

Anan Elayan

Chapter 7

Kinetic Energy work.

* Kinetic Energy (K)

- mass
- speed

$$K = \frac{1}{2} m v^2$$

$K \equiv$ scalar quantity

P5

initial

$$\left[\begin{array}{l} K_F = \frac{1}{2} K_S \\ m_S = \frac{1}{2} m_F \end{array} \right]$$

: f : father
s : son

$$(K_f)_i = \frac{1}{2} (K_f)_f$$

$$\frac{1}{2} m (v_f)_i^2 = \frac{1}{2} \left[\frac{1}{2} m (v_{if} + 1)^2 \right]$$

$v_{if} = ?$

$(v_f)_f = v_{if} + 1 \text{ m/s}$

$K_f = K_s ?$

سبب الفعل هو القوة المؤثرة

* work

① force \equiv constant [gravitational force]
قوة الجاذبية الأرضية

② force [variable] [spring]

$\vec{f} \equiv$ constant $\Rightarrow W = \vec{f} \cdot \Delta \vec{x}$

$\vec{f} =$ force
 $\Delta \vec{x} =$ displacement
 $W =$ work

$[W] = N \cdot m : \text{Joule}$

work scalar quantity

$$W = |\vec{f}| |\Delta x| \cos \theta$$

Δx و f و $\cos \theta$

$$\frac{f}{\Delta a}$$

Δt

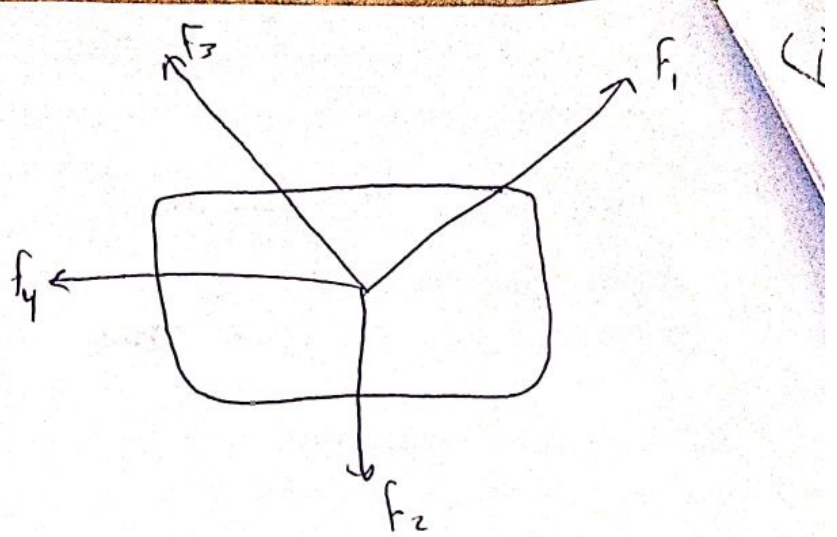
$W = 0$

- $\vec{f} = 0$
- $\Delta \vec{x} = 0$
- $\vec{f} \perp \Delta \vec{x}$

$$f = \frac{m v^2}{r}$$

$$① W_{net} = \vec{f}_{net} \cdot \Delta \vec{r}$$

$$② W_{net} = W_1 + W_2 + W_3 + \dots$$



P14

$$f_1 = 3N \quad \theta_2 = 50^\circ$$

$$f_2 = 4 \quad \theta_3 = 35^\circ$$

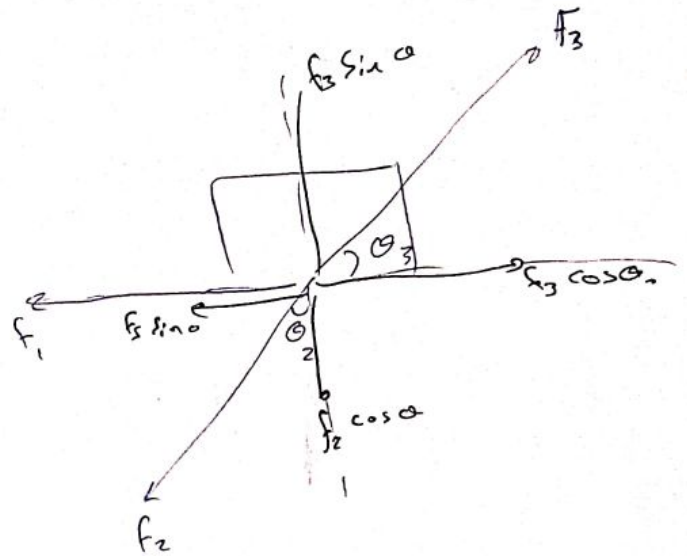
$$f_3 = 9N$$

$$W = ?$$

$$W_{net} = \vec{f}_{net} \cdot \Delta \vec{r}$$

$$\begin{aligned} \sum f_x &= f_3 \cos \theta_3 - f_2 \sin \theta_2 - f_1 \\ &= 9 \cos 35 - 4 \sin 50 - 3 \\ &= \boxed{1.24N} \end{aligned}$$

$$\begin{aligned} \sum f_y &= f_3 \sin \theta_3 - f_2 \cos \theta_2 \\ &= 9 \sin 35 - 4 \cos 50 \\ &= \boxed{2.6N} \end{aligned}$$



$$f_{net} = \sqrt{(1.24)^2 + (2.6)^2} = \boxed{2.88}$$

$$\begin{aligned} W &= f \cdot \Delta r \\ &= 2.88 \cdot 4 = \boxed{11.5} \text{ Joule} \end{aligned}$$

from 1st r
نصف المسافة

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Exp: gravitational force

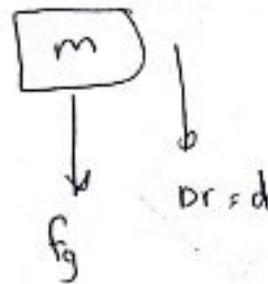
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① $\vec{f}_g = mg$

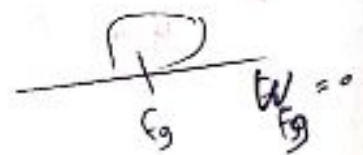
① $W_{\vec{f}_g} = \vec{f}_g \cdot \vec{\Delta r}$
 $= f_g d \cos \theta$

$W_{\vec{f}_g} = f_g d$

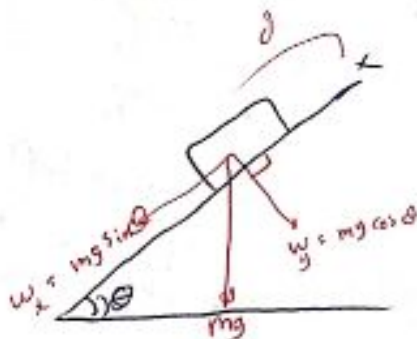
② $W_{\vec{f}_g} = -f_g d$



$f_g \perp \Delta r$



Case 3



① downward

$W_{W_y} = 0$

$W_{W_x} = W_x \cdot d$

② upward $W_{W_x} = -W_x \cdot d$

constant speed (up) ②
 (no net work)

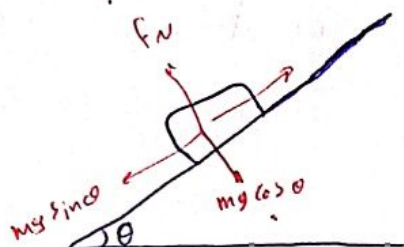
تأثير الجاذبية

* Kientic energy - work theorem :-

$$W_{\text{net}} = \Delta K$$

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P20

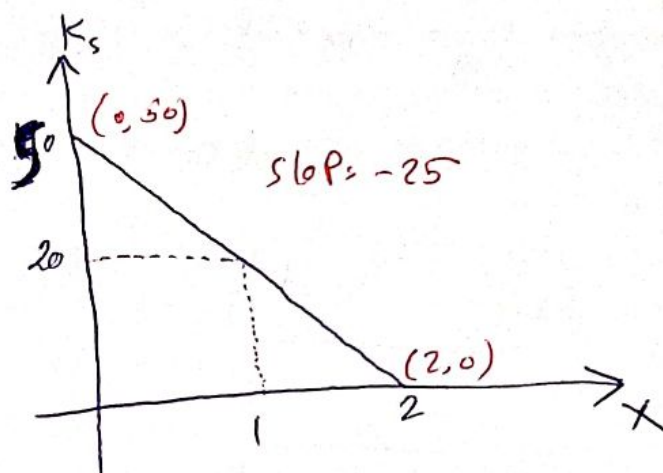


$$v_i = 5 \text{ m/s}$$

$$F_N = ??$$

$$F_N = mg \cos \theta$$

$$= 4(9.8) \cos 39.6^\circ$$



$$W_{\text{net}} = f_{\text{net}} \cdot \Delta x = \Delta K$$

$$\text{slope} = \frac{\Delta K}{\Delta x} = f_{\text{net}} \cdot x = -mg \sin \theta$$

$$25 = 4(9.8) \sin \theta$$

$$\theta = 39.6^\circ$$

$$K_i = 50 \text{ J}$$

$$50 = \frac{1}{2} m (5)^2$$

$$m = 4 \text{ kg}$$

Part 2

* work done by variable

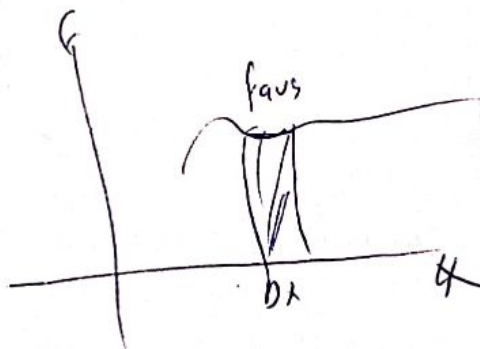
$$\Delta x \rightarrow f_{\text{avg}}$$

$$W_i = f_{\text{avg}} \cdot \Delta x$$

$$W = \sum_i f_{\text{avg}} \cdot \Delta x$$

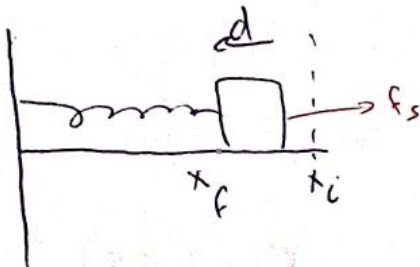
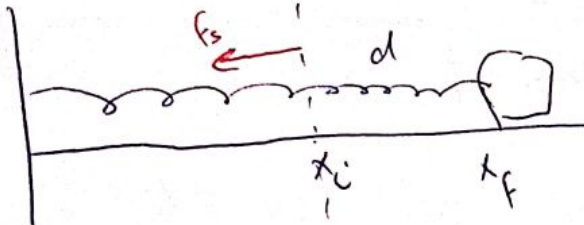
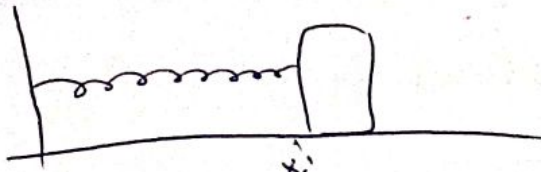
$$W = \lim_{\Delta x \rightarrow 0} \sum f_{\text{avg}} \Delta x$$

$$W = \int_{x_i}^{x_f} F dx$$



* Spring force و عمل

فردی، عملی، و



Hook's law,

$$\vec{f} \propto \vec{d}$$

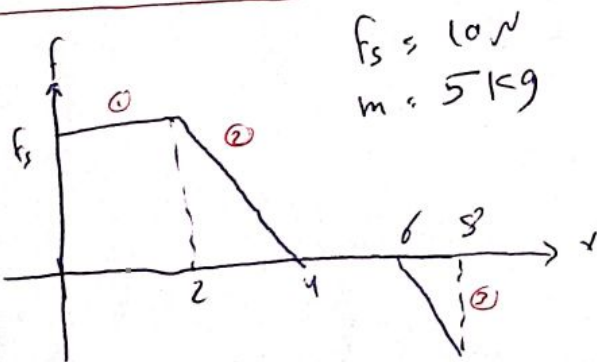
$$f_s = -Kd$$

$K \equiv$ spring constant
ثابت فن

$$W_{\text{applied}} = -W_{\text{spring}}$$

$$= \frac{1}{2}K(x_f^2 - x_i^2)$$

P36



$f_s = 10 \text{ N}$
 $m = 5 \text{ kg}$

$$W = \int_{x_i}^{x_f} f dx = \text{Area}(f-x)$$

$$W_{\text{spring}} = \int_{x_i}^{x_f} f_s dx$$

$$= \int_{x_i}^{x_f} -Kx dx$$

$$= \frac{1}{2}Kx \Big|_{x_f}^{x_i}$$

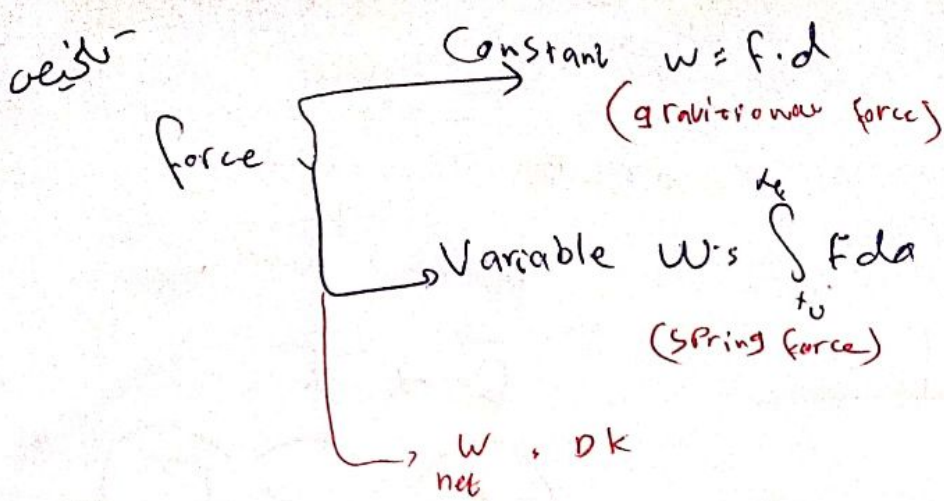
$$= \frac{1}{2}Kx_i^2 - \frac{1}{2}Kx_f^2$$

$$= \frac{1}{2}K(x_i^2 - x_f^2)$$

$$F \xrightarrow{\int} W$$

$$x_f = \sqrt{\frac{mv_i^2}{K}}$$

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* \vec{F} in 2D, 3D

$$\vec{F} = F_x \hat{i} + F_y \hat{j} + F_z \hat{k}$$

$$W = \int_{x_i}^{x_f} F_x dx + \int_{y_i}^{y_f} F_y dy + \int_{z_i}^{z_f} F_z dz$$

EXPL $\vec{F} = 2x\hat{i} - 3y\hat{j} + \hat{k}$ $\vec{r}_1 = (0, 2, 4) \rightarrow \vec{r}_2 = (2, 3, 5)$

$$W = \int_0^2 2x dx - \int_2^3 3y dy + \int_4^5 1 dz$$

$$= x^2 \Big|_0^2 - \frac{3y^2}{2} \Big|_2^3 + z \Big|_4^5$$

* Work - Kinetic theorem

$$W_{net} = \int F_{net} dx, F_{net} = ma, a = \frac{dv}{dt}$$

$$= \int m a dx = \int m \frac{dv}{dt} dx$$

$$\frac{dv}{dt} = \frac{dv}{dx} \cdot \frac{dx}{dt} = v \frac{dv}{dx}$$

$$\int m v \frac{dv}{dx} dx$$

$$m \int_{v_i}^{v_f} v dv$$

$$W_{net} = m \left[\frac{v^2}{2} \right]_{v_i}^{v_f}$$

$$= \frac{1}{2} m (v_f^2 - v_i^2) = \Delta K$$

Power (P)

$$P_{avg} = \frac{\Delta W}{\Delta t} = \frac{J}{s}$$

$$[P] = \frac{J}{s} = \text{watt}$$

$$W = mgh$$

$$\Rightarrow P_{inst} = \frac{dW}{dt}$$

$$P_{avg} = \vec{F} \cdot \vec{v}$$

$$P = F \cdot v \text{ (if } \vec{F} \text{ and } \vec{v} \text{ are in the same direction)}$$

Discussion

$$P_i: m_p = 1,67 \times 10^{-27} \text{ kg}$$

$$\alpha = 3,6 \times 10^{15} \text{ m/s}^2$$

$$v_i = 2,4 \times 10^7 \text{ m/s}$$

$$\Delta x = 3,5 \text{ cm}$$

$$① v_f^2 = v_i^2 + 2a\Delta x$$

$$v_f = \sqrt{(2,4 \times 10^7)^2 + 2(3,6 \times 10^{15})(3,5 \times 10^{-2})}$$

$$= 2,9 \times 10^7 \text{ m/s}$$

② final speed ③ ΔK

$$④ \Delta K = K_f - K_i$$

$$= \frac{1}{2}m(v_f^2 - v_i^2)$$

$$= \frac{1}{2} [1,67 \times 10^{-27}] [(2,9 \times 10^7)^2 - (2,4 \times 10^7)^2]$$

$$= 2,1 \times 10^{-13} \text{ J}$$

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P15

$$① W_{net} = ?$$

$$W_{net} = \vec{F} \cdot \Delta \vec{x}$$

$$W_1 = \vec{F} \cdot \Delta \vec{x} = |F| |\Delta x| \cos \theta$$

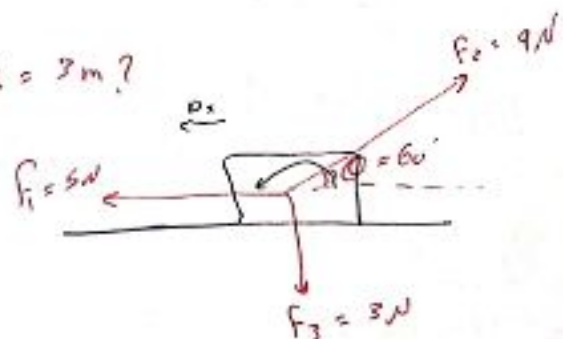
$$= 5(3) \cos 0 = 15 \text{ J}$$

$$W_2 = fd \cos \theta = 9(3) \cos 120 = -13,5 \text{ J}$$

$$W_3 = fd \cos \theta = 3(3) \cos 90 = 0 \text{ J}$$

$$W_{net} = 1,5$$

$$\Delta x = 3 \text{ m?}$$

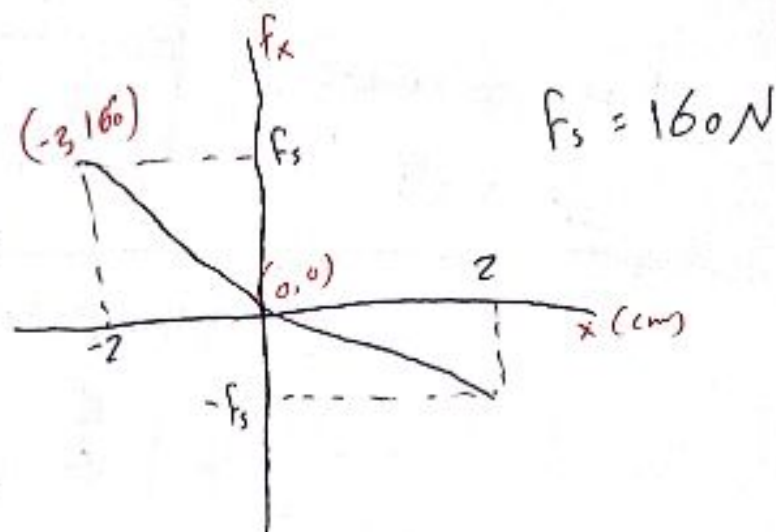


⑥ $\Delta K = W_{\text{net}}$
 $= 1.5 \text{ J}$

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P32

how much work does the spring do on the block moves from $x_i = 8 \text{ cm}$ to



① $x_f = 5 \text{ cm}$ ② $x_f = -5 \text{ cm}$

③ $x_f = -8 \text{ cm}$

$W_s = \frac{1}{2} K (x_i^2 - x_f^2)$

$F_s = -Kx$
 slope $= \frac{y}{x}$

slope $= -K = \frac{160 - 0}{-2 - 0} = -80$

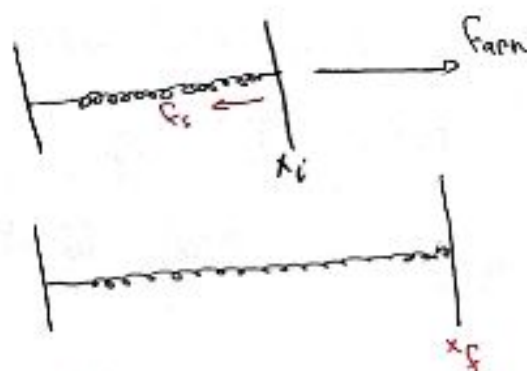
$K = 80 \text{ N/cm} \times 10^{-2}$

$K = 8000 \frac{\text{N}}{\text{m}}$

① $W_s = \frac{1}{2} (8000) [(8 \times 10^{-2})^2 - (5 \times 10^{-2})^2] = 15.6 \text{ J}$

