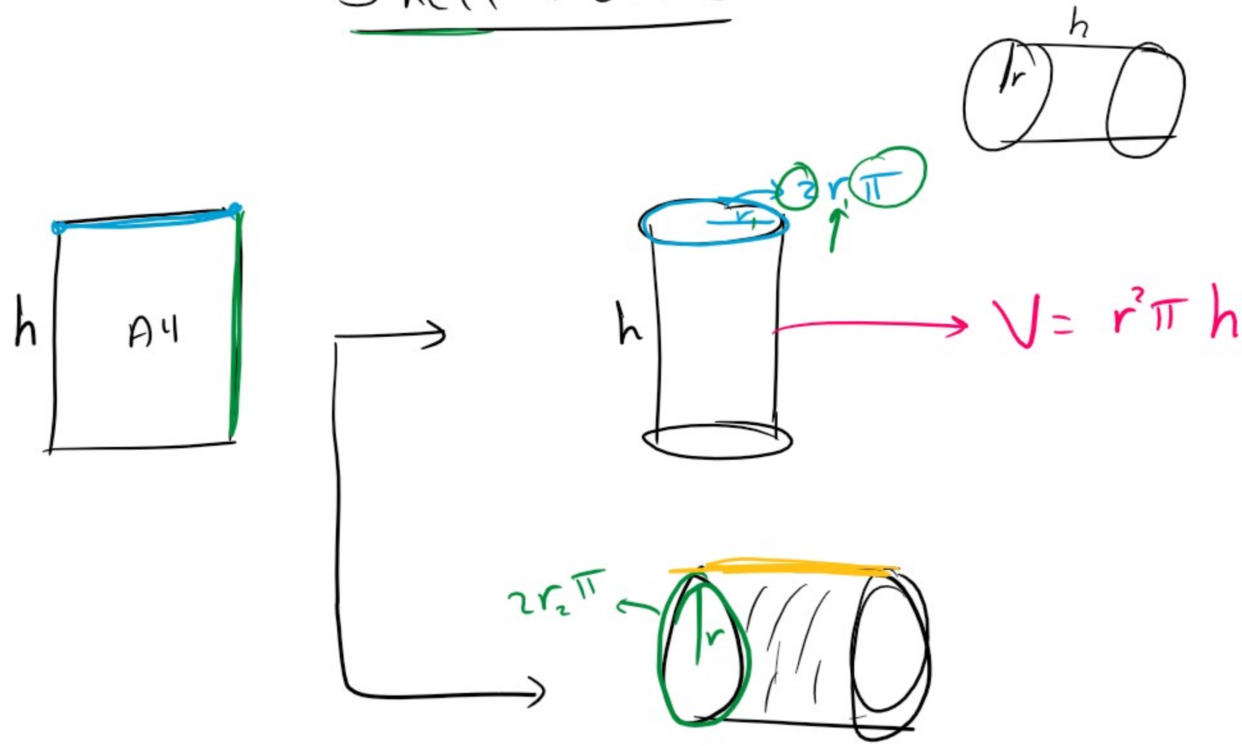
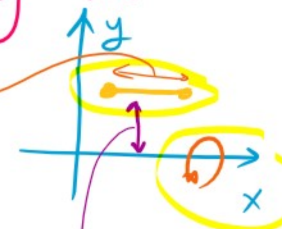
6.2

Monday, November 8, 2021 12:34 PM

Shell Method



(A) The volume of solid results by revolving the region about **x-axis** is



$$V = \int_c^d 2\pi \begin{pmatrix}\text{shell}\\ \text{radius}\end{pmatrix} \begin{pmatrix}\text{shell}\\ \text{length}\end{pmatrix} dy$$

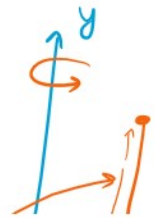
موازية لمحور الدوران (shell length)

البعد عن محور الدوران (shell radius)

