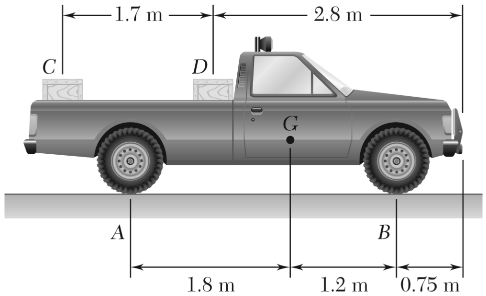
CHAPTER 4

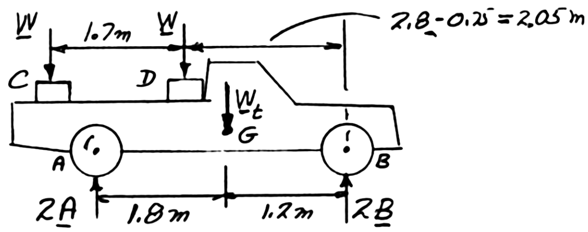


PROBLEM 4.1

Two crates, each of mass 350 kg, are placed as shown in the bed of a 1400-kg pickup truck. Determine the reactions at each of the two (*a*) rear wheels *A*, (*b*) front wheels *B*.

SOLUTION

Free-Body Diagram:



$$W = (350\text{ kg})(9.81\text{ m/s}^2) = 3.4335\text{ kN}$$

$$W_t = (1400\text{ kg})(9.81\text{ m/s}^2) = 13.7340\text{ kN}$$

(*a*) <u>Rear wheels</u>:

$$+\circlearrowleft \Sigma M_B = 0: \quad W(1.7\text{ m} + 2.05\text{ m}) + W(2.05\text{ m}) + W_t(1.2\text{ m}) - 2A(3\text{ m}) = 0$$

$$(3.4335\text{ kN})(3.75\text{ m}) + (3.4335\text{ kN})(2.05\text{ m}) + (13.7340\text{ kN})(1.2\text{ m}) - 2A(3\text{ m}) = 0$$

$$A = +6.0659\text{ kN}$$

$$\mathbf{A} = 6.07\text{ kN}\uparrow \blacktriangleleft$$

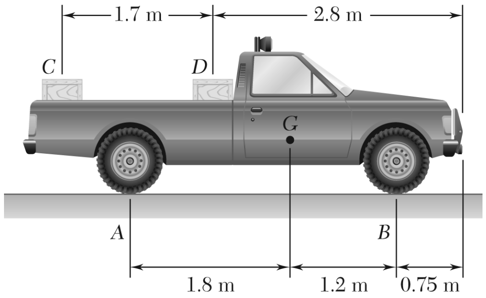
(*b*) <u>Front wheels</u>:

$$+\uparrow \Sigma F_y = 0: \quad -W - W - W_t + 2A + 2B = 0$$

$$-3.4335\text{ kN} - 3.4335\text{ kN} - 13.7340\text{ kN} + 2(6.0659\text{ kN}) + 2B = 0$$

$$B = +4.2346\text{ kN}$$

$$\mathbf{B} = 4.23\text{ kN}\uparrow \blacktriangleleft$$



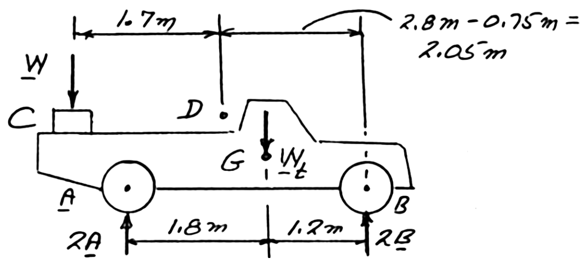
PROBLEM 4.2

Solve Problem 4.1, assuming that crate D is removed and that the position of crate C is unchanged.

PROBLEM 4.1 Two crates, each of mass 350 kg, are placed as shown in the bed of a 1400-kg pickup truck. Determine the reactions at each of the two (*a*) rear wheels A, (*b*) front wheels B.

SOLUTION

Free-Body Diagram:



$$W = (350\text{ kg})(9.81\text{ m/s}^2) = 3.4335\text{ kN}$$

$$W_t = (1400\text{ kg})(9.81\text{ m/s}^2) = 13.7340\text{ kN}$$

(*a*) Rear wheels: $+\circlearrowleft \Sigma M_B = 0$: $W(1.7\text{ m} + 2.05\text{ m}) + W_t(1.2\text{ m}) - 2A(3\text{ m}) = 0$

$$(3.4335\text{ kN})(3.75\text{ m}) + (13.7340\text{ kN})(1.2\text{ m}) - 2A(3\text{ m}) = 0$$

$$A = +4.8927\text{ kN}$$

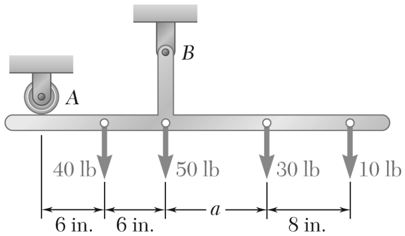
$\mathbf{A} = 4.89\text{ kN}\uparrow$ ◀

(*b*) Front wheels: $+\uparrow \Sigma M_y = 0$: $-W - W_t + 2A + 2B = 0$

$$-3.4335\text{ kN} - 13.7340\text{ kN} + 2(4.8927\text{ kN}) + 2B = 0$$

$$B = +3.6911\text{ kN}$$

$\mathbf{B} = 3.69\text{ kN}\uparrow$ ◀

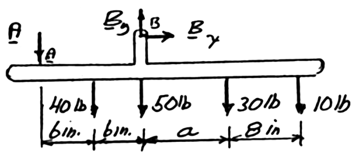


PROBLEM 4.3

A T-shaped bracket supports the four loads shown. Determine the reactions at A and B (a) if $a = 10$ in., (b) if $a = 7$ in.

SOLUTION

Free-Body Diagram:



$\xrightarrow{+} \Sigma F_x = 0$: $\quad B_x = 0$

$+\circlearrowleft \Sigma M_B = 0$: $\quad (40\text{ lb})(6\text{ in.}) - (30\text{ lb})a - (10\text{ lb})(a + 8\text{ in.}) + (12\text{ in.})A = 0$

$$A = \frac{(40a - 160)}{12} \qquad (1)$$

$+\circlearrowleft \Sigma M_A = 0$: $\quad -(40\text{ lb})(6\text{ in.}) - (50\text{ lb})(12\text{ in.}) - (30\text{ lb})(a + 12\text{ in.})$
$\quad - (10\text{ lb})(a + 20\text{ in.}) + (12\text{ in.})B_y = 0$

$$B_y = \frac{(1400 + 40a)}{12}$$

Since

$$B_x = 0, \quad B = \frac{(1400 + 40a)}{12} \qquad (2)$$

(a) For $a = 10$ in.,

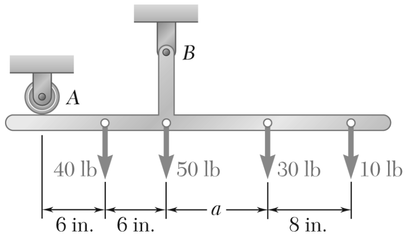
Eq. (1): $\quad A = \dfrac{(40 \times 10 - 160)}{12} = +20.0\text{ lb} \qquad \mathbf{A} = 20.0\text{ lb}\downarrow$ ◀

Eq. (2): $\quad B = \dfrac{(1400 + 40 \times 10)}{12} = +150.0\text{ lb} \qquad \mathbf{B} = 150.0\text{ lb}\uparrow$ ◀

(b) For $a = 7$ in.,

Eq. (1): $\quad A = \dfrac{(40 \times 7 - 160)}{12} = +10.00\text{ lb} \qquad \mathbf{A} = 10.00\text{ lb}\downarrow$ ◀

Eq. (2): $\quad B = \dfrac{(1400 + 40 \times 7)}{12} = +140.0\text{ lb} \qquad \mathbf{B} = 140.0\text{ lb}\uparrow$ ◀



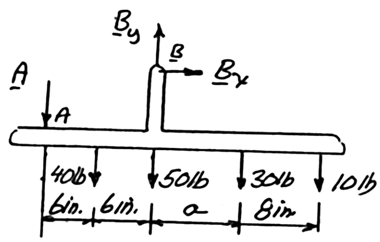
PROBLEM 4.4

For the bracket and loading of Problem 4.3, determine the smallest distance a if the bracket is not to move.

PROBLEM 4.3 A T-shaped bracket supports the four loads shown. Determine the reactions at A and B (a) if $a = 10$ in., (b) if $a = 7$ in.

SOLUTION

Free-Body Diagram:

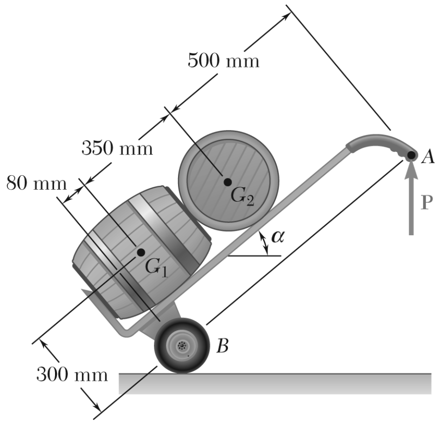


For no motion, reaction at A must be downward or zero; smallest distance a for no motion corresponds to $\mathbf{A} = 0$.

$$+\circlearrowleft \Sigma M_B = 0: \quad (40 \text{ lb})(6 \text{ in.}) - (30 \text{ lb})a - (10 \text{ lb})(a + 8 \text{ in.}) + (12 \text{ in.})A = 0$$

$$A = \frac{(40a - 160)}{12}$$

$$A = 0: \quad (40a - 160) = 0 \qquad a = 4.00 \text{ in.} \blacktriangleleft$$

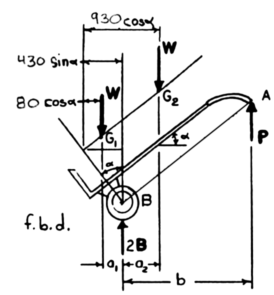


PROBLEM 4.5

A hand truck is used to move two kegs, each of mass 40 kg. Neglecting the mass of the hand truck, determine (*a*) the vertical force **P** that should be applied to the handle to maintain equilibrium when $\alpha = 35°$, (*b*) the corresponding reaction at each of the two wheels.

SOLUTION

Free-Body Diagram:



Dimensions in mm

$$W = mg = (40 \text{ kg})(9.81 \text{ m/s}^2) = 392.40 \text{ N}$$
$$a_1 = (300 \text{ mm})\sin\alpha - (80 \text{ mm})\cos\alpha$$
$$a_2 = (430 \text{ mm})\cos\alpha - (300 \text{ mm})\sin\alpha$$
$$b = (930 \text{ mm})\cos\alpha$$

From free-body diagram of hand truck,

$$+\circlearrowleft \Sigma M_B = 0: \quad P(b) - W(a_2) + W(a_1) = 0 \qquad (1)$$

$$+\uparrow \Sigma F_y = 0: \quad P - 2W + 2B = 0 \qquad (2)$$

For $\alpha = 35°$

$$a_1 = 300\sin 35° - 80\cos 35° = 106.541 \text{ mm}$$
$$a_2 = 430\cos 35° - 300\sin 35° = 180.162 \text{ mm}$$
$$b = 930\cos 35° = 761.81 \text{ mm}$$

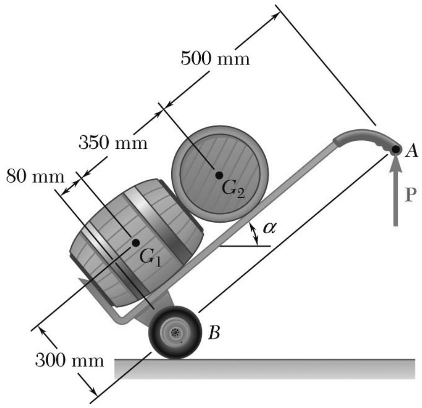
(*a*) From Equation (1):

$$P(761.81 \text{ mm}) - 392.40 \text{ N}(180.162 \text{ mm}) + 392.40 \text{ N}(106.54 \text{ mm}) = 0$$

$$P = 37.921 \text{ N}$$ or $\mathbf{P} = 37.9 \text{ N}\uparrow$ ◀

(*b*) From Equation (2):

$$37.921 \text{ N} - 2(392.40 \text{ N}) + 2B = 0$$ or $\mathbf{B} = 373 \text{ N}\uparrow$ ◀



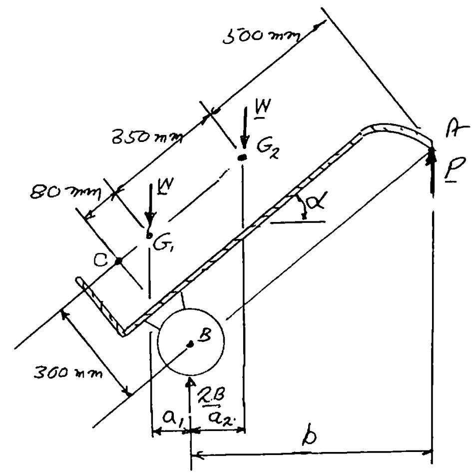
PROBLEM 4.6

Solve Problem 4.5 when $\alpha = 40°$.

PROBLEM 4.5 A hand truck is used to move two kegs, each of mass 40 kg. Neglecting the mass of the hand truck, determine (*a*) the vertical force **P** that should be applied to the handle to maintain equilibrium when $\alpha = 35°$, (*b*) the corresponding reaction at each of the two wheels.

SOLUTION

Free-Body Diagram:



$$W = mg = (40\text{ kg})(9.81\text{ m/s}^2)$$
$$W = 392.40\text{ N}$$
$$a_1 = (300\text{ mm})\sin\alpha - (80\text{ mm})\cos\alpha$$
$$a_2 = (430\text{ mm})\cos\alpha - (300\text{ mm})\sin\alpha$$
$$b = (930\text{ mm})\cos\alpha$$

From F.B.D.:

$$+\circlearrowleft \Sigma M_B = 0: \quad P(b) - W(a_2) + W(a_1) = 0$$
$$P = W(a_2 - a_1)/b \qquad (1)$$

$$+\uparrow \Sigma F_y = 0: \quad -W - W + P + 2B = 0$$
$$B = W - \frac{1}{2}P \qquad (2)$$

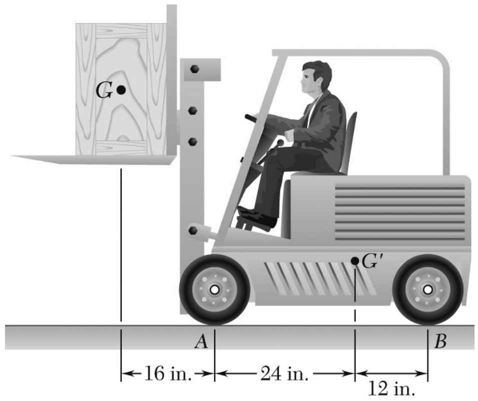
For $\alpha = 40°$:

$$a_1 = 300\sin 40° - 80\cos 40° = 131.553\text{ mm}$$
$$a_2 = 430\cos 40° - 300\sin 40° = 136.563\text{ mm}$$
$$b = 930\cos 40° = 712.42\text{ mm}$$

(*a*) From Equation (1): $P = \dfrac{392.40\text{ N }(0.136563\text{ m} - 0.131553\text{ m})}{0.71242\text{ m}}$

$P = 2.7595\text{ N}$ $\qquad$ $\mathbf{P} = 2.76\text{ N}\uparrow$ ◀

(*b*) From Equation (2): $B = 392.40\text{ N} - \dfrac{1}{2}(2.7595\text{ N})$ $\qquad$ $\mathbf{B} = 391\text{ N}\uparrow$ ◀

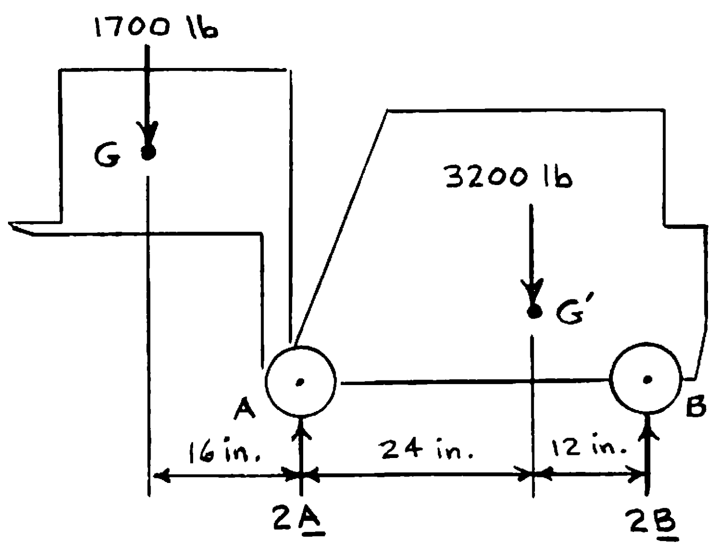


PROBLEM 4.7

A 3200-lb forklift truck is used to lift a 1700-lb crate. Determine the reaction at each of the two (*a*) front wheels *A*, (*b*) rear wheels *B*.

SOLUTION

Free-Body Diagram:

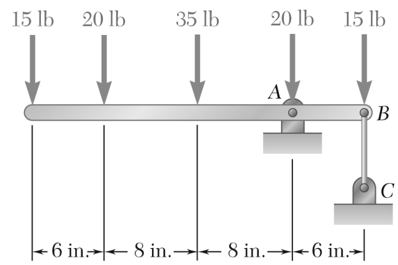


(*a*) Front wheels: $+\circlearrowleft \Sigma M_B = 0$: $(1700\text{ lb})(52\text{ in.}) + (3200\text{ lb})(12\text{ in.}) - 2A(36\text{ in.}) = 0$

$A = +1761.11\text{ lb}$ $\qquad\qquad$ $\mathbf{A} = 1761\text{ lb}\uparrow$ ◀

(*b*) Rear wheels: $+\uparrow \Sigma F_y = 0$: $-1700\text{ lb} - 3200\text{ lb} + 2(1761.11\text{ lb}) + 2B = 0$

$B = +688.89\text{ lb}$ $\qquad\qquad$ $\mathbf{B} = 689\text{ lb}\uparrow$ ◀

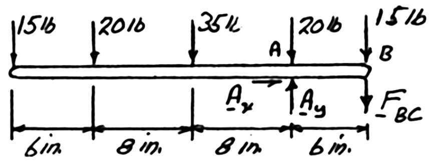


PROBLEM 4.8

For the beam and loading shown, determine (*a*) the reaction at *A*, (*b*) the tension in cable *BC*.

SOLUTION

Free-Body Diagram:



(*a*) <u>Reaction at *A*</u>: $\Sigma F_x = 0$: $A_x = 0$

$$+\circlearrowleft \Sigma M_B = 0: \quad (15\text{ lb})(28\text{ in.}) + (20\text{ lb})(22\text{ in.}) + (35\text{ lb})(14\text{ in.}) + (20\text{ lb})(6\text{ in.}) - A_y(6\text{ in.}) = 0$$

$$A_y = +245\text{ lb}$$

$\mathbf{A} = 245\text{ lb}\uparrow$ ◀

(*b*) <u>Tension in *BC*</u>:

$$+\circlearrowleft \Sigma M_A = 0: \quad (15\text{ lb})(22\text{ in.}) + (20\text{ lb})(16\text{ in.}) + (35\text{ lb})(8\text{ in.}) - (15\text{ lb})(6\text{ in.}) - F_{BC}(6\text{ in.}) = 0$$

$$F_{BC} = +140.0\text{ lb}$$

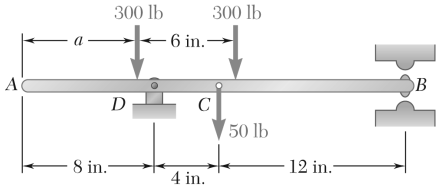
$F_{BC} = 140.0\text{ lb}$ ◀

Check:

$$+\uparrow \Sigma F_y = 0: \quad -15\text{ lb} - 20\text{ lb} = 35\text{ lb} - 20\text{ lb} + A - F_{BC} = 0$$

$$-105\text{ lb} + 245\text{ lb} - 140.0 = 0$$

$$0 = 0 \quad \text{(Checks)}$$

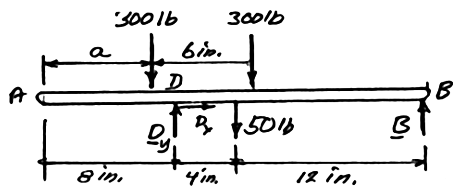


PROBLEM 4.9

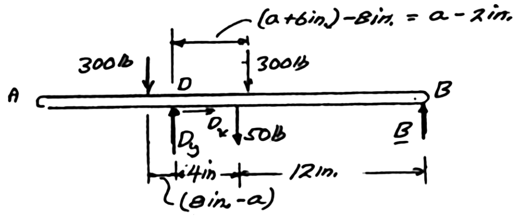
For the beam and loading shown, determine the range of the distance a for which the reaction at B does not exceed 100 lb downward or 200 lb upward.

SOLUTION

Assume B is positive when directed $\uparrow$.



Sketch showing distance from D to forces.



$$+\circlearrowleft \Sigma M_D = 0: \quad (300 \text{ lb})(8 \text{ in.} - a) - (300 \text{ lb})(a - 2 \text{ in.}) - (50 \text{ lb})(4 \text{ in.}) + 16B = 0$$

$$-600a + 2800 + 16B = 0$$

$$a = \frac{(2800 + 16B)}{600} \qquad (1)$$

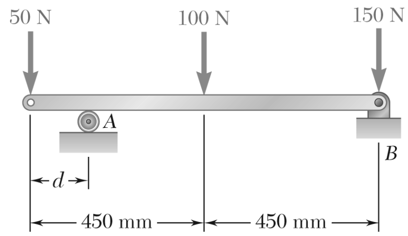
For $B = 100 \text{ lb}\downarrow = -100 \text{ lb}$, Eq. (1) yields:

$$a \geq \frac{[2800 + 16(-100)]}{600} = \frac{1200}{600} = 2 \text{ in.} \qquad a \geq 2.00 \text{ in.} \ \triangleleft$$

For $B = 200\uparrow = +200 \text{ lb}$, Eq. (1) yields:

$$a \leq \frac{[2800 + 16(200)]}{600} = \frac{6000}{600} = 10 \text{ in.} \qquad a \leq 10.00 \text{ in.} \ \triangleleft$$

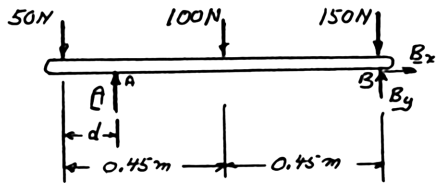
Required range: $2.00 \text{ in.} \leq a \leq 10.00 \text{ in.}$ ◀



PROBLEM 4.10

The maximum allowable value of each of the reactions is 180 N. Neglecting the weight of the beam, determine the range of the distance d for which the beam is safe.

