dot{\theta}) = \frac{1}{\sqrt{3}d}\left[-\frac{1}{2}d\omega^2 - (2)\left(-\frac{1}{2}d\omega\right)\left(\frac{1}{2}\omega\right)\right] \qquad \ddot{\theta} = 0 \blacktriangleleft$$

PROBLEM 11.171

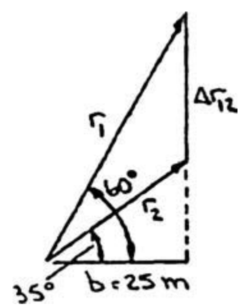


For the racecar of Problem 11.167, it was found that it took 0.5 s for the car to travel from the position $\theta = 60°$ to the position $\theta = 35°$. Knowing that $b = 25$ m, determine the average speed of the car during the 0.5-s interval.

PROBLEM 11.167 To study the performance of a racecar, a high-speed camera is positioned at Point *A*. The camera is mounted on a mechanism which permits it to record the motion of the car as the car travels on straightway *BC*. Determine (*a*) the speed of the car in terms of b, θ, and $\dot{\theta}$, (*b*) the magnitude of the acceleration in terms of b, θ, $\dot{\theta}$, and $\ddot{\theta}$.

SOLUTION

From the diagram:



$$\Delta r_{12} = 25 \tan 60° - 25 \tan 35° = 25.796 \text{ m}$$

Now

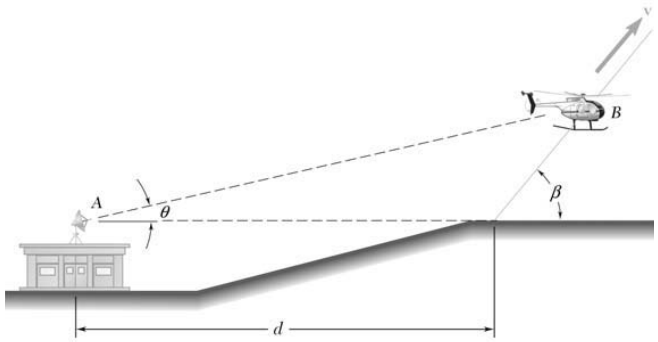
$$v_{ave} = \frac{\Delta r_{12}}{\Delta t_{12}} = \frac{25.796 \text{ m}}{0.5 \text{ s}} = 51.592 \text{ m/s}$$

or $v_{ave} = 185.7$ km/h ◀

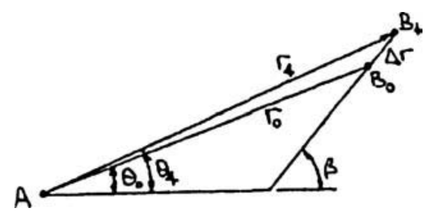
PROBLEM 11.172

For the helicopter of Problem 11.168, it was found that when the helicopter was at B, the distance and the angle of elevation of the helicopter were $r = 3000$ ft and $\theta = 20°$, respectively. Four seconds later, the radar station sighted the helicopter at $r = 3320$ ft and $\theta = 23.1°$. Determine the average speed and the angle of climb β of the helicopter during the 4-s interval.

PROBLEM 11.168 After taking off, a helicopter climbs in a straight line at a constant angle β. Its flight is tracked by radar from Point A. Determine the speed of the helicopter in terms of d, β, θ, and $\dot{\theta}$.



SOLUTION



We have $r_0 = 3000$ ft $\quad \theta_0 = 20°$

$r_4 = 3320$ ft $\quad \theta_4 = 23.1°$

From the diagram:

$$\Delta r^2 = 3000^2 + 3320^2 - 2(3000)(3320)\cos(23.1° - 20°)$$

or $\Delta r = 362.70$ ft

Now

$$v_{ave} = \frac{\Delta r}{\Delta t} = \frac{362.70 \text{ ft}}{4 \text{ s}} = 90.675 \text{ ft/s}$$

or $v_{ave} = 61.8$ mi/h ◀

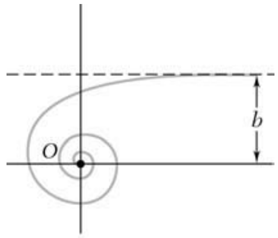
Also, $\Delta r \cos\beta = r_4 \cos\theta_4 - r_0 \cos\theta_0$

or

$$\cos\beta = \frac{3320\cos 23.1° - 3000\cos 20°}{362.70}$$

or $\beta = 49.7°$ ◀

PROBLEM 11.173



A particle moves along the spiral shown; determine the magnitude of the velocity of the particle in terms of b, θ, and $\dot{\theta}$.

SOLUTION

Hyperbolic spiral.

$$r = \frac{b}{\theta}$$

$$\dot{r} = \frac{dr}{dt} = -\frac{b}{\theta^2}\frac{d\theta}{dt} = -\frac{b}{\theta^2}\dot{\theta}$$

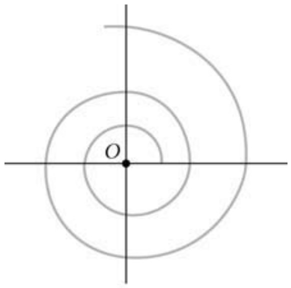
$$v_r = \dot{r} = -\frac{b}{\theta^2}\dot{\theta} \qquad v_\theta = r\dot{\theta} = \frac{b}{\theta}\dot{\theta}$$

$$v = \sqrt{v_r^2 + v_\theta^2} = b\dot{\theta}\sqrt{\left(-\frac{1}{\theta^2}\right)^2 + \left(\frac{1}{\theta}\right)^2}$$

$$= \frac{b\dot{\theta}}{\theta^2}\sqrt{1+\theta^2}$$

$$v = \frac{b}{\theta^2}\sqrt{1+\theta^2}\,\dot{\theta} \blacktriangleleft$$

PROBLEM 11.174



A particle moves along the spiral shown; determine the magnitude of the velocity of the particle in terms of b, θ, and $\dot{\theta}$.

SOLUTION

Logarithmic spiral.

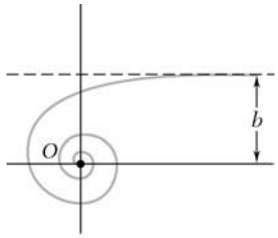
$$r = e^{b\theta}$$

$$\dot{r} = \frac{dr}{dt} = be^{b\theta}\frac{d\theta}{dt} = be^{b\theta}\dot{\theta}$$

$$v_r = \dot{r} = be^{b\theta}\dot{\theta} \quad v_\theta = r\dot{\theta} = e^{b\theta}\dot{\theta}$$

$$v = \sqrt{v_r^2 + v_\theta^2} = e^{b\theta}\dot{\theta}\sqrt{b^2 + 1}$$

$$v = e^{b\theta}\sqrt{1+b^2}\ \dot{\theta} \blacktriangleleft$$



Hyperbolic spiral $r\theta = b$

PROBLEM 11.175

A particle moves along the spiral shown. Knowing that $\dot{\theta}$ is constant and denoting this constant by ω, determine the magnitude of the acceleration of the particle in terms of b, θ, and ω.

SOLUTION

Hyperbolic spiral. $$r = \frac{b}{\theta}$$

From Problem 11.173 $$\dot{r} = -\frac{b}{\theta^2}\dot{\theta}$$

$$\ddot{r} = -\frac{b}{\theta^2}\ddot{\theta} + \frac{2b}{\theta^3}\dot{\theta}^2$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = -\frac{b}{\theta^2}\ddot{\theta} + \frac{2b}{\theta^3}\dot{\theta}^2 - \frac{b}{\theta}\dot{\theta}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = \frac{b}{\theta}\ddot{\theta} + 2\left(-\frac{b}{\theta^2}\dot{\theta}\right)\dot{\theta} = \frac{b}{\theta}\ddot{\theta} - 2\frac{b}{\theta^2}\dot{\theta}^2$$

Since $\dot{\theta} = \omega = \text{constant}$, $\ddot{\theta} = 0$, and we write:

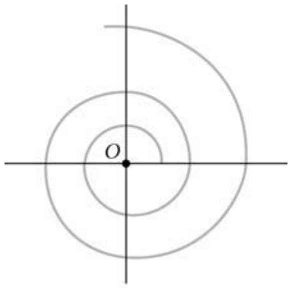
$$a_r = +\frac{2b}{\theta^3}\omega^2 - \frac{b}{\theta}\omega^2 = \frac{b\omega^2}{\theta^3}(2-\theta^2)$$

$$a_\theta = -2\frac{b}{\theta^2}\omega^2 = -\frac{b\omega^2}{\theta^3}(2\theta)$$

$$a = \sqrt{a_r^2 + a_\theta^2} = \frac{b\omega^2}{\theta^3}\sqrt{(2-\theta^2)^2 + (2\theta)^2} = \frac{b\omega^2}{\theta^3}\sqrt{4 - 4\theta^2 + \theta^4 + 4\theta^2}$$

$$a = \frac{b\omega^2}{\theta^3}\sqrt{4+\theta^4}$$ ◀

PROBLEM 11.176



A particle moves along the spiral shown. Knowing that $\dot{\theta}$ is constant and denoting this constant by ω, determine the magnitude of the acceleration of the particle in terms of b, θ, and ω.