② $W_s = 15.6$

③ $W_s = 0$



$F_s = -Kx$

$F_a = -F_s$

$W_s = \int_{x_i}^{x_f} F_s dx$

$W_s = -W_a$

P 37

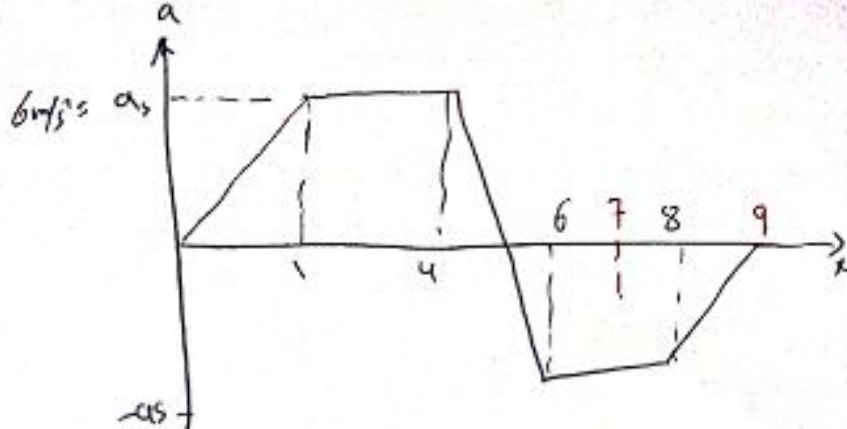
$m = 2 \text{ kg}$

$\int_{\text{net}} F \cdot dx$

$$W = \int_{\text{net}} F(x) dx$$

$$= \int m a(x) dx$$

$$W_{\text{net}} = m \int a(x) dx \rightarrow \text{area}$$



a) $W_{\text{net}(4)} = 2 \left[\frac{1}{2}(1)(6) + (3)(6) \right] = 42 \text{ J}$

b) $W_{\text{net}(7)} = 30 \text{ J}$

c) $W_{\text{net}(9)} = 12 \text{ J}$

d) V at $x = 4$

$$W_{\text{net}} = \Delta K$$

$$= K_f - K_i$$

$$42 = \frac{1}{2} m v_f^2 \Rightarrow v_f = \boxed{6.5 \text{ m/s}}$$

V at $x = 7$

$$30 = K_f - K_i \Rightarrow 30 = \frac{1}{2} (2) v_f^2 \Rightarrow v_f = \boxed{5.5}$$

V at $x = 9$

$$12 = \frac{1}{2} (2) v_f^2 \Rightarrow v_f = \boxed{3.5 \text{ m/s}}$$

$$P_{\text{avg}} = \frac{dW}{dt}, P_{\text{inst}} = \frac{dW}{dt}, P = \vec{F} \cdot \vec{v}$$

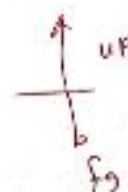
P 46 $m = 3 \times 10^3 \text{ kg}, \Delta y = 210 \text{ m}^{\uparrow}, \Delta t = 23 \text{ sec}$

$$P = \vec{F} \cdot \vec{v} = F v \cos \theta$$

$$= m g v$$

$$|P| = m g \frac{dy}{dt}$$

$$= 3 \times 10^3 (9.8) \left(\frac{210}{23} \right) = 2.68 \times 10^3$$



Good Luck
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Chapter 8

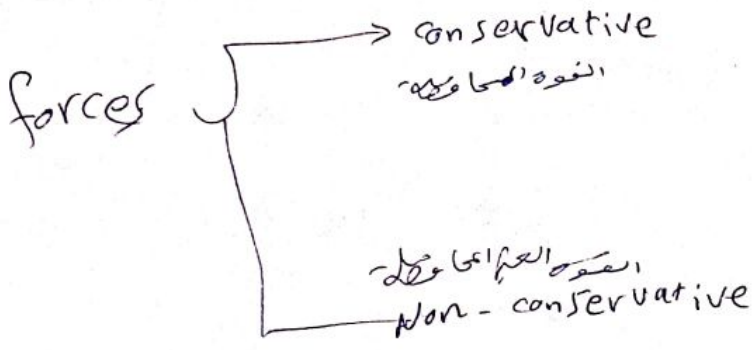
$E_{tot} = E_{kin} + E_{pot}$
 $m = 2 \text{ kg}$
 $v = 3 \text{ m/s}$
 $\mu_k = 0.5$

Potential energy (U)

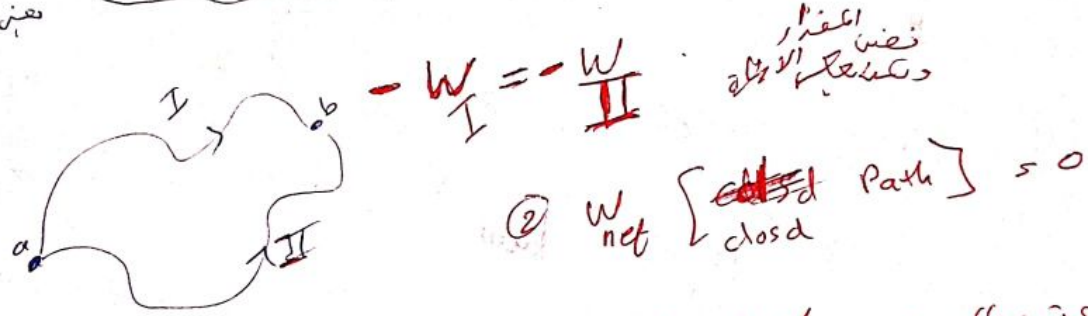
$\Delta U = U_2 - U_1$ $U_1 \leq 0$

$\Delta U = -W$

- ① Isolated system
- ② Conservative force



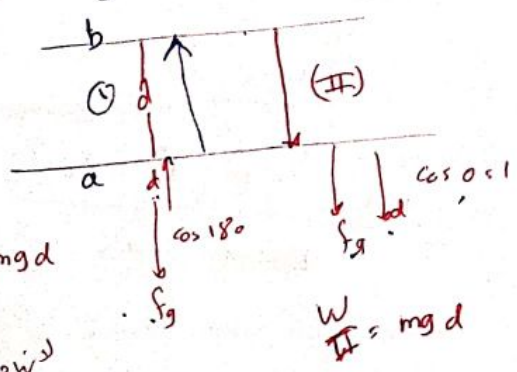
Conservative force: Work does not depend on the path



Non-Conservative forces: Work depends on the path

Conservative forces

- ① gravitational force



K. friction
 Drag force

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$$W = \int_{x_i}^{x_f} f(x) dx$$

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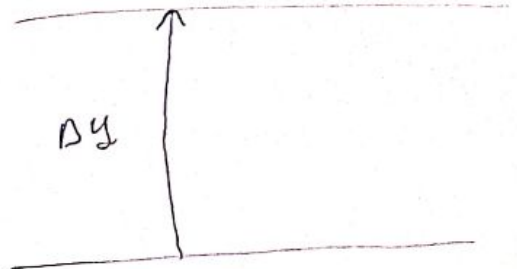
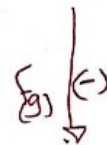
$$\Delta U = -W = - \int_{x_i}^{x_f} f(x) dx$$

$$U_2 - U_1 = - \int_{x_0}^{x_f} f(x) dx$$

Case (I) : gravitational force

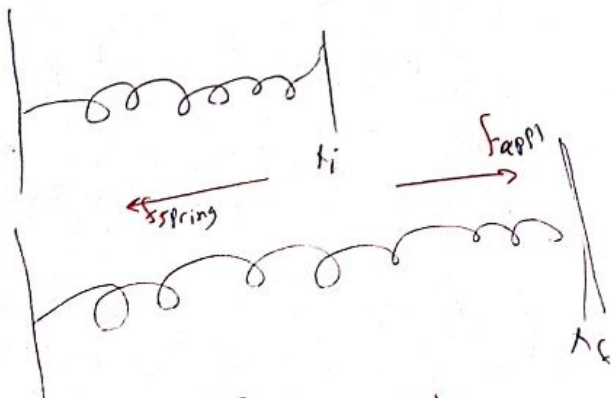
$$\Delta U = - \int_{y_i}^{y_f} f(y) dy$$

$$= \int_{y_i}^{y_f} mg dy \Rightarrow \Delta U = mg(y_f - y_i)$$



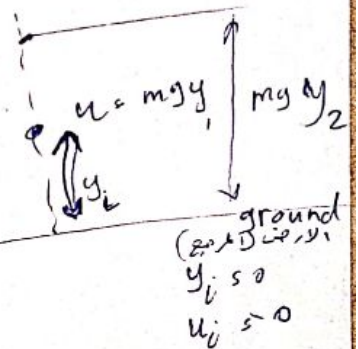
Case (II) spring force

$F_{spring} = -Kx$ Hooke's law



$$W_s = \frac{1}{2}K(x_i^2 - x_f^2)$$

$$W_{app} = -W_s$$



$$\Delta U = - \int_{x_i}^{x_f} f(x) dx$$

$$= \int_{x_i}^{x_f} Kx dx$$

$$\Delta U = \frac{1}{2}K(x_f^2 - x_i^2)$$

$$\Delta U = -W_s$$

* Mechanical Energy (E)

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$$\boxed{E = K + U}$$

- Isolated [no external forces]

- Conservative forces

$\int_{mcc} F = \text{Conserved}$

$$W = \Delta K \Rightarrow \Delta K = -\Delta U$$

$$W = -\Delta U$$

$$K_f - K_i = -(U_f - U_i)$$

$$\boxed{K_i + U_i = K_f + U_f}$$

$$\boxed{E_i = E_f} \quad \Delta E_{mcc} = 0$$

$$E = \frac{1}{2} m \omega^2 a^2$$

$$\omega = \sqrt{\frac{k}{m}}$$

$$E = \frac{1}{2} k a^2$$

$$a = \sqrt{\frac{2E}{k}}$$

$$v = \omega a$$

$$h = \frac{v^2 \sin^2 \theta}{2g}$$

$$v = \sqrt{2gh} = \sqrt{2 \cdot 9.8 \cdot 5.4}$$

P(29)

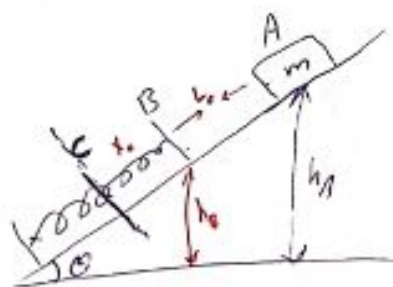
$$m = 12 \text{ kg}$$

$$v_i = 0$$

$$\theta = 30^\circ, x_c = 5.5 \text{ m}$$

$$x = 2 \text{ cm}, F = 270 \text{ N}$$

$$l_0 + x_c = ??$$



$$\text{Since } \frac{h_A}{l_0 + x_c}$$

$$\sin 30 = \frac{0.174}{l_0 + 5.5 \times 10^{-2}}$$

$$l_0 = 0.292 \text{ m}$$

$$E_A = E_C$$

$$K_A + U_A = K_C + U_C$$

$$0 + mgh_A = 0 + \frac{1}{2} k x_c^2$$

$$mgh_A = \frac{1}{2} k x_c^2$$

$$12(9.8)h_A = \frac{1}{2} (1.35 \times 10^4) (5.5)^2$$

$$h_A = 0.174 \text{ m}$$

$$F = -kx \Rightarrow k = \frac{F}{x}$$

$$= \frac{270}{2 \times 10^{-2}}$$

$$= 1.35 \times 10^4$$

$$E_A = E_B$$

$$\cancel{K_A} + u_A = K_B + u_B$$

$$mgh_{(A)} = \frac{1}{2}mv_B^2 + mgh_{(B)}$$

$$\Delta y = h_A - h_B$$

$$L_0 \sin \theta = \Delta y$$

$$L_0 \sin \theta = h_A - h_B$$

$$h_B =$$

ويعطى موف و شرميه (م)

$$v_B = 1.7 \text{ m/s}$$

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Reading a potential energy curve

$$\Delta u(x)$$

$$\Delta u(x) = -W = -f(x) \cdot \Delta x$$

$$f(x) = -\frac{\Delta u}{\Delta x}$$

$$f(x) = -\frac{du(x)}{dx}$$

$$E_{mec} = 5.5$$

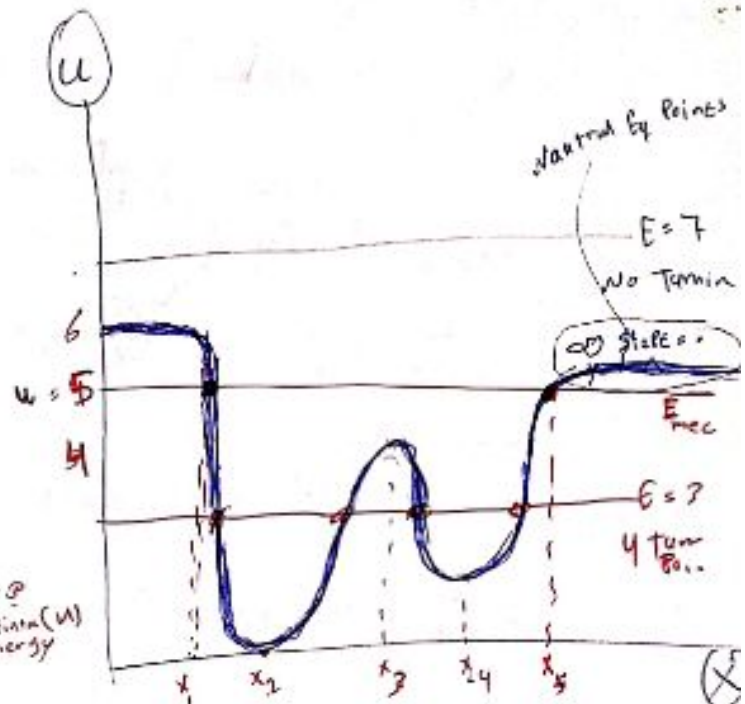
$$E = K(x) + u(x)$$

* turning point

$$E = u$$

$$K(x) = 0$$

نقطة التوقف
حيث تكون الطاقة الحركية $K=0$
الطاقة الكامنة (u)
Energy



$x_1, x_2 =$ turning points
 $u = E$
 $K(x) = 0$
 $V(x) = 0$
 $V = 0$

* Equilibrium Points

نقطة التوازن
حيث تكون القوة صفر
في نقطة التوازن

$$\vec{F}_{net} = 0, f = -\frac{du}{dx}$$

↓
slope(u-x)

$$-\frac{du}{dx} = 0 \Rightarrow \left[\text{local min, local max} \right]$$

if $\vec{F} = 0$, slope $= 0$

$x_1, x_3, x_4 \Rightarrow E_q \text{ points}$

E_q Points

ثبات
نقطة التوازن

① Stable Eq Points

x_2, x_4 (stable)

② un-stable Eq Points
 x_3 (un-stable)

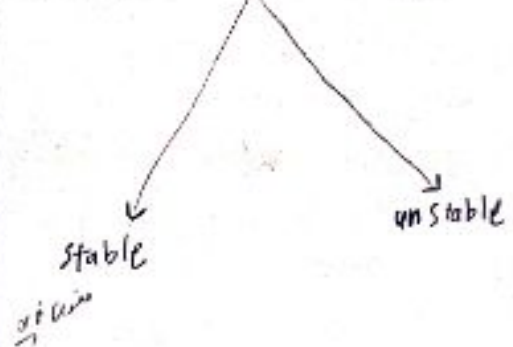
Exo $f(x) = x^2 - 2x$

$$\left. \frac{df}{dx} \right|_{x_0} > 0 \rightarrow \text{max}$$

$$\left. \frac{df}{dx} \right|_{x_0} < 0 \rightarrow \text{min}$$

نقطة التوازن
 $u(x)$

① Eq. Points ($f=0 \Rightarrow \frac{du}{dx} = 0$)



② turning Points
($K=0$) ($u=0$)

$E = u(x)$; $x \in \text{turning points}$

Exp: if $u(x) = x^2 - 2x$, and $E_{mec} = 4$ Find

① Turning Points, ② Eq. Points.

