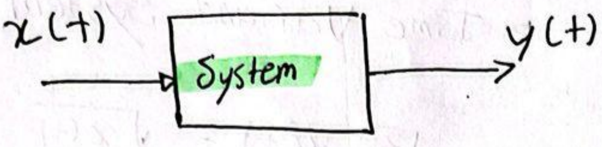
chapter 2: **System Modeling in the time domain**



properties of system:

1 continuos-Time and Discreat-Time

طبيعة النظام

2 Fixed and Time-Variant system

Fixed → Time-invariant

بدنا ندرس تأثير **dealy** طبعًا احنا بنشتغل على ideal case
بحيث لو صار عندي dealy بمكان معين بدي يحط مكان ماساس يدخل
ماشي الاشي بشكل مثالي

example if $y(t) = x(t^2)$

To check - if $y(t)$ is fixed or not fixed → Time-invariant or Time-Variant

- Delay of input "function of Time"

$$y_1(t-t_0) = x_1(t^2 - t_0) \longrightarrow ①$$

هون الديلي ل input فعمل shift بمقدار t_0 للانبت كامل

- Time delay

$$y_2(t-t_0) = x_2((t-t_0)^2) \longrightarrow ②$$

هون delay ل time فزبط في تايم بعمل shift

هسا بدي أعمل عملية مقارنة، اذا كنت متساويات بحكي عندي النظام (Time-invariant) → fixed

يعني ما عندي تأثير ل delay او وجود عندي

أما لو كانت $eq(1) \neq eq(2)$ ومادحل المثال الاول

→ Time Variant System

$\boxed{2}$ $y(t) = x(t)$

- Delay of input "function of time"

$$y_1(t - t_0) = x_1(t - t_0) \longrightarrow ①$$

- Delay - Time

$$y_2(t - t_0) = x_2(t - t_0) \longrightarrow ②$$

$$eq(1) = eq(2) \longrightarrow \underline{\text{Time-invariant}}$$

fixed

$\boxed{3}$ $y(t) = \sqrt{x(t)}$

1- $y_1(t - \tau) = \sqrt{x(t - \tau)}$

2- $y_2(t - \tau) = \sqrt{x(t - \tau)}$

Time-invariant

تكملة الخصائص

3) Causal and non-causal

دراسة علاقة القيم الموجودة عندي في output في القيم الموجودة عندي في input

e.g if the system

$$3y(t) + \int_{-\infty}^{t} y(\lambda)\, d\lambda = x(t)$$

عشان احكي النظام causal لازم القيم الموجودة في output تكون اكبر او تساوي القيم الموجودة في input

اما non-causal قيم ال output كأنه تتنبؤ رقم مستقبلية موجودة في input

→ causal system

عشان احل check ل causality

يروح بقارن القيم يلي يعتمد عليها output اكبر او تساوي القيم يلي بده يعتمد عليها input

يعني من ارقام (القيم الموجودة بين الاقواس في output تساوي القيم الموجودة بين الاقواس ل input

ال causal ما يعتمد على ... مستقبل، يعتمد على ارقام حاضرة في الوقت الحالي

2 the System $y(t) = x(t^2)$

- $\rightarrow 0 \leq t \leq 1$ causal — القيم بتحمص — $y(\frac{1}{2}) = \frac{1}{4}$
- $\rightarrow t > 1$ — $y(2) = 4$ — Non-causal

e.g $y(t) = x(3t) \rightarrow$ Non-causal
لانه بيعتمد على قيم مستقبلية

اذا السؤال ماشرّط قيم t بقدر أحكي إنه النظام ككل non-causal

3 the system $y(t) = 10\,x(t+2) + 5$

بشوف قيم t يلي بداخل الاقواس للـ input او output

non-causal since the values of output depends on the future values of input. بسبب اعتمد على قيم مستقبلية

$$y(1) = 10\,x(3) + 5$$

output < input

4 Dynaimiac (Memory) and Instanenious (Memory less)

Instanenious: Static لحظي

وكانه كل قيمة في input تكافئ القيمة في output

e.g: ① $y(t) = x(t)$ or $y(t+2) = x(t+2)$

Instaneous or Memory less

يلي بين الاقواس في input و output نفسه ويكون لحظي ، غير هيك otherwise

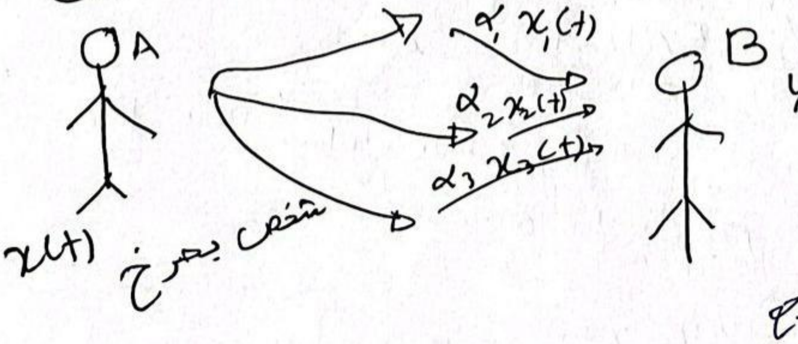
② $$2\frac{\partial y(t)}{\partial t} + 3y(t) = \frac{\partial^2 x(t)}{\partial t^2} + x(t)$$

لما يكون عندي مشتقة او تكامل بكون عبارة عن Dynaimic لانه بكون تشتغل على اكثر من لحظة

constant coffisent

تغيرات النظام Time-invarant

5 Linear and non-linear System



α_1 Constant الهارطل

نوع اشارة يلي اصطدمت

يلي لو صفر وصول الاشارات ارجع أحصل على الاشارة الاصلية (واسمع الصوت)

Linearity → **Scaling**, **Adding**

eg: the system $y(t) = 2\pi x(t)$

Scaling بعمل عملية

$$\begin{cases} \alpha_1 y_1(t) = 2\pi x_1(t)\,\alpha_1 \\ \alpha_2 y_2(t) = 2\pi x_2(t)\,\alpha_2 \end{cases}$$

Adding هسا بعمل

$$\alpha_1 y_1(t) + \alpha_2 y_2(t) = 2\pi(\alpha_1 x_1(t) + \alpha_2 x_2(t))$$

بدي افترض لو عندي اشارة ثالثة تكون عبارة عن حاصل جمعهم

$$\alpha_1 y_1(t) + \alpha_2 y_2(t) = 2\pi(\alpha_1 x_1(t) + \alpha_2 x_2(t)) \longrightarrow (1)$$

Assume
$$\alpha_3 y_3(t) = \alpha_1 y_1(t) + \alpha_2 y_2(t)$$
$$\alpha_3 x_3(t) = \alpha_1 x_1(t) + \alpha_2 x_2(t)$$

$$\alpha_3 y_3(t) = 2\pi \alpha_3 x_3(t)$$
$$\alpha_1 y_1(t) + \alpha_2 y_2(t) = 2\pi(\alpha_1 x_1(t) + \alpha_2 x_2(t)) \longrightarrow (2)$$

هسا بعمل مقارنة

اذا كانوا متساويات بكونوا $\boxed{\text{linear}}$

ex: $\dfrac{d^2y}{dt^2} + 5\dfrac{dy}{dt} + 5y(t) + 6 = x(t) \longrightarrow$ non-linear

عشان احكي لينيار لازم كل الحدود يكون فيها y بقوة واحد

هون 6 خلت السستم مش لينيار

$\dfrac{d^2y}{dt} + \sin(t)\,y(t) = x(t) \longrightarrow$ Time-Variant

لانو اضروب y(t) هي sin
متغير مع الزمن

Lecture 2

مراجعة

Time-varant and Time-invarant

Delay → function of time

e.g: $y(t) = x(3t)$

وين معندي t مضروبة برا بني مجموعة لاشي هاد بعملو كله شيفت بمقدار t_0

$y_1(t+t_0) = x_2(3t-t_0)$

[بعمل لكل الفنكشن شيفت]

Delay of time

$y_2(t-t_0) = x(3(t-t_0))$

هون فقط t بعمل شيفت

$y_1(t-t_0) \neq y_2(t-t_0)$

← لأنهم مش متساويا time-Varant

Example:

بدي أعمل لكل property لحالها وأدققها

① $y(t) = x(t-2) + x(2-t)$

— linearity

$$\alpha_1 y_1(t) = \alpha_1 x_1(t-2) + \alpha_1 x_1(2-t)$$
$$\alpha_2 y_2(t) = \alpha_2 x_2(t-2) + \alpha_2 x_2(2-t)$$

$\boxed{1} \leftarrow \alpha_1 y_1(t) + \alpha_2 y_2(t) = \alpha_1 x_1(t-2) + \alpha_2 x_2(t-2) + \alpha_1 x_1(2-t) + \alpha_2 x_2(2-t)$

Assume:

$$\alpha_3 x_3(t) = \alpha_1 x_1(t) + \alpha_2 x_2(t) \quad \& \quad \alpha_3 y_3(t) = \alpha_1 y_1(t) + \alpha_2 y_2(t)$$

$$\alpha_3 y_3(t) = \alpha_3 x_3(t-2) + \alpha_3 x_3(2-t)$$

$\boxed{2} \leftarrow \alpha_1 y_1(t) + \alpha_2 y_2(t) = \alpha_1 x_1(t-2) + \alpha_2 x_2(t-2) + \alpha_1 x_1(2-t) + \alpha_2 x_2(2-t)$

Since eq $\boxed{1}$ = eq $\boxed{2}$ ⟶ linear

Causality

$$y(0) = x(-2) + x(2)$$

present ← $y(0)$, past ← $x(-2)$, $x(2)$ → future

Non - causal

- Time - Invariant or variant

① — Delay of function time

$$y_1(t-t_0) = x_1(t-2-t_0) + x_1(2-t-t_0)$$

Delay of time

② — $y_2(t-t_0) = x_2((t-t_0)-2) + x_2(2-(t-t_0))$

Since eq ≠ eq

the system is $\boxed{\text{Time Varaint}}$

Dynaimiac and instanenious

$$y(0) = x(-2) + x(2) \longrightarrow \text{Dynaimiac}$$

Dynaimiac كل فاليو غير لذلك

مشتقة او تكامل طواي بحكي Dynaimial

Neso Ac causal and Non-Causal System

① $y(t) = x(t)$

output just depend on present Value of input

يحكي انه cassal
ازا يعتمد على present
او present وباست
تع بعض

② $y(t) = x(t) + x(t-1) \longrightarrow$ causal

present, past

Non causal → depends on future values of input at any instant of time

(2) $y(t) = \cos(3t)\, x(t)$

linearity

$\alpha_1 y_1(t) = \cos(3t)\, \alpha_1 x_1(t)$

$\alpha_2 y_2(t) = \cos(3t)\, \alpha_2 x_2(t)$

$\alpha_1 y_1(t) + \alpha_2 y_2(t) = \cos(3t)\left[\alpha_1 x_1(t) + \alpha_2 x_2(t)\right] \longrightarrow ①$

Assume:

$\alpha_3 y_3(t) = \alpha_2 y_2(t) + \alpha_2 y_2(t)$ and $\alpha_3 x_3(t) = \alpha_1 x_1(t) + \alpha_2 x_2(t)$

$\cancel{\alpha_3 y_3(t) = \cos 3t\, \alpha_3 x_3(t) \rightarrow}$

$\alpha_3 y_3(t) = \cos(3t)\, \alpha_3 x_3(t) \rightarrow \alpha_1 y_1(t) + \alpha_2 y_2(t) = \cos(3t)\left[\alpha_1 x_1(t) + \alpha_2 x_2(t)\right]$

eq(1) = eq(2) $\longrightarrow$ linear

causilty

$y(t) = \cos(3t)\, x(t) \longrightarrow$ causal

Instaneous or dynamic $\longrightarrow$ instaneous $\longrightarrow$ Memory less

$\boxed{y(0)} = \cos(3\cdot t)\, \boxed{x(0)}$

طالما يلي بين الاقواس نفس
بحكي instan

Time invairant

Delay function of time

$y_1(t - t_0) = \cos(3t - t_0)\, x_1(t - t_0) \rightarrow ①$

Delay of time

$y_2(t - t_0) = \cos(3(t - t_0))\, x_1(t - t_0) \rightarrow ②$

eq (1) $\neq$ eq (2) $\rightarrow$ time-Variant

[3] $y(t) = \int_{-\infty}^{2t} x(t)\, dt$

$\alpha_1 y_1(t) = \int_{-\infty}^{2t} \alpha_1 x_1(t)\, dt$ and $\alpha_2 y_2(t) = \int_{-\infty}^{2t} \alpha_2 x_2(t)\, dt$

linearity:

$\alpha_1 y(t) + \alpha_2 y_2(t) = \int_{-\infty}^{2t} \left(\alpha_1 x_1(t) + \alpha_2 x_2(t)\right) dt \longrightarrow ①$

Assume $\alpha_3 x_3(t) = \alpha_1 x_1(t) + \alpha_2 x_2(t)$ & $\alpha_3 y_3(t) = \alpha_1 y_1(t) + \alpha_2 y_2(t)$

$$\alpha_3 y_3(t) = \int_{-\infty}^{2t} \alpha_3 x_3(t)\, dt \longrightarrow \alpha_1 y_1(t) + \alpha_2 y_2(t) = \int_{-\infty}^{2t} \alpha_1 x_1(t) + \alpha_2 x_2(t)\, dt$$

Since eq (1) = eq (2) $\longrightarrow$ linear

$\boxed{\text{causality}}$

$$y(t) = \int_{-\infty}^{2t} x(t)\, dt \longrightarrow \text{non-causal}$$

$$v(t)\Big|_{-\infty}^{2t} \longrightarrow$$

يعني لو عوضت واحد الجواب 2

$\boxed{\text{time-invariant}}$

Delay function of time

$$y_1(t-t_0) = \int_{-\infty}^{2t-t_0} x_1(\tau)\, d\tau \longrightarrow ①$$

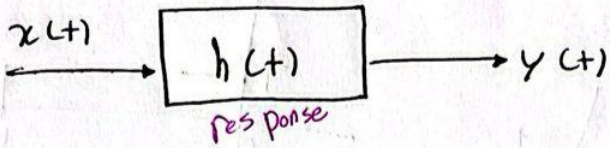
$$y_2(t-t_0) = \int_{-\infty}^{2(t-t_0)} x_2(\tau)\, d\tau \longrightarrow ②$$

$eq_1 \neq eq_2$

time-variant

Since I deal with $\int$ so the system is $\boxed{\text{Dynaimiac}}$

LTI System



For LTI

$$y(t) = x(t) * h(t)$$

$$= \int_{-\infty}^{\infty} x(t-\lambda)\, h(\lambda)\, d\lambda$$

$\boxed{OR}$

$$= \int_{-\infty}^{\infty} x(\lambda)\, h(t-\lambda)\, d\lambda$$

by using change of variable

let $u = t - \lambda \longrightarrow du = -d\lambda$

when $\lambda = -\infty : u = \infty$

when $\lambda = \infty \quad u = -\infty$

$$\int_{\infty}^{-\infty} x(u)\, h(t-u)\, -du$$

$$= \int_{-\infty}^{\infty} x(u)\, h(t-u)\, du$$

Example: For LTI system if

$x(t) = 2\Pi\left(\frac{t-5}{2}\right)$ and $h(t) = \Pi\left(\frac{t-2}{4}\right)$

$= 2\Pi\left(\frac{1}{2}(t-5)\right)$

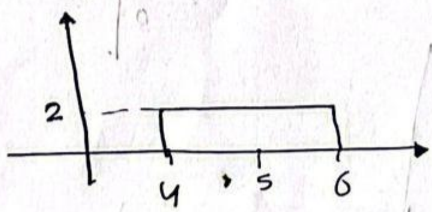
$\alpha = 2 \quad \beta = 5$

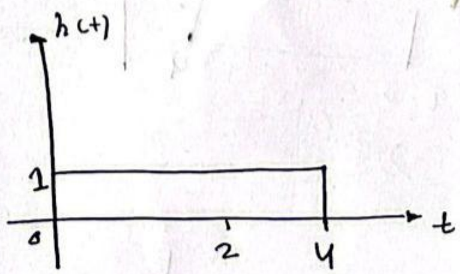
Find $y(t)$

Ans: For LTI system

① $y(t) = x(t) * h(t)$

②





③ $[4, 6] \qquad [0, 4]$

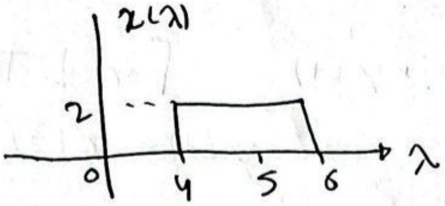
$[4+0,\ 4+4,\ 6+0,\ 6+4] \longrightarrow [4, 8, 6, 10]$