SOLUTION

Logarithmic spiral.

$$r = e^{b\theta}$$

$$\dot{r} = \frac{dr}{dt} = be^{b\theta}\dot{\theta}$$

$$\ddot{r} = be^{b\theta}\ddot{\theta} + b^2 e^{b\theta}\dot{\theta}^2 = be^{b\theta}(\ddot{\theta} + b\dot{\theta}^2)$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = be^{b\theta}(\ddot{\theta} + b\dot{\theta}^2) - e^{b\theta}\dot{\theta}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = e^{b\theta}\ddot{\theta} + 2(be^{b\theta}\dot{\theta})\dot{\theta}$$

Since $\dot{\theta} = \omega =$ constant, $\ddot{\theta} = \theta$, and we write

$$a_r = be^{b\theta}(b\omega^2) - e^{b\theta}\omega^2 = e^{b\theta}(b^2 - 1)\omega^2$$

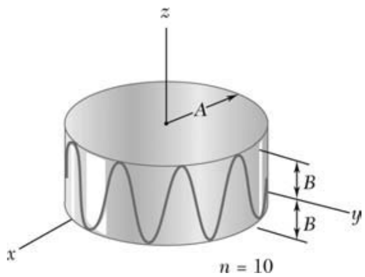
$$a_\theta = 2be^{b\theta}\omega^2$$

$$a = \sqrt{a_r^2 + a_\theta^2} = e^{b\theta}\omega^2\sqrt{(b^2-1)^2 + (2b)^2}$$

$$= e^{b\theta}\omega^2\sqrt{b^4 - 2b^2 + 1 + 4b^2} = e^{b\theta}\omega^2\sqrt{b^4 + 2b^2 + 1}$$

$$= e^{b\theta}\omega^2\sqrt{(b^2+1)^2} = e^{b\theta}\omega^2(b^2+1)$$

$$a = (1 + b^2)\omega^2 e^{b\theta} \blacktriangleleft$$



PROBLEM 11.177

The motion of a particle on the surface of a right circular cylinder is defined by the relations $R = A$, $\theta = 2\pi t$, and $z = B\sin 2\pi nt$, where A and B are constants and n is an integer. Determine the magnitudes of the velocity and acceleration of the particle at any time t.

SOLUTION

$$R = A \qquad \theta = 2\pi t \qquad z = B\sin 2\pi nt$$
$$\dot{R} = 0 \qquad \dot{\theta} = 2\pi \qquad \dot{z} = 2\pi n\, B\cos 2\pi nt$$
$$\ddot{R} = 0 \qquad \ddot{\theta} = 0 \qquad \ddot{z} = -4\pi^2 n^2\, B\sin 2\pi nt$$

Velocity (Eq. 11.49)

$$\mathbf{v} = \dot{R}\mathbf{e}_R + R\dot{\theta}\mathbf{e}_\theta + \dot{z}\mathbf{k}$$
$$\mathbf{v} = \quad + A(2\pi)\mathbf{e}_\theta + 2\pi n\, B\cos 2\pi nt\,\mathbf{k}$$

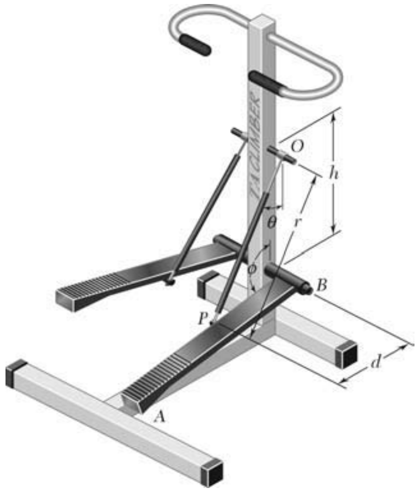
$$v = 2\pi\sqrt{A^2 + n^2B^2\cos^2 2\pi nt} \ ◀$$

Acceleration (Eq. 11.50)

$$\mathbf{a} = (\ddot{R} - R\dot{\theta}^2)\mathbf{e}_R + (R\ddot{\theta} + 2\dot{R}\dot{\theta})\mathbf{e}_\theta + \ddot{z}\mathbf{k}$$
$$\mathbf{a} = -4\pi^2 A\mathbf{e}_k - 4\pi^2 n^2\ B\sin 2\pi nt\,\mathbf{k}$$

$$a = 4\pi^2\sqrt{A^2 + n^4B^2\sin^2 2\pi nt} \ ◀$$

PROBLEM 11.178

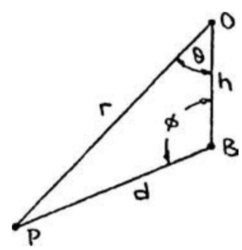


Show that $\dot{r} = h\dot{\phi}\sin\theta$ knowing that at the instant shown, step AB of the step exerciser is rotating counterclockwise at a constant rate $\dot{\phi}$.

SOLUTION

From the diagram

$$r^2 = d^2 + h^2 - 2dh\cos\phi$$



Then $$2r\dot{r} = 2dh\dot{\phi}\sin\phi$$

Now $$\frac{r}{\sin\phi} = \frac{d}{\sin\theta}$$

or $$r = \frac{d\sin\phi}{\sin\theta}$$

Substituting for r in the expression for $\dot{r}$

$$\left(\frac{d\sin\phi}{\sin\theta}\right)\dot{r} = dh\dot{\phi}\sin\phi$$

or $$\dot{r} = h\dot{\phi}\sin\theta \qquad \text{Q.E.D.}$$ ◀

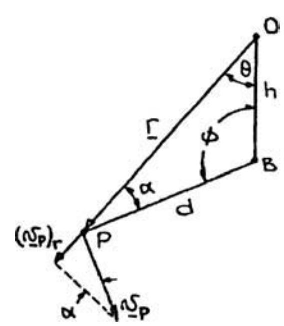
Alternative solution.

First note $$\alpha = 180° - (\phi + \theta)$$

Now $$\mathbf{v} = \mathbf{v}_r + \mathbf{v}_\theta = \dot{r}\mathbf{e}_r + r\dot{\theta}\mathbf{e}_\theta$$

With B as the origin

$$v_P = d\dot{\phi} \qquad (d = \text{constant} \Rightarrow \dot{d} = 0)$$



PROBLEM 11.178 (Continued)

With O as the origin $(v_P)_r = \dot{r}$

where $(v_P)_r = v_P \sin\alpha$

Then $\dot{r} = d\dot{\phi}\sin\alpha$

Now $$\frac{h}{\sin\alpha} = \frac{d}{\sin\theta}$$

or $d\sin\alpha = h\sin\theta$

substituting $\dot{r} = h\dot{\phi}\sin\theta$ Q.E.D.

PROBLEM 11.179

The three-dimensional motion of a particle is defined by the relations $R = A(1-e^{-t})$, $\theta = 2\pi t$, and $z = B(1-e^{-t})$. Determine the magnitudes of the velocity and acceleration when (*a*) $t = 0$, (*b*) $t = \infty$.

SOLUTION

$$R = A(1-e^{-t}) \qquad \theta = 2\pi t \qquad z = B(1-e^{-t})$$
$$\dot{R} = Ae^{-t} \qquad \dot{\theta} = 2\pi \qquad \dot{z} = Be^{-t}$$
$$\ddot{R} = -Ae^{-t} \qquad \ddot{\theta} = 0 \qquad \ddot{z} = -Be^{-t}$$

Velocity (Eq. 11.49)

$$\mathbf{v} = \dot{R}\mathbf{e}_R + R\dot{\theta}\mathbf{e}_\theta + \dot{z}\mathbf{k}$$
$$\mathbf{v} = Ae^{-t}\mathbf{e}_R + 2\pi A(1-e^{-t})\mathbf{e}_\theta + Be^{-t}\mathbf{k}$$

(*a*) When $t = 0$: $e^{-t} = e^0 = 1$; $\mathbf{v} = A\mathbf{e}_R + B\mathbf{k}$ $\qquad v = \sqrt{A^2 + B^2}$ ◀

(*b*) When $t = \infty$: $e^{-t} = e^{-\infty} = 0$ $\quad \mathbf{v} = 2\pi A\mathbf{e}_\theta$ $\qquad v = 2\pi A$ ◀

Acceleration (Eq. 11.50)

$$\mathbf{a} = (\ddot{R} - R\dot{\theta}^1)\mathbf{e}_R + (R\ddot{\theta} + 2\dot{R}\dot{\theta})\mathbf{e}_\theta + \ddot{z}\mathbf{k}$$
$$= [-Ae^{-t} - A(1-e^{-t})4\pi^2]\mathbf{e}_R + [0 + 2Ae^{-t}(2\pi)]\mathbf{e}_\theta - Be^{-t}\mathbf{k}$$

(*a*) When $t = 0$: $e^{-t} = e^0 = 1$

$$\mathbf{a} = -A\mathbf{e}_R + 4\pi A\mathbf{e}_\theta - B\,\mathbf{k}$$

$$a = \sqrt{A^2 + (4\pi A)^2 + B^2} \qquad a = \sqrt{(1+16\pi^2)A^2 + B^2} \; ◀$$

(*b*) When $t = \infty$: $e^{-t} = e^{-\infty} = 0$

$$\mathbf{a} = -4\pi^2 A\mathbf{e}_R \qquad a = 4\pi^2 A \; ◀$$

PROBLEM 11.180*

For the conic helix of Problem 11.95, determine the angle that the osculating plane forms with the y axis.

