جامعة بيرزيت

BIRZEIT UNIVERSITY

Mechatronics System Design

Chapter 7: Case Studies.

Instructor: Dr. Ahmad Al-Balasie

First Semester - 2021/2022

Controller Realization

- In this chapter, we establish an orderly sequence for the design of feedback control systems that will be followed as we progress through the rest of the book. Figure 1 shows the described process as well as the chapters in which the steps are discussed

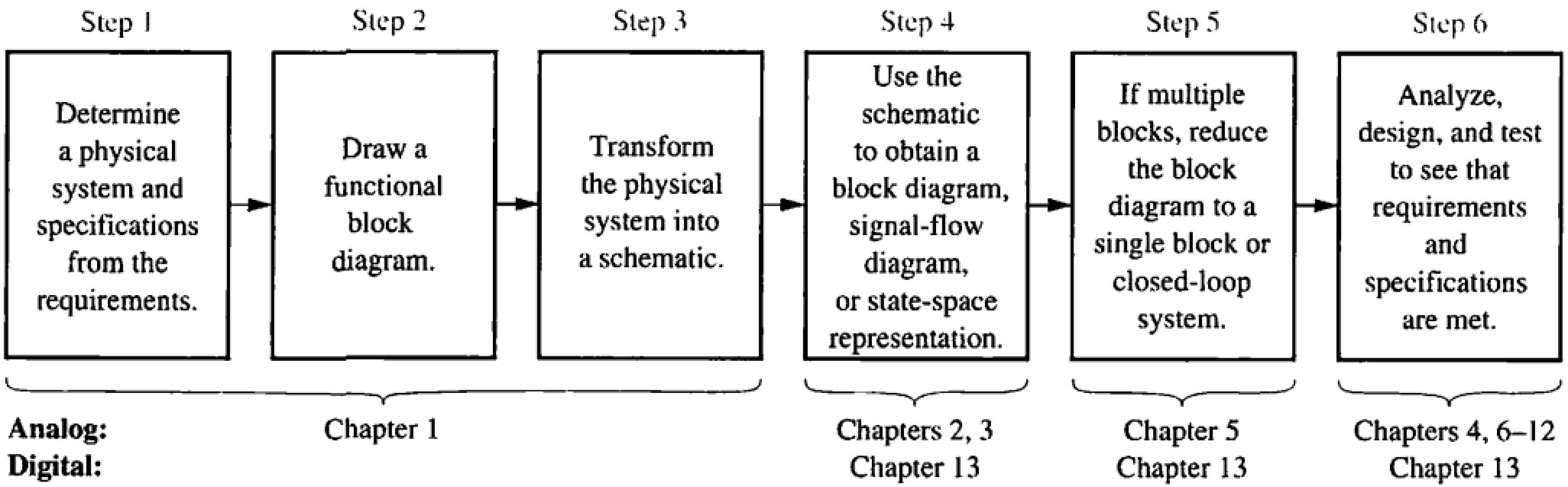

FIGURE 1.11 The control system design process

Antenna Azimuth: An Introduction to Position Control Systems

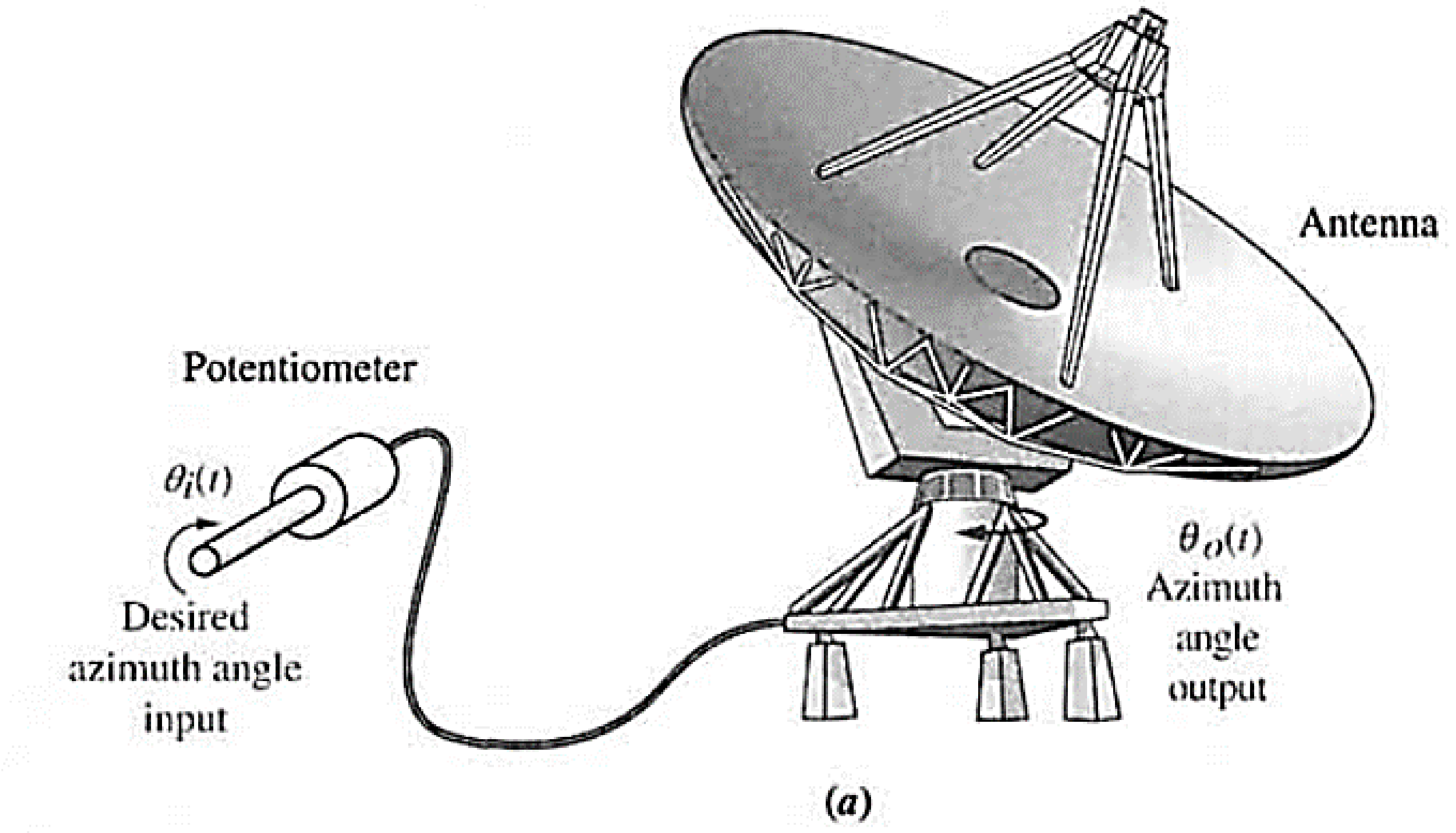

Figure 1: Simplified Antenna Azimuth Position Control System

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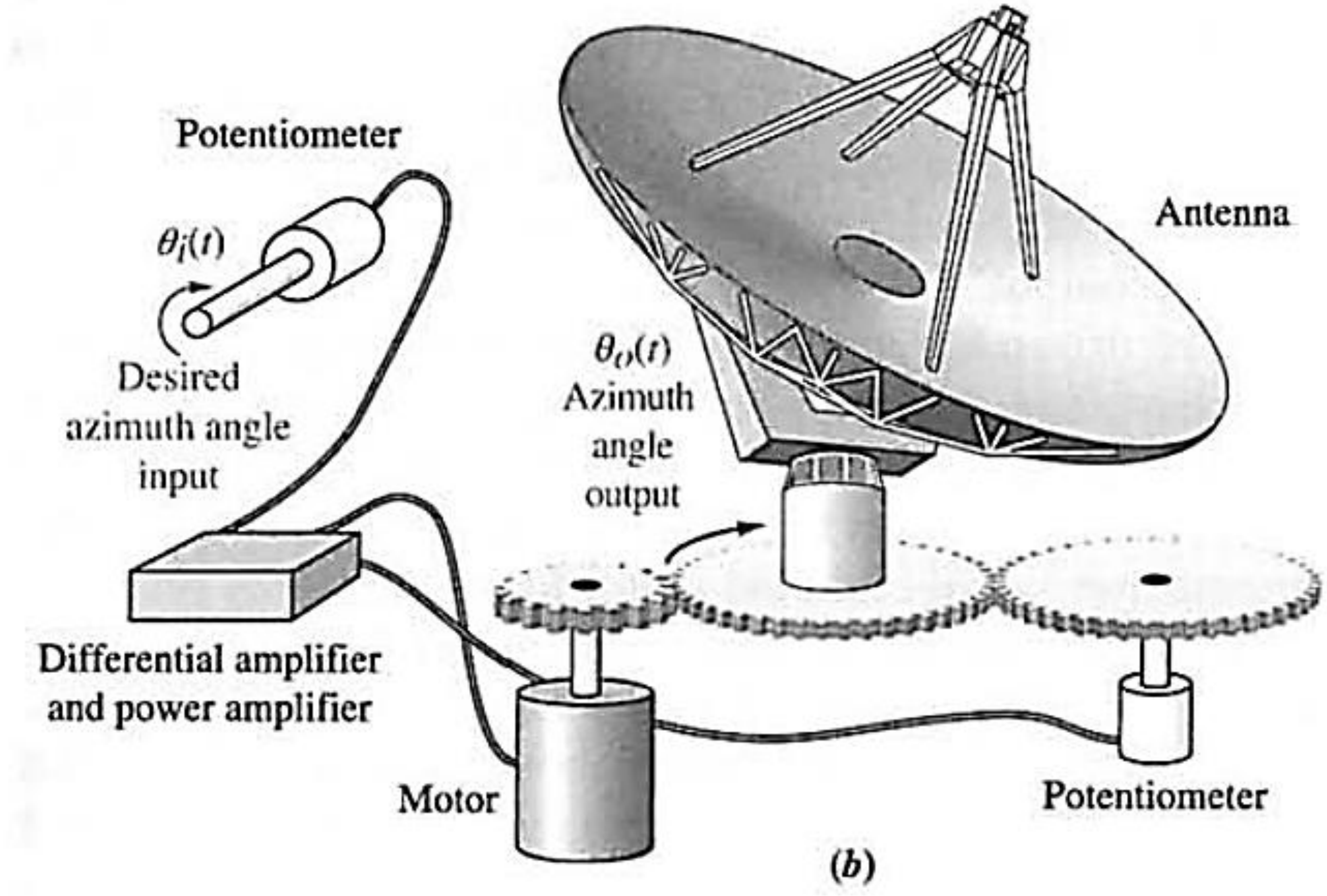

