MEASUREMENTS
ENME 321

Ahmed Abu Hanieh

PREPARATION OF LABORATORY REPORTS
FORMAL REPORT

FORMAL REPORT

The report should be typewritten including graphs and figures by a computer. Use double spacing with 12 font. Spell check your report.

The following sections are to be included in this order:

TITLE PAGE

- Course and section Number
- Number and title of the experiment
- Student's name
- Names of group members
- Date the experiment was performed
- Date the report is submitted.

ABSTRACT

This is a stand-alone summary of the report. It should include objective, what was done, results and conclusions. It should be clear, informative, concise and short. It should not make any references to the body of the report or to the appendices. An abstract should not exceed one page.

OBJECTIVE

The objective of the experiment should be stated on a separate heading OBJECTIVE. State the objective clearly, and make it relevant to the carried experiment

THEORY

In this section state and explain any equations or theoretical principles and assumptions that were used in the experiment and the analysis. Define all parameters used. To find this information refer to textbooks, notes etc. Write equations using equation writer in the word processor.

EXPERIMENTAL

Give a detailed description of how you accomplished the experiment. This should include equipment used in the experiment as well as how it was used. The description should have sufficient detail so that another experimenter could duplicate your efforts. Name all measured quantities and how they are measured. Use sketches, diagrams, or photos to describe the experimental set-up. Label the main components.

CALCULATIONS

Demonstrate how you performed the calculation made in the experiment. Include a detailed sample calculation including used formulas and constants. Show the generic calculations to support all your work. Provide any computer or calculator program listings, along with sample input and output.

RESULTS

Summarize your results in an introductory sentence. Relate your results to your objective. Present the results in the easiest way for your reader to understand and how the experiment's hand out requires:

Graphs, tables, figures, etc. Spreadsheets are often a good approach. See section on preparation of graphs. All tables and figures must be referenced in the text; use a numbering system for identification of each one and each should have a distinguishable title.

DISCUSSION OF RESULTS

Explain the results of the experiment; comment on the shapes of the curves; compare results with expected results; give probable reasons for discrepancies from the theory; answer any questions outlined in the instructions and solve any problems that may have been presented. Tell why things happened, not only that they did happen.

Comparisons should include numerical values and corresponding error percentages where relevant.

Do not present calculations and formulas in this section. Your calculations should be detailed in the Appendices under SAMPLE CALCULATIONS. Formulas should be discussed in the THEORY section.

CONCLUSIONS

State your discoveries, judgments and opinions from the results of this experiment. Summarize your primary results in comparison with theory in two or three sentences. These should answer the objective of the experiment. Make recommendation for further study. Suggest ways to improve the experiment.

Consider that in the real world. Information like that in the RESULTS and CONCLUSIONS will be all that upper management will want to receive.

APPENDICES

- **DATA TABLES**

Data tables are for the convenience of the extremely interested reader. These tables may contain any additional comparisons or calculations that you have prepared and were not included in the RESULTS section which may contain only summaries of your work. Data Tables are the place to show everything that you did.

- **RAW DATA SHEETS**

Data sheets must be completed in ink and signed by the instructor at the completion of the laboratory period.

In the case of an error, line through the mistake, initial the mistake, and continue. Record the name of the recorder and the group members on the raw data sheets.

REFERENCES

- List any book or publication that you have referenced in your report. Provide titles, authors, publisher, date of publication, page number, Website addresses etc.
- Book reference: Author last name, Author first name. Book's title. Publisher and city of publication, year of publication.
- Journal reference: Author last name, Author first name. Paper title, Name of journal, volume, pages, year.
- Internet reference: Site location (http://www.) and retrieved date.

PREPARATION OF GRAPHS

- **COORDINATE AXES**

Draw the axes of coordinates on the cross- sectioned part of the sheet, far enough in from the margin to leave room for inserting the scales and their identifications between the edges of the cross- sectioning and the axes.

- **SCALES**

Start all linear scales at (0,0) unless such a procedure would obscure the presentation of data. Of course, this is not possible when using log scales. The units on the major divisions of log scales should be powers of 10. Choose scales of 1, 2, 4, or 5 units per centimeter, or any decimal multiples, such as 0.1, .002 and 400. Proper choice of scales is important.

Guiding principles

- Utilize a good portion of the graph sheet area; do not squeeze curves into one corner.
- Do not unduly extend the scales. Have the scales readable to the precision of the instruments from which data was taken. Further extension of the scales only scatters the data points, emphasizing the experimental error.
- Keep in mind the purpose of the graph. Avoid using scales that hide the real meaning or fail to show the intended relationships.
- Letter in the scale numerals along the axes, putting the abscissa scale beneath the horizontal axis at appropriate intervals. Set all numerals on either axis in a vertical position as viewed from the bottom page.
- Put major and minor ticks in the axis, use as much as needed to make graph clear and understandable.

SCALE LEGEND

Place legend beneath the abscissa scale so as to be read from the bottom of the page. Place name of the ordinate legend to the left of the ordinate scale so as to be read from the right hand of the page. If more than one ordinate scale is used, place each ordinate legend immediately adjacent to the corresponding scale. Use descriptive titles followed by dimensional units, e.g., Temperature (ºC).

RULES

- **DATA POINTS**

Indicate data points by small circles or appropriate geometric symbols, except in the case of correction curves for instruments where the plotted points are not emphasized.

- **CURVES**

With EXCEL Software, draw smooth curves whose positions are governed by the plotted points. The curves should not necessarily pass through every point; you may use trend curves from the EXCEL. Only in the case of perfectly smooth data will all the points lie on the curve. Draw curves with ink or with computer printer and software such as **EXCEL**.

On the graphs show both the experimental data point and the trend curves or data fitting curves.

INDEPENDENT VERSUS DEPENDENT VARIABLES

Plot the independent variable horizontally along bottom of the graph. Plot the dependent variable vertically. The dependent variable is usually mentioned first, e.g., "PRESSURE VERSUS TEMPERATURE" where pressure is the ordinate. Label axis as normally they appear in literature or books, does not exchange the axis.

PREPARATION OF TABLES

Give a number and a title for each table, place them above the table. Give names including units for all columns or rows in the table. Use table forms with white background.

TECHNICAL REPORT

In the technical report include only the following items:

- Title page
- Abstract
- Objectives
- Calculations
- Results
- Discussion of results

Error Calculation

Xm = measured value

Xt = true value

Error = e = Xm – Xt

u = uncertainty

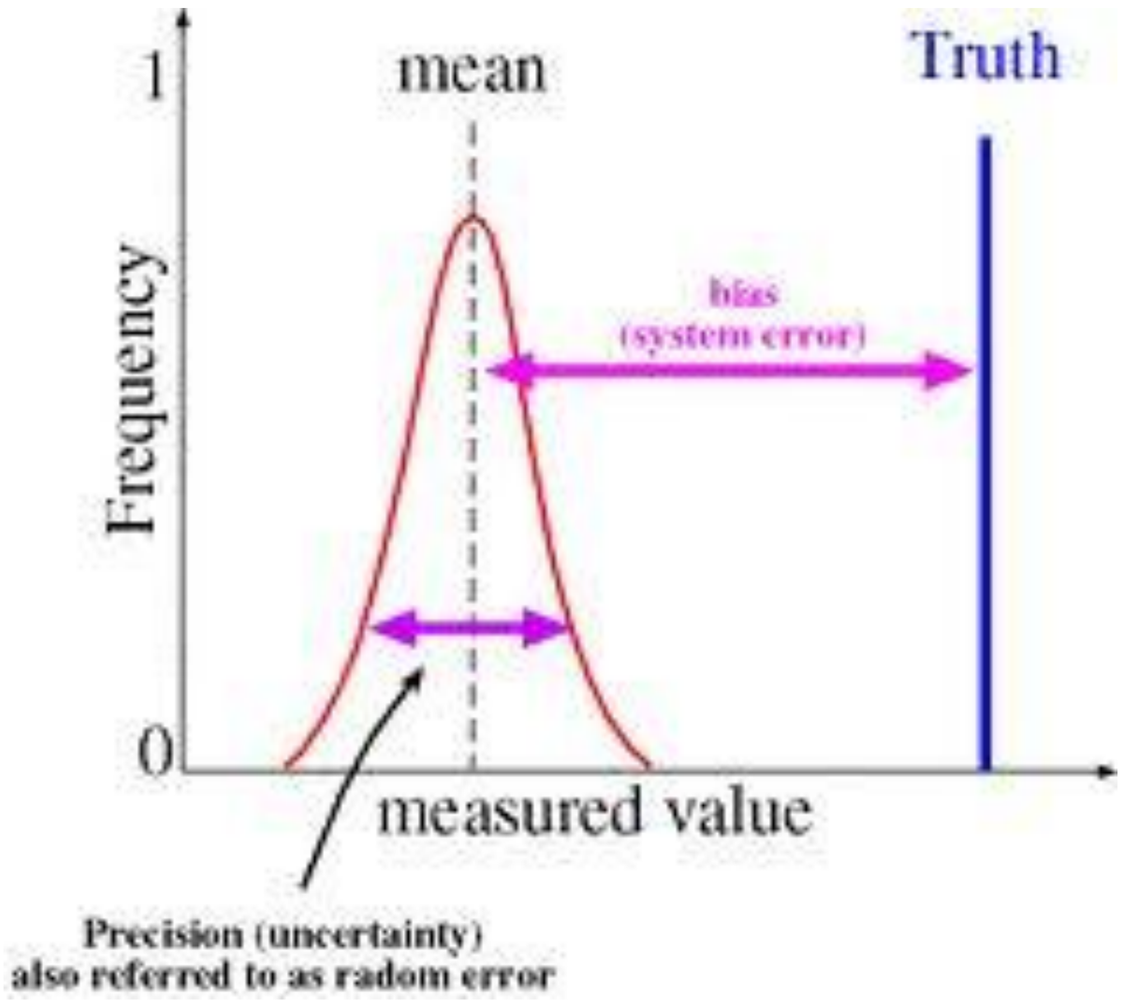
$-u < e < +u$

$Xm - u < Xt < Xm + u$

Error Types

- Bias error (Systematic error)
- Precision error (Random error)
- Illegitimate error
- Mixed error

Error Types



1
0
mean
Truth
Frequency
bias
(system error)
measured value
Precision (uncertainty)
also referred to as radom error

Error sources

Bias (systematic) error

1. Calibration
2. Human
3. Defective equipment
4. Loading
5. Limited resolution

Error sources

Precision (Random) error

1. Human
2. Disturbance
3. Change of conditions
4. Insufficient sensitivity

Error sources

Illegitimate error

1. Mistakes during experiment
2. Computational error after experiment

Mixed error

1. Friction, backlash and Hysterisis
2. Calibration and environment conditions
3. Change of procedure

Error calculation

- x = measured value
- X' = mean

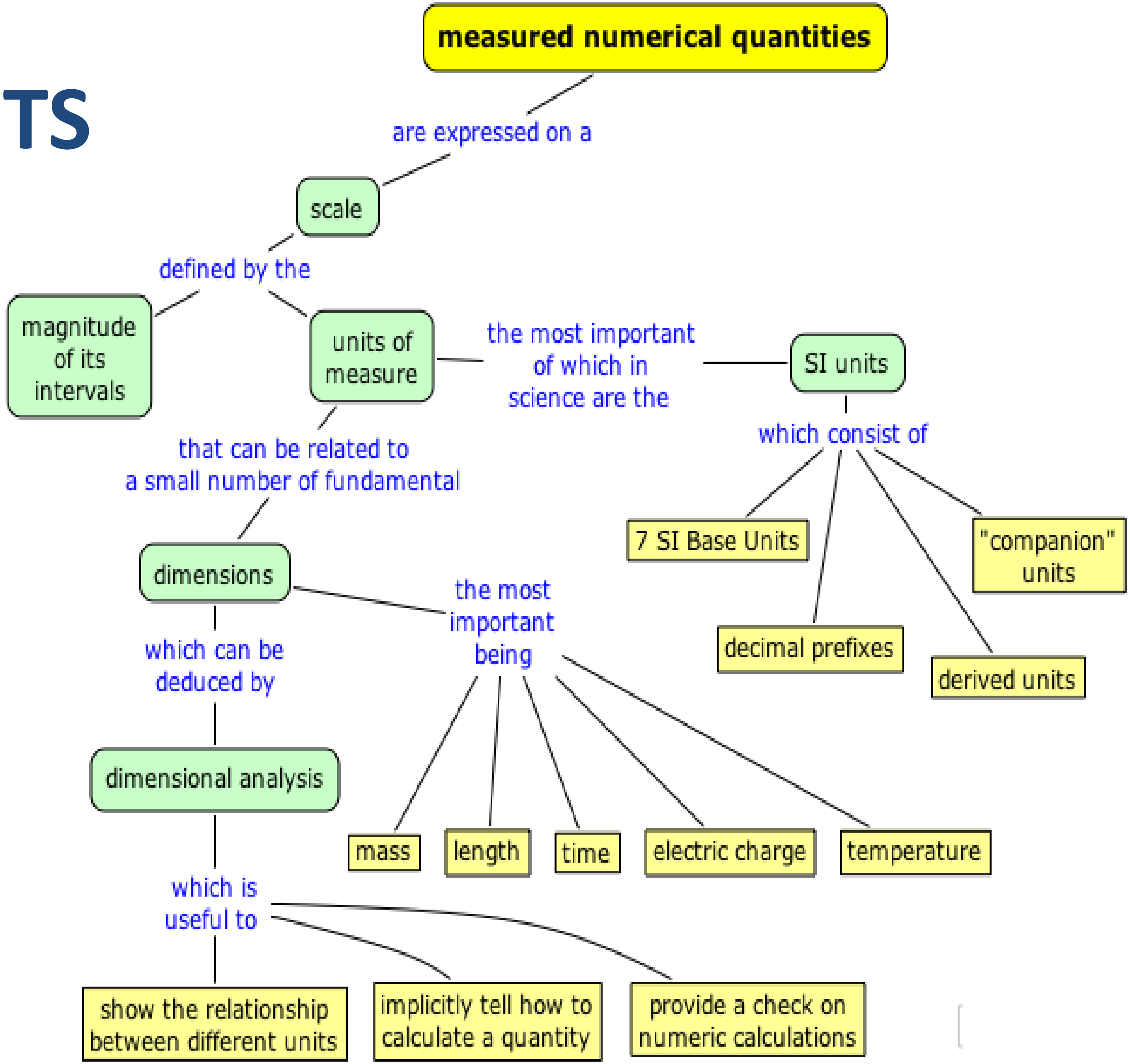
- Xrms = Root Mean Square

$$Xrms = \sqrt{\frac{1}{N}\sum x^2}$$

- STDEV = Standard deviation

$$STDEV = \sqrt{\frac{1}{N}\sum (x - X')^2}$$

UNITS


measured numerical quantities
are expressed on a
scale
defined by the
magnitude of its intervals
units of measure
the most important of which in science are the
SI units
which consist of
7 SI Base Units
decimal prefixes
derived units
"companion" units
that can be related to a small number of fundamental
dimensions
the most important being
mass
length
time
electric charge
temperature
which can be deduced by
dimensional analysis
which is useful to
show the relationship between different units
implicitly tell how to calculate a quantity
provide a check on numeric calculations

SI Base Units

In principle, any physical quantity can be expressed in terms of only seven ***base units***

length	**meter**	**m**
mass	kilogram	kg
time	second	s
temperature (absolute)	kelvin	K
amount of substance	mole	mol
electric curent	ampere	A
luminous intensity	candela	cd

Decimal Fractions

prefix	abbreviation	multiplier	--	prefix	abbreviation	multiplier
peta	P	10^{18}		deci	s	10^{-1}
tera	T	10^{12}		centi	c	10^{-2}
giga	G	10^{9}		milli	m	10^{-3}
mega	M	10^{6}		micro	µ	10^{-6}
kilo	k	10^{3}		nano	n	10^{-9}
hecto	h	10^{2}		pico	p	10^{-12}
deca	da	10		femto	f	10^{-15}

Derived Units

The ***dimensions*** of a unit are the powers which M, L, T, Q and Θ must be given in order to express the unit

Volume:

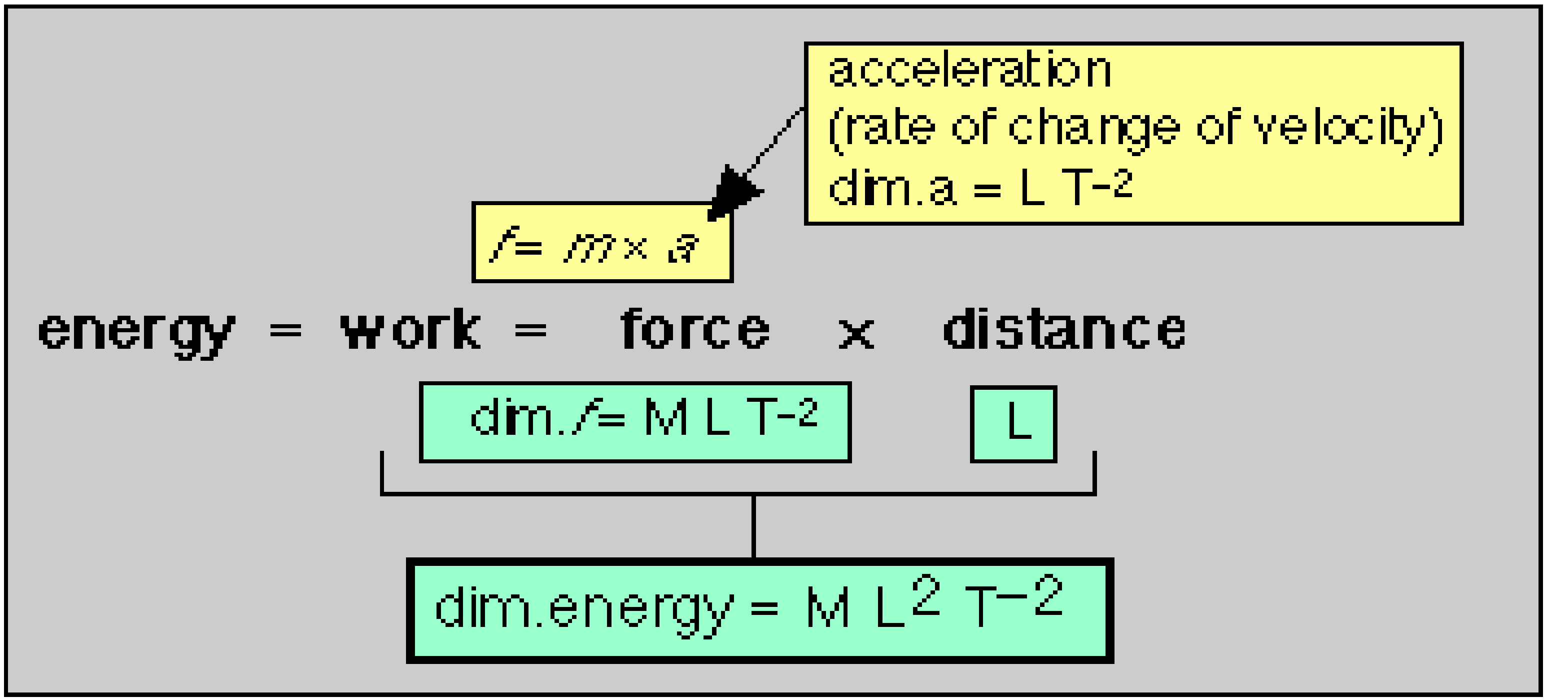
$V = x \cdot y \cdot z$

dim.$V = M^0 L^3 T^0 Q^0 \, \Theta^0 \; = \; L^3$

Example

Find the dimensions of ***energy***

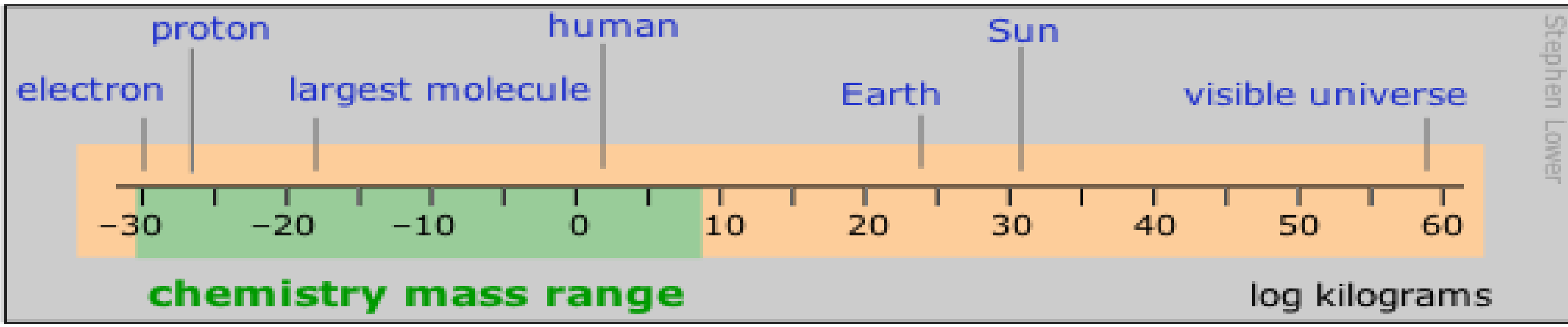
Solution:

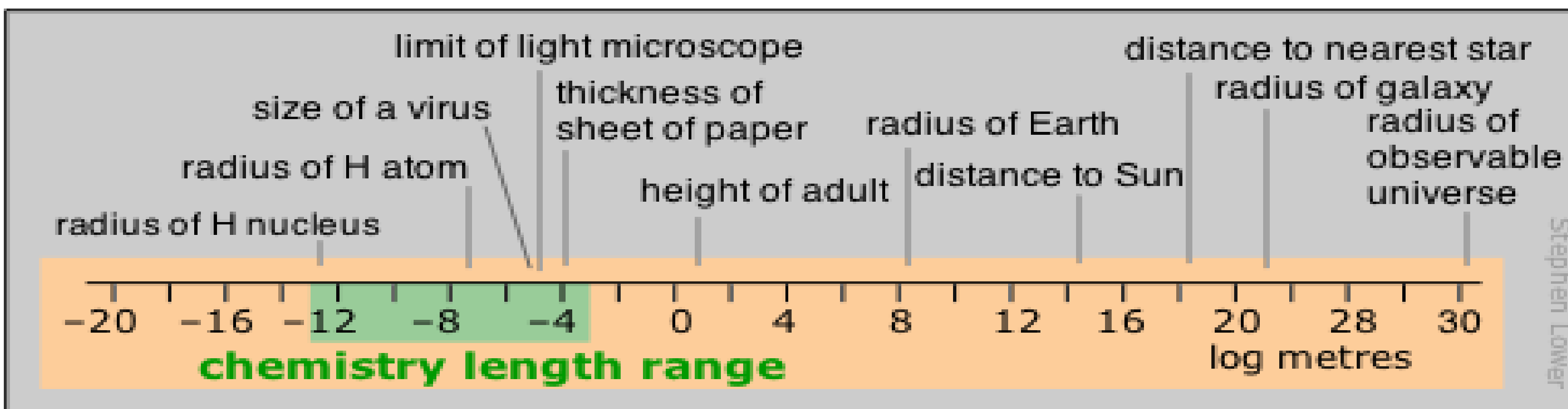


Some derived units

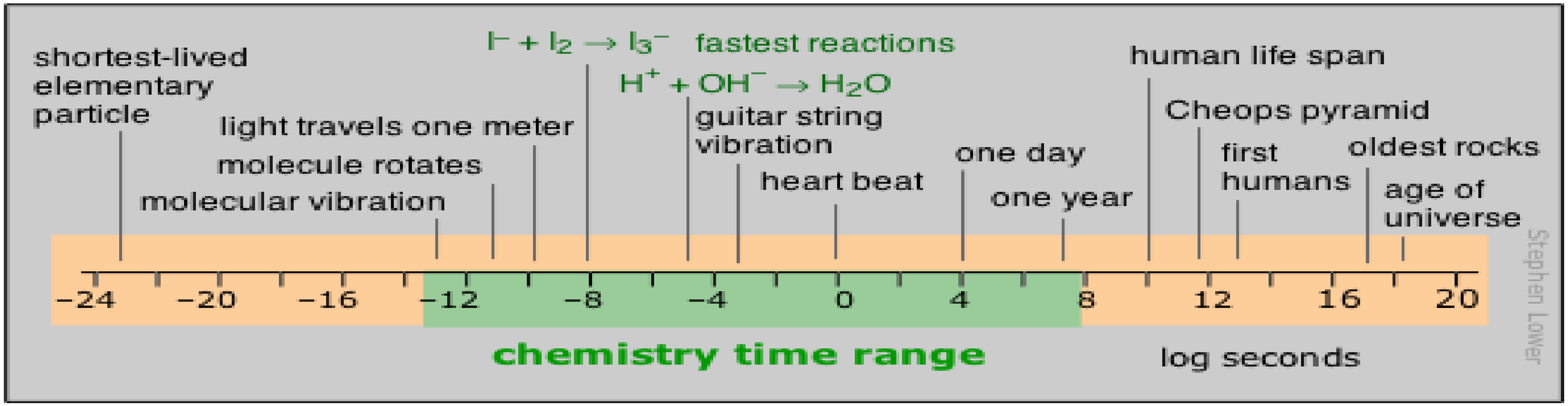
quantity	SI unit, other typical units
electric charge	coulomb
mass	kilogram, gram, metric ton, pound
length	meter, foot, mile
time	second, day, year
volume	liter, cm^3, quart, fluidounce
density	kg m^{-3}, g cm^{-3}
force	newton, dyne
pressure	pascal, atmosphere, torr
energy	joule, erg, calorie, electron-volt
power	watt
electric potential	volt
electric current	ampere

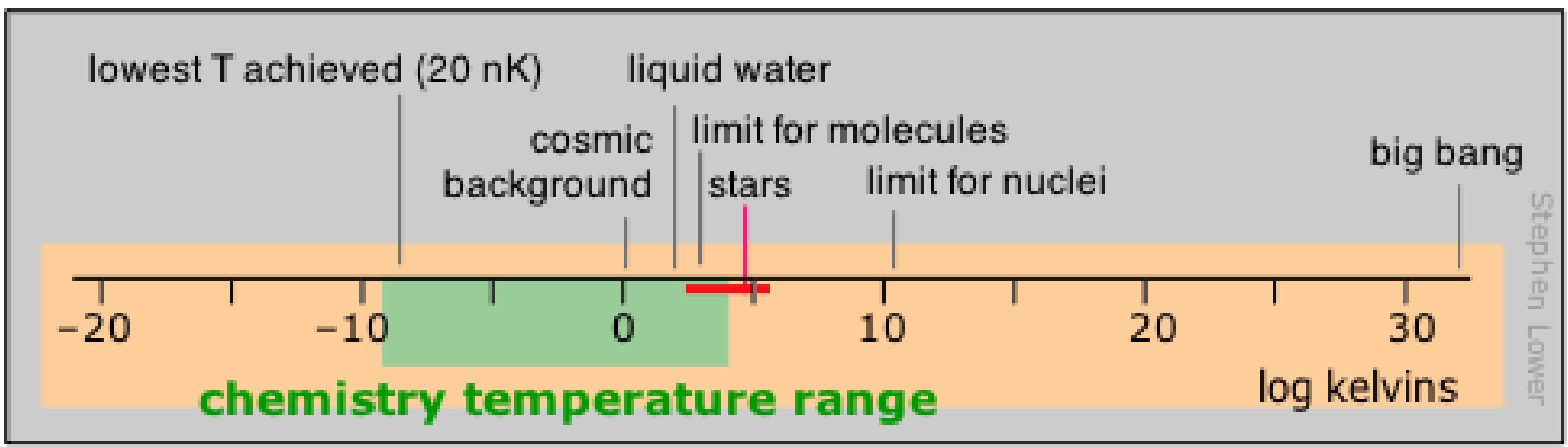
Ranges of Units



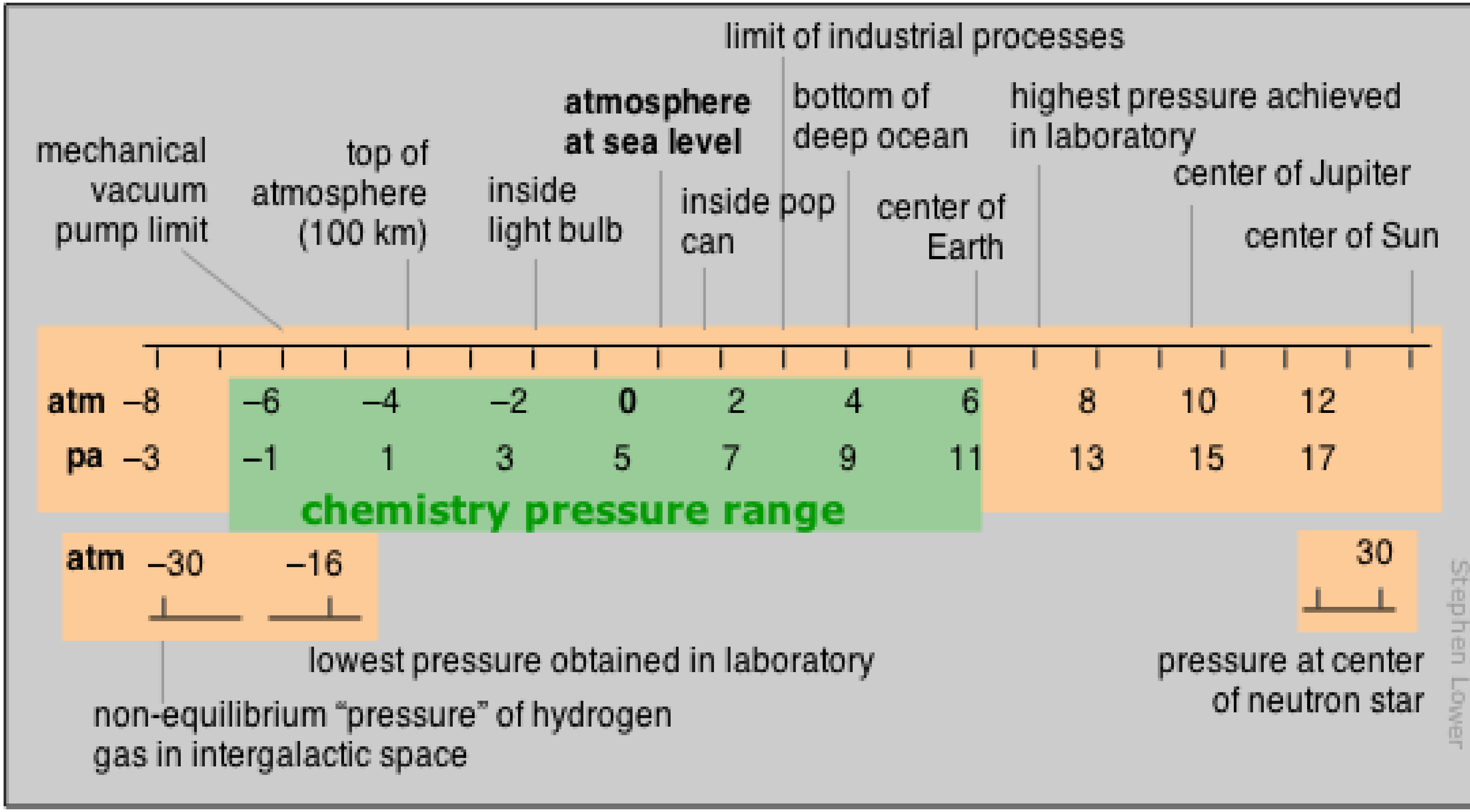


Ranges of Units





Ranges of Units



Measurement Theory

Measurement: A quantitative comparison between a predefined standard and a measurement.

NIST = National Institute of Standard and Technology

NBS = National bureau of Standards

ISO = International Standardization Organization

ANSI = American National Standard Institute

General Measurement

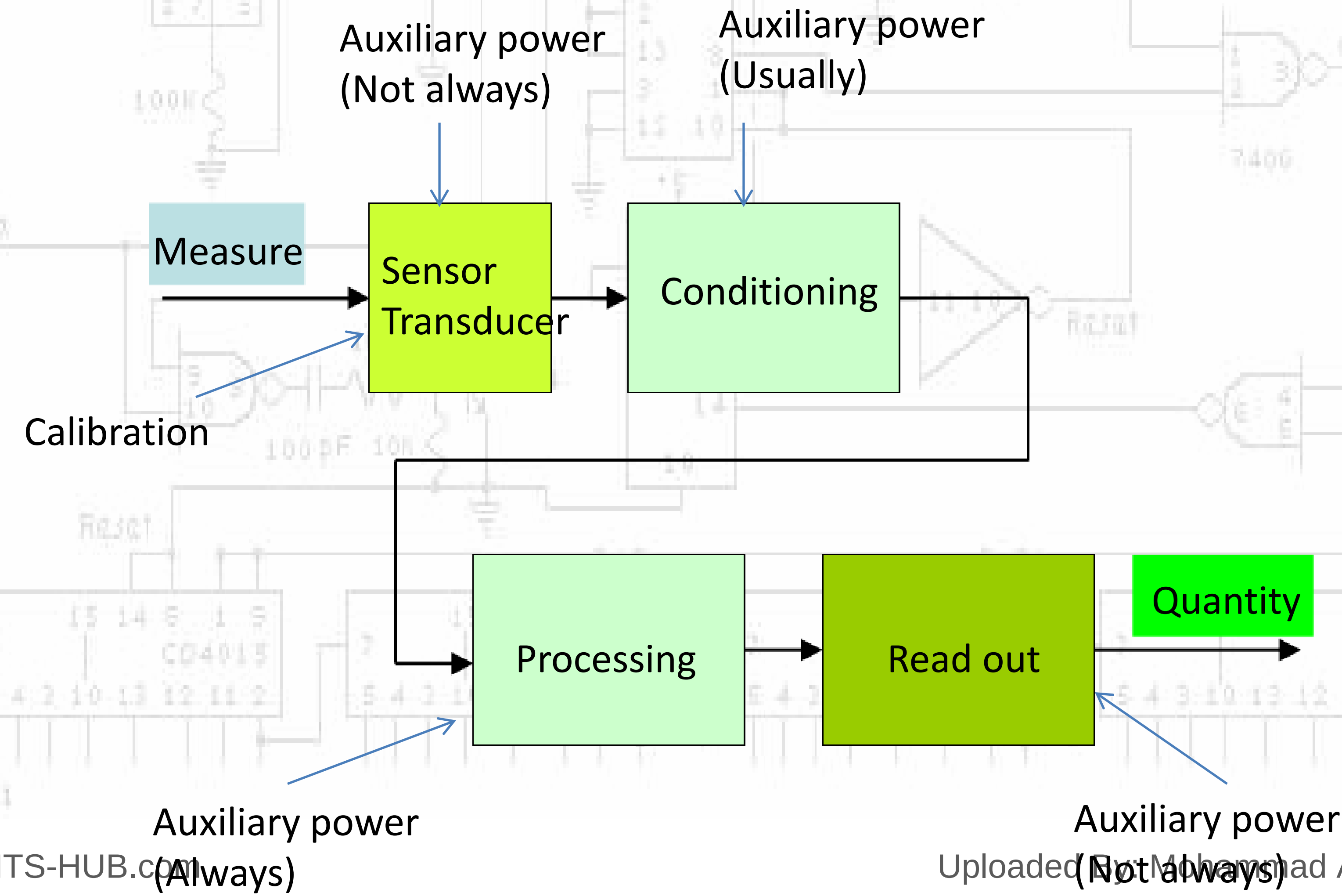
Three Phase stage measurement system:

1. Sensing (Transducing)

2. Conditioning and Processing

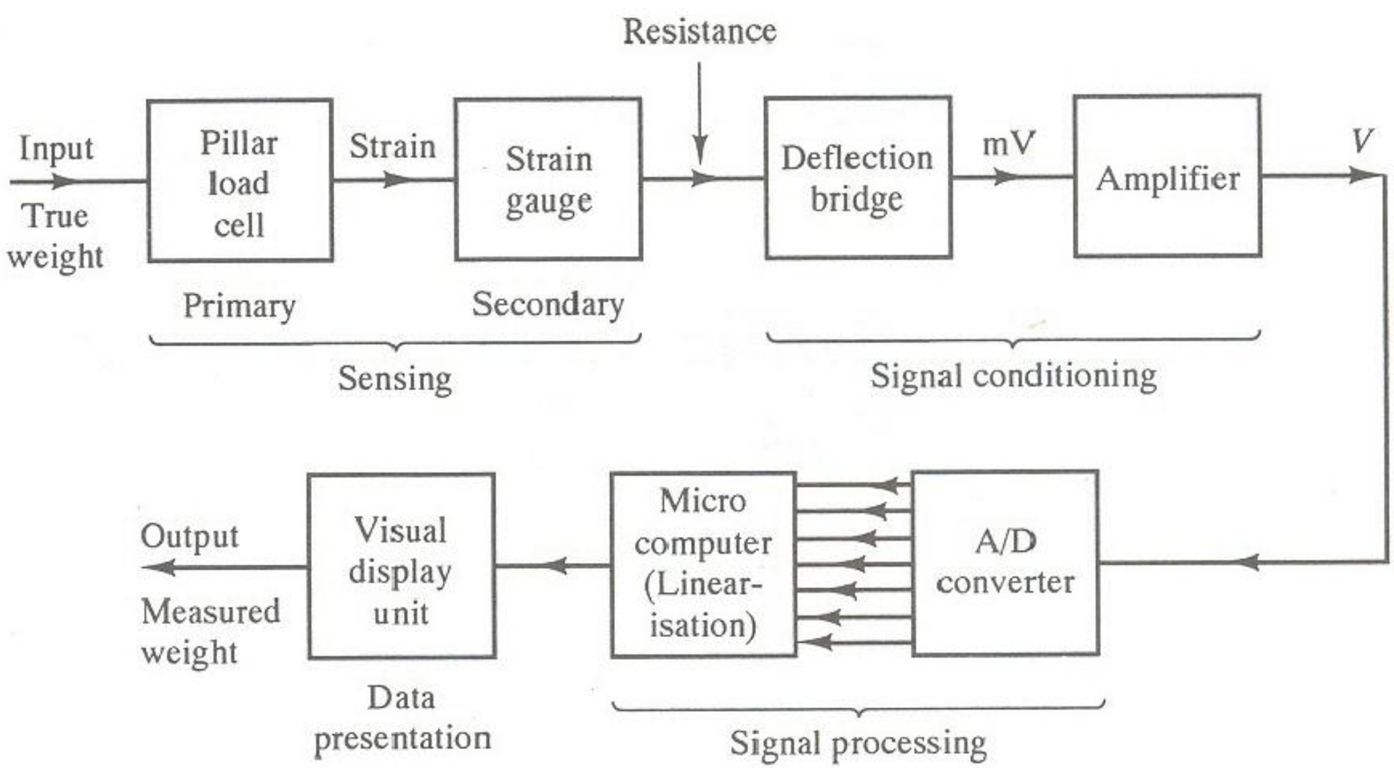
3. Read out

General measurement



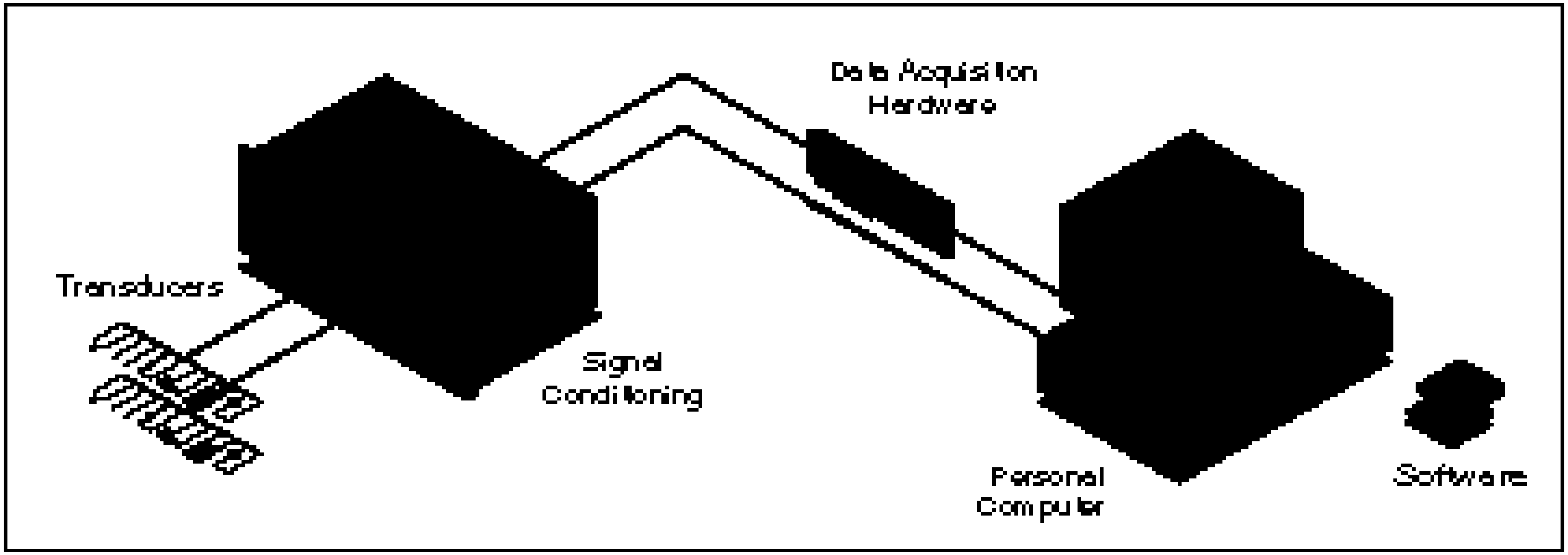
Example

Weight measurement by load cell

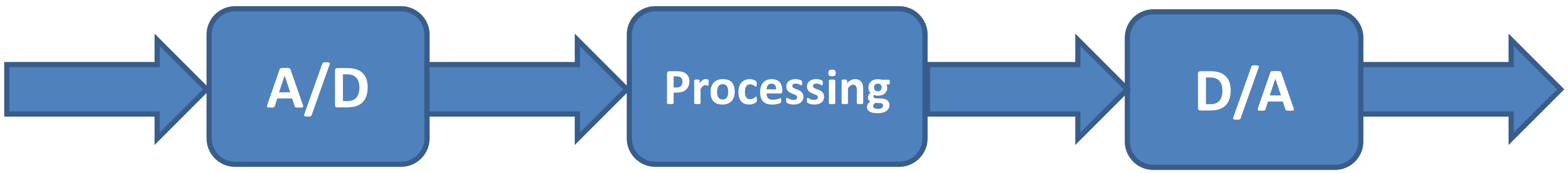


Data Acquisition

- The personal computer
- Transducers
- Signal conditioning
- DAQ hardware
- Software



Data processing



Analog Input

Digital signal

Digital signal

Analog output

Static
Dynamic

Nyquist–Shannon sampling theorem

Sampling:

is the process of converting a signal (for example, a function of continuous time or space) into a numeric sequence (a function of discrete time or space).