SOLUTION



$$\Sigma F_x = 0: \quad B_x = 0$$

$$B = B_y$$

$$+\circlearrowleft \Sigma M_A = 0: \quad (50 \text{ N})d - (100 \text{ N})(0.45 \text{ m} - d) - (150 \text{ N})(0.9 \text{ m} - d) + B(0.9 \text{ m} - d) = 0$$

$$50d - 45 + 100d - 135 + 150d + 0.9B - Bd = 0$$

$$d = \frac{180 \text{ N}\cdot\text{m} - (0.9 \text{ m})B}{300A - B} \qquad (1)$$

$$+\circlearrowleft \Sigma M_B = 0: \quad (50 \text{ N})(0.9 \text{ m}) - A(0.9 \text{ m} - d) + (100 \text{ N})(0.45 \text{ m}) = 0$$

$$45 - 0.9A + Ad + 45 = 0$$

$$d = \frac{(0.9 \text{ m})A - 90 \text{ N}\cdot\text{m}}{A} \qquad (2)$$

Since $B \le 180$ N, Eq. (1) yields

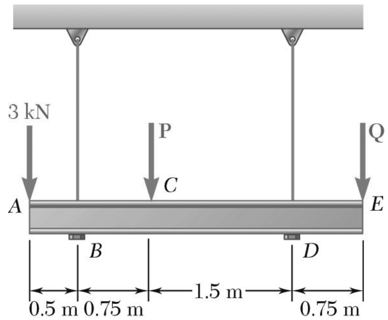
$$d \ge \frac{180 - (0.9)180}{300 - 180} = \frac{18}{120} = 0.15 \text{ m} \qquad d \ge 150.0 \text{ mm} \triangleleft$$

Since $A \le 180$ N, Eq. (2) yields

$$d \le \frac{(0.9)180 - 90}{180} = \frac{72}{180} = 0.40 \text{ m} \qquad d \le 400 \text{ mm} \triangleleft$$

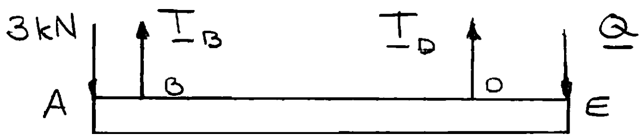
Range: $150.0 \text{ mm} \le d \le 400 \text{ mm}$ ◀

PROBLEM 4.11



Three loads are applied as shown to a light beam supported by cables attached at B and D. Neglecting the weight of the beam, determine the range of values of Q for which neither cable becomes slack when $P = 0$.

SOLUTION



$$+\circlearrowleft \Sigma M_B = 0: \quad (3.00 \text{ kN})(0.500 \text{ m}) + T_D(2.25 \text{ m}) - Q(3.00 \text{ m}) = 0$$

$$Q = 0.500 \text{ kN} + (0.750)\, T_D \qquad (1)$$

$$+\circlearrowleft \Sigma M_D = 0: \quad (3.00 \text{ kN})(2.75 \text{ m}) - T_B(2.25 \text{ m}) - Q(0.750 \text{ m}) = 0$$

$$Q = 11.00 \text{ kN} - (3.00)\, T_B \qquad (2)$$

For cable B not to be slack, $T_B \geq 0$, and from Eq. (2),

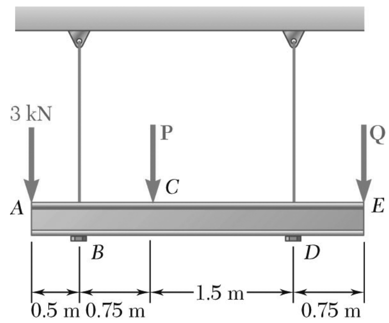
$$Q \leq 11.00 \text{ kN}$$

For cable D not to be slack, $T_D \geq 0$, and from Eq. (1),

$$Q \geq 0.500 \text{ kN}$$

For neither cable to be slack,

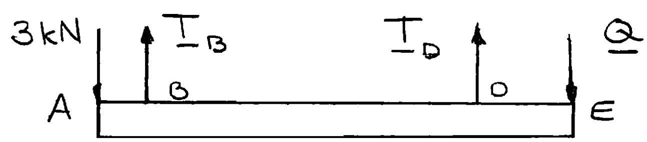
$$0.500 \text{ kN} \leq Q \leq 11.00 \text{ kN} \blacktriangleleft$$



PROBLEM 4.12

Three loads are applied as shown to a light beam supported by cables attached at B and D. Knowing that the maximum allowable tension in each cable is 4 kN and neglecting the weight of the beam, determine the range of values of Q for which the loading is safe when $P = 0$.

SOLUTION



$$+\circlearrowleft \Sigma M_B = 0: \quad (3.00 \text{ kN})(0.500 \text{ m}) + T_D(2.25 \text{ m}) - Q(3.00 \text{ m}) = 0$$

$$Q = 0.500 \text{ kN} + (0.750)\, T_D \qquad (1)$$

$$+\circlearrowleft \Sigma M_D = 0: \quad (3.00 \text{ kN})(2.75 \text{ m}) - T_B(2.25 \text{ m}) - Q(0.750 \text{ m}) = 0$$

$$Q = 11.00 \text{ kN} - (3.00)\, T_B \qquad (2)$$

For $T_B \le 4.00$ kN, Eq. (2) yields

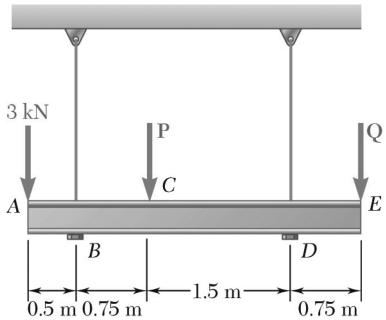
$$Q \ge 11.00 \text{ kN} - 3.00(4.00 \text{ kN}) \qquad Q \ge -1.000 \text{ kN}$$

For $T_D \le 4.00$ kN, Eq. (1) yields

$$Q \le 0.500 \text{ kN} + 0.750(4.00 \text{ kN}) \qquad Q \le 3.50 \text{ kN}$$

For loading to be safe, cables must also not be slack. Combining with the conditions obtained in Problem 4.11,

$$0.500 \text{ kN} \le Q \le 3.50 \text{ kN} \blacktriangleleft$$

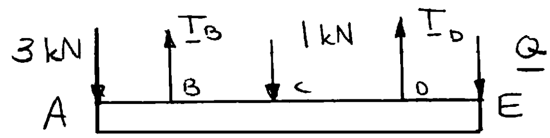


PROBLEM 4.13

For the beam of Problem 4.12, determine the range of values of Q for which the loading is safe when $P = 1$ kN.

PROBLEM 4.12 Three loads are applied as shown to a light beam supported by cables attached at B and D. Knowing that the maximum allowable tension in each cable is 4 kN and neglecting the weight of the beam, determine the range of values of Q for which the loading is safe when $P = 0$.

SOLUTION



$$+\circlearrowleft \Sigma M_B = 0: \quad (3.00 \text{ kN})(0.500 \text{ m}) - (1.000 \text{ kN})(0.750 \text{ m}) + T_D(2.25 \text{ m}) - Q(3.00 \text{ m}) = 0$$

$$Q = 0.250 \text{ kN} + 0.75\, T_D \qquad (1)$$

$$+\circlearrowleft \Sigma M_D = 0: \quad (3.00 \text{ kN})(2.75 \text{ m}) + (1.000 \text{ kN})(1.50 \text{ m}) - T_B(2.25 \text{ m}) - Q(0.750 \text{ m}) = 0$$

$$Q = 13.00 \text{ kN} - 3.00\ T_B \qquad (2)$$

For the loading to be safe, cables must not be slack and tension must not exceed 4.00 kN.

Making $0 \le T_B \le 4.00$ kN in Eq. (2), we have

$$13.00 \text{ kN} - 3.00(4.00 \text{ kN}) \le Q \le 13.00 \text{ kN} - 3.00(0)$$

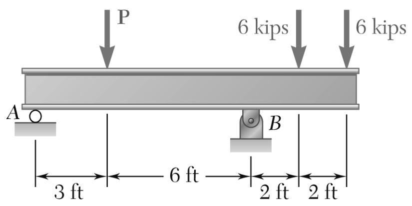
$$1.000 \text{ kN} \le Q \le 13.00 \text{ kN} \qquad (3)$$

Making $0 \le T_D \le 4.00$ kN in Eq. (1), we have

$$0.250 \text{ kN} + 0.750(0) \le Q \le 0.250 \text{ kN} + 0.750(4.00 \text{ kN})$$

$$0.250 \text{ kN} \le Q \le 3.25 \text{ kN} \qquad (4)$$

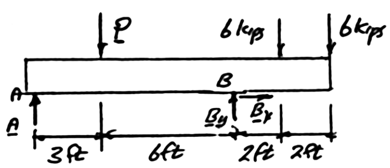
Combining Eqs. (3) and (4), $1.000 \text{ kN} \le Q \le 3.25 \text{ kN}$ ◀



PROBLEM 4.14

For the beam of Sample Problem 4.2, determine the range of values of P for which the beam will be safe, knowing that the maximum allowable value of each of the reactions is 30 kips and that the reaction at A must be directed upward.

SOLUTION



$$\Sigma F_x = 0: \quad B_x = 0$$

$$\mathbf{B} = B_y \uparrow$$

$$+\circlearrowleft \Sigma M_A = 0: \quad -P(3\text{ ft}) + B(9\text{ ft}) - (6\text{ kips})(11\text{ ft}) - (6\text{ kips})(13\text{ ft}) = 0$$

$$P = 3B - 48\text{ kips} \qquad (1)$$

$$+\circlearrowleft \Sigma M_B = 0: \quad -A(9\text{ ft}) + P(6\text{ ft}) - (6\text{ kips})(2\text{ ft}) - (6\text{ kips})(4\text{ ft}) = 0$$

$$P = 1.5A + 6\text{ kips} \qquad (2)$$

Since $B \le 30$ kips, Eq. (1) yields

$$P \le (3)(30\text{ kips}) - 48\text{ kips} \qquad P \le 42.0\text{ kips} \quad \triangleleft$$

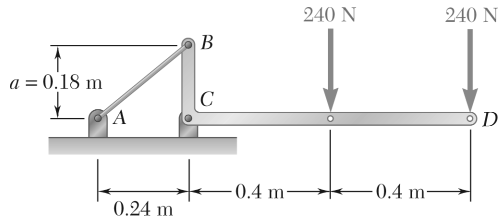
Since $0 \le A \le 30$ kips, Eq. (2) yields

$$0 + 6\text{ kips} \le P \le (1.5)(30\text{ kips})1.6\text{ kips}$$

$$6.00\text{ kips} \le P \le 51.0\text{ kips} \quad \triangleleft$$

Range of values of P for which beam will be safe:

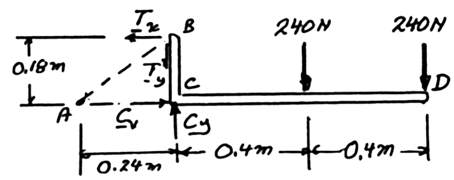
$$6.00\text{ kips} \le P \le 42.0\text{ kips} \quad \blacktriangleleft$$



PROBLEM 4.15

The bracket BCD is hinged at C and attached to a control cable at B. For the loading shown, determine (*a*) the tension in the cable, (*b*) the reaction at C.

SOLUTION



At B:

$$\frac{T_y}{T_x} = \frac{0.18 \text{ m}}{0.24 \text{ m}}$$

$$T_y = \frac{3}{4} T_x \qquad (1)$$

(*a*)

$$+\circlearrowleft \Sigma M_C = 0: \quad T_x(0.18 \text{ m}) - (240 \text{ N})(0.4 \text{ m}) - (240 \text{ N})(0.8 \text{ m}) = 0$$

$$T_x = +1600 \text{ N}$$

From Eq. (1):

$$T_y = \frac{3}{4}(1600 \text{ N}) = 1200 \text{ N}$$

$$T = \sqrt{T_x^2 + T_y^2} = \sqrt{1600^2 + 1200^2} = 2000 \text{ N} \qquad T = 2.00 \text{ kN} \blacktriangleleft$$

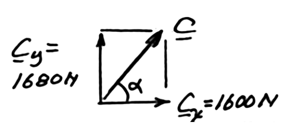
(*b*)

$$\xrightarrow{+} \Sigma F_x = 0: \quad C_x - T_x = 0$$

$$C_x - 1600 \text{ N} = 0 \quad C_x = +1600 \text{ N} \qquad \mathbf{C}_x = 1600 \text{ N} \longrightarrow$$

$$+\uparrow \Sigma F_y = 0: \quad C_y - T_y - 240 \text{ N} - 240 \text{ N} = 0$$

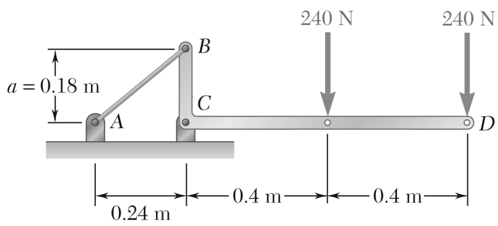
$$C_y - 1200 \text{ N} - 480 \text{ N} = 0$$



$$C_y = +1680 \text{ N} \qquad \mathbf{C}_y = 1680 \text{ N} \uparrow$$

$$\alpha = 46.4°$$

$$C = 2320 \text{ N} \qquad \mathbf{C} = 2.32 \text{ kN} \measuredangle 46.4° \blacktriangleleft$$

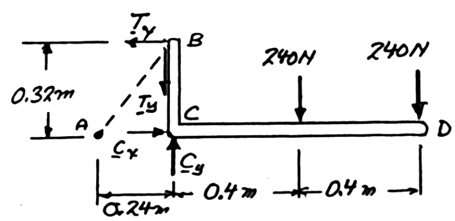


PROBLEM 4.16

Solve Problem 4.15, assuming that $a = 0.32$ m.

PROBLEM 4.15 The bracket *BCD* is hinged at *C* and attached to a control cable at *B*. For the loading shown, determine (*a*) the tension in the cable, (*b*) the reaction at *C*.

SOLUTION



At *B*:

$$\frac{T_y}{T_x} = \frac{0.32 \text{ m}}{0.24 \text{ m}}$$

$$T_y = \frac{4}{3} T_x$$

$$+\circlearrowleft \Sigma M_C = 0: \quad T_x(0.32 \text{ m}) - (240 \text{ N})(0.4 \text{ m}) - (240 \text{ N})(0.8 \text{ m}) = 0$$

$$T_x = 900 \text{ N}$$

From Eq. (1):

$$T_y = \frac{4}{3}(900 \text{ N}) = 1200 \text{ N}$$

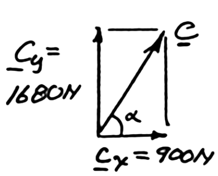
$$T = \sqrt{T_x^2 + T_y^2} = \sqrt{900^2 + 1200^2} = 1500 \text{ N} \qquad T = 1.500 \text{ kN} \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad C_x - T_x = 0$$

$$C_x - 900 \text{ N} = 0 \quad C_x = +900 \text{ N} \qquad \mathbf{C}_x = 900 \text{ N} \longrightarrow$$

$$+\uparrow \Sigma F_y = 0: \quad C_y - T_y - 240 \text{ N} - 240 \text{ N} = 0$$

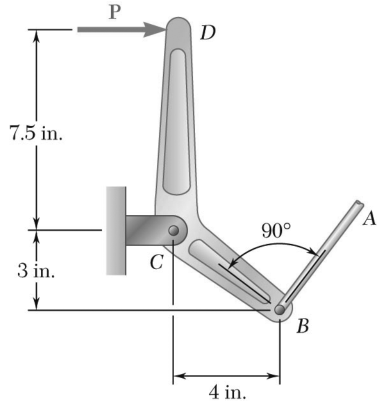
$$C_y - 1200 \text{ N} - 480 \text{ N} = 0$$



$$C_y = +1680 \text{ N} \qquad \mathbf{C}_y = 1680 \text{ N} \uparrow$$

$$\alpha = 61.8°$$

$$C = 1906 \text{ N} \qquad \mathbf{C} = 1.906 \text{ kN} \measuredangle 61.8° \blacktriangleleft$$

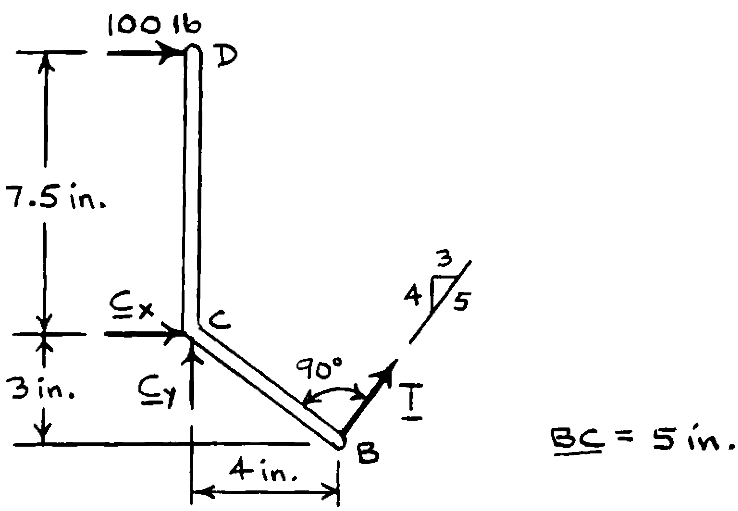


PROBLEM 4.17

The lever *BCD* is hinged at *C* and attached to a control rod at *B*. If $P = 100$ lb, determine (*a*) the tension in rod *AB*, (*b*) the reaction at *C*.

SOLUTION

Free-Body Diagram:

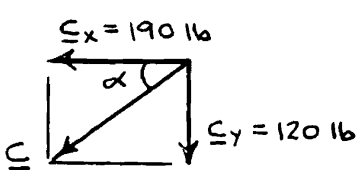


(*a*) $+\circlearrowleft \Sigma M_C = 0$: $T(5 \text{ in.}) - (100 \text{ lb})(7.5 \text{ in.}) = 0$

$T = 150.0$ lb ◀

(*b*) $\xrightarrow{+} \Sigma F_x = 0$: $C_x + 100 \text{ lb} + \frac{3}{5}(150.0 \text{ lb}) = 0$

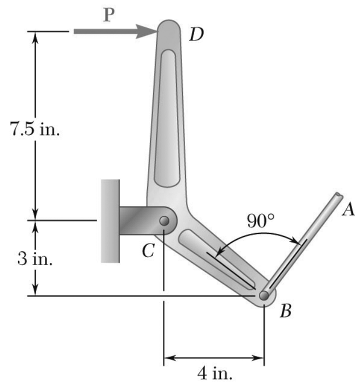
$C_x = -190$ lb $\qquad \mathbf{C}_x = 190 \text{ lb} \longleftarrow$



$+\uparrow \Sigma F_y = 0$: $C_y + \frac{4}{5}(150.0 \text{ lb}) = 0$

$C_y = -120$ lb $\qquad \mathbf{C}_y = 120 \text{ lb} \downarrow$

$\alpha = 32.3°$ $\qquad C = 225$ lb $\qquad \mathbf{C} = 225 \text{ lb} \measuredangle 32.3°$ ◀

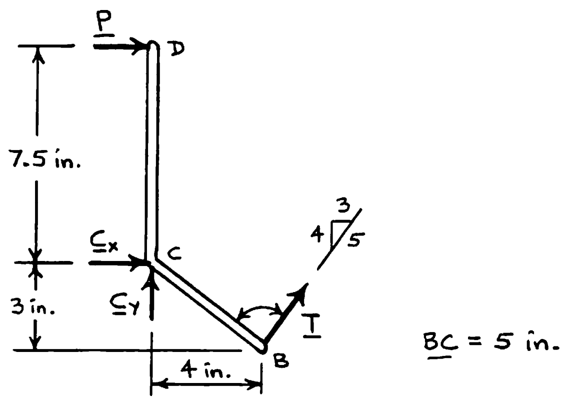


PROBLEM 4.18

The lever *BCD* is hinged at *C* and attached to a control rod at *B*. Determine the maximum force **P** that can be safely applied at *D* if the maximum allowable value of the reaction at *C* is 250 lb.

SOLUTION

Free-Body Diagram:

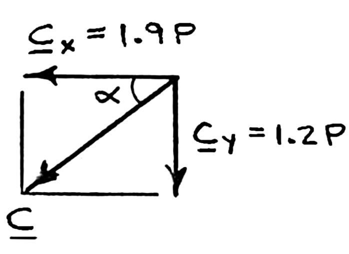


$$+\circlearrowleft \Sigma M_C = 0: \quad T(5\text{ in.}) - P(7.5\text{ in.}) = 0$$

$$T = 1.5P$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad P + C_x + \frac{3}{5}(1.5P) = 0$$

$$C_x = -1.9P \qquad \mathbf{C}_x = 1.9P \longleftarrow$$



$$+\uparrow \Sigma F_y = 0: \quad C_y + \frac{4}{5}(1.5P) = 0$$

$$C_y = -1.2P \qquad \mathbf{C}_y = 1.2P \downarrow$$

$$C = \sqrt{C_x^2 + C_y^2}$$

$$= \sqrt{(1.9P)^2 + (1.2P)^2}$$

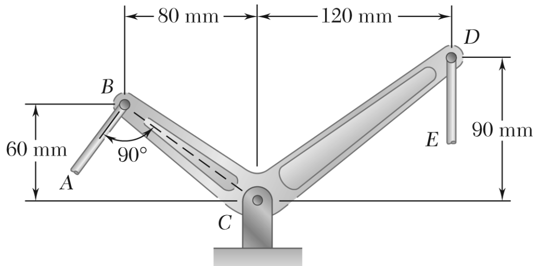
$$C = 2.2472P$$

For $C = 250$ lb,

$$250\text{ lb} = 2.2472P$$

$$P = 111.2\text{ lb}$$

$\mathbf{P} = 111.2\text{ lb} \longrightarrow$ ◀

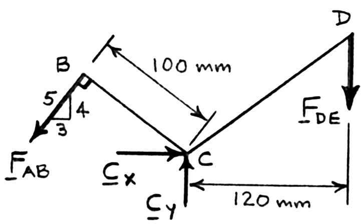


PROBLEM 4.19

Two links *AB* and *DE* are connected by a bell crank as shown. Knowing that the tension in link *AB* is 720 N, determine (*a*) the tension in link *DE*, (*b*) the reaction at *C*.

SOLUTION

Free-Body Diagram:



$$+\circlearrowleft \Sigma M_C = 0: \quad F_{AB}(100 \text{ mm}) - F_{DE}(120 \text{ mm}) = 0$$

$$F_{DE} = \frac{5}{6} F_{AB} \qquad (1)$$

(*a*) For

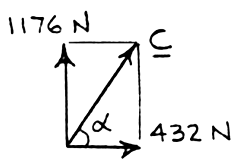
$$F_{AB} = 720 \text{ N}$$

$$F_{DE} = \frac{5}{6}(720 \text{ N}) \qquad F_{DE} = 600 \text{ N} \blacktriangleleft$$

(*b*)

$$\xrightarrow{+} \Sigma F_x = 0: \quad -\frac{3}{5}(720 \text{ N}) + C_x = 0$$

$$C_x = +432 \text{ N}$$



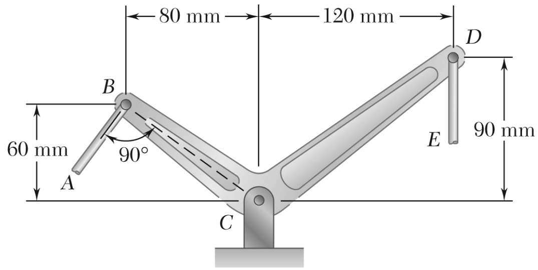
$$+\uparrow \Sigma F_y = 0: \quad -\frac{4}{5}(720 \text{ N}) + C_y - 600 \text{ N} = 0$$

$$C_y = +1176 \text{ N}$$

$$C = 1252.84 \text{ N}$$

$$\alpha = 69.829°$$

$$\mathbf{C} = 1253 \text{ N} \measuredangle 69.8° \blacktriangleleft$$



PROBLEM 4.20

Two links *AB* and *DE* are connected by a bell crank as shown. Determine the maximum force that may be safely exerted by link *AB* on the bell crank if the maximum allowable value for the reaction at *C* is 1600 N.