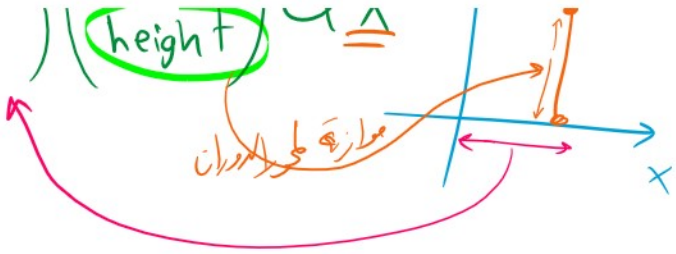
(B) = = = = = = = =

= = **y-axis** is

$$V = \int^b 2\pi \begin{pmatrix}\text{shell}\\ \text{radius}\end{pmatrix} \begin{pmatrix}\text{shell}\\ \text{height}\end{pmatrix} dx$$



$V = \int_a^b 2\pi \,(\text{radius})\,(\text{height})\,dy$



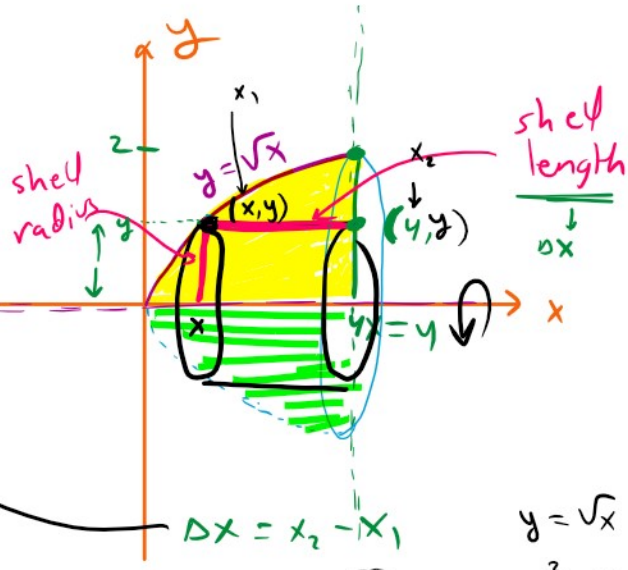
Use Shell Method to

Exp Find the volume of the solid generated by revolving the region bounded by

1) $y = \sqrt{x}$, x-axis, $x = 4$ about x-axis

$\Rightarrow$ y-axis

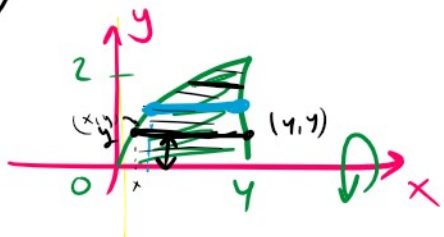
$V = \int_c^d 2\pi \,(\text{shell radius})\,(\text{shell length})\,dy$



$= \int_0^2 2\pi\,(y)(4 - y^2)\,dy$

$= 2\pi \int_0^2 (4y - y^3)\,dy$

$= 2\pi \left(2y^2 - \frac{y^4}{4}\right)\Big|_0^2 = \cdots = 8\pi$



2) $y = \sqrt{x}$, x-axis, $x = 4$ about y-axis

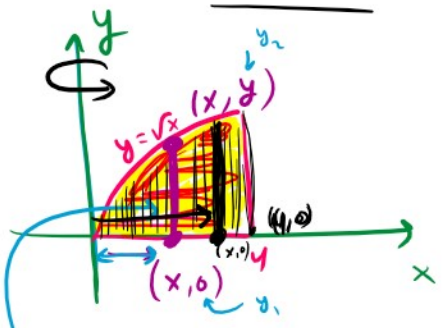
$V = \int_a^b (\text{shell})\,(\text{shell})\,dx$

$$V = \int_a^b 2\pi \left(\begin{matrix}\text{shell}\\ \text{radius}\end{matrix}\right)\left(\begin{matrix}\text{shell}\\ \text{height}\end{matrix}\right) dx$$

$$= \int_0^4 2\pi (x)\left(\sqrt{x}\right) dx$$

$$= 2\pi \int_0^4 x^{\frac{3}{2}}\, dx$$

$$= 2\pi \left.\frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1}\right|_0^4 = \cdots = \frac{128\pi}{5}$$



