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Q6 $\frac{5}{1\cdot 2}+\frac{5}{2\cdot 3}+\frac{5}{3\cdot 4}+\dots+\frac{5}{n(n+1)}+\dots$

$\frac{5}{n(n+1)} \Rightarrow$ كسور جزئية $\Rightarrow \frac{5}{n(n+1)}=\frac{A}{n}+\frac{B}{n+1} \Rightarrow 5=A(n+1)+B(n) \Rightarrow n=0 \to A=5$

$\Rightarrow n=-1 \to B=-5$

$\frac{5}{n(n+1)}=\frac{5}{n}-\frac{5}{n+1} \Rightarrow S_n=\left(5-\cancel{\frac{5}{2}}\right)+\left(\cancel{\frac{5}{2}}-\cancel{\frac{5}{3}}\right)+\left(\cancel{\frac{5}{3}}-\cancel{\frac{5}{4}}\right)+\cancel{\dots}+\left(\cancel{\frac{5}{n}}-\frac{5}{n+1}\right)$

$\Rightarrow S_n=5-\frac{5}{n+1} \Rightarrow \lim_{n\to\infty} S_n=\lim_{n\to\infty}\left(5-\frac{5}{n+1}\right)=5-5\boxed{\overset{\to 0}{\lim \frac{1}{n+1}}}=5$

Hence, $\sum_{n=1}^{\infty}\left(\frac{5}{n(n+1)}\right)$ converge to 5 by n^{th} partial sum Test

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Q14 $\sum_{n=0}^{\infty}\left(\frac{2^{n+1}}{5^n}\right) \Rightarrow S_n=2+\frac{4}{5}+\frac{8}{25}+\frac{16}{125}+\dots$

$=2\left[\underset{a}{1}+\frac{2}{5}+\frac{4}{25}+\frac{8}{125}+\dots\right]$

$r=\frac{2}{5}\div 1=\frac{4}{25}\div\frac{2}{5}=\frac{2}{5}$

$\therefore \sum_{n=0}^{\infty}\left(\frac{2^{n+1}}{5^n}\right)=2\left(\frac{a}{1-r}\right)=2\left(\frac{1}{1-\frac{2}{5}}\right)=2\left(\frac{1}{\frac{3}{5}}\right)=\frac{10}{3}$

Q18 $\underbrace{\left(\frac{-2}{3}\right)^2}_{a}+\left(\frac{-2}{3}\right)^3+\left(\frac{-2}{3}\right)^4+\left(\frac{-2}{3}\right)^5+\left(\frac{-2}{3}\right)^6+\dots$ الجيومترك يبلش من $n=0$

$r=\frac{-2}{3}\in(-1,1)$ $\therefore$ The Geometric series is converg by Geometric series Test

$Sum=\frac{a}{1-r}=\frac{\left(\frac{-2}{3}\right)^2}{1+\frac{2}{3}}$

Q24 $1.\overline{414}=1.414\,414\,414\dots$

$=1+\left[\overset{a}{\frac{414}{1000}}+\frac{414}{(1000)^2}+\frac{414}{(1000)^3}+\dots\right]$

$r=\frac{1}{1000} \Rightarrow Sum=1+\frac{a}{1-r}=1+\frac{\frac{414}{1000}}{1-\frac{1}{1000}}=1+\frac{\frac{414}{1000}}{\frac{999}{1000}}=1+\frac{414}{999}=\frac{1413}{999}$

Q32 $\sum_{n=0}^{\infty}\frac{e^n}{e^n+n} \Rightarrow \lim_{n\to\infty} a_n=\lim_{n\to\infty}\frac{e^n}{e^n+n}=\frac{\infty}{\infty} \xrightarrow{\text{لوبيتال}} \lim\frac{e^n}{e^n+1} \to \frac{\infty}{\infty} \to \lim\frac{e^n}{e^n}=\lim 1=\underset{1\neq 0}{1}$ $\therefore \sum_{n=0}^{\infty}\frac{e^n}{e^n+n}$ diverge by n^{th}-Term Test

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Q39 $\sum_{n=1}^{\infty}\left(\cos^{-1}\left(\frac{1}{n+1}\right)-\cos^{-1}\left(\frac{1}{n+2}\right)\right)$