The theorem states that exact reconstruction of a continuous-time baseband signal from its samples is possible if the signal is band limited and the sampling frequency is greater than twice the signal bandwidth. The theorem also leads to an effective reconstruction formula.

Sampling Theory

If X(*f*) is the continuous Fourier transform of that signal:

$$X(f) \stackrel{\text{def}}{=} \int_{-\infty}^{\infty} x(t)\, e^{-i2\pi f t}\, \mathrm{d}t.$$

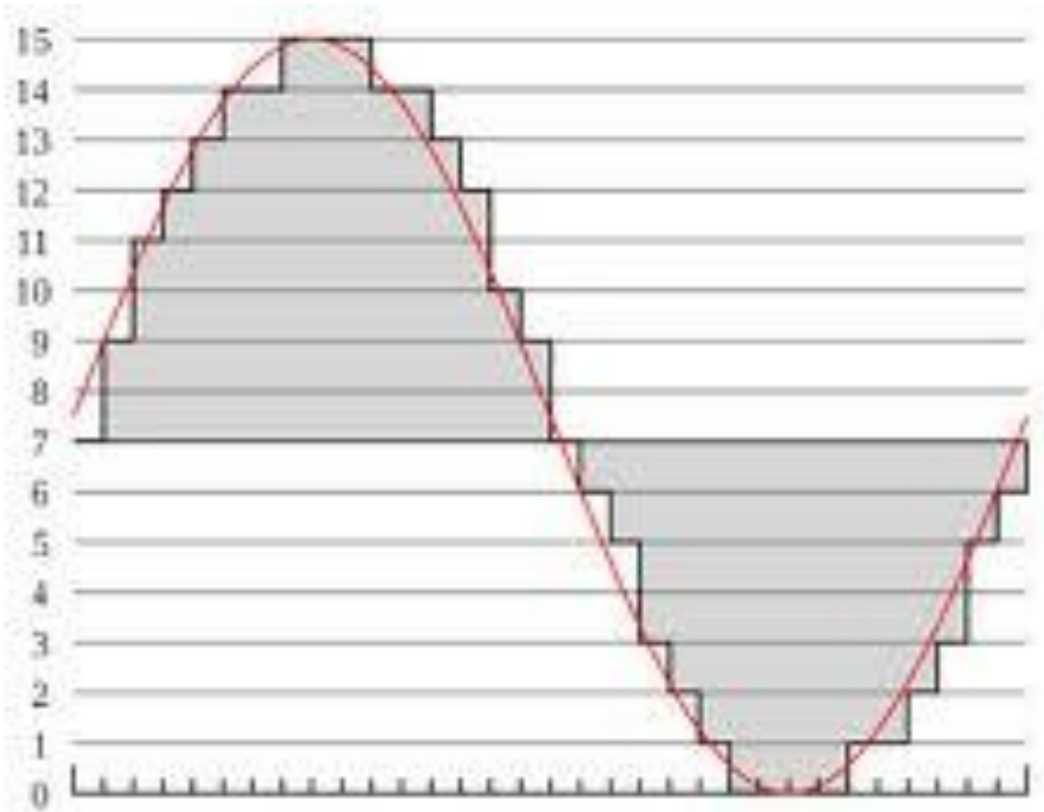
The signal $x(t)$ is said to be bandlimited to a one-sided baseband bandwidth, B, if

$$X(f) = 0 \quad \text{for all} \quad |f| > B,$$

a uniform sampling rate f_s (in samples per unit time)

$$f_s > 2B. \qquad\qquad T \stackrel{\text{def}}{=} \frac{1}{f_s},$$

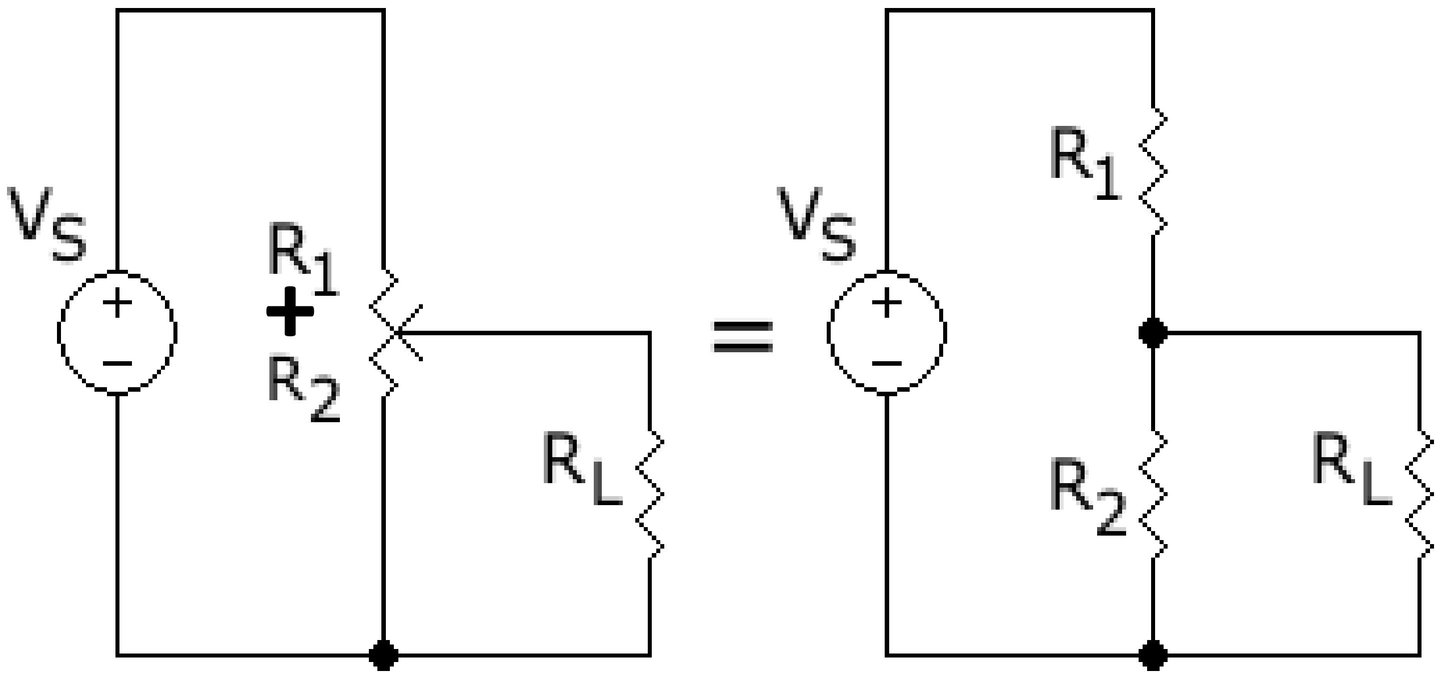
Sampling frequency (time)



Each digitized sample of audio is assigned a value that corresponds to the amplitude of the analog wave.

Potentiometer

Potentiometer



Potentiometer

The voltage across R_L can be calculated by:

$$V_\mathrm{L} = \frac{R_2 R_\mathrm{L}}{R_1 R_\mathrm{L} + R_2 R_\mathrm{L} + R_1 R_2} \cdot V_s.$$

If R_L is large compared to the other resistances (like the input to an operational amplifier), the output voltage can be approximated by the simpler equation:

$$V_\mathrm{L} = \frac{R_2}{R_1 + R_2} \cdot V_s.$$

(dividing throughout by R_L and cancelling terms with R_L as denominator)

Linear Variable Differential Transformer (LVDT)

- **Linear variable differential transformer**
- **Electrical transformer measuring linear displacement**

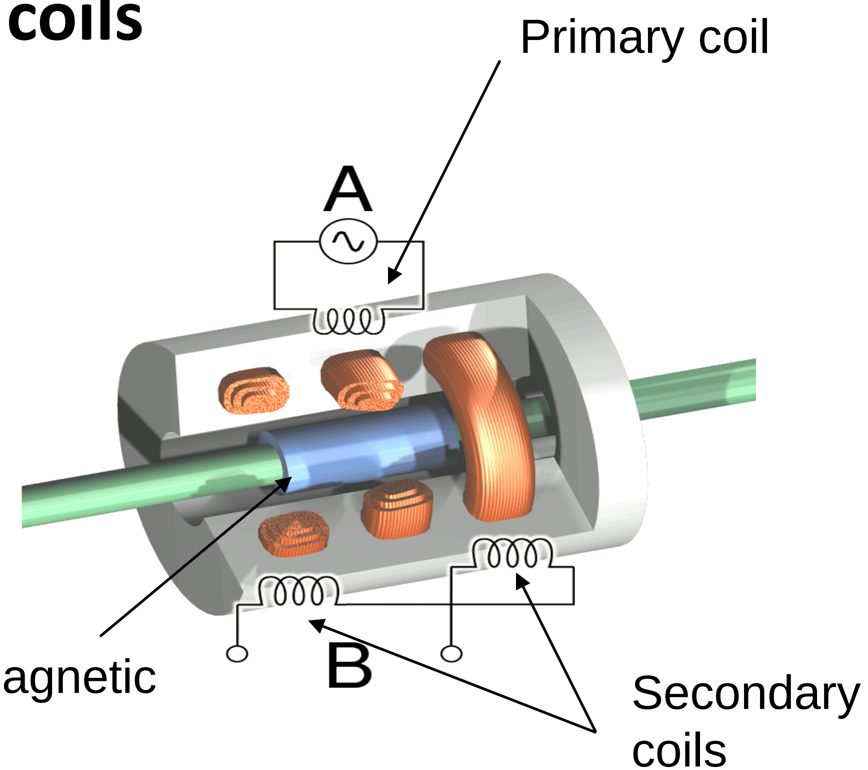
Construction of LVDT

- **One Primary coil**
- **Two symmetric secondary coils**
- **Ferromagnetic core**

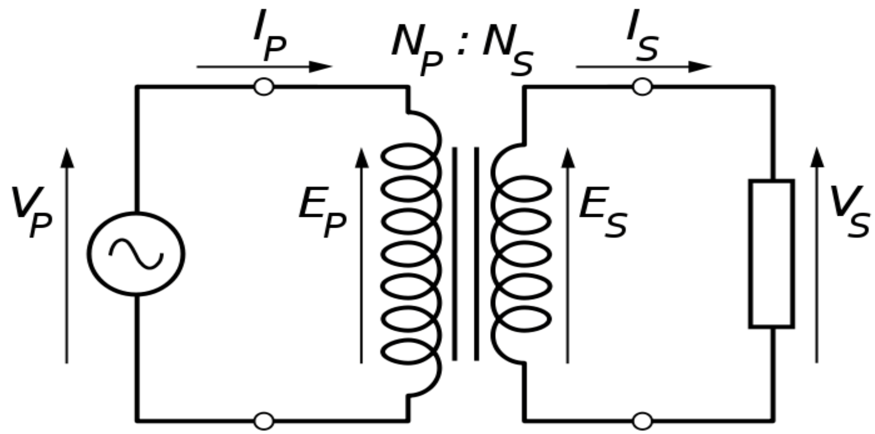
•The primary coil is energized with a A.C.

•The two secondary coils are identical, symmetrically distributed.

•The two secondary coils are connected in opposition



Recall of conventional transformer



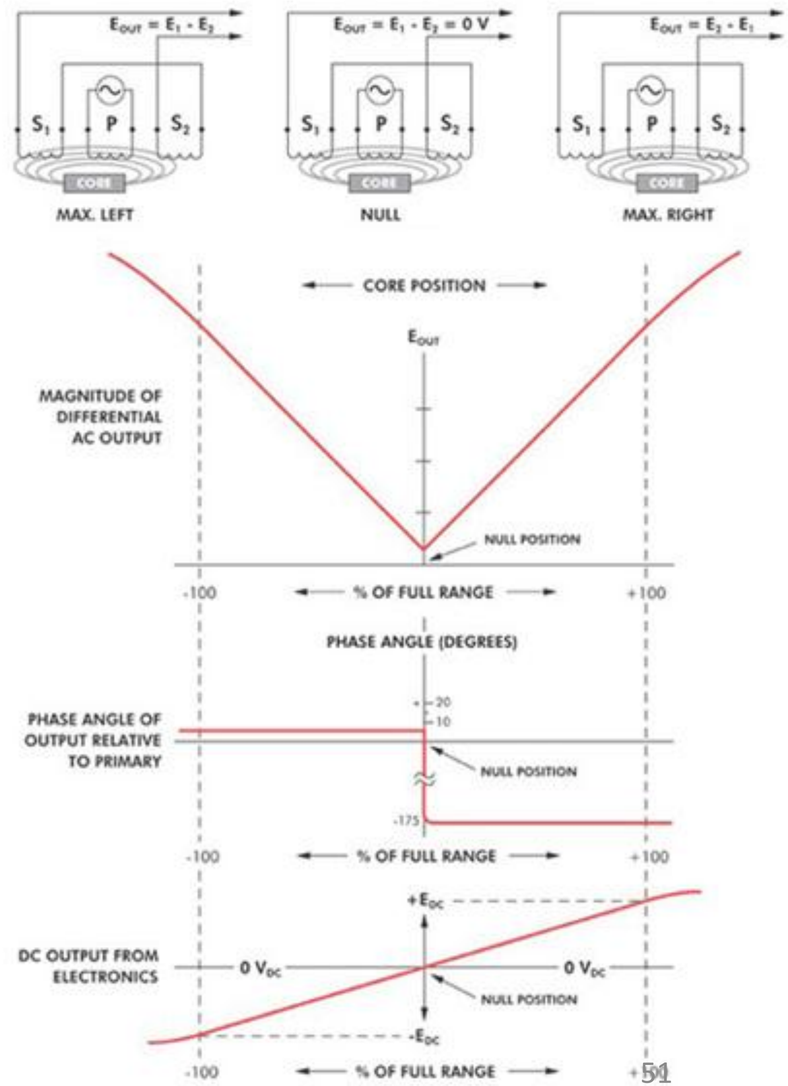
$$\frac{V_P}{V_S} = \frac{I_S}{I_P} = \frac{N_P}{N_S}$$

- Mutual induction
- the secondary voltage proportional to the primary voltage
- The transformer core is fixed
- Energy transferred is high

How LVDT works

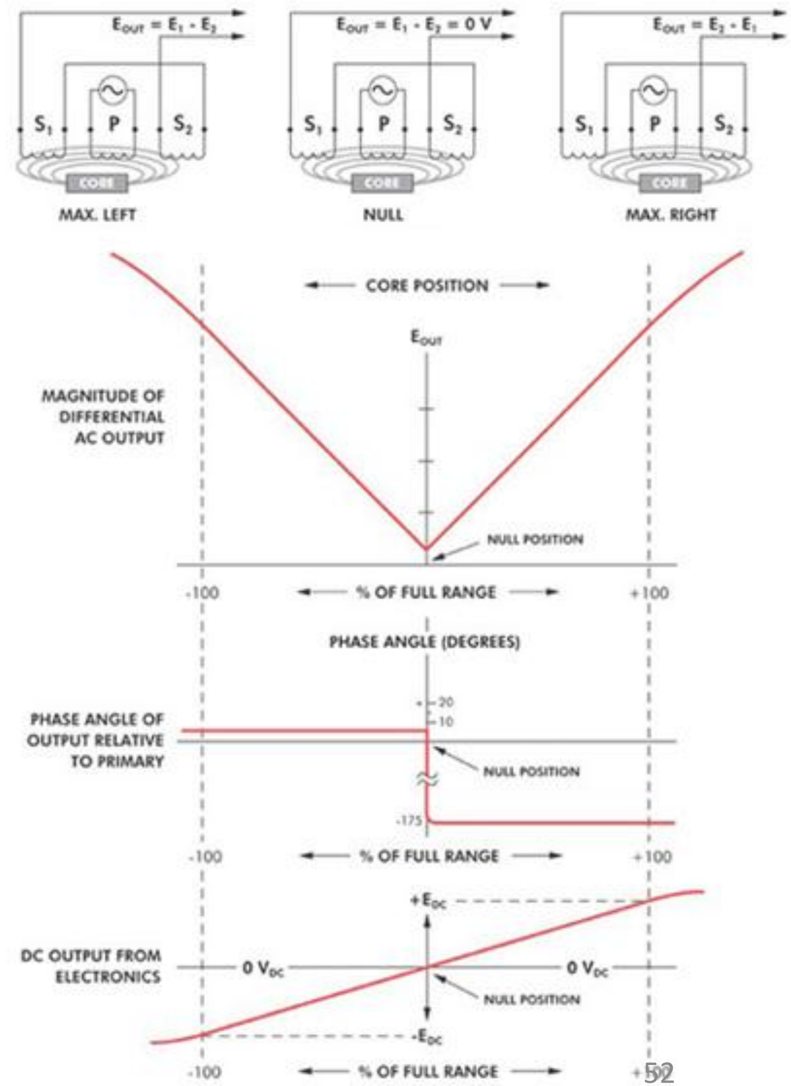
- If the core is located midway between S1 and S2

- Equal flux is coupled to each secondary.
- Voltage E1 and E2 are equal.
- The differential voltage output, (E1 - E2), is zero.
- This core position is called null point.