Figure 2: Detailed Antenna Azimuth Position Control System

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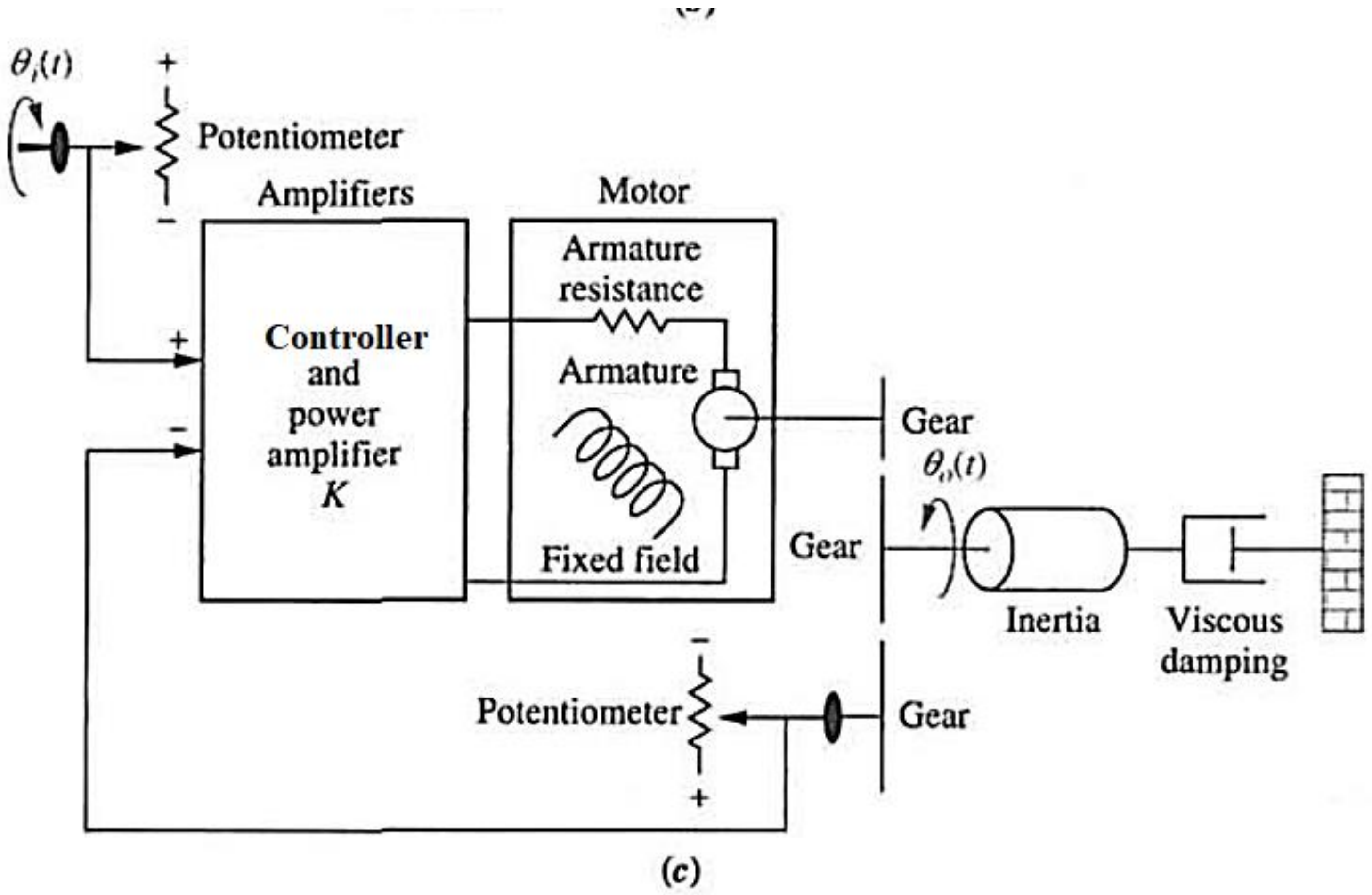

Figure 3: Schematic Antenna Azimuth Position Control System

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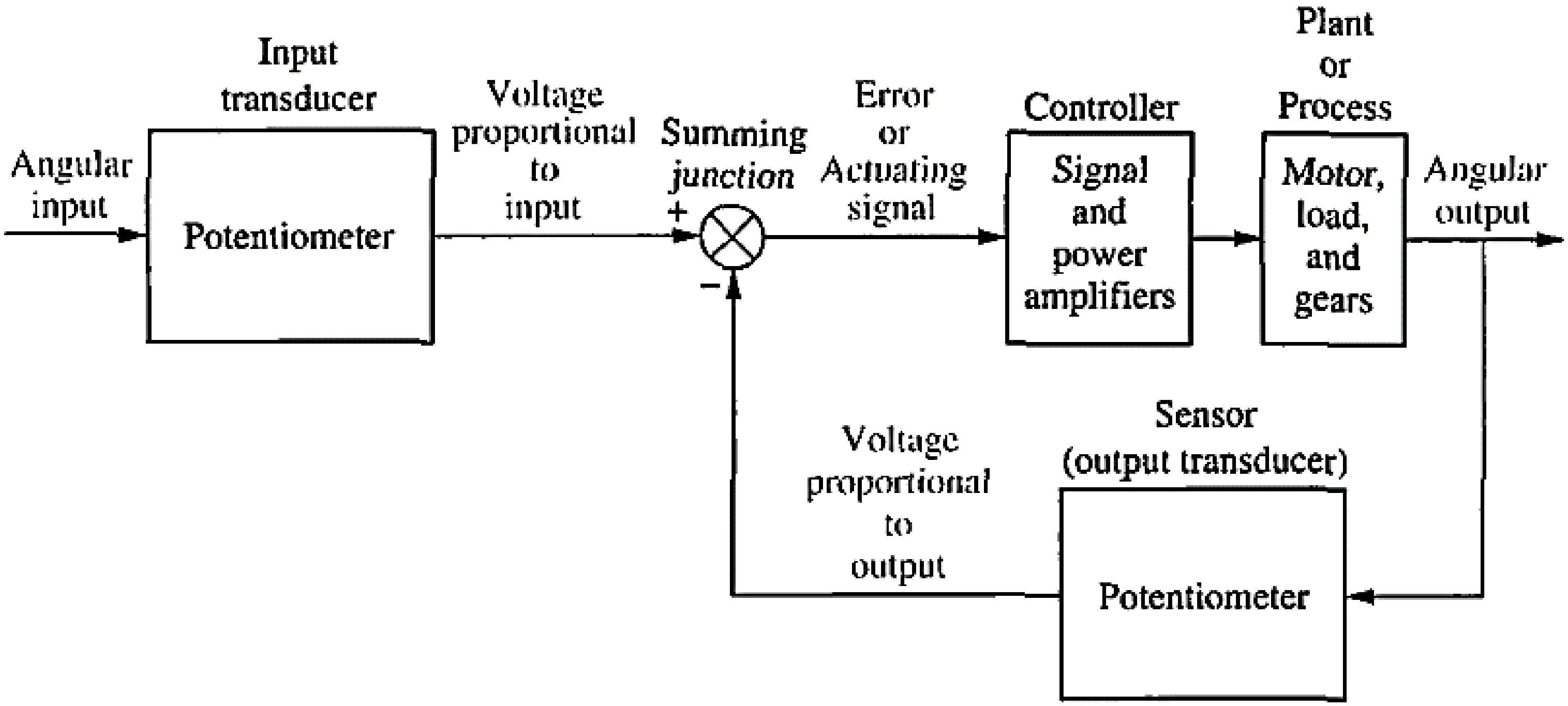

Figure 4: Functional Block Diagram

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TABLE 2.6 Subsystems of the antenna azimuth position control system

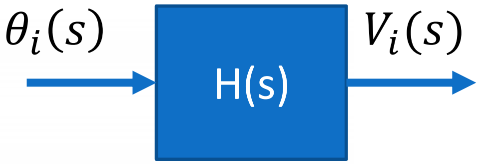
Subsystem	Input	Output
Input potentiometer	Angular rotation from user, $\theta_i(t)$	Voltage to preamp, $v_i(t)$
Preamp	Voltage from potentiometers, $v_e(t) = v_i(t) - v_0(t)$	Voltage to power amp, $v_p(t)$
Power amp	Voltage from preamp, $v_p(t)$	Voltage to motor, $e_a(t)$
Motor	Voltage from power amp, $e_a(t)$	Angular rotation to load, $\theta_0(t)$
Output potentiometer	Angular rotation from load, $\theta_0(t)$	Voltage to preamp, $v_0(t)$

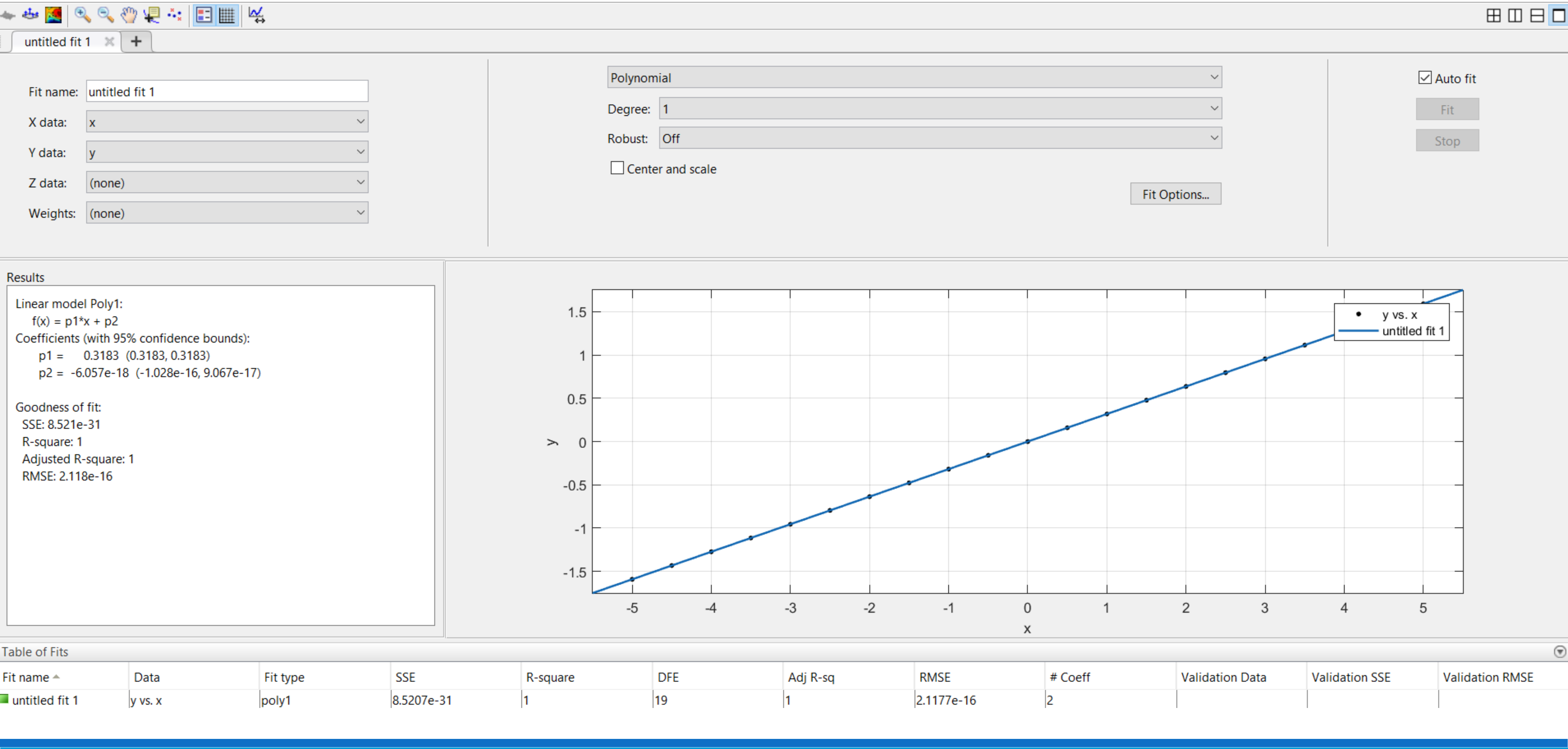
Input Potentiometer; Output Potentiometer

$\theta_i(s)$ tunrns	$V_i(s)$ volts	$\theta_i(s)$ tunrns	$V_i(s)$ volts
-5	-1.5915	0.5	0.1592
-4.5	-1.4324	1	0.3183
-4	-1.2732	1.5	0.4775
-3.5	-1.1141	2	0.6366
-3	-0.9549	2.5	0.7958
-2.5	-0.7958	3	0.9549
-2	-0.6366	3.5	1.1141
-1.5	-0.4775	4	1.2732
-1	-0.3183	4.5	1.4324
-0.5	-0.1592	5	1.5915
0	0		

