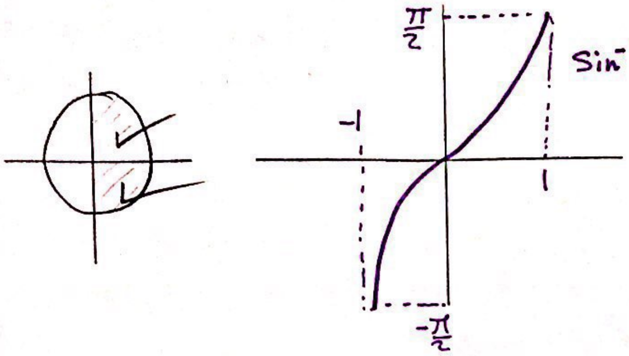
7.6 Inverse of Trigonometric functions

$f(x) = \sin x$ on $[-\frac{\pi}{2}, \frac{\pi}{2}]$ then

$f^{-1}(x) = \sin^{-1} x = \arcsin x$ on $[-1, 1]$

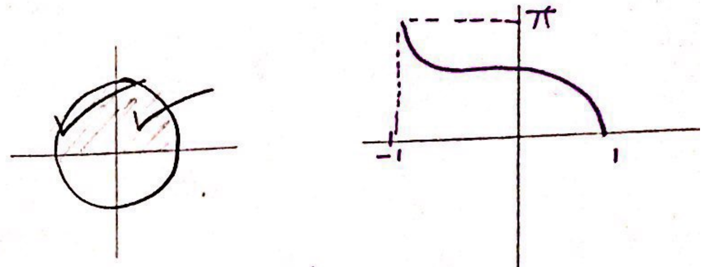


$\sin(x) + \sin^{-1}(x) = 0$

لانه متماثل حول نقطة الاصل

if $f(x) = \cos x$ on $[0, \pi]$ then

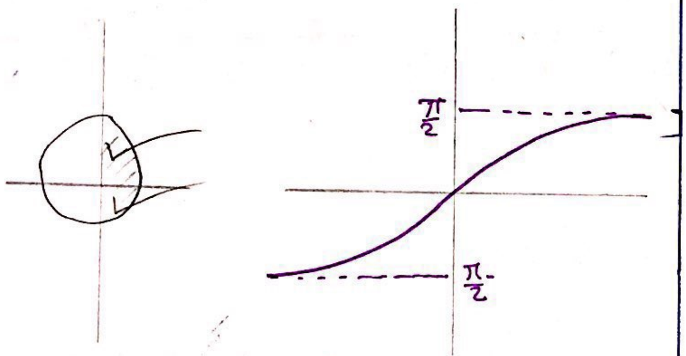
$f^{-1}(x) = \cos^{-1} x = \arccos x$ on $[-1, 1]$