PROBLEM 11.95 The three-dimensional motion of a particle is defined by the position vector $\mathbf{r} = (Rt\cos\omega_n t)\mathbf{i} + ct\mathbf{j} + (Rt\sin\omega_n t)\mathbf{k}$. Determine the magnitudes of the velocity and acceleration of the particle. (The space curve described by the particle is a conic helix.)

SOLUTION

First note that the vectors **v** and **a** lie in the osculating plane.

Now $$\mathbf{r} = (Rt\cos\omega_n t)\mathbf{i} + ct\mathbf{j} + (Rt\sin\omega_n t)\mathbf{k}$$

Then $$\mathbf{v} = \frac{dr}{dt} = R(\cos\omega_n t - \omega_n t\sin\omega_n t)\mathbf{i} + c\mathbf{j} + R(\sin\omega_n t + \omega_n t\cos\omega_n t)\mathbf{k}$$

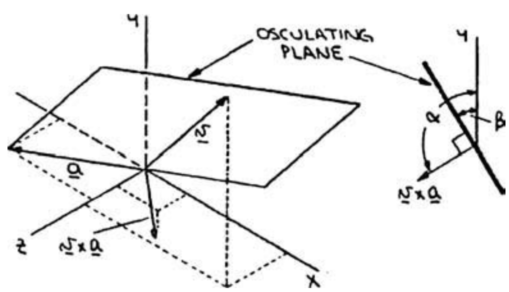
and $$\mathbf{a} = \frac{dv}{dt}$$

$$= R\left(-\omega_n\sin\omega_n t - \omega_n\sin\omega_n t - \omega_n^2 t\cos\omega_n t\right)\mathbf{i}$$

$$+ R\left(\omega_n\cos\omega_n t + \omega_n\cos\omega_n t - \omega_n^2 t\sin\omega_n t\right)\mathbf{k}$$

$$= \omega_n R[-(2\sin\omega_n t + \omega_n t\cos\omega_n t)\mathbf{i} + (2\cos\omega_n t - \omega_n t\sin\omega_n t)\mathbf{k}]$$

It then follows that the vector $(\mathbf{v}\times\mathbf{a})$ is perpendicular to the osculating plane.



$$(\mathbf{v}\times\mathbf{a}) = \omega_n R\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ R(\cos\omega_n t - \omega_n t\sin\omega_n t) & c & R(\sin\omega_n t + \omega_n t\cos\omega_n t) \\ -(2\sin\omega_n t + \omega_n t\cos\omega_n t) & 0 & (2\cos\omega_n t - \omega_n t\sin\omega_n t) \end{vmatrix}$$

$$= \omega_n R\{c(2\cos\omega_n t - \omega_n t\sin\omega_n t)\mathbf{i} + R[-(\sin\omega_n t + \omega_n t\cos\omega_n t)(2\sin\omega_n t + \omega_n t\cos\omega_n t)$$

$$-(\cos\omega_n t - \omega_n t\sin\omega_n t)(2\cos\omega_n t - \omega_n t\sin\omega_n t)]\mathbf{j} + c(2\sin\omega_n t + \omega_n t\cos\omega_n t)\mathbf{k}$$

$$= \omega_n R\left[c(2\cos\omega_n t - \omega_n t\sin\omega_n t)\mathbf{i} - R\left(2 + \omega_n^2 t^2\right)\mathbf{j} + c(2\sin\omega_n t + \omega_n t\cos\omega_n t)\mathbf{k}\right]$$

The angle α formed by the vector $(\mathbf{v}\times\mathbf{a})$ and the y axis is found from

$$\cos\alpha = \frac{(\mathbf{v}\times\mathbf{a})\cdot\mathbf{j}}{|(\mathbf{v}\times\mathbf{a})||\mathbf{j}|}$$

Where $$|\mathbf{j}| = 1$$

$$(\mathbf{v}\times\mathbf{a})\cdot\mathbf{j} = -\omega_n R^2\left(2+\omega_n^2 t^2\right)$$

$$|(\mathbf{v}\times\mathbf{a})| = \omega_n R\Big[c^2(2\cos\omega_n t - \omega_n t\sin\omega_n t)^2 + R^2\left(2+\omega_n^2 t^2\right)^2 + c^2(2\sin\omega_n t + \omega_n t\cos\omega_n t)^2\Big]^{1/2}$$

$$= \omega_n R\left[c^2\left(4+\omega_n^2 t^2\right) + R^2\left(2+\omega_n^2 t^2\right)^2\right]^{1/2}$$

Then $$\cos\alpha = \frac{-\omega_n R^2\left(2+\omega_n^2 t^2\right)}{\omega_n R\left[c^2\left(4+\omega_n^2 t^2\right) + R^2\left(2+\omega_n^2 t^2\right)^2\right]^{1/2}}$$

$$= \frac{-R\left(2+\omega_n^2 t^2\right)}{\left[c^2\left(4+\omega_n^2 t^2\right) + R^2\left(2+\omega_n^2 t^2\right)^2\right]^{1/2}}$$

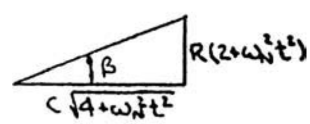
The angle β that the osculating plane forms with y axis (see the above diagram) is equal to

$$\beta = \alpha - 90^\circ$$

Then $$\cos\alpha = \cos(\beta + 90^\circ) = -\sin\beta$$

$$-\sin\beta = \frac{-R\left(2+\omega_n^2 t^2\right)}{\left[c^2\left(4+\omega_n^2 t^2\right) + R^2\left(2+\omega_n^2 t^2\right)^2\right]^{1/2}}$$

Then $$\tan\beta = \frac{R\left(2+\omega_n^2 t^2\right)}{c\sqrt{4+\omega_n^2 t^2}}$$



or $$\beta = \tan^{-1}\left[\frac{R\left(2+\omega_n^2 t^2\right)}{c\sqrt{4+\omega_n^2 t^2}}\right] \blacktriangleleft$$

PROBLEM 11.181*

Determine the direction of the binormal of the path described by the particle of Problem 11.96 when (*a*) $t = 0$, (*b*) $t = \pi/2$ s.

SOLUTION

<u>Given</u>: $\mathbf{r} = (At\cos t)\mathbf{i} + \left(A\sqrt{t^2+1}\right)\mathbf{j} + (Bt\sin t)\mathbf{k}$

$r - \text{ft}, \quad t - \text{s}; \quad A = 3, \quad B - 1$

First note that $\mathbf{e}_b$ is given by

$$\mathbf{e}_b = \frac{\mathbf{v}\times\mathbf{a}}{|\mathbf{v}\times\mathbf{a}|}$$

Now $\mathbf{r} = (3t\cos t)\mathbf{i} + \left(3\sqrt{t^2+1}\right)\mathbf{j} + (t\sin t)\mathbf{k}$

Then
$$\mathbf{v} = \frac{d\mathbf{r}}{dt}$$
$$= 3(\cos t - t\sin t)\mathbf{i} + \frac{3t}{\sqrt{t^2+1}}\mathbf{j} + (\sin t + t\cos t)\mathbf{k}$$

and
$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = 3(-\sin t - \sin t - t\cos t)\mathbf{i} + 3\frac{\sqrt{t^2+1} - t\left(\frac{t}{\sqrt{t^2+1}}\right)}{t^2+1}\mathbf{j}$$
$$+ (\cos t + \cos t - t\sin t)\mathbf{k}$$
$$= -3(2\sin t + t\cos t)\mathbf{i} + \frac{3}{(t^2+1)^{3/2}}\mathbf{j} + (2\cos t - t\sin t)\mathbf{k}$$