By using cftool in Matlab the transfer function can be found

$$\frac{V_i(s)}{\theta_i(s)} = \frac{10}{10\pi} = \frac{1}{\pi}$$




untitled fit 1
Fit name: untitled fit 1
X data: x
Y data: y
Z data: (none)
Weights: (none)
Polynomial
Degree: 1
Robust: Off
Center and scale
Fit Options...
Auto fit
Fit
Stop
Results
Linear model Poly1:
f(x) = p1*x + p2
Coefficients (with 95% confidence bounds):
p1 = 0.3183 (0.3183, 0.3183)
p2 = -6.057e-18 (-1.028e-16, 9.067e-17)
Goodness of fit:
SSE: 8.521e-31
R-square: 1
Adjusted R-square: 1
RMSE: 2.118e-16
y vs. x
untitled fit 1
y
x
Table of Fits
Fit name | Data | Fit type | SSE | R-square | DFE | Adj R-sq | RMSE | # Coeff | Validation Data | Validation SSE | Validation RMSE
untitled fit 1 | y vs. x | poly1 | 8.5207e-31 | 1 | 19 | 1 | 2.1177e-16 | 2

Design Requirements and Dynamic Parameters

1	Use Bang-Bang profile	move from $[0 - 2\pi]$ in 30 seconds
2	Settling time	0.5 seconds
3	%OS	10%
4	$e_{ss}(\infty)$=0	For step input
5	$N_1 = 20$	Driver gear
6	$N_2 = 100$	Driven gear
7	J_L=0.5 $kg.m^2$	The total inertia of the satellite around the rotating axis
8	D_L=2 $N.m.s/rad$	The damping into the satellite

On the satellite side:

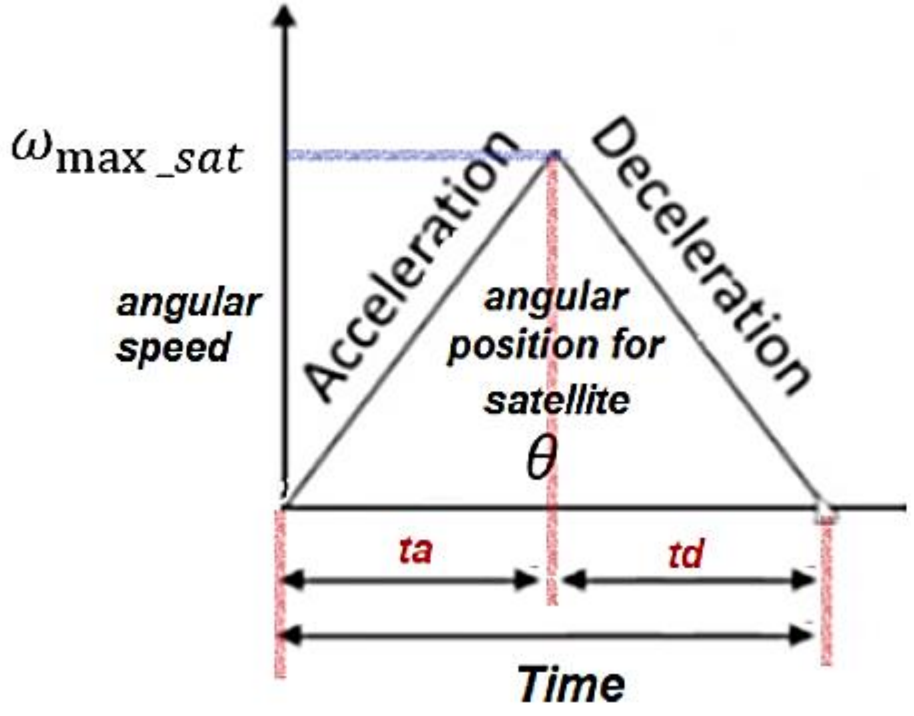
$$\theta_{total} = 0.5 * t_a * \omega_{max} + 0.5 * t_d * \omega_{max}$$

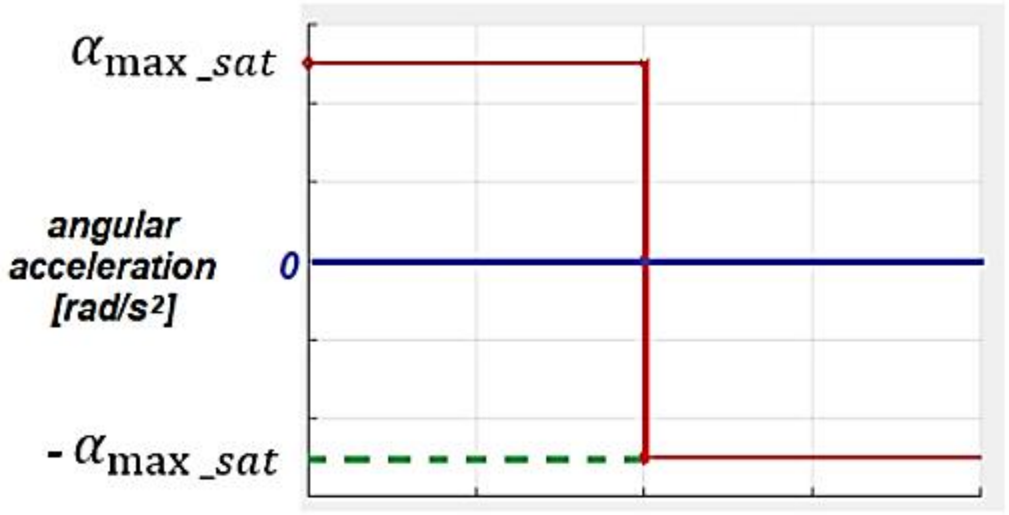
$$t_a = t_d$$

$$\theta_{total} = t_a * \omega_{max}$$

$$\omega_{\max\_sat} = \frac{2\pi}{15} = 0.4189\ rad/s$$

$$\alpha_{\max\_sat} = \frac{\omega_{\max\_sat}}{t_a} = \frac{0.4189}{15} = 0.0279\ rad/s^2$$





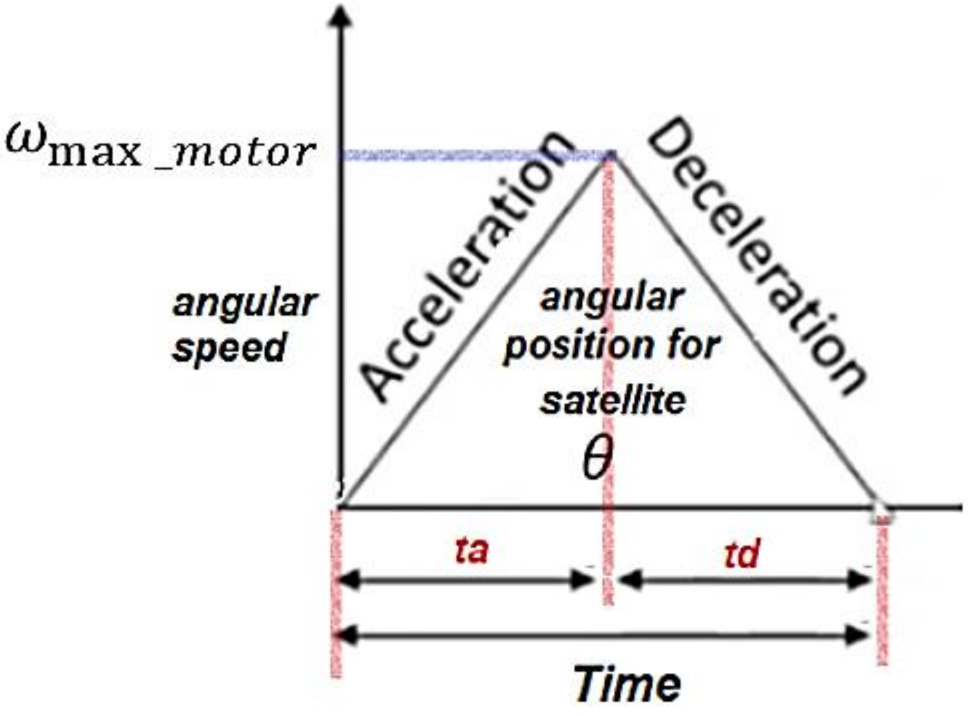
On the motor side

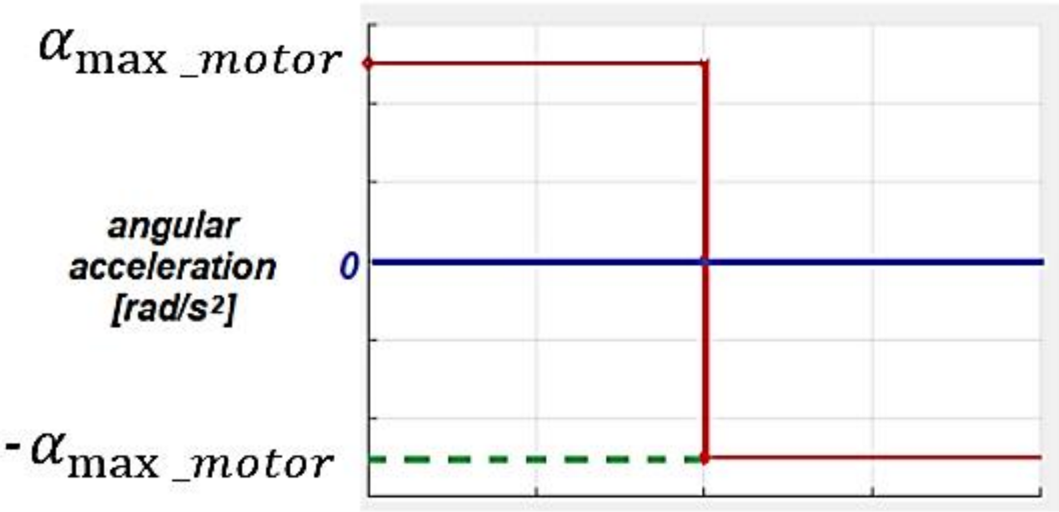
$$r_g = \frac{\omega_{HSS}}{\omega_{Lss}} = \frac{\omega_{\max\_motor}}{\omega_{\max\_sat}} = \frac{N_2}{N_1} = \frac{100}{20} = 5$$

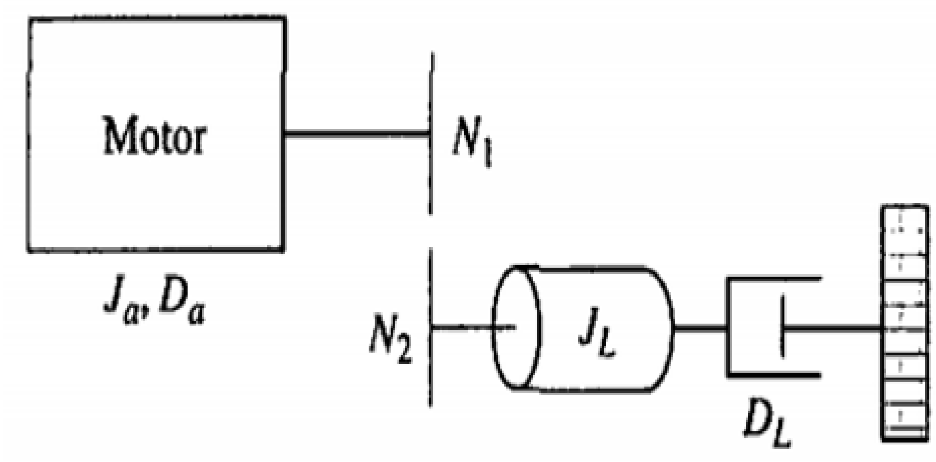
$$\omega_{\max\_motor} = r_g\ \omega_{\max\_sat} = 5 * 0.4189 = 2.0945\ \text{rad/s}$$

$$r_g = \frac{\alpha_{HSS}}{\alpha_{Lss}} = \frac{\alpha_{\max\_motor}}{\alpha_{\max\_sat}} = \frac{N_2}{N_1} = \frac{100}{20} = 5$$

$$\alpha_{\max\_motor} = r_g\ \alpha_{\max\_sat} = 5 * 0.0279 = 0.1395\ rad/s^2$$







$$J_m = J_a + J_L\left(\frac{N_1}{N_2}\right)^2;\ D_m = D_a + D_L\left(\frac{N_1}{N_2}\right)^2$$