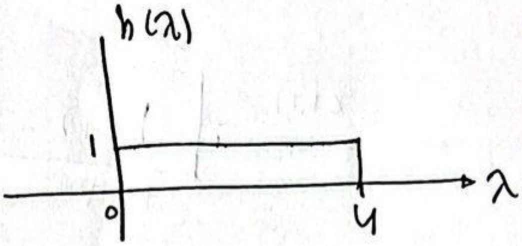
$\longrightarrow [4, 6, 8, 10]$

بحدود range يلي بدو

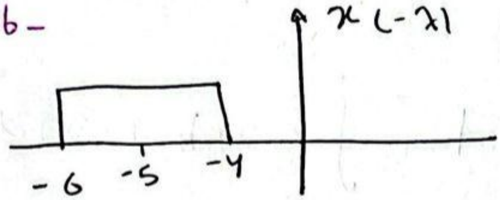
④ $y(t) = \int_{-\infty}^{\infty} x(t-\lambda)\, h(\lambda)\, d\lambda$

a-

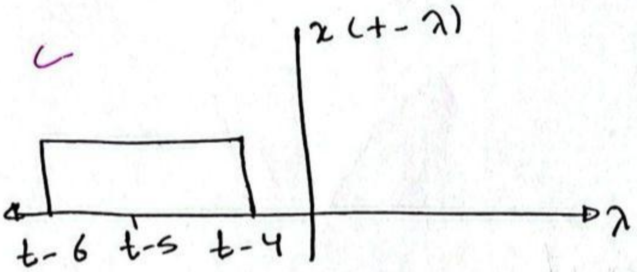


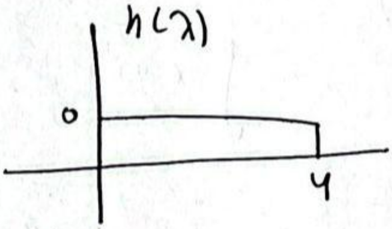


b-

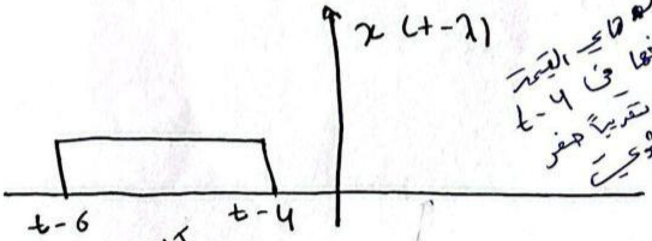


c



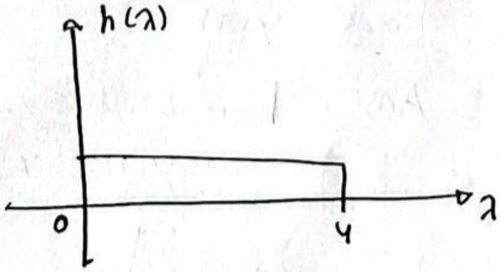


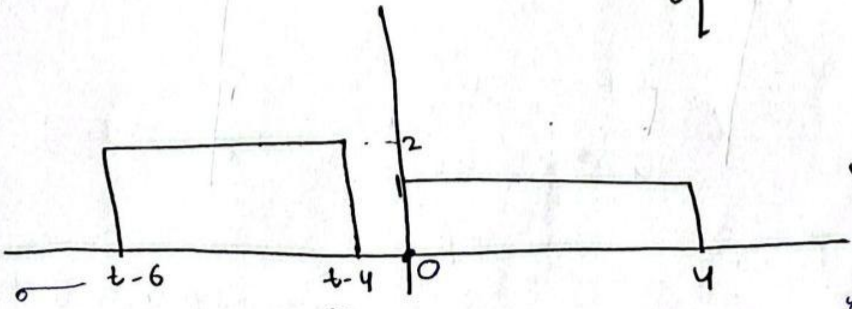
⑤ for $-\infty < t < 4$



لو كانت بعوضها في القيمة t-4 فيعطي تقريباً صفر ولا شوي

كأنه يتحرك





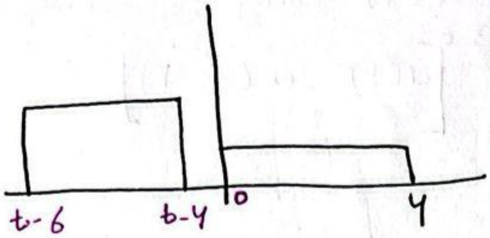
$y(t) = 0$

فش إشي مشترك

4-6
-2

وقف قبل الصفر بشوي

for $4 < t < 6$

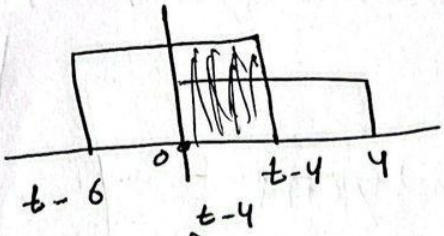


$t-4$

$6-4=2$

$t-6$

$\cancel{4-6 = -2}$



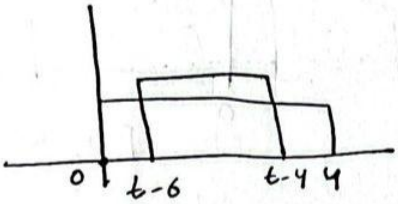
$$y(t) = \int_0^{t-4} (2)(1)\, d\lambda$$

$$= 2\lambda \Big|_0^{t-4}$$

$$= 2(t-4) - 0$$

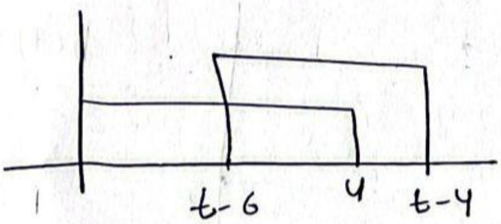
$$= 2t - 8$$

for $6 < t < 8$



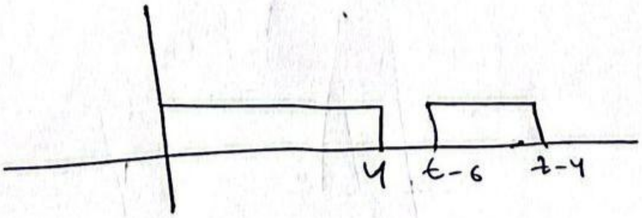
$$y(t) = \int_{t-6}^{t-4} (2)(1)\, d\lambda$$