How LVDT works

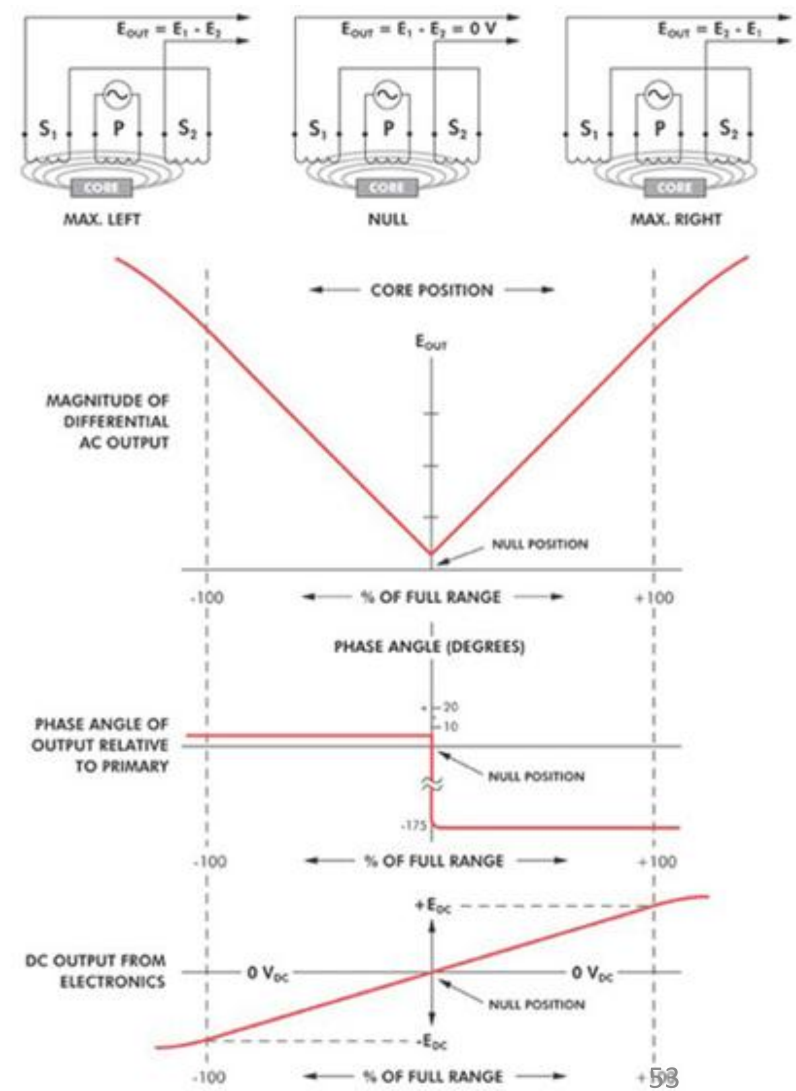
- If the core is moved closer to S1 than to S2
- More flux is coupled to S1 than S2 .
- The induced voltage E1 is increased while E2 is decreased.
- The differential voltage is (E1 - E2).



How LVDT works

- If the core is moved closer to S2 than to S1

- More flux is coupled to S2 than to S1 .
- The induced E2 is increased as E1 is decreased.
- The differential voltage is (E2 - E1).

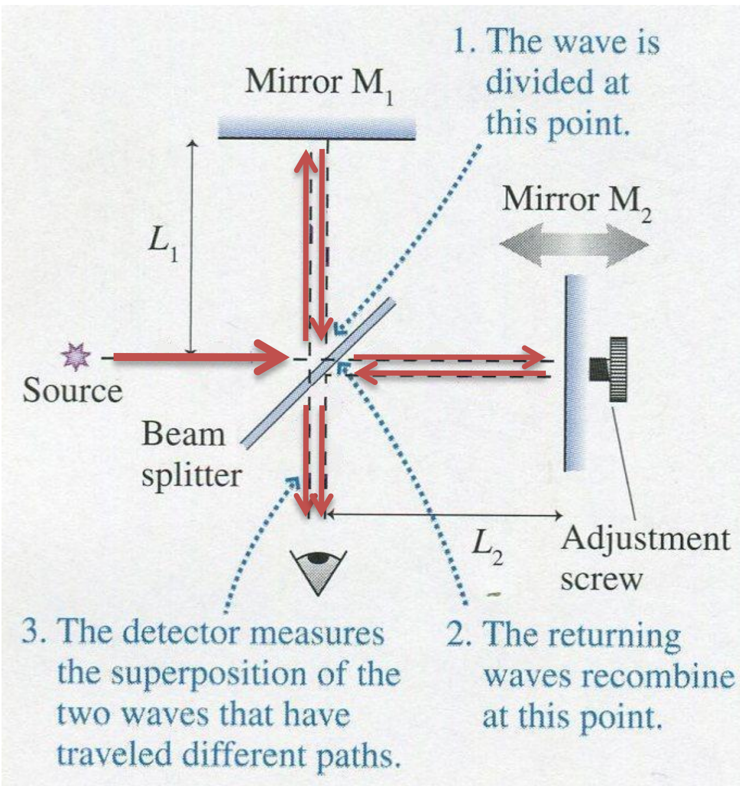


Laser Interferometer

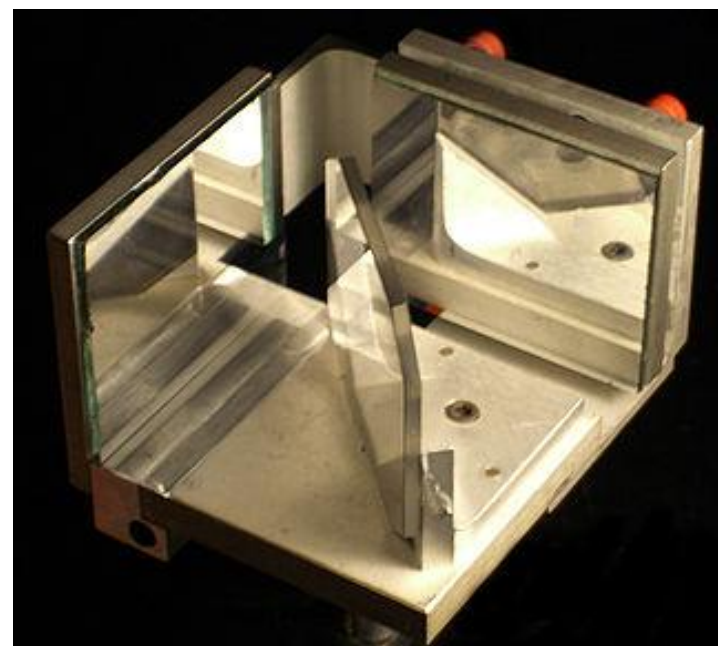
- Laser Interferometer:
  - the instrument used for high precision measurements (distance, angles…. etc.)
  - it uses interferometry as the basis for measurement.
  - it uses the very small, stable and accurately defined wavelength of laser as a unit of measure.

Principle of Michelson Interferometer

- Michelson Interferometer



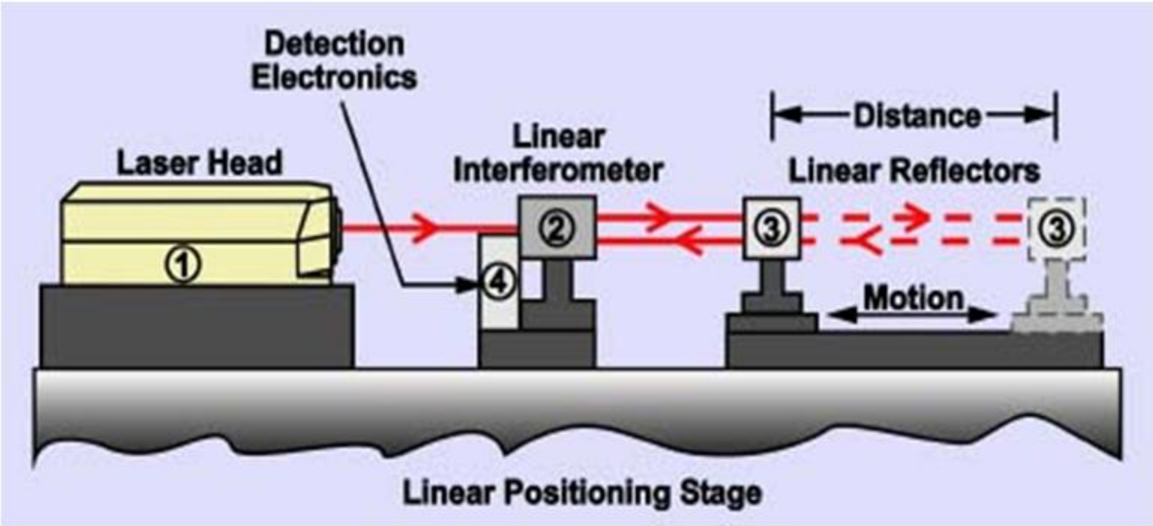
1) Separation
2) Recombination
3) Interference



A Michelson Interferometer for use on an optical table

Applications

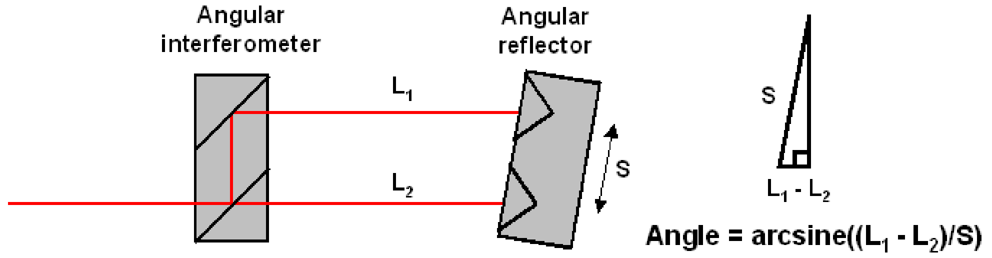
- Measurement of Distance
  - 1) frequency stabilized He-Ne laser tube
  - 2) combination of beam-splitter and retroreflector
  - 3) a moving retroreflector
  - 4) detection electronics



Aerotech's LZR3000 Series Laser Interferometer System

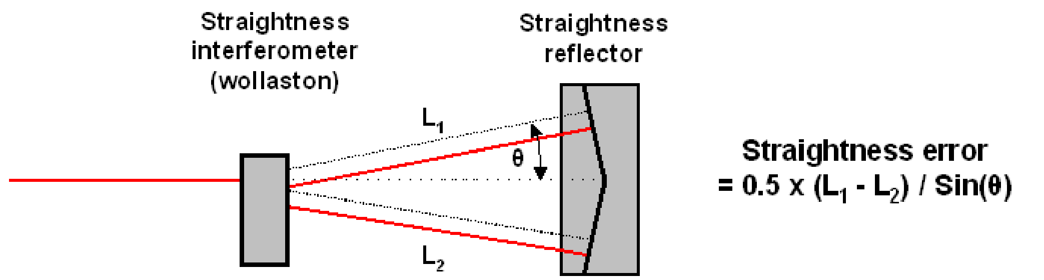
Applications

- Other Applications
  - Measure angles, flatness, straightness, velocity and vibrations, etc.



Angular measurement with an interferometer.

Rearrangements of the light paths

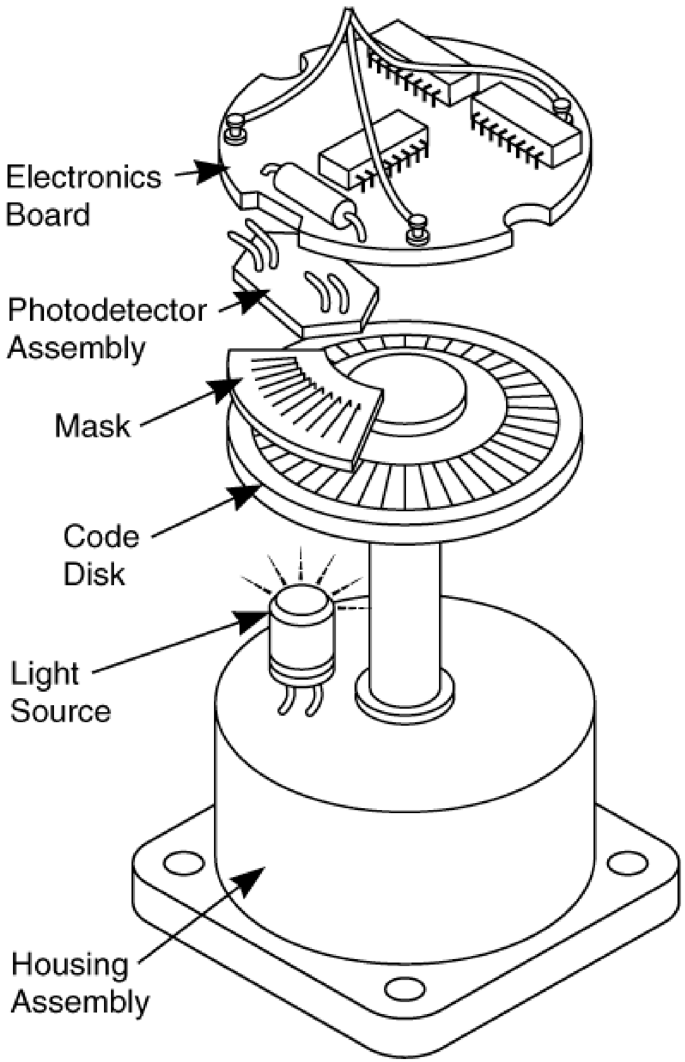


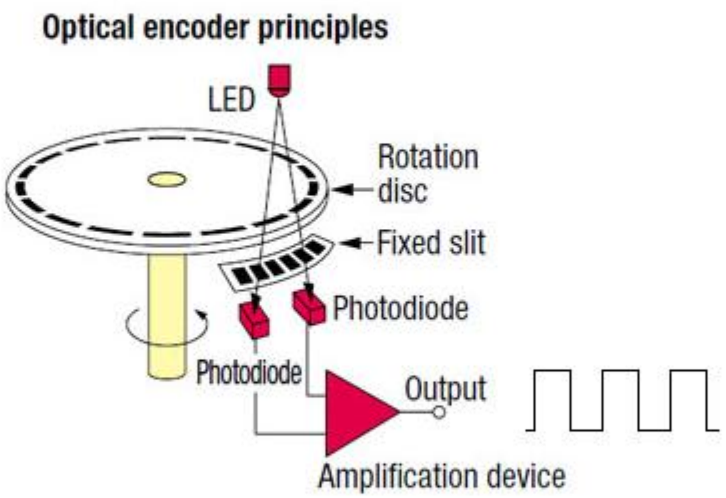
Straightness measurement with an interferometer.

Encoders

- An accessory to a mechanical device that translates mechanical motion into a measurable electrical signal Digital or Analog (preferably digital).
- Optical Encoders
  - Use light & photosensors to produce digital code
  - Most popular type of encoder.
- Can be linear or rotary.

Optical Encoders: Components



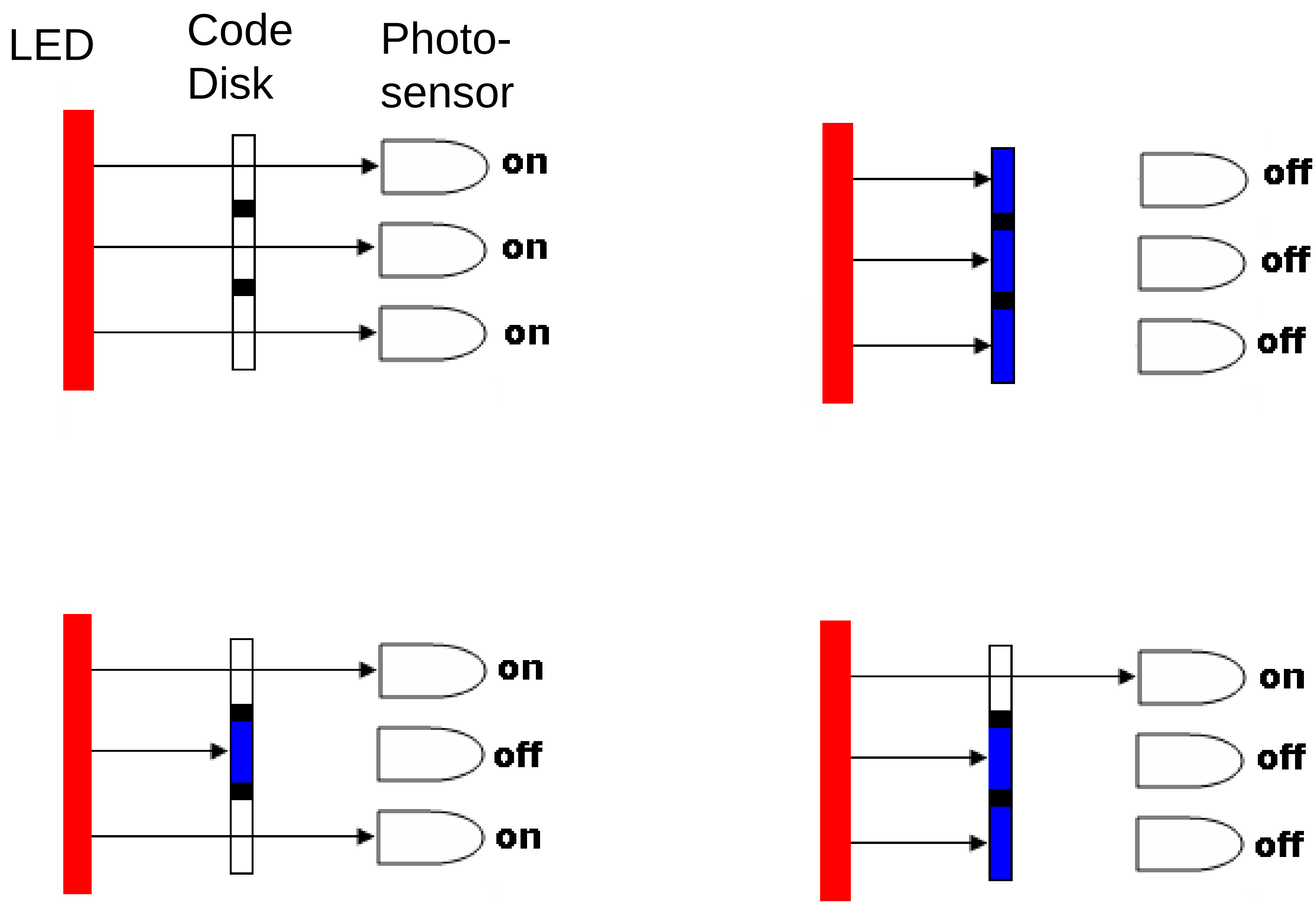


- Code Disk: Used to produce different light patterns on a photo detector assembly from a stationary light source.
- Code Disk: Determines the Optical Encoder type.

Optical Encoders: Components

- Light source(s)
  - LEDs or IR LEDs provide light source.
  - Light is collimated using a lens to make the beams parallel.
- Photodetector(s)
  - Either Photodiodes or Phototransistors.
- Opaque disk (Code Disk)
  - One or more “tracks” with slits to allow light to pass through.

Optical Encoders: Theory

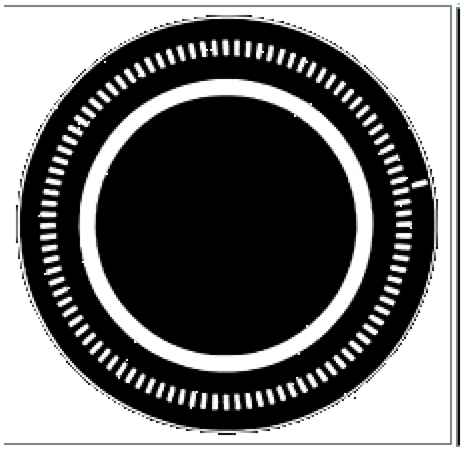


on = 0 **off = 1**

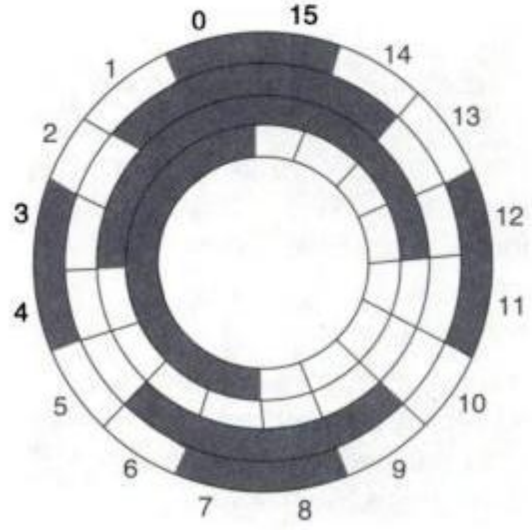
Optical Encoder Types

- Incremental Encoders: Mechanical motion computed by measuring consecutive “on” states.
- Absolute Encoders: Digital data produced by code disk, which carries position information.