$\Delta y = y_2 - y_1$

$= y - 0$ $\quad y = \sqrt{x}$

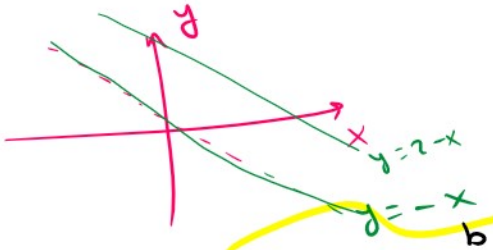
$= y$

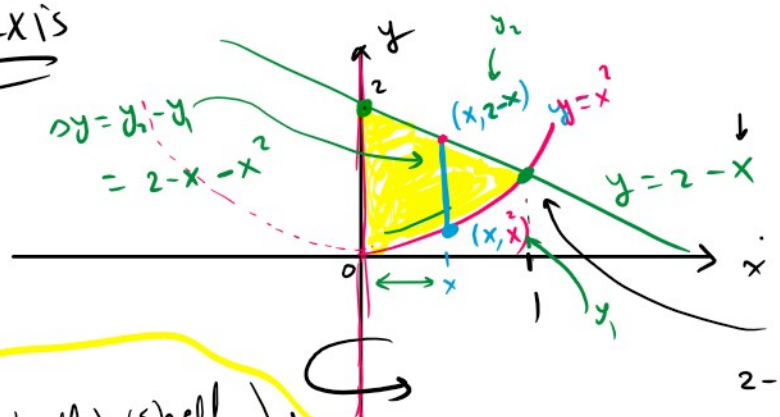
$= \sqrt{x}$

Q9

(3) $y = x^2$, $y = 2 - x$, $x = 0$, $x \geq 0$

about y-axis





$y = y$

$2 - x = x^2$

$x^2 + x - 2 = 0$

$(x + 2)(x - 1) = 0$

$x = -2$, $x = 1$

$\Rightarrow V = 2\pi \int_a^b \left(\begin{matrix}\text{shell}\\ \text{radius}\end{matrix}\right)\left(\begin{matrix}\text{shell}\\ \text{height}\end{matrix}\right) dx$ $\quad x = 0$

$$= 2\pi \int_0^1 x\left(2 - x - x^2\right) dx$$

$$= 2\pi \int_0^1 \left(2x - x^2 - x^3\right) dx = 2\pi \left(x^2 - \frac{x^3}{3} - \frac{x^4}{4}\right)\Big|_0^1$$

$= 2\pi \int_0^1 (2x - x^2 - x^2)\,dx = 2\pi \left(x - \frac{x^3}{3} - \frac{x^4}{4}\right)\Big|_0^1$

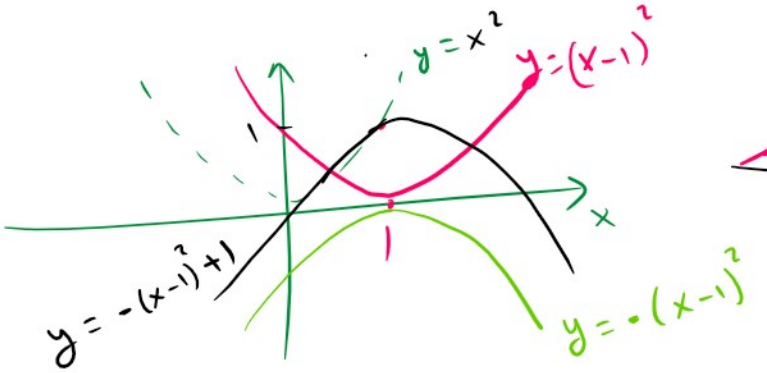
$= 2\pi \left(1 - \frac{1}{3} - \frac{1}{4}\right) = \frac{5\pi}{6}$