(*a*) At $t = 0$: $\mathbf{v} = (3\text{ ft/s})\mathbf{i}$

$$\mathbf{a} = (3\text{ ft/s}^2)\mathbf{j} + (2\text{ ft/s}^2)\mathbf{k}$$

Then
$$\mathbf{v}\times\mathbf{a} = 3\mathbf{i}\times(3\mathbf{j} + 2\mathbf{k})$$
$$= 3(-2\mathbf{j} + 3\mathbf{k})$$

and
$$|\mathbf{v}\times\mathbf{a}| = 3\sqrt{(-2)^2 + (3)^2} = 3\sqrt{13}$$

Then
$$\mathbf{e}_b = \frac{3(-2\mathbf{j}+3\mathbf{k})}{3\sqrt{13}} = \frac{1}{\sqrt{13}}(-2\mathbf{j}+3\mathbf{k})$$

$$\cos\theta_x = 0 \qquad \cos\theta_y = -\frac{2}{\sqrt{13}} \qquad \cos\theta_z = \frac{3}{\sqrt{13}}$$

or
$$\theta_x = 90° \qquad \theta_y = 123.7° \qquad \theta_z = 33.7°$$ ◀

(*b*) At $t = \frac{\pi}{2}$ s:

$$\mathbf{v} = -\left(\frac{3\pi}{2} \text{ ft/s}\right)\mathbf{i} + \left(\frac{3\pi}{\sqrt{\pi^2+4}} \text{ ft/s}\right)\mathbf{j} + (1 \text{ ft/s})\mathbf{k}$$

$$\mathbf{a} = -(6 \text{ ft/s}^2)\mathbf{i} + \left[\frac{24}{(\pi^2+4)^{3/2}} \text{ ft/s}^2\right]\mathbf{j} - \left(\frac{\pi}{2} \text{ ft/s}^2\right)\mathbf{k}$$

Then

$$\mathbf{v}\times\mathbf{a} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -\frac{3\pi}{2} & \frac{3\pi}{(\pi^2+4)^{1/2}} & 1 \\ -6 & \frac{24}{(\pi^2+4)^{3/2}} & -\frac{\pi}{2} \end{vmatrix}$$

$$= -\left[\frac{3\pi^2}{2(\pi^2+4)^{1/2}} + \frac{24}{(\pi^2+4)^{3/2}}\right]\mathbf{i} - \left(6 + \frac{3\pi^2}{4}\right)\mathbf{j}$$

$$+ \left[-\frac{36\pi}{(\pi^2+4)^{3/2}} + \frac{18\pi}{(\pi^2+4)^{1/2}}\right]\mathbf{k}$$

$$= -4.43984\mathbf{i} - 13.40220\mathbf{j} + 12.99459\mathbf{k}$$

and

$$|\mathbf{v}\times\mathbf{a}| = [(-4.43984)^2 + (-13.40220)^2 + (12.99459)^2]^{1/2}$$

$$= 19.18829$$

Then

$$\mathbf{e}_b = \frac{1}{19.1829}(-4.43984\mathbf{i} - 13.40220\mathbf{j} + 12.99459\mathbf{k})$$

$$\cos\theta_x = -\frac{4.43984}{19.18829} \quad \cos\theta_y = -\frac{13.40220}{19.18829} \quad \cos\theta_z = \frac{12.99459}{19.18829}$$

or $\theta_x = 103.4°$ $\theta_y = 134.3°$ $\theta_z = 47.4°$ ◀

PROBLEM 11.182

The motion of a particle is defined by the relation $x = 2t^3 - 15t^2 + 24t + 4$, where x and t are expressed in meters and seconds, respectively. Determine (*a*) when the velocity is zero, (*b*) the position and the total distance traveled when the acceleration is zero.

SOLUTION

$$x = 2t^3 - 15t^2 + 24t + 4$$

so $$v = \frac{dx}{dt} = 6t^2 - 30t + 24$$

$$a = \frac{dv}{dt} = 12t - 30$$

(*a*) Times when $v = 0$. $\quad 0 = 6t^2 - 30t + 24 = 6\,(t^2 - 5t + 4)$

$$(t-4)(t-1) = 0 \qquad t = 1.00 \text{ s}, \quad t = 4.00 \text{ s} \blacktriangleleft$$

(*b*) Position and distance traveled when $a = 0$.

$$a = 12t - 30 = 0 \qquad t = 2.5 \text{ s}$$

so $$x_2 = 2(2.5)^3 - 15(2.5)^2 + 24(2.5) + 4$$

Final position $\quad x = 1.50 \text{ m} \blacktriangleleft$

For $\quad 0 \le t \le 1 \text{ s}, \quad v > 0.$

For $\quad 1 \text{ s} \le t \le 2.5 \text{ s}, \quad v \le 0.$

At $\quad t = 0, \quad x_0 = 4 \text{ m}.$

At $\quad t = 1 \text{ s}, \quad x_1 = (2)(1)^3 - (15)(1)^2 + (24)(1) + 4 = 15 \text{ m}$

Distance traveled over interval: $x_1 - x_0 = 11 \text{ m}$

For $\quad 1 \text{ s} \le t \le 2.5 \text{ s}, \quad v \le 0$

Distance traveled over interval

$$|x_2 - x_1| = |1.5 - 15| = 13.5 \text{ m}$$

Total distance: $\quad d = 11 + 13.5 \qquad d = 24.5 \text{ m} \blacktriangleleft$

PROBLEM 11.183

A particle starting from rest at $x = 1$ m is accelerated so that its velocity doubles in magnitude between $x = 2$ m and $x = 8$ m. Knowing that the acceleration of the particle is defined by the relation $a = k[x - (A/x)]$, determine the values of the constants A and k if the particle has a velocity of 29 m/s when $x = 16$ m.

SOLUTION

We have $$v\frac{dv}{dx} = a = k\left(x - \frac{A}{x}\right)$$

When $x = 1$ ft, $v = 0$: $$\int_0^v v\,dv = \int_1^x k\left(x - \frac{A}{x}\right)dx$$

or $$\frac{1}{2}v^2 = k\left[\frac{1}{2}x^2 - A\ln x\right]_1^x$$

$$= k\left(\frac{1}{2}x^2 - A\ln x - \frac{1}{2}\right)$$

At $x = 2$ ft: $$\frac{1}{2}v_2^2 = k\left[\frac{1}{2}(2)^2 - A\ln 2 - \frac{1}{2}\right] = k\left(\frac{3}{2} - A\ln 2\right)$$

$x = 8$ ft: $$\frac{1}{2}v_8^2 = k\left[\frac{1}{2}(8)^2 - A\ln 8 - \frac{1}{2}\right] = k(31.5 - A\ln 8)$$

Now $\frac{v_8}{v_2} = 2$: $$\frac{\frac{1}{2}v_8^2}{\frac{1}{2}v_2^2} = (2)^2 = \frac{k(31.5 - A\ln 8)}{k\left(\frac{3}{2} - A\ln 2\right)}$$

$$6 - 4A\ln 2 = 31.5 - A\ln 8$$

$$25.5 = A(\ln 8 - 4\ln 2) = A(\ln 8 - \ln 2^4) = A\ln\left(\frac{1}{2}\right)$$

$$A = \frac{25.5}{\ln\frac{1}{2}}$$

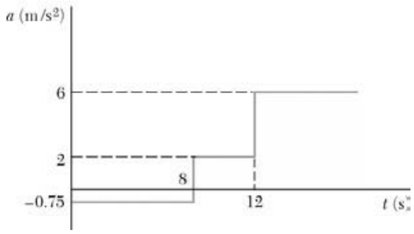
$A = -36.8\ \text{m}^2$ ◀

When $x = 16$ m, $v = 29$ m/s: $$\frac{1}{2}(29)^2 = k\left[\frac{1}{2}(16)^2 - \frac{25.5}{\ln\left(\frac{1}{2}\right)}\ln(16) - \frac{1}{2}\right]$$

$$420.5k = k\left[128 + 102 - \frac{1}{2}\right] = 230.5k$$

$$= 230.5k$$

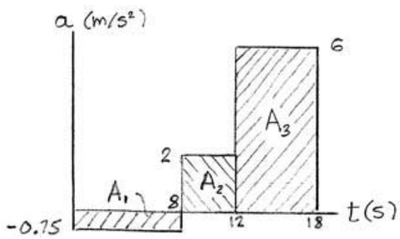
$k = 1.832\ \text{s}^{-2}$ ◀



PROBLEM 11.184

A particle moves in a straight line with the acceleration shown in the figure. Knowing that the particle starts from the origin with $v_0 = -2$ m/s, (*a*) construct the $v-t$ and $x-t$ curves for $0 < t < 18$ s, (*b*) determine the position and the velocity of the particle and the total distance traveled when $t = 18$ s.