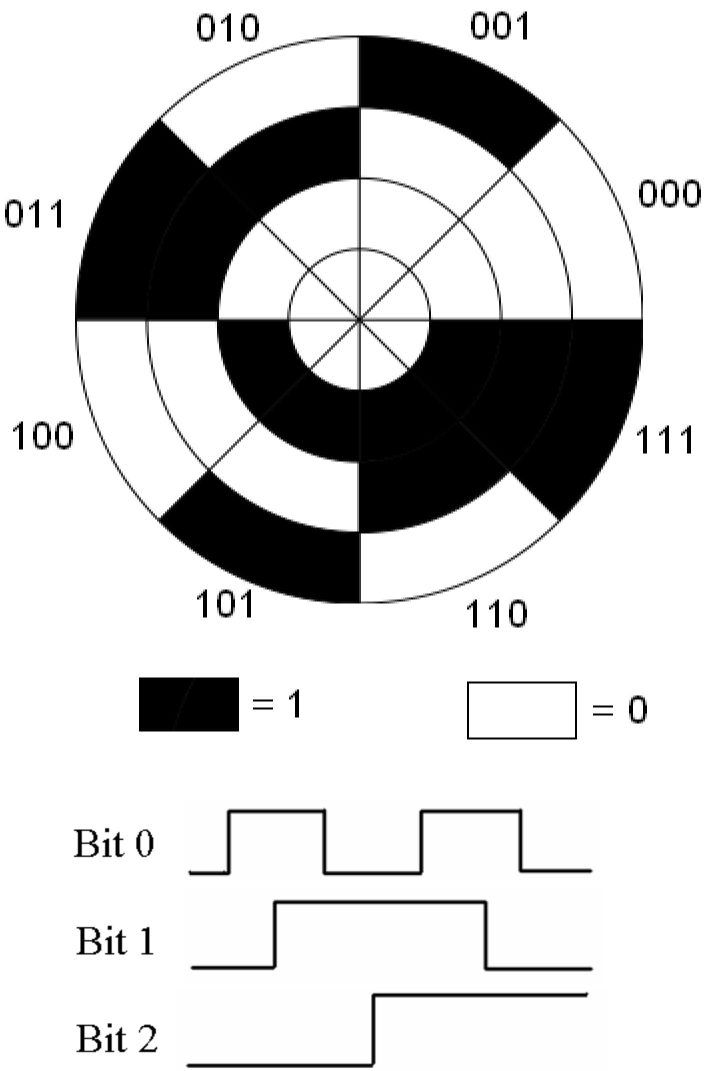
Incremental Encoder code Disk



Absolute Encoder code Disk



Standard Binary Encoding



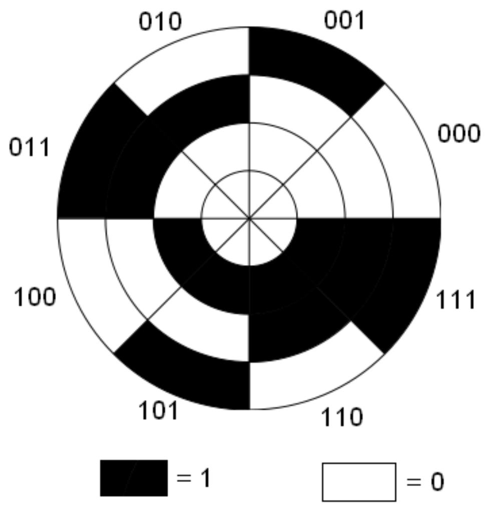
Angle	Binary	Decimal
0-45	000	0
45-90	001	1
90-135	010	2
135-180	011	3
180-225	100	4
225-270	101	5
270-315	110	6
315-360	111	7

Encoder Resolution

- Absolute Optical Encoder
  - Resolution = $360º/(2^n)$
  - n = number of encoder bits
  - Measures the rotational displacement that can be measured per bit change.
- Incremental Optical Encoder
  - Resolution = 360/N
  - N = number of windows on code disk
  - Resolution can be increased by reading both rising and falling edges

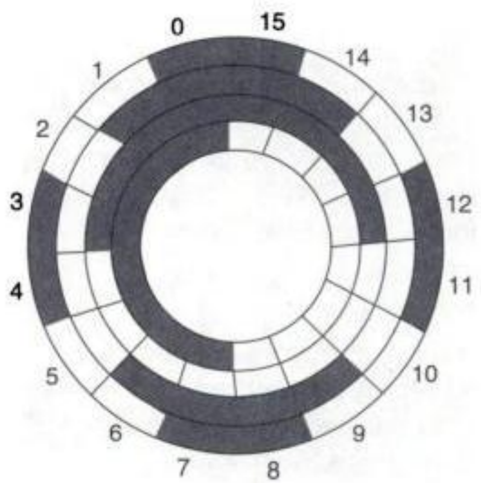
$$\Delta\theta = \frac{360}{2N}$$

Example



Number of bits on encoder code disk n = 3

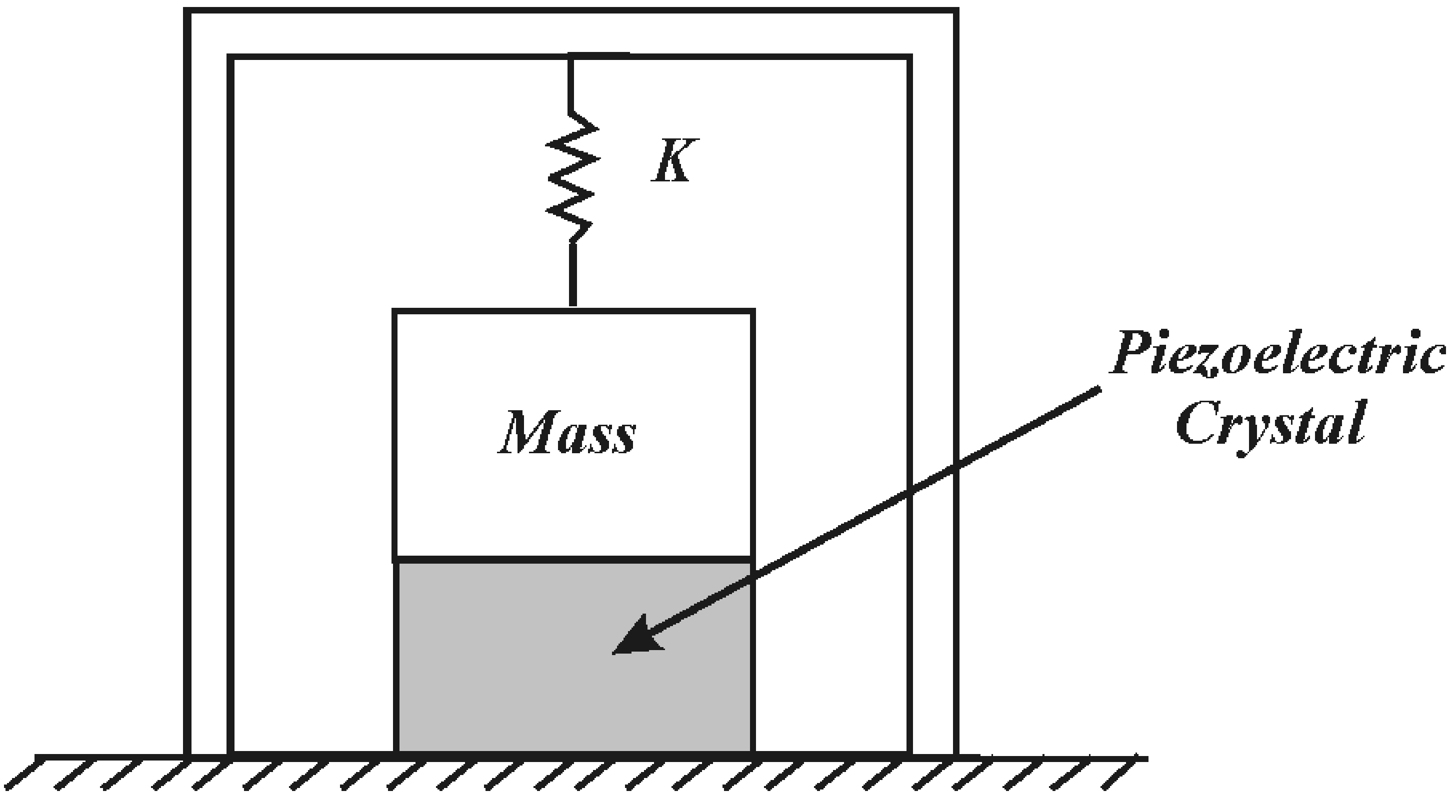
Resolution = $360^o/2^3 = 45^o$



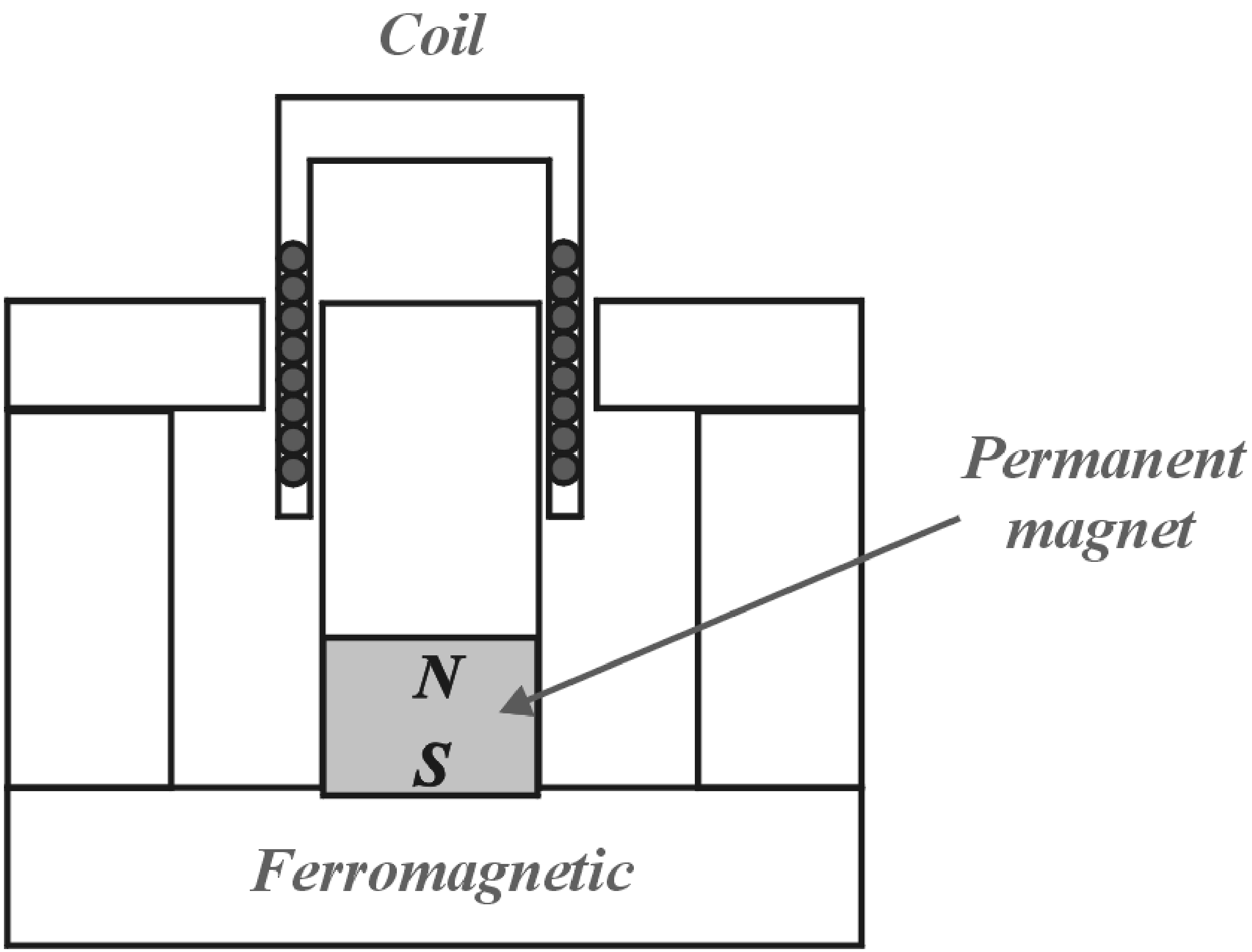
Number of bits on encoder code disk n = 4

Resolution = $360^o/2^4 = 22.5^o$

Accelerometer



Geophone

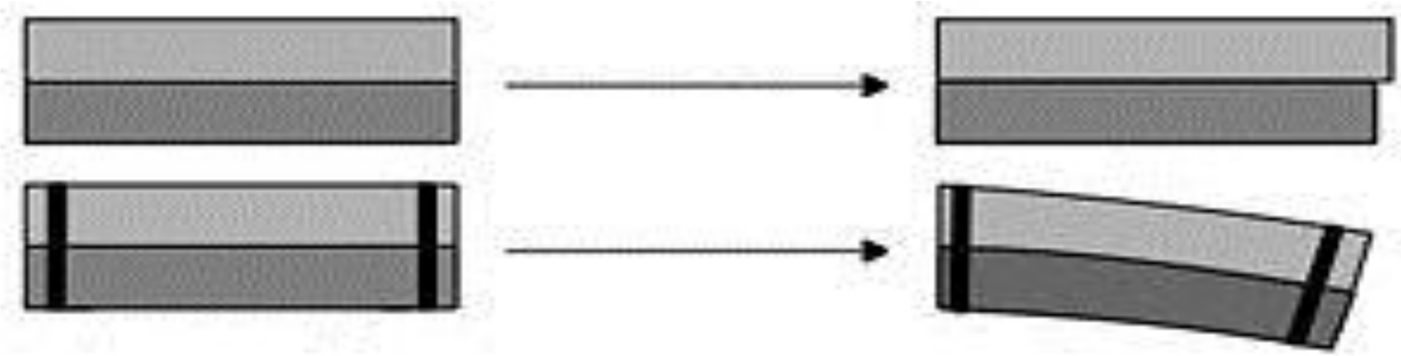


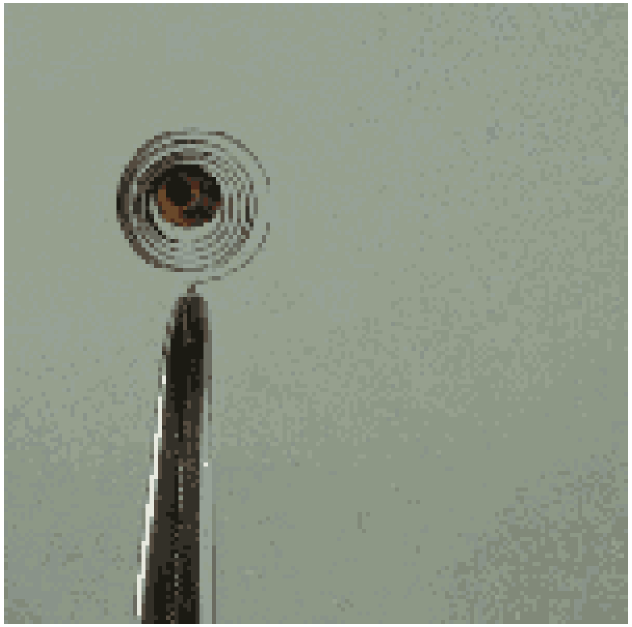
Mercury in glass thermometer

Calibration of the thermometer:

1. Place the cylinder of the thermometer in melting ice made of pure water and mark the point where the fluid in the thermometer stabilizes. This point is the freeze/thaw point of water.
2. In the same manner mark the point where the fluid stabilizes when the thermometer is placed in boiling water vapor.
3. Divide the length between the two marks into 100 equal pieces.

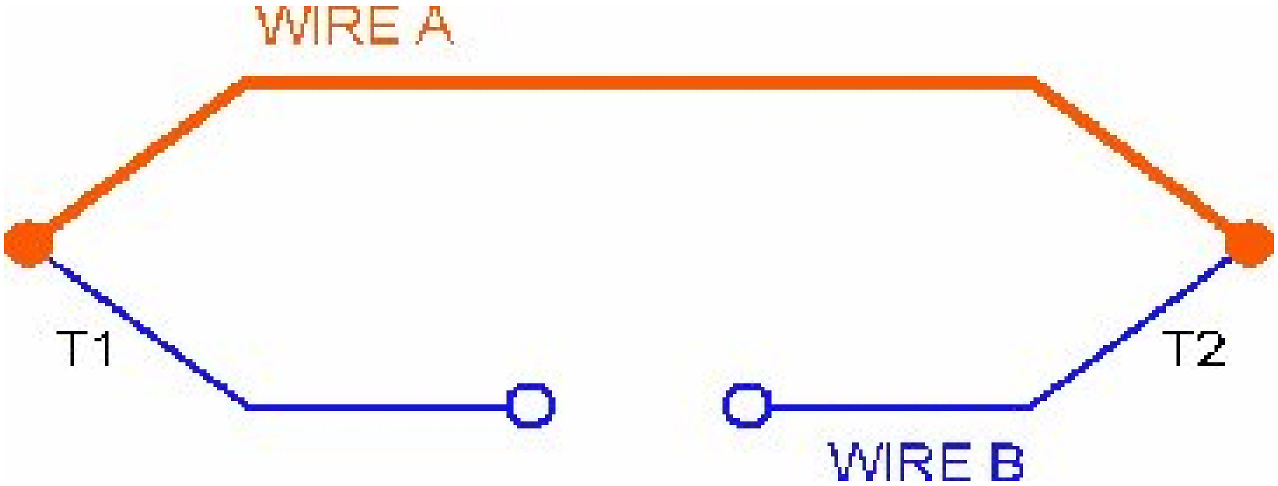
Bimetallic strip





Thermocouple

Thomas Seebeck



Thermocouple

Thermocouple	Maximum Temperature (^oC)	
	Continuous	Spot
Copper-Constantan (T)	400	500
Iron-Constantan (J)	850	1,100
Chromel-Constantan (E)	700	1,000
Chromel-Alumel (K)	1,100	1,300
Nicrosil-Nisil (N)	1,250	-
Tungsten-Molybdenum*	2,600	2,650