- $\sum M = J_m\, \alpha_{\max\_motor}$
- $T_m - \dot{\theta}_m * D_m = J_m * \alpha_{\max\_motor}$

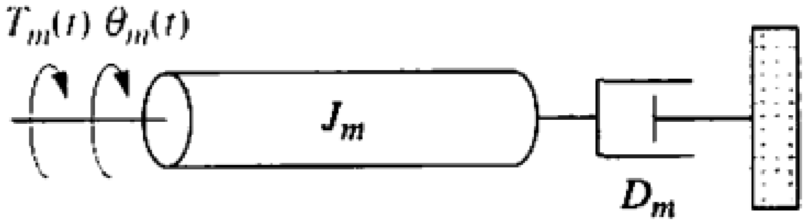


FIGURE 2.36 Typical equivalent mechanical loading on a motor

- Acceleration phase: (maximum point)
- $T_{m\_acc} - \omega_{\max\_motor} * D_m = J_m * \alpha_{max-motor}$
- $T_{m\_acc} = \left(J_a + \frac{J_L}{r_g^2}\right)\alpha_{max-motor} + \left(D_a + \frac{D_L}{r_g^2}\right)\omega_{max-motor}$
- $T_{m\_acc} = \left(J_a + \frac{0.5}{25}\right)0.1395 + \left(D_a + \frac{2}{25}\right)2.0945$
- $T_{m\_acc} = (0.1395\, J_a + 0.00279) + (2.0945\, D_a + 0.16756)$
- $T_{m\_acc} = (0.1395\, J_a + 2.0945\, D_a + 0.17035)$

- Deceleration phase: (maximum point)

- $T_{m\_dec} - \omega_{\max\_motor} * D_m = J_m * \alpha_{max-motor}$

- $T_{m\_dec} = \left(J_a + \frac{J_L}{r_g^2}\right)(-\alpha_{max-motor}) + \left(D_a + \frac{D_L}{r_g^2}\right)(-\omega_{max-motor})$

- $T_{m\_dec} = \left(J_a + \frac{0.5}{25}\right)(-0.1395) + \left(D_a + \frac{2}{25}\right)(-2.0945\ )$

- $T_{m\_dec} = (-0.1395\, J_a - 0.00279) + (-2.0945\, D_a - 0.16756)$

- $T_{m\_dec} = (-0.1395\, J_a - 2.0945\, D_a - 0.17035)$

Part Number*		C23-L45					C23-L50					C23-L55				
Winding Code**		10	20	30	40	50	10	20	30	40	50	10	20	30	40	50
L = Length	inches	4.5					5					5.45				
	millimeters	114.3					127.0					138.4				
Peak Torque	oz-in	310.0	310.0	310.0	310.0	310.0	360.0	360.0	360.0	360.0	360.0	430.0	430.0	430.0	430.0	430.0
	Nm	2.189	2.189	2.189	2.189	2.189	2.542	2.542	2.542	2.542	2.542	3.037	3.037	3.037	3.037	3.037
Continuous Stall Torque	oz-in	34.0	34.0	34.0	34.0	34.0	42.0	42.0	42.0	42.0	42.0	50.0	50.0	50.0	50.0	50.0
	Nm	0.240	0.240	0.240	0.240	0.240	0.297	0.297	0.297	0.297	0.297	0.353	0.353	0.353	0.353	0.353
Rated Terminal Voltage	volts DC	12 -24	12 - 48	12 -60	12 - 60	12 - 60	12 - 24	12 - 60	12 - 60	18 - 60	24 - 60	12 - 24	12 - 60	12 - 60	18 - 60	24 - 60
Terminal Voltage	volts DC	12	24	36	48	60	12	24	36	48	60	12	24	36	48	60
Rated Speed	RPM	1950	2600	2600	2100	1555	1600	2150	2150	1800	1283	1350	1800	1700	1300	887
	rad/sec	204	272	272	220	163	168	225	225	188	134	141	188	178	136	93
Rated Torque	oz-in	25.3	26.5	25.8	23.3	23	27.1	30.1	32	31.5	34.3	36.4	39.3	40.5	40.9	43.5
	Nm	0.18	0.19	0.18	0.16	0.16	0.19	0.21	0.23	0.22	0.24	0.26	0.28	0.29	0.29	0.31
Rated Current	Amps	5.8	3.75	2.4	1.4	0.95	5.1	3.5	2.4	1.5	1.05	5.6	3.75	2.5	1.6	1.1
Rated Power	Watts	36.5	51.0	49.6	36.2	26.5	32.1	47.9	50.9	42.0	32.6	36.4	52.3	50.9	39.3	28.6
	Horsepower	0.05	0.07	0.07	0.05	0.04	0.04	0.06	0.07	0.06	0.04	0.05	0.07	0.07	0.05	0.04
Torque Senstivity	oz-in/amp	6.06	9.75	14.9	23.5	36	7.32	11.7	18	28.3	43.4	8.78	14.04	21.6	34	52.1
	Nm/amp	0.0428	0.0689	0.1052	0.1659	0.2542	0.0517	0.0826	0.1271	0.1998	0.3065	0.0620	0.0991	0.1525	0.2401	0.3679
Back EMF	volts/KRPM	4.5	7.2	11	17.25	26.5	5.41	8.65	13.3	20.9	32	6.49	10.38	16	25.14	38.5
	volts/rad/sec	0.0430	0.0688	0.1050	0.1647	0.2531	0.0517	0.0826	0.1270	0.1996	0.3056	0.0620	0.0991	0.1528	0.2401	0.3676
Terminal Resistance	ohms	0.54	1.40	3.27	8.13	19.0	0.63	1.60	3.20	7.00	16.50	0.56	1.43	3.39	8.40	19.10
Terminal Inductance	mH	0.72	1.75	4.26	10.24	24.20	0.77	1.96	4.66	11.44	27.00	0.97	2.38	5.50	13.73	32.28
Motor Constant	oz-in/watt^1/2	8.2	8.2	8.2	8.2	8.2	9.3	9.2	10.1	10.7	10.7	11.7	11.7	11.7	11.7	11.7
	Nm/watt	0.058	0.058	0.058	0.058	0.058	0.065	0.065	0.071	0.076	0.075	0.083	0.083	0.083	0.083	0.083
Rotor Inertia	oz-in-sec²	0.0052	0.0052	0.0052	0.0052	0.0052	0.0065	0.0065	0.0065	0.0065	0.0065	0.0078	0.0078	0.0078	0.0078	0.0078
	g-cm²	367.2	367.2	367.2	367.2	367.2	459.0	459.0	459.0	459.0	459.0	550.8	550.8	550.8	550.8	550.8
Friction Torque	oz-in	5	5	5	5	5	5	5	5	5	5	6	6	6	6	6
	Nm	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04	0.04
Thermal Resistance	°C/watt	4.7	4.7	4.7	4.7	4.7	4.3	4.3	4.3	4.3	4.3	3.9	3.9	3.9	3.9	3.9
Damping Factor	oz-in/KRPM	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.3	0.3	0.3	0.3	0.3
	Nm s/rad	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.002	0.002	0.002	0.002	0.002
Weight	oz	46	46	46	46	46	56	56	56	56	56	65	65	65	65	65
	g	1304	1304	1304	1304	1304	1588	1588	1588	1588	1588	1843	1843	1843	1843	1843
Electrical Time Constant	millisecond	1.3309	1.2500	1.3028	1.2595	1.2670	1.2300	1.2250	1.4563	1.6343	1.6364	1.7321	1.6643	1.6224	1.6345	1.6386
Mech. Time Constant	millisecond	10.80095	10.85778	10.86223	10.91902	10.90021	10.75786	10.75915	9.096742	8.054255	8.085451	8.025833	8.013327	8.010641	8.025579	8.020641
Speed/Torque Gradient	rpm/oz-in	-19.83865	-19.94302	-19.95119	-20.0555	-20.02096	-15.8076	-15.8095	-13.36675	-11.83492	-11.88076	-9.82763	-9.812617	-9.809028	-9.82732	-9.821273

The selected motor is DC motor which has a model C23-L55 winding code 50

- $T_{m\_acc} = (0.1395 * 0.000055 + 2.0945 * 0.002 + 0.17035) = 0.1745 N.m.$

$$T_r = 0.31 > T_m$$

- $\omega_{\max\_motor} = 2.094$ rad/s

$$\omega_{rated} = 93 > \omega_{\max\_motor}$$

- $P_{out} = 0.1745 * 2.0945 = 0.3655$watt

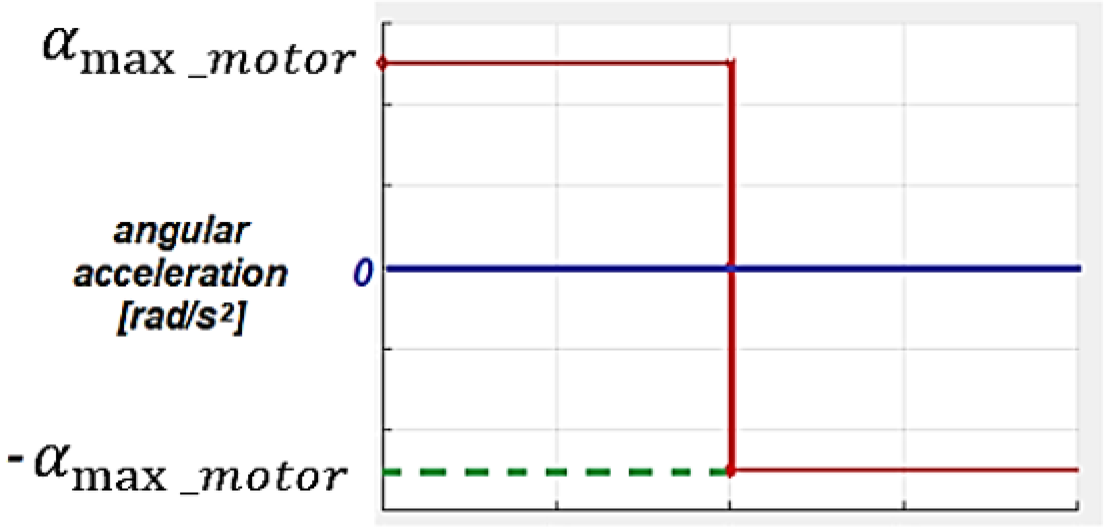
$$P_{rated} = 28.6 > P_{out}$$

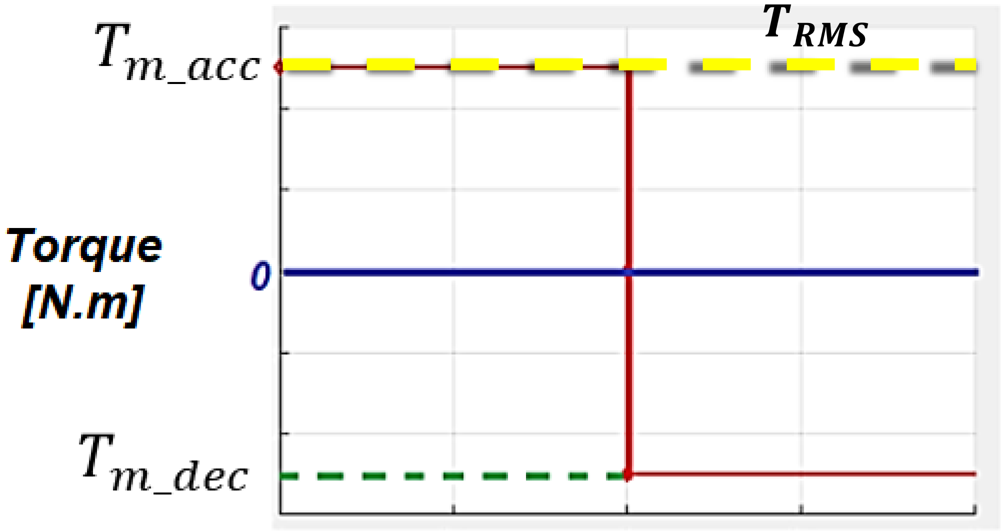
- Thus the selected motor is suitable
- For the deceleration phase, similar results will be computed