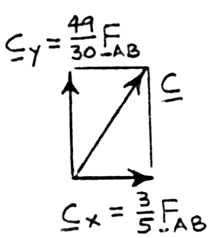
SOLUTION

See solution to Problem 4.15 for F.B.D. and derivation of Eq. (1).

$$F_{DE} = \frac{5}{6}F_{AB} \qquad (1)$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad -\frac{3}{5}F_{AB} + C_x = 0 \qquad C_x = \frac{3}{5}F_{AB}$$

$$+\uparrow \Sigma F_y = 0: \quad -\frac{4}{5}F_{AB} + C_y - F_{DE} = 0$$



$$-\frac{4}{5}F_{AB} + C_y - \frac{5}{6}F_{AB} = 0$$

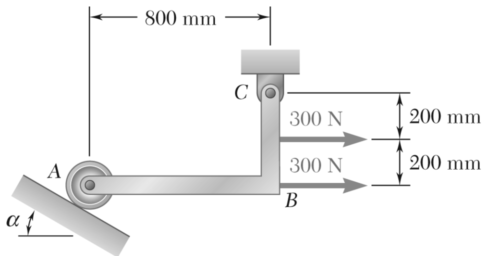
$$C_y = \frac{49}{30}F_{AB}$$

$$C = \sqrt{C_x^2 + C_y^2}$$

$$= \frac{1}{30}\sqrt{(49)^2 + (18)^2}\,F_{AB}$$

$$C = 1.74005F_{AB}$$

For $C = 1600$ N, $\quad 1600 \text{ N} = 1.74005F_{AB}$ $\qquad F_{AB} = 920 \text{ N}$ ◀



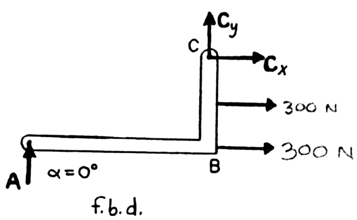
PROBLEM 4.21

Determine the reactions at A and C when (a) $\alpha = 0$, (b) $\alpha = 30°$.

SOLUTION

(a) $\alpha = 0$

From F.B.D. of member ABC:



$$+\circlearrowleft \Sigma M_C = 0: \quad (300 \text{ N})(0.2 \text{ m}) + (300 \text{ N})(0.4 \text{ m}) - A(0.8 \text{ m}) = 0$$

$$A = 225 \text{ N} \qquad \text{or} \qquad \mathbf{A} = 225 \text{ N}\uparrow \blacktriangleleft$$

$$+\uparrow \Sigma F_y = 0: \quad C_y + 225 \text{ N} = 0$$

$$C_y = -225 \text{ N} \quad \text{or} \quad \mathbf{C}_y = 225 \text{ N}\downarrow$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad 300 \text{ N} + 300 \text{ N} + C_x = 0$$

$$C_x = -600 \text{ N} \quad \text{or} \quad \mathbf{C}_x = 600 \text{ N} \longleftarrow$$

Then $$C = \sqrt{C_x^2 + C_y^2} = \sqrt{(600)^2 + (225)^2} = 640.80 \text{ N}$$

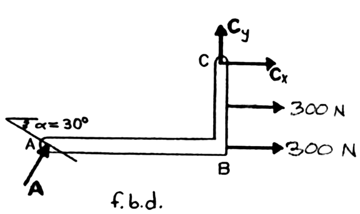
and $$\theta = \tan^{-1}\left(\frac{C_y}{C_x}\right) = \tan^{-1}\left(\frac{-225}{-600}\right) = 20.556°$$

$$\text{or} \quad \mathbf{C} = 641 \text{ N} \measuredangle 20.6° \blacktriangleleft$$

(b) $\alpha = 30°$

From F.B.D. of member ABC:

$$+\circlearrowleft \Sigma M_C = 0: \quad (300 \text{ N})(0.2 \text{ m}) + (300 \text{ N})(0.4 \text{ m}) - (A\cos 30°)(0.8 \text{ m})$$
$$+ (A\sin 30°)(20 \text{ in.}) = 0$$



$$A = 365.24 \text{ N} \qquad \text{or} \qquad \mathbf{A} = 365 \text{ N} \measuredangle 60.0° \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad 300 \text{ N} + 300 \text{ N} + (365.24 \text{ N})\sin 30° + C_x = 0$$

$$C_x = -782.62$$

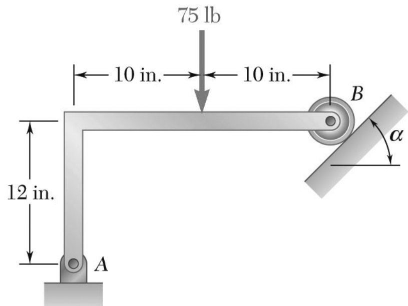
$$+\uparrow \Sigma F_y = 0: \quad C_y + (365.24 \text{ N})\cos 30° = 0$$

$$C_y = -316.31 \text{ N} \quad \text{or} \quad \mathbf{C}_y = 316 \text{ N} \downarrow$$

Then $$C = \sqrt{C_x^2 + C_y^2} = \sqrt{(782.62)^2 + (316.31)^2} = 884.12 \text{ N}$$

and $$\theta = \tan^{-1}\left(\frac{C_y}{C_x}\right) = \tan^{-1}\left(\frac{-316.31}{-782.62}\right) = 22.007°$$

or $$\mathbf{C} = 884 \text{ N} \measuredangle\, 22.0° \blacktriangleleft$$

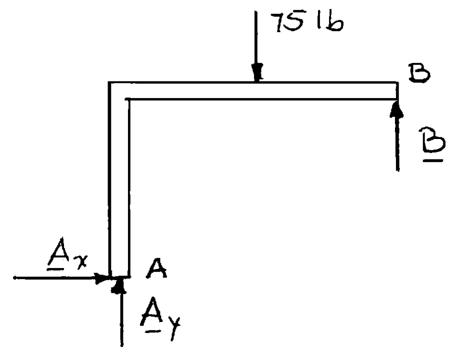


PROBLEM 4.22

Determine the reactions at A and B when (a) $\alpha = 0$, (b) $\alpha = 90°$, (c) $\alpha = 30°$.

SOLUTION

(a) $\alpha = 0$



$$+\circlearrowleft \Sigma M_A = 0: \quad B(20\text{ in.}) - 75\text{ lb}(10\text{ in.}) = 0$$
$$B = 37.5\text{ lb}$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x = 0$$

$$+\uparrow \Sigma F_y = 0: \quad A_y - 75\text{ lb} + 37.5\text{ lb} = 0$$
$$A_y = 37.5\text{ lb}$$

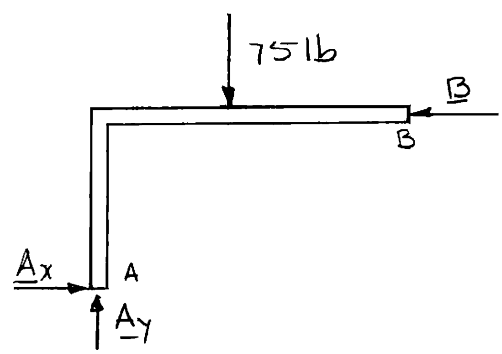
$\mathbf{A} = \mathbf{B} = 37.5\text{ lb}\uparrow$ ◀

(b) $\alpha = 90°$

$$+\circlearrowleft \Sigma M_A = 0: \quad B(12\text{ in.}) - 75\text{ lb}(10\text{ in.}) = 0$$
$$B = 62.5\text{ lb}$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x - B = 0$$
$$A_x = 62.5\text{ lb}$$

$$+\uparrow \Sigma F_y = 0: \quad A_y - 75\text{ lb} = 0$$
$$A_y = 75\text{ lb}$$

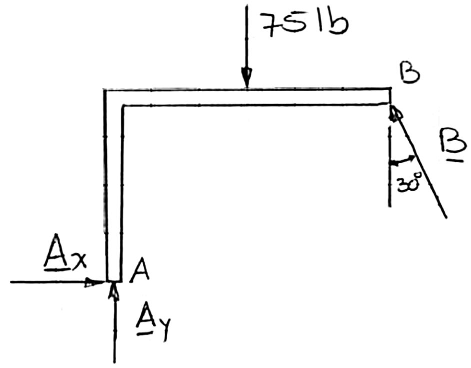


$$A = \sqrt{A_x^2 + A_y^2}$$
$$= \sqrt{(62.5\text{ lb})^2 + (75\text{ lb})^2}$$
$$= 97.6\text{ lb}$$

$$\tan\theta = \frac{75}{62.5}$$
$$\theta = 50.2°$$

$\mathbf{A} = 97.6\text{ lb} \measuredangle 50.2°; \ \mathbf{B} = 62.51\text{ lb} \longleftarrow$ ◀

(*c*) $\alpha = 30°$



$$+\circlearrowleft \Sigma M_A = 0: \; (B\cos 30°)(20 \text{ in.}) + (B\sin 30°)(12 \text{ in.}) - (75 \text{ lb})(10 \text{ in.}) = 0$$

$$B = 32.161 \text{ lb}$$

$$\xrightarrow{+} \Sigma F_x = 0: \; A_x - (32.161)\sin 30° = 0$$

$$A_x = 16.0805 \text{ lb}$$

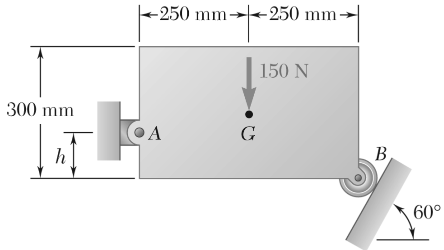
$$+\uparrow \Sigma F_y = 0: \; A_y + (32.161)\cos 30° - 75 = 0$$

$$A_y = 47.148 \text{ lb}$$

$$A = \sqrt{A_x^2 + A_y^2}$$
$$= \sqrt{(16.0805)^2 + (47.148)^2}$$
$$= 49.8 \text{ lb}$$

$$\tan\theta = \frac{47.148}{16.0805}$$
$$\theta = 71.2°$$

$\mathbf{A} = 49.8$ lb $\measuredangle$ $71.2°$; $\mathbf{B} = 32.2$ lb $\measuredangle$ $60.0°$ ◀

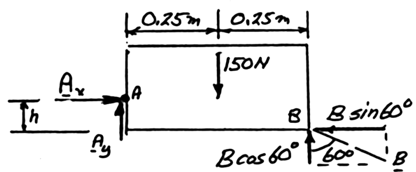


PROBLEM 4.23

Determine the reactions at A and B when (*a*) $h = 0$, (*b*) $h = 200$ mm.

SOLUTION

Free-Body Diagram:



$$+\circlearrowleft \Sigma M_A = 0: \quad (B\cos 60°)(0.5\text{ m}) - (B\sin 60°)h - (150\text{ N})(0.25\text{ m}) = 0$$

$$B = \frac{37.5}{0.25 - 0.866h} \qquad (1)$$

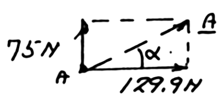
(*a*) When $h = 0$,

From Eq. (1): $B = \dfrac{37.5}{0.25} = 150$ N $\qquad$ $\mathbf{B} = 150.0$ N $\searrow$ 30.0° ◀

$\xrightarrow{+} \Sigma F_y = 0: \quad A_x - B\sin 60° = 0$

$A_x = (150)\sin 60° = 129.9$ N $\qquad$ $\mathbf{A}_x = 129.9$ N $\longrightarrow$

$+\uparrow \Sigma F_y = 0: \quad A_y - 150 + B\cos 60° = 0$



$A_y = 150 - (150)\cos 60° = 75$ N $\qquad$ $\mathbf{A}_y = 75$ N $\uparrow$

$\alpha = 30°$

$A = 150.0$ N $\qquad$ $\mathbf{A} = 150.0$ N $\nearrow$ 30.0° ◀

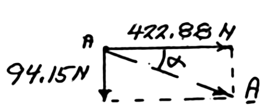
(*b*) When $h = 200$ mm $= 0.2$ m,

From Eq. (1): $B = \dfrac{37.5}{0.25 - 0.866(0.2)} = 488.3$ N $\qquad$ $\mathbf{B} = 488$ N $\searrow$ 30.0° ◀

$\xrightarrow{+} \Sigma F_x = 0: \quad A_x - B\sin 60° = 0$

$A_x = (488.3)\sin 60° = 422.88$ N $\qquad$ $\mathbf{A}_x = 422.88$ N $\longrightarrow$

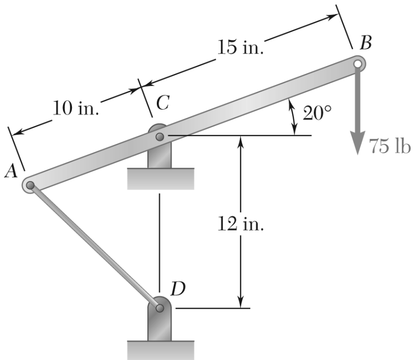
$+\uparrow \Sigma F_y = 0: \quad A_y - 150 + B\cos 60° = 0$



$A_y = 150 - (488.3)\cos 60° = -94.15$ N $\qquad$ $\mathbf{A}_y = 94.15$ N $\downarrow$

$\alpha = 12.55°$

$A = 433.2$ N $\qquad$ $\mathbf{A} = 433$ N $\searrow$ 12.55° ◀

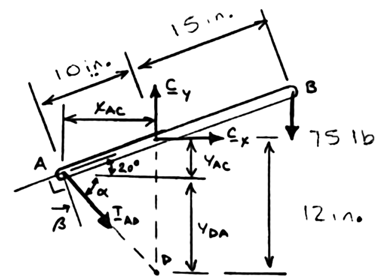


PROBLEM 4.24

A lever AB is hinged at C and attached to a control cable at A. If the lever is subjected to a 75-lb vertical force at B, determine (a) the tension in the cable, (b) the reaction at C.

SOLUTION

Free-Body Diagram:



Geometry:

$$x_{AC} = (10\text{ in.})\cos 20° = 9.3969\text{ in.}$$
$$y_{AC} = (10\text{ in.})\sin 20° = 3.4202\text{ in.}$$
$$\Rightarrow y_{DA} = 12\text{ in.} - 3.4202\text{ in.} = 8.5798\text{ in.}$$
$$\alpha = \tan^{-1}\left(\frac{y_{DA}}{x_{AC}}\right) = \tan^{-1}\left(\frac{8.5798}{9.3969}\right) = 42.397°$$
$$\beta = 90° - 20° - 42.397° = 27.603°$$

Equilibrium for lever:

(a) $\quad +\circlearrowleft \Sigma M_C = 0$: $\quad T_{AD}\cos 27.603°(10\text{ in.}) - (75\text{ lb})[(15\text{ in.})\cos 20°] = 0$

$$T_{AD} = 119.293\text{ lb}$$

$T_{AD} = 119.3\text{ lb}$ ◀

(b) $\quad \xrightarrow{+} \Sigma F_x = 0$: $\quad C_x + (119.293\text{ lb})\cos 42.397° = 0$

$$C_x = -88.097\text{ lb}$$

$+\uparrow \Sigma F_y = 0$: $\quad C_y - 75\text{ lb} - (119.293\text{ lb})\sin 42.397° = 0$

$$C_y = 155.435$$

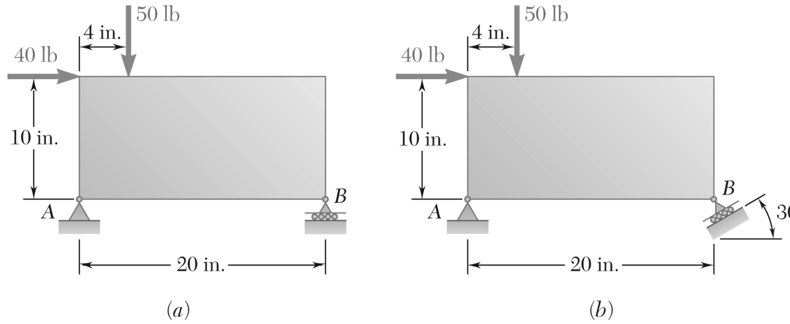
Thus, $\quad C = \sqrt{C_x^2 + C_y^2} = \sqrt{(-88.097)^2 + (155.435)^2} = 178.665\text{ lb}$

and $\quad \theta = \tan^{-1}\frac{C_y}{C_x} = \tan^{-1}\frac{155.435}{88.097} = 60.456°$

$\mathbf{C} = 178.7\text{ lb} \measuredangle\ 60.5°$ ◀

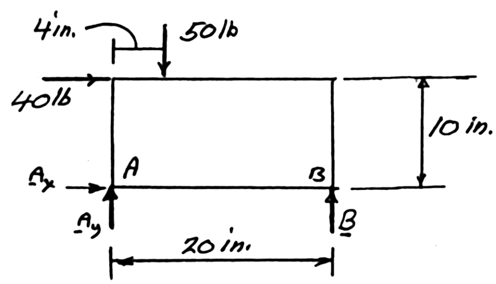
PROBLEM 4.25

For each of the plates and loadings shown, determine the reactions at A and B.



SOLUTION

(*a*) **Free-Body Diagram:**



$$+\circlearrowleft \Sigma M_A = 0: \quad B(20 \text{ in.}) - (50 \text{ lb})(4 \text{ in.}) - (40 \text{ lb})(10 \text{ in.}) = 0$$

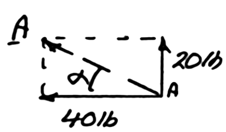
$B = +30$ lb $\qquad$ $\mathbf{B} = 30.0 \text{ lb} \uparrow$ ◀

$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x + 40 \text{ lb} = 0$$

$A_x = -40$ lb $\qquad$ $\mathbf{A}_x = 40.0 \text{ lb} \leftarrow$

$$+\uparrow \Sigma F_y = 0: \quad A_y + B - 50 \text{ lb} = 0$$

$$A_y + 30 \text{ lb} - 50 \text{ lb} = 0$$

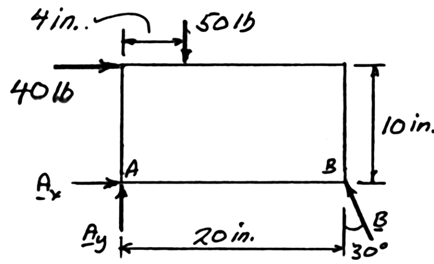


$A_y = +20$ lb $\qquad$ $\mathbf{A}_y = 20.0 \text{ lb} \uparrow$

$\alpha = 26.56°$

$A = 44.72$ lb $\qquad$ $\mathbf{A} = 44.7 \text{ lb} \searrow 26.6°$ ◀

(*b*) **Free-Body Diagram:**



$+\circlearrowleft \Sigma M_A = 0$: $(B\cos 30°)(20 \text{ in.}) - (40 \text{ lb})(10 \text{ in.}) - (50 \text{ lb})(4 \text{ in.}) = 0$

$B = 34.64 \text{ lb}$ $\qquad$ $\mathbf{B} = 34.6 \text{ lb} \measuredangle 60.0°$ ◀

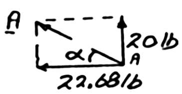
$\xrightarrow{+} \Sigma F_x = 0$: $A_x - B\sin 30° + 40 \text{ lb}$

$A_x - (34.64 \text{ lb})\sin 30° + 40 \text{ lb} = 0$

$A_x = -22.68 \text{ lb}$ $\qquad$ $\mathbf{A}_x = 22.68 \text{ lb} \longleftarrow$

$+\uparrow \Sigma F_y = 0$: $A_y + B\cos 30° - 50 \text{ lb} = 0$

$A_y + (34.64 \text{ lb})\cos 30° - 50 \text{ lb} = 0$



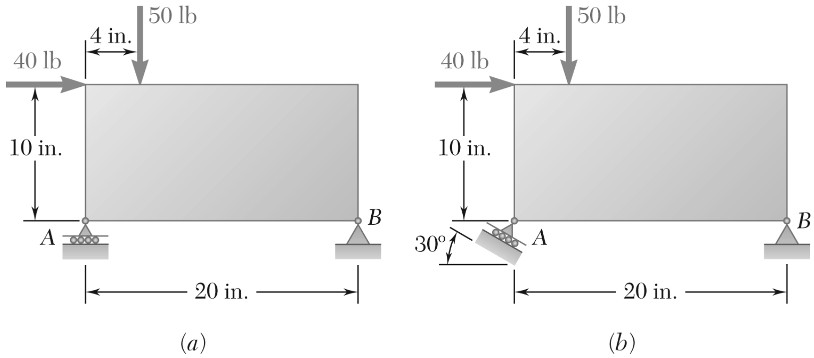
$A_y = +20 \text{ lb}$ $\qquad$ $\mathbf{A}_y = 20.0 \text{ lb} \uparrow$

$\alpha = 41.4°$

$A = 30.24 \text{ lb}$ $\qquad$ $\mathbf{A} = 30.2 \text{ lb} \measuredangle 41.4°$ ◀

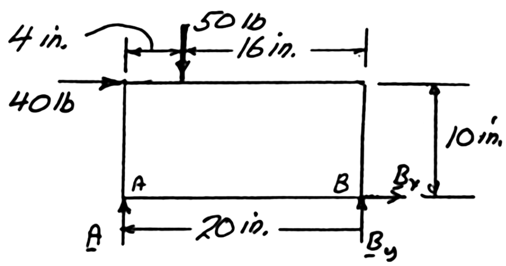
PROBLEM 4.26

For each of the plates and loadings shown, determine the reactions at A and B.



SOLUTION

(*a*) **Free-Body Diagram:**



$$+\circlearrowleft \Sigma M_B = 0: \quad A(20 \text{ in.}) + (50 \text{ lb})(16 \text{ in.}) - (40 \text{ lb})(10 \text{ in.}) = 0$$

$$A = +20 \text{ lb}$$

$\mathbf{A} = 20.0 \text{ lb} \uparrow$ ◀

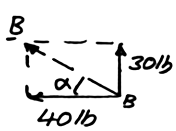
$$\xrightarrow{+} \Sigma F_x = 0: \quad 40 \text{ lb} + B_x = 0$$

$$B_x = -40 \text{ lb}$$

$\mathbf{B}_x = 40 \text{ lb} \longleftarrow$

$$+\uparrow \Sigma F_y = 0: \quad A + B_y - 50 \text{ lb} = 0$$

$$20 \text{ lb} + B_y - 50 \text{ lb} = 0$$



$$B_y = +30 \text{ lb}$$

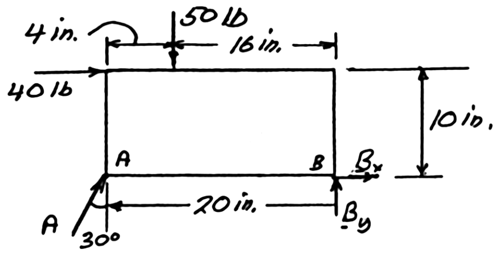
$\mathbf{B}_y = 30 \text{ lb} \uparrow$

$\alpha = 36.87°$ $\quad B = 50 \text{ lb}$

$\mathbf{B} = 50.0 \text{ lb} \searrow 36.9°$ ◀

PROBLEM 4.26 (Continued)

(*b*)



$+\circlearrowleft \Sigma M_A = 0: \quad -(A\cos 30°)(20 \text{ in.}) - (40 \text{ lb})(10 \text{ in.}) + (50 \text{ lb})(16 \text{ in.}) = 0$

$A = 23.09 \text{ lb}$ $\quad\quad$ $\mathbf{A} = 23.1 \text{ lb} \angle 60.0°$ ◀

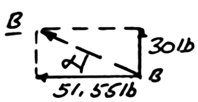
$\xrightarrow{+} \Sigma F_x = 0: \quad A\sin 30° + 40 \text{ lb} + B_x = 0$

$(23.09 \text{ lb})\sin 30° + 40 \text{ lb} + 8_x = 0$

$B_x = -51.55 \text{ lb}$ $\quad\quad$ $\mathbf{B}_x = 51.55 \text{ lb} \longleftarrow$

$+\uparrow \Sigma F_y = 0: \quad A\cos 30° + B_y - 50 \text{ lb} = 0$

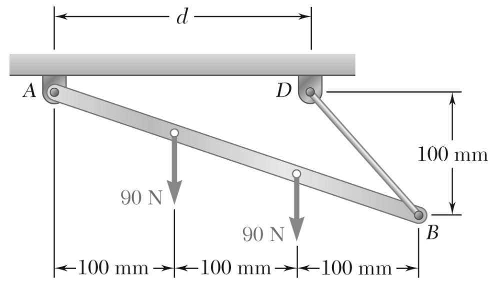
$(23.09 \text{ lb})\cos 30° + B_y - 50 \text{ lb} = 0$



$B_y = +30 \text{ lb}$ $\quad\quad$ $\mathbf{B}_y = 30 \text{ lb} \uparrow$

$\alpha = 30.2°$

$B = 59.64 \text{ lb}$ $\quad\quad$ $\mathbf{B} = 59.6 \text{ lb} \searrow 30.2°$ ◀

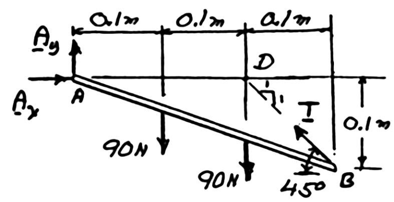


PROBLEM 4.27

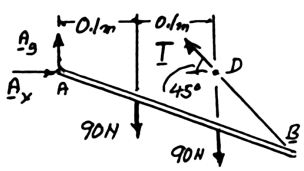
A rod AB hinged at A and attached at B to cable BD supports the loads shown. Knowing that $d = 200$ mm, determine (*a*) the tension in cable BD, (*b*) the reaction at A.