Infrared Thermometer

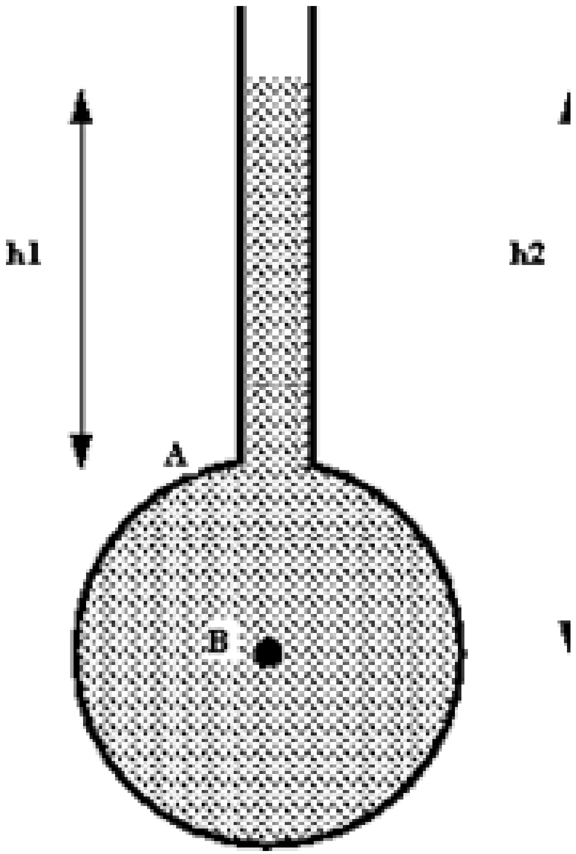
Piezometer tube manometer

pressure at A = pressure due to column of liquid above A

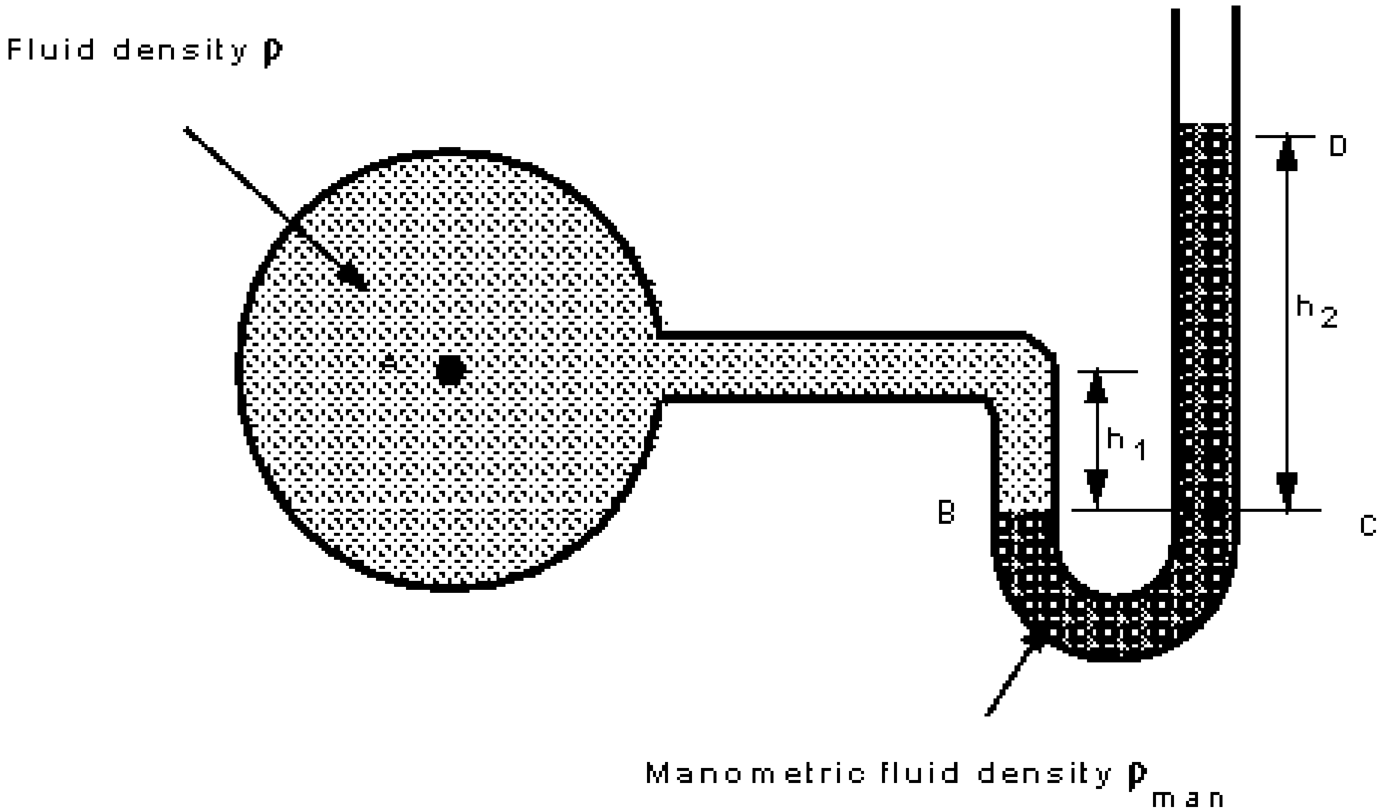
$$p_A = \rho g h_1$$

pressure at B = pressure due to column of liquid above B

$$p_B = \rho g h_2$$



U – tube Manometer



U – tube Manometer

$$\text{pressure at B} = \text{pressure at C}$$

$$p_B = p_C$$

$$\text{pressure at B} = \text{pressure at A} + \text{pressure due to hei ght h}_1 \text{ of fluid being meas ured}$$

$$p_B = p_A + \rho g h_1$$

$$\text{pressure at C} = \text{pressure at D} + \text{pressure due to hei ght h}_2 \text{ of manome tric fluid}$$

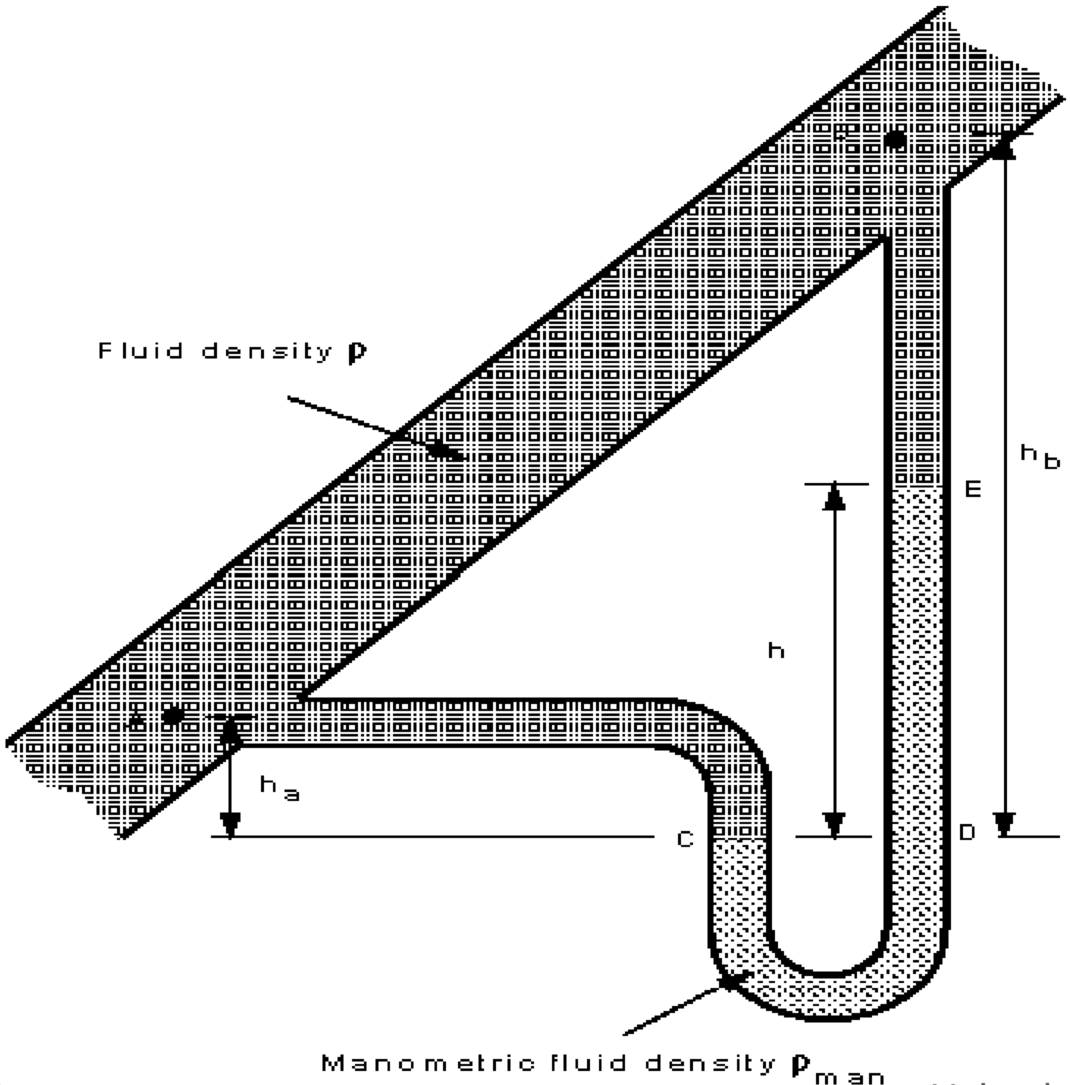
$$p_C = p_{\text{Amospheric}} + \rho_{\text{man}} g h_2$$

$$p_B = p_C$$

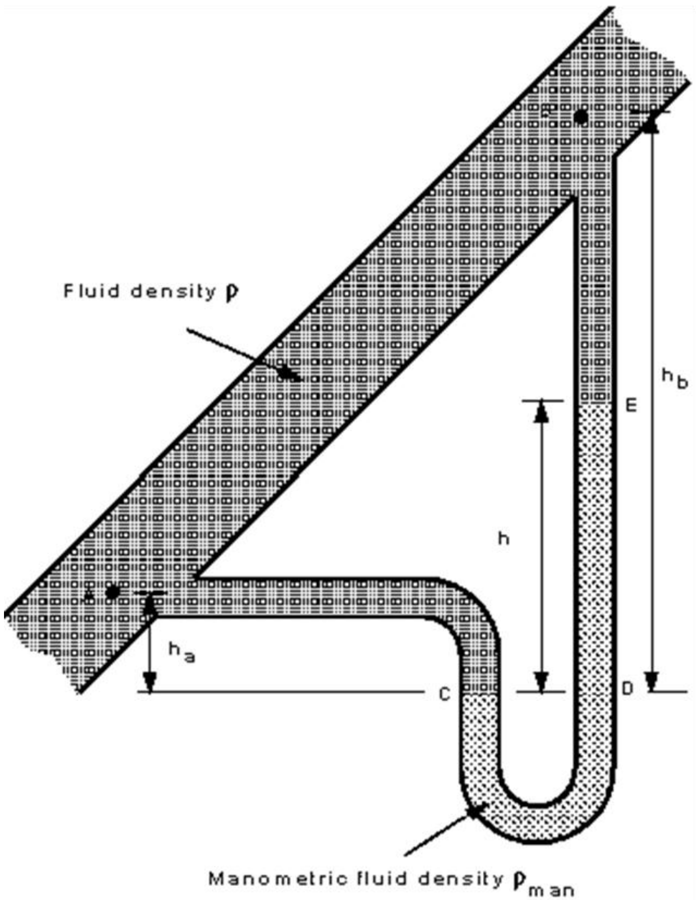
$$p_A = \rho_{\text{man}} g h_2 - \rho g h_1$$

$$p_A = \rho_{\text{man}} g h_2$$

U – tube Manometer



U – tube Manometer



$$\text{pressure at C} = \text{pressure at C}$$

$$p_C = p_D$$

$$p_C = p_A + \rho g h_a$$

$$p_D = p_B + \rho g\left(h_b - h\right) + \rho_{man} g h$$

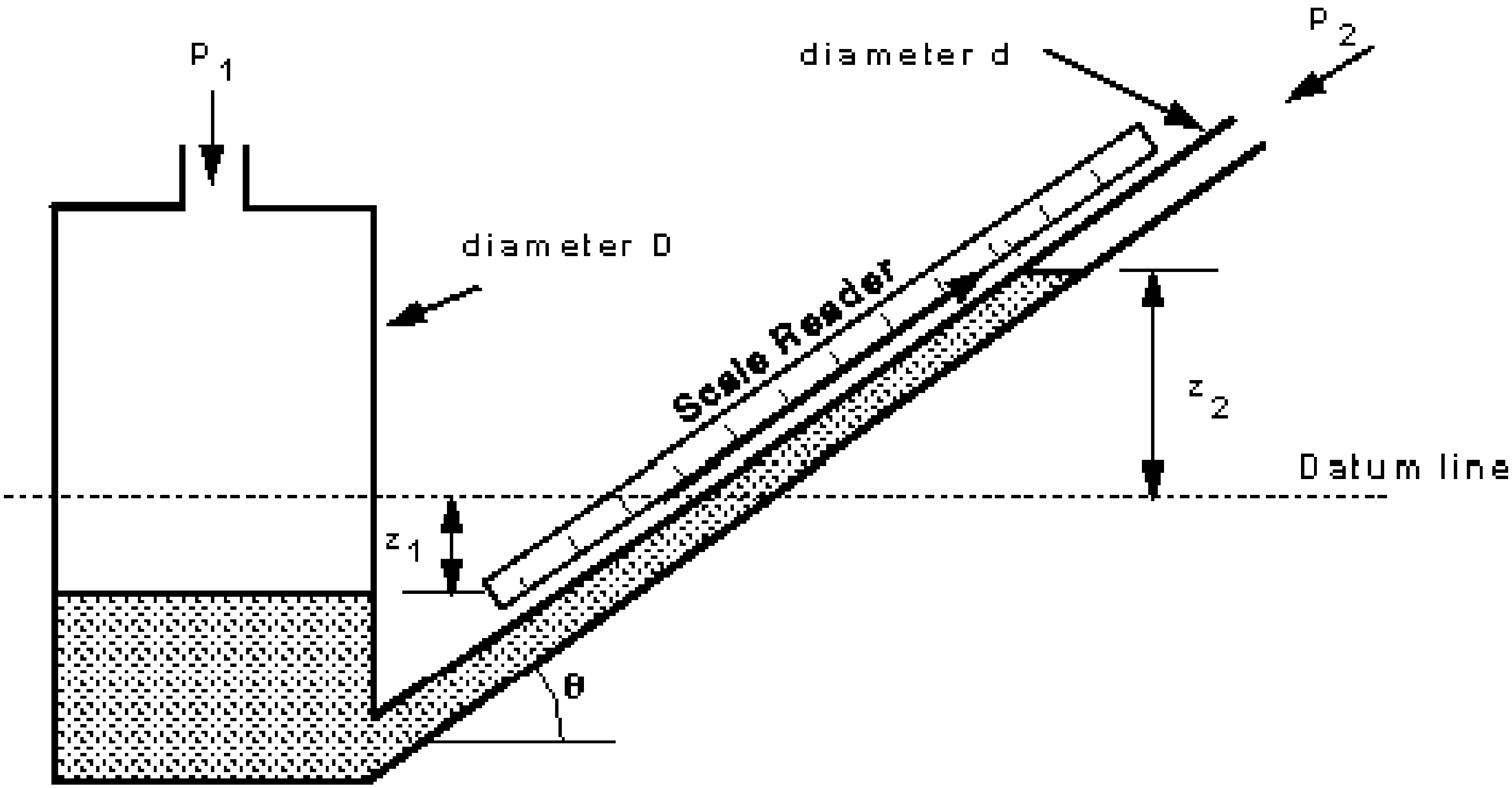
$$p_A + \rho g h_a = p_B + \rho g\left(h_b - h\right) + \rho_{man} g h$$

$$p_A - p_B = \rho g\left(h_b - h_a\right) + \left(\rho_{man} - \rho\right) g h$$