$\cos(x) + \cos^{-1} x = \pi$

if $f(x) = \tan x$ on $(-\frac{\pi}{2}, \frac{\pi}{2})$ then

$\tan^{-1} x = \arctan x$ on $(-\infty, \infty)$ $\Rightarrow \mathbb{R}$

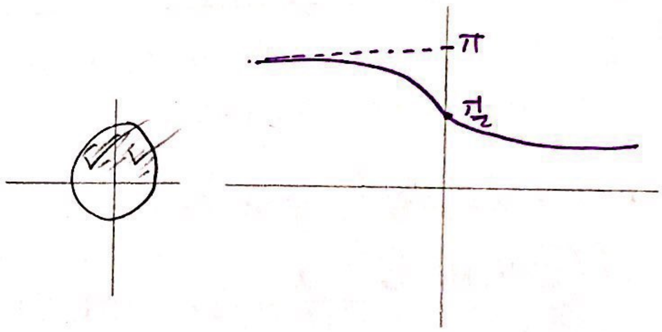


$\tan x + \tan^{-1} x = 0$

لانه متماثل حول نقطة الاصل

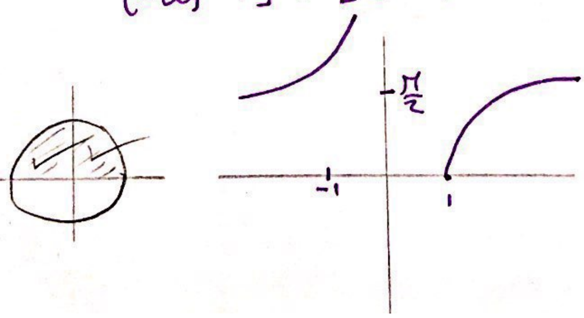
if $f(x) = \cot x$ on $(0, \pi)$ then

$f^{-1}(x) = \cot^{-1} x$ on $\mathbb{R}$



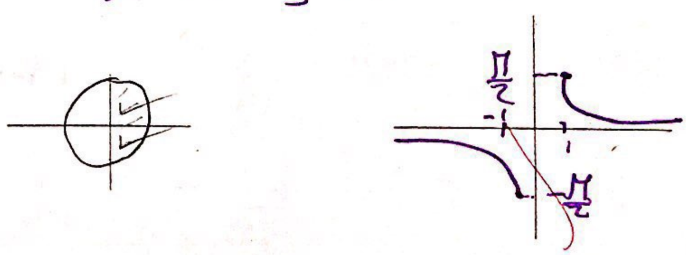
if $f(x) = \sec x$ on $[0, \frac{\pi}{2}) \cup (\frac{\pi}{2}, \pi]$ then

$f^{-1}(x) = \sec^{-1} x = \text{arc} \sec x$ on $(-\infty, -1] \cup [1, \infty)$

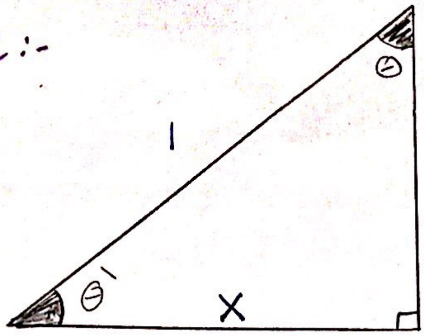


if $f(x) = \csc x$ on $[-\frac{\pi}{2}, 0) \cup (0, \frac{\pi}{2}]$ then

$f^{-1}(x) = \csc^{-1} x = \text{arc} \csc$ on $(-\infty, -1] \cup [-1, \infty)$



Note :-



$\sin^{-1}(x) = \csc^{-1}\frac{1}{x}$

$\cos^{-1}x = \sec^{-1}\frac{1}{x}$

Note that

الوتر/المجاور $\leftarrow \cos\theta = x \Rightarrow \sec\theta = \frac{1}{x}$ ← الوتر على المجاور

الوتر/المقابل $\leftarrow \sin\theta = x \Rightarrow \csc\theta = \frac{1}{x}$ ← الوتر على المقابل

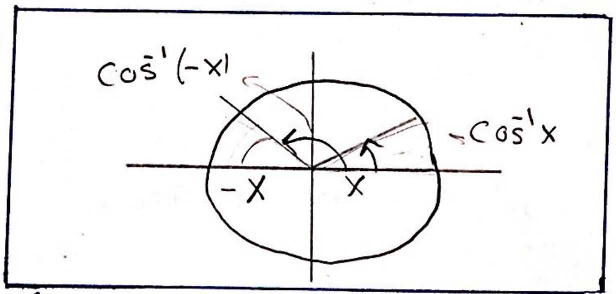
$$\sec^{-1}\frac{1}{x} = \cos^{-1}x$$

$$\csc^{-1}\frac{1}{x} = \sin^{-1}x$$

$$\cos^{-1}x + \sin^{-1}x = \frac{\pi}{2}$$

$$\sec^{-1}\frac{1}{x} + \csc^{-1}\frac{1}{x} = \frac{\pi}{2}$$

$$\cos^{-1}x + \cos^{-1}(-x) = \pi$$



* Derivatives for the inverse of Trigonometric functions

- $u(x)$ is diff function of x

1- $\dfrac{d(\sin^{-1}u(x))}{dx} = \dfrac{u'}{\sqrt{1-u^2}}$ $\quad |u|<1$

2- $\dfrac{d(\cos^{-1}u(x))}{dx} = \dfrac{-u'}{\sqrt{1-u^2}}$ $\quad |u|<1$

3- $\dfrac{d(\tan^{-1}u(x))}{dx} = \dfrac{u'}{1+u^2}$

4- $\dfrac{d(\cot^{-1}u(x))}{dx} = \dfrac{-u'}{1+u^2}$

5- $\dfrac{d(\sec^{-1}u(x))}{d(x)} = \dfrac{u'}{|u|\sqrt{u^2-1}}$ $\quad |u|>1$

6- $\dfrac{d(\csc^{-1}u(x))}{dx} = \dfrac{-u'}{|u|\sqrt{u^2-1}}$ $\quad |u|>1$

* Integrals for the inverse of Trigonometric functions

→ $a \neq 0$

1- $\displaystyle\int \frac{du}{\sqrt{a^2-u^2}} = \sin^{-1}\frac{u}{a} + C$

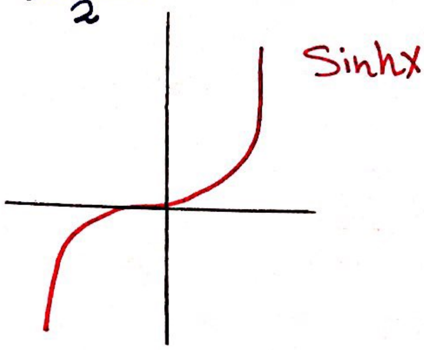
2- $\displaystyle\int \frac{du}{a^2+u^2} = \frac{1}{a}\tan^{-1}\frac{u}{a} + C$

3- $\displaystyle\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\sec^{-1}\left|\frac{u}{a}\right| + C$

7.7 hyperbolic functions

- $\sinh x = \dfrac{e^x - e^{-x}}{2}$

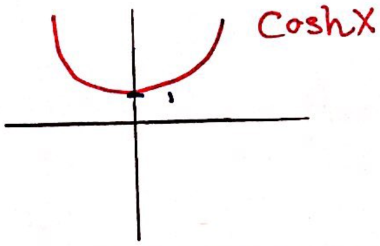
D: $\mathbb{R}$
R: $\mathbb{R}$
odd



- $\cosh x = \dfrac{e^x + e^{-x}}{2}$

• عشان الاقتران عبارة عن جمع بين اقترانين موجبين ولا يقطعانه الصفر اذن الاقتران موجب ولا يقطع الصفر

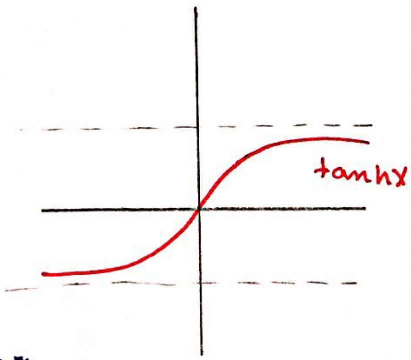
D: $\mathbb{R}$
R: $[1, \infty)$
Even



- $\tanh x = \dfrac{\sinh x}{\cosh x} = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$