SOLUTION

Free-Body Diagram:



(*a*) Move **T** along BD until it acts at Point D.



$$+\circlearrowleft \Sigma M_A = 0: \quad (T \sin 45°)(0.2 \text{ m}) + (90 \text{ N})(0.1 \text{ m}) + (90 \text{ N})(0.2 \text{ m}) = 0$$

$$T = 190.919 \text{ N} \qquad T = 190.9 \text{ N} ◀$$

(*b*)

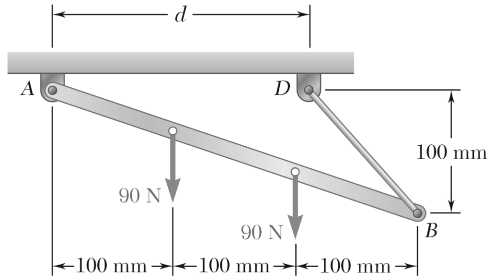
$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x - (190.919 \text{ N}) \cos 45° = 0$$

$$A_x = +135.0 \text{ N} \qquad \mathbf{A}_x = 135.0 \text{ N} \longrightarrow$$

$$+\uparrow \Sigma F_y = 0: \quad A_y - 90 \text{ N} - 90 \text{ N} + (190.919 \text{ N}) \sin 45° = 0$$

$$A_y = +45.0 \text{ N} \qquad \mathbf{A}_y = 45.0 \text{ N} \uparrow$$

$$\mathbf{A} = 142.3 \text{ N} \measuredangle 18.43° ◀$$

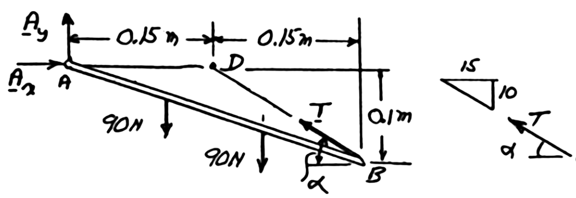


PROBLEM 4.28

A rod AB, hinged at A and attached at B to cable BD, supports the loads shown. Knowing that $d = 150$ mm, determine (*a*) the tension in cable BD, (*b*) the reaction at A.

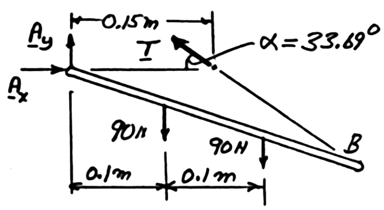
SOLUTION

Free-Body Diagram:



$$\tan\alpha = \frac{10}{15}; \quad \alpha = 33.690°$$

(*a*) Move **T** along BD until it acts at Point D.



$$+\circlearrowleft \Sigma M_A = 0: \quad (T \sin 33.690°)(0.15 \text{ m}) - (90 \text{ N})(0.1 \text{ m}) - (90 \text{ N})(0.2 \text{ m}) = 0$$

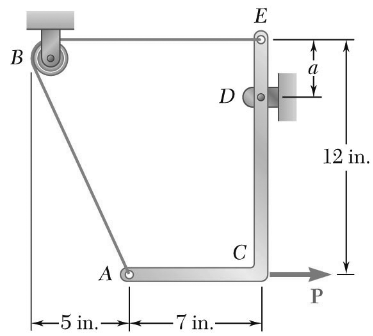
$T = 324.50$ N $\qquad$ $T = 324$ N ◀

(*b*) $\xrightarrow{+} \Sigma F_x = 0: \quad A_x - (324.50 \text{ N})\cos 33.690° = 0$

$A_x = +270$ N $\qquad$ $\mathbf{A}_x = 270 \text{ N} \longrightarrow$

$+\uparrow \Sigma F_y = 0: \quad A_y - 90 \text{ N} - 90 \text{ N} + (324.50 \text{ N})\sin 33.690° = 0$

$A_y = 0$ $\qquad$ $\mathbf{A} = 270 \text{ N} \longrightarrow$ ◀

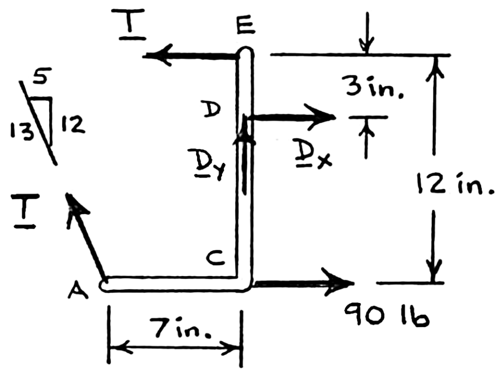


PROBLEM 4.29

A force **P** of magnitude 90 lb is applied to member *ACDE*, which is supported by a frictionless pin at *D* and by the cable *ABE*. Since the cable passes over a small pulley at *B*, the tension may be assumed to be the same in portions *AB* and *BE* of the cable. For the case when $a = 3$ in., determine (*a*) the tension in the cable, (*b*) the reaction at *D*.

SOLUTION

Free-Body Diagram:



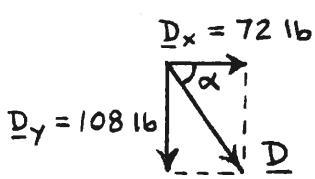
(*a*) $+\circlearrowright \Sigma M_D = 0: \quad (90 \text{ lb})(9 \text{ in.}) - \frac{5}{13}T(9 \text{ in.}) - \frac{12}{13}T(7 \text{ in.}) + T(3 \text{ in.}) = 0$

$T = 117 \text{ lb}$

$T = 117.0 \text{ lb}$ ◀

(*b*) $\xrightarrow{+} \Sigma F_x = 0: \quad D_x - 117 \text{ lb} - \frac{5}{13}(117 \text{ lb}) + 90 = 0$

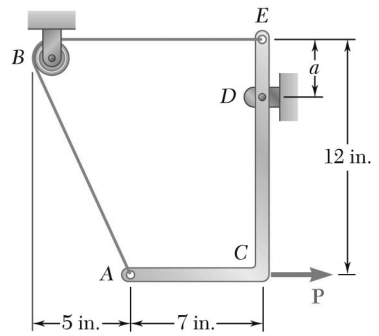
$D_x = +72 \text{ lb}$



$+\uparrow \Sigma F_y = 0: \quad D_y + \frac{12}{13}(117 \text{ lb}) = 0$

$D_y = -108 \text{ lb}$

$\mathbf{D} = 129.8 \text{ lb} \measuredangle 56.3°$ ◀



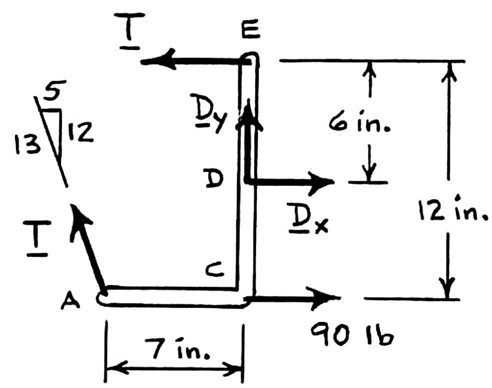
PROBLEM 4.30

Solve Problem 4.29 for $a = 6$ in.

PROBLEM 4.29 A force **P** of magnitude 90 lb is applied to member *ACDE*, which is supported by a frictionless pin at *D* and by the cable *ABE*. Since the cable passes over a small pulley at *B*, the tension may be assumed to be the same in portions *AB* and *BE* of the cable. For the case when $a = 3$ in., determine (*a*) the tension in the cable, (*b*) the reaction at *D*.

SOLUTION

Free-Body Diagram:



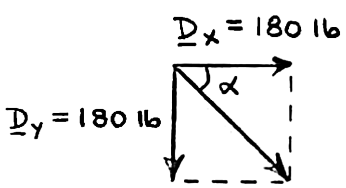
(*a*) $$+\circlearrowleft \Sigma M_D = 0: \quad (90 \text{ lb})(6 \text{ in.}) - \frac{5}{13}T(6 \text{ in.}) - \frac{12}{13}T(7 \text{ in.}) + T(6 \text{ in.}) = 0$$

$$T = 195 \text{ lb}$$

$T = 195.0$ lb ◀

(*b*) $$\xrightarrow{+} \Sigma F_x = 0: \quad D_x - 195 \text{ lb} - \frac{5}{13}(195 \text{ lb}) + 90 = 0$$

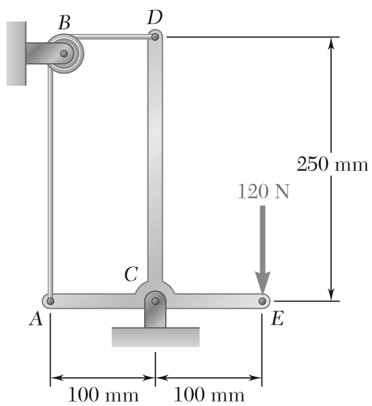
$$D_x = +180 \text{ lb}$$



$$+\!\uparrow \Sigma F_y = 0: \quad D_y + \frac{12}{13}(195 \text{ lb}) = 0$$

$$D_y = -180 \text{ lb}$$

$\mathbf{D} = 255 \text{ lb} \searrow 45.0°$ ◀

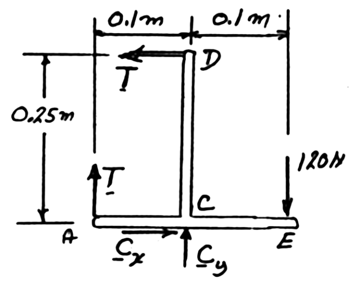


PROBLEM 4.31

Neglecting friction, determine the tension in cable *ABD* and the reaction at support *C*.

SOLUTION

Free-Body Diagram:

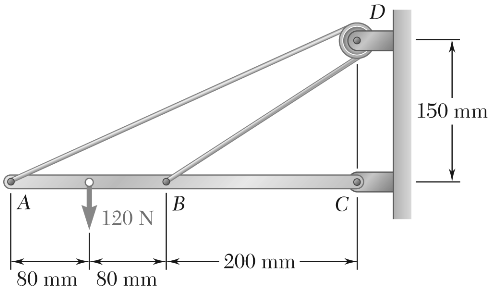


$$+\circlearrowleft \Sigma M_C = 0: \quad T(0.25 \text{ m}) - T(0.1 \text{ m}) - (120 \text{ N})(0.1 \text{ m}) = 0 \qquad T = 80.0 \text{ N} \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad C_x - 80 \text{ N} = 0 \qquad C_x = +80 \text{ N} \qquad \mathbf{C}_x = 80.0 \text{ N} \longrightarrow$$

$$+\uparrow \Sigma F_y = 0: \quad C_y - 120 \text{ N} + 80 \text{ N} = 0 \qquad C_y = +40 \text{ N} \qquad \mathbf{C}_y = 40.0 \text{ N} \uparrow$$

$$\mathbf{C} = 89.4 \text{ N} \measuredangle 26.6° \blacktriangleleft$$



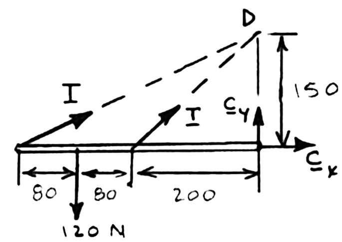
PROBLEM 4.32

Neglecting friction and the radius of the pulley, determine (*a*) the tension in cable *ADB*, (*b*) the reaction at *C*.

SOLUTION

Free-Body Diagram:

Dimensions in mm



Geometry:

Distance: $AD = \sqrt{(0.36)^2 + (0.150)^2} = 0.39 \text{ m}$

Distance: $BD = \sqrt{(0.2)^2 + (0.15)^2} = 0.25 \text{ m}$

Equilibrium for beam:

(*a*) $+\circlearrowleft \Sigma M_C = 0$: $(120 \text{ N})(0.28 \text{ m}) - \left(\dfrac{0.15}{0.39}T\right)(0.36 \text{ m}) - \left(\dfrac{0.15}{0.25}T\right)(0.2 \text{ m}) = 0$

$T = 130.000 \text{ N}$

or $T = 130.0 \text{ N}$ ◀

(*b*) $\xrightarrow{+} \Sigma F_x = 0$: $C_x + \left(\dfrac{0.36}{0.39}\right)(130.000 \text{ N}) + \left(\dfrac{0.2}{0.25}\right)(130.000 \text{ N}) = 0$

$C_x = -224.00 \text{ N}$

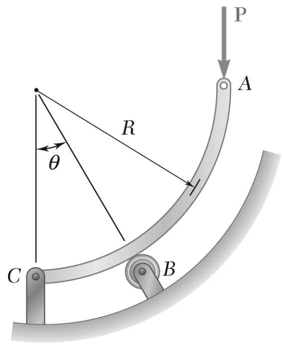
$+\uparrow \Sigma F_y = 0$: $C_y + \left(\dfrac{0.15}{0.39}\right)(130.00 \text{ N}) + \left(\dfrac{0.15}{0.25}\right)(130.00 \text{ N}) - 120 \text{ N} = 0$

$C_y = -8.0000 \text{ N}$

Thus, $C = \sqrt{C_x^2 + C_y^2} = \sqrt{(-224)^2 + (-8)^2} = 224.14 \text{ N}$

and $\theta = \tan^{-1}\dfrac{C_y}{C_x} = \tan^{-1}\dfrac{8}{224} = 2.0454°$

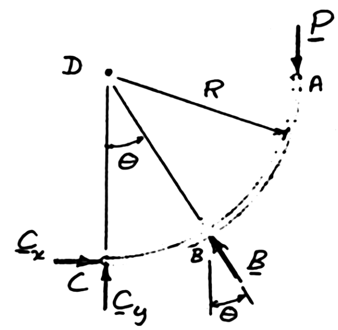
$\mathbf{C} = 224 \text{ N} \measuredangle 2.05°$ ◀



PROBLEM 4.33

Rod ABC is bent in the shape of an arc of circle of radius R. Knowing the $\theta = 30°$, determine the reaction (a) at B, (b) at C.

SOLUTION



Free-Body Diagram:

$$+\circlearrowleft \Sigma M_D = 0: \quad C_x(R) - P(R) = 0$$

$$C_x = +P$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad C_x - B\sin\theta = 0$$

$$P - B\sin\theta = 0$$

$$B = P/\sin\theta$$

$$\mathbf{B} = \frac{P}{\sin\theta} \measuredangle \theta$$

$$+\uparrow \Sigma F_y = 0: \quad C_y + B\cos\theta - P = 0$$

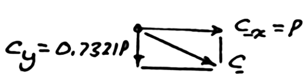
$$C_y + (P/\sin\theta)\cos\theta - P = 0$$

$$C_y = P\left(1 - \frac{1}{\tan\theta}\right)$$

For $\theta = 30°$,

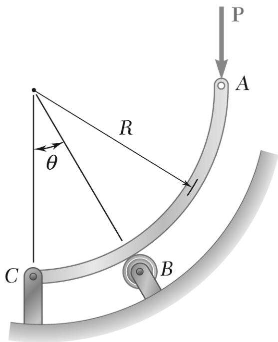
(a) $\quad B = P/\sin 30° = 2P$ $\qquad \mathbf{B} = 2P \measuredangle 60.0°$ ◀

(b) $\quad C_x = +P \qquad C_x = P \longrightarrow$



$C_y = P(1 - 1/\tan 30°) = -0.7321P$ $\qquad \mathbf{C}_y = 0.7321P \downarrow$

$\mathbf{C} = 1.239P \measuredangle 36.2°$ ◀



PROBLEM 4.34

Rod ABC is bent in the shape of an arc of circle of radius R. Knowing that $\theta = 60°$, determine the reaction (*a*) at B, (*b*) at C.

SOLUTION

See the solution to Problem 4.33 for the free-body diagram and analysis leading to the following expressions:

$$C_x = +P$$

$$C_y = P\left(1 - \frac{1}{\tan\theta}\right)$$

$$B = \frac{P}{\sin\theta}$$

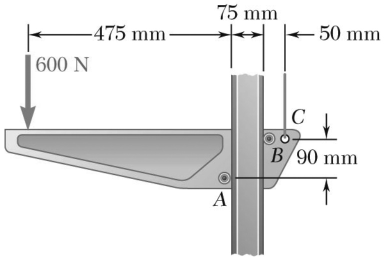
For $\theta = 60°$,

(*a*) $B = P/\sin 60° = 1.1547P$ $\qquad$ $\mathbf{B} = 1.155P \searrow 30.0°$ ◀

(*b*) $C_x = +P \qquad C_x = P \longrightarrow$

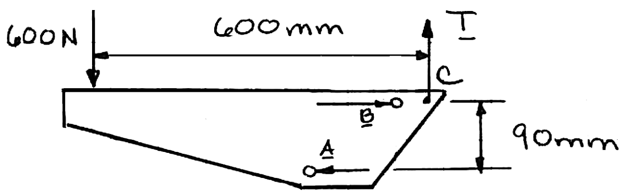
$C_y = P(1 - 1/\tan 60°) = +0.4226P$ $\qquad$ $\mathbf{C}_y = 0.4226P \downarrow$

$\mathbf{C} = 1.086P \measuredangle 22.9°$ ◀

	**PROBLEM 4.35** A movable bracket is held at rest by a cable attached at C and by frictionless rollers at A and B. For the loading shown, determine (a) the tension in the cable, (b) the reactions at A and B.

SOLUTION

Free-Body Diagram:



(a) $+\uparrow\Sigma F_y = 0: \quad T - 600\text{ N} = 0$

$T = 600$ N ◀

(b) $\xrightarrow{+} \Sigma F_x = 0: \quad B - A = 0 \qquad \therefore \quad B = A$

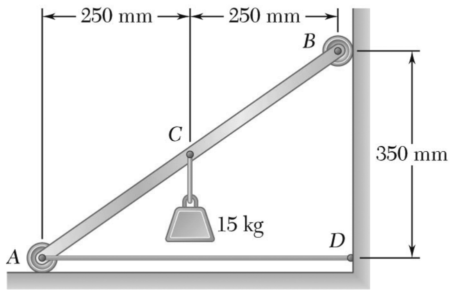
Note that the forces shown form two couples.

$+\circlearrowleft \Sigma M = 0: \quad (600\text{ N})(600\text{ mm}) - A(90\text{ mm}) = 0$

$A = 4000$ N

$\therefore \quad B = 4000$ N

A $= 4.00$ kN $\longleftarrow$; **B** $= 4.00$ kN $\longrightarrow$ ◀

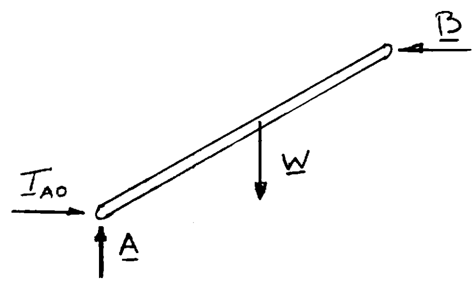


PROBLEM 4.36

A light bar AB supports a 15-kg block at its midpoint C. Rollers at A and B rest against frictionless surfaces, and a horizontal cable AD is attached at A. Determine (*a*) the tension in cable AD, (*b*) the reactions at A and B.

SOLUTION

Free-Body Diagram:



$$W = (15 \text{ kg})(9.81 \text{ m/s}^2)$$
$$= 147.150 \text{ N}$$

(*a*) $\xrightarrow{+} \Sigma F_x = 0: \quad T_{AD} - 105.107 \text{ N} = 0$

$$T_{AD} = 105.1 \text{ N} \blacktriangleleft$$

(*b*) $+\uparrow \Sigma F_y = 0: \quad A - W = 0$

$$A - 147.150 \text{ N} = 0$$

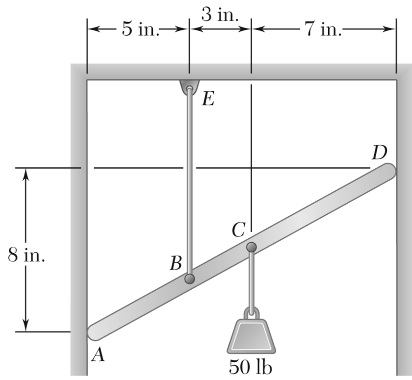
$$\mathbf{A} = 147.2 \text{ N} \uparrow \blacktriangleleft$$

$+\circlearrowleft \Sigma M_A = 0: \quad B(350 \text{ mm}) - (147.150 \text{ N})(250 \text{ mm}) = 0$

$$B = 105.107 \text{ N}$$

$$\mathbf{B} = 105.1 \text{ N} \leftarrow \blacktriangleleft$$

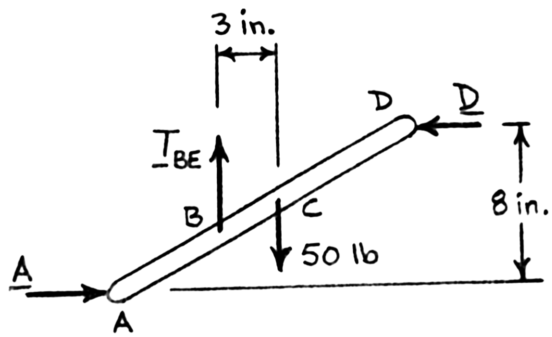
PROBLEM 4.37



A light bar AD is suspended from a cable BE and supports a 50-lb block at C. The ends A and D of the bar are in contact with frictionless vertical walls. Determine the tension in cable BE and the reactions at A and D.

SOLUTION

Free-Body Diagram:



$\Sigma F_x = 0$: $\quad A = D$

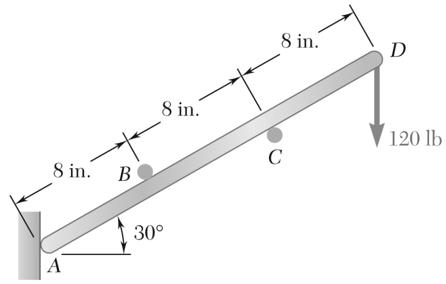
$\Sigma F_y = 0$: $\qquad T_{BE} = 50.0\text{ lb}$ ◀

We note that the forces shown form two couples.

$$+\circlearrowleft \Sigma M = 0: \quad A(8\text{ in.}) - (50\text{ lb})(3\text{ in.}) = 0$$

$$A = 18.75\text{ lb}$$

$\mathbf{A} = 18.75\text{ lb} \longrightarrow \qquad \mathbf{D} = 18.75\text{ lb} \longleftarrow$ ◀

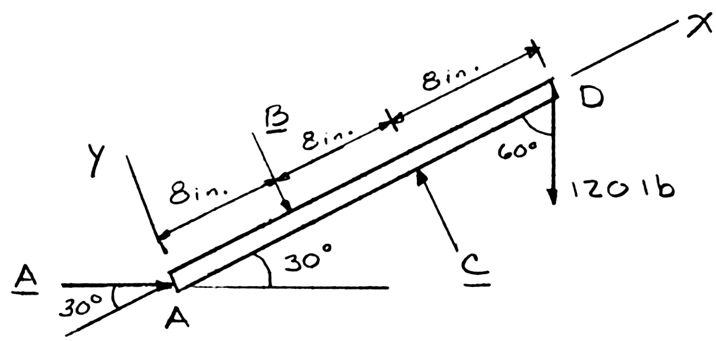


PROBLEM 4.38

A light rod *AD* is supported by frictionless pegs at *B* and *C* and rests against a frictionless wall at *A*. A vertical 120-lb force is applied at *D*. Determine the reactions at *A*, *B*, and *C*.