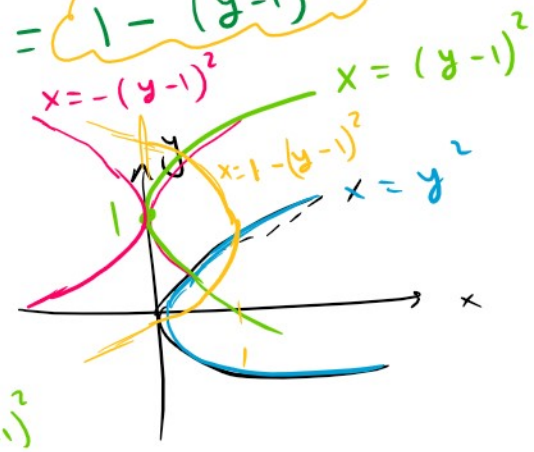
(4) $x = 2y - y^2$, y-axis about x-axis

$\Rightarrow V = 2\pi \int_c^d (\text{shell radius})(\text{shell length})\,dy$

$x = 2y - y^2 = -[y^2 - 2y] = -[y^2 - 2y + 1 - 1]$

$= -[(y-1)^2 - 1] = 1 - (y-1)^2$

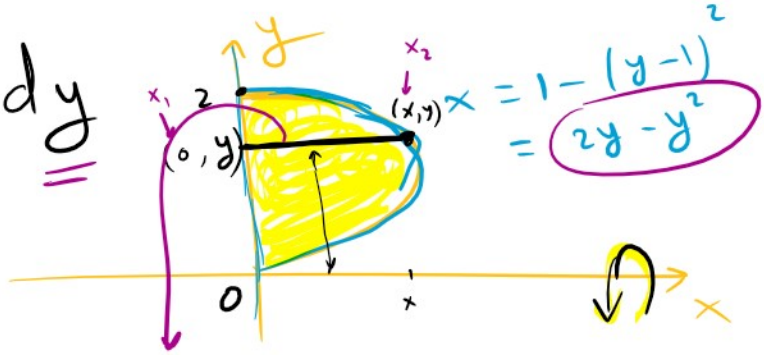




$x = 1 - (y-1)^2$

$V = 2\pi \int_0^2 (y)(2y - y^3)\,dy$ (shell length)