for $8 < t < 10$



$$y(t) = \int_{t-6}^{4} (2)(1)\, d\lambda$$

for $10 < t < \infty$



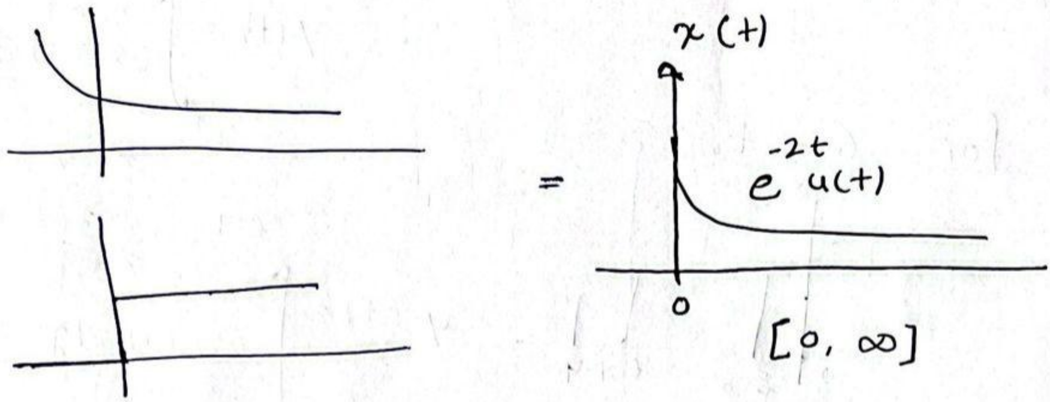
$y(t) = 0$

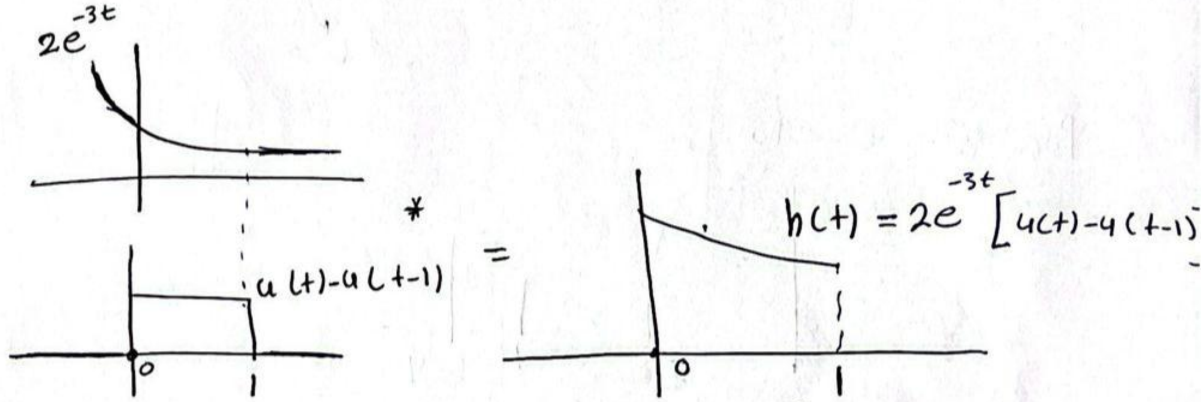
مش اشي مشترك

For LTI system if $x(t) = e^{-2t} u(t)$ and

$$h(t) = 2e^{-3t}[u(t) - u(t-1)]$$

Find $y(t)$?



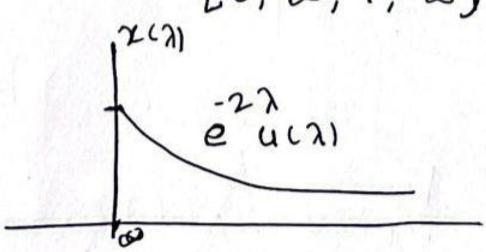


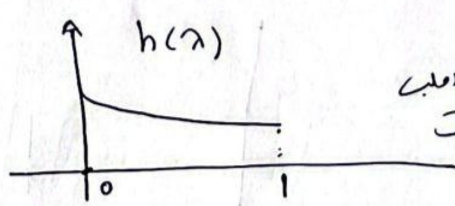
[0,1]

لما اضرب يلي بالازرق بيردوا

[0, ∞)   [0, 1]

$[0, \infty, 1, \infty] = [0, 1, \infty)$



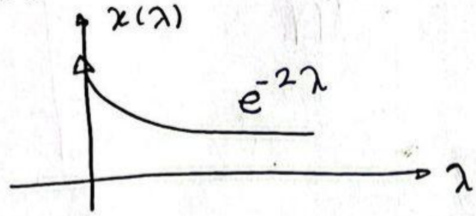


الا وقت اطلب الثابتات

تكملة السؤال صفحة 15

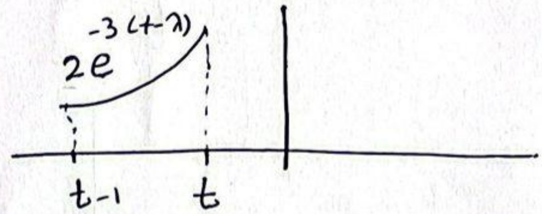
تكملة السؤال
صفحة 12

$$y(t) = \int_{-\infty}^{\infty} x(\lambda)\, h(t-\lambda)\, d\lambda$$

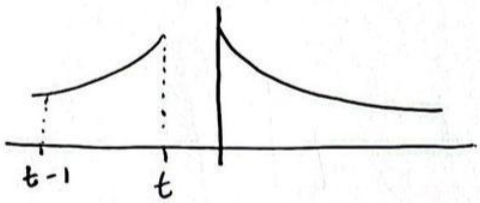


$$h(\lambda) = 2e^{-3\lambda}$$

$$h(t-\lambda) = 2e^{-3(t-\lambda)}$$

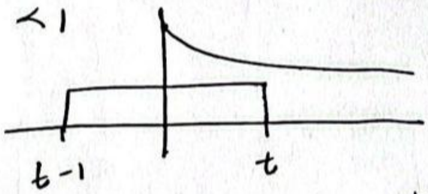


for $-\infty < t < 0$



$$y(t) = 0$$

for $0 < t < 1$



$$y(t) = \int_{0}^{t} e^{-2\lambda}\, 2e^{-3(t-\lambda)}\, d\lambda$$

$$y(t) = \int_{0}^{t} e^{-2\lambda} . 2\, e^{-3(t-\lambda)}\, d\lambda$$

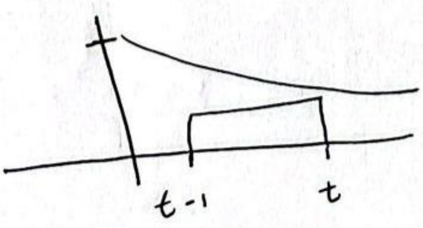
$$= \int_{0}^{t} 2e^{-3t} . e^{-2\lambda} . e^{+3\lambda}\, d\lambda$$

$$= 2e^{-3t} \int_{0}^{t} e^{\lambda}\, d\lambda = 2e^{-3t} e^{\lambda} \Big|_{0}^{t}$$

$$= 2e^{-3t} e^{t} - 2e^{-3t} e^{0}$$

$$= 2e^{-2t} - 2e^{-3t}$$

for $1 < t < \infty$

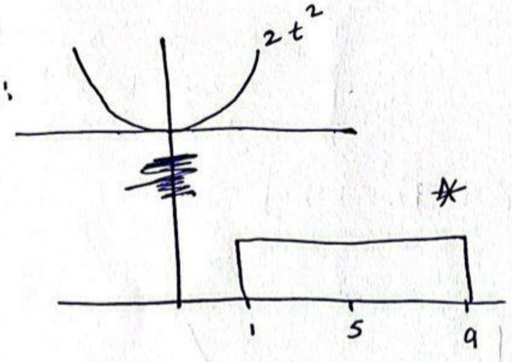


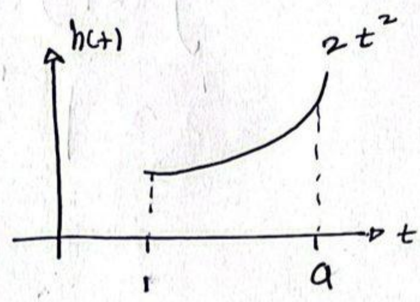
$$y(t) = 2e^{-3t} \int_{t-1}^{t} e^{-\lambda}\, d\lambda$$

Example: compute using the convolution Integral, the response of the LTI with impulse response $h(t) = 2t^2 \Pi\left(\frac{t-5}{8}\right)$ do the input $x(t) = \Pi\left(\frac{t-10}{2}\right)$ Find $y(t)$

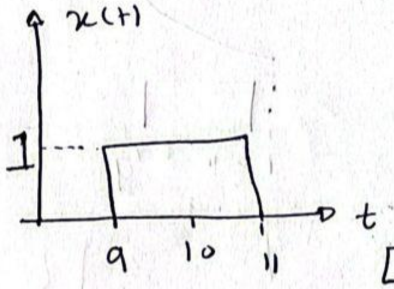
Ans: LTI $\rightarrow y(t) = x(t) * h(t)$

h(t):