❑RMS Torque:

$$T_{rms} = \sqrt{\frac{(T_a + T_L)^2 \cdot t_1 + (T_d - T_L)^2 \cdot t_2}{t_f}}$$

$$T_{RMS} = \sqrt{\frac{(0.1745)^2 * 15 + (-0.1745)^2 * 15}{30}} = 0.1745 \text{ N.m.}$$





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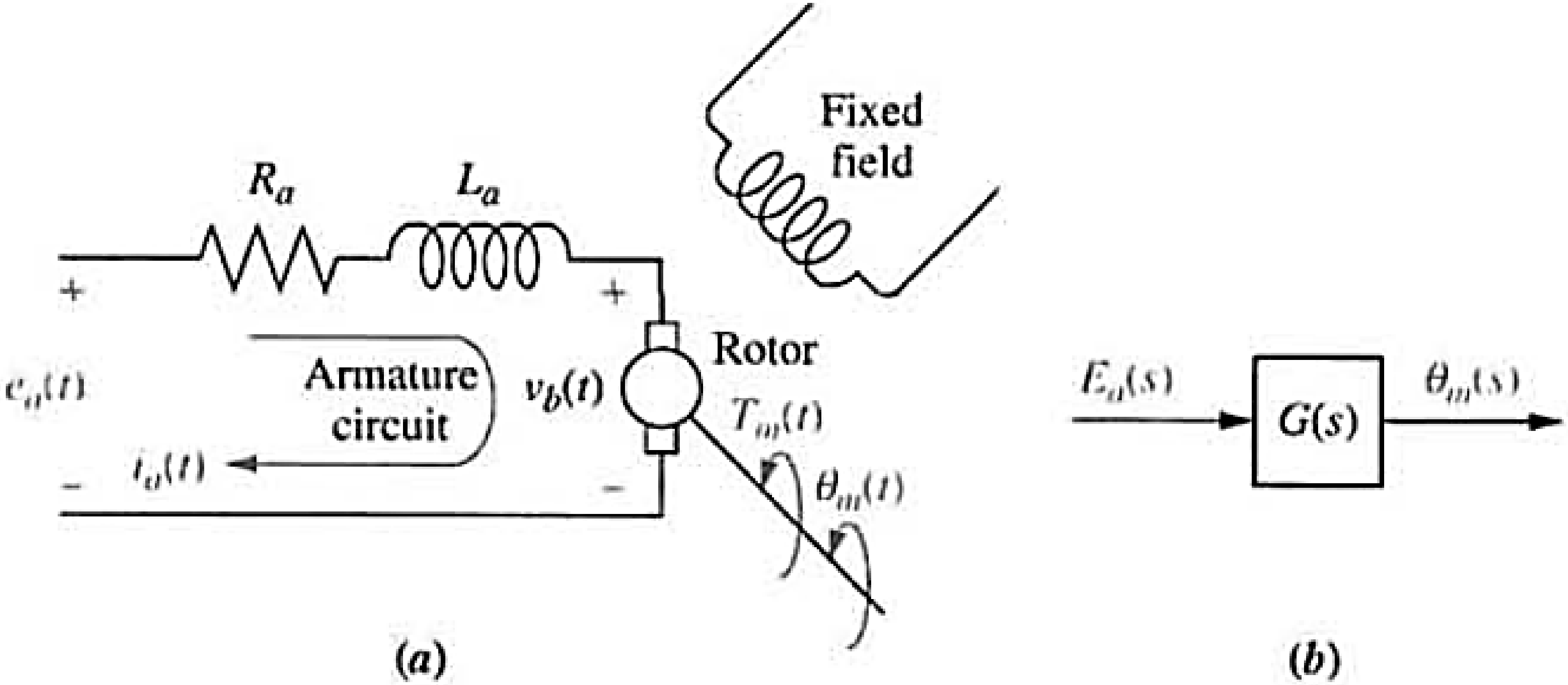

Antenna Azimuth: An Introduction to Position Control Systems

$$v_b(t) = K_b \frac{d\theta_m(t)}{dt} \tag{2.144}$$

We call $v_b(t)$ the *back electromotive force (back emf)*; K_b is a constant of proportionality called the back emf constant; and $d\theta_m(t)/dt = \omega_m(t)$ is the angular velocity of the motor. Taking the Laplace transform, we get

$$V_b(s) = K_b s\theta_m(s) \tag{2.145}$$

The relationship between the armature current, $i_a(t)$, the applied armature voltage, $e_a(t)$, and the back emf, $v_b(t)$, is found by writing a loop equation around the Laplace transformed armature circuit :

$$R_a I_a(s) + L_a s I_a(s) + V_b(s) = E_a(s) \tag{2.146}$$

The torque developed by the motor is proportional to the armature current; thus,

$$T_m(s) = K_t I_a(s) \tag{2.147}$$

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where T_m is the torque developed by the motor, and K_t is a constant of proportionality, called the motor torque constant, which depends on the motor and magnetic field characteristics. In a consistent set of units, the value of K_t is equal to the value of K_b. Rearranging Eq. (2.147) yields

$$I_a(s) = \frac{1}{K_t} T_m(s) \tag{2.148}$$

To find the transfer function of the motor, we first substitute Eqs. (2.145) and (2.148) into (2.146), yielding

$$\frac{(R_a + L_a s)T_m(s)}{K_t} + K_b s\theta_m(s) = E_a(s) \tag{2.149}$$

Now we must find $T_m(s)$ in terms of $\theta_m(s)$ if we are to separate the input and output variables and obtain the transfer function, $\theta_m(s)/E_a(s)$.

Figure 2.36 shows a typical equivalent mechanical loading on a motor. J_m is the equivalent inertia at the armature and includes both the armature inertia and, as we will see later, the load inertia reflected to the armature.

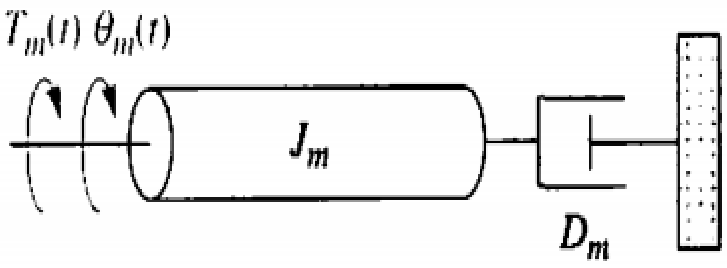

FIGURE 2.36 Typical equivalent mechanical loading on a motor

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D_m is the equivalent viscous damping at the armature and includes both the armature viscous damping and, as we will see later, the load viscous damping reflected to the armature. From Figure 2.36,

$$T_m(s) = (J_m s^2 + D_m s)\theta_m(s) \tag{2.150}$$

Substituting Eq. (2.150) into Eq. (2.149) yields

$$\frac{(R_a + L_a s)(J_m s^2 + D_m s)\theta_m(s)}{K_t} + K_b s\theta_m(s) = E_a(s) \tag{2.151}$$

If we assume that the armature inductance, L_a, is small compared to the armature resistance, R_a, which is usual for a dc motor, Eq. (2.151) becomes

$$\left[\frac{R_a}{K_t}(J_m s + D_m) + K_b\right] s\theta_m(s) = E_a(s) \tag{2.152}$$

After simplification, the desired transfer function, $\theta_m(s)/E_a(s)$, is found to be

$$\boxed{\frac{\theta_m(s)}{E_a(s)} = \frac{K_t/(R_a J_m)}{s\left[s + \frac{1}{J_m}\left(D_m + \frac{K_t K_b}{R_a}\right)\right]}} \qquad (2.153)^{13}$$

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the reader may be concerned about how to evaluate the constants.

Let us first discuss the mechanical constants, J_m and D_m. Consider Figure 2.37, which shows a motor with inertia J_a and damping D_a at the armature driving a load consisting of inertia J_L and damping D_L. Assuming that all inertia and damping values shown are known, J_L and D_L can be reflected back to the armature as some equivalent inertia and damping to be added to J_a and D_a, respectively. Thus, the equivalent inertia, J_m, and equivalent damping, D_m, at the armature are

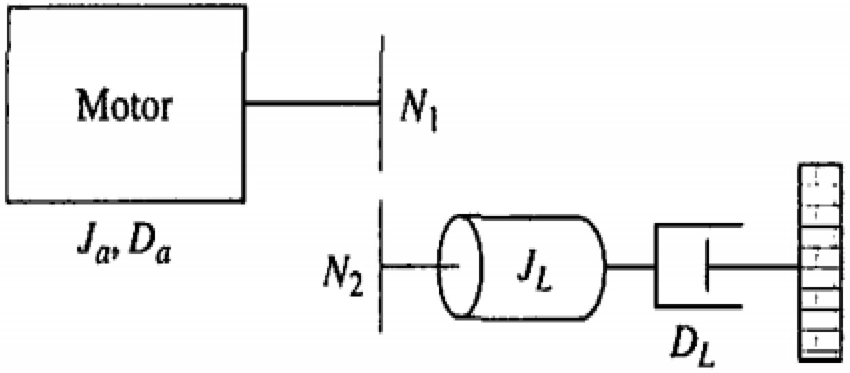

FIGURE 2.37 DC motor driving a rotational mechanical load

$$J_m = J_a + J_L\left(\frac{N_1}{N_2}\right)^2; \; D_m = D_a + D_L\left(\frac{N_1}{N_2}\right)^2 \qquad (2.155)^{14}$$

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$R_a = 19.1\ ohm$ (resistance)
$K_b = 0.3676$ volts.s/rad (Back EMF)
J_a =0.00005508 $\text{kg}.m^2$