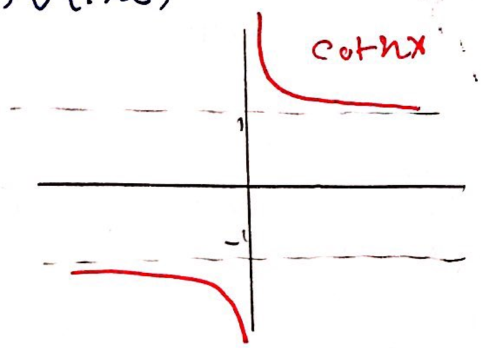
D: $\mathbb{R}$
R: $(-1, 1)$
odd
It has h. Asy at $y = 1$ & $y = -1$

if $e^x > e^{-x} \Rightarrow$ موجب
if $e^x < e^{-x} \Rightarrow$ سالب



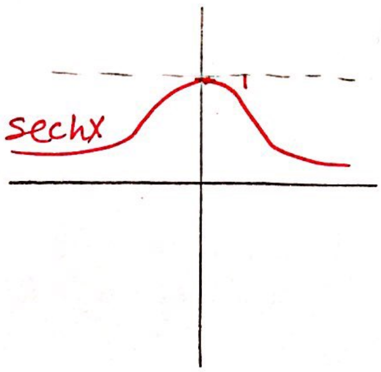
- $\coth x = \dfrac{\cosh x}{\sinh x} = \dfrac{e^x + e^{-x}}{e^x - e^{-x}}$

D: $\mathbb{R} \setminus \{0\}$
R: $(-\infty, -1) \cup (1, \infty)$
odd



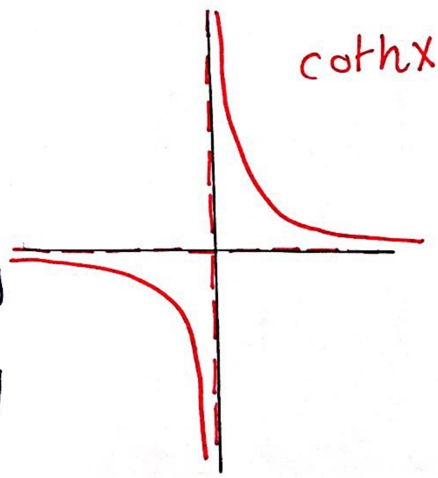
- $\operatorname{sech} x = \dfrac{1}{\cosh x} = \dfrac{2}{e^x + e^{-x}}$

D: $\mathbb{R}$
R: $(0, 1]$
even
has a h. Asy at $y = 0$



- $\operatorname{csch} x = \dfrac{1}{\sinh x} = \dfrac{2}{e^x - e^{-x}}$

D: $\mathbb{R} \setminus \{0\}$
R: $\mathbb{R} \setminus \{0\}$
odd.
it has a V. Asy at $x = 0$ and a h. Asy at $y = 0$



Derivatives of hyperbolic functions

Assume $u(x)$ is differentiable

- If $y = \sinh u(x) \longrightarrow y' = \cosh u \dfrac{du}{dx}$
- If $y = \cosh u(x) \longrightarrow y' = \sinh u \dfrac{du}{dx}$
- If $y = \tanh u(x) \longrightarrow y' = \operatorname{sech}^2 u \dfrac{du}{dx}$
- If $y = \coth u(x) \longrightarrow y' = -\operatorname{csch}^2 u \dfrac{du}{dx}$
- If $y = \operatorname{sech} u(x) \longrightarrow y' = -\operatorname{sech}^2 u \dfrac{du}{dx}$
- If $y = \operatorname{csch} u \longrightarrow y' = -\operatorname{csch} u \coth u \dfrac{du}{dx}$

Integrals of hyperbolic functions

- $\int \sinh x \, dx = \cosh x + C$
- $\int \cosh x \, dx = \sinh x + c$
- $\int \operatorname{sech}^2 x \, dx = \tanh x + c$
- $\int \operatorname{csch}^2 x \, dx = -\coth x + c$
- $\int \operatorname{sech} x \tanh x \, dx = -\operatorname{sech} x + c$
- $\int \operatorname{csch} x \coth x \, dx = -\operatorname{csch} x + c$