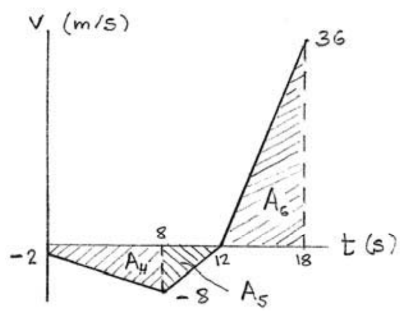
SOLUTION



Compute areas under $a - t$ curve.

$$A_1 = (-0.75)(8) = -6 \text{ m/s}$$
$$A_2 = (2)(4) = 8 \text{ m/s}$$
$$A_3 = (6)(6) = 36 \text{ m/s}$$

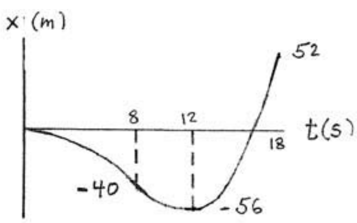
$$v_0 = -2 \text{ m/s}$$
$$v_8 = v_0 + A_1 = -8 \text{ m/s}$$
$$v_{12} = v_8 + A_2 = 0$$
$$v_{18} = v_{12} + A_3 \qquad v_{18} = 36 \text{ m/s} \blacktriangleleft$$



Sketch $v-t$ curve using straight line portions over the constant acceleration periods.

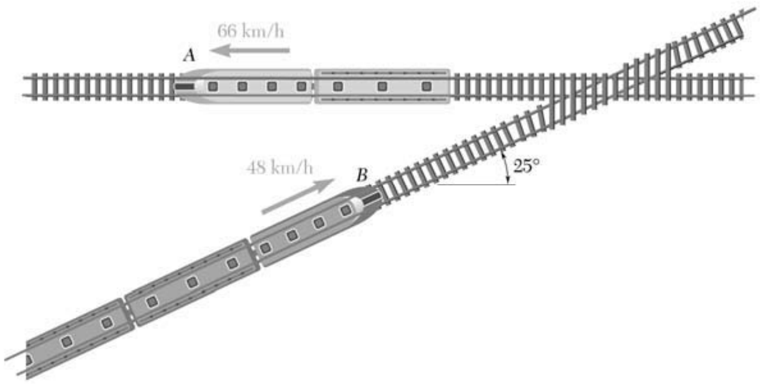
Compute areas under the $v-t$ curve.

$$A_4 = \frac{1}{2}(-2 - 8)(8) = -40 \text{ m}$$
$$A_5 = \frac{1}{2}(-8)(4) = -16 \text{ m}$$
$$A_6 = \frac{1}{2}(36)(6) = 108 \text{ m}$$



$$x_0 = 0$$
$$x_8 = x_0 + A_4 = -40 \text{ m}$$
$$x_{12} = x_8 + A_5 = -56 \text{ m}$$
$$x_{18} = x_{12} + A_6 \qquad x_{18} = 52 \text{ m} \blacktriangleleft$$

Total distance traveled $= 56 + 108 \qquad d = 164 \text{ m} \blacktriangleleft$



PROBLEM 11.185

The velocities of commuter trains *A* and *B* are as shown. Knowing that the speed of each train is constant and that *B* reaches the crossing 10 min after *A* passed through the same crossing, determine (*a*) the relative velocity of *B* with respect to *A*, (*b*) the distance between the fronts of the engines 3 min after *A* passed through the crossing.

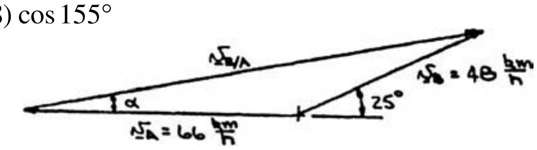
SOLUTION

(*a*) We have $\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$

The graphical representation of this equation is then as shown.

Then $v_{B/A}^2 = 66^2 + 48^2 - 2(66)(48)\cos 155°$

or $v_{B/A} = 111.366$ km/h



and $$\frac{48}{\sin\alpha} = \frac{111.366}{\sin 155°}$$

or $\alpha = 10.50°$

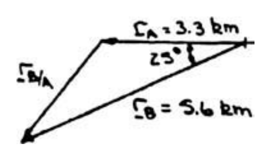
$\mathbf{v}_{B/A} = 111.4$ km/h ⦢ 10.50° ◀

(*b*) First note that

at $t = 3$ min, *A* is $(66 \text{ km/h})\left(\frac{3}{60}\right) = 3.3$ km west of the crossing.

at $t = 3$ min, *B* is $(48 \text{ km/h})\left(\frac{7}{60}\right) = 5.6$ km southwest of the crossing.

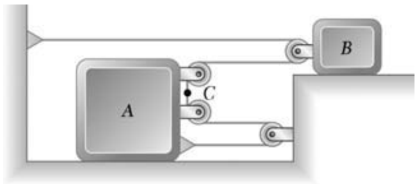
Now $\mathbf{r}_B = \mathbf{r}_A + \mathbf{r}_{B/A}$



Then at $t = 3$ min, we have

$$r_{B/A}^2 = 3.3^2 + 5.6^2 - 2(3.3)(5.6)\cos 25°$$

or $r_{B/A} = 2.96$ km ◀



PROBLEM 11.186

Slider block B starts from rest and moves to the right with a constant acceleration of $1\ \text{ft/s}^2$. Determine (*a*) the relative acceleration of portion C of the cable with respect to slider block A, (*b*) the velocity of portion C of the cable after 2 s.

SOLUTION

Let d be the distance between the left and right supports.

Constraint of entire cable: $x_B + (x_B - x_A) + 2(d - x_A) = \text{constant}$

$$2v_B - 3v_A = 0 \quad \text{and} \quad 2a_B - 3a_A = 0$$

$$a_A = \frac{2}{3}a_B = \frac{2}{3}(1) = 0.667\ \text{ft/s}^2 \quad \text{or} \quad a_A = 0.667\ \text{ft/s}^2 \longrightarrow$$

Constraint of Point C: $\quad 2(d - x_A) + y_{C/A} = \text{constant}$

$$-2v_A + v_{C/A} = 0 \quad \text{and} \quad -2a_A + a_{C/A} = 0$$

(*a*) $\quad a_{C/A} = 2a_A = 2(0.667) = 1.333\ \text{ft/s}^2$

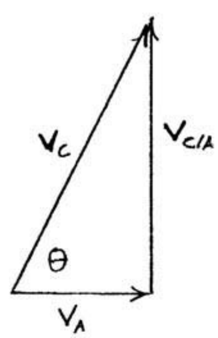
$$\mathbf{a}_{C/A} = 1.333\ \text{ft/s}^2 \uparrow \blacktriangleleft$$

Velocity vectors after 2s: $\mathbf{v}_A = (0.667)(2) = 1.333\ \text{ft/s} \longrightarrow$

$$\mathbf{v}_{C/A} = (1.333)(2) = 2.666\ \text{ft/s} \uparrow$$

$$\mathbf{v}_C = \mathbf{v}_A + \mathbf{v}_{C/A}$$

Sketch the vector addition.



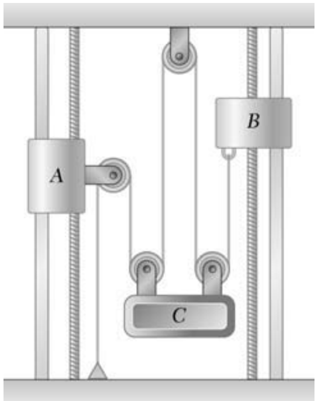
$$v_C^2 = v_A^2 + v_{C/A}^2 = (1.333)^2 + (2.666)^2 = 8.8889(\text{ft/s})^2$$

$$v_C = 2.981\ \text{ft/s}$$

$$\tan\theta = \frac{v_{C/A}}{v_A} = \frac{2.666}{1.333} = 2, \quad \theta = 63.4°$$

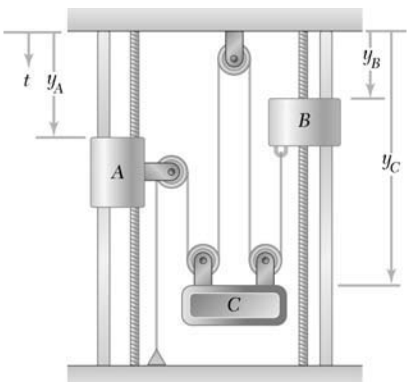
(*b*) $\quad \mathbf{v}_C = 2.98\ \text{ft/s} \measuredangle\ 63.4° \blacktriangleleft$

PROBLEM 11.187



Collar A starts from rest at $t = 0$ and moves downward with a constant acceleration of 7 in./s^2. Collar B moves upward with a constant acceleration, and its initial velocity is 8 in./s. Knowing that collar B moves through 20 in. between $t = 0$ and $t = 2$ s, determine (*a*) the accelerations of collar B and block C, (*b*) the time at which the velocity of block C is zero, (*c*) the distance through which block C will have moved at that time.