$L_a = 0.03228$ mH (inductance)
$K_m = 0.083$ Nm/watt (motor constant)
D_a =0.002 N.m.s/rad

$$J_m = \left(J_a + \frac{0.5}{25}\right) = \left(0.00005508 + \frac{0.5}{25}\right) = 0.02005508\,\text{kg}.m^2$$

$$D_m = \left(0.002 + \frac{2}{25}\right) = 0.082\text{N.m.s/rad}$$

$$K_M = \frac{K_T I}{\sqrt{P}} = \frac{K_T I}{\sqrt{I^2 R}} = \frac{K_T}{\sqrt{R}}$$

where

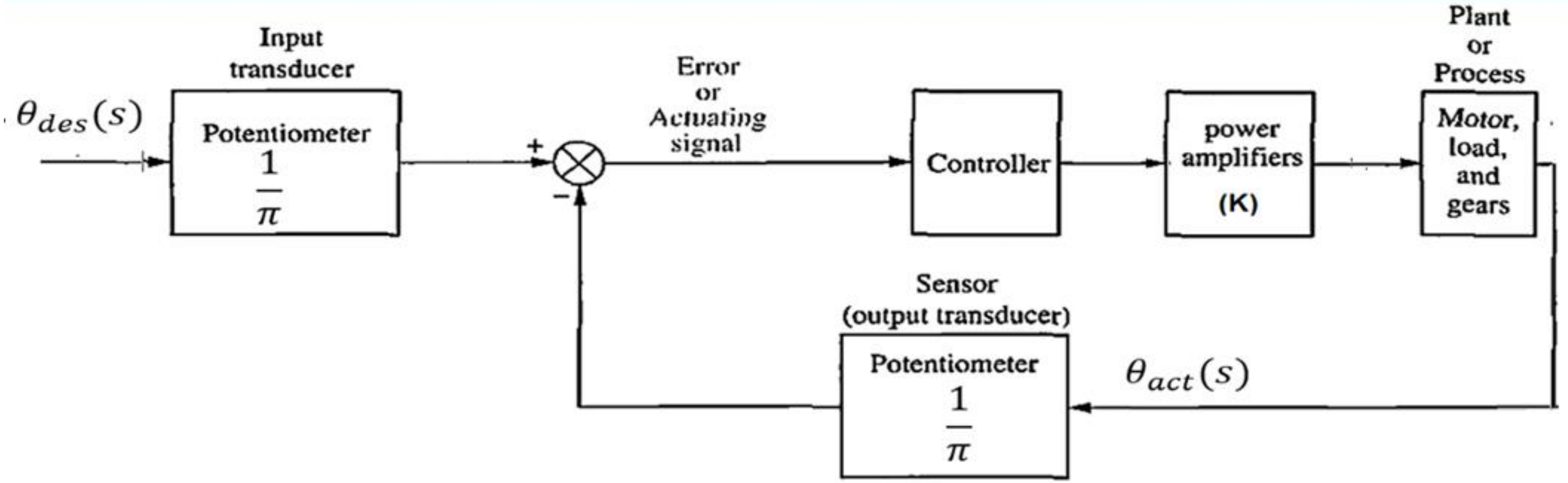
- I is the current (SI unit, ampere)
- R is the resistance (SI unit, ohm)
- K_T is the motor torque constant (SI unit, newton–metre per ampere, N·m/A), see below

$$K_T = \sqrt{R}\,K_M$$ = 0.3627

Antenna Azimuth: An Introduction to Position Control Systems

$$\frac{\theta_m(s)}{E_a(s)} = \frac{K_t/(R_a J_m)}{s\left[s + \frac{1}{J_m}\left(D_m + \frac{K_t K_b}{R_a}\right)\right]}$$

$$\frac{\theta_m(s)}{E_a(s)} = \frac{0.3627/(19.1 * 0.02005508)}{s\left[s + \frac{1}{0.02005508}\left(0.082 + \frac{0.3627 * 0.3676}{19.1}\right)\right]} = \frac{0.9469}{s + 4.437s}$$



worst case

Preamplifier

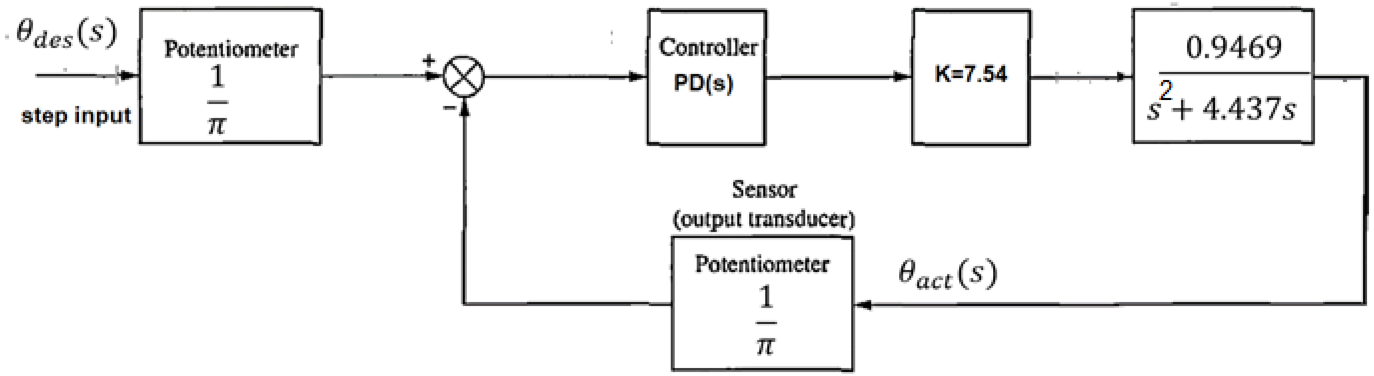
The transfer functions of the amplifiers are given in the problem statement. Two phenomena are *neglected*. First, we *assume* that saturation is never reached. Second, the dynamics of the preamplifier are *neglected*, since its speed of response is typically much greater than that of the power amplifier.

preamplifier,

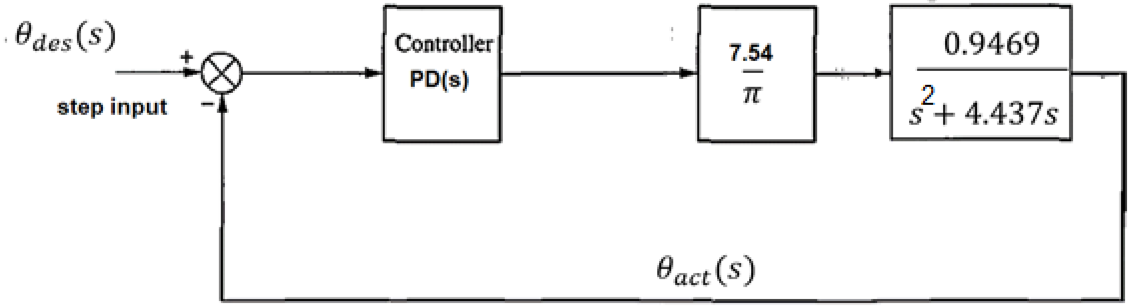
$$\frac{V_p(s)}{V_e(s)} = K = 7.54 \qquad (2.203)$$

Settling time	0.5 seconds
%OS	10%
$e_{ss}(\infty)=0$	For step input

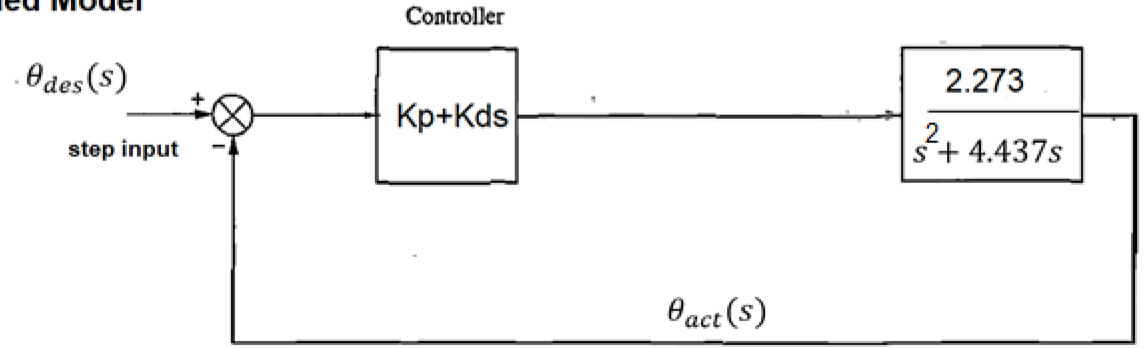
Real Model

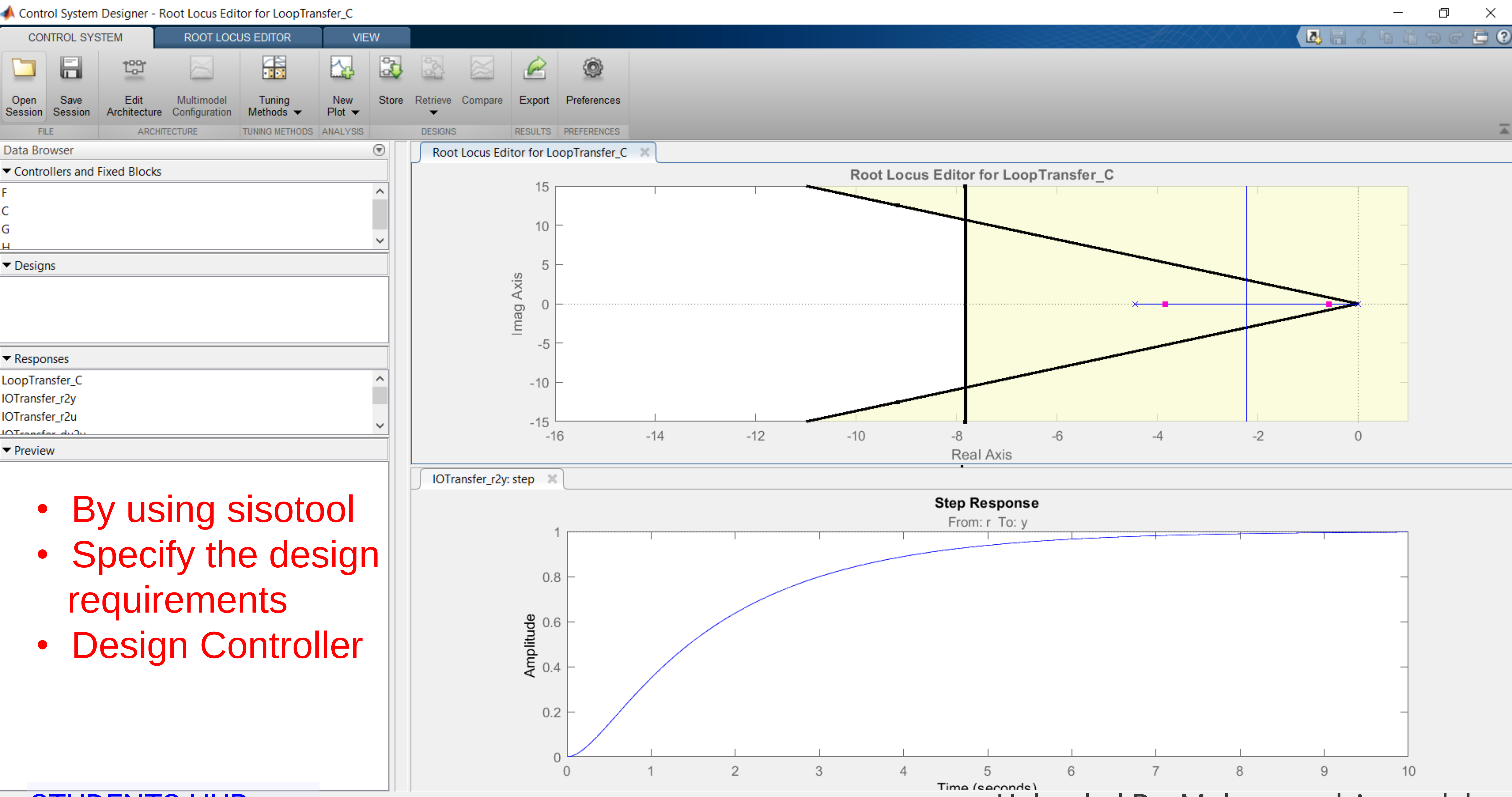


(by using block reduction)