$$\rho_{man} >> \rho$$

$$p_A - p_B = \rho_{man} g h$$

Tilted tube manometer



Tilted tube manometer

$$z_1 = \frac{\text{Volume moved}}{\text{Area of left side}}$$

$$= \frac{z_2\left(\pi d^2/4\right)}{\pi D^2/4}$$

$$= z_2\left(\frac{d}{D}\right)^2$$

$$p_1 - p_2 = \rho g\left[z_2 + z_2\left(\frac{d}{D}\right)^2\right]$$

$$= \rho g z_2\left[1 + \left(\frac{d}{D}\right)^2\right]$$

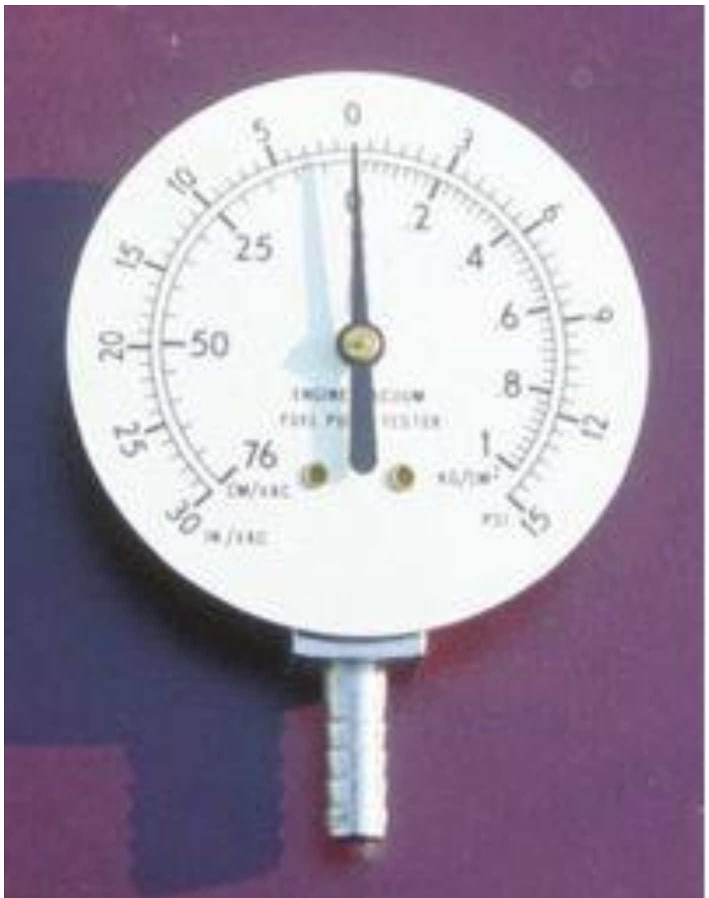
If $(d/D)^2$ is very small

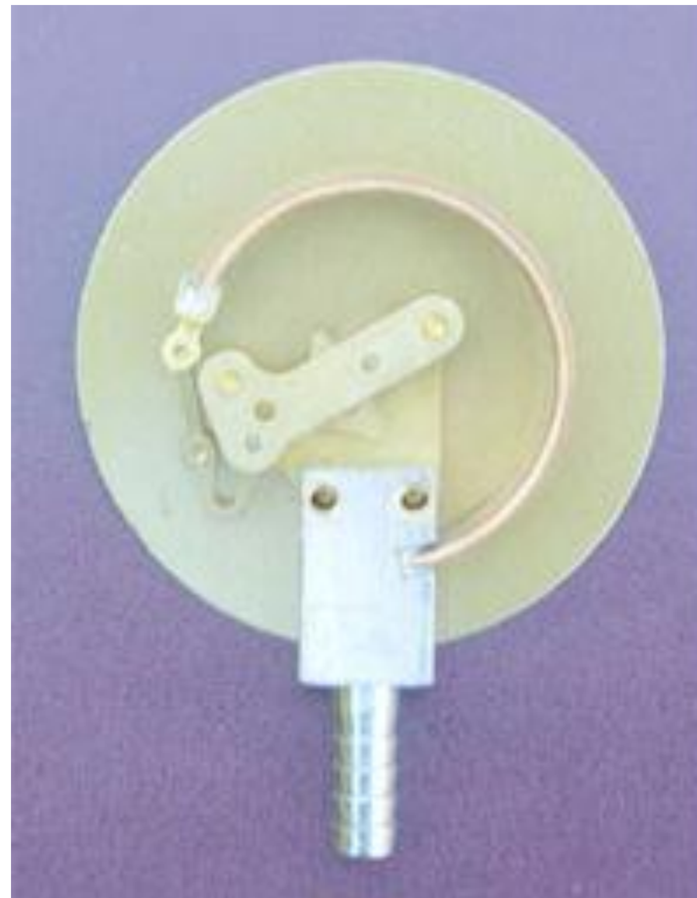
$$p_1 - p_2 = \rho g z_2$$

$$p_1 - p_2 = \rho g z_2$$

$$= \rho g x \sin\theta$$

Bourdon tube manometer

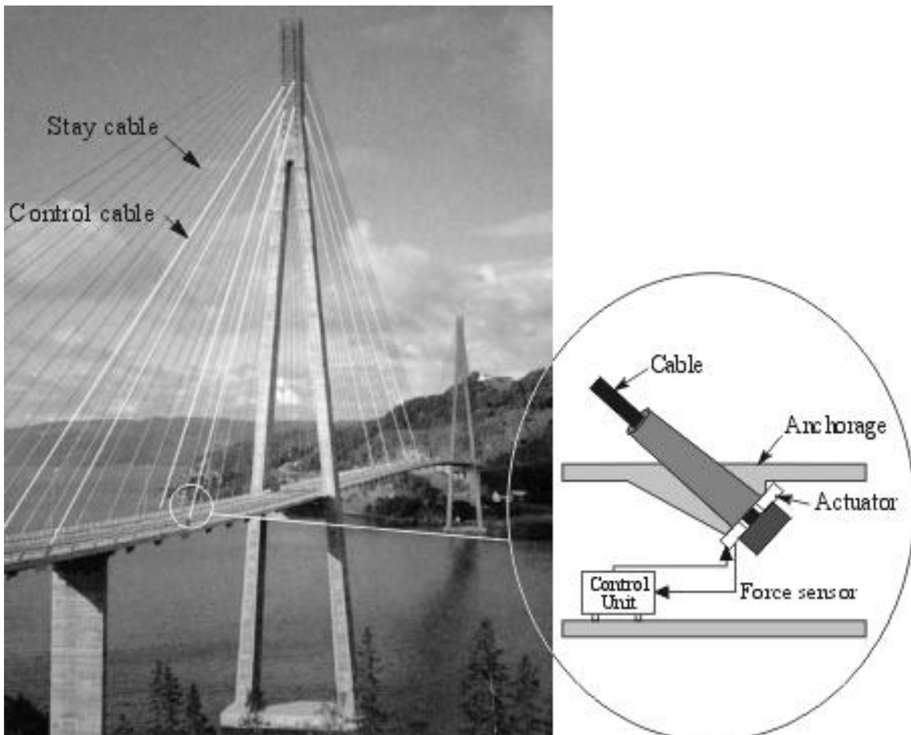




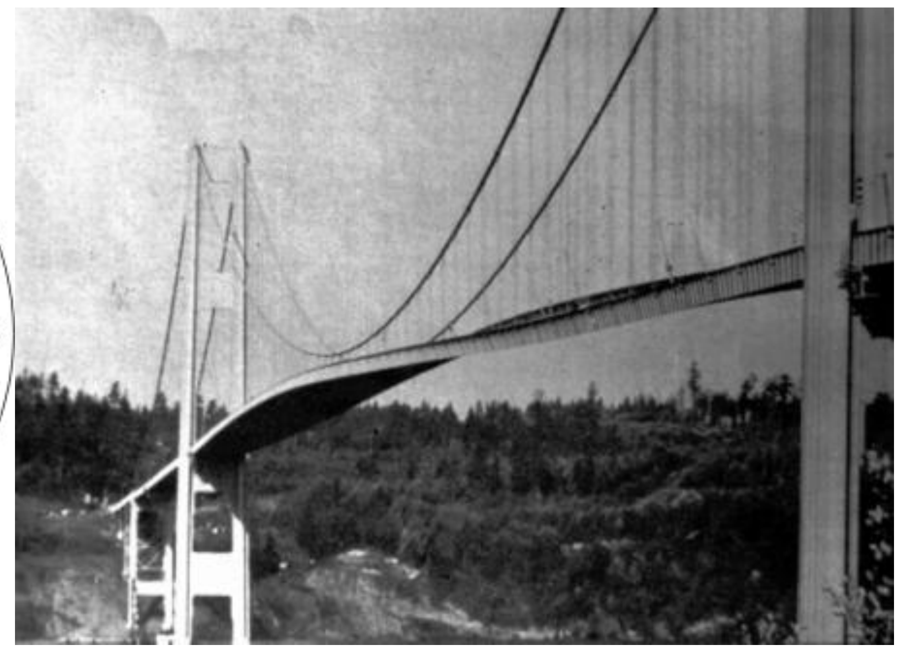
Diaphragm manometer

MECHANICAL VIBRATIONS

Bridges

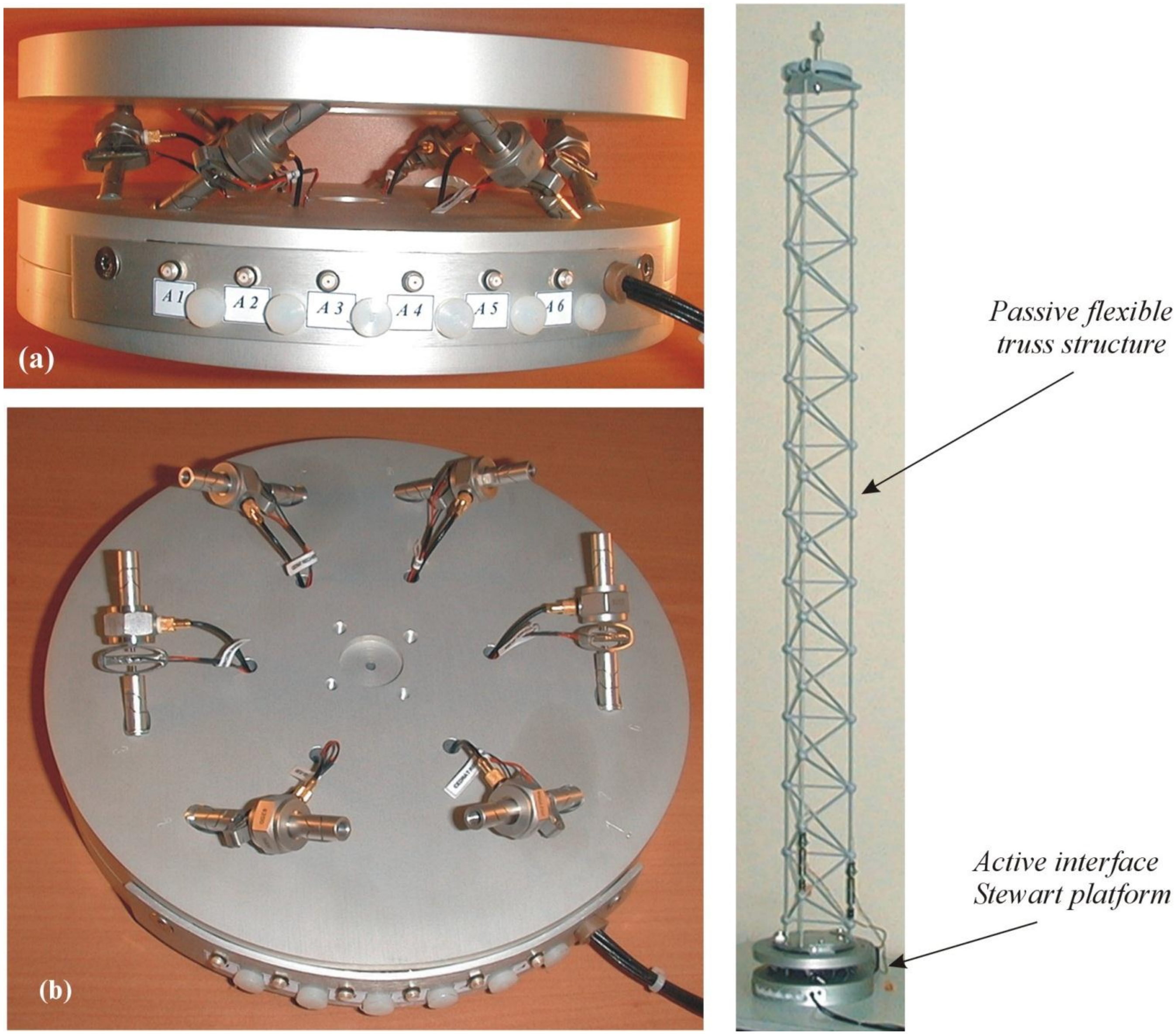


Suspension bridge



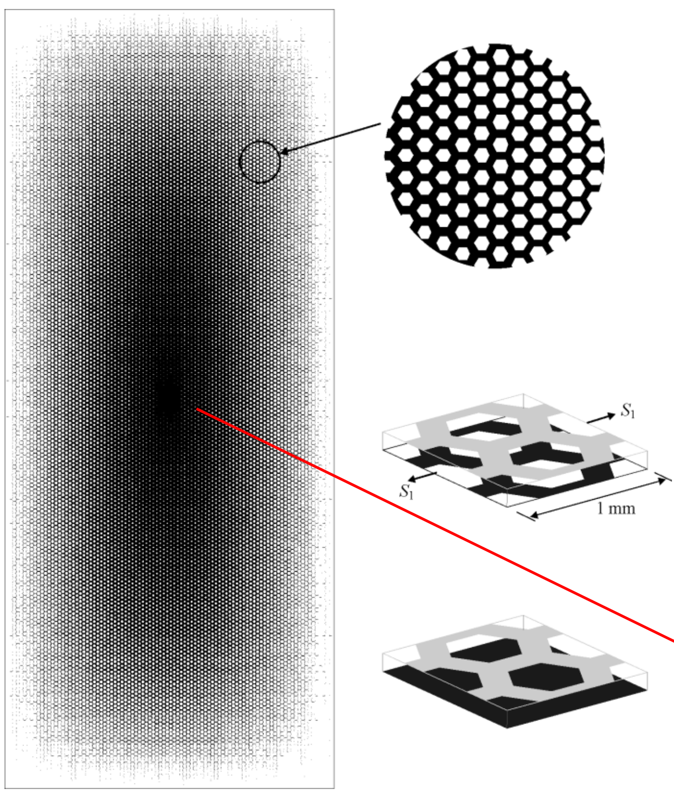
Cable stayed bridge

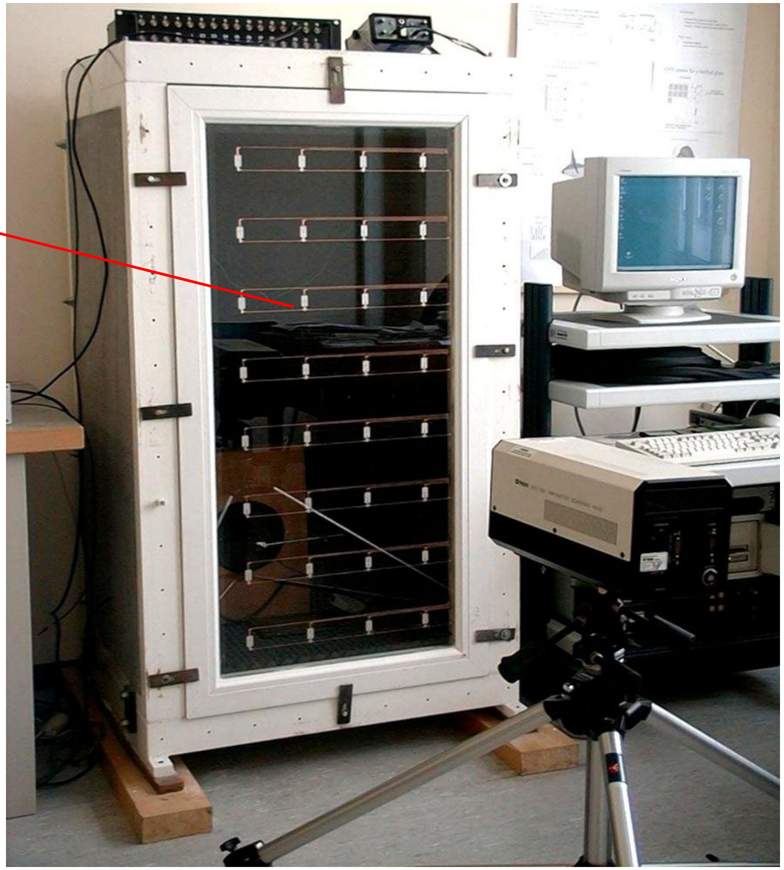
Hard interface for truss structure - ULB



Active window Experimental set-up

4 x 8 PZT patches
Glass plate





Honey comb

Vibrations

Types of vibrations

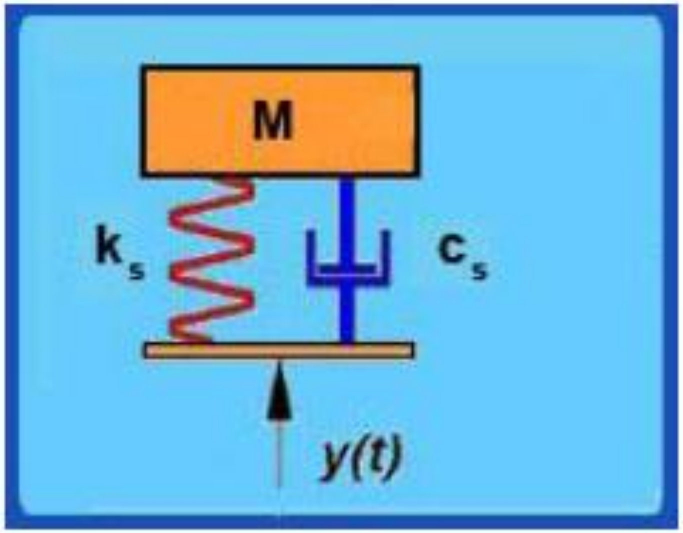
- Free Vibrations
- Forced Vibrations
- Damped Vibrations

Methods of vibration reduction

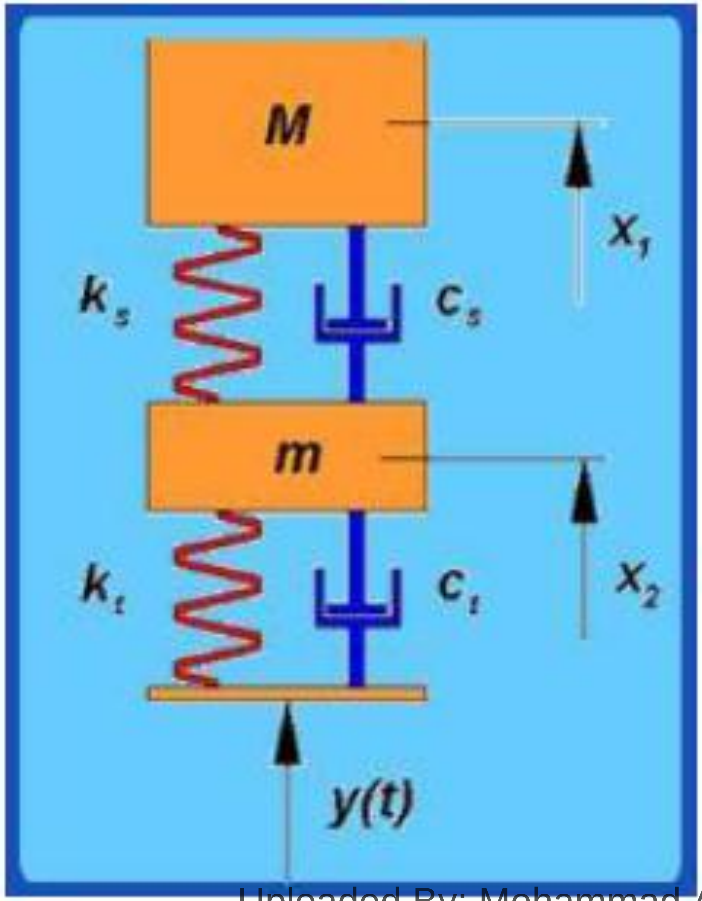
- Passive
- Active

Degrees of freedom

Single degree of freedom
(SDOF)

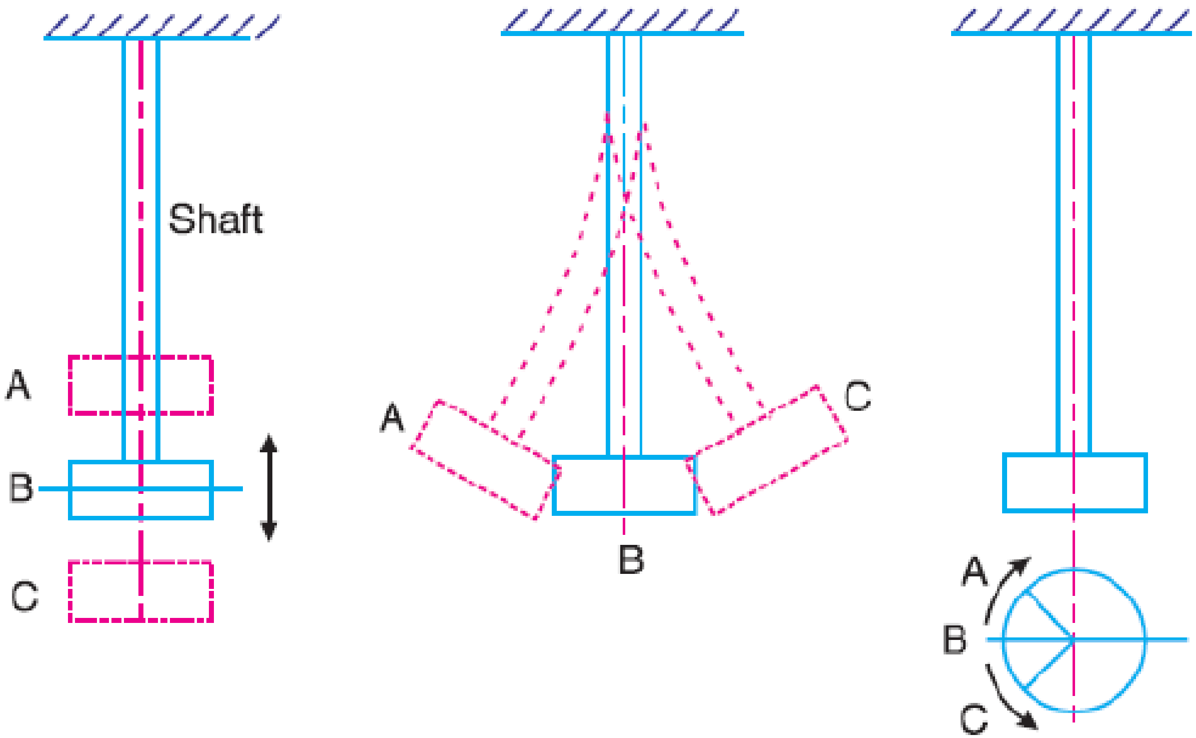


Two degrees of freedom
(2 DOF)



Types of vibrations

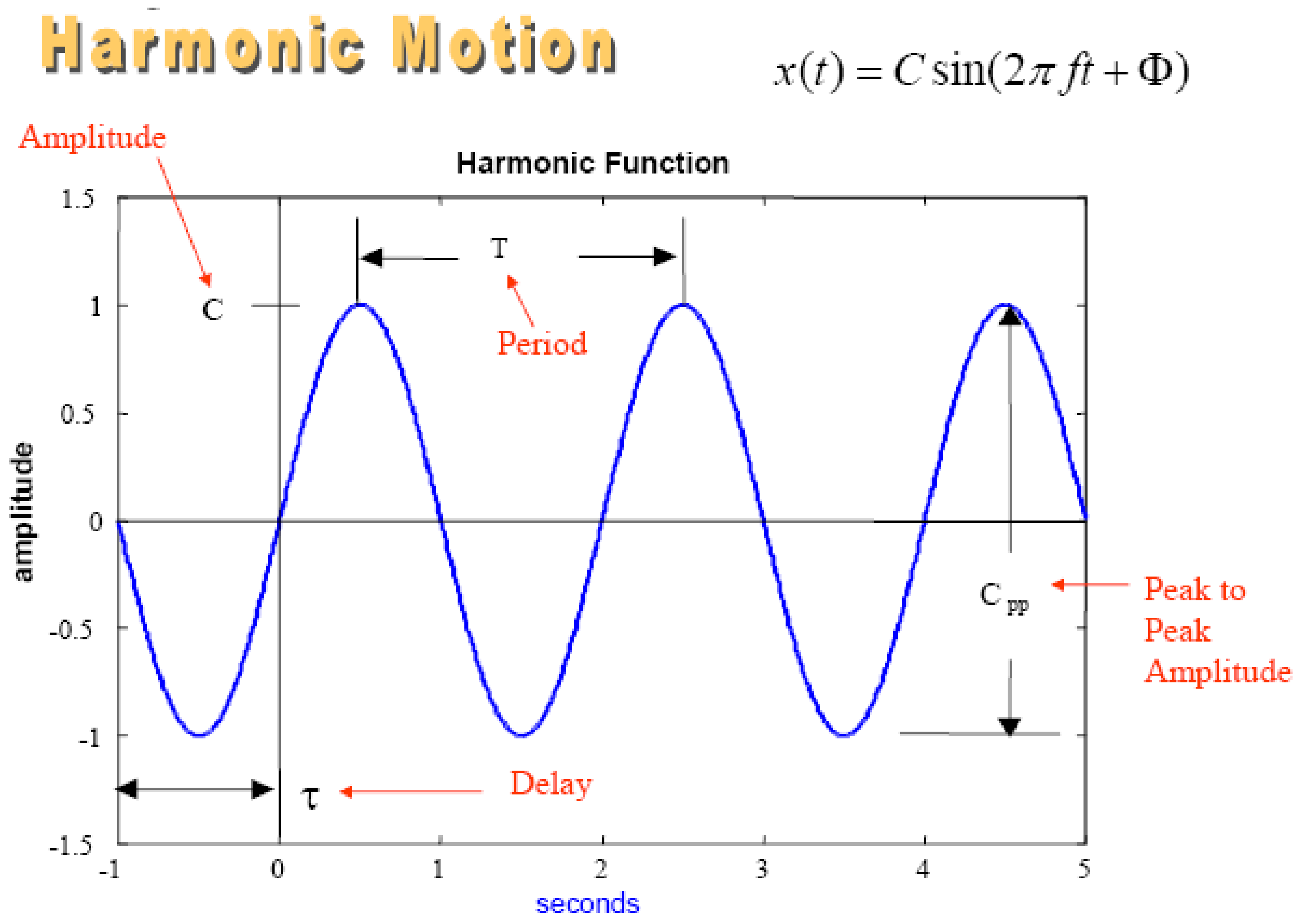
(a)Longitudinal vibration (b)Transverse Vibration (c)Torsional Vibration.