SOLUTION

Free-Body Diagram:



$\nearrow \Sigma F_x = 0$: $A\cos 30° - (120 \text{ lb})\cos 60° = 0$

$A = 69.28 \text{ lb}$ $\qquad$ $\mathbf{A} = 69.3 \text{ lb} \longrightarrow$ ◀

$+\circlearrowleft \Sigma M_B = 0$: $C(8 \text{ in.}) - (120 \text{ lb})(16 \text{ in.})\cos 30° + (69.28 \text{ lb})(8 \text{ in.})\sin 30° = 0$

$C = 173.2 \text{ lb}$ $\qquad$ $\mathbf{C} = 173.2 \text{ lb} \measuredangle 60.0°$ ◀

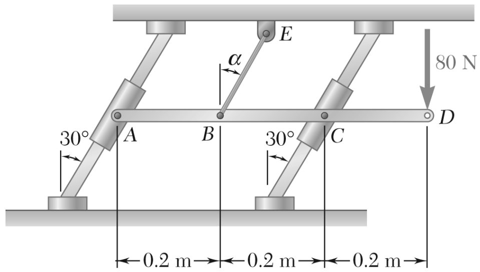
$+\circlearrowleft \Sigma M_C = 0$: $B(8 \text{ in.}) - (120 \text{ lb})(8 \text{ in.})\cos 30° + (69.28 \text{ lb})(16 \text{ in.})\sin 30° = 0$

$B = 34.6 \text{ lb}$ $\qquad$ $\mathbf{B} = 34.6 \text{ lb} \measuredangle 60.0°$ ◀

Check: $\nwarrow \Sigma F_y = 0$: $173.2 - 34.6 - (69.28)\sin 30° - (120)\sin 60° = 0$

$0 = 0$ (check)

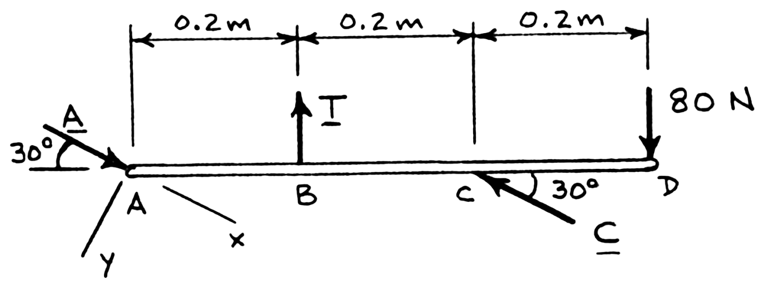
PROBLEM 4.39



Bar AD is attached at A and C to collars that can move freely on the rods shown. If the cord BE is vertical ($\alpha = 0$), determine the tension in the cord and the reactions at A and C.

SOLUTION

Free-Body Diagram:



$$+\swarrow \Sigma F_y = 0: \quad -T\cos 30° + (80\ \text{N})\cos 30° = 0$$

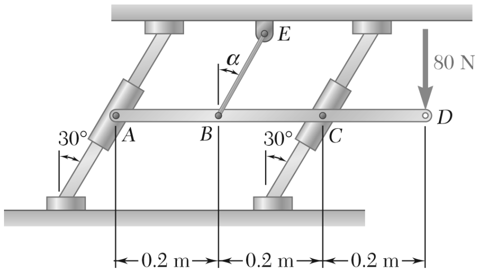
$$T = 80\ \text{N} \qquad\qquad T = 80.0\ \text{N} \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_C = 0: \quad (A\sin 30°)(0.4\ \text{m}) - (80\ \text{N})(0.2\ \text{m}) - (80\ \text{N})(0.2\ \text{m}) = 0$$

$$A = +160\ \text{N} \qquad \mathbf{A} = 160.0\ \text{N} \angle 30.0° \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_A = 0: \quad (80\ \text{N})(0.2\ \text{m}) - (80\ \text{N})(0.6\ \text{m}) + (C\sin 30°)(0.4\ \text{m}) = 0$$

$$C = +160\ \text{N} \qquad \mathbf{C} = 160.0\ \text{N} \angle 30.0° \blacktriangleleft$$



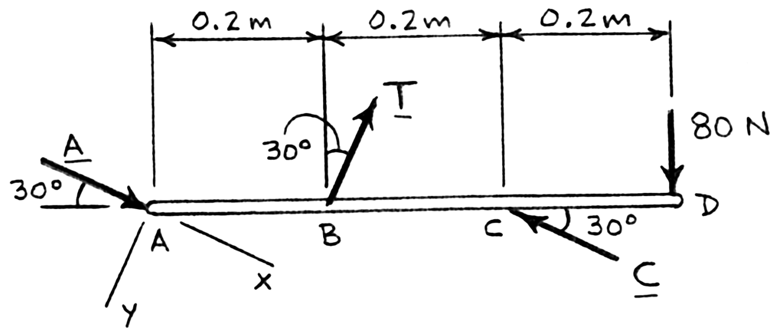
PROBLEM 4.40

Solve Problem 4.39 if the cord BE is parallel to the rods ($\alpha = 30°$).

PROBLEM 4.39 Bar *AD* is attached at *A* and *C* to collars that can move freely on the rods shown. If the cord *BE* is vertical ($\alpha = 0$), determine the tension in the cord and the reactions at *A* and *C*.

SOLUTION

Free-Body Diagram:



$$+\swarrow \Sigma F_y = 0: \quad -T + (80\text{ N})\cos 30° = 0$$

$$T = 69.282\text{ N} \qquad T = 69.3\text{ N} \blacktriangleleft$$

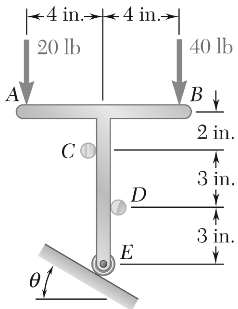
$$+\circlearrowleft \Sigma M_C = 0: \quad -(69.282\text{ N})\cos 30°(0.2\text{ m})$$
$$-(80\text{ N})(0.2\text{ m}) + (A\sin 30°)(0.4\text{ m}) = 0$$

$$A = +140.000\text{ N} \qquad \mathbf{A} = 140.0\text{ N} \measuredangle 30.0° \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_A = 0: \quad +(69.282\text{ N})\cos 30°(0.2\text{ m})$$
$$-(80\text{ N})(0.6\text{ m}) + (C\sin 30°)(0.4\text{ m}) = 0$$

$$C = +180.000\text{ N} \qquad \mathbf{C} = 180.0\text{ N} \measuredangle 30.0° \blacktriangleleft$$

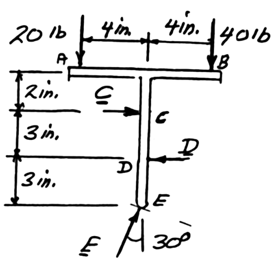
PROBLEM 4.41



The T-shaped bracket shown is supported by a small wheel at E and pegs at C and D. Neglecting the effect of friction, determine the reactions at C, D, and E when $\theta = 30°$.

SOLUTION

Free-Body Diagram:



$$+\uparrow \Sigma F_y = 0: \quad E\cos 30° - 20 - 40 = 0$$

$$E = \frac{60 \text{ lb}}{\cos 30°} = 69.282 \text{ lb} \qquad \mathbf{E} = 69.3 \text{ lb} \measuredangle 60.0° \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_D = 0: \quad (20 \text{ lb})(4 \text{ in.}) - (40 \text{ lb})(4 \text{ in.})$$

$$- C(3 \text{ in.}) + E\sin 30°(3 \text{ in.}) = 0$$

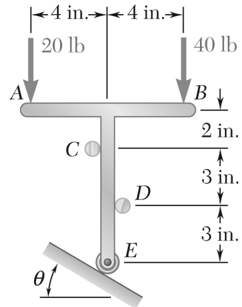
$$-80 - 3C + 69.282(0.5)(3) = 0$$

$$C = 7.9743 \text{ lb} \qquad \mathbf{C} = 7.97 \text{ lb} \longrightarrow \blacktriangleleft$$

$$\Sigma F_x = 0: \quad E\sin 30° + C - D = 0$$

$$(69.282 \text{ lb})(0.5) + 7.9743 \text{ lb} - D = 0$$

$$D = 42.615 \text{ lb} \qquad \mathbf{D} = 42.6 \text{ lb} \longleftarrow \blacktriangleleft$$

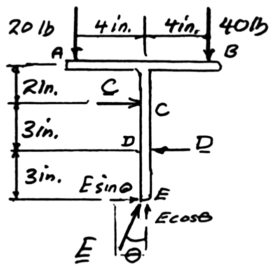


PROBLEM 4.42

The T-shaped bracket shown is supported by a small wheel at E and pegs at C and D. Neglecting the effect of friction, determine (a) the smallest value of θ for which the equilibrium of the bracket is maintained, (b) the corresponding reactions at C, D, and E.

SOLUTION

Free-Body Diagram:



$$+\uparrow \Sigma F_y = 0: \quad E\cos\theta - 20 - 40 = 0$$

$$E = \frac{60}{\cos\theta} \qquad (1)$$

$$+\circlearrowleft \Sigma M_D = 0: \quad (20 \text{ lb})(4 \text{ in.}) - (40 \text{ lb})(4 \text{ in.}) - C(3 \text{ in.}) + \left(\frac{60}{\cos\theta}\sin\theta\right) 3 \text{ in.} = 0$$

$$C = \frac{1}{3}(180\tan\theta - 80)$$

(a) For $C = 0$, $\quad 180\tan\theta = 80$

$$\tan\theta = \frac{4}{9} \quad \theta = 23.962°$$

$\theta = 24.0°$ ◀

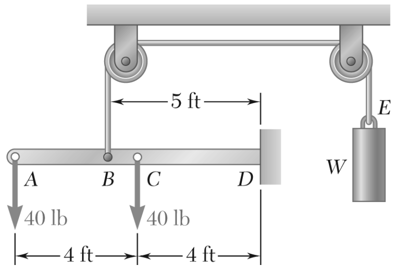
From Eq. (1): $\quad E = \dfrac{60}{\cos 23.962°} = 65.659$

$$\xrightarrow{+} \Sigma F_x = 0: \quad -D + C + E\sin\theta = 0$$

$$D = (65.659)\sin 23.962 = 26.666 \text{ lb}$$

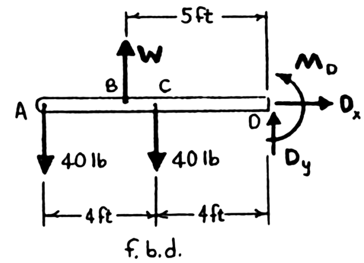
(b) $\quad \mathbf{C} = 0 \quad \mathbf{D} = 26.7 \text{ lb} \leftarrow \qquad \mathbf{E} = 65.7 \text{ lb} \measuredangle\ 66.0°$ ◀

PROBLEM 4.43



Beam AD carries the two 40-lb loads shown. The beam is held by a fixed support at D and by the cable BE that is attached to the counterweight W. Determine the reaction at D when (*a*) $W = 100$ lb, (*b*) $W = 90$ lb.

SOLUTION



(*a*) $W = 100$ lb

From F.B.D. of beam AD:

$$\xrightarrow{+} \Sigma F_x = 0: \quad D_x = 0$$

$$+\uparrow \Sigma F_y = 0: \quad D_y - 40 \text{ lb} - 40 \text{ lb} + 100 \text{ lb} = 0$$

$$D_y = -20.0 \text{ lb} \qquad \text{or} \qquad \mathbf{D} = 20.0 \text{ lb} \downarrow \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_D = 0: \quad M_D - (100 \text{ lb})(5 \text{ ft}) + (40 \text{ lb})(8 \text{ ft}) + (40 \text{ lb})(4 \text{ ft}) = 0$$

$$M_D = 20.0 \text{ lb} \cdot \text{ft} \qquad \text{or} \qquad \mathbf{M}_D = 20.0 \text{ lb} \cdot \text{ft} \circlearrowleft \blacktriangleleft$$

(*b*) $W = 90$ lb

From F.B.D. of beam AD:

$$\xrightarrow{+} \Sigma F_x = 0: \quad D_x = 0$$

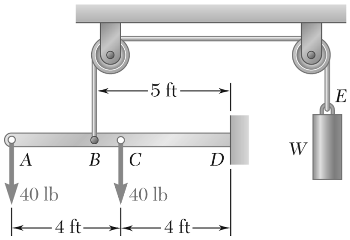
$$+\uparrow \Sigma F_y = 0: \quad D_y + 90 \text{ lb} - 40 \text{ lb} - 40 \text{ lb} = 0$$

$$D_y = -10.00 \text{ lb} \qquad \text{or} \qquad \mathbf{D} = 10.00 \text{ lb} \downarrow \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_D = 0: \quad M_D - (90 \text{ lb})(5 \text{ ft}) + (40 \text{ lb})(8 \text{ ft}) + (40 \text{ lb})(4 \text{ ft}) = 0$$

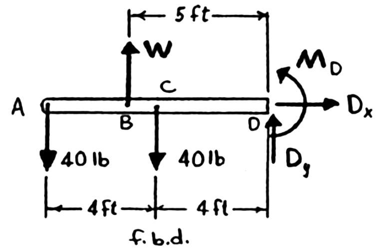
$$M_D = -30.0 \text{ lb} \cdot \text{ft} \qquad \text{or} \qquad \mathbf{M}_D = 30.0 \text{ lb} \cdot \text{ft} \circlearrowright \blacktriangleleft$$

PROBLEM 4.44



For the beam and loading shown, determine the range of values of W for which the magnitude of the couple at D does not exceed 40 lb · ft.

SOLUTION



For $W_{\min}$, $\quad M_D = -40 \text{ lb} \cdot \text{ft}$

From F.B.D. of beam AD: $\quad +\circlearrowleft \Sigma M_D = 0\text{:} \quad (40 \text{ lb})(8 \text{ ft}) - W_{\min}(5 \text{ ft})$

$$+ (40 \text{ lb})(4 \text{ ft}) - 40 \text{ lb} \cdot \text{ft} = 0$$

$$W_{\min} = 88.0 \text{ lb}$$

For $W_{\max}$, $\quad M_D = 40 \text{ lb} \cdot \text{ft}$

From F.B.D. of beam AD: $\quad +\circlearrowleft \Sigma M_D = 0\text{:} \quad (40 \text{ lb})(8 \text{ ft}) - W_{\max}(5 \text{ ft})$

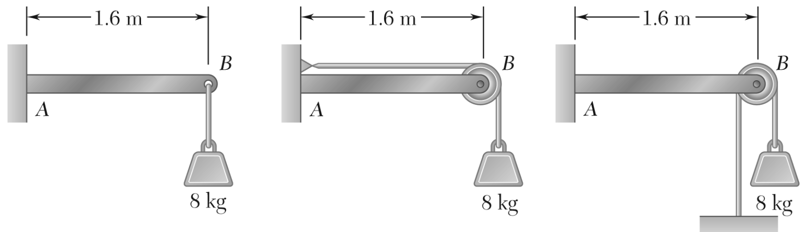
$$+ (40 \text{ lb})(4 \text{ ft}) + 40 \text{ lb} \cdot \text{ft} = 0$$

$$W_{\max} = 104.0 \text{ lb}$$

or $88.0 \text{ lb} \le W \le 104.0 \text{ lb}$ ◀

PROBLEM 4.45

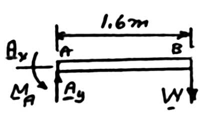
An 8-kg mass can be supported in the three different ways shown. Knowing that the pulleys have a 100-mm radius, determine the reaction at *A* in each case.



(*a*) (*b*) (*c*)

SOLUTION

$$W = mg = (8\text{ kg})(9.81\text{ m/s}^2) = 78.480\text{ N}$$



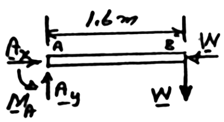
(*a*) $\Sigma F_x = 0$: $A_x = 0$

$+\uparrow \Sigma F_y = 0$: $A_y - W = 0$ $\qquad \mathbf{A}_y = 78.480\text{ N}\uparrow$

$+\circlearrowleft \Sigma M_A = 0$: $M_A - W(1.6\text{ m}) = 0$

$M_A = +(78.480\text{ N})(1.6\text{ m})$ $\qquad \mathbf{M}_A = 125.568\text{ N}\cdot\text{m} \circlearrowleft$

$\mathbf{A} = 78.5\text{ N}\uparrow$ $\qquad \mathbf{M}_A = 125.6\text{ N}\cdot\text{m} \circlearrowleft$ ◀



(*b*) $\xrightarrow{+} \Sigma F_x = 0$: $A_x - W = 0$ $\qquad \mathbf{A}_x = 78.480\uparrow$

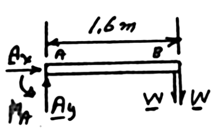
$+\uparrow \Sigma F_y = 0$: $A_y - W = 0$ $\qquad \mathbf{A}_y = 78.480 \leftarrow$

$$\mathbf{A} = (78.480\text{ N})\sqrt{2} = 110.987\text{ N} \measuredangle 45^\circ$$

$+\circlearrowleft \Sigma M_A = 0$: $M_A - W(1.6\text{ m}) = 0$

$M_A = +(78.480\text{ N})(1.6\text{ m})$ $\qquad \mathbf{M}_A = 125.568\text{ N}\cdot\text{m} \circlearrowleft$

$\mathbf{A} = 111.0\text{ N} \measuredangle 45^\circ$ $\qquad \mathbf{M}_A = 125.6\text{ N}\cdot\text{m} \circlearrowleft$ ◀



(*c*) $\Sigma F_x = 0$: $A_x = 0$

$+\uparrow \Sigma F_y = 0$: $A_y - 2W = 0$

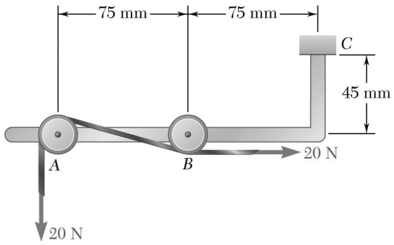
$$A_y = 2W = 2(78.480\text{ N}) = 156.960\text{ N}\uparrow$$

$+\circlearrowleft \Sigma M_A = 0$: $M_A - 2W(1.6\text{ m}) = 0$

$M_A = +2(78.480\text{ N})(1.6\text{ m})$ $\qquad \mathbf{M}_A = 251.14\text{ N}\cdot\text{m} \circlearrowleft$

$\mathbf{A} = 157.0\text{ N}\uparrow$ $\qquad \mathbf{M}_A = 251\text{ N}\cdot\text{m} \circlearrowleft$ ◀

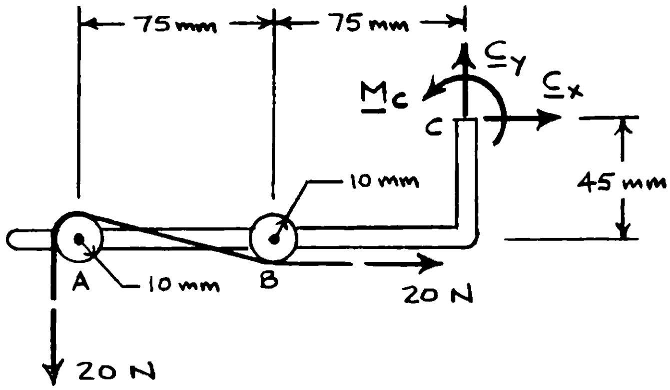
PROBLEM 4.46



A tension of 20 N is maintained in a tape as it passes through the support system shown. Knowing that the radius of each pulley is 10 mm, determine the reaction at *C*.

SOLUTION

Free-Body Diagram:



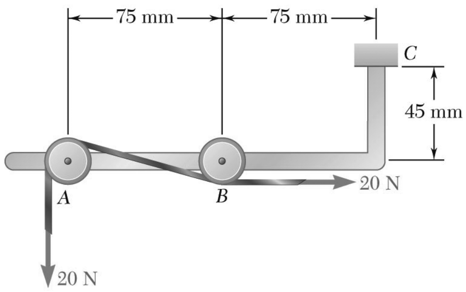
$\xrightarrow{+} \Sigma F_x = 0: \quad C_x + (20 \text{ N}) = 0 \qquad C_x = -20 \text{ N}$

$+\uparrow \Sigma F_y = 0: \quad C_y - (20 \text{ N}) = 0 \qquad C_y = +20 \text{ N}$

$\mathbf{C} = 28.3 \text{ N} \measuredangle 45.0° \blacktriangleleft$

$+\circlearrowleft \Sigma M_C = 0: \quad M_C + (20 \text{ N})(0.160 \text{ m}) + (20 \text{ N})(0.055 \text{ m}) = 0$

$M_C = -4.30 \text{ N} \cdot \text{m} \qquad \mathbf{M}_C = 4.30 \text{ N} \cdot \text{m} \circlearrowright \blacktriangleleft$



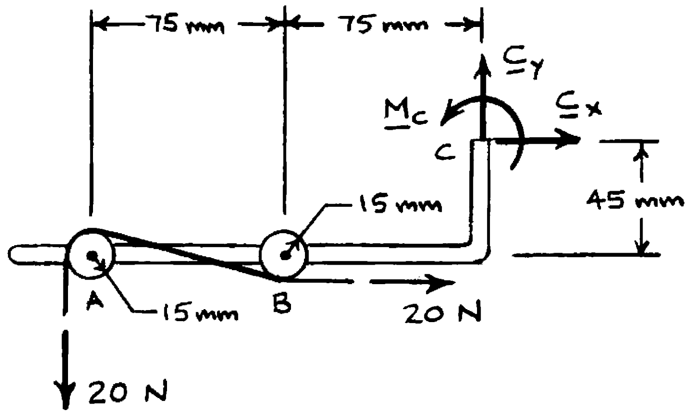
PROBLEM 4.47

Solve Problem 4.46, assuming that 15-mm-radius pulleys are used.

PROBLEM 4.46 A tension of 20 N is maintained in a tape as it passes through the support system shown. Knowing that the radius of each pulley is 10 mm, determine the reaction at *C*.

SOLUTION

Free-Body Diagram:



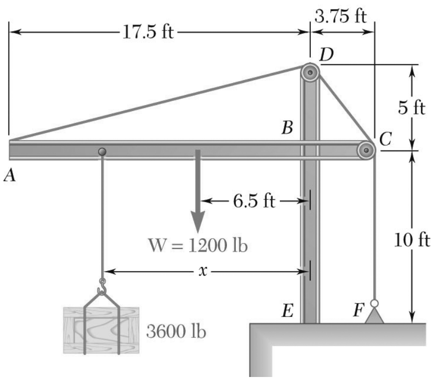
$$\xrightarrow{+} \Sigma F_x = 0: \quad C_x + (20 \text{ N}) = 0 \qquad C_x = -20 \text{ N}$$

$$+\uparrow \Sigma F_y = 0: \quad C_y - (20 \text{ N}) = 0 \qquad C_y = +20 \text{ N}$$

$$\mathbf{C} = 28.3 \text{ N} \measuredangle 45.0° \blacktriangleleft$$

$$+\circlearrowright \Sigma M_C = 0: \quad M_C + (20 \text{ N})(0.165 \text{ m}) + (20 \text{ N})(0.060 \text{ m}) = 0$$

$$M_C = -4.50 \text{ N} \cdot \text{m} \qquad \mathbf{M}_C = 4.50 \text{ N} \cdot \text{m} \circlearrowleft \blacktriangleleft$$

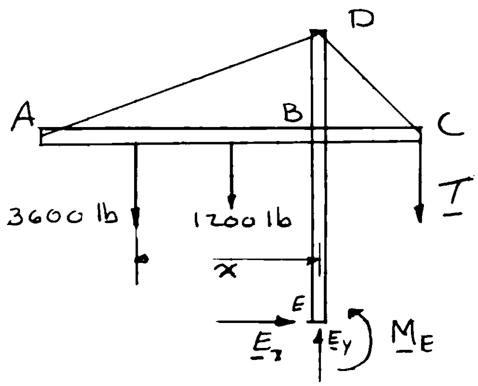


PROBLEM 4.48

The rig shown consists of a 1200-lb horizontal member *ABC* and a vertical member *DBE* welded together at *B*. The rig is being used to raise a 3600-lb crate at a distance $x = 12$ ft from the vertical member *DBE*. If the tension in the cable is 4 kips, determine the reaction at *E*, assuming that the cable is (*a*) anchored at *F* as shown in the figure, (*b*) attached to the vertical member at a point located 1 ft above *E*.