$= 2\pi \int_0^2 (2y^2 - y^3)\,dy$



$x = 1 - (y-1)^2 = 2y - y^2$

$\Delta x = x_2 - x_1$

when $x = 0 \Rightarrow 0 = 2y - y^2$

$= 2\pi \int_0^2 (\quad)$

$= 2\pi \left[2\frac{y^3}{3} - \frac{y^4}{4} \right]_0^2$

$= \dots = \frac{8\pi}{3}$

$\Delta x = x_2 - x_1$

$= x - 0$

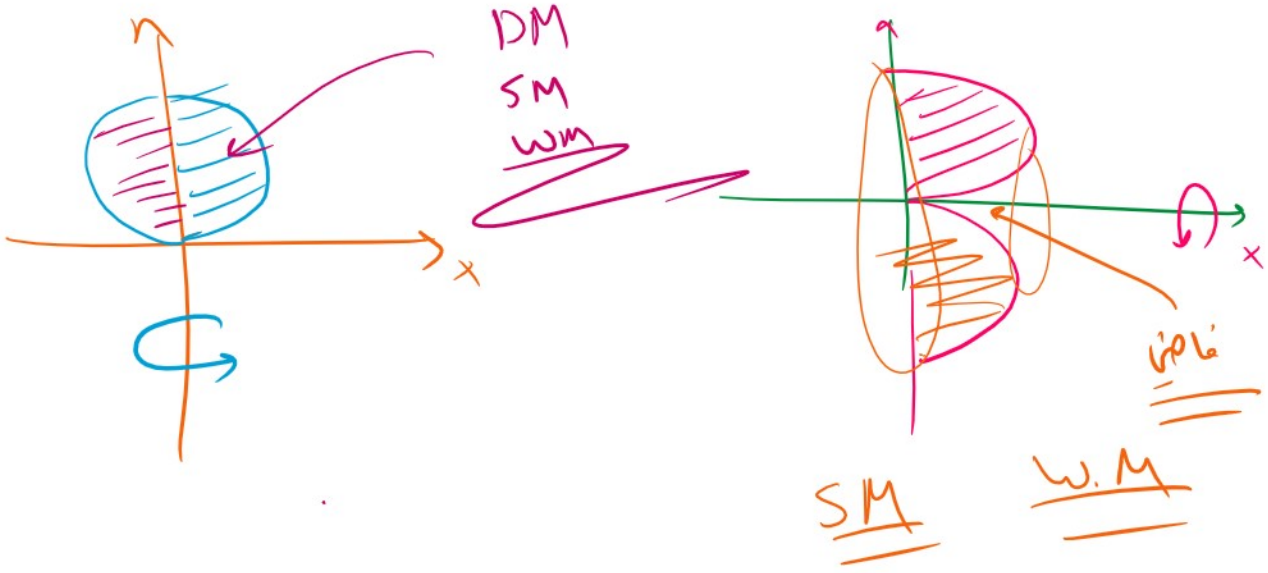
$= x$

$= 2y - y^2$

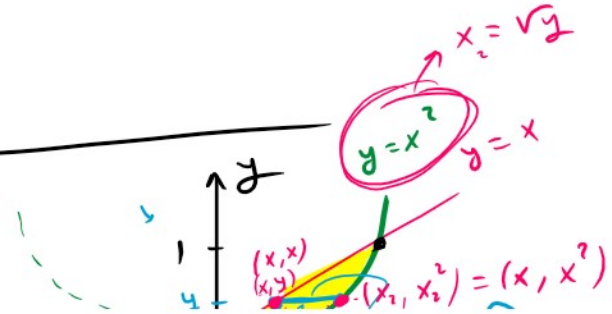
when $x = 0 \Rightarrow 0 = 2y - y^2$

$0 = y(2-y)$

$y = 0, \quad y = 2$



Q29 Find V of solid generated by rotating the region bounded by $y = x$, $y = x^2$ about x-axis and y-axis using SM and WM



$y = y$

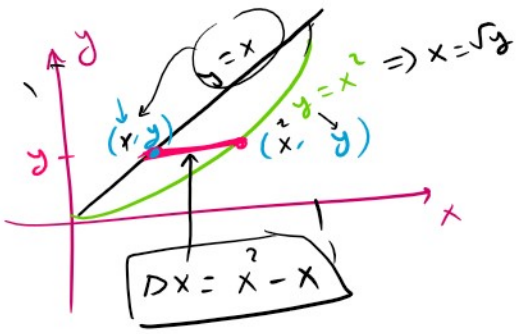
$y = y$

$x^2 = x$

$x^2 - x = 0$

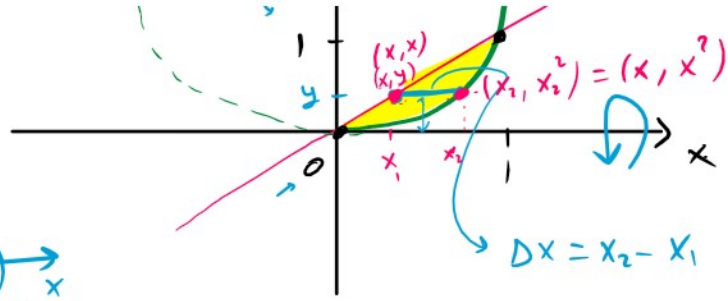
$x(x-1) = 0$

$x = 0, \quad x = 1$



$\Rightarrow x = \sqrt{y}$

$= \sqrt{y} - y$



$$V = 2\pi \int_c^d \left(\text{shell radius}\right)\left(\text{shell length}\right) dy$$