① $E = u(x)$

$$4 = x^2 - 2x$$

$$x^2 - 2x - 4 = 0$$

$$x = \frac{b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{2 \pm \sqrt{4 - 4(1)(-4)}}{2}$$

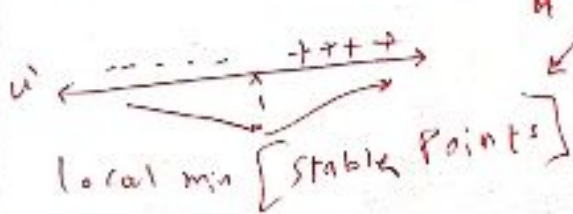
$$\frac{2 \pm \sqrt{20}}{2} \Rightarrow x_1 = \frac{2 + \sqrt{20}}{2} = 3.23m$$

$$x_2 = \frac{2 - \sqrt{20}}{2} = -1.23m$$

② Eq. Points

$$f=0 \Rightarrow \frac{du}{dx} = 0$$

$$2x - 2 = 0 \Rightarrow x = 1m$$



4
او بتحول في
صانوج

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$$f(x) = -\frac{du}{dx}$$

$$\Delta u = \int f dx = -W$$

$$\boxed{\Delta u = -W} \text{ - Isolated conservative forces.}$$

Exp $u(x,y) = x^2 y + 2x$

$$f(x) = -\frac{du}{dx} = -(2xy + 2) \quad \vec{f} = (-2xy + 2)\hat{i} - x^2\hat{j}$$

$$f(y) = -\frac{du}{dy} = -(x^2 + 0) \quad \text{at } (x=1, y=2) \quad \vec{f} = -4\hat{i} - \hat{j}$$

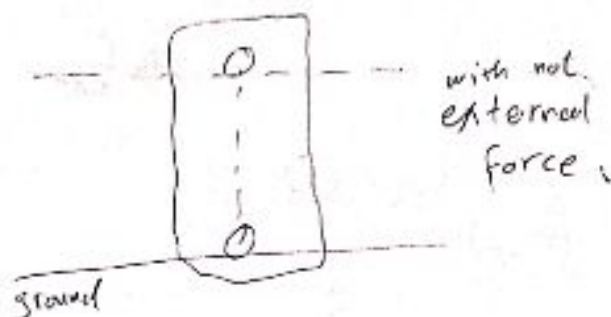
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review go

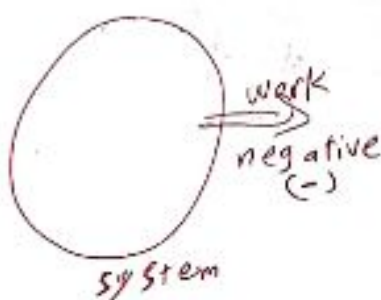
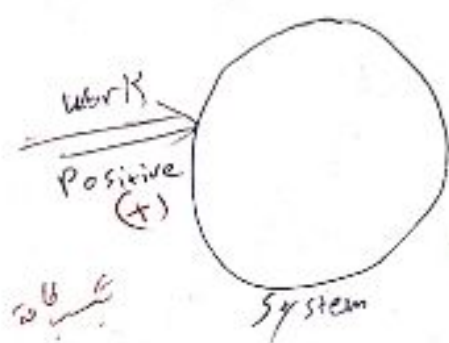
* Isolated system
[conservative force]

$$E_{\text{mec}} = K + U \quad \text{w.s.o}$$

$$\Delta E_{\text{mec}} = \Delta K + \Delta U = 0$$

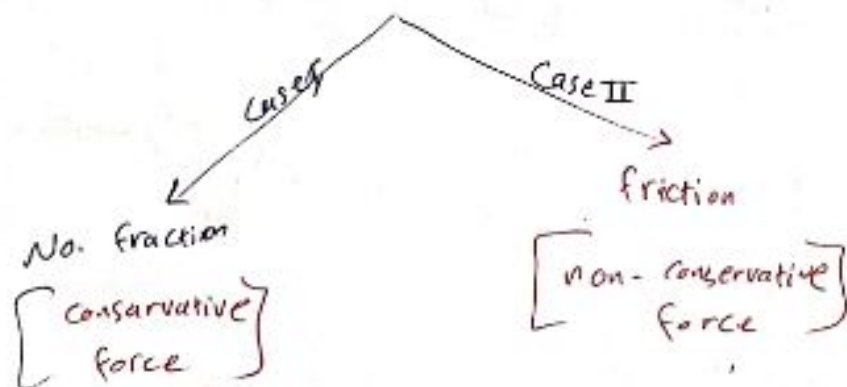


* work : is energy transferred to or from the system
when external force acts on that system:

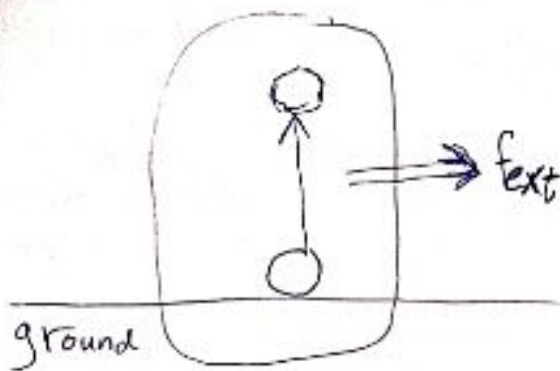


* non-isolated system

(external)



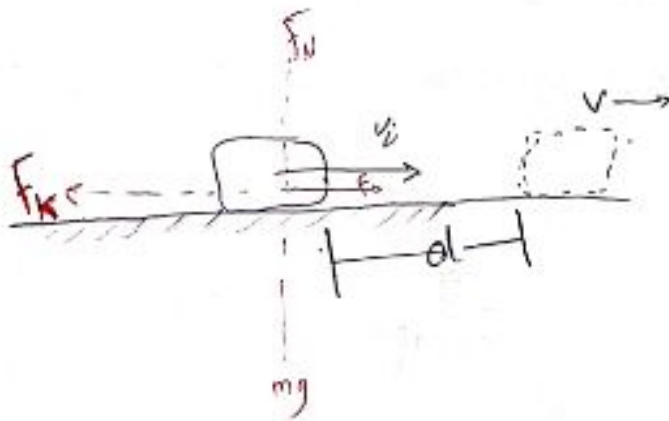
⇒ Non-Isolated system (without friction):
غير معزول



$$W = \Delta E_{mec} \neq 0$$

$$= \Delta K + \Delta U$$

⇒ Non-Isolated system with friction:



$$F_{net} = ma$$

$$F - F_k = ma$$

to find $a \rightarrow$ use SK

$$v^2 = v_0^2 + 2ad$$

$$2ad = v^2 - v_0^2$$

$$a = \frac{1}{2d} (v^2 - v_0^2)$$

$$Fd = \left(\frac{1}{2} m v^2 - m v_0^2 \right) + F_k d$$

$$Fd = \Delta K + F_k d$$

$$\Delta E_{mec} = \Delta K$$

$$Fd = \Delta E_{mec} + F_k d$$

$$W = \Delta E_{mec} + F_k d$$

← work done by friction

← ΔE_{th} thermal energy

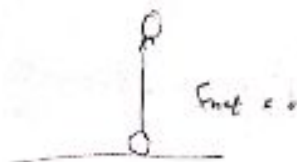
Uesib~

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Isolated system ($w = 0$)

No friction

$$\Delta E_{mec} = 0$$



friction

$$\Delta E_{mec} + \Delta E_{th} = 0$$

Non-Isolated system

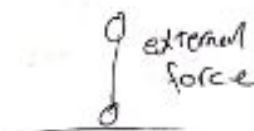
$$w \neq 0$$

$$\Delta E_{int} = 0$$

No friction

$$w = \Delta E_{mec}$$

$$= \Delta K + \Delta U$$



friction

$$w = \Delta E_{mec} + \Delta E_{th}$$

* Conservation of energy

$$w = \Delta E_{mec} + \Delta E_{th} + \Delta E_{int}$$

* Isolated system (friction)

$$w = 0$$

$$\Delta E_{mec} + \Delta E_{th} = 0$$

$$\Delta U + \Delta K + f_k d = 0$$

$$U_f - U_i + K_f - K_i + f_k d = 0$$

$$U_f + K_f = U_i + K_i - f_k d \rightarrow \text{friction}$$

$$\text{Power} = P [\text{watt}]$$

$$P_{\text{int}} = \frac{dE}{dt}$$

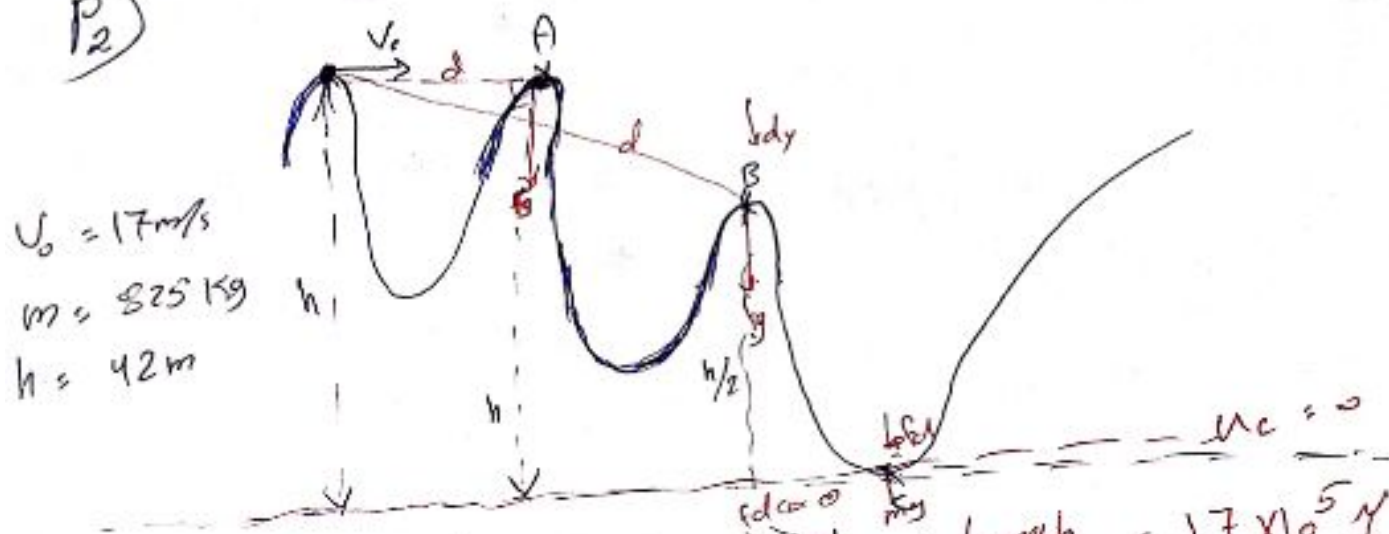
$$P = \frac{Dw}{Dt}$$

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$$P_{\text{avg}} = \frac{D E}{D t}$$

Dissectione

P₂)



$$v_0 = 17 \text{ m/s}$$

$$m = 825 \text{ kg}$$

$$h = 42 \text{ m}$$

a) w_A, w_B, w_C

$$w = F d \cos 90^\circ = 0$$

$$w_B = mg \frac{h}{2} = \frac{1}{2} mgh = 1.7 \times 10^5 \text{ J}$$

$$w_C = F d \cos 0 = mgh = 3.4 \times 10^5 \text{ J}$$

b) $u_B \rightarrow u_A$ والنقطة
التي هي

$$u_C = 0$$

$$\Delta u_{BC} = -W \quad \text{or} \quad u = mg \Delta y$$

$$u_B = mgh \frac{1}{2} = 1.7 \times 10^5 \text{ J}$$

$$u_A = mgh = 3.4 \times 10^5 \text{ J}$$

Isolated system ($F_{ext} = 0$)

$$W_{ext} = 0$$

no-friction

$$W = \Delta E_{mec} = 0$$

$$K_i + U_i = K_f + U_f$$

friction $f_k \cdot d$

$$W = \Delta E_{mec} + \Delta E_{th} = 0$$

$$K_i + U_i = K_f + U_f + f_k d$$

Non Isolated ($F_{ext} \neq 0$)

$$W_{ext} \neq 0$$

No friction

$$W = \Delta E_{mec}$$

$$= \Delta K + \Delta U$$

friction

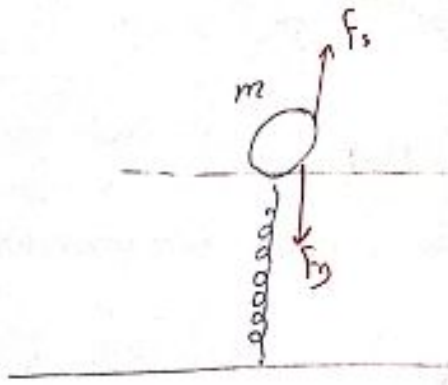
$$W = \Delta E_{mec} + \Delta E_{th}$$

$$= \Delta K + \Delta U + f_k d$$

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P19

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$$m = 8 \text{ kg}$$

$$v_i = 0 \text{ (at rest at 0)}$$

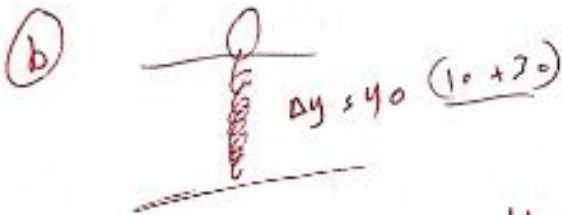
$$\Delta y = 10 \text{ cm}$$

$$\textcircled{a} \quad K = ?$$

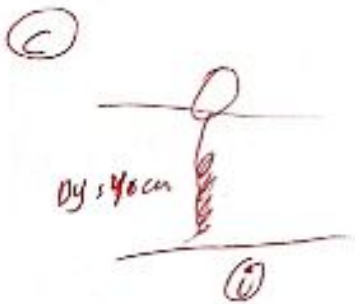
$$F_s - F_g = 0$$

$$K \Delta y = mg$$

$$K = \frac{mg}{\Delta y} = 784 \text{ N/m}$$



$$u_s = \frac{1}{2} K y^2 = \boxed{62.7 \text{ J}}$$



$$\cancel{K_i} + u_i = \cancel{K_f} + u_f$$

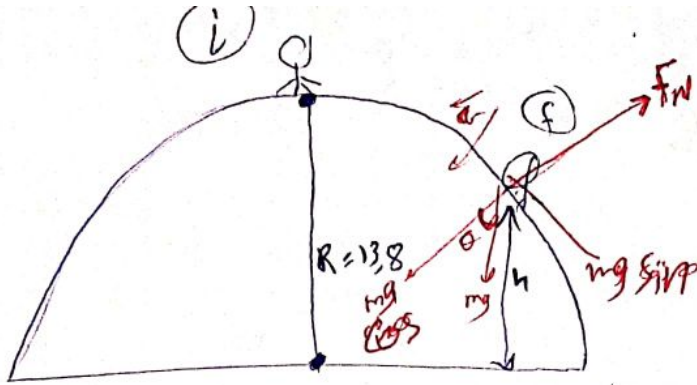
Energy
conservation

$$62.7 = mgh$$

$$h = \frac{62.7}{mg} = \boxed{9.8 \text{ m}}$$

34

at rest



$$K_i + U_i = K_f + U_f$$

$$0 + mgR = \frac{1}{2}mv_f^2 + mgh$$

$$R = \frac{1}{2}R \cos \theta + h$$

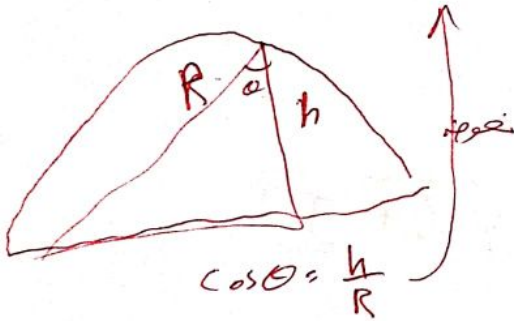
$$R = \frac{1}{2}R \cos \theta + h$$

or $\frac{v^2}{R}$ (f=0) بعد منطلق

$$mg \cos \theta - F_N = \frac{mv^2}{R}$$

$$mg \cos \theta = \frac{mv^2}{R}$$

$$v^2 = Rg \cos \theta$$



$$h = 9.2 \text{ m}$$

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Potential Energy & Conservation of Energy

Forces in Nature :

① Gravitational force = mg

② Spring force = $-Kx$

③ Normal force = $F_n = N$

④ Friction force $f_s, f_{sm}, f_{sl}, f_k, f_{kl}$

⑤ Air Drag force = $\frac{1}{2} C_d \rho A v^2$

قوة الحافظة
Conservative forces

U Potential Energy
طاقة الوضع

$U_s = \frac{1}{2} K x^2$ Joule
الزنبرك

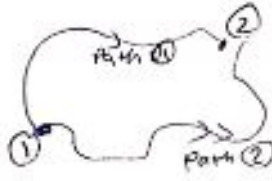
$U_g = mgy$
Earth

Non conservative force

Properties of conservative forces :-

① Work done by F_{cons} is Path independent :

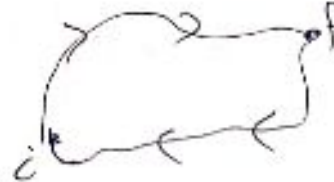
$W_{con1} = W_{con2}$



شغل القوة الحافظة لا يعتمد على المسار

② Work done by $F_{con} = 0$ $i \rightarrow i$

$W_{cons} = 0$ around a closed path



③ Work done against F_{cons} do not ~~lost~~ but it ~~stored~~ as ~~energy~~ \rightarrow

الشغل المبذول ضد القوة الحافظة لا يضيع بل يخزن في شكل طاقة مخزنة

Energy called Potential Energy.