B = Mean position ; A and C = Extreme positions.

(a) Longitudinal vibrations. (b) Transverse vibrations. (c) Torsional vibrations.

Harmonic motion



Harmonic motion

$$T = \text{period} = \frac{1}{f}(\text{sec}) \quad \Longrightarrow \quad f = \frac{1}{T}\left(\frac{cycles}{\text{sec}}\right) \rightarrow (Hz) \quad \Longrightarrow \quad \omega = 2\pi f\left(\frac{rad}{\text{sec}}\right)$$

$C =$ amplitude

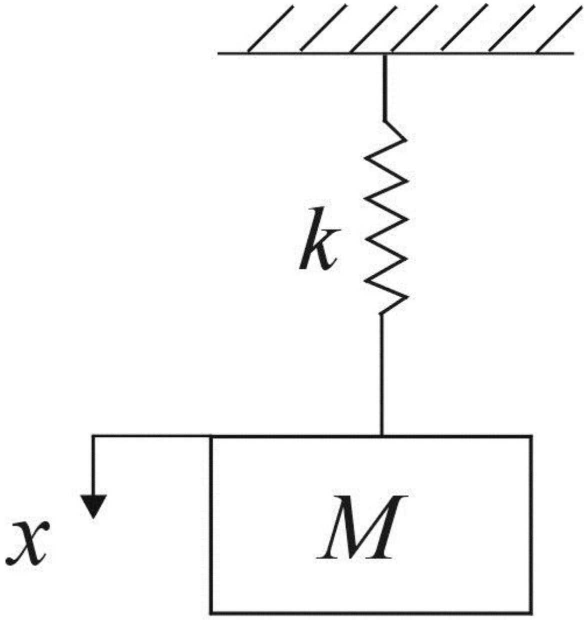
$C_{pp} =$ peak to peak amplitude

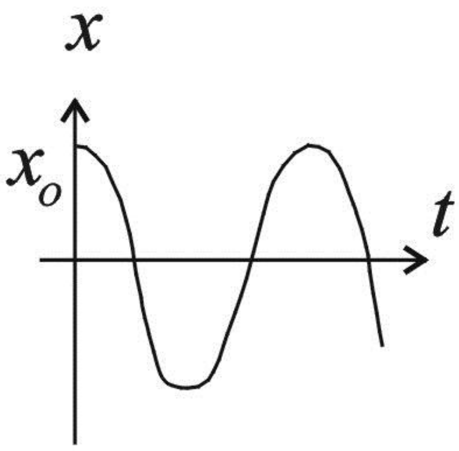
$$C_{\text{rms}} = \left[\frac{1}{T}\int_0^T C^2 \sin^2(2\pi f t + \Phi)dt\right]^{1/2} = \frac{\sqrt{2}}{2}C = 0.707C$$

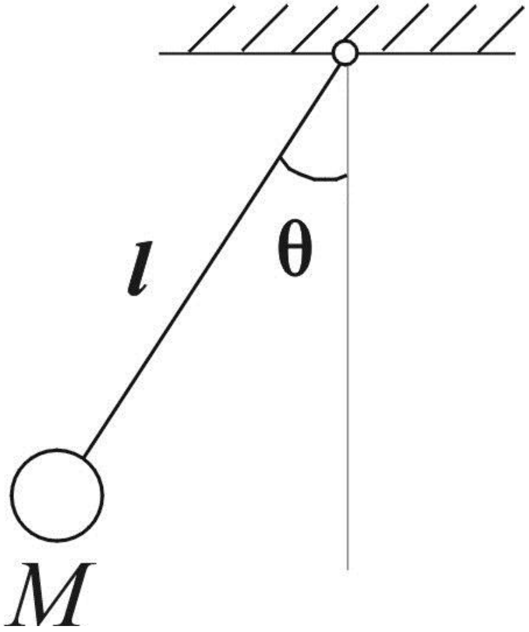
rms=root mean square

$$\Phi = \text{phase} = \omega\tau = 2\pi f \tau$$

Fundamental concepts of vibrations







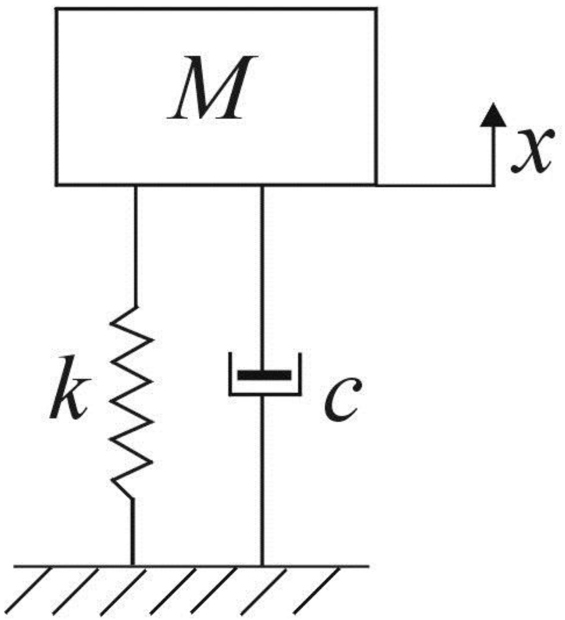
$$x = x_o \cos \omega t$$

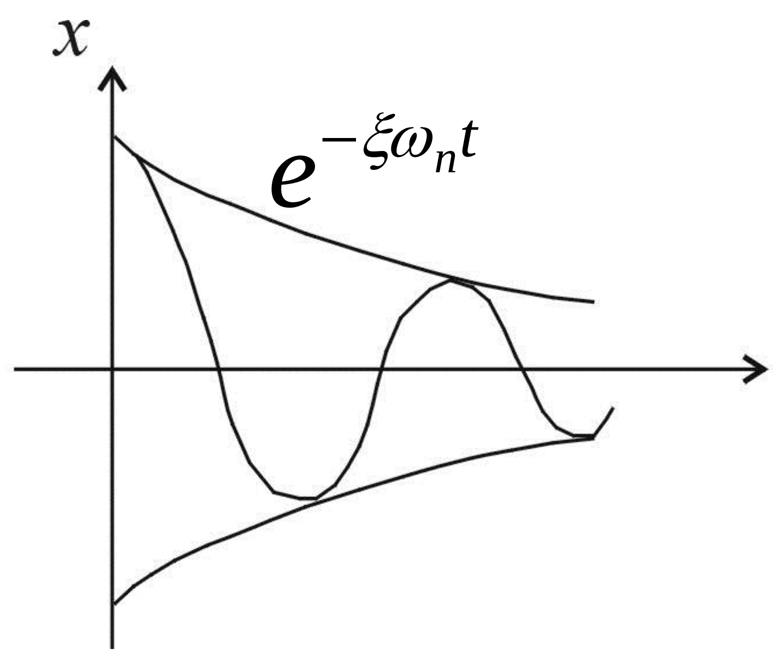
$$\theta = \theta_o \cos \omega t$$

$$\omega = \sqrt{\frac{k}{M}}$$

$$\omega = \sqrt{\frac{g}{l}}$$

Viscous damping

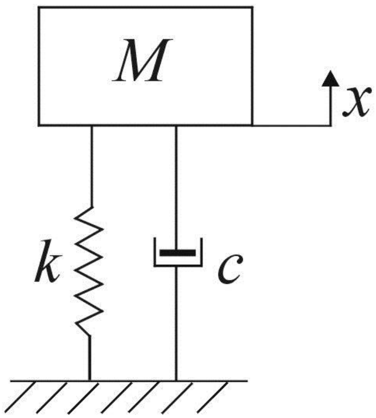




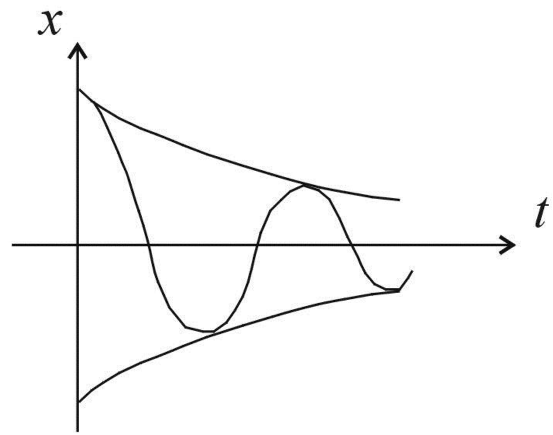
Damping force

$$f_c = -c\,\dot{x}$$

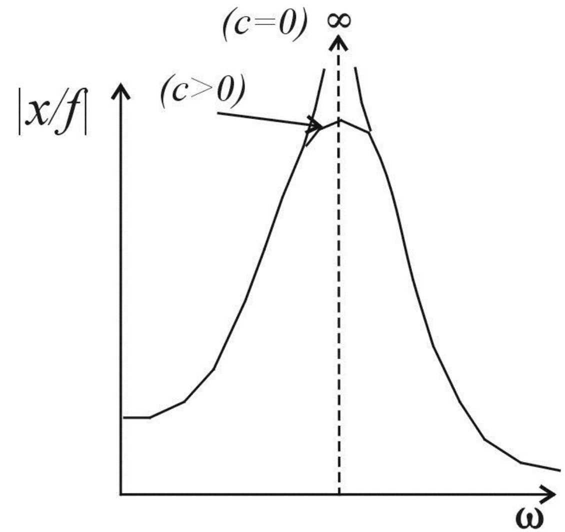
Single degree of freedom oscillator



SDOF oscillator

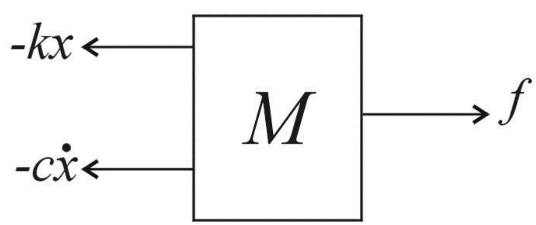


Time response



Frequency response

$$\omega_n^2 = \frac{k}{M}$$



$$2\xi\omega_n = \frac{c}{M}$$

Equations of motion

$$m\ddot{x} = f - c\dot{x} - kx$$

$$m\ddot{x} + c\dot{x} + kx = f$$

$$\omega_n^2 = \frac{k}{m} \qquad and \qquad 2\xi\omega_n = \frac{c}{m}$$

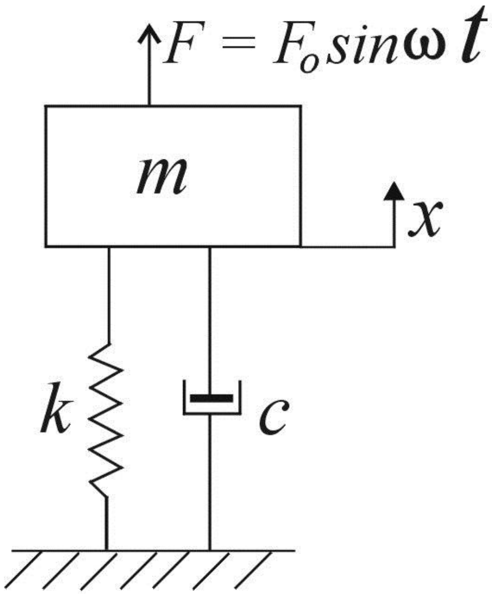
$$\ddot{x} + 2\xi\omega_n\dot{x} + \omega_n^2 x = \frac{f}{m}$$

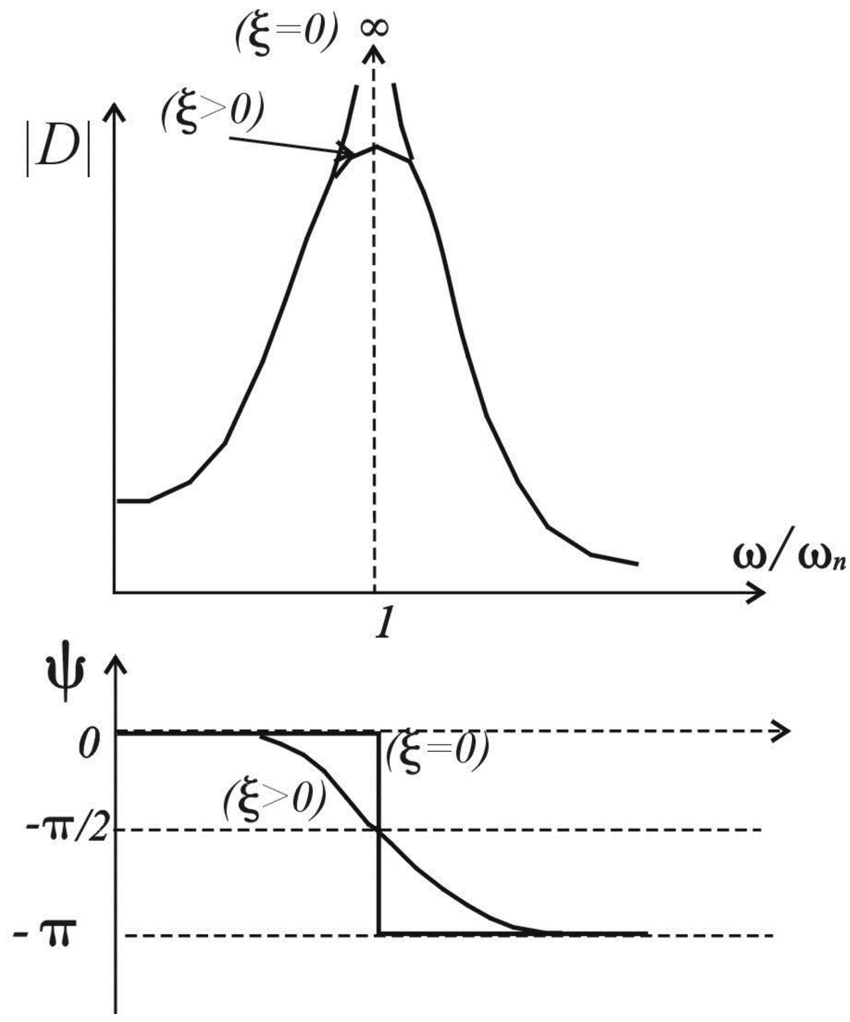
Free response

$$x = e^{-\xi\omega_n t}(A\cos\omega_d t + B\sin\omega_d t)$$

$$\omega_d = \omega_n\sqrt{1-\xi^2}$$

Forced vibrations





$$D = \left|D\right| e^{j\psi}$$

$$m\ddot{x}+c\dot{x}+kx=F_{\circ}e^{j\omega t}$$

$$\ddot{x}+2\xi\omega_n\dot{x}+\omega_n^2 x=\frac{F_{\circ}}{m}e^{j\omega t}$$

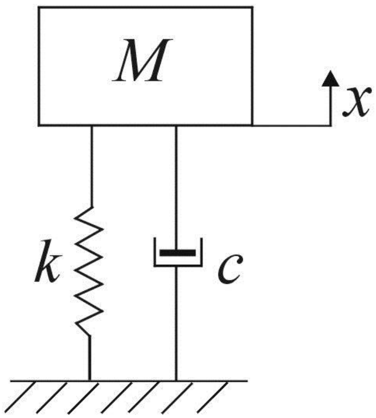
$$x=Xe^{j\omega t} \qquad \textit{Particular} \quad \textit{solution}$$

$$x=\frac{F_{\circ}}{k}\left\{\frac{1}{(1-\frac{\omega^2}{\omega_n^2})+2j\xi\frac{\omega}{\omega_n}}\right\}$$

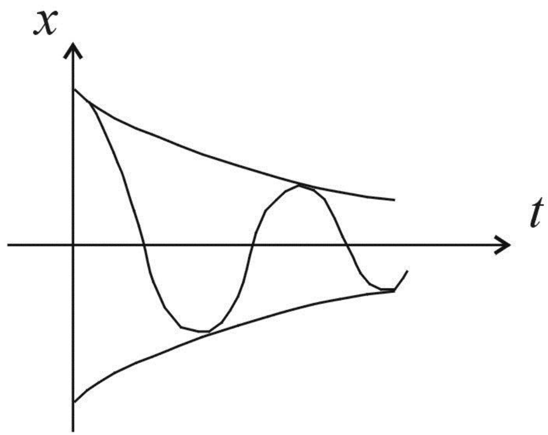
Static deformation

Dynamic amplification

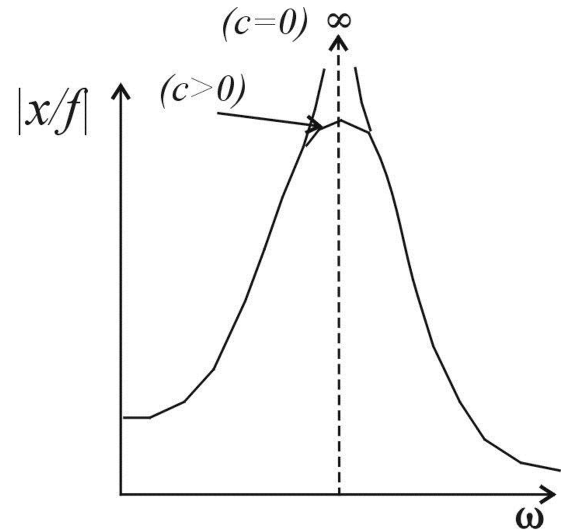
Dynamic modeling



SDOF oscillator

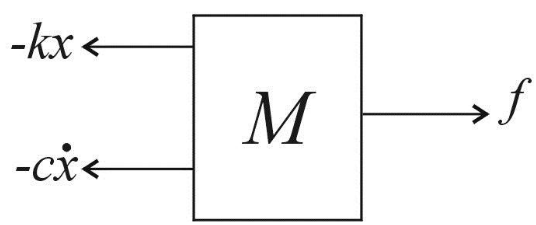


Time response



Frequency response

$$\omega_n^2 = \frac{k}{M}$$



$$2\xi\omega_n = \frac{c}{M}$$

Transfer Function
(Frequency response Function [FRF])

$$m\ddot{x} = f - c\dot{x} - kx$$

$$m\ddot{x} + c\dot{x} + kx = f$$ Equation of motion

Or $$ms^2x + csx + kx = f$$ In Laplace transform

$$x(ms^2 + cs + k) = f$$

$$\frac{x}{f} = \frac{1}{ms^2 + cs + k}$$ Transfer function

$$\omega_n^2 = \frac{k}{m} \qquad and \qquad 2\xi\omega_n = \frac{c}{m}$$

$$\frac{x}{f} = \frac{1}{s^2 + 2\xi\omega_n s + \omega_n^2}$$ Transfer function

Bode plot

$$\frac{x}{f} = \frac{1}{\frac{s^2}{\omega_n^2} + 2\xi \frac{s}{\omega_n} + 1}$$