$$= 2\pi \int_0^1 y\,(\sqrt{y} - y)\,dy$$

$$= 2\pi \int_0^1 \left(y^{\frac{3}{2}} - y^2\right) dy$$

$$= 2\pi \left[\frac{y^{\frac{3}{2}+1}}{\frac{3}{2}+1} - \frac{y^3}{3}\right]_0^1 = \cdots = \frac{2\pi}{15}$$

$R(x) = DY$

$= y_2 - y_1$

$= 0 - 0$

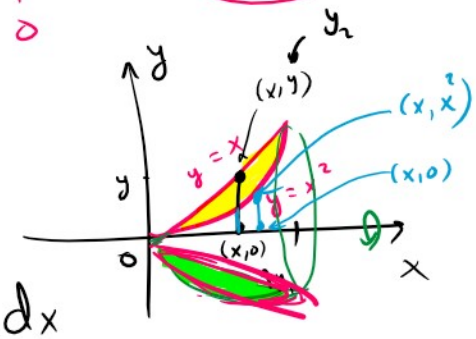
$= y$

$= x$

$r(x) = DY = y_2 - y_1$

$= x^2 - 0$

$= x^2$

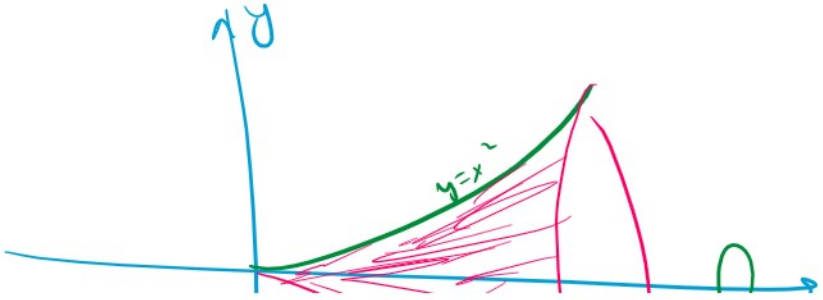


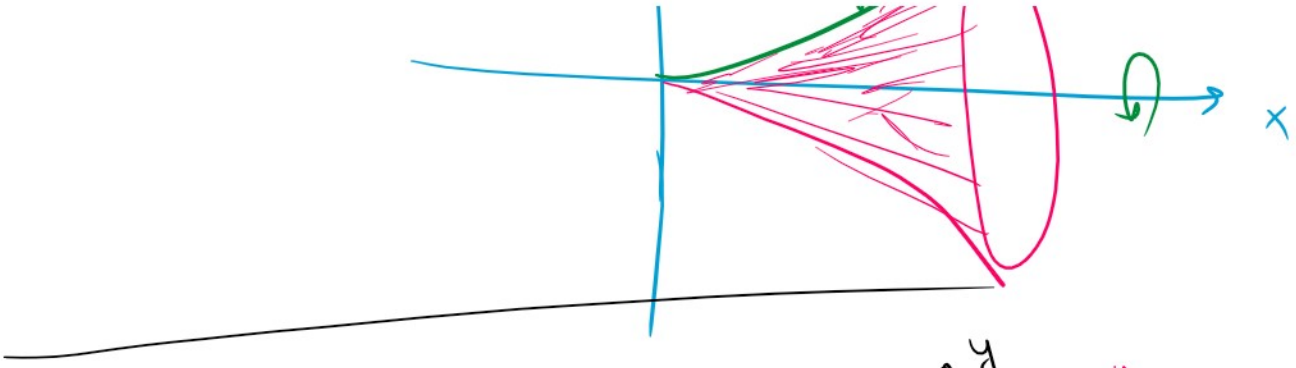
$$V = \pi \int_a^b \left[R^2(x) - r^2(x)\right] dx$$