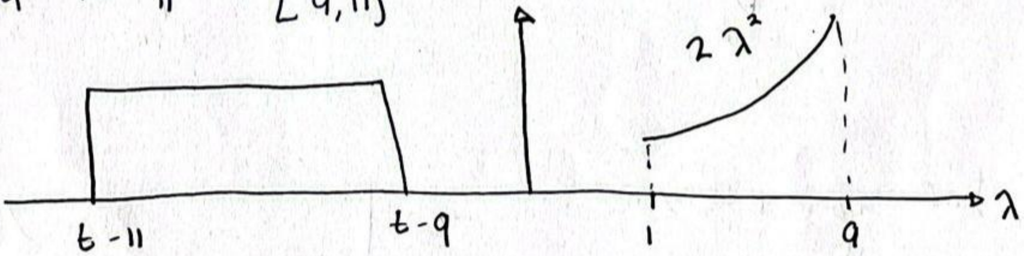


[1, 9]



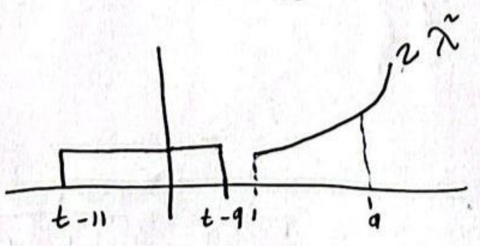
[9, 11]

③ $y(t) = \int_{-\infty}^{\infty} x(t-\lambda)\, h(\lambda)\, d\lambda$



$[1+9, 1+11, 9+9, 9+11] = [10, 12, 18, 20]$

- for $-\infty < t < 10$

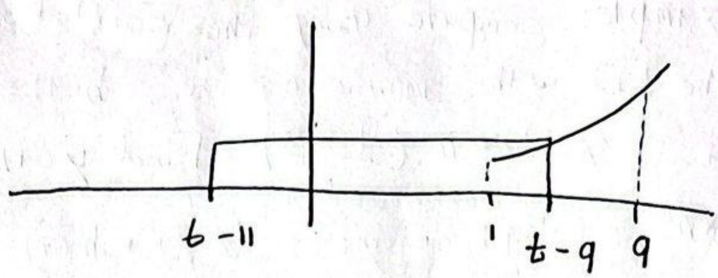


$y(t) = 0$

ما مروا على بعض

for $10 < t < 12$

$10 - 11 = -1$
$12 - 9 = 3$

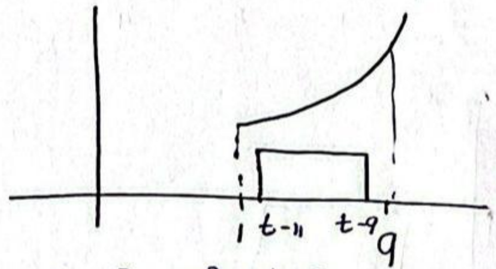


$$y(t) = \int_{1}^{t-9} (2\lambda^2)(1)\, d\lambda$$

اذا جبلي دالة متغيرات وواحد ثابت
بختار اللي اعمل شيفت ل الثابتة
عشان يكون التكامل اسهل

$$= \frac{2\lambda^3}{3}\Big|_{1}^{t-9} = \frac{2}{3}\left[(t-9)^3 - (1)^3\right]$$

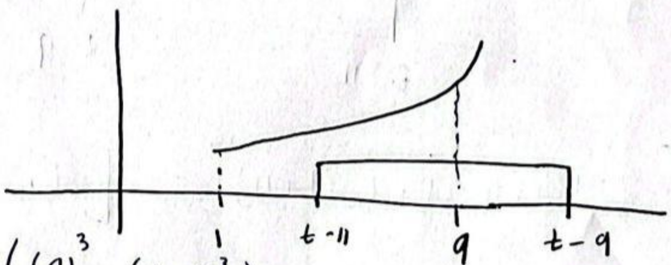
for $12 < t < 18$



≤ 1 9

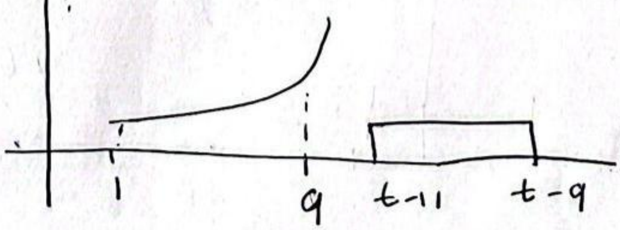
$$y(t) = \int_{t-11}^{t-9} (2\lambda^2)(1)\, d\lambda = \frac{2}{3}\lambda^3\Big|_{t-11}^{t-9} = \frac{2}{3}\left[(t-9)^3 - (t-11)^3\right]$$

for $18 < t < 20$



$$y(t) = \int_{t-11}^{9} (2\lambda^2)(1)\, d\lambda = \frac{2}{3}\lambda^3\Big|_{t-11}^{9} = \frac{2}{3}\left((9)^3 - (t-11)^3\right)$$

for $20 < t < \infty$



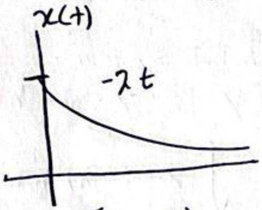
$$y(t) = 0$$

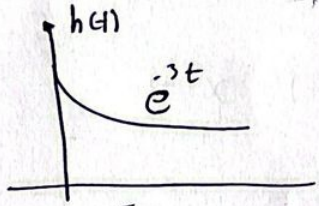
Example: for LTI system $x(t) = e^{-2t} u(t)$

$h(t) = e^{-3t} u(t)$

Find $y(t)$.

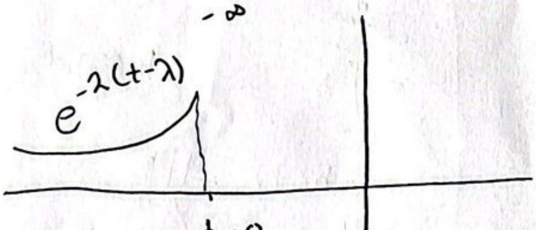
Ans: for LTI system $\longrightarrow y(t) = x(t) * h(t)$

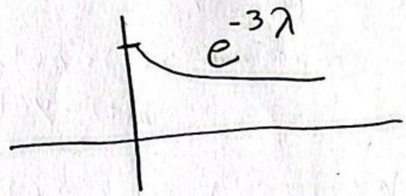




$\longrightarrow [0, \infty)$

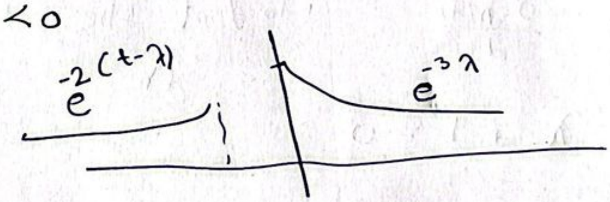
$$y(t) = \int_{-\infty}^{\infty} x(t-\lambda)\, h(\lambda)\, d\lambda$$





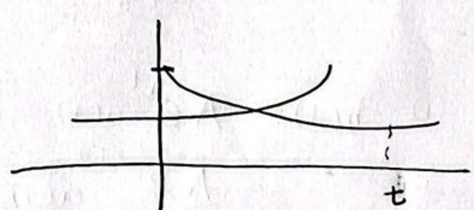
for $-\infty < t < 0$

$y(t) = 0$



for $0 < t < \infty$

$$y(t) = \int_{0}^{\infty} e^{-2(t-\lambda)} \cdot e^{-3\lambda}\, d\lambda$$



$$= \int_{0}^{\infty} e^{-2t} \cdot e^{+2t} \cdot e^{-3\lambda}\, d\lambda$$

$$= e^{-2t} \int_{0}^{\infty} e^{-\lambda}\, d\lambda$$

$$= -e^{-2t} \left[e^{-\lambda} \Big|_0^{\infty} \right]$$

$$= -e^{-2t} [0 - 1]$$

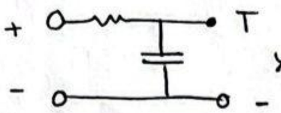
$$= e^{-2t}$$

Impulse Response of a fixed linear system $\longrightarrow$ LTI

$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$$

هذا الشكل النظام ح احصل على معادلة تفاضلية

طبيعة input يلي بدي ادخلها على النظام عبارة عن impulse



$y(t)$ DFE

$$RC\frac{dy(t)}{dt} + y(t) = x(t)$$

Impulse Response

$$x(t) = \delta(t)$$

for LTI system

$$y(t) = x(t) * h(t)$$

$$= \int_{-\infty}^{\infty} x(t-\lambda)\, h(\lambda)\, d\lambda$$

$$= \int_{-\infty}^{\infty} \delta(t-\lambda)\, h(\lambda)\, d\lambda$$

$$= \int_{-\infty}^{\infty} \delta(-(\lambda-t))\, h(\lambda)\, d\lambda$$

$$= \int_{-\infty}^{\infty} \delta(\lambda-t)\, h(\lambda)\, d\lambda$$

$$= h(t) \quad \#$$

يعني $y(t)$ هي $h(t)$

Example: find the impulse response of a system modeled by the differentiel eqution