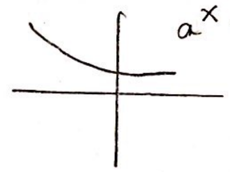
Identities of hyperbolic functions

- $\cosh^2 x - \sinh^2 x = 1$
- $\sinh 2x = 2\sinh x \cosh x$
- $\cosh 2x = \cosh^2 x + \sinh^2 x$
  - $= 2\cosh^2 x - 1$
  - $= 1 + \sinh^2 x$
- $\tanh^2 x + \operatorname{sech}^2 x = 1$
- $\coth^2 x - \operatorname{csch}^2 x = 1$

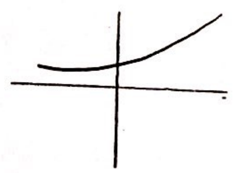
7.8 : Relative Rates of Growth

Important Notes

$$\lim_{x \to \infty} a^x = \begin{cases} 0 & \text{if } 0 < a < 1 \\ \infty & \text{if } a > 1 \end{cases}$$



$a^x \Rightarrow 0 < a < 1$



$a^x \Rightarrow a > 1$

e^x is faster than $\ln x$

Definition

If $f(x)$ and $g(x)$ are positive functions
let $0 < L < \infty$ be a positive contant

If $\lim_{x \to \infty} \dfrac{f(x)}{g(x)} = \begin{cases} \infty, & f \text{ grows faster than } g \text{ as } x \to \infty \\ 0, & f \text{ grows slower than } g \text{ as } x \to \infty \\ L, & f \text{ grows at the same rate as } g \text{ as } x \to \infty \end{cases}$

- If you forgot The Derivatives of the inverse of Trigonometric functions go back to the

Proof

① $f(x) = \sin x \Rightarrow f^{-1}(x) = \sin^{-1}(x)$

$f'(x) = \cos x$

$$(f^{-1}(x))' = \frac{1}{f'(f^{-1}(x))} = \frac{1}{f'(\sin^{-1}x)}$$

$$= \frac{1}{\cos(\sin^{-1}x)}$$

But:

$\cos^2 x + \sin^2 x = 1$

$\cos x = \sqrt{1-\sin^2 x}$

$$= \frac{1}{\sqrt{1-\sin^2(\sin^{-1}(x))}}$$

$$= \frac{1}{\sqrt{1-x^2}}$$

Same Way for $(\cos^{-1}x)'$

② $f(x) = \tan x \Rightarrow f^{-1}(x) = \tan^{-1} x$

$f'(x) = \sec^2 x$

$$\left(f^{-1}(x)\right)' = \frac{1}{f'(f^{-1}(x))} = \frac{1}{f'(\tan^{-1} x)}$$

$$= \frac{1}{\sec^2(\tan^{-1} x)}$$

But

$\tan^2 x = \sec^2 x - 1$

$\sec^2 x = \tan^2 x + 1$

$$= \frac{1}{\left(\tan(\tan^{-1} x)\right)^2 + 1}$$

But it's not the Same for $\sec^{-1} x$

$$= \frac{1}{x^2 + 1}$$

③ for $\sec^{-1} x$:-

let $y = \sec^{-1} x$

أخذت sec للطرفين

① --- $\sec y = \sec(\sec^{-1} x)$ نشتق الطرفين

$\sec y \tan y \, y' = 1$

$$y' = \frac{1}{\sec y \tan y}$$

Back to A
$\sec y = X$

$$y' = \frac{1}{X \tan y}$$

$\tan y = \sqrt{\sec^2 y - 1}$
$= \sqrt{X^2 - 1}$

$$y' = \frac{1}{X \pm \left(\sqrt{X^2 - 1}\right)}$$

$$y' = \frac{1}{|X| \sqrt{X^2 - 1}}$$ ##

- If you get this you will never have a mistake in you exam

(But never say never :P)