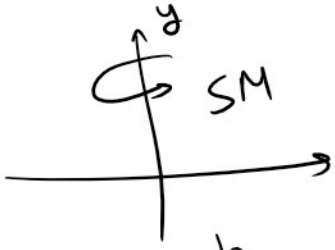
$$= \pi \int_0^1 \left[(x)^2 - (x^2)^2\right] dx$$

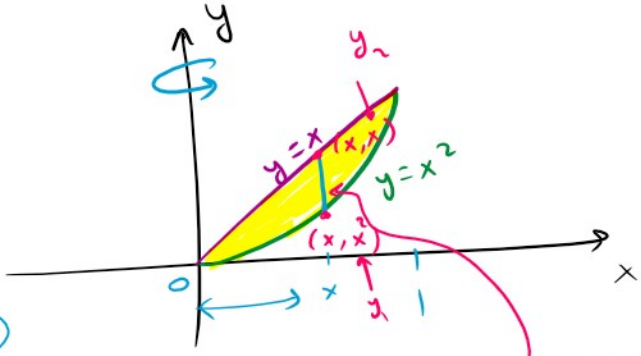
$$= \pi \int_0^1 (x^2 - x^4)\,dx = \pi \left(\frac{x^3}{3} - \frac{x^5}{5}\right)\Big|_0^1$$

$$= \cdots = \frac{2\pi}{15}$$









$$V = 2\pi \int_a^b \left(\text{shell radius}\right)\left(\text{shell height}\right) dx$$

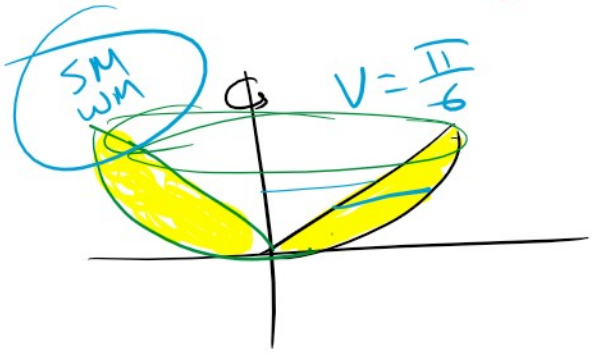
$$= 2\pi \int_0^1 (x)(x - x^2)\, dx$$

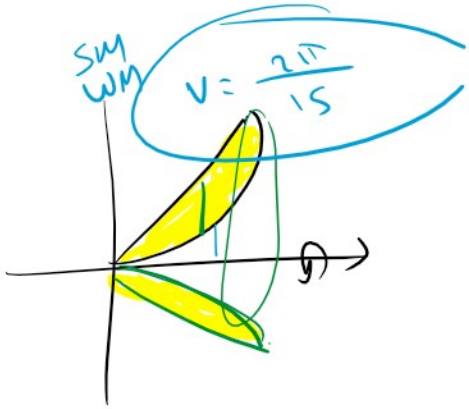
$$= 2\pi \int_0^1 (x^2 - x^3)\, dx$$

$$= 2\pi \left(\frac{x^3}{3} - \frac{x^4}{4}\right)\Big|_0^1 = \cdots = \frac{\pi}{6}$$