SOLUTION



From the diagram

$$-y_A + (y_C - y_A) + 2y_C + (y_C - y_B) = \text{constant}$$

Then $$-2v_A - v_B + 4v_C = 0 \qquad (1)$$

and $$-2a_A - a_B + 4a_C = 0 \qquad (2)$$

Given: $$(v_A)_0 = 0$$

$$(\mathbf{a}_A) = 7 \text{ in./s}^2 \downarrow$$

$$(\mathbf{v}_B)_0 = 8 \text{ in./s} \uparrow$$

$$\mathbf{a}_B = \text{constant} \uparrow$$

At $t = 2$ s $$y - (y_B)_0 = 20 \text{ in.} \uparrow$$

(*a*) We have $$y_B = (y_B)_0 + (v_B)_0 t + \frac{1}{2} a_B t^2$$

At $t = 2$ s: $$-20 \text{ in.} = (-8 \text{ in./s})(2 \text{ s}) + \frac{1}{2} a_B (2 \text{ s})^2$$

$$a_B = -4 \text{ in./s}^2 \quad \text{or} \quad \mathbf{a}_B = 2 \text{ in./s}^2 \uparrow \blacktriangleleft$$

Then, substituting into Eq. (2)

$$-2(7 \text{ in./s}^2) - (-2 \text{ in./s}^2) + 4a_C = 0$$

$$a_C = 3 \text{ in./s}^2 \quad \text{or} \quad \mathbf{a}_C = 3 \text{ in./s}^2 \downarrow \blacktriangleleft$$

PROBLEM 11.187 (Continued)

(*b*) Substituting into Eq. (1) at $t = 0$

$$-2(0) - (-8 \text{ in./s}) + 4(v_C)_0 = 0 \quad \text{or} \quad (v_C)_0 = -2 \text{ in./s}$$

Now $$v_C = (v_C)_0 + a_C t$$

When $v_C = 0$: $$0 = (-2 \text{ in./s}) + (3 \text{ in./s}^2)t$$

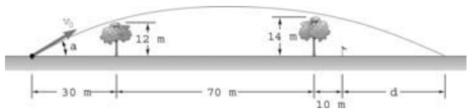
or $$t = \frac{2}{3} \text{ s}$$ $t = 0.667$ s ◀

(*c*) We have $$y_C = (y_C)_0 + (v_C)_0 t + \frac{1}{2} a_C t^2$$

At $t = \frac{2}{3}$ s: $$y_C - (y_C)_0 = (-2 \text{ in./s})\left(\frac{2}{3} \text{ s}\right) + \frac{1}{2}(3 \text{ in./s}^2)\left(\frac{2}{3} \text{ s}\right)^2$$

$$= -0.667 \text{ in.}$$ or $\mathbf{y}_C - (\mathbf{y}_C)_0 = 0.667 \text{ in.}\uparrow$ ◀

PROBLEM 11.188



A golfer hits a ball with an initial velocity of magnitude v_0 at an angle α with the horizontal. Knowing that the ball must clear the tops of two trees and land as close as possible to the flag, determine v_0 and the distance d when the golfer uses (*a*) a six-iron with $\alpha = 31°$, (*b*) a five-iron with $\alpha = 27°$.

SOLUTION

The horizontal and vertical motions are

$$x = (v_0 \cos\alpha)t \quad \text{or} \quad v_0 = \frac{x}{t\cos\alpha} \tag{1}$$

$$y = (v_0 \sin\alpha)t - \frac{1}{2}gt^2 = x\tan\alpha - \frac{1}{2}gt^2$$

or

$$t^2 = \frac{2(x\tan\alpha - y)}{g} \tag{2}$$

At the landing Point C: $\quad y_C = 0, \quad t = \dfrac{2v_0 \sin\alpha}{g}$

And

$$x_C = (v_0\cos\alpha)t = \frac{2v_0^2 \sin\alpha\cos\alpha}{g} \tag{3}$$

(*a*) $\alpha = 31°$

To clear tree A: $\quad x_A = 30 \text{ m}, \ y_A = 12 \text{ m}$

From (2), $\quad t_A^2 = \dfrac{2(30\tan 31° - 12)}{9.81} = 1.22851 \text{ s}^2, \qquad t_A = 1.1084 \text{ s}$

From (1), $\quad (v_0)_A = \dfrac{30}{1.1084\cos 31°} = 31.58 \text{ m/s}$

To clear tree B: $\quad x_B = 100 \text{ m}, \quad y_B = 14 \text{ m}$

From (2), $\quad (t_B)^2 = \dfrac{2(100\tan 31° - 14)}{9.81} = 9.3957 \text{ s}^2, \qquad t_B = 3.0652 \text{ s}$

From (1), $\quad (v_0)_B = \dfrac{100}{3.0652\cos 31°} = 38.06 \text{ m/s}$

The larger value governs, $\quad v_0 = 38.06 \text{ m/s} \qquad\qquad v_0 = 38.1 \text{ m/s}$ ◀

From (3), $\quad x_C = \dfrac{(2)(38.06)^2 \sin 31° \cos 31°}{9.81} = 130.38 \text{ m}$

$d = x_C - 110 \qquad\qquad d = 20.4 \text{ m}$ ◀

PROBLEM 11.188 (Continued)

(*b*) $\alpha = 27°$

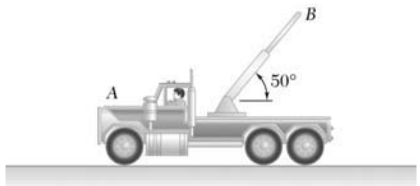
By a similar calculation, $t_A = 0.81846$ s, $(v_0)_A = 41.138$ m/s,

$t_B = 2.7447$ s, $(v_0)_B = 40.890$ m/s,

$v_0 = 41.138$ m/s $\qquad v_0 = 41.1$ m/s ◀

$x_C = 139.56$ m, $\qquad d = 29.6$ m ◀

PROBLEM 11.189



As the truck shown begins to back up with a constant acceleration of 4 ft/s^2, the outer section B of its boom starts to retract with a constant acceleration of 1.6 ft/s^2 relative to the truck. Determine (a) the acceleration of section B, (b) the velocity of section B when $t = 2$ s.

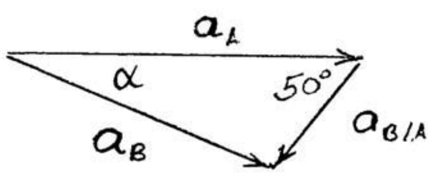
SOLUTION

For the truck, $\mathbf{a}_A = 4\ \text{ft/s}^2 \longrightarrow$

For the boom, $\mathbf{a}_{B/A} = 1.6\ \text{ft/s}^2 \swarrow 50°$

(a) $\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$

Sketch the vector addition.



By law of cosines:

$$a_B^2 = a_A^2 + a_{B/A}^2 - 2a_A a_{B/A} \cos 50°$$

$$= 4^2 + 1.6^2 - 2(4)(1.6)\cos 50°$$

$$a_B = 3.214\ \text{ft/s}^2$$

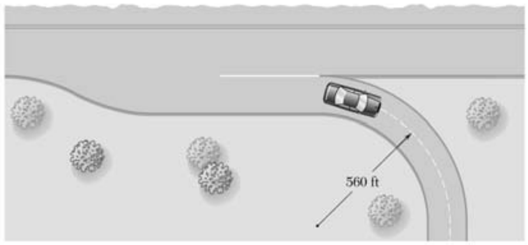
Law of sines: $\sin\alpha = \dfrac{a_{B/A}\sin 50°}{a_B} = \dfrac{1.6\sin 50°}{3.214} = 0.38131$

$$\alpha = 22.4°, \qquad \mathbf{a}_B = 3.21\ \text{ft/s}^2 \searrow 22.4° \blacktriangleleft$$

(b) $\mathbf{v}_B = (v_B)_0 + a_B t = 0 + (3.214)(2)$

$$\mathbf{v}_B = 6.43\ \text{ft/s}^2 \searrow 22.4° \blacktriangleleft$$

PROBLEM 11.190



A motorist traveling along a straight portion of a highway is decreasing the speed of his automobile at a constant rate before exiting from the highway onto a circular exit ramp with a radius of 560-ft. He continues to decelerate at the same constant rate so that 10 s after entering the ramp, his speed has decreased to 20 mi/h, a speed which he then maintains. Knowing that at this constant speed the total acceleration of the automobile is equal to one-quarter of its value prior to entering the ramp, determine the maximum value of the total acceleration of the automobile.

SOLUTION

First note $$v_{10} = 20 \text{ mi/h} = \frac{88}{3} \text{ ft/s}$$

While the car is on the straight portion of the highway.

$$a = a_{\text{straight}} = a_t$$

and for the circular exit ramp

$$a = \sqrt{a_t^2 + a_n^2}$$

where $$a_n = \frac{v^2}{\rho}$$

By observation, $a_{\max}$ occurs when v is maximum, which is at $t = 0$ when the car first enters the ramp.

For uniformly decelerated motion

$$v = v_0 + a_t t$$

and at $t = 10$ s: $$v = \text{constant} \Rightarrow a = a_n = \frac{v_{10}^2}{\rho}$$

$$a = \frac{1}{4} a_{\text{st.}}$$

Then $$a_{\text{straight}} = a_t \Rightarrow \frac{1}{4} a_t = \frac{v_{10}^2}{\rho} = \frac{\left(\frac{88}{3} \text{ ft/s}\right)^2}{560 \text{ ft}}$$

or $$a_t = -6.1460 \text{ ft/s}^2$$

(The car is decelerating; hence the minus sign.)