$W_{app} = \Delta U = U_f - U_i$

④ W done by $F_{\text{cons}} = -\Delta U$

$$W_{\text{cons}} = -\Delta U$$

$$i \rightarrow f = -[U_f - U_i]$$

Gravitational Potential Energy.

$$W_{\text{cons}} = -\Delta U = -[U_f - U_i]$$

$$\int_{r_i}^{r_f} \vec{F}_{\text{cons}} \cdot d\vec{r} = -\Delta U$$

تغير الطاقة (اعمالية)

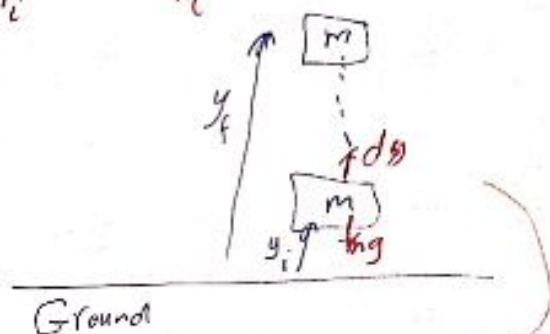
تغير القوة المتماثلة

$$\Delta U = - \int_{r_i}^{r_f} \vec{F}_{\text{cons}} \cdot d\vec{r}$$

$$U_f - U_i = - \int_{r_i}^{r_f} \vec{F}_{\text{cons}} \cdot d\vec{r}$$

⑤

$$\int_{r_i}^{r_f} \vec{F} \cdot d\vec{r} = \int_{x_i}^{x_f} F_x dx + \int_{y_i}^{y_f} F_y dy + \int_{z_i}^{z_f} F_z dz$$



$$U_f - U_i = - \int_{r_i}^{r_f} \vec{F}_g \cdot d\vec{r}$$

$$= - \int_{y_i}^{y_f} (-mg dy)$$

$$= mg \int_{y_i}^{y_f} dy$$

$$U_f - U_i = mgy_f - mgy_i$$

let $y_i = 0, U_i = 0$

$$U_g = mgy$$

Joule

y is above zero level

⑥ Spring Potential Energy:

$$U_f - U_i = - \int_{r_i}^{r_f} \vec{F}_s \cdot d\vec{r}$$

$$U_f - U_i = - \int_{x_i}^{x_f} (-kx) dx = k \int_{x_i}^{x_f} x dx$$

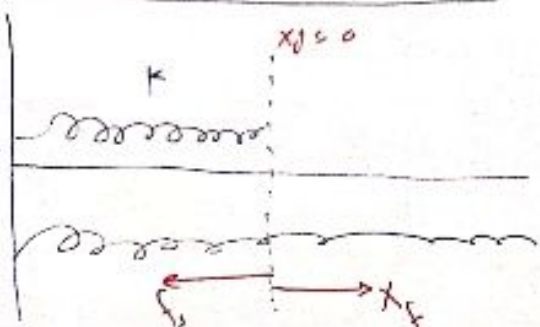
$$U_f - U_i = \frac{1}{2} k x_f^2 - \frac{1}{2} k x_i^2$$

let $x_i = 0 \Rightarrow U_i = 0$

$$U_s = \frac{1}{2} k x^2$$

$x_i = 0$

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$$E_{\text{mechanical}} = \text{Kinetic Energy} + \text{Potential Energy}$$

$$E_m = \frac{1}{2}mv^2 + U$$

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★ Conservation of Mechanical Energy :-
قانون حفظ الطاقة الميكانيكية

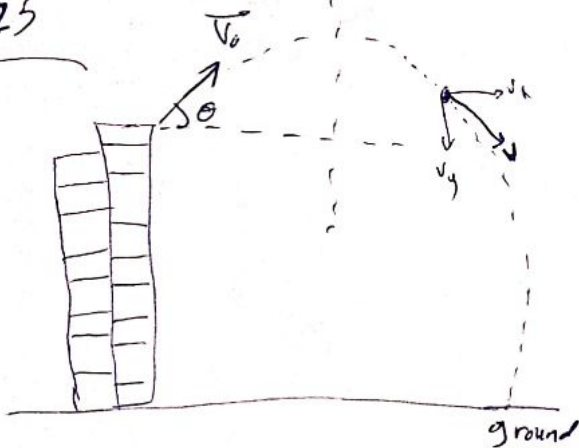
If the only force Acting on the system is conservative force.

$$\begin{aligned} W_{\text{cons}} &= -\Delta U \\ W_{\text{net}} &= \Delta K \end{aligned}$$

$$\begin{aligned} \Delta K &= -\Delta U \\ \Delta K + \Delta U &= 0 \end{aligned}$$

$$\begin{aligned} \Delta(K+U) &= 0 \quad \text{قانون حفظ الطاقة الميكانيكية} \\ K+U &= \text{constant} \\ (K+U)_i &= (K+U)_f \end{aligned}$$

Q25



at 6s

$$\begin{aligned} v_x &= 18 \text{ m/s} \\ v_y &= v_{0y} + a_y t \\ &= 24 + (-10)(6) \\ &= 24 - 60 \\ &= -36 \text{ m/s} \\ \vec{v}_6 &= 18\hat{i} - 36\hat{j} \text{ m/s} \end{aligned}$$

$m = 1 \text{ kg}$ At $t=0$, $\vec{v}_0 = 18\hat{i} + 24\hat{j} \text{ m/s}$
After 6s find ΔU ?

$$\begin{aligned} (K+U)_i &= (K+U)_f = K_0 + U_0 = K_f + U_f \\ K_0 - K_f &= U_f - U_0 \Rightarrow U_f - U_0 = 450 - 768 = -318 \text{ J} \end{aligned}$$

$$\begin{aligned} K_0 &= \frac{1}{2}mv_0^2 \\ &= \frac{1}{2}(1)(18^2 + 24^2) = 450 \text{ J} \\ K_f &= \frac{1}{2}mv_f^2 \\ &= \frac{1}{2}(1)(18^2 + (-36)^2) = 768 \text{ J} \end{aligned}$$

$$W_{\text{net}} = -\Delta U = 318 \text{ J}$$

Chapter (8) Lecture (2)

$F_{\text{Conservation}} \rightarrow mg = U_g = mgy \quad (y)$
 $-Kx \Rightarrow U_s = \frac{1}{2} Kx^2 \quad (x)$

$$E_{\text{initial}} = E_{\text{final}}$$

$$\frac{1}{2} m v_i^2 + U_i = \frac{1}{2} m v_f^2 + U_f$$

F is
Conservative

$$W_{\text{Conservative}} = -\Delta U$$

* Finding F_{cons} From U :-

$$W_{\text{con}} = -\Delta U$$

$$dU = -F_x dx$$

U is a function of x
 $f_{\text{cons}} = -\frac{dU}{dx}$

a) $W_g (P \rightarrow Q)$

$$W_g (P \rightarrow Q) = -\Delta U$$

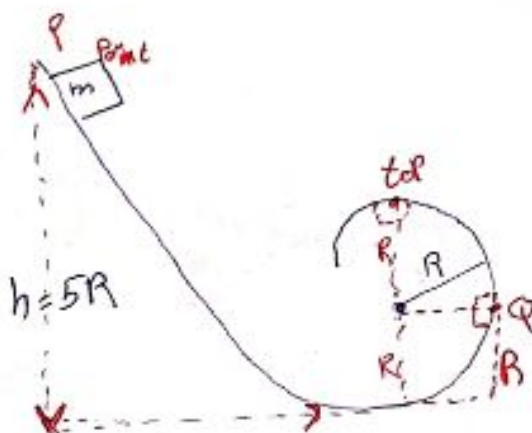
$$= -[U_Q - U_P]$$

$$= -[mgR - mg(5R)]$$

$$= +4mgR = 0.150 \text{ J}$$

Problem 8, 6

$m = 0.32 \text{ kg}$
 $R = 12 \text{ cm}$



b) $W_g (P \rightarrow R) = -\Delta U$

$$= -[U_{\text{top}} - U_P]$$

$$= -[mg(2R) - mg(5R)]$$

$$= +3mgR = 0.113 \text{ J}$$

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Problem 8-17

مسألة السقوط الحر

(a) At Q find \vec{F}_N ? \vec{F}_V ?

$$F_y = -mg$$

$$K = \frac{1}{2} m v_p^2 + U_p = \frac{1}{2} m v_Q^2 + U_Q$$

$$0 + 5mgR = \frac{1}{2} m v_Q^2 + mgR$$

$$\frac{1}{2} v_Q^2 = 4Rg \Rightarrow v_Q = \sqrt{8Rg}$$

$$F_N = \frac{m v_Q^2}{R} = \frac{m(8Rg)}{R} = 8mg \text{ () } (-\hat{i})$$

$$\vec{F}_Q = -8mg\hat{i} - mg\hat{j}$$

find the normal force acting on (m) at the top point at the top

$$F_N + mg = \frac{m v_t^2}{R}$$

$$(K + U)_p = (K + U)_t$$

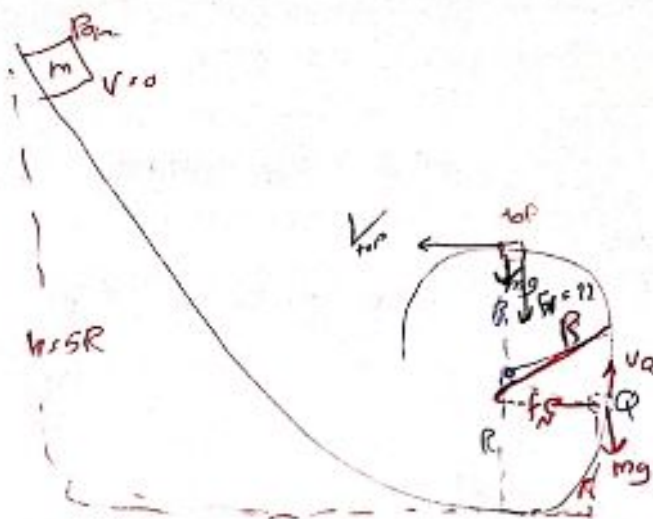
$$(0 + 5mgR) = \frac{1}{2} m v_t^2 + mg(2R) \Rightarrow v_t = \sqrt{6Rg}$$

at the top

$$F_N = \frac{m v_t^2}{R} - mg$$

$$F_N = \frac{m}{R} 6Rg - mg \Rightarrow F_N = 5mg \text{ at the top}$$

$$F = -5mg\hat{j}$$

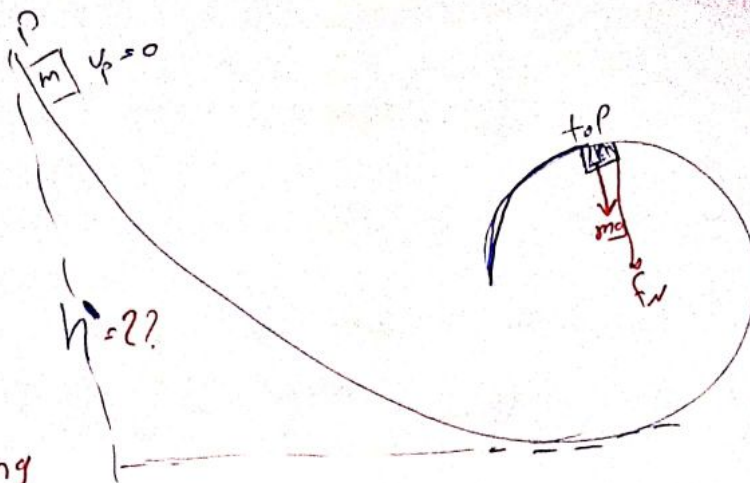


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© find h ??

At the top

$$\frac{mv^2}{R} = mg + f_N$$



on the verge of losing contact at the top

means $f_N \rightarrow 0$

$$\frac{mv_t^2}{R} = mg \Rightarrow v_t = \sqrt{Rg}$$

$$K_p + U_p = K_t + U_t$$

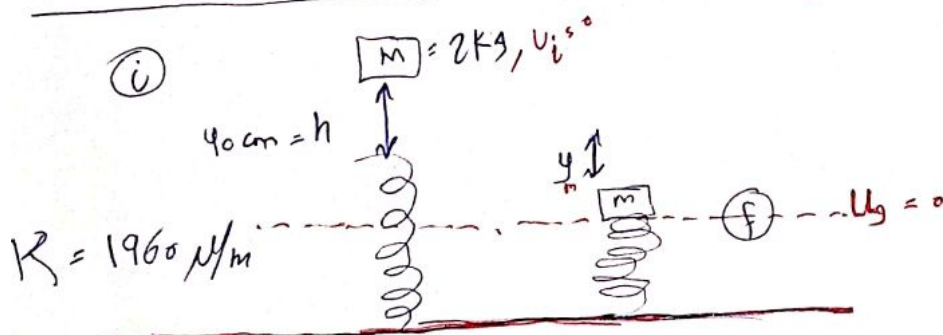
$$0 + mgh = \frac{1}{2}mv_t^2 + 2mgR$$

$$mgh = \frac{1}{2}m(Rg) + 2mgR$$

$$mgh = 2.5mgR \Rightarrow h = 2.5R$$

Problem 8-24

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$$(K + U)_i = (K + U)_f$$

$$(0 + mg(h + y_m)) = (0 + \frac{1}{2}ky_m^2)$$

$$2(9.8)[0.4 + y_m] = \frac{1}{2}(1960)y_m^2 = \boxed{}$$

Problem 8 - 104

$$m = 20 \text{ kg}, F_{\text{con}} = -3x - 5x^2$$

$$\text{at } x=0, U_0 = 0$$

a) Find U at $x = 2 \text{ m}$?

$$W_{\text{con}} = -\Delta U \Rightarrow \Delta U = - \int_i^f F_{\text{con}} dx$$

$$U_f - U_i = - \int (-3x - 5x^2) dx$$

$$U = \frac{3}{2}x^2 + \frac{5}{3}x^3 + C$$

$$0 = 0 + 0 + C$$

$$\boxed{C = 0}$$

$$U = \frac{3}{2}x^2 + \frac{5}{3}x^3$$

$$U(2) = \frac{3}{2}(2)^2 + \frac{5}{3}(2)^3 = 19.6 \text{ J}$$

b) At $x = 5 \text{ m}$, $U_x = -4 \text{ m/s}$

find V_x at $x = 0$??

$$(K + U)_{x=0} = (K + U)_{x=5}$$

$$\frac{1}{2}mV_0^2 + 0 = \frac{1}{2}m(-4)^2 + \left[\frac{3}{2}(5)^2 + \frac{5}{3}(5)^3 \right]$$

$$V_0 = \sqrt{\quad}$$
$$V_0 = -6.37 \text{ m/s}$$

c) Repeat a & b for $U = -8 \text{ J}$ at $x = 0$

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Problem $m = 0,2 \text{ kg}$, $U(x) = 8x^2 + 2x^4$ Joule
at $x = 1 \text{ m}$, $v = 5 \text{ m/s}$

Find v at the origin?

$$(K + U)_i = (K + U)_o$$

$$\frac{1}{2}(0,2)(5)^2 + (8(1) + 2(1)) = \left(\frac{1}{2}(0,2)v_o^2 + 0\right)$$

b) Find F_{cons} ?

$$F_{\text{cons}} = -\frac{dU}{dx} = -[6x + 8x^3]$$

$$F_{\text{cons}} = -16x - 8x^3$$

End ch8
Good Luck
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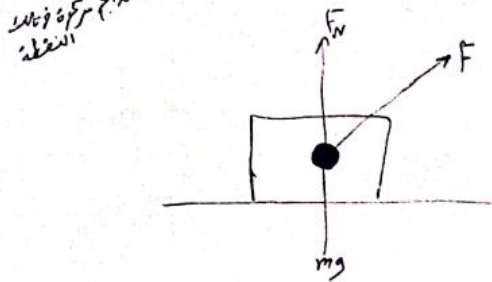
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Chapter 9

Center of mass, linear momentum:
مركز الكتلة كمية التمرار الخطية (الزخم)

* center of mass: is the points that moves as though, all of the system's mass concentrated there, and

all external force applied there.



* Two cases

→ system of Particles

→ Rigid (solid) bodies.

Case I \circ system of Particles

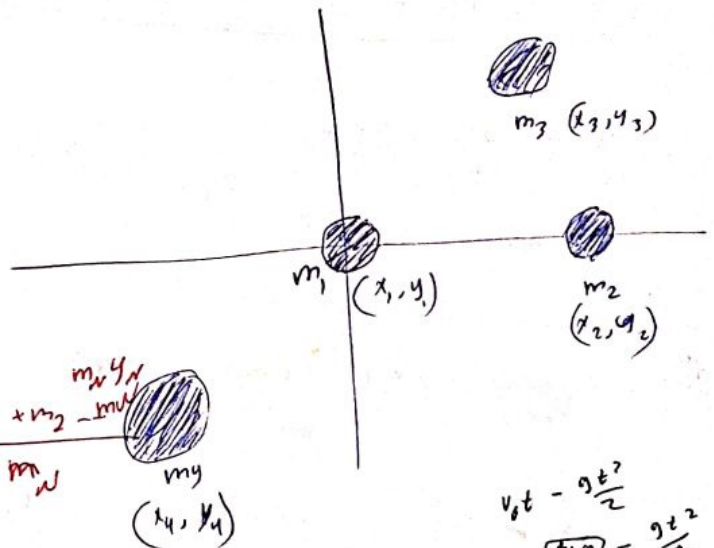
$$X_{com} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3 + m_4 x_4}{m_1 + m_2 + m_3 + m_4}$$

$$\Rightarrow X_{com} = \frac{1}{M} \sum_{i=1}^N m_i x_i$$

total mass

$$\Rightarrow Y_{com} = \frac{1}{M} \sum_{i=1}^N m_i y_i = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3 + m_4 y_4}{m_1 + m_2 + m_3 + m_4}$$

$$\Rightarrow Z_{com} = \frac{1}{M} \sum_{i=1}^N m_i z_i$$



$$v_{0t} - \frac{gt^2}{2}$$

$$25 + 0 - \frac{9t^2}{2}$$

$$(17) \quad u = gt$$

$$\vec{r}_{com} = x_{com} \hat{i} + y_{com} \hat{j} + z_{com} \hat{k}$$

$$\vec{r}_{com} = \frac{1}{M} \sum_{i=1}^N m_i \vec{r}_i$$

P1 $m_1 = 2 \text{ kg} \rightarrow (-1, 5) \rightarrow \mathbf{r}_1 = -\hat{i} + 5\hat{j}$

$m_2 = 4 \text{ kg} \rightarrow (6, -7, 5)$

$m_3 = 3 \text{ kg} \rightarrow ?? (x_3, y_3)$

$\mathbf{r}_{\text{com}} = (-0,5, -0,7)$
 $\downarrow \quad \downarrow$
 $x_{\text{com}} \quad y_{\text{com}}$

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$x_{\text{com}} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3} \quad ??$

$-0,5 = \frac{-2 + 24 + 3x_3}{2 + 4 + 3} \Rightarrow x_3 = -1,5 \text{ m}$
 $-4,5 = 24 + 3x_3$

$y_{\text{com}} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{9}$

$-0,7 = \frac{10 + 30 + 3y_3}{9} \Rightarrow y_3 = -1,43 \text{ m}$

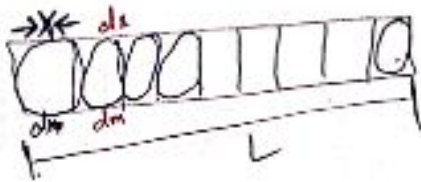
$-6,3 = 40 + 3y_3$
 $-46,3 = 3y_3$
 $y_3 = -15,43$

OR $\mathbf{r}_{\text{com}} = \frac{m_1 \mathbf{r}_1 + m_2 \mathbf{r}_2 + m_3 \mathbf{r}_3}{M}$

$-0,5\hat{i} + 0,7\hat{j} = \frac{2(-\hat{i} + 5\hat{j}) + 4(6\hat{i} - 7,5\hat{j}) + 3(x_3\hat{i} + y_3\hat{j})}{9}$

Case II: Solid bodies (Rigid)

$$X_{com} = \frac{1}{M} \int x \, dm, \quad Y_{com} = \frac{1}{M} \int y \, dm, \quad Z_{com} = \frac{1}{M} \int z \, dm$$