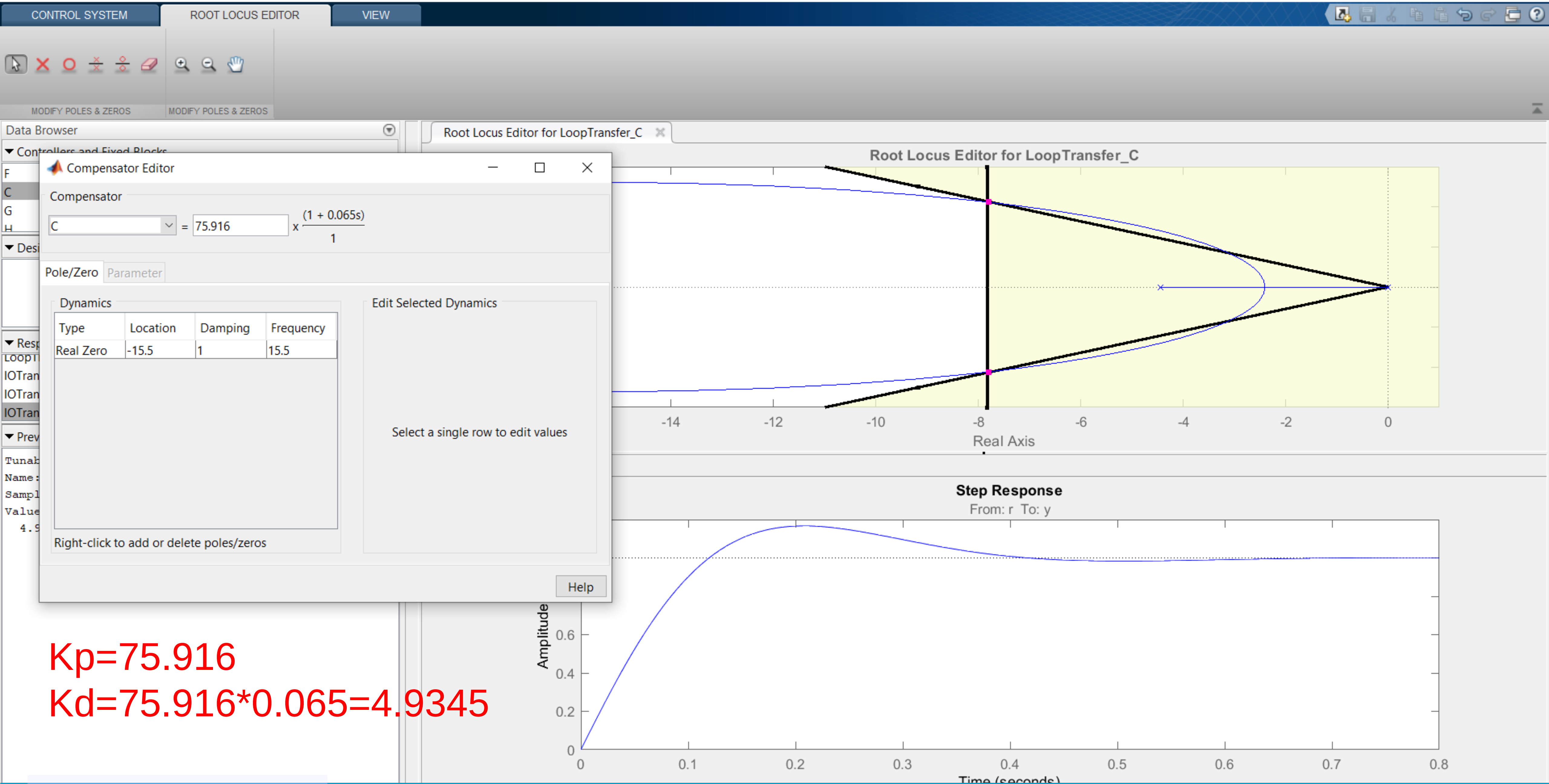


Simplified Model





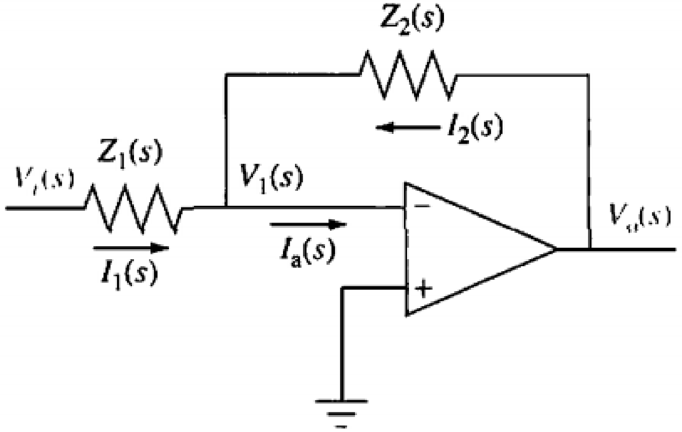
- By using sisotool
- Specify the design requirements
- Design Controller



Kp=75.916

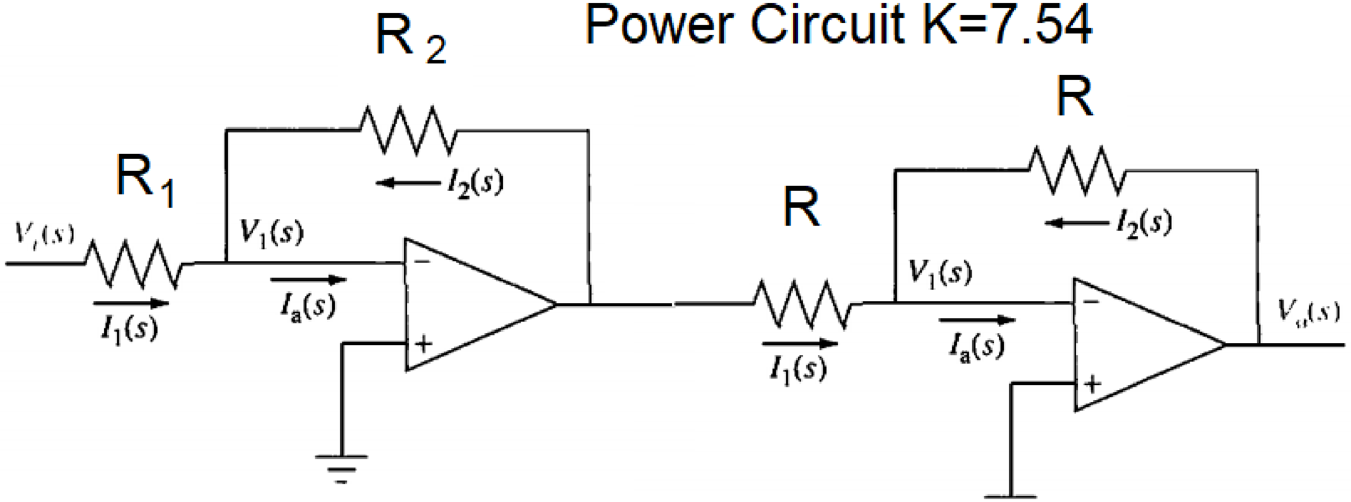
Kd=75.916*0.065=4.9345

Realization of Power Circuit



$$\boxed{\frac{V_o(s)}{V_i(s)} = -\frac{Z_2(s)}{Z_1(s)}}$$

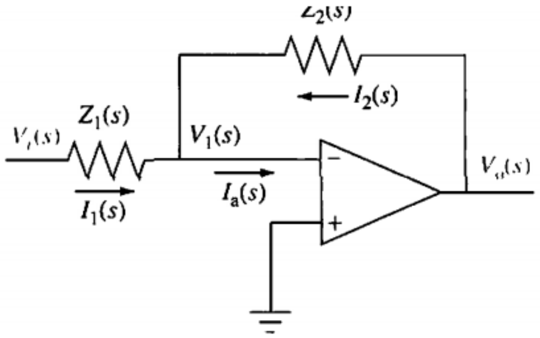
Power Circuit K=7.54



$\frac{V_o(s)}{V_i(s)} = 7.54$ ⇒ $R_2 = 7.54\, R_1$

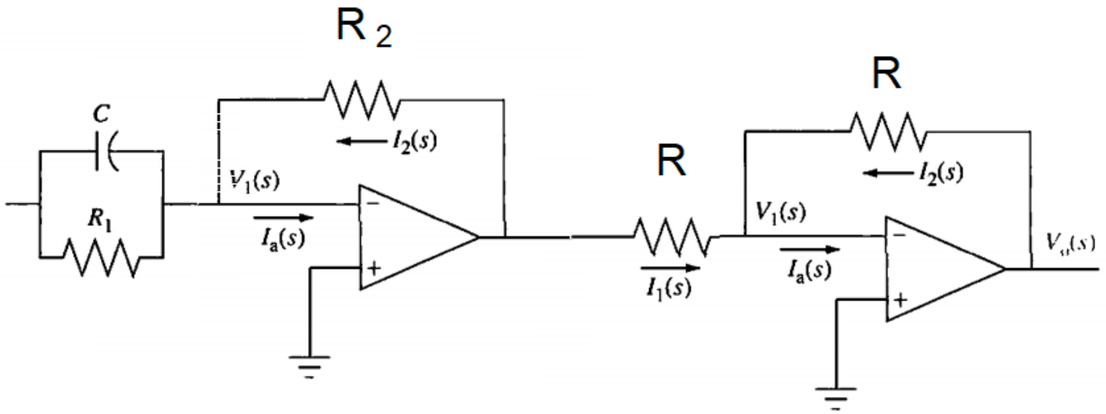
Let R_1 =100 ohm

so R_2 =754 ohm



$$\boxed{\frac{V_o(s)}{V_i(s)} = -\frac{Z_2(s)}{Z_1(s)}}$$

Realization of PD Controller



$$\frac{V_o(s)}{V_i(s)} = R_2C\left(s + \frac{1}{R_1C}\right) = 75.916 + 4.9345\,s$$

$\Rightarrow$ $R_2C = 75.916$

Let C= 100 Micro Farad

R_2 =759160 ohm

$\Rightarrow$ $\dfrac{R_2}{R_1}$ =4.9345

R_1 =153847.40 ohm

If all resistors are equal in value, then the output voltage can be derived by using superposition principle.

Subtractor:

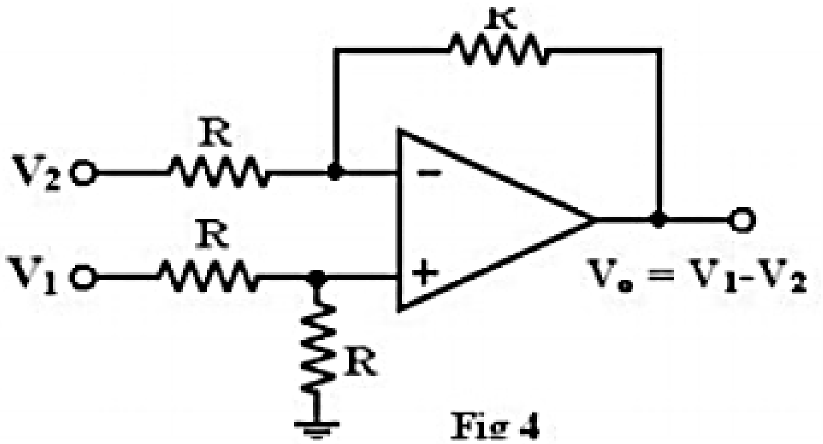
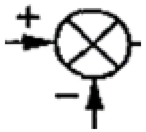


Fig 4



Realization by using Arduino kit:

Arduino Uno is a microcontroller board based on the ATmega328P shown in Fig. 2. It has 14 digital input/output pins (of which 6 can be used as PWM outputs), 6 analog inputs, a 16 MHz quartz crystal, a USB connection, a power jack, an ICSP header and a reset button. It contains everything needed to support the microcontroller; simply connect it to a computer with a USB cable or power it with a AC-to-DC adapter or battery to get started. In this project , the Arduino microcontroller is very well-suited to drive the PWM signal for DC motor for the improvement of the output response for the DC motor position control system.

Figure 2: Arduino Uno Microcontroller

Realization by using Arduino kit:

B. L298N Dual H-Bridge Controller

The L298N H-bridge IC shows in Fig. 3 that can allow to control the speed and direction of two DC motors. This module can be used with motors that have a voltage of between 5 and 35V DC with a peak current up to 2A. The module has two screw terminal blocks for the motor A and B, and another screw terminal block for the Ground pin, the VCC for motor and a 5V pin which can either be an input or output. Pin assignments for L298N dual H-Bridge Module is shown in table 1. The digital pin assign from HIGH to LOW or LOW to HIGH is used IN1and IN2 on the L298N board to control the direction. And the controller output PWM signal is send to ENA or ENB to control the position. The forward and reverse speed or position controlling for the motor has done by using PWM signal.