$$t_0 \frac{dy(t)}{dt} + y(t) = x(t)$$

Ans: ① impulse respone $\to$ $x(t) = \delta(t)$

$h(t) = y(t)$

② $$\tau_0 \frac{dh(t)}{d(t)} + h(t) = \delta(t)$$

③ let $\lambda^n = \dfrac{\partial^n h(t)}{dt}$ $\longrightarrow$ to find the roots of DFE

$$\frac{dh(t)}{dt} = \lambda^1$$

$$h(t) = \lambda^0 = 1$$

$$t_0 \lambda + 1 = 0$$

$$\boxed{\lambda = \frac{-1}{\tau_0}}$$

$$g(t) = A e^{-t/\tau_0}$$

④ $$h(t) = A e^{\lambda t} u(t)$$

نفس مكنا نحل في الديفرنشال مسكنداورار

$$= A e^{-t/\tau_0} u(t) \qquad = g(t)\,u(t)$$

فرضت

عشان يصير اسهل لقدام

هذا معادلة 2 ومعادلة 4

يصير اشارك بينهم واحدة ذستك $h(t)$ مرة بعد بين اقربها د

واصبها $h(t)$

⑤ $h(t) = g(t)\,u(t)$ and $\dfrac{dh(t)}{dt} = g(t)\dfrac{\partial u(t)}{\partial t} + g'(t)\,u(t)$

دايما مشتقة u.s عبارة عن delta

$$= g(t)\,\delta(t) + g'(t)\,u(t)$$

$$\tau_0\left[g(t)\delta(t) + g'(t)u(t)\right] + g(t)u(t) = \delta(t)$$

$$\tau_0 g(t)\delta(t) + t_0 g'(t)u(t) + g(t)u(t) = \delta(t)$$

$$\tau_0 g(0)\delta(t) + \tau_0 g'(t)u(t) + g(t)u(t) = \delta(t)$$

Since $g(t) = Ae^{-t/\tau_0}$

$g(0) = Ae^{0} = A$

⑥ to find A

بساوي معاملات $\delta(t)$

$\tau_0 g(0) = 1$

$t_0 A = 1$

$$\boxed{A = \frac{1}{\tau_0}}$$

⑦ $h(t) = Ae^{\lambda t}u(t)$

$$= \frac{1}{\tau_0}e^{-t/\tau_0}u(t)$$

Example: find the impulse response of the system $t_0\dfrac{dy(t)}{dt} + y(t) = x(t-2)$

① Impulse response $x(t) = \delta(t)$ and $h(t) = y(t)$

$$t_0\frac{\partial h_1(t)}{dt} + h(t) = \delta(t-2)$$

② To find the Solution

وحل السؤال كأنه ما عندي شيفت

بس طبعا بحل السؤال كامل. $h_1(t) = \dfrac{1}{T_0}e^{-t/t_0}u(t)$

الشفت بس بسويها بآخر خطوة

③ $h(t) = h_1(t-2)$

$$h(t) = \frac{1}{T_0}e^{-(t-2)/T_0}u(t-2)$$

Superposition Integral interms of response of LTI system

$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$$

For LTI:

$$y(t) = \int_{-\infty}^{\infty} x(t-\lambda)\, h(\lambda)\, d\lambda$$

By using Integral by parts

$u = x(t-\lambda)$ $\qquad dv = h(\lambda)\, d\lambda$

$du = -\dot{x}(t-\lambda)$ $\qquad v = \int_{-\infty}^{\infty} h(\lambda)\, d\lambda = a(t)$

$$x(t-\lambda)\, a(\lambda) \Big|_{-\infty}^{\lambda} + \int_{-\infty}^{t} x(t-\lambda)\, a(\lambda)\, d\lambda$$

مش مقيدين
نحل بالاشتقاق

In general:-

$$y(t) = \int_{-\infty}^{\infty} \dot{x}(t-\lambda)\, a(\lambda)\, d\lambda$$

Duhamel's theorem

if $x(t) = u(t)$

$$y_s(t) = \int_{-\infty}^{\infty} \cancel{\delta(t)}\, \delta(t-\lambda)\, a(\lambda)\, d\lambda = a(t)$$

$$= \int_{-\infty}^{t} h(\lambda)\, d\lambda$$

لما يجيني في السؤال اوجدلي
Step response
يعني بكامل Impulse response

طب لو ram بكامل step
(بكامله مرتين)

Example: consider the LTI system defined by the following DFE

$$t_0 \frac{\partial y(t)}{\partial t} + y(t) = x(t)$$

Find ① the impulse response ② the step response ③ the ramp response ④ the response of the system if

$$x_\Delta(t) = r(t) - 2r(t-1) + r(t-2)$$

Ans: the **impulse response** ① $h(t) = \frac{1}{t_0} e^{-t/t_0} u(t)$ شكل ما حلينا قبل

② the **step response** $y_s(t) = \int_{-\infty}^{t} h(\lambda)\, d\lambda = \int_{-\infty}^{t} \frac{1}{T_0} e^{-\lambda/t_0} u(\lambda)\, d\lambda$ بقدر اشيل u(λ) بس بالمقابل بغير حدود التكامل

$$= \int_{0}^{t} \frac{1}{t_0} e^{-\lambda/t_0}\, d\lambda$$

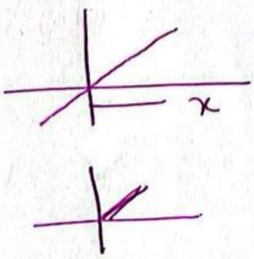
$$= \left[1 - e^{-t/t_0}\right] u(t)$$ دايما بضرب الجواب ب u(t)

③ the ramp response

$$y_r(t) = \int_{-\infty}^{t} y_s(\lambda)\, d\lambda = \int_{-\infty}^{t} (1 - e^{-\lambda/t_0})\, u(\lambda)\, d\lambda$$

$$= \int_{0}^{t} (1 - e^{-\lambda/t_0})\, d\lambda = t\,u(t) - t_0\left[1 - e^{-t/t_0}\right] u(t)$$

دايما بضرب ب u(t) بالجواب النهائي



$\rightarrow r(t) - t_0\left[1 - e^{-t/t_0}\right] u(t)$

④ the impulse for the $x_\Delta(t) = r(t) - 2r(t-1) + r(t-2)$

$$h_\Delta(t) = y_r(t) - 2y_r(t-1) + y_r(t-2)$$

where $y_r(t) = r(t) - \tau_0\left[1 - e^{-t/\tau_0}\right]u(t)$

$$y_r(t-1) = r(t-1) - \tau_0\left[1 - e^{-(t-1)/\tau_0}\right]u(t)$$

$$y_r(t-2) = r(t-2) - \tau_0\left[1 - e^{-(t-2)/\tau_0}\right]u(t)$$

Example: consider the following LTI system defined by DEF

$$2\frac{dy(t)}{dt} + y(t) = x(t)$$

① find impulse response of the system
② Find step response of the system
③ find ramp response of the System
④ find for response of $x_\Delta(t) = 3u(t-1) + 2r(t-1) + 4\delta(t-2)$