Uniform λ $\Rightarrow \lambda = \text{constant}$
 الكثافة الخطية $\lambda = \frac{dm}{dx}$

$$\frac{dm}{dx} = \frac{M}{L} = \lambda$$

uniform linear density

بمعنى: كثافة مادية موحدة

$$\frac{dm}{dx} = \frac{M}{L} \Rightarrow dm = \frac{M}{L} dx$$

$$dm = \lambda dx$$

1-D
 كثافة خطية

$$\left. \begin{aligned} X_{com} &= \frac{1}{M} \int x \, dm \\ Y_{com} &= \frac{1}{M} \int y \, dm \\ Z_{com} &= \frac{1}{M} \int z \, dm \end{aligned} \right\} \Rightarrow r_{com} = \frac{1}{M} \int \vec{r} \, dm$$

$$dm = \lambda dx, \quad \lambda = \text{uniform}, \quad \lambda = \frac{M}{L}, \quad \lambda = \text{linear density}$$

2-D
 كثافة سطحية

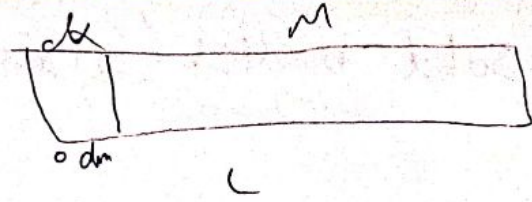
$$dm = \sigma dA, \quad \sigma = \text{surface density}, \quad \sigma = \frac{M}{A}$$

3-D

$$dm = \rho dV, \quad \rho = \text{volume density}, \quad \rho = \frac{M}{V_{\text{total}}}$$

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$$x_{com} = \frac{1}{M} \int x dm$$



$$dm = \lambda dx = \frac{M}{L} dx$$

$$\Rightarrow x_{com} = \frac{1}{M} \int_0^L x \left(\frac{M}{L} \right) dx = \frac{1}{L} \int_0^L x dx$$

rod is
(uniform rod)

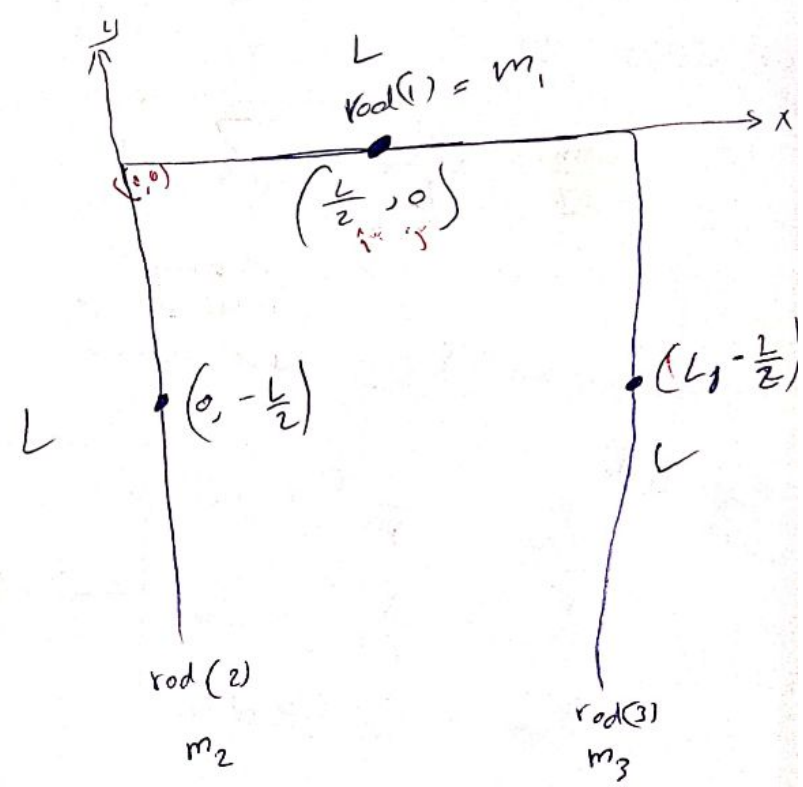
$$= \frac{1}{L} \left[\frac{x^2}{2} \right]_0^L = \frac{1}{L} \cdot \frac{L^2}{2} = \frac{L}{2}$$

center of mass

L = locum

$$x_{com} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3}$$

$$y_{com} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{m_1 + m_2 + m_3}$$



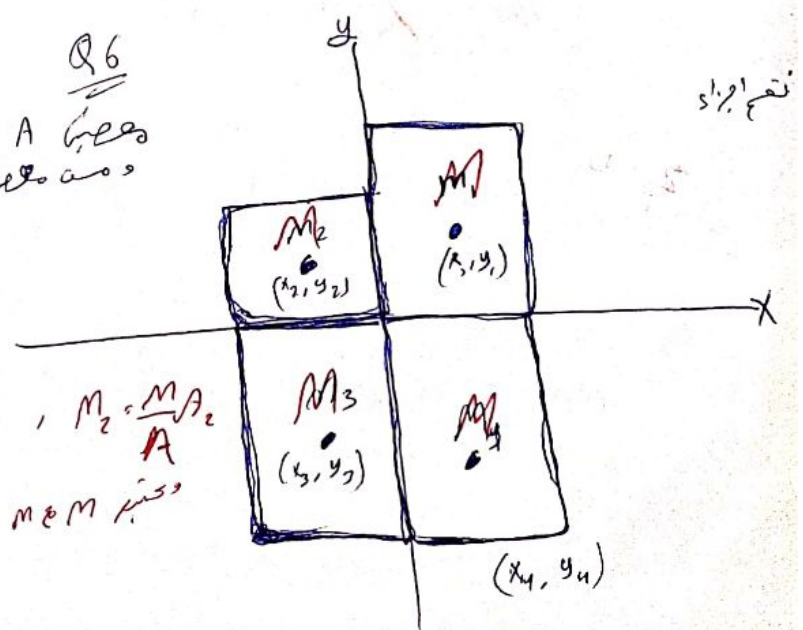
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Q6
A Green
rectangle

$$\frac{M_1}{A_1} = \frac{M}{A}$$

$$M_1 = \frac{M}{A} A_1, \quad M_2 = \frac{M}{A} A_2$$

m & M are mass



نصف المساحة

Newton's 2nd Law for system of particles.

$$\boxed{\vec{F}_{\text{net}} = M \vec{a}_{\text{com}}}$$

$$\vec{r}_{\text{com}} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2 + \dots + m_N \vec{r}_N}{M}$$

$$M \vec{r}_{\text{com}} = m_1 \vec{r}_1 + m_2 \vec{r}_2 + \dots + m_N \vec{r}_N$$

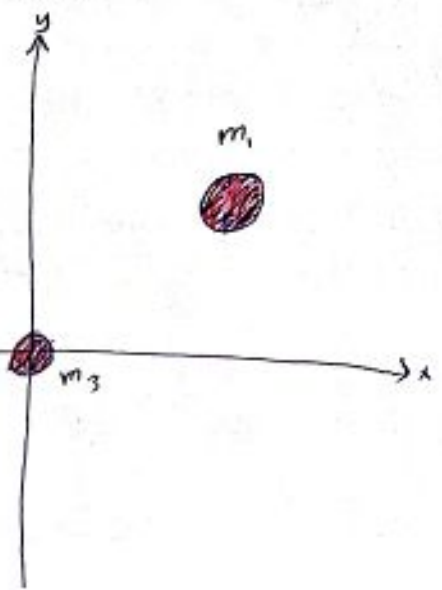
$$M \vec{v}_{\text{com}} = m_1 \vec{v}_1 + m_2 \vec{v}_2 + \dots + m_N \vec{v}_N$$

$$M \vec{a}_{\text{com}} = m_1 \vec{a}_1 + m_2 \vec{a}_2 + \dots + m_N \vec{a}_N$$

$$\boxed{M \vec{a}_{\text{com}} = \vec{F}_{\text{net}}}$$

نتائج التفاضل
المشتق

system



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Chapter 9, 2

Center of mass

system of particles

$$\begin{aligned} x_{\text{com}} &= \frac{1}{M} \sum_{i=1}^N m_i x_i \\ y_{\text{com}} &= \frac{1}{M} \sum_{i=1}^N m_i y_i \\ z_{\text{com}} &= \frac{1}{M} \sum_{i=1}^N m_i z_i \\ \vec{r}_{\text{com}} &= \frac{1}{M} \sum_{i=1}^N m_i \vec{r}_i \end{aligned}$$

Rigid bodies

$$\begin{aligned} x_{\text{com}} &= \frac{1}{M} \int x \, dm \\ dm &= \lambda \, dx \\ \lambda &= \frac{M}{L} \\ \sigma &= \frac{M}{A} \\ \rho &= \frac{M}{V} \end{aligned}$$

* linear momentum كمية الزخم (\vec{p})

* $\vec{p} = m \vec{v}$, $m = \text{mass}$
 $v = \text{velocity}$

* p is in the same direction of \vec{v} .
 بنفس اتجاه v يكون p

$$\frac{dp}{dt} = F_{\text{net}}$$

مجموع القوى

* Newton's 2nd law

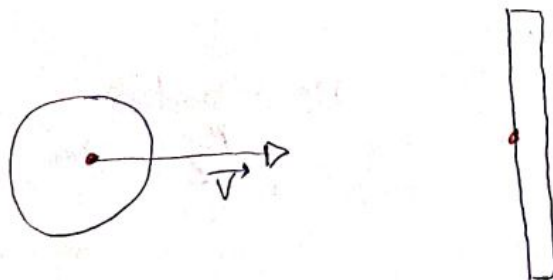
$$F_{\text{net}} = \frac{dp}{dt} , \vec{p} = m\vec{v}$$

$$= \frac{d}{dt} [mv] = m \frac{dv}{dt} + v \frac{dm}{dt}$$

if $m = \text{constant}$ $\Rightarrow \frac{dm}{dt} = 0$

$$F_{\text{net}} = m \frac{dv}{dt} = \boxed{m\vec{a}}$$

* Collision and Impulse الدفع



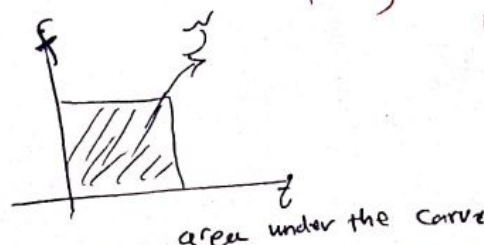
* Impulse (\vec{J})

$$\vec{J} = \vec{F}_{\text{Avg}} \cdot \Delta t$$

, \vec{F} constant

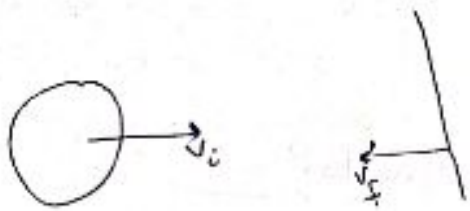
$$\vec{J} = \int_{t_1}^{t_2} \vec{F}(t) dt$$

(F variable)
 تعتمد على
 time
 الزمن



area under the curve

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$$J = \Delta P$$

$$J = m \Delta v$$

$$= m(\vec{v}_f - \vec{v}_i)$$

$$F_{\text{avg}} = \frac{dP}{dt} \quad \text{تباين}$$

$$F_{\text{avg}} = \frac{\Delta P}{\Delta t}$$

$$\vec{J} = F_{\text{avg}} \cdot \Delta t = \Delta \vec{P} = m(\vec{v}_f - \vec{v}_i) \quad , F \text{ Constant}$$

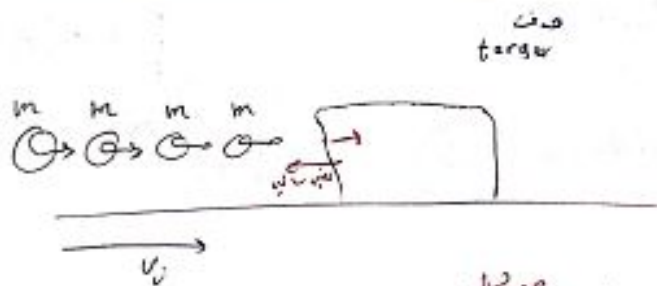
$$J = \int F dt = \Delta \vec{P} \quad , \text{not constant.}$$

Special case

$$\vec{J} = -n \Delta \vec{P}$$

$n = \# \text{ of particles}$

$\Delta P = \text{change in linear momentum of 1 particle.}$



تارگت
تارگت

$$J = -n \Delta P = n m \Delta v$$

$$F_{\text{avg}} \Delta t = -n m \Delta v$$

$$F_{\text{avg}} = \frac{-n m \Delta v}{\Delta t}$$

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حفظ كمية الزخم
Conservation of linear momentum

from Newton's 2nd law

$$F_{net} = \frac{dp}{dt} \Rightarrow p \text{ is conserved if } \boxed{\Delta p = 0}$$

\vec{P} is conserved (حفظ كمية الزخم) ($\Delta \vec{P} = 0$), if the system is isolated and closed (منزوح و مغلق)

$$\Delta \vec{P} = 0$$

$$\vec{P}_f - \vec{P}_i = 0$$

$$\boxed{\vec{P}_f = \vec{P}_i}$$

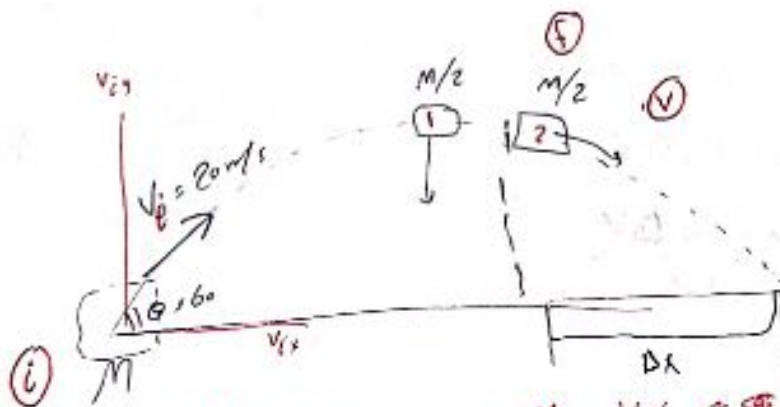
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Isolated system
 $\Delta p = 0 \Rightarrow$

$$\boxed{P_{ix} = P_{fx}}$$

$$\boxed{P_{iy} = P_{fy}}$$

P13



$$P_{ix} = P_{f,x(2)} + P_{f,x(1)}$$

$$M v_i \cos \theta = \frac{M}{2} V$$

$$V = 2 v_i \cos \theta$$

$$= 2(20) \cos 60 = \boxed{20 \text{ m/s}}$$

$$\Delta x = v_i t \cos \theta$$

$$v_f^2 = v_{iy}^2 + 2a \Delta y$$

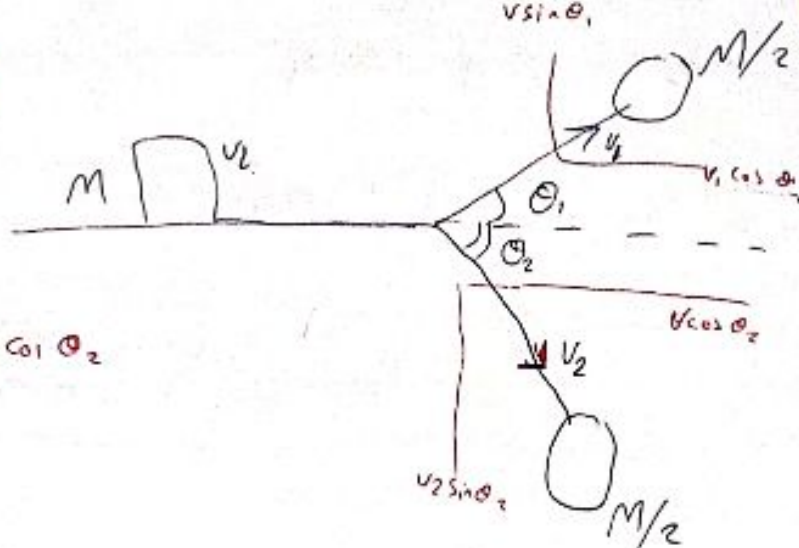
$$= v_i^2 \sin^2 \theta + 2g \Delta y$$

$$\Delta y = \frac{v_i^2 \sin^2 \theta}{2g} = \boxed{15.3}$$

$$\Delta y = v_{iy} t + \frac{1}{2} g t^2$$

$$15.3 = \frac{1}{2} g t^2 \Rightarrow t = \boxed{1.75 \text{ s}}$$

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$$\vec{P}_{ix} = \vec{P}_{fx}$$

$$M v_i = \frac{M}{2} v_1 \cos \theta_1 + \frac{M}{2} v_2 \cos \theta_2$$

$$\vec{P}_{iy} = \vec{P}_{fy}$$

$$0 = \frac{M}{2} v_1 \sin \theta_1 - \frac{M}{2} v_2 \sin \theta_2$$

chapter 9, 3

momentum and kinetic energy in collision

النوع التصادمات
* type of collisions.