Then using the analogWrite() function and send the PWM signal to the Enable pin of the L298N board, which actually drives the motor.

Table.1 Pins assignment for L298

Out 1:	Motor A lead out
Out 2:	Motor A lead out
Out 3:	Motor B lead out
Out 4:	Motor B lead out
GND:	Ground
5V :	5V input
ENA:	Enables PWM signal for Motor A
IN1:	Enable Motor A
IN2:	Enable Motor A
IN3:	Enable Motor B
IN4:	Enable Motor B
ENB:	Enables PWM signal for Motor B

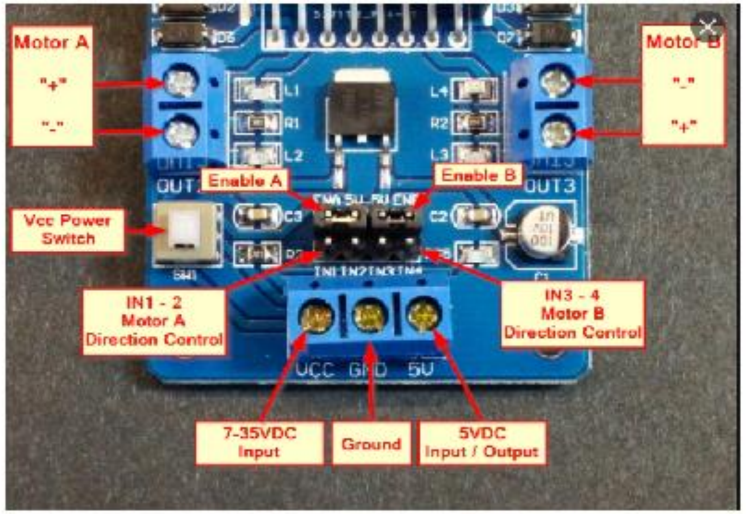
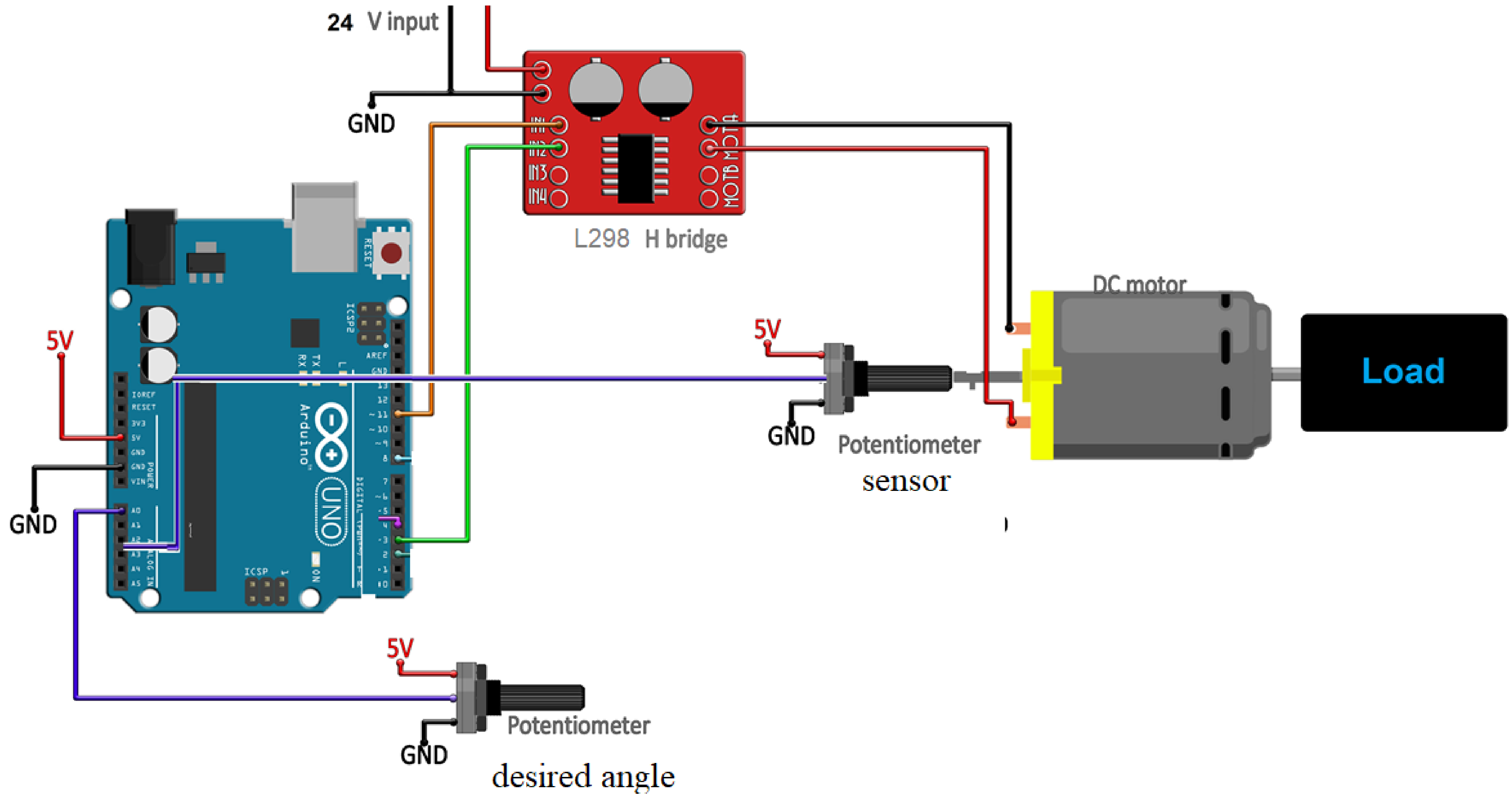
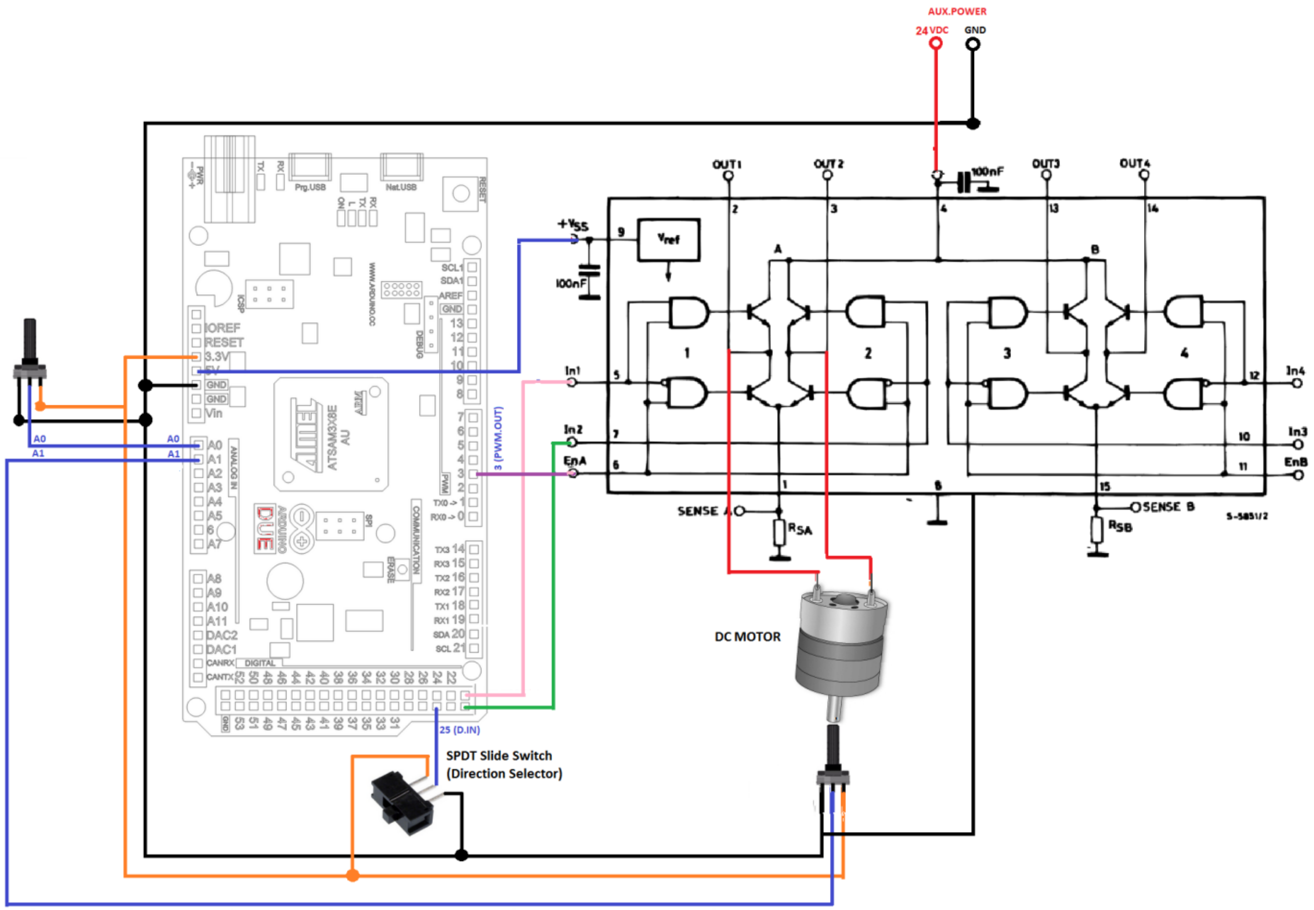


Figure 3: L298N dual H-Bridge Controller Module

Realization by using Arduino kit:



Realization by using Arduino kit:



The Enable A and Enable B pins are used for enabling and controlling the speed of the motor. If a jumper is present on this pin, the motor will be enabled and work at maximum speed, and if we remove the jumper we can connect a PWM input to this pin and in that way control the speed of the motor. If we connect this pin to a Ground the motor will be disabled.

Realization by using Arduino kit:

Settling time	0.5 seconds
%OS	10%
$e_{ss}(\infty)=0$	For step input

$$\%OS \Longrightarrow \zeta = 0.59$$

$$\omega_n = \frac{4}{\zeta T_s} = \frac{4}{0.5*0.59} = 13.6 \text{ rad/s}$$

- Now select the required sampling time:

$$\omega_s = 10 * \omega_n = 136 \text{ rad/s}$$

$$\tau_s = \frac{2\pi}{\omega_s} = \frac{2\pi}{136} = 0.046 \text{ second}$$

Realization by using Arduino kit:

- Now find the discrete PD controller based on Trapezoidal method:

$$PD(s) = K_p + K_d * s$$

K_p =75.916 K_d =4.9345 $\tau_s = 0.046$ second

Now to discretize the controller remember the following equations:

- $s \approx \frac{z-1}{T}$ (Forward difference or Euler's method)
- $s \approx \frac{z-1}{zT}$ (Backward difference)
- $s \approx \frac{2}{T}\frac{z-1}{z+1}$ (Trapezoidal method, or Tustin's approximation)

$$PD(z) = \frac{U(z)}{E(z)} = K_p + K_d * \frac{2}{\tau_s}\frac{(z-1)}{(z+1)} = 75.916 + 4.9345 * \frac{2}{0.046}\frac{(z-1)}{(z+1)}$$

$$= 75.916 + 214.54\frac{(z-1)}{(z+1)}$$

Realization by using Arduino kit:

$$\frac{U(z)}{E(z)}=75.916\frac{(z+1)}{(z+1)}+214.54\frac{(z-1)}{(z+1)}=\frac{290.45\,z-138.62}{z+1}$$

$$U(z)z+U(z)=290.45\,z\,E(z)-138.62\,E(z)$$

$$u(k+1)=290.45\,e(k+1)-138.62\,e(k)-u(k)$$

or this equation can be used

$$u(k)=290.45\,e(k)-138.62\,e(k-1)-u(k-1)$$

Realization by using Arduino kit: