

Problem

Find exact values for each of the following quantities. Do not use a calculator.

a. $\log_3 81$

b. $\log_2 1024$

c. $\log_3 \left(\frac{1}{27} \right)$

d. $\log_2 1$

e. $\log_{10} \left(\frac{1}{10} \right)$

f. $\log_3 3$

g. $\log_2(2k)$

Step-by-step solution

Step 1 of 7

$$\begin{aligned} \text{(a)} \quad \log_3 81 &= \log_3 3^4 \\ &= 4 \log_3 3 \\ &= 4(1) = 4 \end{aligned}$$

Step 2 of 7

$$\begin{aligned} \text{(b)} \quad \log_2 1024 &= \log_2 2^{10} \\ &= 10 \log_2 2 \\ &= 10(1) = 10 \end{aligned}$$

Step 3 of 7

$$\begin{aligned} \text{(c)} \quad \log_3 \left(\frac{1}{27} \right) &= \log_3 27^{-1} \\ &= \log_3 3^{-3} \\ &= -3(\log_3 3) \\ &= -3 \times 1 = -3 \end{aligned}$$

Step 4 of 7

$$\text{(d)} \quad \log_2 1 = 0 \text{ since } \log 1 = 0$$

Step 5 of 7

$$\begin{aligned}
 \text{(e)} \quad \log_{10} \left(\frac{1}{10} \right) &= \log_{10} 10^{-1} \\
 &= -1 \cdot \log_{10} 10 \\
 &= -1 \times 1 = -1
 \end{aligned}$$

Step 6 of 7

$$\text{(f)} \quad \log_3 3 = 1$$

Step 7 of 7

$$\begin{aligned}
 \text{(g)} \quad \log_2 (2^k) &= k (\log_2 2) \\
 &= k (1) \\
 &= k
 \end{aligned}$$