① elastic collision • $\vec{P}_i = \vec{P}_f$ ($\Delta \vec{P} = 0$) ($\Delta K = 0$)

② inelastic collision ($\Delta K \neq 0$)

③ completely inelastic collision. ($\Delta K \neq 0$)

1 Elastic collision

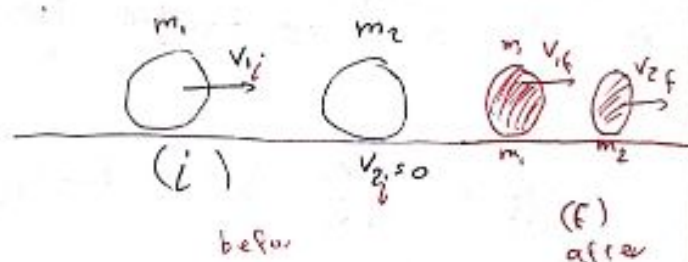
$$\vec{P}_i = \vec{P}_f$$

$$K_i = K_f$$

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

$$m_1 v_{1i} = m_1 v_{1f} + m_2 v_{2f} \rightarrow \text{①}$$

$$\frac{1}{2} m_1 v_{1i}^2 + 0 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2 \rightarrow \text{②}$$



$$m_1 v_{1i} = m_1 v_{1f} + m_2 v_{2f} \quad \text{--- (1)}$$

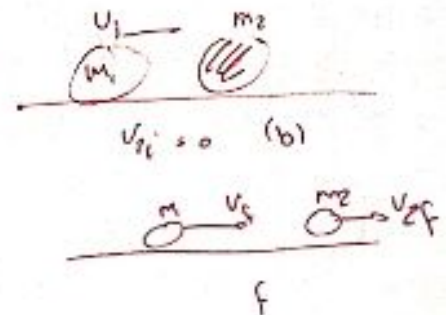
since f_{2f} from eqn (1)
substitute in eqn (2)

$$\frac{1}{2} m_1 v_{1i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2 \quad \text{--- (2)}$$

at time t_0 v_{1i}

$$v_{1f} = \frac{m_1 + m_2}{m_1 + m_2} v_{1i}$$

$$f_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i}$$



$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

$$\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$

ان غير m_2 تترك

thrust (F) = mass rate of fuel exhaust velocity
 $\dot{m} \cdot v_e$

if $m_1 = m_2 \Rightarrow v_{1f} = 0$

$$\Rightarrow v_{2f} = \frac{2m}{m_1 + m_2} v_{1i}$$

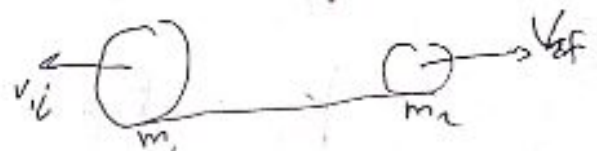
$$v_{2f} = v_{1i}$$

السرعة التي بها الوقود يخرج
السرعة التي بها الوقود يخرج

(2) if massive target ($m_2 \gg m_1$) neglect $\frac{m_1}{m_2}$

$$v_{1f} = \frac{-m_2}{m_2} v_{1i} = -v_{1i}$$

$$v_{2f} = \frac{2m_1}{m_2} v_{1i}$$

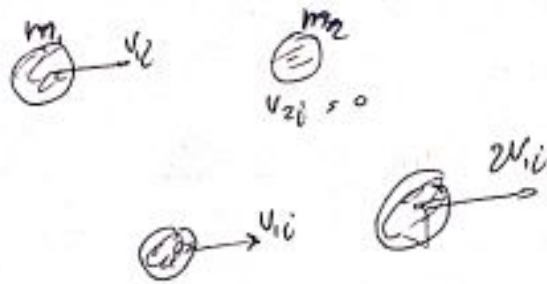


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3) massive projectile ($m_1 \gg m_2$)

$$v_{1f} = v_{1i}$$

$$v_{2f} = \frac{2m_1}{m_1} v_{1i} = 2v_{1i}$$



ملاحظة -

* target at rest ($v_{2i} = 0$)

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i}$$

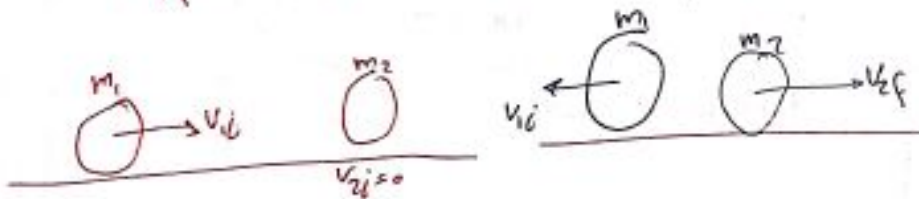
$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i}$$

Case 1) $m_1 = m_2$ ($v_{1f} = 0$, $v_{2f} = v_{1i}$)



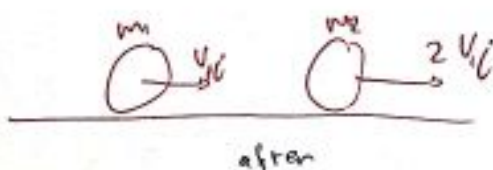
Case 2) massive target ($m_2 \gg m_1$)

$$v_{1f} = -v_{1i}, \quad v_{2f} \approx \frac{2m_1}{m_2} v_{1i}$$



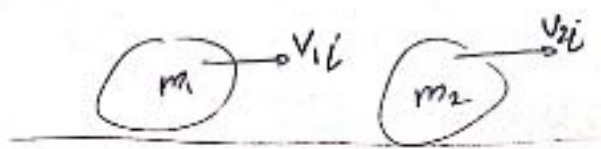
Case 3) massive projectile ($m_1 \gg m_2$)

$$v_{1f} \approx v_{1i}, \quad v_{2f} \approx 2v_{1i}$$

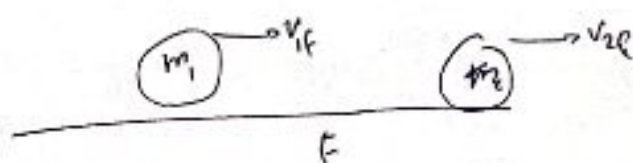


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مصادف
moving target



(i)



$$\vec{p}_i = \vec{p}_f$$

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

$$K_i = K_f$$

$$\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$

moving target

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} + \frac{2m_2}{m_1 + m_2} v_{2i}$$

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} + \frac{m_2 - m_1}{m_1 + m_2} v_{2i}$$

Elastic collision

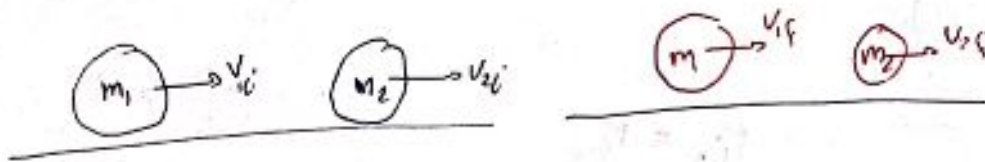
- target at rest
- moving target

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② Inelastic collision

$$\vec{P}_i = \vec{P}_f, \quad K_i \neq K_f \Rightarrow K_i \neq K_f + \overset{\substack{\Delta E \\ \uparrow}}{E_{\text{diss}}}$$

$$\Delta K = K_i - K_f$$



$$P_i = P_f$$

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

$$\Delta E = \left(\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 \right) - \left(\frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2 \right) \quad \text{--- (2)}$$

③ Completely Inelastic Collision



$$\vec{P}_i = \vec{P}_f \Rightarrow m_1 v_{1i} + m_2 v_{2i} = M V_f$$

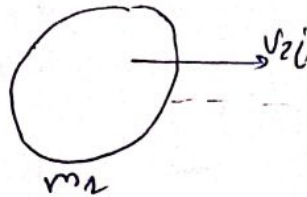
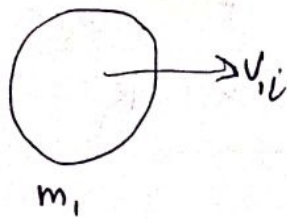
$$\Delta E = K_i - K_f \Rightarrow \Delta E = \left(\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 \right) - \frac{1}{2} M V_f^2$$

$$P = M \vec{V}_{\text{com}}$$

$$V_{\text{com}} = \frac{\vec{P}}{M} = \frac{m_1 v_{1i} + m_2 v_{2i}}{M}$$

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* Collision in 2D :



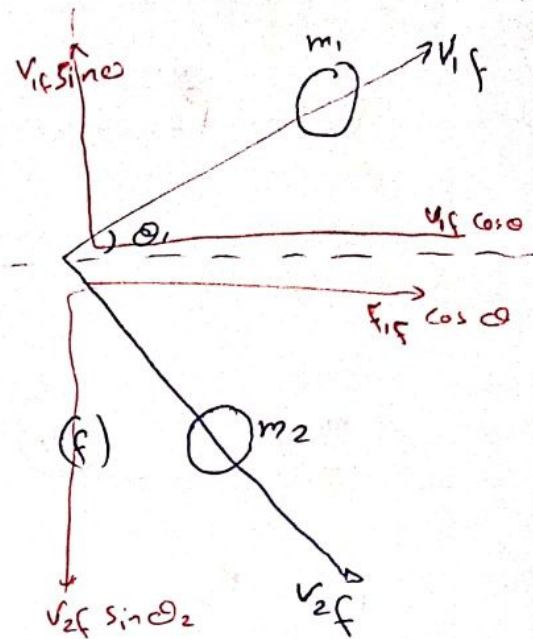
(i)

$$\vec{P}_i = \vec{P}_f$$

$$P_{ix} = P_{fx}$$

$$P_{iy} = P_{fy}$$

لا ينفصل



$$P_{ix} = P_{fx}$$

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} \cos \theta_1 + m_2 v_{2f} \cos \theta_2 \quad \text{--- (1)}$$

$$P_{iy} = P_{fy}$$

$$0 = m_1 v_{1f} \sin \theta_1 - m_2 v_{2f} \sin \theta_2 \quad \text{--- (2)}$$

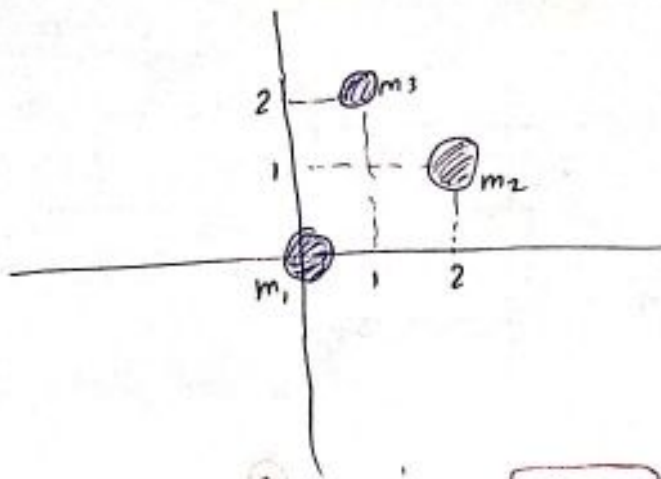
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P2

$$m_1 = 3 \text{ kg}$$

$$m_2 = 4 \text{ kg}$$

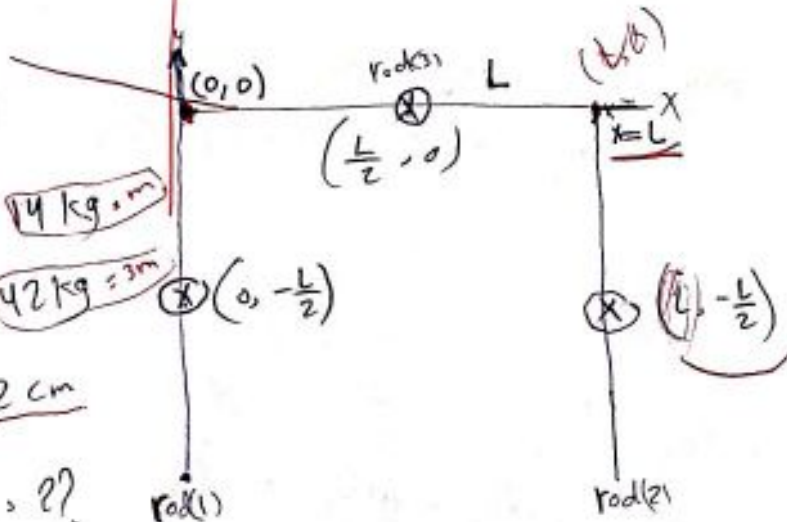
$$m_3 = 8 \text{ kg}$$



$$x_{\text{com}} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3} = 1,1 \text{ m}$$

$$y_{\text{com}} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{m_1 + m_2 + m_3} = 1,3 \text{ m}$$

P4



$$m_1 = m_2 = 4 \text{ kg}$$

$$m_3 = 42 \text{ kg}$$

$$L = 22 \text{ cm}$$

$$y_{\text{com}}, x_{\text{com}} = ??$$

$$x_{\text{com}} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{M} = \frac{4(0) + 4(L/2) + 42(L)}{50} = \frac{46L}{50} = \frac{23L}{25}$$

$$y_{\text{com}} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{M} = \frac{4(0) + 4(0) + 42(-L/2)}{50} = \frac{-42L}{50} = -\frac{21L}{25} = -4,4 \text{ cm}$$

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P22

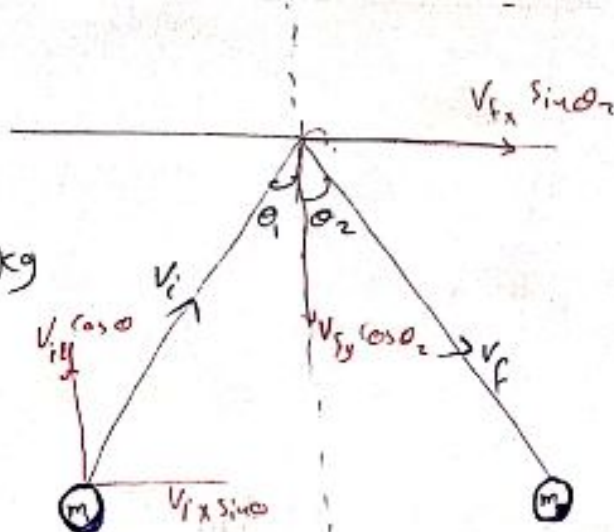
$$\theta_1 = 30^\circ$$

$$m = 0.165 \text{ kg}$$

$$v_i = 2 \text{ m/s}$$

$$v_{xi} = v_{xf}$$

y = variable



@ angle $\theta_2 = ?$

⑥ $\Delta \vec{P} = ??$

⑦ $\vec{P}_x = m\vec{v}_x$
 $\vec{P}_{x(i)} = \vec{P}_{x(f)}$

$$m v_i \sin \theta_1 = m v_f \sin \theta_2$$

$$\sin \theta_1 = \sin \theta_2 \Rightarrow \theta_2 = \theta_1 = 30^\circ$$

⑧ $\Delta \vec{P} \begin{cases} \Delta P_x = 0 \\ \Delta P_y = ? \end{cases}$

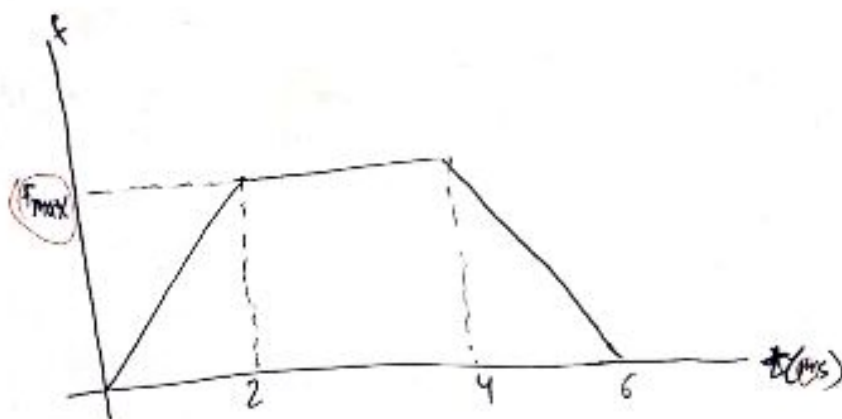
$$\Delta P_y = P_{fy} - P_{iy}$$

$$= -m v_f \cos \theta_2 - m v_i \cos \theta_1$$

$$= -2m v_i \cos \theta_1$$

$$\Delta P = -0.572 \text{ kg m/s}$$

P35



$$m = 58 \text{ g}$$

$$v_i = 34 \text{ m/s}$$

$$v_f = 34 \text{ m/s}$$

wall

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$\vec{J} = \Delta P = \int \vec{F} dt$

$$\Delta P = \overline{P_f} - \overline{P_i}$$

$$= m v_f - m v_i$$

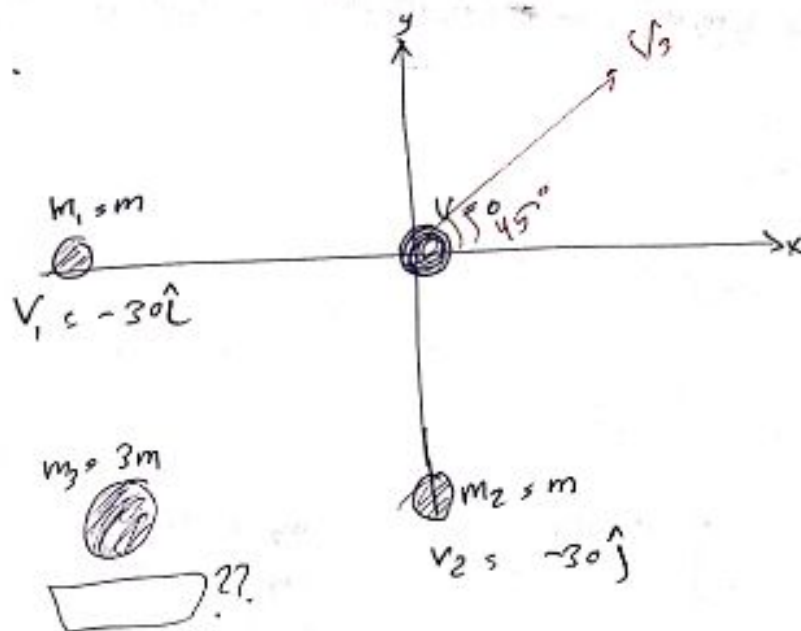
$$\Delta P = 2 m v_i$$

$$\int f dt = \frac{1}{2} (0,002) f_{\max} + 0,002 f_{\max} + \frac{1}{2} (0,002) f_{\max} = 0,004 f_{\max}$$

$$2mV_i = 0.004 f_{\text{smax}}$$

$$f_{\text{spring}} = 9.9 \times 10^2 \text{ N}$$

P 47



$$\overrightarrow{P_i} = \overrightarrow{P_f}$$

$$0 = m_1 \vec{v}_1 + m_2 \vec{v}_2 + m_3 \vec{v}_3$$

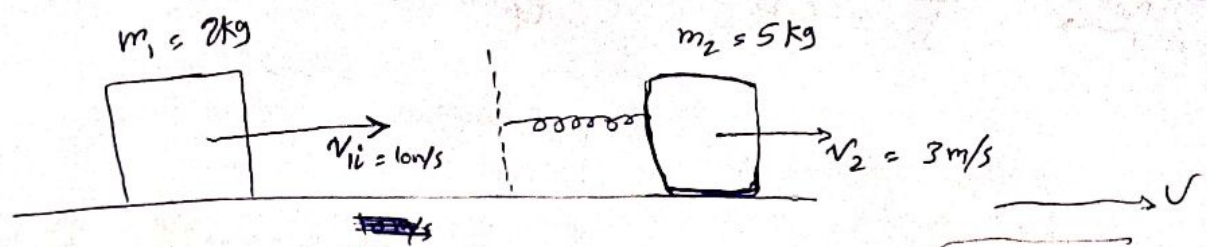
$$V_3 = \frac{-m_1 \vec{v}_1 - m_2 \vec{v}_2}{m_3} = \frac{30 \text{ m/s } \hat{i} + 30 \text{ m/s } \hat{j}}{3 \text{ m}} = \boxed{10 \hat{i} + 10 \hat{j} = V_3}$$

$\tan \theta = \frac{10}{10} = 1 \Rightarrow 45^\circ$
 $\times 225$
 $\theta = 45^\circ$

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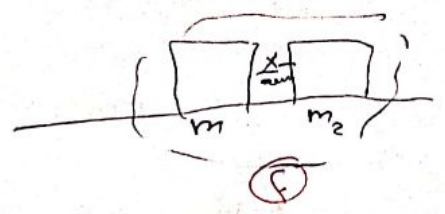
P59

$k = 1120 \text{ N/m}$



$x_{\text{max}} = ??$

"completely inelastic collision"



$$K_i + U_{\text{spring}} = K_f + U_{\text{spring}} \quad U_s = \frac{1}{2} k x^2$$

$$K_i - K_f = U_s$$

$$\left(\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 \right) - \frac{1}{2} (m_1 + m_2) v_f^2 = \frac{1}{2} k x^2 \Rightarrow x = 0.25 \text{ m}$$

