# 11.4 Graphing in Polar Coordinates

* Symmetry Tests for Polar Graphs.

a) Symmetry about x-axis :-

if $(r, \theta)$ on the graph, then $(r, -\theta)$ or $(-r, \pi - \theta)$ on the graph

b) Symmetry about the y-axis :-

if $(r, \theta)$ on the graph, then $(r, \pi - \theta)$ or $(-r, -\theta)$ on the graph

c) Symmetry about the origin:

if $(r, \theta)$ on the graph, then $(-r, \theta)$ or $(r, \theta + \pi)$ lies on the graph

* slope :-

the slope of the curve $r = f(\theta)$ is

$$\left.\frac{dy}{dx}\right|_{(r,\theta)} = \frac{f'(\theta)\sin\theta + f(\theta)\cos\theta}{f'(\theta)\cos\theta - f(\theta)\sin\theta}$$

$$\left.\frac{dy}{dx}\right|_{(0,\theta_0)} = \tan\theta_0$$

Identify the symmetric of the curves. Then sketch the curves.

1 $r = 1 + \cos\theta$

$1 + \cos(-\theta) = 1 + \cos\theta = r$

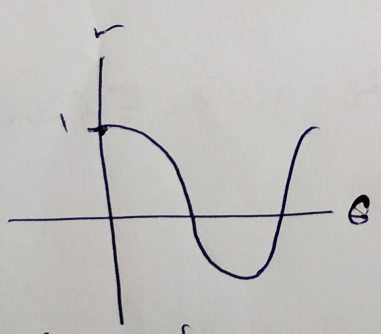
$(r, -\theta)$ $\quad$ $(r, \theta)$

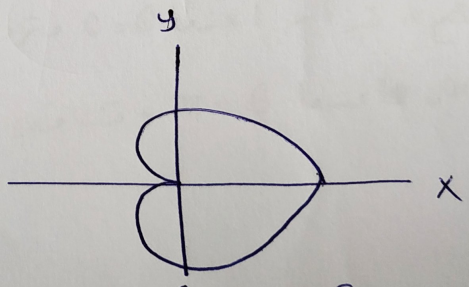
symmetric about the x-axis

$-r \neq 1 + \cos(-\theta)$

$r \neq 1 + \cos(\pi - \theta)$

} not symmetric about the y-axis




قيم r تبدأ من 2 وتتناقص حتى 0 ثم تبدأ تتزايد حتى 2

6 $r = 1 + 2\sin\theta$

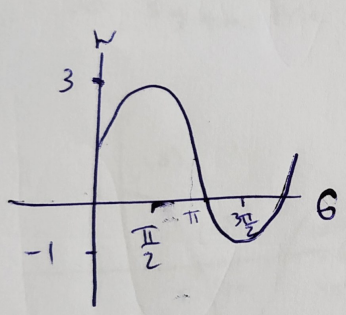
$1 + 2\sin\theta = 1 - 2\sin\theta \neq r$

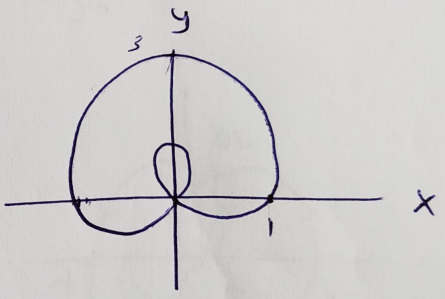
$1 + 2\sin(\pi - \theta) = 1 + 2\sin\theta \neq -r$ } not symmetric about the x-axis

$1 + 2\sin(\pi - \theta) = 1 + 2\sin\theta = r$

symmetric about the y-axis.

not symmetric about the origin




قيم r تبدأ من 1 عندما $\theta = 0$ وتزداد حتى تصل 3 عندما $\theta = \frac{\pi}{2}$ ثم تتناقص

حتى 0 عندما $\theta = \pi$ وتتناقص حتى تصل 0

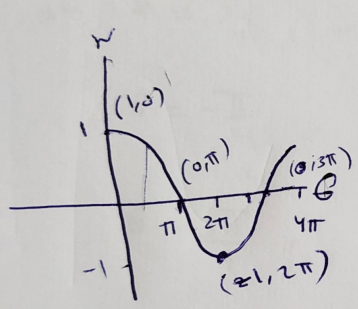
8 $r = \cos\left(\frac{\theta}{2}\right)$