SOLUTION

Free-Body Diagram:



$$+\circlearrowleft M_E = 0: \quad M_E + (3600 \text{ lb})x + (1200 \text{ lb})(6.5 \text{ ft}) - T(3.75 \text{ ft}) = 0$$

$$M_E = 3.75T - 3600x - 7800 \qquad (1)$$

(*a*) For $x = 12$ ft and $T = 4000$ lbs,

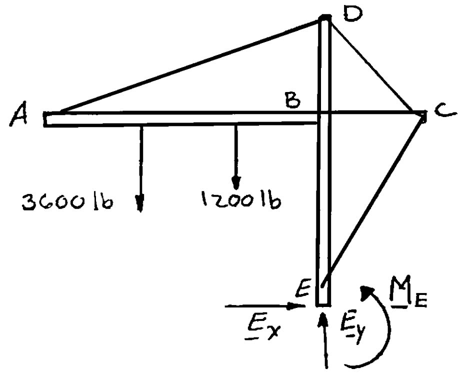
$$M_E = 3.75(4000) - 3600(12) - 7800$$
$$= 36{,}000 \text{ lb} \cdot \text{ft}$$

$$\xrightarrow{+} \Sigma F_x = 0 \quad \therefore \quad E_x = 0$$

$$+\uparrow \Sigma F_y = 0: \quad E_y - 3600 \text{ lb} - 1200 \text{ lb} - 4000 = 0$$

$$E_y = 8800 \text{ lb}$$

$$\mathbf{E} = 8.80 \text{ kips} \uparrow; \ \mathbf{M}_E = 36.0 \text{ kip} \cdot \text{ft} \circlearrowright \blacktriangleleft$$



(*b*) $+\circlearrowleft \Sigma M_E = 0: \quad M_E + (3600 \text{ lb})(12 \text{ ft}) + (1200 \text{ lb})(6.5 \text{ ft}) = 0$

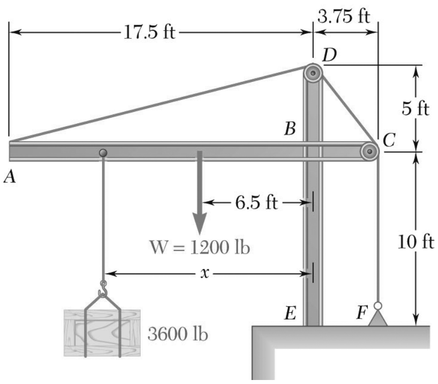
$$M_E = -51{,}000 \text{ lb} \cdot \text{ft}$$

$$\xrightarrow{+} \Sigma F_x = 0 \quad \therefore \quad E_x = 0$$

$$+\uparrow \Sigma F_y = 0: \qquad E_y - 3600 \text{ lb} - 1200 \text{ lb} = 0$$

$$E_y = 4800 \text{ lb}$$

$\mathbf{E} = 4.80 \text{ kips} \uparrow; \; \mathbf{M}_E = 51.0 \text{ kip} \cdot \text{ft} \circlearrowright$ ◀

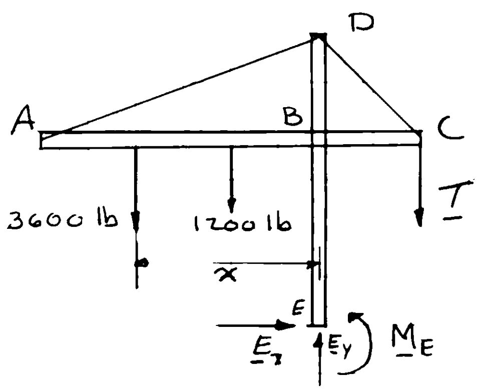


PROBLEM 4.49

For the rig and crate of Prob. 4.48, and assuming that cable is anchored at F as shown, determine (*a*) the required tension in cable *ADCF* if the maximum value of the couple at E as x varies from 1.5 to 17.5 ft is to be as small as possible, (*b*) the corresponding maximum value of the couple.

SOLUTION

Free-Body Diagram:



$$+\circlearrowleft M_E = 0: \quad M_E + (3600\text{ lb})x + (1200\text{ lb})(6.5\text{ ft}) - T(3.75\text{ ft}) = 0$$

$$M_E = 3.75T - 3600x - 7800 \qquad (1)$$

For $x = 1.5$ ft, Eq. (1) becomes

$$(M_E)_1 = 3.75T - 3600(1.5) - 7800 \qquad (2)$$

For $x = 17.5$ ft, Eq. (1) becomes

$$(M_E)_2 = 3.75T - 3600(17.5) - 7800$$

(*a*) For smallest max value of $|M_E|$, we set

$$(M_E)_1 = -(M_E)_2$$

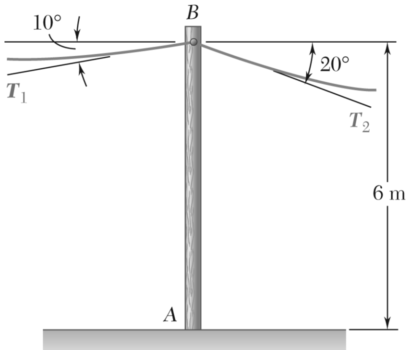
$$3.75T - 13{,}200 = -3.75T + 70{,}800$$

$\mathbf{T} = 11.20$ kips ◀

(*b*) From Equation (2), then

$$M_E = 3.75(11.20) - 13.20$$

$|M_E| = 28.8$ kip·ft ◀

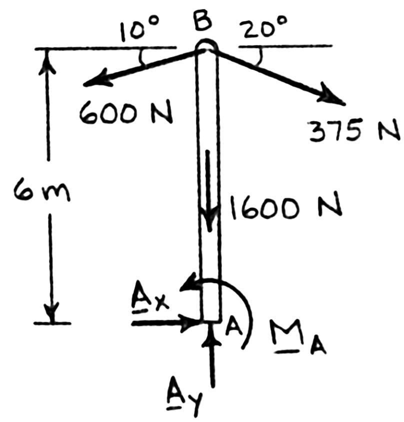


PROBLEM 4.50

A 6-m telephone pole weighing 1600 N is used to support the ends of two wires. The wires form the angles shown with the horizontal axis and the tensions in the wires are, respectively, $T_1 = 600$ N and $T_2 = 375$ N. Determine the reaction at the fixed end A.

SOLUTION

Free-Body Diagram:



$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x + (375 \text{ N})\cos 20° - (600 \text{ N})\cos 10° = 0$$

$$A_x = +238.50 \text{ N}$$

$$+\uparrow \Sigma F_y = 0: \quad A_y - 1600 \text{ N} - (600 \text{ N})\sin 10° - (375 \text{ N})\sin 20° = 0$$

$$A_y = +1832.45 \text{ N}$$

$$A = \sqrt{238.50^2 + 1832.45^2}$$

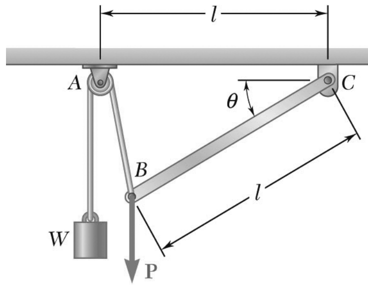
$$\theta = \tan^{-1}\frac{1832.45}{238.50}$$

$$\mathbf{A} = 1848 \text{ N} \measuredangle 82.6° \blacktriangleleft$$

$$+\circlearrowleft \Sigma M_A = 0: \quad M_A + (600 \text{ N})\cos 10°(6 \text{ m}) - (375 \text{ N})\cos 20°(6 \text{ m}) = 0$$

$$M_A = -1431.00 \text{ N}\cdot\text{m}$$

$$\mathbf{M}_A = 1431 \text{ N}\cdot\text{m} \circlearrowright \blacktriangleleft$$

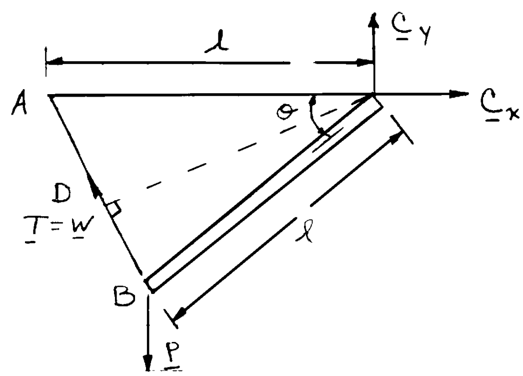


PROBLEM 4.51

A vertical load **P** is applied at end *B* of rod *BC*. (*a*) Neglecting the weight of the rod, express the angle θ corresponding to the equilibrium position in terms of *P*, *l*, and the counterweight *W*. (*b*) Determine the value of θ corresponding to equilibrium if $P = 2W$.

SOLUTION

Free-Body Diagram:



(*a*) Triangle *ABC* is isosceles. We have

$$CD = (BC)\cos\left(\frac{\theta}{2}\right) = l\cos\left(\frac{\theta}{2}\right)$$

$$+\circlearrowleft \Sigma M_C = 0: \quad P(l\cos\theta) - W\left(l\cos\frac{\theta}{2}\right) = 0$$

Setting $\cos\theta = 2\cos^2\frac{\theta}{2} - 1$:

$$Pl\left(2\cos^2\frac{\theta}{2} - 1\right) - Wl\cos\frac{\theta}{2} = 0$$

$$\cos^2\frac{\theta}{2} - \left(\frac{W}{2P}\right)\cos\frac{\theta}{2} - \frac{1}{2} = 0$$

$$\cos\frac{\theta}{2} = \frac{1}{4}\left(\frac{W}{P} \pm \sqrt{\frac{W^2}{P^2} + 8}\right)$$

$$\theta = 2\cos^{-1}\left[\frac{1}{4}\left(\frac{W}{P} \pm \sqrt{\frac{W^2}{P^2} + 8}\right)\right] \blacktriangleleft$$

PROBLEM 4.51 (Continued)

(*b*) For $P = 2W$,

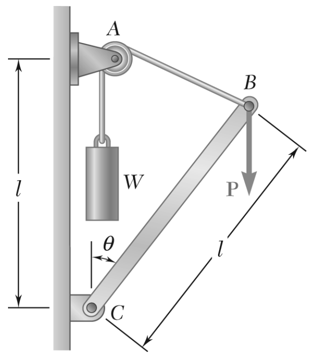
$$\cos\frac{\theta}{2} = \frac{1}{4}\left(\frac{1}{2} \pm \sqrt{\frac{1}{4} + 8}\right) = \frac{1}{8}\left(1 \pm \sqrt{33}\right)$$

$$\cos\frac{\theta}{2} = 0.84307 \quad \text{and} \quad \cos\frac{\theta}{2} = -0.59307$$

$$\frac{\theta}{2} = 32.534° \qquad \frac{\theta}{2} = 126.375°$$

$$\theta = 65.1° \qquad \theta = 252.75° \quad \text{(discard)}$$

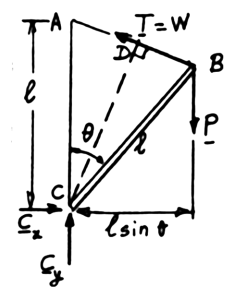
$\theta = 65.1°$ ◀



PROBLEM 4.52

A vertical load **P** is applied at end B of rod BC. (*a*) Neglecting the weight of the rod, express the angle θ corresponding to the equilibrium position in terms of P, l, and the counterweight W. (*b*) Determine the value of θ corresponding to equilibrium if $P = 2W$.

SOLUTION



(*a*) Triangle ABC is isosceles. We have

$$CD = (BC)\cos\frac{\theta}{2} = l\cos\frac{\theta}{2}$$

$$+\circlearrowleft \Sigma M_C = 0: \quad W\left(l\cos\frac{\theta}{2}\right) - P(l\sin\theta) = 0$$

Setting $\sin\theta = 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$: $\quad Wl\cos\frac{\theta}{2} - 2Pl\sin\frac{\theta}{2}\cos\frac{\theta}{2} = 0$

$$W - 2P\sin\frac{\theta}{2} = 0 \qquad \theta = 2\sin^{-1}\left(\frac{W}{2P}\right) \blacktriangleleft$$

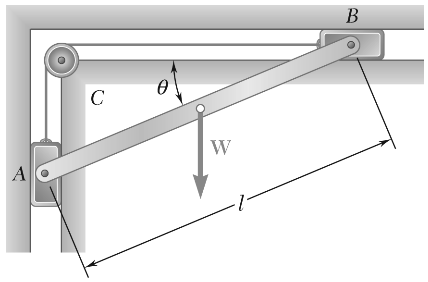
(*b*) For $P = 2W$,

$$\sin\frac{\theta}{2} = \frac{W}{2P} = \frac{W}{4W} = 0.25$$

$$\frac{\theta}{2} = 14.5° \qquad \theta = 29.0° \blacktriangleleft$$

or

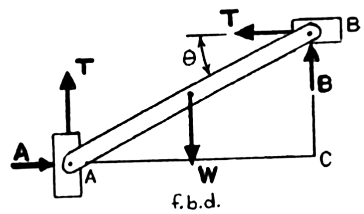
$$\frac{\theta}{2} = 165.5° \quad \theta = 331° \quad \text{(discard)}$$



PROBLEM 4.53

A slender rod *AB*, of weight *W*, is attached to blocks *A* and *B*, which move freely in the guides shown. The blocks are connected by an elastic cord that passes over a pulley at *C*. (*a*) Express the tension in the cord in terms of *W* and θ. (*b*) Determine the value of θ for which the tension in the cord is equal to 3*W*.

SOLUTION



(*a*) From F.B.D. of rod *AB*:

$$+\circlearrowleft \Sigma M_C = 0: \quad T(l\sin\theta) + W\left[\left(\frac{1}{2}\right)\cos\theta\right] - T(l\cos\theta) = 0$$

$$T = \frac{W\cos\theta}{2(\cos\theta - \sin\theta)}$$

Dividing both numerator and denominator by cos θ,

$$T = \frac{W}{2}\left(\frac{1}{1-\tan\theta}\right) \qquad \text{or} \quad T = \frac{\left(\frac{W}{2}\right)}{(1-\tan\theta)} \blacktriangleleft$$

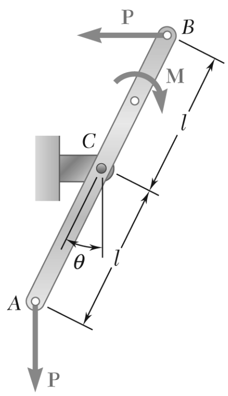
(*b*) For $T = 3W$,

$$3W = \frac{\left(\frac{W}{2}\right)}{(1-\tan\theta)}$$

$$1 - \tan\theta = \frac{1}{6}$$

or

$$\theta = \tan^{-1}\left(\frac{5}{6}\right) = 39.806° \qquad \text{or} \quad \theta = 39.8° \blacktriangleleft$$

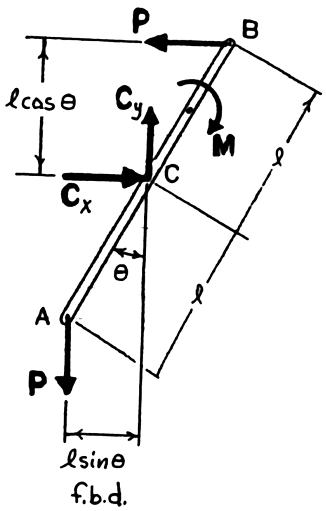


PROBLEM 4.54

Rod AB is acted upon by a couple **M** and two forces, each of magnitude P. (*a*) Derive an equation in θ, P, M, and l that must be satisfied when the rod is in equilibrium. (*b*) Determine the value of θ corresponding to equilibrium when $M = 150 \text{ N} \cdot \text{m}$, $P = 200 \text{ N}$, and $l = 600 \text{ mm}$.

SOLUTION

Free-Body Diagram:



(*a*) From free-body diagram of rod AB:

$$+\circlearrowleft \Sigma M_C = 0: \quad P(l\cos\theta) + P(l\sin\theta) - M = 0$$

$$\text{or } \sin\theta + \cos\theta = \frac{M}{Pl} \quad \blacktriangleleft$$

(*b*) For $M = 150 \text{ lb} \cdot \text{in.}$, $P = 20 \text{ lb}$, and $l = 6 \text{ in.}$,

$$\sin\theta + \cos\theta = \frac{150 \text{ lb} \cdot \text{in.}}{(20 \text{ lb})(6 \text{ in.})} = \frac{5}{4} = 1.25$$

Using identity $\quad \sin^2\theta + \cos^2\theta = 1$

$$\sin\theta + (1 - \sin^2\theta)^{1/2} = 1.25$$

$$(1 - \sin^2\theta)^{1/2} = 1.25 - \sin\theta$$

$$1 - \sin^2\theta = 1.5625 - 2.5\sin\theta + \sin^2\theta$$

$$2\sin^2\theta - 2.5\sin\theta + 0.5625 = 0$$

Using quadratic formula

$$\sin\theta = \frac{-(-2.5) \pm \sqrt{(625) - 4(2)(0.5625)}}{2(2)}$$

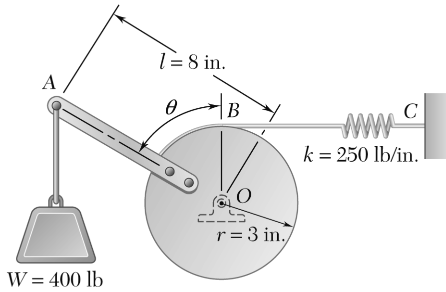
$$= \frac{2.5 \pm \sqrt{1.75}}{4}$$

or $\quad \sin\theta = 0.95572 \quad \text{and} \quad \sin\theta = 0.29428$

$$\theta = 72.886° \quad \text{and} \quad \theta = 17.1144°$$

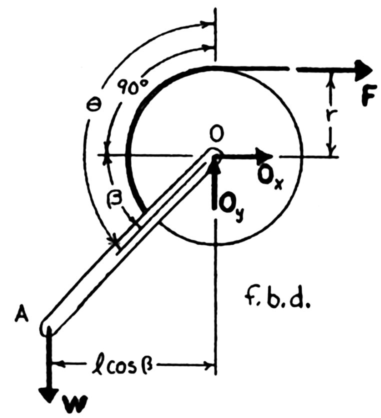
$$\text{or } \theta = 17.11° \quad \text{and} \quad \theta = 72.9° \quad \blacktriangleleft$$

PROBLEM 4.55



Solve Sample Problem 4.5, assuming that the spring is unstretched when $\theta = 90°$.

SOLUTION



First note: $T = \text{tension in spring} = ks$

where $s = \text{deformation of spring}$

$$= r\beta$$

$$F = kr\beta$$

From F.B.D. of assembly: $+\circlearrowleft \Sigma M_0 = 0$: $W(l\cos\beta) - F(r) = 0$

or $$Wl\cos\beta - kr^2\beta = 0$$

$$\cos\beta = \frac{kr^2}{Wl}\beta$$

For $k = 250$ lb/in.

$r = 3$ in.

$l = 8$ in.

$W = 400$ lb

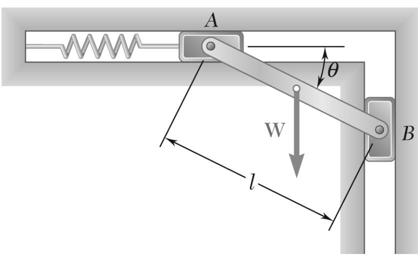
$$\cos\beta = \frac{(250\text{ lb/in.})(3\text{ in.})^2}{(400\text{ lb})(8\text{ in.})}\beta$$

or $$\cos\beta = 0.703125\beta$$

Solving numerically, $$\beta = 0.89245 \text{ rad}$$

or $$\beta = 51.134°$$

Then $\theta = 90° + 51.134° = 141.134°$ or $\theta = 141.1°$ ◀

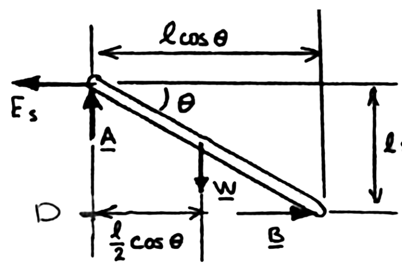


PROBLEM 4.56

A slender rod AB, of weight W, is attached to blocks A and B that move freely in the guides shown. The constant of the spring is k, and the spring is unstretched when $\theta = 0$. (*a*) Neglecting the weight of the blocks, derive an equation in W, k, l, and θ that must be satisfied when the rod is in equilibrium. (*b*) Determine the value of θ when $W = 75$ lb, $l = 30$ in., and $k = 3$ lb/in.

SOLUTION

Free-Body Diagram:



Spring force: $$F_s = ks = k(l - l\cos\theta) = kl(1-\cos\theta)$$

(*a*) $$+\circlearrowleft \Sigma M_D = 0: \quad F_s(l\sin\theta) - W\left(\frac{l}{2}\cos\theta\right) = 0$$

$$kl(1-\cos\theta)l\sin\theta - \frac{W}{2}l\cos\theta = 0$$

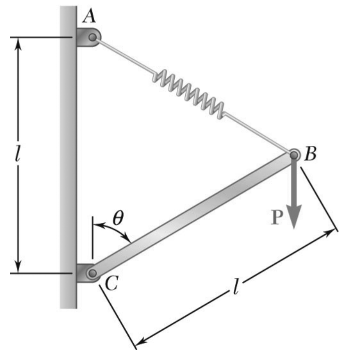
$$kl(1-\cos\theta)\tan\theta - \frac{W}{2} = 0 \qquad \text{or} \quad (1-\cos\theta)\tan\theta = \frac{W}{2kl} \blacktriangleleft$$

(*b*) For given values of

$$W = 75 \text{ lb}$$
$$l = 30 \text{ in.}$$
$$k = 3 \text{ lb/in.}$$

$$(1-\cos\theta)\tan\theta = \tan\theta - \sin\theta = \frac{75 \text{ lb}}{2(3 \text{ lb/in.})(30 \text{ in.})} = 0.41667$$

Solving numerically, $\theta = 49.710°$ or $\theta = 49.7° \blacktriangleleft$

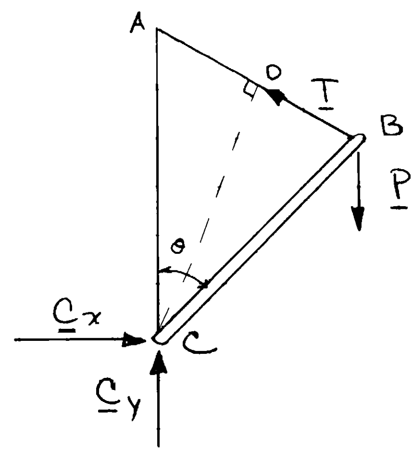


PROBLEM 4.57

A vertical load **P** is applied at end *B* of rod *BC*. The constant of the spring is *k*, and the spring is unstretched when $\theta = 60°$. (*a*) Neglecting the weight of the rod, express the angle θ corresponding to the equilibrium position terms of *P*, *k*, and *l*. (*b*) Determine the value of θ corresponding to equilibrium if $P = \frac{1}{4}kl$.