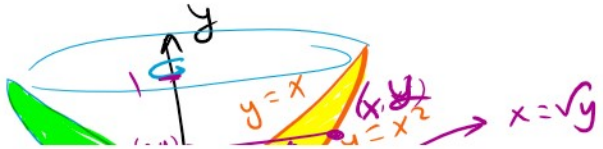
$dy = y_2 - y_1$

$= x - x^2$

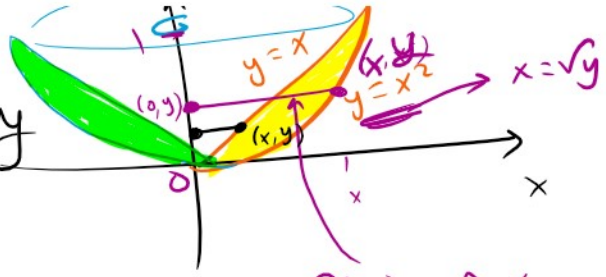




$$V = \pi \int_c^d \left[R^2(y) - r^2(y)\right] dy$$



$$V = \pi \int_0^1 \left[(\sqrt{y})^2 - (y)^2 \right] dy$$



$$= \pi \int_0^1 (y - y^2)\, dy$$

$$= \pi \left(\frac{y^2}{2} - \frac{y^3}{3} \right) \Big|_0^1 = \frac{\pi}{6}$$

$R(y) = \Delta x$

$= x_2 - x_1$

$= \sqrt{y} - 0$

$= \sqrt{y}$

$r(y) = \Delta x$

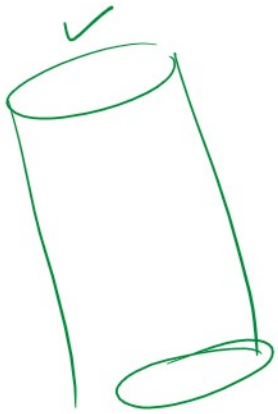
$= x_2 - x_1$

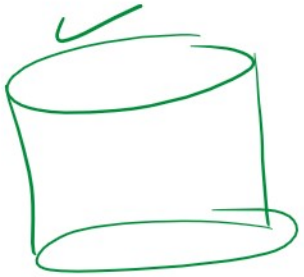
$= y - 0$

سؤال العلامة

العمودي

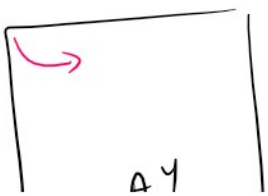
الأفقي





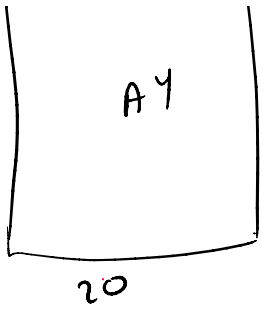
$20 = \text{المحيط} = 2r\pi \Rightarrow r\pi = 10$

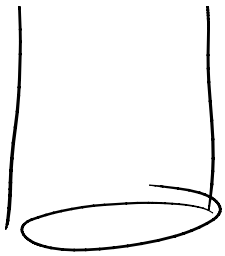
$r = \frac{10}{\pi}$



$V = (r^2 \pi)\, 40$

$(100 \pi)\, 40$





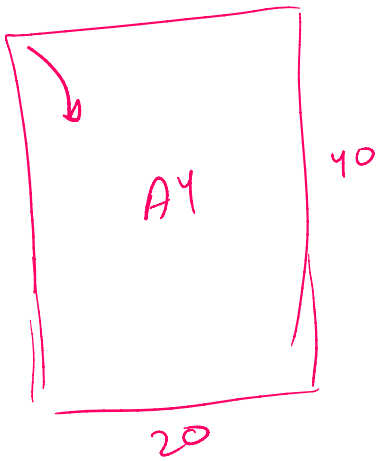
$V = \left(\frac{100}{\pi^2}\pi\right) 40$

$= \frac{4000}{\pi}$

$40 = \text{المحيط} = 2r\pi$

$r\pi = 20$

$r = \frac{20}{\pi}$



$V_2 = \left(r^2\pi\right) 20$

$= \left(\frac{400}{\pi^2}\right)\pi\ 20$

$= \frac{8000}{\pi}$