PROBLEM 11.190 (Continued)

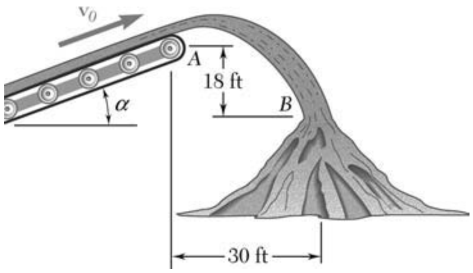
Then at $t = 10$ s: $$\frac{88}{3}\text{ ft/s} = v_0 + (-6.1460\text{ ft/s}^2)(10\text{ s})$$

or $$v_0 = 90.793\text{ ft/s}$$

Then at $t = 0$: $$a_{max} = \sqrt{a_t^2 + \left(\frac{v_0^2}{\rho}\right)^2}$$

$$= \left\{(-6.1460\text{ ft/s}^2)^2 + \left[\frac{(90.793\text{ ft/s})^2}{560\text{ ft}}\right]^2\right\}^{1/2}$$

or $$a_{max} = 15.95\text{ ft/s}^2 \blacktriangleleft$$



PROBLEM 11.191

Sand is discharged at A from a conveyor belt and falls onto the top of a stockpile at B. Knowing that the conveyor belt forms an angle $\alpha = 25°$ with the horizontal, determine (*a*) the speed v_0 of the belt, (*b*) the radius of curvature of the trajectory described by the sand at Point B.

SOLUTION

The motion is projectile motion. Place the origin at Point A. Then $x_0 = 0$ and $y_0 = 0$.

The coordinates of Point B are $x_B = 30$ ft and $y_B = -18$ ft.

Horizontal motion:
$$v_x = v_0 \cos 25° \qquad (1)$$
$$x = v_0 t \cos 25° \qquad (2)$$

Vertical motion:
$$v_y = v_0 \sin 25° - gt \qquad (3)$$
$$y = v_0 t \sin 25° - \frac{1}{2} g t^2 \qquad (4)$$

At Point B, Eq. (2) gives

$$v_0 t_B = \frac{x_B}{\cos 25°} = \frac{30}{\cos 25°} = 33.101 \text{ ft}$$

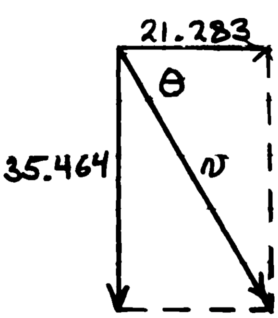
Substituting into Eq. (4),

$$-18 = (33.101)(\sin 25°) - \frac{1}{2}(32.2) t_B^2$$
$$t_B = 1.40958 \text{ s}$$

(*a*) Speed of the belt.
$$v_0 = \frac{v_0 t_B}{t_B} = \frac{33.101}{1.40958} = 23.483$$

$v_0 = 23.4$ ft/s ◀

Eqs. (1) and (3) give



$$v_x = 23.483 \cos 25° = 21.283 \text{ ft/s}$$
$$v_y = (23.483)\sin 25° - (32.2)(1.40958) = -35.464 \text{ ft/s}$$

$$\tan\theta \frac{-v_y}{v_x} = 1.66632 \qquad \theta = 59.03°$$
$$v = 41.36 \text{ ft/s}$$

PROBLEM 11.191 (Continued)

Components of acceleration.

$$\mathbf{a} = 32.2 \text{ ft/s}^2 \downarrow \qquad a_t = 32.2 \sin\theta$$

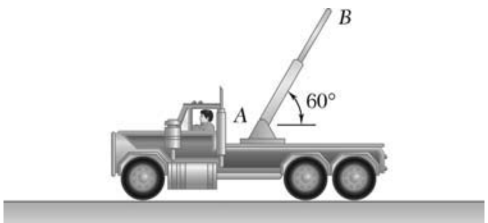
$$a_n = 32.2\cos\theta = 32.2\cos 59.03° = 16.57 \text{ ft/s}^2$$

(*b*) Radius of curvature at *B*.

$$a_n = \frac{v^2}{\rho}$$

$$\rho = \frac{v^2}{a_n} = \frac{(41.36)^2}{16.57}$$

$\rho = 103.2$ ft ◀



PROBLEM 11.192

The end Point B of a boom is originally 5 m from fixed Point A when the driver starts to retract the boom with a constant radial acceleration of $\ddot{r} = -1.0 \text{ m/s}^2$ and lower it with a constant angular acceleration $\ddot{\theta} = -0.5 \text{ rad/s}^2$. At $t = 2$ s, determine (*a*) the velocity of Point B, (*b*) the acceleration of Point B, (*c*) the radius of curvature of the path.

SOLUTION

Radial motion. $\quad r_0 = 5 \text{ m}, \quad \dot{r}_0 = 0, \quad \ddot{r} = -1.0 \text{ m/s}^2$

$$r = r_0 + \dot{r}_0 t + \frac{1}{2}\ddot{r}t^2 = 5 + 0 - 0.5t^2$$

$$\dot{r} = \dot{r}_0 + \ddot{r}t = 0 - 1.0t$$

At $t = 2$ s,

$$r = 5 - (0.5)(2)^2 = 3 \text{ m}$$

$$\dot{r} = (-1.0)(2) = -2 \text{ m/s}$$

Angular motion. $\quad \theta_0 = 60° = \frac{\pi}{3}\text{rad}, \quad \dot{\theta}_0 = 0, \quad \ddot{\theta} = -0.5 \text{ rad/s}^2$

$$\theta = \theta_0 + \dot{\theta}_0 + \frac{1}{2}\ddot{\theta}t^2 = \frac{\pi}{3} + 0 - 0.25t^2$$

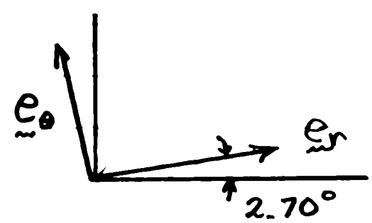
$$\dot{\theta} = \dot{\theta}_0 + \ddot{\theta}t = 0 - 0.5t$$

At $t = 2$ s,

$$\theta = \frac{\pi}{3} + 0 - (0.25)(2)^2 = 0.047198 \text{ rad} = 2.70°$$

$$\dot{\theta} = -(0.5)(2) = -1.0 \text{ rad/s}$$

Unit vectors $\mathbf{e}_r$ and $\mathbf{e}_\theta$.



(*a*) Velocity of Point B at $t = 2$ s.

$$\mathbf{v}_B = \dot{r}\mathbf{e}_r + r\dot{\theta}\mathbf{e}_\theta$$

$$= -(2 \text{ m/s})\mathbf{e}_r + (3 \text{ m})(-1.0 \text{ rad/s})\mathbf{e}_\theta$$

$$\mathbf{v}_B = (-2.00 \text{ m/s})\mathbf{e}_r + (-3.00 \text{ m/s})\mathbf{e}_\theta \blacktriangleleft$$

$$\tan\alpha = \frac{v_\theta}{v_r} = \frac{-3.0}{-2.0} = 1.5 \qquad \alpha = 56.31°$$

$$v = \sqrt{v_r^2 + v_\theta^2} = \sqrt{(-2)^2 + (-3)^2} = 3.6055 \text{ m/s}$$

Direction of velocity.

$$\mathbf{e}_t = \frac{\mathbf{v}}{v} = \frac{-2\mathbf{e}_r - 3\mathbf{e}_\theta}{3.6055} = -0.55470\mathbf{e}_r - 0.83205\mathbf{e}_\theta$$

$$\theta + \alpha = 2.70 + 56.31° = 59.01°$$

$$\mathbf{v}_B = 3.61 \text{ m/s} \measuredangle 59.0° \blacktriangleleft$$

(*b*) Acceleration of Point *B* at $t = 2$ s.