SOLUTION

Free-Body Diagram:



(*a*) Triangle *ABC* is isosceles. We have

$$AB = 2(AD) = 2l\sin\left(\frac{\theta}{2}\right);\ CD = l\cos\left(\frac{\theta}{2}\right)$$

Elongation of spring: $x = (AB)_\theta - (AB)_\theta = 60°$

$$= 2l\sin\left(\frac{\theta}{2}\right) - 2l\sin 30°$$

$$T = kx = 2kl\left(\sin\frac{\theta}{2} - \frac{1}{2}\right)$$

$$\circlearrowleft\Sigma M_C = 0:\quad T\left(l\cos\frac{\theta}{2}\right) - P(l\sin\theta) = 0$$

PROBLEM 4.57 (Continued)

$$2kl\left(\sin\frac{\theta}{2}-\frac{1}{2}\right)l\cos\frac{\theta}{2}-Pl\left(2\sin\frac{\theta}{2}\cos\frac{\theta}{2}\right)=0$$

$$\cos\frac{\theta}{2}=0 \quad \text{or} \quad 2(kl-P)\sin\frac{\theta}{2}-kl=0$$

$$\theta=180° \text{ (trivial)} \qquad \sin\frac{\theta}{2}=\frac{\frac{1}{2}kl}{kl-P}$$

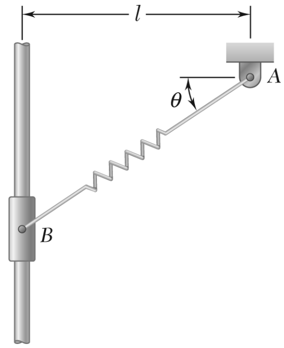
$$\theta=2\sin^{-1}\left[\frac{1}{2}kl/(kl-P)\right] ◀$$

(*b*) For $P=\dfrac{1}{4}kl,$

$$\sin\frac{\theta}{2}=\frac{\frac{1}{2}kl}{\frac{3}{4}kl}=\frac{2}{3}$$

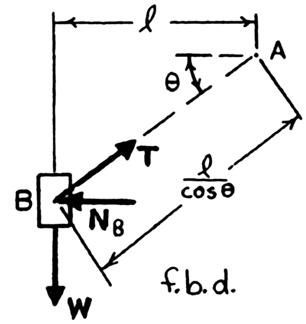
$$\frac{\theta}{2}=41.8° \qquad \theta=83.6° ◀$$

PROBLEM 4.58



A collar B of weight W can move freely along the vertical rod shown. The constant of the spring is k, and the spring is unstretched when $\theta = 0$. (*a*) Derive an equation in θ, W, k, and l that must be satisfied when the collar is in equilibrium. (*b*) Knowing that $W = 300$ N, $l = 500$ mm, and $k = 800$ N/m, determine the value of θ corresponding to equilibrium.

SOLUTION



First note: $T = ks$

where $k =$ spring constant

$s =$ elongation of spring

$$= \frac{l}{\cos\theta} - l$$

$$= \frac{l}{\cos\theta}(1 - \cos\theta)$$

$$T = \frac{kl}{\cos\theta}(1 - \cos\theta)$$

(*a*) From F.B.D. of collar *B*: $+\uparrow \Sigma F_y = 0: \quad T\sin\theta - W = 0$

or $\frac{kl}{\cos\theta}(1 - \cos\theta)\sin\theta - W = 0$ or $\tan\theta - \sin\theta = \frac{W}{kl}$ ◀

(*b*) For

$$W = 3 \text{ lb}$$

$$l = 6 \text{ in.}$$

$$k = 8 \text{ lb/ft}$$

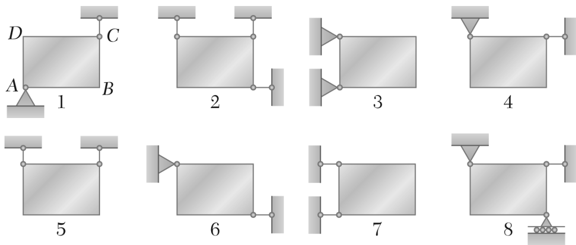
$$l = \frac{6 \text{ in.}}{12 \text{ in./ft}} = 0.5 \text{ ft}$$

$$\tan\theta - \sin\theta = \frac{3 \text{ lb}}{(8 \text{ lb/ft})(0.5 \text{ ft})} = 0.75$$

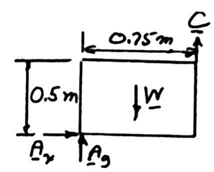
Solving numerically, $\theta = 57.957°$ or $\theta = 58.0°$ ◀

PROBLEM 4.59

Eight identical 500×750-mm rectangular plates, each of mass $m = 40$ kg, are held in a vertical plane as shown. All connections consist of frictionless pins, rollers, or short links. In each case, determine whether (*a*) the plate is completely, partially, or improperly constrained, (*b*) the reactions are statically determinate or indeterminate, (*c*) the equilibrium of the plate is maintained in the position shown. Also, wherever possible, compute the reactions.



SOLUTION



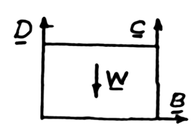
1. Three non-concurrent, non-parallel reactions:

   (*a*) Plate: completely constrained

   (*b*) Reactions: determinate

   (*c*) Equilibrium maintained

$$\mathbf{A} = \mathbf{C} = 196.2\text{ N}\uparrow$$



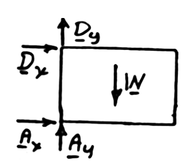
2. Three non-concurrent, non-parallel reactions:

   (*a*) Plate: completely constrained

   (*b*) Reactions: determinate

   (*c*) Equilibrium maintained

$$\mathbf{B} = 0, \quad \mathbf{C} = \mathbf{D} = 196.2\text{ N}\uparrow$$



3. Four non-concurrent, non-parallel reactions:

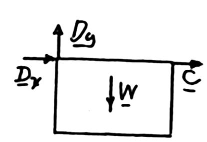
   (*a*) Plate: completely constrained

   (*b*) Reactions: indeterminate

   (*c*) Equilibrium maintained

$$\mathbf{A}_x = 294\text{ N}\longrightarrow, \quad \mathbf{D}_x = 294\text{ N}\longleftarrow$$

$$(\mathbf{A}_y + \mathbf{D}_y = 392\text{ N}\uparrow)$$



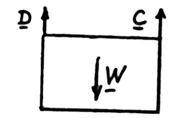
4. Three concurrent reactions (through D):

   (*a*) Plate: improperly constrained

   (*b*) Reactions: indeterminate

   (*c*) No equilibrium $(\Sigma M_D \neq 0)$

5. Two reactions:



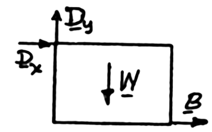
(*a*) Plate: partial constraint

(*b*) Reactions: determinate

(*c*) Equilibrium maintained

$$\mathbf{C} = \mathbf{D} = 196.2 \text{ N}\uparrow$$

6. Three non-concurrent, non-parallel reactions:



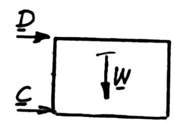
(*a*) Plate: completely constrained

(*b*) Reactions: determinate

(*c*) Equilibrium maintained

$$\mathbf{B} = 294 \text{ N}\longrightarrow, \quad \mathbf{D} = 491 \text{ N} \measuredangle 53.1^\circ$$

7. Two reactions:

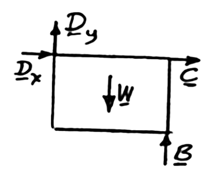


(*a*) Plate: improperly constrained

(*b*) Reactions determined by dynamics

(*c*) No equilibrium $(\Sigma F_y \neq 0)$

8. Four non-concurrent, non-parallel reactions:



(*a*) Plate: completely constrained

(*b*) Reactions: indeterminate

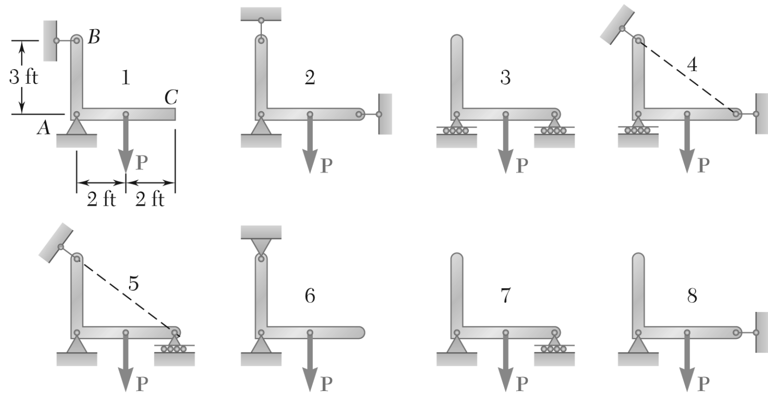
(*c*) Equilibrium maintained

$$\mathbf{B} = \mathbf{D}_y = 196.2 \text{ N}\uparrow$$

$$(\mathbf{C} + \mathbf{D}_x = 0)$$

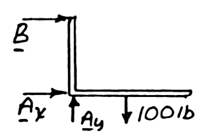
PROBLEM 4.60

The bracket *ABC* can be supported in the eight different ways shown. All connections consist of smooth pins, rollers, or short links. For each case, answer the questions listed in Problem 4.59, and, wherever possible, compute the reactions, assuming that the magnitude of the force **P** is 100 lb.



SOLUTION

1. Three non-concurrent, non-parallel reactions:



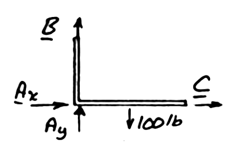
   (*a*) Bracket: complete constraint

   (*b*) Reactions: determinate

   (*c*) Equilibrium maintained

$$\mathbf{A} = 120.2 \text{ lb} \measuredangle 56.3°, \quad \mathbf{B} = 66.7 \text{ lb} \longleftarrow$$

2. Four concurrent, reactions (through *A*):

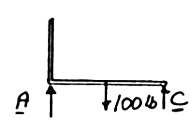


   (*a*) Bracket: improper constraint

   (*b*) Reactions: indeterminate

   (*c*) No equilibrium $(\Sigma M_A \neq 0)$

3. Two reactions:



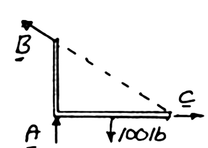
   (*a*) Bracket: partial constraint

   (*b*) Reactions: indeterminate

   (*c*) Equilibrium maintained

$$\mathbf{A} = 50 \text{ lb} \uparrow, \quad \mathbf{C} = 50 \text{ lb} \uparrow$$

4. Three non-concurrent, non-parallel reactions:

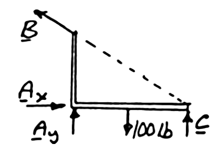


   (*a*) Bracket: complete constraint

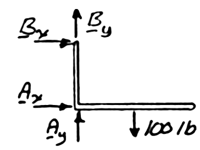
   (*b*) Reactions: determinate

   (*c*) Equilibrium maintained

$$\mathbf{A} = 50 \text{ lb} \uparrow, \quad \mathbf{B} = 83.3 \text{ lb} \searrow 36.9°, \quad \mathbf{C} = 66.7 \text{ lb} \longrightarrow$$



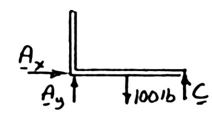
5. Four non-concurrent, non-parallel reactions:
   (*a*) Bracket: complete constraint
   (*b*) Reactions: indeterminate
   (*c*) Equilibrium maintained $(\Sigma M_C = 0)$ $\mathbf{A}_y = 50 \text{ lb}\uparrow$



6. Four non-concurrent, non-parallel reactions:
   (*a*) Bracket: complete constraint
   (*b*) Reactions: indeterminate
   (*c*) Equilibrium maintained

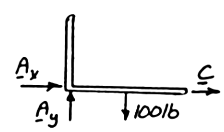
$$\mathbf{A}_x = 66.7 \text{ lb} \longrightarrow \quad \mathbf{B}_x = 66.7 \text{ lb} \longleftarrow$$

$$(\mathbf{A}_y + \mathbf{B}_y = 100 \text{ lb}\uparrow)$$

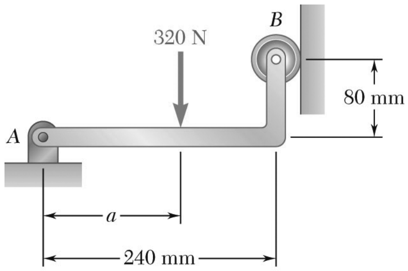


7. Three non-concurrent, non-parallel reactions:
   (*a*) Bracket: complete constraint
   (*b*) Reactions: determinate
   (*c*) Equilibrium maintained

$$\mathbf{A} = \mathbf{C} = 50 \text{ lb}\uparrow$$



8. Three concurrent, reactions (through *A*)
   (*a*) Bracket: improper constraint
   (*b*) Reactions: indeterminate
   (*c*) No equilibrium $(\Sigma M_A \neq 0)$

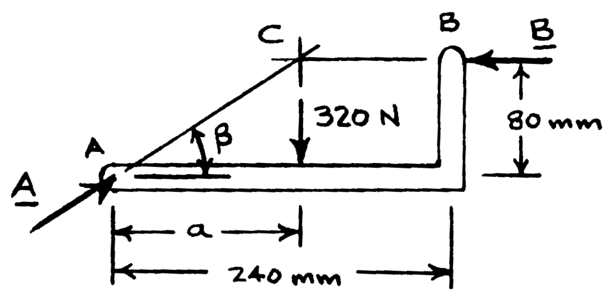


PROBLEM 4.61

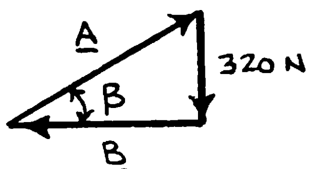
Determine the reactions at A and B when $a = 150$ mm.

SOLUTION

Free-Body Diagram:



Force triangle



$$\tan\beta = \frac{80 \text{ mm}}{a} = \frac{80 \text{ mm}}{150 \text{ mm}}$$

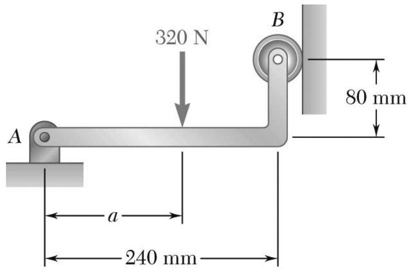
$$\beta = 28.072°$$

$$A = \frac{320 \text{ N}}{\sin 28.072°}$$

A = 680 N ⦣ 28.1° ◀

$$B = \frac{320 \text{ N}}{\tan 28.072°}$$

B = 600 N ⟵ ◀

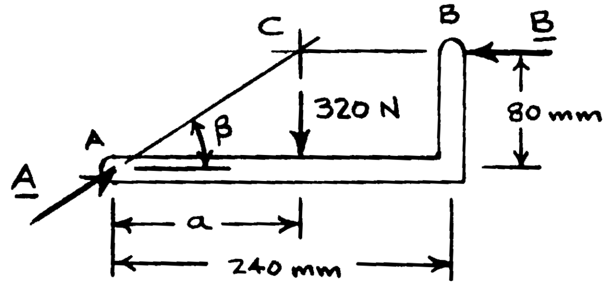


PROBLEM 4.62

Determine the value of a for which the magnitude of the reaction at B is equal to 800 N.

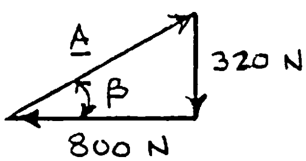
SOLUTION

Free-Body Diagram:



Force triangle

$$\tan\beta = \frac{80 \text{ mm}}{a} \qquad a = \frac{80 \text{ mm}}{\tan\beta} \tag{1}$$



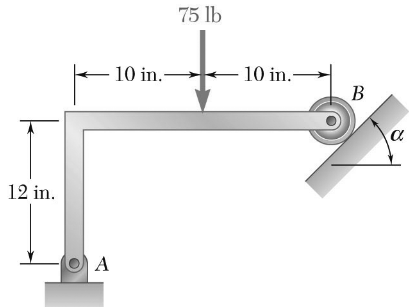
From force triangle:

$$\tan\beta = \frac{320 \text{ N}}{800 \text{ N}} = 0.4$$

From Eq. (1):

$$a = \frac{80 \text{ mm}}{0.4}$$

$a = 200$ mm ◀



PROBLEM 4.63

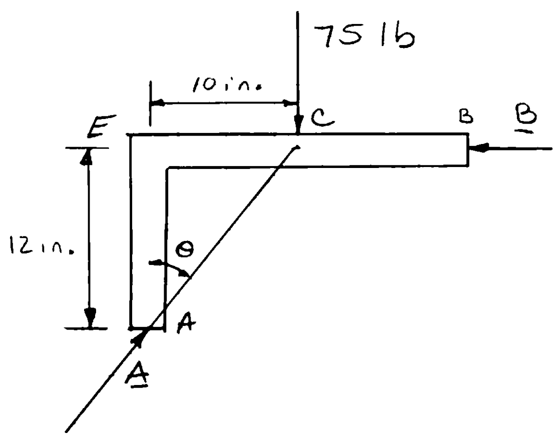
Using the method of Sec. 4.7, solve Problem 4.22*b*.

PROBLEM 4.22 Determine the reactions at *A* and *B* when (*a*) $\alpha = 0$, (*b*) $\alpha = 90°$, (*c*) $\alpha = 30°$.

SOLUTION

Free-Body Diagram:

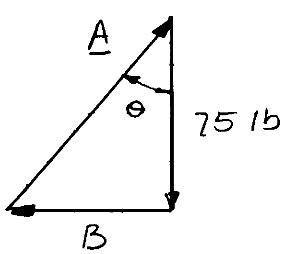
(Three-force body)



The line of action at *A* must pass through *C*, where **B** and the 75-lb load intersect.

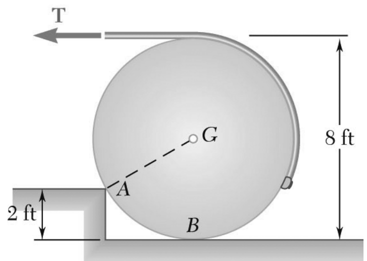
In triangle *ACE*: $\tan\theta = \dfrac{10 \text{ in.}}{12 \text{ in.}}$ $\qquad \theta = 39.806°$

Force triangle



$$B = (75 \text{ lb}) \tan 39.806°$$
$$= 62.5 \text{ lb}$$
$$A = \frac{75 \text{ lb}}{\cos 39.806°} = 97.6°$$

$\mathbf{A} = 97.6 \text{ lb} \measuredangle\ 50.2°; \ \mathbf{B} = 62.5 \text{ lb} \longleftarrow$ ◀

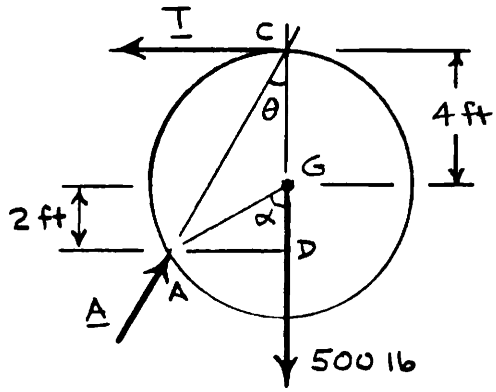


PROBLEM 4.64

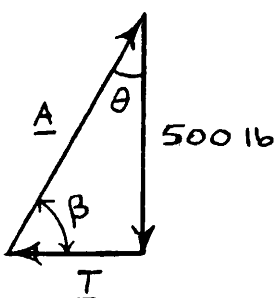
A 500-lb cylindrical tank, 8 ft in diameter, is to be raised over a 2-ft obstruction. A cable is wrapped around the tank and pulled horizontally as shown. Knowing that the corner of the obstruction at A is rough, find the required tension in the cable and the reaction at A.

SOLUTION

Free-Body Diagram:



Force triangle

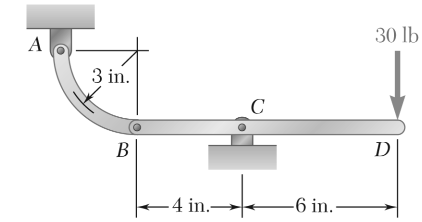


$$\cos\alpha = \frac{GD}{AG} = \frac{2\text{ ft}}{4\text{ ft}} = 0.5 \qquad \alpha = 60°$$

$$\theta = \frac{1}{2}\alpha = 30° \qquad (\beta = 60°)$$

$$T = (500\text{ lb})\tan 30° \qquad T = 289\text{ lb}$$

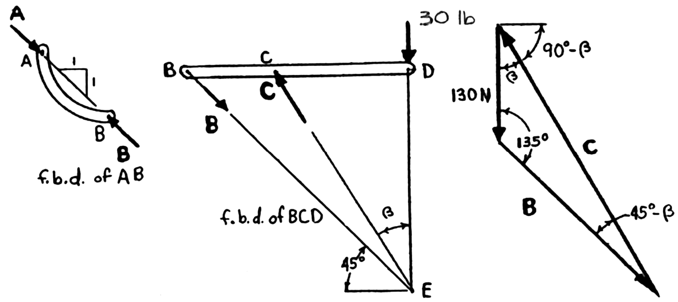
$$A = \frac{500\text{ lb}}{\cos 30°} \qquad \mathbf{A} = 577\text{ lb} \measuredangle\, 60.0°$$



PROBLEM 4.65

For the frame and loading shown, determine the reactions at A and C.

SOLUTION



Since member AB is acted upon by two forces, **A** and **B**, they must be colinear, have the same magnitude, and be opposite in direction for AB to be in equilibrium. The force **B** acting at B of member BCD will be equal in magnitude but opposite in direction to force **B** acting on member AB. Member BCD is a three-force body with member forces intersecting at E. The F.B.D.'s of members AB and BCD illustrate the above conditions. The force triangle for member BCD is also shown. The angle β is found from the member dimensions:

$$\beta = \tan^{-1}\left(\frac{6 \text{ in.}}{10 \text{ in.}}\right) = 30.964°$$

Applying the law of sines to the force triangle for member BCD,

$$\frac{30 \text{ lb}}{\sin(45° - \beta)} = \frac{B}{\sin\beta} = \frac{C}{\sin 135°}$$

or

$$\frac{30 \text{ lb}}{\sin 14.036°} = \frac{B}{\sin 30.964°} = \frac{C}{\sin 135°}$$

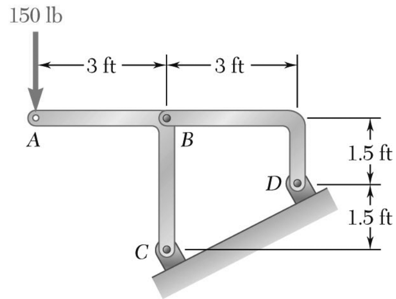
$$A = B = \frac{(30 \text{ lb})\sin 30.964°}{\sin 14.036°} = 63.641 \text{ lb}$$

or $\mathbf{A} = 63.6 \text{ lb} \nwarrow 45.0°$ ◀

and

$$C = \frac{(30 \text{ lb})\sin 135°}{\sin 14.036°} = 87.466 \text{ lb}$$

or $\mathbf{C} = 87.5 \text{ lb} \searrow 59.0°$ ◀



PROBLEM 4.66

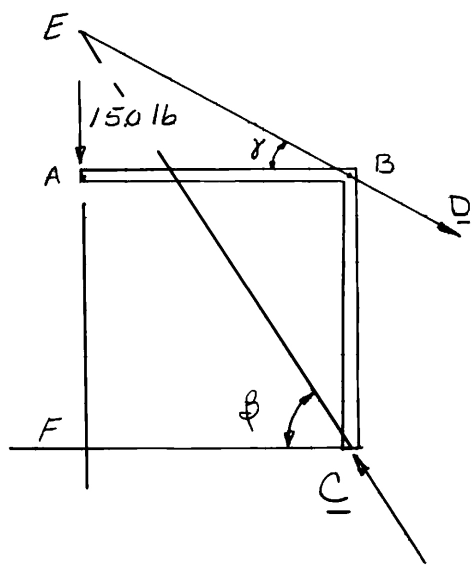
For the frame and loading shown, determine the reactions at C and D.

SOLUTION

Since BD is a two-force member, the reaction at D must pass through Points B and D.

Free-Body Diagram:

(Three-force body)



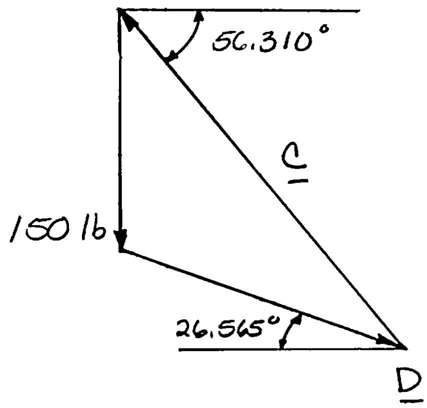
Reaction at C must pass through E, where the reaction at D and the 150-lb load intersect.

