

0.4 Radicals and Rational Exponents.

الجزء

النسبة

if we have $b^3 = 8$

$$b = \sqrt[3]{8} = \sqrt[3]{2 \cdot 2 \cdot 2} = 2$$

$$b = 2$$

In general, the n^{th} root of a is $\sqrt[n]{a}$

$$\sqrt[n]{a} = b \quad \text{only if} \quad b^n = a$$

n : index, a : radicand

Examples: ① $\sqrt{4} = \sqrt[2]{4} = \sqrt[2]{2 \cdot 2} = 2$

② $\sqrt[3]{64} = \sqrt[3]{4 \cdot 4 \cdot 4} = 4$

③ $\sqrt[3]{-27} = \sqrt[3]{-3 \cdot -3 \cdot -3} = -3$

④ $\sqrt{-9} = \sqrt{-3 \cdot 3}$ not real

If n is even and $a < 0$, then $\sqrt[n]{a}$ is not real

Definition: For a positive integer n

$$\sqrt[n]{a} = a^{\frac{1}{n}}$$

Example: $9^{\frac{1}{2}} = \sqrt{9} = \sqrt[2]{3 \cdot 3} = 3$

$(16)^{\frac{1}{4}} = \sqrt[4]{16} = \sqrt[4]{2 \cdot 2 \cdot 2 \cdot 2} = 2$

$$(-25)^{\frac{1}{2}} = \sqrt[2]{-25} \quad \text{not real.}$$

Definition. For a positive integer n

$$\begin{aligned} a^{\frac{m}{n}} &= (a^{\frac{1}{n}})^m = (\sqrt[n]{a})^m \\ \text{or} \quad a^{\frac{m}{n}} &= (a^m)^{\frac{1}{n}} = \sqrt[n]{a^m} \end{aligned}$$

Example: Write the following in radical form and simplify
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$$\textcircled{1} \quad 16^{\frac{3}{4}} = (16^{\frac{1}{4}})^3 = (\sqrt[4]{16})^3 = (2)^3 = 8$$

$$\textcircled{2} \quad y^{-3/2} = \frac{1}{y^{3/2}} = \frac{1}{(y^3)^{1/2}} = \frac{1}{\sqrt{y^3}}$$

$$\begin{aligned} \textcircled{3} \quad \sqrt[3]{(ab)^3} &\neq ((ab)^3)^{1/3} = (ab)^{3/3} = ab \\ (6m)^{2/3} &= \sqrt[3]{(6m)^2} = \sqrt[3]{(6)^2 m^2} = \sqrt[3]{36 m^2} \end{aligned}$$

Example: Write the following without radical signs:

$$\textcircled{1} \quad \sqrt{x^3} = (x^3)^{1/2} = x^{3/2}$$

$$\textcircled{2} \quad \frac{1}{\sqrt[3]{b^2}} = \frac{1}{(b^2)^{1/3}} = \frac{1}{b^{2/3}} = b^{-2/3}$$

$$\textcircled{3} \quad \sqrt[3]{(ab)^3} = (ab)^{3/3} = ab$$

Rules for radicals: If $\sqrt[n]{a}$ and $\sqrt[n]{b}$ are real

$$\textcircled{1} \sqrt[n]{a^n} = a$$

This does not apply to $\sqrt[2]{(-2)^2}$ because $\sqrt{-2}$ not real
so $\sqrt{a^2} = |a|$

$$\textcircled{2} \sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$$

$$\text{Ex: } \sqrt[3]{2} \cdot \sqrt[3]{4} = \sqrt[3]{2 \cdot 4} = \sqrt[3]{8} = 2$$

$$\textcircled{3} \frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}, \quad b \neq 0$$

$$\text{Ex: } \frac{\sqrt[3]{54}}{\sqrt[3]{2}} = \sqrt[3]{\frac{54}{2}} = \sqrt[3]{27} = \sqrt[3]{3 \cdot 3 \cdot 3} = 3$$

Simplifying Radicals:

A radical is considered simplified if its radicand has no factors with powers $\geq$ index.

$$\text{Ex: } \textcircled{1} \sqrt[3]{x^3} = x^{3/3} = x$$

$$\textcircled{2} \sqrt{49x} = \sqrt{7^2 x} = \sqrt{7 \cdot 7 \cdot x} = 7\sqrt{x}$$

$$\textcircled{3} \sqrt{6x^4} = \sqrt{6x^2 \cdot x^2} = \sqrt{6} x^2$$

$$\text{or } \sqrt{6} \cdot \sqrt{x^4} = \sqrt{6} \cdot x^{4/2} = \sqrt{6} \cdot x^2$$

Question. Find the following and simplify the answer

$$\begin{aligned}
 \textcircled{1} \quad & \sqrt{8xy^3z} \cdot \sqrt{4x^2y^3z^2} \\
 &= \sqrt{8 \cdot 4 \cdot x \cdot x^2 \cdot y^3 \cdot y^3 \cdot z \cdot z^2} \\
 &= \sqrt{16 \cdot 2 \cdot x \cdot x^2 \cdot y^6 \cdot z \cdot z^2} \\
 &= \sqrt{16x^2y^6z^2} \cdot \sqrt{2xz} = 4xy^3z\sqrt{2xz}
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{2} \quad & \frac{\sqrt[3]{-16x^3y^4}}{\sqrt[3]{128y^2}} = \sqrt[3]{\frac{-16x^3y^4}{128y^2}} = \sqrt[3]{\frac{-\cancel{16}x^3y^4}{8 \cdot \cancel{16} \cdot y^2}} \\
 &= \sqrt[3]{\frac{-x^3y^4}{8y^2}} = \sqrt[3]{\frac{-x^3y^{4-2}}{8}} = \sqrt[3]{\frac{-x^3y^2}{8}} \\
 &= \frac{\sqrt[3]{-x^3} \sqrt[3]{y^2}}{\sqrt[3]{8}} = \frac{-x \sqrt[3]{y^2}}{2}
 \end{aligned}$$