7.6 Sampling Distribution of $\bar{p}$

- Recall that the sample proportion $\bar{p}$ is the point estimator of the population proportion p.
- We compute $\bar{p}$ by the formula $\bar{p} = \frac{x}{n}$ where

  x = number of elements of intrest in the sample

  n = sample size

- Note that the sample proportion $\bar{p}$ is a random variable and it's prob. distribution is called the sampling distribution of $\bar{p}$.

- The sampling distribution of $\bar{p}$ is the probability distribution of all possible values of the sample proportion $\bar{p}$.

- To determine how close the sample proportion $\bar{p}$ is to the population proportion p, we study the properties of the sampling distribution of $\bar{p}$ in terms of the following characteristics:
  1. Expected value of $\bar{p}$
  2. Standard deviation of $\bar{p}$
  3. the shap (Form) of the sampling distribution of $\bar{p}$.

1 Expected value of $\bar{p}$

$E(\bar{p}) = p$ Hence, $\bar{p}$ is an unbiased estimator of p.

2 Standard deviation of $\bar{p}$

* For Finite Population: the standard deviation of $\bar{p}$ is

$$\sigma_{\bar{p}} = \sqrt{\frac{N-n}{N-1}} \sqrt{\frac{p(1-p)}{n}}$$ where

$\sqrt{\frac{N-n}{N-1}}$ is the finite population correction factor.

* For infinite population "process" or when $\frac{n}{N} \leq 0.05$
the standard deviation of $\bar{p}$ is

$$\sigma_{\bar{p}} = \sqrt{\frac{p(1-p)}{n}}$$

* Note that $\sigma_{\bar{p}}$ is also called the standard error of proportion. Thus, $\sigma_{\bar{x}}$ and $\sigma_{\bar{p}}$ are called standard errors.

Recall Example* where $p = 0.6$, $N = 2500$, $n = 30$

$\frac{n}{N} = \frac{30}{2500} = 0.012 \leq 0.05$ so we ignore the finite population correction factor $\sqrt{\frac{N-n}{N-1}}$

Hence, the standard error of $\bar{p}$ is $\Rightarrow$ $\sigma_{\bar{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.6(1-0.6)}{30}} = \sqrt{\frac{(0.6)(0.4)}{30}} = 0.0894$

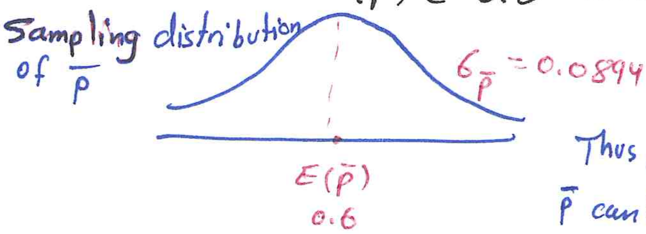
$\boxed{3}$ Shap (Form) of the Sampling Distribution of $\bar{p}$

$\bar{p} = \frac{x}{n}$ where x is binomial random variable that indicates the number of elements of intrest in the sample n which is taken from Large population. "n is constant"

- Since n is constant, it follows that $P(\frac{x}{n}) = f(x) =$ binomial prob. of x. Hence $\bar{p}$ is a discrete prob. distribution.
- But, as we showed in ch6, a binomial distribution can be approximated by a normal distribution if $np \geq 5$ and $n(1-p) \geq 5$.

- The sampling distribution of $\bar{p}$ can be approximated by a normal distribution if $np \geq 5$ and $n(1-p) \geq 5$.

Recall the Example* where $E(\bar{p}) = 0.6$ and $\sigma_{\bar{p}} = 0.0894$



$np = 30(0.6) = 18 \geq 5$

$n(1-p) = 30(0.4) = 12 \geq 5$

Thus, the sampling distribution of $\bar{p}$ can be approximated by normal Dist.

Example (Q 31 page 282) A simple random sample of size 100 is selected from a population with $p = 0.4$.

[a] what is the expected value of $\bar{p}$? $E(\bar{p}) = p = 0.4$

[b] what is the standard error of $\bar{p}$? $\sigma_{\bar{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{(0.4)(0.6)}{100}} = 0.049$

[c] show the sampling distribution of $\bar{p}$?

$np = 100(0.4) = 40 \geq 5$ and

$n(1-p) = 100(0.6) = 60 \geq 5$

Hence, the sampling distribution is approximated by a normal distribution with $E(\bar{p}) = 0.4$ and $\sigma_{\bar{p}} = 0.049$

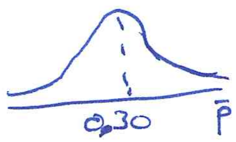
[d] What does the sampling distribution of $\bar{p}$ show?

The probability distribution of $\bar{p}$.

Example (Q 35 page 283) $p = 30\%$ and $n = 100$.

What is the sampling distribution of $\bar{p}$?

[a] Assume $p = 0.30$.



$np = 100(0.30) = 30 \geq 5$ and

$n(1-p) = 100(0.70) = 70 \geq 5$.

$\Rightarrow$ The normal distribution is appropriate with $E(\bar{p}) = 0.30$

and $\sigma_{\bar{p}} = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.3(0.7)}{100}} = 0.0458$

[b] What is the prob. that the sample proportion $\bar{p}$ will be between 0.20 and 0.40?

$P(0.20 \leq \bar{p} \leq 0.40) = P(-2.18 \leq z \leq 2.18) = 0.9854 - 0.0146$
$= 0.9708$

$z = \frac{\boxed{0.20} - E(\bar{p})}{\sigma_{\bar{p}}} = \frac{0.20 - 0.30}{0.0458} = -2.18$

$z = \frac{\boxed{0.40} - E(\bar{p})}{\sigma_{\bar{p}}} = \frac{0.40 - 0.30}{0.0458} = 2.18$

[c] what is the prob. that the sample proportion $\bar{p}$ will be between 0.25 and 0.35?

$P(0.25 \leq \bar{p} \leq 0.35) = P(-1.09 \leq z \leq 1.09) = 0.8621 - 0.1379 = 0.7242$

$z = \frac{\boxed{0.25} - E(\bar{p})}{\sigma_{\bar{p}}} = \frac{0.25 - 0.30}{0.0458} = -1.09$

$z = \frac{\boxed{0.35} - E(\bar{p})}{\sigma_{\bar{p}}} = \frac{0.35 - 0.30}{0.0458} = 1.09$