المعادن

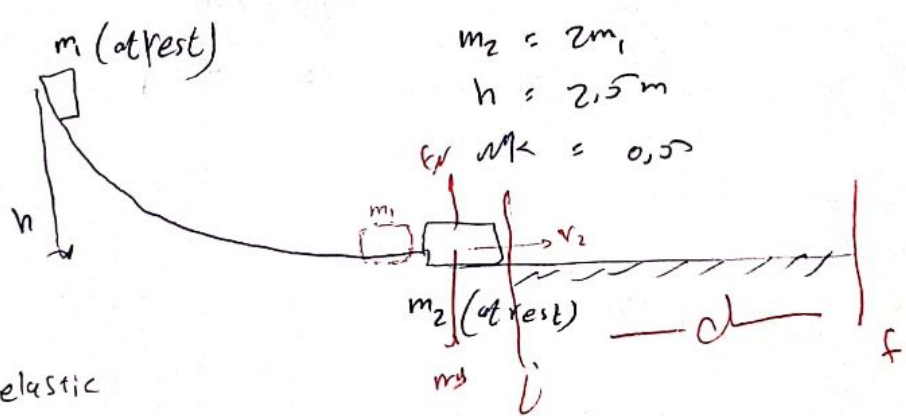
$P_i = P_f$

$$m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v_f$$

$v_f = \boxed{}$

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P68



a) elastic

b) Completely inelastic

$$v_2 = \frac{2m_1}{m_1 + m_2} v_{1i}$$

$$v_{1i} = \sqrt{2gh} = \sqrt{2(9.8)(2.5)} = 7 \text{ m/s}$$

$$\frac{2m_1}{m_1 + 2m_2} (7) = \frac{2}{3} (7) = \boxed{4.67 \text{ m/s}}$$

$$K_i + U_i = K_f + U_f + \Delta E_{th} \rightarrow W_{friction}$$

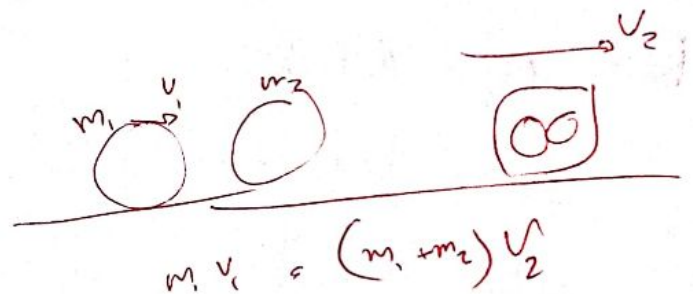
$$\frac{1}{2} m v_2^2 = \Delta E_{th}$$

$$\frac{1}{2} m v_2^2 = f_k \cdot d$$

$$\frac{1}{2} m v_2^2 = \mu_k (mg) d$$

$$d = 2,22 \text{ m}$$

$$(2) \quad v_2 = \frac{m_1}{m_1 + m_2} v_{1i} \Rightarrow d = 0,556 \text{ m}$$



$$F_{net} = \frac{dP}{dt} \Rightarrow F_{net} = m \vec{a}$$

$$P = mv$$

$$F_{net} = \frac{d}{dt} [mv] = m \frac{dv}{dt} + v \frac{dm}{dt}$$

$$= ma + v \frac{dm}{dt} \rightarrow \text{if } m \text{ is constant}$$

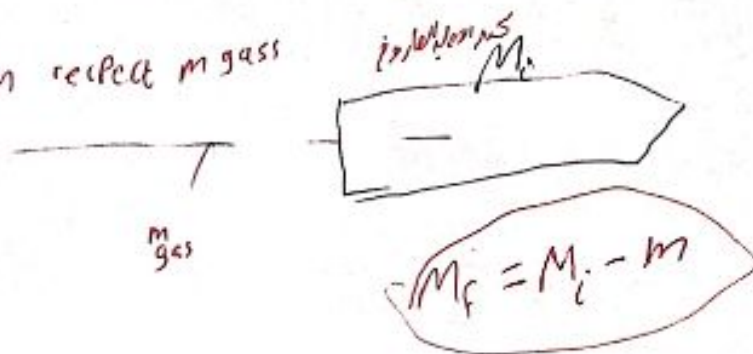
$$F_{net} = ma$$

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$\phi \quad Ma = R V_{rel}$ 'is equation of rocket'

* $V_f - V_i = V_{rel} \ln \frac{M_i}{M_f}$ '2nd rocket equation'
initial / final velocity

V_{rel} = velocity of gas with respect to the rocket.



R: $\frac{dm}{dt}$ *rate of change of mass*

P76 $M_i = 6090 \text{ kg}$
 $V_i = 105 \text{ m/s}$
 $m_{\text{gas}} = 80 \text{ kg}$
 $V_{rel} = 253$
 $V_f = ??$

$$V_f - V_i = V_{rel} \ln \frac{M_i}{M_f}$$

$$V_f - 105 = 253 \ln \frac{6090}{6090 - 80}$$

$$V_f = \boxed{}$$

End ch9
 Good Luck
 Anan Elayan

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Chapter 10

Rotation الحركة الدورانية

Motion الحركة

Translation

انتقالية (Position) , $\Delta x = x_2 - x_1$, $u_{avg} = \frac{dx}{dt}$
 $\Delta v_{avg} = \frac{dv}{dt}$, speed

Rotation

دائرية (Angle) , $\Delta \theta = \theta_2 - \theta_1$, ω_{avg} , α_{avg}

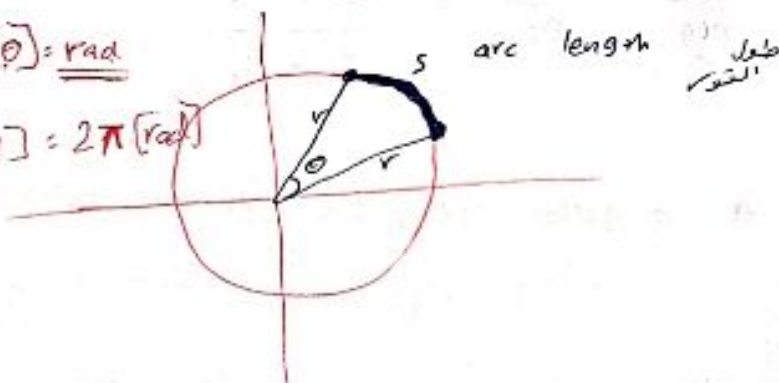
* Rotational Variables : متغيرات الحركة الدورانية

① angular position : الموضع الزاوي / الزاوي (θ)

$$\theta = \frac{s}{r}$$

$$[\theta] = \text{rad}$$

$$1 \text{ rev} = 2\pi \text{ rad}$$

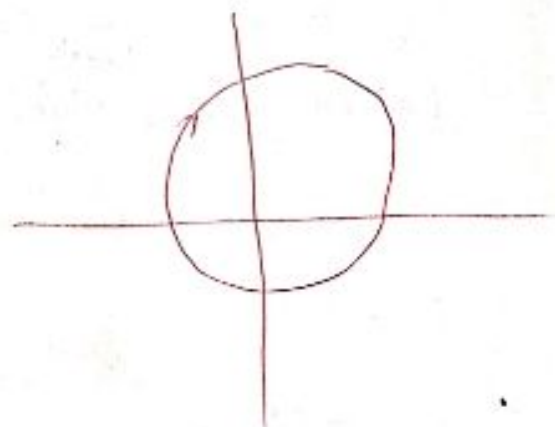


② angular displacement : الإزاحة الزاوية ($\Delta \theta$)

$$\Delta \theta = \theta_2 - \theta_1$$

تغير في الزاوية
وإزاحة

$$[\Delta \theta] = \text{rad}$$

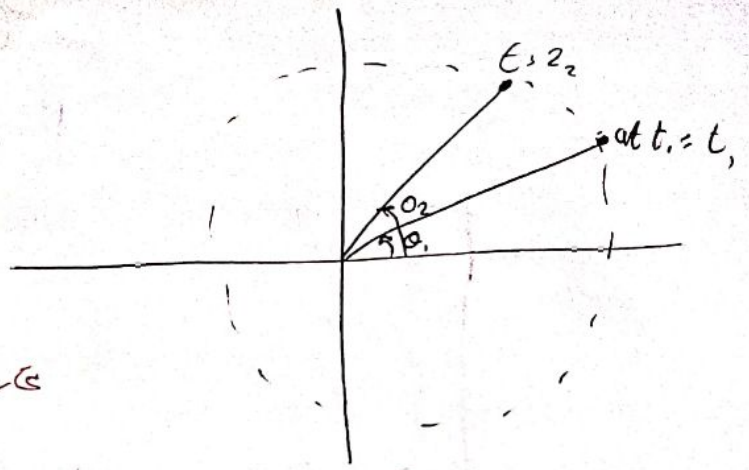


* Clock wise $\Rightarrow \Delta \theta$ is negative (-)

* Counter clock wise $\Rightarrow \Delta \theta$ is positive (+)

3 angular Velocity (ω_{avg})

$$\omega_{avg} = \frac{\Delta \theta}{\Delta t} = \frac{\theta_2 - \theta_1}{t_2 - t_1}$$



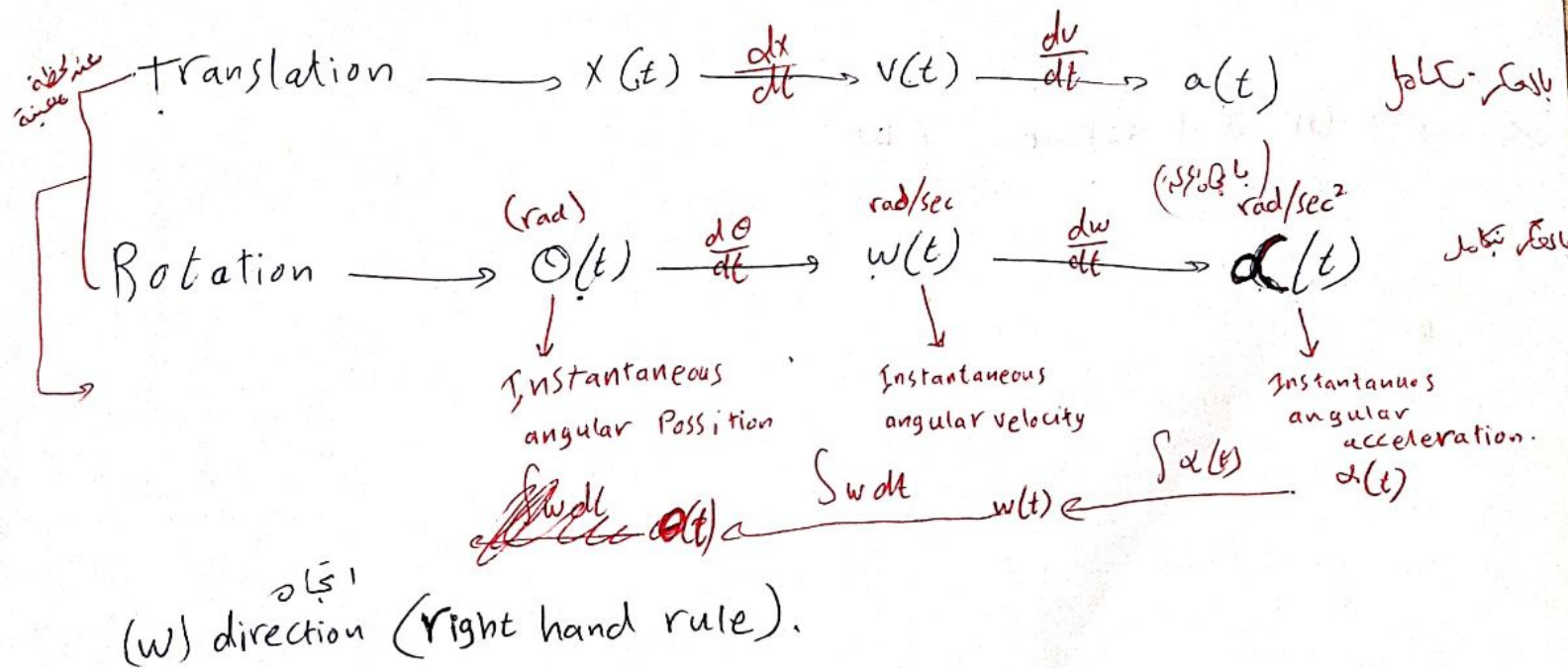
$\omega_{avg} \rightarrow \oplus$ counter clockwise عقارب عقارب

$\omega_{avg} \rightarrow \ominus$ clockwise مع عقارب

4 angular acceleration (α_{avg})

$$\alpha = \frac{\Delta \omega}{\Delta t} = \frac{\omega_2 - \omega_1}{t_2 - t_1}$$

* angular speed = $|\omega|$ angular velocity



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P(u)₂

$$\theta(t) = 2 + 4t + 2t^3$$

at $t=0 \Rightarrow$ (a) angular Position

(b) angular velocity

(c) ω at $t=4$ sec

(d) α at $t=2$ sec?

A $\theta|_{t=0} = 2 \text{ rad}$

(b) $\omega(t) = \frac{d\theta}{dt} = 4 + 6t^2 = \omega(0) = 4 \text{ rad/sec}$

(c) $\omega|_{t=4} = 4 + 6(4)^2 = 100 \text{ rad/s}$

(d) $\alpha = \frac{d\omega}{dt} = 12t = \alpha|_{t=2} = 24 \text{ rad/sec}^2$

Prob (b) ω_{avg} $t=1$ to $t=4$

$$\omega_{avg} = \frac{\theta_2 - \theta_1}{t_2 - t_1}$$

* Rotation motion at constant α :- (w) *proof*

translation ($a \equiv \text{cons}$)

$$V = V_0 + at$$

$$V^2 = V_0^2 + 2a \Delta x$$

$$\Delta x = V_0 t + \frac{1}{2} a t^2$$

$\Delta x, V, a$

Rotation ($\alpha \equiv \text{cons}$)

$$\omega = \omega_0 + \alpha t$$

$$\omega^2 = \omega_0^2 + 2\alpha \Delta \theta$$

$$\Delta \theta = \omega_0 t + \frac{1}{2} \alpha t^2$$

$\Delta \theta, \omega, \alpha$

$$\omega = \frac{2\pi R}{T}$$

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P(9)

$$\omega_0 = 12,6 \text{ rad/sec}, \quad \alpha = -4,2 \text{ rad/sec}^2$$

(a) t until t stop ?

$$\omega = \omega_0 + \alpha t$$

$$t = \frac{-\omega_0}{\alpha} = \frac{-12,6}{-4,2} = \boxed{3 \text{ sec}}$$

(b) $\omega^2 = \omega_0^2 + 2\alpha \Delta\theta$

$$\Delta\theta = \frac{-\omega_0^2}{2\alpha} = \frac{-(12,6)^2}{-2(4,2)} = \boxed{18,9 \text{ rad}}$$

$$\begin{aligned} 1 \text{ rev} &\rightarrow 2\pi \text{ rad} \\ \times &= 18,9 \text{ rad} \end{aligned}$$

translation

Rotation

$x, v, a \longleftrightarrow \odot, \omega, \alpha$

$$s = r\theta \quad \text{angular position.}$$

linear position $\frac{ds}{dt} = r \frac{d\theta}{dt} \Rightarrow \boxed{v = r\omega}$

$$\frac{ds^2}{dt^2} = r \frac{d\omega^2}{dt^2}$$

$$\boxed{a = r\alpha}$$

$$\boxed{\frac{a}{v} = \frac{v}{r}}$$

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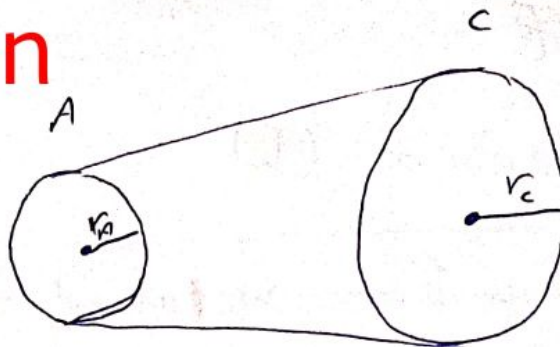
P28

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$$r_A = 10 \text{ cm}, \quad r_C = 25 \text{ cm}$$

$$\omega_{A(i)} = 0, \quad \alpha_A = 1.6 \text{ rad/sec}^2$$

$$v_A = v_C$$



(a) $t \rightarrow \omega_C = 100 \text{ rev/min} \quad ?? \Rightarrow 100 \times \frac{2\pi}{60} = 10.67 \text{ rad/s}$

Yes: $v_A = v_C$

$$\omega_A r_A = \omega_C r_C$$

$$\omega_A = \frac{\omega_C r_C}{r_A} = (10.67) \left(\frac{25}{10} \right) = 26.67 \text{ rad/sec}$$

$$\omega_A = \omega_{0A} + \alpha t$$

$$26.67 = 0 + 1.6(t) \Rightarrow t = 16.67 \text{ sec}$$

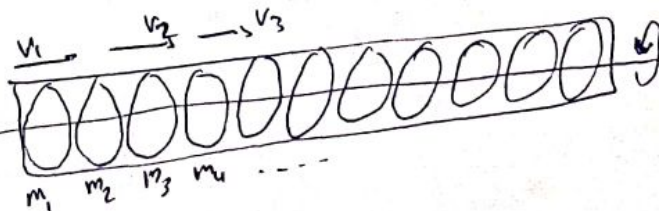
* Kinetic energy (K.E) in Rotation :-

$$K.E = \sum_{i=1}^N \frac{1}{2} m v_i^2$$

$$v_i = r_i \omega_i \Rightarrow K.E = \sum_{i=1}^N \frac{1}{2} m_i (r_i \omega_i)^2$$

$$= \frac{1}{2} \left(\sum_{i=1}^N m_i r_i^2 \right) \omega_i^2 = \frac{1}{2} I \omega_i^2 \quad (m \propto I)$$

\downarrow
 Inertia (I)



~~KE~~

Inertia (I)