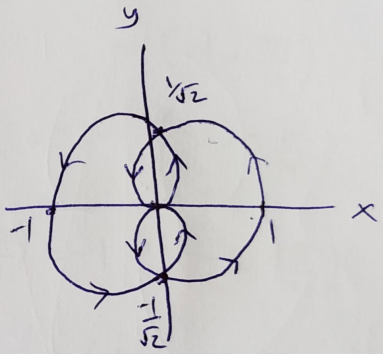
$\cos\left(-\frac{\theta}{2}\right) = \cos\left(\frac{\theta}{2}\right) = r$ → symmetric about the x-axis

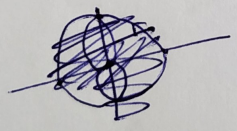
$\cos\left(\pi - \frac{\theta}{2}\right) = \cos\left(\frac{2\pi - \theta}{2}\right)$ → symmetric about the y-axis.

$= -\cos\left(\frac{\theta}{2}\right) = -r$

So symmetric about the origin



14 What symmetric do curve has?

$r^2 = 4\sin 2\theta$

$4\sin(2)(-\theta) = -4\sin 2\theta \neq r^2$

$4\sin 2(\pi - \theta) = 4\sin(2\pi - 2\theta) = 4\left(\sin 2\pi \cos(-2\theta) + \cos(2\pi)\sin(-2\theta)\right)$

$= -4\sin 2\theta \neq r^2$

not symmetric about the x-ax

$(-r)^2 = r^2 = 4\sin 2\theta$

the graph is symmetric about the origin

so it is not symmetric about the y-axis

**19** Find the slope of the curve

$r = \sin 2\theta$ , at $\theta = \pm\frac{\pi}{4}, \pm\frac{3\pi}{4}$

at $\theta = \frac{\pi}{4}$, $r = \sin 2(\frac{\pi}{4}) = 1 \rightarrow (1, \frac{\pi}{4})$

$\theta = -\frac{\pi}{4}$, $r = \sin 2(-\frac{\pi}{4}) = -1$ $(-1, -\frac{\pi}{4})$

$\theta = \frac{3\pi}{4}$, $r = \sin 2(\frac{3\pi}{4}) = -1$ $(-1, \frac{3\pi}{4})$

$\theta = -\frac{3\pi}{4}$, $r = 1$ $(1, -\frac{3\pi}{4})$

$r = \sin 2\theta = f(\theta)$

$f'(\theta) = 2\cos 2\theta$

$$\text{slope} = \frac{f'(\theta)\sin\theta + f(\theta)\cos\theta}{f'(\theta)\cos\theta - f(\theta)\sin\theta}$$

$$= \frac{2\cos 2\theta \sin\theta + \sin 2\theta \cos\theta}{2\cos 2\theta \cos\theta - \sin 2\theta \sin\theta}$$

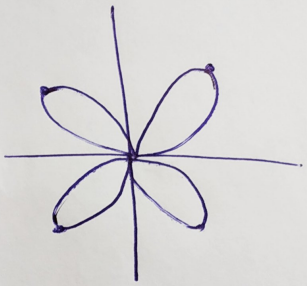
slope at $\theta = \frac{\pi}{4}$: $\dfrac{2\cos(\frac{\pi}{2})\sin(\frac{\pi}{4}) + \sin(\frac{\pi}{2})\cos(\frac{\pi}{4})}{2\cos(\frac{\pi}{2})\cos(\frac{\pi}{4}) - \sin(\frac{\pi}{2})\sin(\frac{\pi}{4})}$

$= -1$

slope at $\theta = \frac{-\pi}{4}$: 1

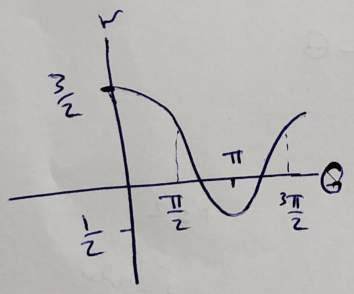
slope at $\theta = \frac{3\pi}{4}$: 1

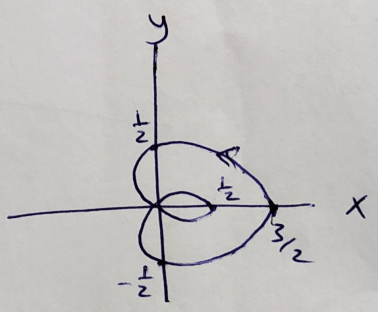
slope at $\theta = -\frac{3\pi}{4}$: $-1$



**21** Graph

a) $r = \frac{1}{2} + \cos\theta$




b) $r = \frac{1}{2} + \sin\theta$