1 impulse response $x(t) = \delta(t)$ and $y(t) = h(t)$

$$2\frac{\partial h(t)}{\partial t} + h(t) = \delta(t)$$

let $\lambda^n = \frac{\partial^n h(t)}{\partial t}$

$$\frac{\partial h(t)}{\partial t} = g(t)\frac{\partial u(t)}{\partial t} + g'(t)u(t)$$

$$= g(t)\delta(t) + g'(t)u(t)$$

$$= g(0)\delta(t) + g'(t)u(t)$$

$2\lambda + 1 = 0$

$$\boxed{\lambda = -\frac{1}{2}}$$

$h(t) = A e^{\lambda t} u(t)$

$= A e^{-t/2} u(t)$

$= g(t)\, u(t)$

21 ذكرة

$2g(0)\delta(t) + 2g'(t)u(t) + g(t)u(t) = \delta(t)$

$2g(0) = 1$

$\boxed{g(0) = \frac{1}{2}}$ sinse $g(0) = A$ so $\boxed{A = \frac{1}{2}}$

$h(t) = \frac{1}{2} e^{-t/2} u(t)$

② the step response $y_s = \int_{-\infty}^{t} h(\lambda)\,d\lambda = \int_{-\infty}^{t} \frac{1}{2} e^{-\lambda/2} u(\lambda)\,d\lambda$

$= \int_{0}^{t} \frac{1}{2} e^{-\lambda/2}\,d\lambda = [1 - e^{-t/2}]u(t)$

③ $y_r(t) = \int_{-\infty}^{t} y_s(\lambda)\,d\lambda = \int_{-\infty}^{t} [1 - e^{-\lambda/2}]u(\lambda)\,d\lambda$

$= r(t) - 2[1 - e^{-t/2}]u(t)$

④ $x_\Delta(t) = 3u(t-1) + 2r(t-1) + 4\delta(t-2)$

$h_\Delta(t) = 3y_s(t-1) + 2y_r(t-1) + 4h(t-2)$

e.g $\quad t_0 \dfrac{dy(t)}{dt} + y(t) = 2\,r(t-2) + 3u(t-1) + \delta(t-3)$

ramp response — step response — impulse response

To find the solution, we have to find:-

$$h(t) = \frac{1}{t_0} e^{-t/t_0} u(t)$$

$$y_s(t) = \left(1 - e^{-t/t_0}\right) u(t)$$

$$y_r(t) = r(t) - t_0\left[1 - e^{-t/t_0}\right] u(t)$$

$$h_t(t) = 2y_r(t-2) + 3y_s(t-1) + h(t-3)$$

Frequency Response for LTI system

$$\xrightarrow[e^{j\omega t}]{x(t)} \boxed{h(t)} \longrightarrow y(t)$$

آبغى احول التايم دومين
الى فريكونسي دومين
بحل هيك

For LTI system

$$y(t) = x(t) * h(t)$$

$$= \int_{-\infty}^{\infty} x(t-\lambda)\, h(\lambda)\, d\lambda$$

$$= \int_{-\infty}^{\infty} e^{j\omega(t-\lambda)}\, h(\lambda)\, d\lambda = e^{j\omega t} \underbrace{\int_{-\infty}^{\infty} e^{-j\omega\lambda}\, h(\lambda)\, d\lambda}_{H(\omega)}$$

هني بكامل بالنسبة ل λ

$$= H(\omega)\, e^{j\omega t}$$

Example: Find the frequency response of RC-circuit

where $h(t) = \frac{1}{RC} e^{-t/RC} u(t)$

Ans: $H(\omega) = \int_{-\infty}^{\infty} h(\lambda) e^{-j\omega\lambda} d\lambda = \int_{-\infty}^{\infty} \frac{1}{RC} e^{-\lambda/RC} u(t) e^{-j\omega\lambda} d\lambda$

$$= \int_{0}^{\infty} \frac{1}{RC} e^{-\left(\frac{1}{RC} + j\omega\right)\lambda} d\lambda$$

$$= \frac{1}{RC} \cdot \frac{-1}{\frac{1}{RC} + j\omega} e^{-\left(\frac{1}{RC} + j\omega\right)} \Big|_{0}^{\infty}$$

$$= \frac{1}{RC} \cdot \frac{1}{\frac{1}{RC} + j\omega} = \frac{1}{1 + j\omega RC}$$

complex number

زاوية

$$H(\omega) = \frac{1}{1 + j\omega RC} = \frac{1 \angle 0}{\sqrt{(1)^2 + (\omega RC)^2} \angle \tan^{-1}\left(\frac{\omega RC}{1}\right)}$$

طرحت الزاوية يلي بالمقام عشان يلي بالبسط

In general

قسمت الماجنتيود يلي بالبسط على يلي بالمقام

$$= \frac{1}{\sqrt{1 + (\omega RC)^2}} \angle -\tan^{-1}\left(\frac{\omega RC}{1}\right)$$

$$= \frac{-\sin^{-1}(\omega RC)}{\cos^{-1}(\omega RC)}$$

عشان اطلع magnitude

$$\boxed{Z = x + jy}$$

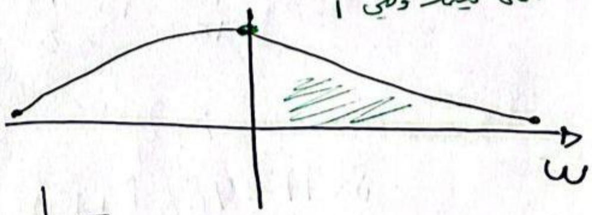
$|Z| = \sqrt{x^2 + y^2}$ — x^2: $(\text{real})^2$, y^2: $(\text{imaginary})^2$

عشان اطلع الزاوية

$\angle \theta_Z = \tan^{-1}\left(\frac{y}{x}\right)$ — y: imaginary, x: real

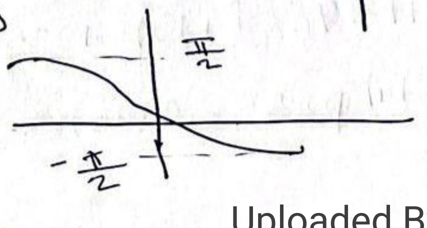
plot $|H(\omega)| = \frac{1}{\sqrt{1 + (\omega RC)^2}}$

لو الاوميجا صفر بيكون عند اعلى قيمة وهي 1



لو الاوميجا ∞ — $\frac{1}{\infty} = 0$

لو الاوميجا $-\infty$ — $\frac{1}{-\infty} = 0$



Power and Energy Spectral Density

- the energy spectral density ($G(f)$) is defined by

$$E = \int_{-\infty}^{\infty} G(f)\, \partial f$$ where E is the total energy

بشتابز لا ما الخوارف

- the power spectral density ($S(f)$) is defined by

$$P = \int_{-\infty}^{\infty} S(f)\, \partial f$$

where p is the average power of the signal:

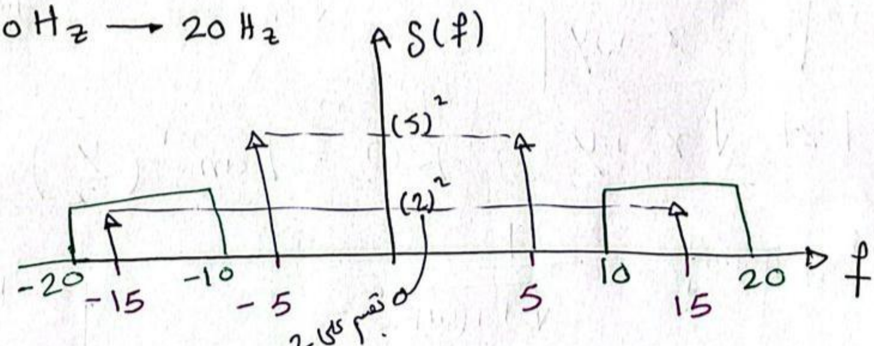
Example:- consider the signal

$$x(t) = 10 \cos\left(10\pi t + \frac{\pi}{7}\right) + 4 \sin\left(30\pi t + \frac{\pi}{8}\right)$$

$w_1 = 10H$, $2\pi f = 10H$, $f = 5$; $w_2 = 30H$, $2\pi f_2 = 30H$, $f_2 = 15$

ⓐ plot its power spectral density

ⓑ compute the power lying within a frequency band from $10\,Hz \rightarrow 20\,Hz$



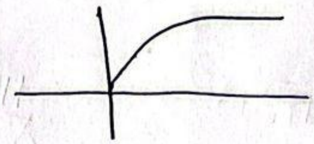
$$S(f) = 4\delta(f+15) + 25\delta(f+5) + 25\delta(f-5) + 4\delta(f-15)$$