$S_n=\left(\cos^{-1}\left(\frac{1}{2}\right)-\cancel{\cos^{-1}\left(\frac{1}{3}\right)}\right)+\left(\cancel{\cos^{-1}\left(\frac{1}{3}\right)}-\cancel{\cos^{-1}\left(\frac{1}{4}\right)}\right)+\left(\cancel{\cos^{-1}\left(\frac{1}{4}\right)}-\cancel{\cos^{-1}\frac{1}{5}}\right)+\cancel{\dots}+\left(\cancel{\cos^{-1}\left(\frac{1}{n+1}\right)}-\cos^{-1}\frac{1}{n+2}\right)$

$S_n=\cos^{-1}\left(\frac{1}{2}\right)-\cos^{-1}\left(\frac{1}{n+2}\right)=\frac{\pi}{3}-\cos^{-1}\left(\frac{1}{n+2}\right) \Rightarrow \lim_{n\to\infty} S_n=\lim_{n\to\infty}\frac{\pi}{3}-\cos^{-1}\left(\frac{1}{n+2}\right)=\frac{\pi}{3}-\lim\cos^{-1}\left(\frac{1}{n+2}\right)=\frac{\pi}{3}-\frac{\pi}{2}=\frac{\pi}{6}=L$ $\therefore$ converge to $\frac{\pi}{6}$ by n^{th} partial sum Test

$\frac{\pi}{2}=\cos^{-1}(0) \leftarrow 0=\frac{1}{\infty}$

Q44 $\sum_{n=1}^{\infty}\frac{2n+1}{n^2(n+1)^2} \xrightarrow{\text{كسور جزئية}} \frac{2n+1}{n^2(n+1)^2}=\frac{A}{n^2}+\frac{B}{(n+1)^2} \Rightarrow 2n+1=A(n+1)+B(n) \Rightarrow n=0 \to A=1$

$n=-1 \to B=-1$

$\frac{2n+1}{n^2(n+1)^2}=\frac{1}{n^2}-\frac{1}{(n+1)^2} \Rightarrow S_n=\left(1-\cancel{\frac{1}{4}}\right)+\left(\cancel{\frac{1}{4}}-\cancel{\frac{1}{9}}\right)+\left(\cancel{\frac{1}{9}}-\cancel{\frac{1}{16}}\right)+\cancel{\dots}+\left(\cancel{\frac{1}{n^2}}-\frac{1}{(n+1)^2}\right)$

$S_n=1-\frac{1}{(n+1)^2} \Rightarrow \lim_{n\to\infty}\left(1-\frac{1}{(n+1)^2}\right)=1-\lim_{n\to\infty}\left(\frac{1}{(n+1)^2}\right)=1-0=1$

Q54 $\sum_{n=0}^{\infty}\frac{\cos n\pi}{5^n} \Rightarrow S_n=\cos 0+\frac{\cos\pi}{5}+\frac{\cos 2\pi}{25}+\frac{\cos 3\pi}{125}+\dots+\frac{\cos n\pi}{5^n}$

$\cos\pi n=(-1)^n$

$=\underset{a}{1}-\frac{1}{5}+\frac{1}{25}-\frac{1}{125}+\dots+\frac{(-1)^n}{5^n}$

$r=\frac{-\frac{1}{5}}{1}=\frac{\frac{1}{25}}{-\frac{1}{5}}=-\frac{1}{5}\in(-1,1)$

Geometric series $\Rightarrow$ converge since $r\in(-1,1)$

$\therefore sum=\frac{a}{1-r}=\frac{1}{1+\frac{1}{5}}=\frac{5}{6}$

Leen Sabri

Q62 $\sum_{n=1}^{\infty} \frac{n^n}{n!} = \lim_{n\to\infty} \frac{n^n}{n!} = \lim \frac{n\cdot n\, n \cdots n}{n(n-1)(n-2)\cdots 1} = \lim_{n\to\infty} \frac{n}{n} \cdot \frac{n}{n-1} \cdot \frac{n}{n-2} \cdots \frac{n}{1} = \infty$ by n^{th} test for diverg

div

Q63 $\sum_{n=1}^{\infty} \frac{2^n+3^n}{4^n} \Rightarrow \sum_{n=1}^{\infty} \frac{2^n}{4^n} + \sum_{n=1} \frac{3^n}{4^n} = \sum_{n=1}^{\infty}\left(\frac{1}{2}\right)^n + \sum_{n=1}^{\infty}\left(\frac{3}{4}\right)^n$

