

4.7 Antiderivatives

(93)

Def A function F is an antiderivative of f on an interval I if $F'(x) = f(x)$ for all $x \in I$.

Example: Find the antiderivative of

① $f(x) = 2x$, $F(x) = x^2$

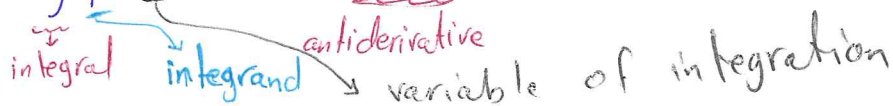
② $g(x) = x^2 - 2x + 1$, $G(x) = \frac{x^3}{3} - x^2 + x$

③ $h(x) = \frac{5}{2\sqrt{x}}$, $H(x) = 5\sqrt{x}$

④ $v(x) = -\pi \sin \pi x$, $R(x) = \cos \pi x$

Theorem: If F is an antiderivative of f on an interval I , then the most general antiderivative of f on I is $F(x) + C$, where C is an arbitrary constant.

Note that the collection of all antiderivatives of f is called the indefinite integral of f with respect to x and defined by $\int f(x) dx = F(x) + C$.



Example: Find the most general antiderivatives (or indefinite integral) of

① $\int (3x^2 + \frac{x}{2}) dx = x^3 + \frac{x^2}{4} + C$

② $\int -2 \cos t dt = -2 \sin t + C$

③ $\int (1 + \tan^2 \theta) d\theta = \int \sec^2 \theta d\theta = \tan \theta + C$

④ $\int (2x^3 - 5x + 7) dx = \frac{2x^4}{4} - \frac{5x^2}{2} + 7x + C$

Example: Find an antiderivative of $f(x) = 3x^2$ (94)
That satisfies $F(1) = -1$

The general antiderivative is $F(x) = x^3 + C$

$$F(1) = (1)^3 + C = -1 \Leftrightarrow C = -2 \quad \text{Thus } F(x) = x^3 - 2$$

So we need to find a function $y(x)$ that satisfies

Initial Value Problem $\left\{ \begin{array}{l} \frac{dy}{dx} = f(x) \\ y(x_0) = y_0 \end{array} \right. \begin{array}{l} \text{--- "differential Equation"} \\ \text{"is equation that has"} \\ \text{derivatives"} \\ \text{--- "initial condition"} \\ \text{To find } C \end{array}$

* The general solution $y(x) = F(x) + C$

* When we find C , we find a particular solution $y(x) = x^3 - 2$

Example: Solve the initial value problem

$$\frac{dy}{dx} = \frac{1}{2\sqrt{x}} + 2\sin 2x, \quad y(0) = 0$$

$$\int dy = \int \left(\frac{1}{2\sqrt{x}} + 2\sin 2x \right) dx$$

$$y(x) = \sqrt{x} - \cos 2x + C \quad \text{--- the general solution}$$

$$y(0) = \sqrt{0} - \cos 0 + C = 0$$

$$0 - 1 + C = 0 \Leftrightarrow \boxed{C = 1}$$

The particular solution is $\boxed{y(x) = \sqrt{x} - \cos 2x + 1}$