4.7 Antiderivatives

Def A function F is an antiderivative of f on an interval I if $F'(x) = f(x)$ for all $x \in I$.

Example: Find the antiderivative of

1. $f(x) = 2x$, $F(x) = x^2$
2. $g(x) = x^2 - 2x + 1$, $G(x) = \frac{x^3}{3} - x^2 + x$
3. $h(x) = \frac{5}{2\sqrt{x}}$, $H(x) = 5\sqrt{x}$
4. $r(x) = -\pi \sin \pi x$, $R(x) = \cos \pi x$

Therorem: If F is an antiderivative of f on an interval I, then the most general antiderivative of f on I is $F(x) + c$, where c is an arbitrary constant.

Note that the collection of all antiderivati of f is called the indefinite integral of f with respect to x and defined by

$$\underbrace{\int}_{\text{integral}} \underbrace{f(x)}_{\text{integrand}} \, d\underbrace{x}_{\text{variable of integration}} = \underbrace{F(x)}_{\text{antiderivative}} + c .$$

 Example: Find the most general antiderivatives (or indefinite integral) of

1. $\int (3x^2 + \frac{x}{2})\, dx = x^3 + \frac{x^2}{4} + c$
2. $\int -2\cos t \, dt = -2\sin t + c$
3. $\int (1 + \tan^2 \theta)\, d\theta = \int \sec^2 \theta \, d\theta = \tan \theta + c$
4. $\int (2x^3 - 5x + 7)\, dx = \frac{x^4}{2} - \frac{5x^2}{2} + 7x + c$

Example: Find an antiderivative of $f(x) = 3x^2$ that satisfies $F(1) = -1$

The general antiderivative is $F(x) = x^3 + c$

$F(1) = (1)^3 + c = -1 \iff c = -2$ Thus $F(x) = x^3 - 2$

So we need to find a function $y(x)$ that satisfies

Initial Value Problem $\Leftarrow \begin{cases} \dfrac{dy}{dx} = f(x) & \text{.... "differential Equation" is equation that has derivatives"} \\ y(x_0) = y_0 & \text{--- "initial condition" To find c} \end{cases}$

* The general solution $y(x) = F(x) + c$
* when we find c, we find a particular solution $y(x) = x^3 - 2$

Example: Solve the initial value problem

$$\frac{dy}{dx} = \frac{1}{2\sqrt{x}} + 2\sin 2x \quad , \quad y(0) = 0$$

$$\int dy = \int\left(\frac{1}{2\sqrt{x}} + 2\sin 2x\right)dx$$

$$y(x) = \sqrt{x} - \cos 2x + c \quad \text{...... the general solution}$$

$$y(0) = \sqrt{0} - \cos 0 + c = 0$$

$$0 - 1 + c = 0 \iff \boxed{c = 1}$$

The particular solution is $\boxed{y(x) = \sqrt{x} - \cos 2x + 1}$