$S_n = \left[\underset{a}{\frac{1}{2}} + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^3 + \dots\right] + \left[\underset{a}{\frac{3}{4}} + \left(\frac{3}{4}\right)^2 + \left(\frac{3}{4}\right)^3 + \dots\right]$

$r_1 = \frac{1}{2} \in (-1,1)$ $\quad r_2 = \frac{3}{4} \in (-1,1)$

so $\sum_{n=1}^{\infty}\left(\frac{1}{2}\right)^n$ and $\sum_{n=1}^{\infty}\left(\frac{3}{4}\right)^n$ both Geometric series, $\sum_{n=1}^{\infty} \frac{2^n+3^n}{4^n} \to$ Geometric series $\Rightarrow$ converges since $r_1, r_2 \in (-1,1)$ by geometric series Test

Sum: $\frac{a_1}{1-r_1} + \frac{a_2}{1-r_2} = \frac{\frac{1}{2}}{1-\frac{1}{2}} + \frac{\frac{3}{4}}{1-\frac{3}{4}} = 1 + \frac{\frac{3}{4}}{\frac{1}{4}} = 1+3 = 4$ $\therefore$ converg to 4

Q78 $\sum_{n=0}^{\infty} (\ln x)^n \Rightarrow S_n = \underset{a}{1} + \ln x + (\ln x)^2 + (\ln x)^3 + \dots + (\ln x)^n$

$r = \ln x$ and because the series converges ther $r \in (-1,1) \Rightarrow |r| < 1 \Rightarrow -1 < r < 1 \Rightarrow -1 < \ln x < 1 \Rightarrow e^{-1} < x < e$

Sum: $\frac{a}{1-r} = \frac{1}{1-\ln x}$

Q90 $\underset{a}{1} + e^b + e^{2b} + e^{3b} + \dots = 9$

$r = e^b$

Sum $= \frac{a}{1-r} = 9 \Rightarrow \frac{1}{1-e^b} = 9 \to 1 = 9(1-e^b) \Rightarrow \frac{1}{9} = 1 - e^b \Rightarrow -\frac{8}{9} = -e^b \Rightarrow \frac{8}{9} = e^b \to \ln\frac{8}{9} = b$

Q22 $0.\bar{d} = 0.ddd\dots$ $\quad r = \frac{1}{10} \to S_n = \frac{a}{1-r} = \frac{\frac{d}{10}}{1-\frac{1}{10}} = \frac{d}{9}$

Q42 $\sum_{n=1}^{\infty} \frac{6}{(2n-1)(2n+1)} \Rightarrow \frac{6}{(2n-1)(2n+1)} = \frac{A}{2n-1} \cdot \frac{B}{2n+1} \Rightarrow 6 = A(2n+1) + B(2n-1)$

When $n = \frac{1}{2} \Rightarrow 6 = 2A \to A = 3$

$\sum_{n=1}^{\infty}\left(\frac{3}{2n-1} - \frac{3}{2n+1}\right) = 3\sum_{n=1}^{\infty}\left(\frac{1}{2n-1} - \frac{1}{2n+1}\right)$ $\quad$ " $n = -\frac{1}{2} \Rightarrow 6 = -2B \to B = -3$

$3\left[1 - \cancel{\frac{1}{3}} + \left(\cancel{\frac{1}{3}} - \cancel{\frac{1}{5}}\right) + \left(\cancel{\frac{1}{5}} - \cancel{\frac{1}{7}}\right) + \left(\cancel{\frac{1}{7}} - \cancel{\frac{1}{9}}\right) \dots + \left(\cancel{\frac{1}{2n-1}} - \frac{1}{2n+1}\right)\right]$

$3\left[1 - \frac{1}{2n+1}\right] \Rightarrow 3\lim\left[1 - \frac{1}{2n+1}\right] = 3(1-0) = 3$