$$\mathbf{a}_B = (\ddot{r} - r\dot{\theta}^2)\mathbf{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\mathbf{e}_\theta$$
$$= [-1.0 - (3)(-1)^2]\mathbf{e}_r + [(3)(-0.5) + (2)(-1.0)(-0.5)]\mathbf{e}_\theta$$

$$\mathbf{a}_B = (-4.00 \text{ m/s}^2)\mathbf{e}_r + (2.50 \text{ m/s}^2)\mathbf{e}_\theta \blacktriangleleft$$

$$\tan\beta = \frac{a_\theta}{a_r} = \frac{2.50}{-4.00} = -0.625 \qquad \beta = -32.00°$$

$$a = \sqrt{a_r^2 + a_\theta^2} = \sqrt{(-4)^2 + (2.5)^2} = 4.7170 \text{ m/s}^2$$

$$\theta + \beta = 2.70° - 32.00° = -29.30°$$

$$\mathbf{a}_B = 4.72 \text{ m/s}^2 \measuredangle 29.3° \blacktriangleleft$$

Tangential component: $\mathbf{a}_t = (\mathbf{a} \cdot \mathbf{e}_t)\mathbf{e}_t$

$$\mathbf{a}_t = (-4\mathbf{e}_r + 2.5\mathbf{e}_\theta) \cdot (-0.55470\mathbf{e}_r - 0.83205\mathbf{e}_\theta)\mathbf{e}_t$$
$$= [(-4)(-0.55470) + (2.5)(-0.83205)]\mathbf{e}_t$$

$$= (0.138675 \text{ m/s}^2)\mathbf{e}_t = 0.1389 \text{ m/s}^2 \measuredangle 59.0°$$

Normal component: $\mathbf{a}_n = \mathbf{a} - \mathbf{a}_t$

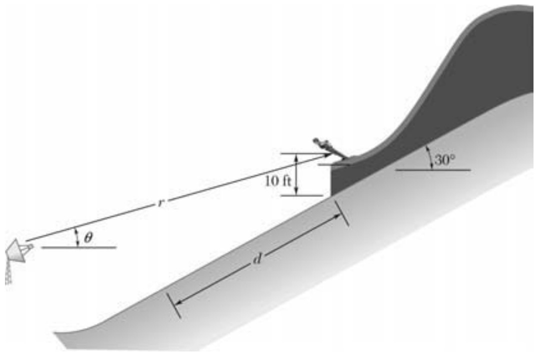
$$\mathbf{a}_n = -4\mathbf{e}_r + 2.5\mathbf{e}_\theta - (0.138675)(-0.55470\mathbf{e}_r - 0.83205\mathbf{e}_\theta)$$
$$= (-3.9231 \text{ m/s}^2)\mathbf{e}_r + (2.6154 \text{ m/s}^2)\mathbf{e}_\theta$$

$$a_n = \sqrt{(3.9231)^2 + (2.6154)^2} = 4.7149 \text{ m/s}^2$$

(*c*) Radius of curvature of the path.

$$a_n = \frac{v^2}{\rho}$$

$$\rho = \frac{v^2}{a_n} = \frac{(3.6055 \text{ m/s})^2}{4.7149 \text{ m/s}} \qquad \rho = 2.76 \text{ m} \blacktriangleleft$$



PROBLEM 11.193

A telemetry system is used to quantify kinematic values of a ski jumper immediately before she leaves the ramp. According to the system $r = 500$ ft, $\dot{r} = -105$ ft/s, $\ddot{r} = -10$ ft/s^2, $\theta = 25°$, $\dot{\theta} = 0.07$ rad/s, $\ddot{\theta} = 0.06$ rad/s^2. Determine (*a*) the velocity of the skier immediately before she leaves the jump, (*b*) the acceleration of the skier at this instant, (*c*) the distance of the jump *d* neglecting lift and air resistance.

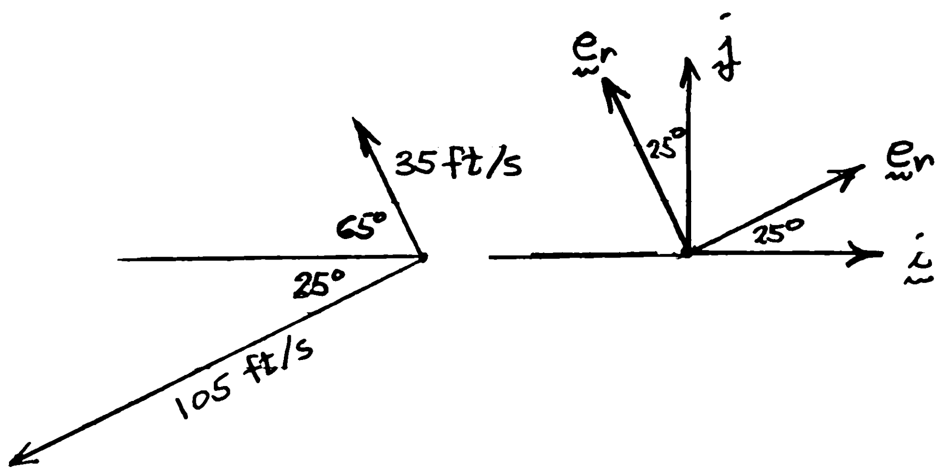
SOLUTION

(*a*) Velocity of the skier. $(r = 500 \text{ ft}, \quad \theta = 25°)$

$$\mathbf{v} = v_r\mathbf{e}_r + v_\theta\mathbf{e}_\theta = \dot{r}\mathbf{e}_r + r\dot{\theta}\mathbf{e}_\theta$$
$$= (-105 \text{ ft/s})\mathbf{e}_r + (500 \text{ ft})(0.07 \text{ rad/s})\mathbf{e}_\theta$$

$$\mathbf{v} = (-105 \text{ ft/s})\mathbf{e}_r + (35 \text{ ft/s})\mathbf{e}_\theta \blacktriangleleft$$

Direction of velocity:



$$\mathbf{v} = (-105\cos 25° - 35\cos 65°)\mathbf{i} + (35\sin 65° - 105\sin 25°)\mathbf{j}$$
$$= (-109.95 \text{ ft/s})\mathbf{i} + (-12.654 \text{ ft/s})\mathbf{j}$$

$$\tan\alpha = \frac{v_y}{v_x} = \frac{-12.654}{-109.95} \qquad \alpha = 6.565°$$

$$v = \sqrt{(105)^2 + (35)^2} = 110.68 \text{ ft/s}$$

$$v = 110.7 \text{ ft/s} \nearrow 6.57° \blacktriangleleft$$

(*b*) Acceleration of the skier.

$$\mathbf{a} = a_r\mathbf{e}_r + a_\theta\mathbf{e}_\theta = (\ddot{r} - r\dot{\theta}^2)\mathbf{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\mathbf{e}_\theta$$

$$a_r = -10 - (500)(0.07)^2 = -12.45 \text{ ft/s}^2$$

$$a_\theta = (500)(0.06) + (2)(-105)(0.07) = 15.30 \text{ ft/s}^2$$

$$\mathbf{a} = (-12.45 \text{ ft/s}^2)\mathbf{e}_r + (15.30 \text{ ft/s}^2)\mathbf{e}_\theta \blacktriangleleft$$

$$\mathbf{a} = (-12.45)(\mathbf{i}\cos 25° + \mathbf{j}\sin 25°) + (15.30)(-\mathbf{i}\cos 65° + \mathbf{j}\sin 65°)$$

$$= (-17.750 \text{ ft/s}^2)\mathbf{i} + (8.6049 \text{ ft/s}^2)\mathbf{j}$$

$$\tan\beta = \frac{a_y}{a_x} = \frac{8.6049}{-17.750} \qquad \beta = -25.9°$$

$$a = \sqrt{(12.45)^2 + (15.30)^2} = 19.725 \text{ ft/s}^2$$

$$a = 19.73 \text{ ft/s}^2 \measuredangle 25.9° \blacktriangleleft$$

(*c*) Distance of the jump *d*.

Projectile motion. Place the origin of the *xy*-coordinate system at the end of the ramp with the *x*-coordinate horizontal and positive to the left and the *y*-coordinate vertical and positive downward.

Horizontal motion: (Uniform motion)

$$x_0 = 0$$

$$\dot{x}_0 = 109.95 \text{ ft/s} \qquad \text{(from Part } a\text{)}$$

$$x = x_0 + \dot{x}_0 t = 109.95t$$

Vertical motion: (Uniformly accelerated motion)

$$y_0 = 0$$

$$\dot{y}_0 = 12.654 \text{ ft/s} \qquad \text{(from Part } a\text{)}$$

$$\ddot{y} = 32.2 \text{ ft/s}^2$$

$$y = y_0 + \dot{y}_0 t + \frac{1}{2}\ddot{y}t^2 = 12.654t - 16.1t^2$$

At the landing point,

$$x = d\cos 30° \qquad (1)$$

$$y = 10 + d\sin 30° \quad \text{or} \quad y - 10 = d\sin 30° \qquad (2)$$

PROBLEM 11.193 (Continued)

Multiply Eq. (1) by sin 30° and Eq. (2) by cos 30° and subtract

$$x\sin 30° - (y - 10)\cos 30° = 0$$

$$(109.95t)\sin 30° - (12.654t + 16.1t^2 - 10)\cos 30° = 0$$

$$-13.943t^2 + 44.016t + 8.6603 = 0$$

$$t = -0.1858 \text{ s} \quad \text{and} \quad 3.3427 \text{ s}$$

Reject the negative root.

$$x = (109.95 \text{ ft/s})(3.3427 \text{ s}) = 367.53 \text{ ft}$$

$$d = \frac{x}{\cos 30°}$$

$d = 424$ ft ◀