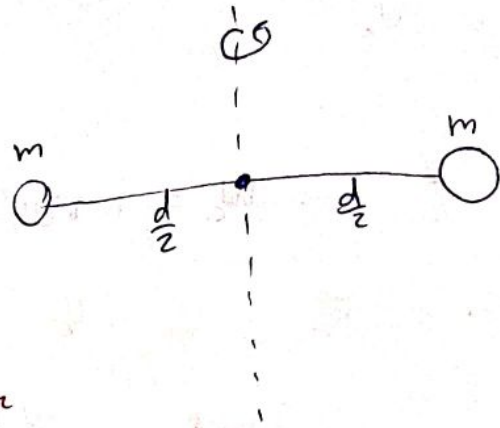
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Case I → Rotational axis through the center of mass

$$I_{com} = \sum_i m_i r_i^2$$

$$= m\left(\frac{d}{2}\right)^2 + m\left(\frac{d}{2}\right)^2$$

$$= \frac{md^2}{2}$$



$\frac{1}{2}md^2$

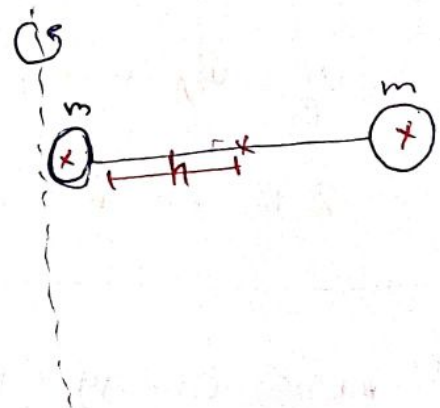
→ Rotational axis through a part from the com

$$I = I_{com} + Mh^2$$

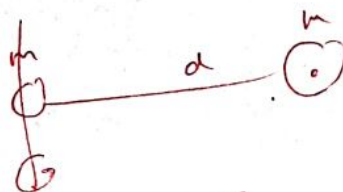
total mass
distance of center of mass
Rotational axis

$$= \frac{md^2}{2} + 2m\left(\frac{d}{2}\right)^2$$

$$= \boxed{md^2}$$



OR



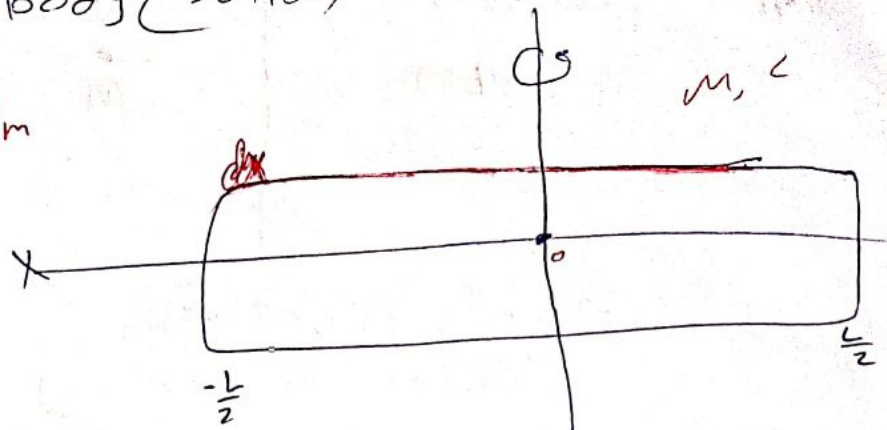
$$I = I_1 + I_2$$

$$= 0 + md^2 = \boxed{md^2}$$

Case II \Rightarrow Rigid body (solid)

$$I = \int r^2 dm = \int x^2 dm$$

$$\lambda = \frac{dm}{dx} = \frac{M}{L}$$



$$I = \int_{-L/2}^{L/2} x^2 \frac{M}{L} dx = \frac{M}{L} \int_{-L/2}^{L/2} x^2 dx = \boxed{\frac{1}{12} M L^2}$$

$$I = \sum m_i r_i^2$$

$$\boxed{I_{rod} = \frac{1}{12} m L^2}$$

محور الدوران
مركز الكتلة

OR

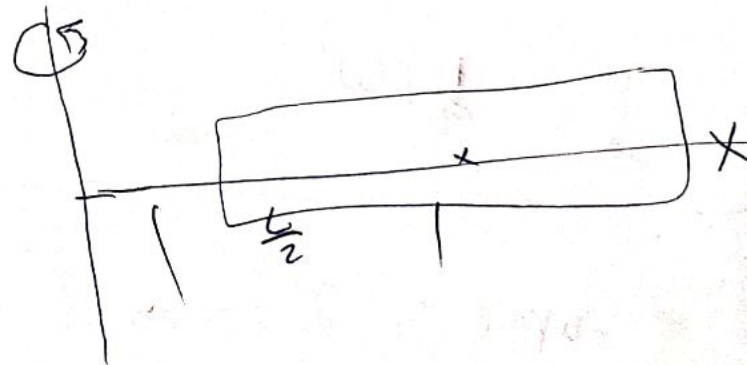
Parallel-axis theorem

$$I = I_{com} + M h^2$$

$$= \frac{1}{12} m L^2 + m \left(\frac{L}{2}\right)^2$$

$$= \frac{1}{12} m L^2 + \frac{m L^2}{4} = \frac{4 m L^2 + 12 m L^2}{12 \times 4} = \frac{16 m L^2}{48}$$

$$= \boxed{\frac{M L^2}{3}}$$



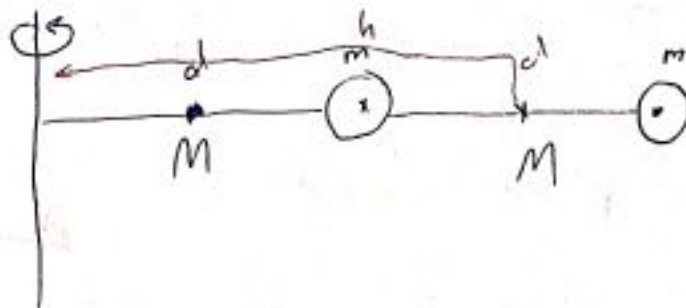
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P41

$$m = 0,85 \text{ kg}$$

$$M = 1,2 \text{ kg}$$

$$\omega = 0,3 \text{ rad/sec}$$



$$I = I_1 + I_2 + I_3 + I_4 = 0,023 \text{ kg/m}^2$$

$$I_1 = I_{\text{cm}} + Mh^2 = \frac{1}{12}ml^2 + M\left(\frac{d}{2}\right)^2 =$$

$$I_2 = \frac{1}{12}Ml^2 + M\left(\frac{3d}{2}\right)^2 =$$

$$I_3 = md^2 =$$

$$I_4 = m(2d)^2 =$$

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$$K_{\text{tot}} = \frac{1}{2}I\omega^2$$

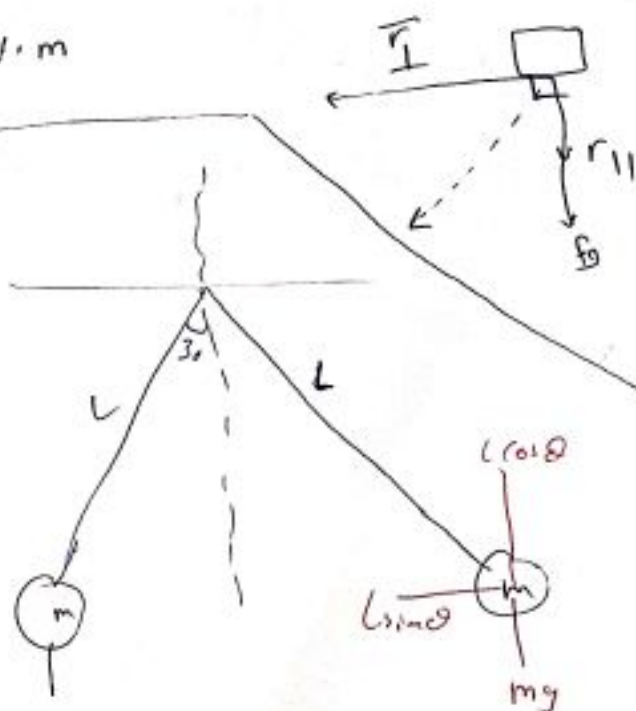
Torque: $(\vec{\tau}) \Rightarrow \vec{\tau} = \vec{r} \times \vec{F} = |\vec{r}| |\vec{F}| \sin \theta$

$$[\tau] = \text{N} \cdot \text{m}$$

P47 $m = 0,75 \text{ kg}$
 $L = 1,25 \text{ m}$

$\tau, \tau_g = ?$

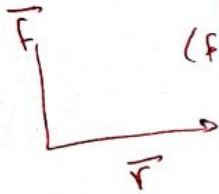
$\tau = mg d \sin \theta = 4,6 \text{ N} \cdot \text{m}$



* Newton's second law: $F_{net} = m \vec{a}$

$$\tau_{net} = I \alpha, \quad \vec{\tau}_{net} \equiv \text{net torque}$$

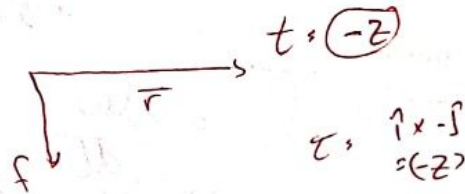
$$\vec{\tau} = \vec{r} \times \vec{F}$$



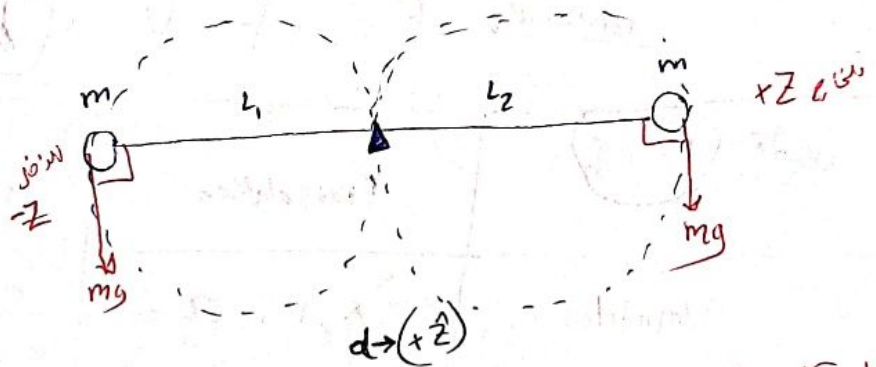
رأس القوة
(F) يتحرك نحو

$I \equiv$ Intertia

$\alpha \equiv$ angular acceleration.



P56) $l_1 = 20 \text{ cm}, l_2 = 80 \text{ cm}$
 a_1, a_2 ??



$$\tau_{net} = I \alpha$$

$$\tau_{net} = \tau_1 + \tau_2 \Rightarrow \tau_1 = mg l_1, \tau_2 = mg l_2 \Rightarrow \tau_{net} = mg (l_2 - l_1)$$

$$\alpha = r \alpha$$

$$a_1 = l_1 \alpha$$

$$a_2 = l_2 \alpha$$

$$I = m l_1^2 + m l_2^2$$

$$mg (l_2 - l_1) = (m l_1^2 + m l_2^2) \alpha \Rightarrow \alpha = 8.65 \text{ rad/sec}^2$$

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$$W_{\text{net}} = \Delta K \Rightarrow K = \frac{1}{2} I \omega^2$$

① Translational motion : $W = \int_{x_0}^{x_f} F dx$

Rotational $\Rightarrow W = \int_{\theta_i}^{\theta_f} \tau d\theta$

$\tau(\theta) = \tau$

$$W = \tau_{\text{avg}} \cdot \Delta\theta$$

② Power $\Rightarrow \vec{P} = \frac{dW}{dt}$

Rotational $\vec{P} = \vec{F} \cdot \vec{v} = \boxed{\vec{\tau} \cdot \vec{\omega}}$

cells Part I:

Translation

Rotation

Variables

$x, v, a \leftarrow r, \omega, \alpha$

Constant acceleration

$v = v_0 + at$
 $v^2 = v_0^2 + 2a\Delta x$
 $\Delta x = v_0 t + \frac{1}{2} at^2$

$\omega = \omega_0 + \alpha t$
 $\omega^2 = \omega_0^2 + 2\alpha \Delta\theta$
 $\Delta\theta = \omega_0 t + \frac{1}{2} \alpha t^2$

Part 2

K

$K = \frac{1}{2} mv^2$

$K = \frac{1}{2} I \omega^2$

Part 3

Part 4

$W = \int F dx$

$W_{\text{net}} = \Delta K$

$P = \vec{F} \cdot \vec{v}$

$F_{\text{net}} = ma$

$W = \int \tau d\theta$

$W_{\text{net}} = \Delta K$

$P = \tau \cdot \omega$

$F_{\text{net}} = I \alpha$

$\omega_f^2 = \omega_i^2 + 2\alpha\theta$

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Discussion chapter 10

net

P2

ω - angular velocity



$$\omega = \frac{d\theta}{dt}$$

2
13
23
34

$$\omega_{\text{second hand}} = \frac{2\pi}{60} = 0.15 \text{ rad/s}$$

$$\omega_{\text{min hand}} = \frac{2\pi}{60 \times 60} = 1.75 \times 10^{-3} \text{ rad/s}$$

$$\omega_{\text{hour hand}} = \frac{2\pi}{60 \times 60 \times 60} = 1.45 \times 10^{-4} \text{ rad/sec}$$

$$\begin{aligned} \theta &= \frac{1}{2}(\omega_i + \omega_f)t \\ \omega &= \omega_i + \alpha t \\ \theta &= \omega_i t + \frac{1}{2}\alpha t^2 \\ \omega_f &= \omega_i + \alpha t \\ \theta &= \omega_i t + \frac{1}{2}\alpha t^2 \end{aligned}$$

P13: $\theta_0 = 0$, $\omega_0 = 1.5 \text{ rad/sec}$, $\theta = 40 \text{ rev}$, $\omega_f = 0$

a) t

b) α

$$\omega_{\text{avg}} = \frac{\Delta\theta}{\Delta t} \Rightarrow \Delta t = \frac{\Delta\theta}{\omega_{\text{avg}}} = \frac{2\Delta\theta}{\omega_0} = 3.4 \times 10^2 \text{ sec}$$

1 rev $\rightarrow 2\pi \text{ rad}$
40 rev $\rightarrow \times$

$$\omega_{\text{avg}} = \frac{1}{2}(\omega_0 + \omega_f) = \frac{\omega_0}{2}$$

$$\omega = \omega_0 + \alpha t$$

$$\alpha = \frac{-\omega_0}{t} = -7.12 \times 10^{-4} \text{ rad/s}^2$$

1 rev $\rightarrow 2\pi \text{ rad}$
20 rev $\rightarrow \times$

c) t for first 20 rev

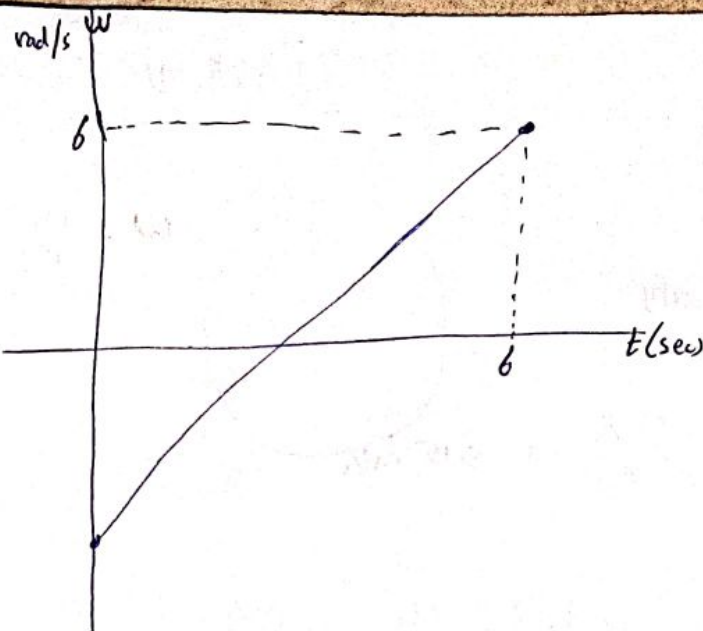
$$\Delta\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

$t = 98 \text{ sec} \checkmark$
 $t = 572 \text{ sec} \times$

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P34

ω



(a) α_{rod}

$$\alpha_{\text{rod}} = \frac{d\omega}{dt} = \text{slope} = 1.5 \text{ rad/s}^2$$

$\frac{4-1}{4-2} = 1.5$

$$\omega \rightarrow \frac{d\omega}{dt} = \alpha$$

(b) $t = 4 \text{ sec} \rightarrow K_{(4)} = 1.6 \text{ J}$ $\omega = 6$

$K_0 \rightarrow (t=0) = ??$

$$K = \frac{1}{2} I \omega^2$$

$$\frac{K_0}{K_4} = \frac{\frac{1}{2} I \omega_0^2}{\frac{1}{2} I \omega_{(4)}^2} =$$

$$K_0 = \frac{\omega_0^2}{\omega_4^2} K_{(4)} = 0.4 \text{ J}$$

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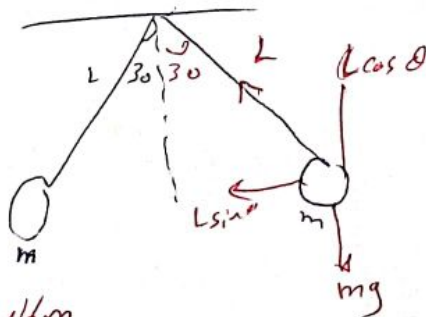
P47

$m = 0.75 \text{ kg}$

$L = 0.25 \text{ m}$

τ (Tangential gravitational force)

$$\tau = mgL \sin \theta = 4.6 \text{ Nm}$$



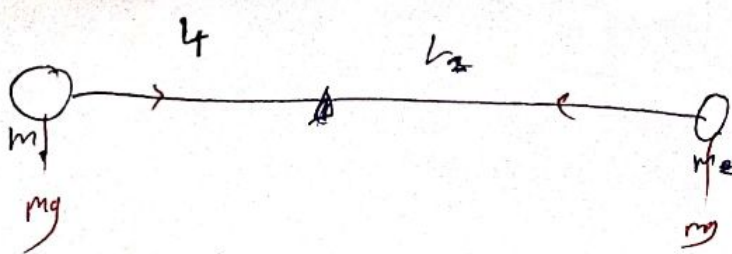
المماس

$$\tau = r \times F$$

$$= r F \sin \theta$$

where $\theta = 90^\circ$
 $180^\circ \rightarrow \tau = 0$

P56



$L_1 = 20 \text{ cm}$
 $L_2 = 80 \text{ cm}$

$\alpha_1, \alpha_2 = ??$

$a = \alpha r$
 $v = r\omega$
 $s = r\theta$

$$\alpha_1 = \alpha L_1$$

$$\alpha_2 = \alpha L_2$$

$$\tau_{\text{net}} = I \alpha = (m_1 L_1^2 + m_2 L_2^2) \alpha$$

$$m_1 g L_1 - m_2 g L_2 = (m_1 L_1^2 + m_2 L_2^2) \alpha$$

$\alpha = \square$

P23

$D = 1.2 \quad r = 0.6 \quad \omega_0 = 200 \text{ rev/min}$

(A) $\omega_0 = \frac{200 \text{ rev} \times 2\pi}{60} = 20.9 \text{ rad/sec}$

(B) $v = \omega_0 r = 20.9(0.6) = 12.5 \text{ m/s}$

(C) $\omega = 1000 \text{ rev/min}, t = 1 \text{ min}$

12 12

$\omega = \omega_0 + \alpha t$

$1000 = 200 + \alpha(1) \rightarrow \alpha = 800 \text{ rev/min}^2$

(D) $\theta_0 = 15 (\omega_0 + \omega) t$

$\Delta \theta = 15 (200 + 1000)(1) \rightarrow \Delta \theta = 600 \text{ rev}$

End ch10
 Good Luck
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