Total power $= 2(25) + 2(4) = 58W$

$P_{(10\rightarrow 20)Hz} = 8W$

Stability

حالة الاستقرار



$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$$

LTI

Bounded يكون output
ما يروح ع ∞

$|y(t)| = \text{Const} < \infty$

bounded output

$$y(t) = x(t) * h(t)$$

$$= \int_{-\infty}^{\infty} x(\lambda)\, h(t-\lambda)\, d\lambda \quad \underline{\underline{\text{OR}}} \quad \int_{-\infty}^{\infty} x(t-\lambda)\, h(\lambda)\, d\lambda$$

$$|y(t)| = \left| \int_{-\infty}^{\infty} x(\lambda)\, h(t-\lambda)\, d\lambda \right| \leq \int_{-\infty}^{\infty} |x(\lambda)|\, |h(t-\lambda)|\, d\lambda$$

$|x(\lambda)| = M < \infty$

Bounded input

To check the stability of the system

$$\int_{-\infty}^{\infty} |h(t-\lambda)|\, d\lambda = \infty \quad \text{BIBO or Stable}$$

OR

$$\int_{-\infty}^{\infty} (h(\lambda))\, d\lambda < \infty \longrightarrow \text{BIBO or Stable}$$

Arithmetic shift left

1110010 $\underline{0}$ → logical

Example: For the following response

$$h(t) = \frac{1}{RC} e^{-t/RC} u(t)$$; check the stability of the system

Ans, To check the stability of the system

$$\int_{-\infty}^{\infty} |h(\lambda)|\, d\lambda = \int_{-\infty}^{\infty} \frac{1}{RC} e^{-\lambda/RC} = 1 < \infty$$

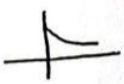
$u(\lambda)$ BIBO or stable

Invertibility and inverse system

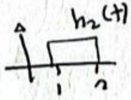
$$x(t) \rightarrow \boxed{h(t)} \rightarrow y(t)$$

Convolution type

• cascade type

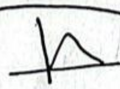


$h_2(t)$



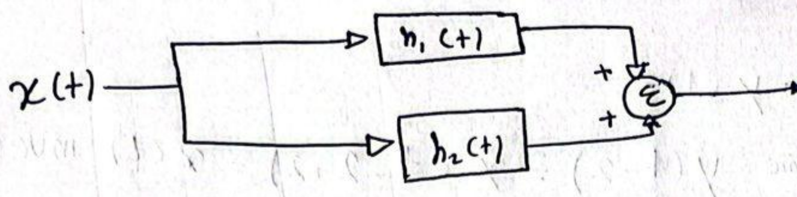
$$x(t) \rightarrow \boxed{h_1(t)} \xrightarrow{r(t)} \boxed{h_2(t)} \rightarrow y(t) = \xrightarrow{x(t)} \boxed{h_1(t) + h_2(t)} \rightarrow y(t)$$

LTI

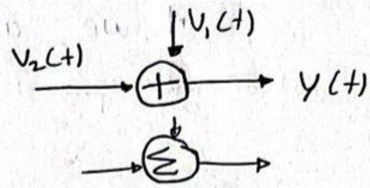


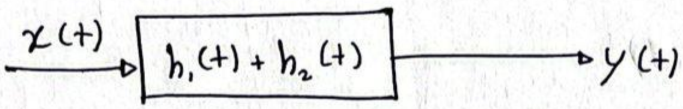
$$V(t) = x(t) * h_1(t) \rightarrow ①$$
$$y(t) = V(t) * h_2(t) \rightarrow ②$$
$$\rightarrow y(t) = x(t) * h_1(t) * h_2(t)$$

2. parallel type



$y(t) \equiv x(t) * [h_1(t) + h_2(t)]$

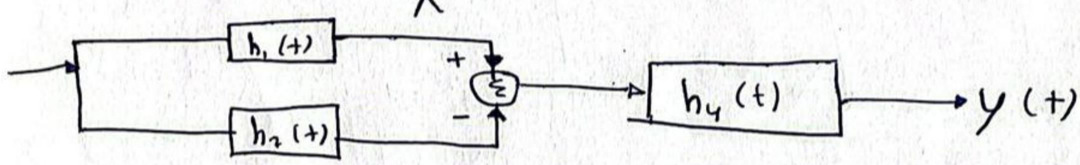




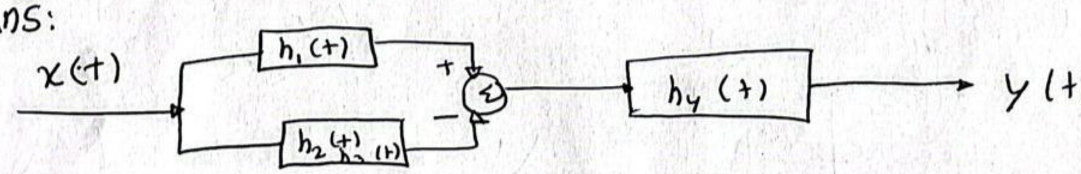
$V_1(t) = x(t) * h_1(t)$ $\qquad$ $V_2(t) = x(t) * h_2(t)$

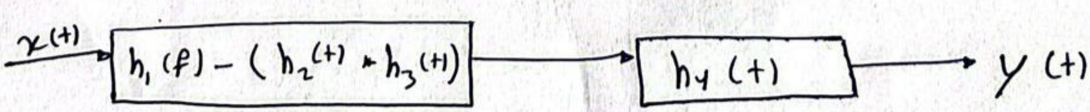
$$
\begin{aligned}
y(t) &= V_1(t) + V_2(t) \\
&= x(t) * h_1(t) + x(t) * h_2(t) \\
&= x(t) * (h_1(t) + h_2(t))
\end{aligned}
$$

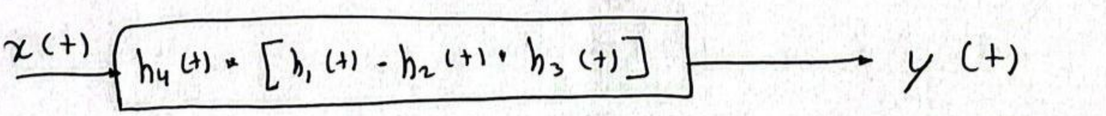
Ex: Reduced the following system



Ans:







Invertibility and inverse of system

e.g: ① $x(t+2) = y(t)$

if we assume $y(t-2) = x(t-2+2) = x(t)$ inver

بدي افحص اكس ادقيق تساوي الاشي المقابل وبعدك بحكم not or inver

② $x(2t) = y(t)$

$y(\frac{t}{2}) = x(2\frac{t}{2}) = x(t)$ inver

③ $y(t) = x^2(t)$ diffrent values of $y(t)$ non inver

ادطوني نفس الفاليو ل $x(t)$

-1 or $1 \rightarrow +1$

دايماً بتعطيني موجب

Example: Determine the observer model of the system defined by

$$\frac{d^4y}{dt^4} + 2\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 5y = 3\frac{d^3x}{dt^3} + 4\frac{dx}{dt} + 2x$$

separate: $$\frac{d^4y}{dt^4} + 2\frac{d^2y}{dt^2} + 3\frac{dy}{dt} - 3\frac{d^3x}{dt^3} - 4\frac{dx}{dt} = 2x - 5y = q_0$$

Integrate + separate $$\frac{d^3y}{dt^3} + 2\frac{dy}{dt} + 3\frac{d^2x}{dt^2} = \int_0^t q_0\, d\sigma + 4x \underbrace{-3y}_{\text{فصلت لاني نزلت درجة المشتقة}} = q_1$$

integrate + separate $$\frac{d^2y}{dt^2} - 3\frac{dx}{dt} = \int_0^t q_1\, d\sigma - 2y = q_2$$

integrate + separate $$\frac{dy}{dt} = \int_0^t q_2\, d\sigma + 3x = q_3$$

intergate $$y = \int_0^t q_3\, d\sigma$$

الخطوات

① يلي فيهم مشتقة بخليهم على طرف لحال ويلي الاورد لحالهم بحطهم على جهة (يلي فيهم y, x فقط)

② بعمل Impl بحيث تعبر عن eq الموجودة باليمين $q_0, q_1, \ldots$

③ بكامل حتى انزل order وبنفصل كمان مرة

④ بعبر عن الكلام ب plot