Triangle *CEF*: $\tan \beta = \dfrac{4.5 \text{ ft}}{3 \text{ ft}}$ $\beta = 56.310°$

Triangle *ABE*: $\tan \gamma = \dfrac{1}{2}$ $\gamma = 26.565°$

PROBLEM 4.66 (Continued)

Force Triangle

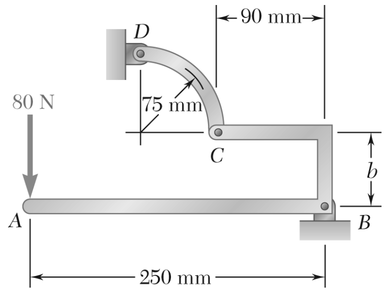


Law of sines:
$$\frac{150\text{ lb}}{\sin 29.745°} = \frac{C}{\sin 116.565°} = \frac{D}{\sin 33.690°}$$

$$C = 270.42\text{ lb},$$

$$D = 167.704\text{ lb}$$

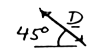
C = 270 lb ⦪ 56.3°; **D** = 167.7 lb ⦩ 26.6° ◀



PROBLEM 4.67

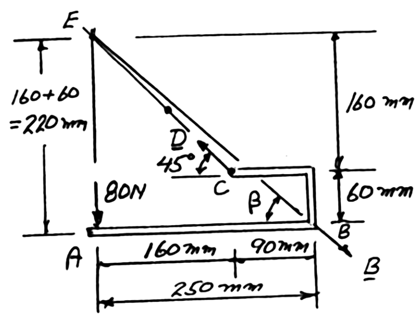
Determine the reactions at B and D when $b = 60$ mm.

SOLUTION

Since CD is a two-force member, the line of action of reaction at D must pass through Points C and D. 

Free-Body Diagram:

(Three-force body)

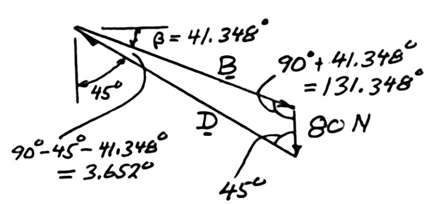


Reaction at B must pass through E, where the reaction at D and the 80-N force intersect.

$$\tan\beta = \frac{220 \text{ mm}}{250 \text{ mm}}$$

$$\beta = 41.348°$$

<u>Force triangle</u>



Law of sines:

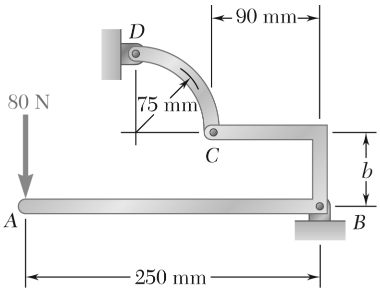
$$\frac{80 \text{ N}}{\sin 3.652°} = \frac{B}{\sin 45°} = \frac{D}{\sin 131.348°}$$

$$B = 888.0 \text{ N}$$

$$D = 942.8 \text{ N}$$

$\mathbf{B} = 888$ N ↖ 41.3° $\mathbf{D} = 943$ N ↘ 45.0° ◀

PROBLEM 4.68



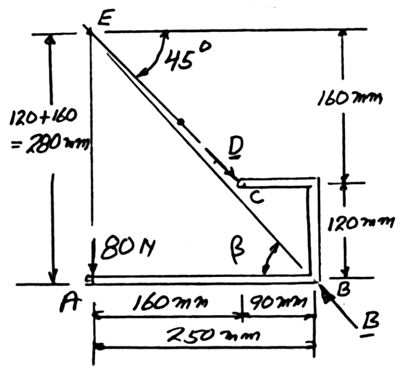
Determine the reactions at B and D when $b = 120$ mm.

SOLUTION

Since CD is a two-force member, line of action of reaction at D must pass through C and D 45°.

Free-Body Diagram:

(Three-force body)

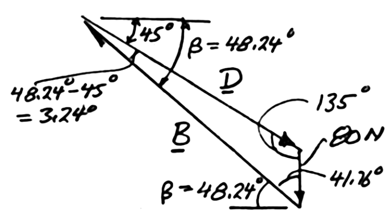


Reaction at B must pass through E, where the reaction at D and the 80-N force intersect.

$$\tan\beta = \frac{280 \text{ mm}}{250 \text{ mm}}$$

$$\beta = 48.24°$$

Force triangle



Law of sines:

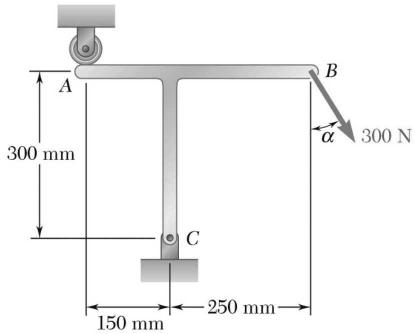
$$\frac{80 \text{ N}}{\sin 3.24°} = \frac{B}{\sin 135°} = \frac{D}{\sin 41.76°}$$

$$B = 1000.9 \text{ N}$$

$$D = 942.8 \text{ N}$$

$\mathbf{B} = 1001$ N 48.2° $\mathbf{D} = 943$ N 45.0° ◀

PROBLEM 4.69

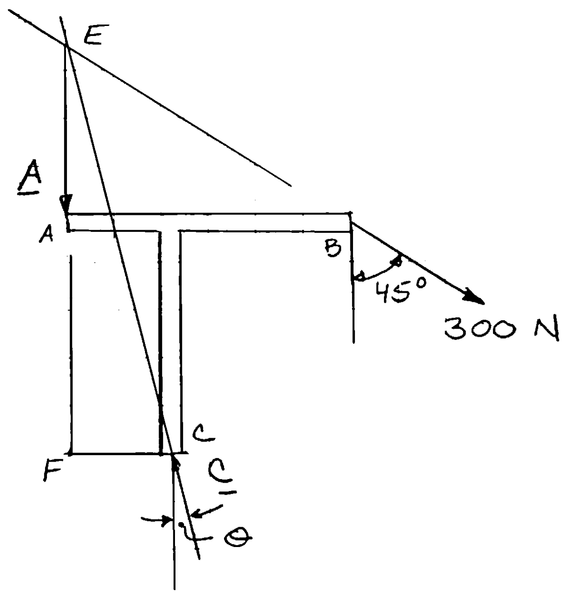


A T-shaped bracket supports a 300-N load as shown. Determine the reactions at A and C when $\alpha = 45°$.

SOLUTION

Free-Body Diagram:

(Three-force body)



The line of action of **C** must pass through E, where **A** and the 300-N force intersect.

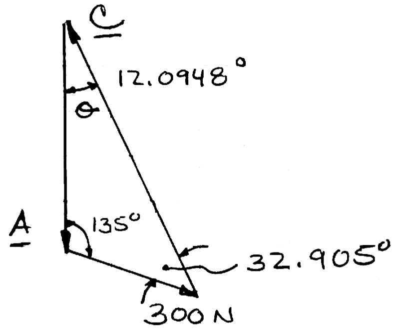
Triangle ABE is isosceles: $EA = AB = 400 \text{ mm}$

In triangle CEF:

$$\tan\theta = \frac{CF}{EF} = \frac{CF}{EA + AF} = \frac{150 \text{ mm}}{700 \text{ mm}} \qquad \theta = 12.0948°$$

PROBLEM 4.69 (Continued)

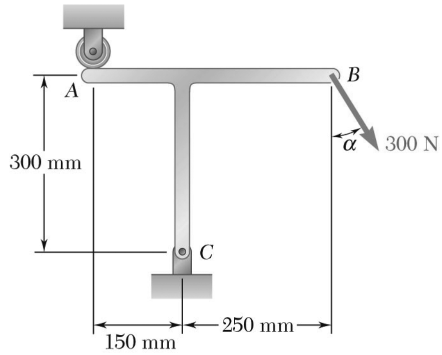
Force Triangle



Law of sines: $$\frac{A}{\sin 32.905°} = \frac{C}{\sin 135°} = \frac{300\text{ N}}{\sin 12.0948°}$$

A = 778 N ↓; **C** = 1012 N ⦪ 77.9° ◀

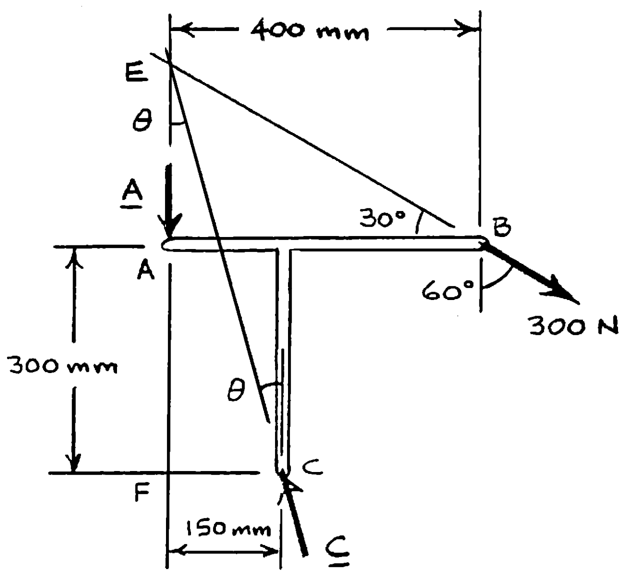
PROBLEM 4.70



A T-shaped bracket supports a 300-N load as shown. Determine the reactions at A and C when $\alpha = 60°$.

SOLUTION

Free-Body Diagram:



$$EA = (400 \text{ mm}) \tan 30°$$
$$= 230.94 \text{ mm}$$

In triangle CEF:

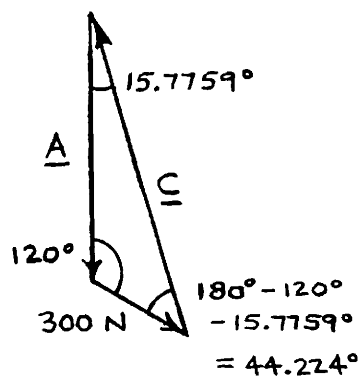
$$\tan\theta = \frac{CF}{EF} = \frac{CF}{EA + AF}$$

$$\tan\theta = \frac{150}{230.94 + 300}$$

$$\theta = 15.7759°$$

PROBLEM 4.70 (Continued)

Force Triangle



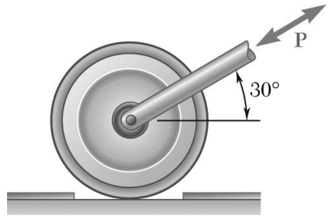
Law of sines:

$$\frac{A}{\sin 44.224^\circ} = \frac{C}{\sin 120^\circ} = \frac{300\text{ N}}{\sin 15.7759^\circ}$$

$$A = 770\text{ N}$$

$$C = 956\text{ N}$$

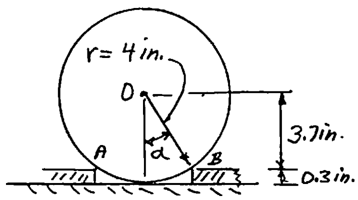
$\mathbf{A} = 770\text{ N}\downarrow$; $\mathbf{C} = 956\text{ N} \searrow 74.2^\circ$ ◀



PROBLEM 4.71

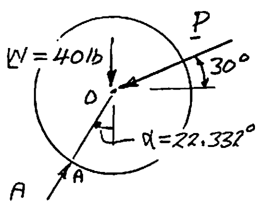
A 40-lb roller, of diameter 8 in., which is to be used on a tile floor, is resting directly on the subflooring as shown. Knowing that the thickness of each tile is 0.3 in., determine the force **P** required to move the roller onto the tiles if the roller is (*a*) pushed to the left, (*b*) pulled to the right.

SOLUTION



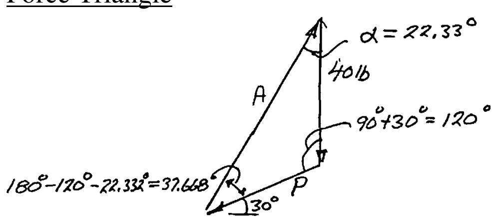
Geometry: For each case as roller comes into contact with tile,

$$\alpha = \cos^{-1}\frac{3.7\text{ in.}}{4\text{ in.}}$$
$$\alpha = 22.332°$$



(*a*) Roller pushed to left (three-force body):
Forces must pass through *O*.

Force Triangle



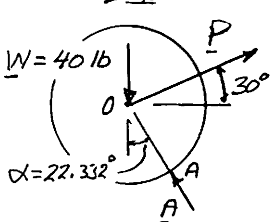
Law of sines: $\dfrac{40\text{ lb}}{\sin 37.668°} = \dfrac{P}{\sin 22.332°}$; $P = 24.87$ lb

$\mathbf{P} = 24.9$ lb ↗ 30.0° ◀

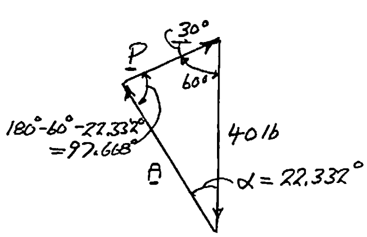
(*b*) Roller pulled to right (three-force body):
Forces must pass through *O*.

Law of sines: $\dfrac{40\text{ lb}}{\sin 97.668°} = \dfrac{P}{\sin 22.332°}$; $P = 15.3361$ lb

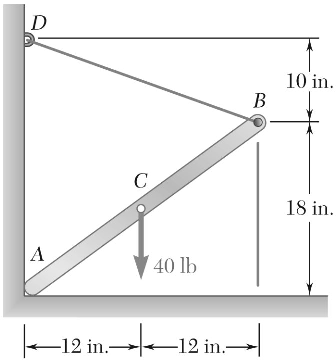
$\mathbf{P} = 15.34$ lb ↗ 30.0° ◀



Force Triangle



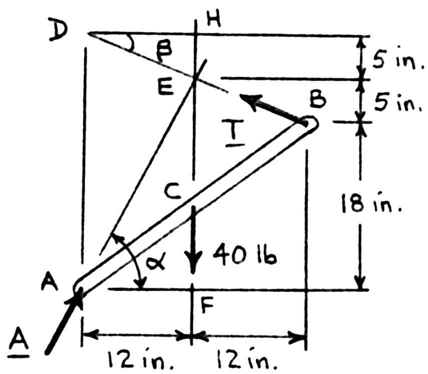
PROBLEM 4.72



One end of rod *AB* rests in the corner *A* and the other end is attached to cord *BD*. If the rod supports a 40-lb load at its midpoint *C*, find the reaction at *A* and the tension in the cord.

SOLUTION

Free-Body Diagram: (Three-force body)



The line of action of reaction at *A* must pass through *E*, where **T** and the 40-lb load intersect.

$$\tan\alpha = \frac{EF}{AF} = \frac{23}{12}$$

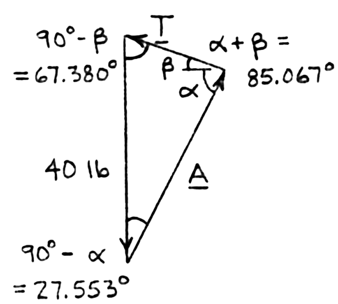
$$\alpha = 62.447°$$

$$\tan\beta = \frac{EH}{DH} = \frac{5}{12}$$

$$\beta = 22.620°$$

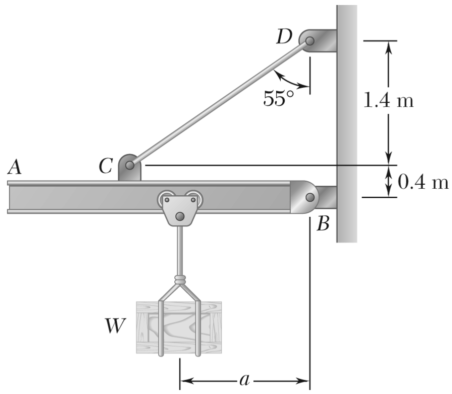
$$\frac{A}{\sin 67.380°} = \frac{T}{\sin 27.553°} = \frac{40\text{ lb}}{\sin 85.067°}$$

Force triangle



A = 37.1 lb ∠ 62.4° ◀

T = 18.57 lb ◀



PROBLEM 4.73

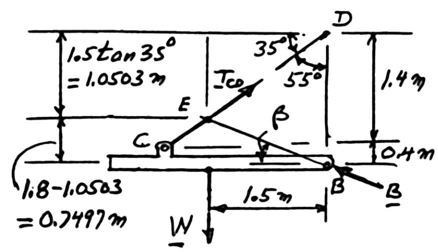
A 50-kg crate is attached to the trolley-beam system shown. Knowing that $a = 1.5$ m, determine (*a*) the tension in cable *CD*, (*b*) the reaction at *B*.

SOLUTION

Three-force body: **W** and $\mathbf{T}_{CD}$ intersect at *E*.

$$\tan\beta = \frac{0.7497 \text{ m}}{1.5 \text{ m}}$$

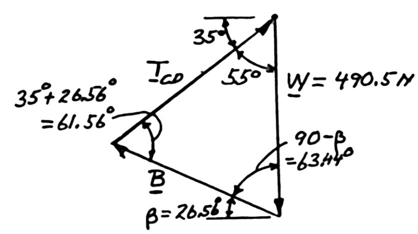
$$\beta = 26.556^\circ$$



Three forces intersect at *E*.

$$W = (50 \text{ kg})\ 9.81 \text{ m/s}^2$$
$$= 490.50 \text{ N}$$

Force triangle



Law of sines:

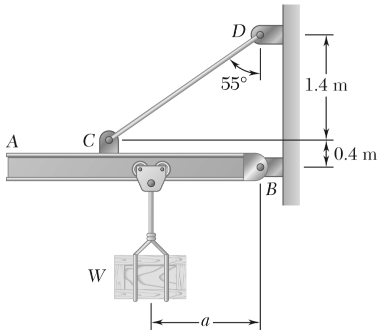
$$\frac{490.50 \text{ N}}{\sin 61.556^\circ} = \frac{T_{CD}}{\sin 63.444^\circ} = \frac{B}{\sin 55^\circ}$$

$$T_{CD} = 498.99 \text{ N}$$

$$B = 456.96 \text{ N}$$

(*a*) $T_{CD} = 499$ N ◀

(*b*) $\mathbf{B} = 457 \text{ N} \measuredangle 26.6^\circ$ ◀



PROBLEM 4.74

Solve Problem 4.73, assuming that $a = 3$ m.

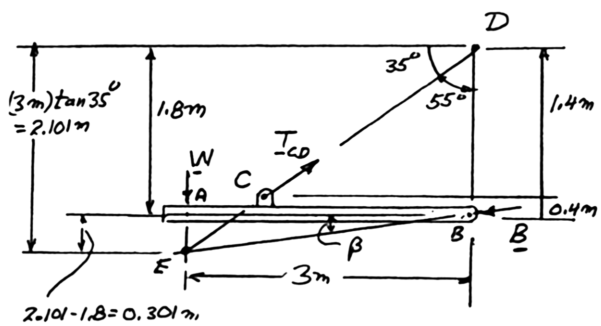
PROBLEM 4.73 A 50-kg crate is attached to the trolley-beam system shown. Knowing that $a = 1.5$ m, determine (*a*) the tension in cable *CD*, (*b*) the reaction at *B*.

SOLUTION

W and $\mathbf{T}_{CD}$ intersect at *E*.

Free-Body Diagram:

Three-Force Body

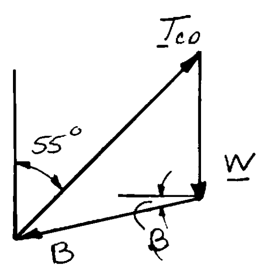


$$\tan \beta = \frac{AE}{AB} = \frac{0.301 \text{ m}}{3 \text{ m}}$$

$$\beta = 5.7295^\circ$$

Three forces intersect at *E*.

Force Triangle



$$W = (50 \text{ kg})\, 9.81 \text{ m/s}^2$$
$$= 490.50 \text{ N}$$

Law of sines:

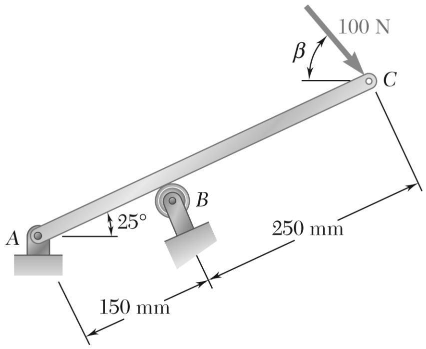
$$\frac{490.50 \text{ N}}{\sin 29.271^\circ} = \frac{T_{CD}}{\sin 95.730^\circ} = \frac{B}{\sin 55^\circ}$$

$$T_{CD} = 998.18 \text{ N}$$

$$B = 821.76 \text{ N}$$

(*a*) $T_{CD} = 998 \text{ N}$ ◀

(*b*) $\mathbf{B} = 822 \text{ N} \measuredangle\ 5.73^\circ$ ◀

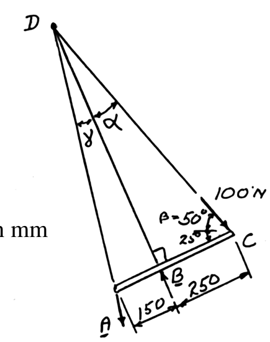


PROBLEM 4.75

Determine the reactions at A and B when $\beta = 50°$.

SOLUTION

Free-Body Diagram: (Three-force body)



Dimensions in mm

Reaction **A** must pass through Point D where the 100-N force and **B** intersect.

In right $\Delta\, BCD$:

$$\alpha = 90° - 75° = 15°$$

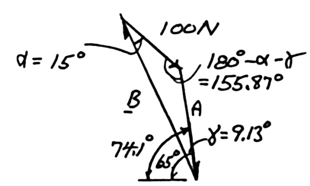
$$BD = 250 \tan 75° = 933.01 \text{ mm}$$

In right $\Delta\, ABD$:

$$\tan\gamma = \frac{AB}{BD} = \frac{150 \text{ mm}}{933.01 \text{ mm}}$$

$$\gamma = 9.1333°$$

Force Triangle

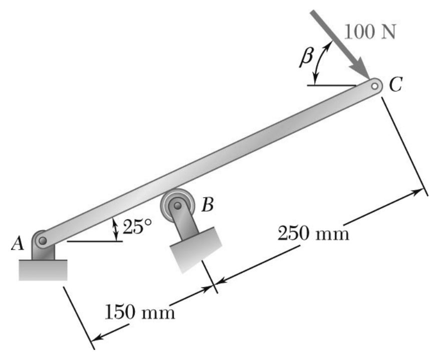


Law of sines:

$$\frac{100 \text{ N}}{\sin 9.1333°} = \frac{A}{\sin 15°} = \frac{B}{\sin 155.867°}$$

$$A = 163.1 \text{ N}; \quad B = 257.6 \text{ N}$$

A $= 163.1$ N ⦨ $74.1°$ **B** $= 258$ N ⦩ $65.0°$ ◀



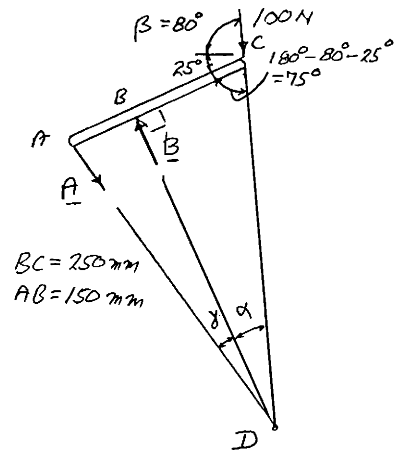
PROBLEM 4.76

Determine the reactions at A and B when $\beta = 80°$.

SOLUTION

Free-Body Diagram:

(Three-force body)



Reaction **A** must pass through D where the 100-N force and **B** intersect.

In right triangle BCD:

$$\alpha = 90° - 75° = 15°$$

$$BD = BC \tan 75° = 250 \tan 75°$$

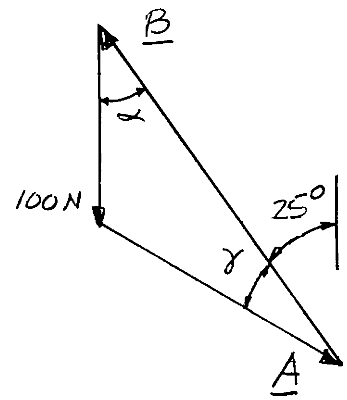
$$BD = 933.01 \text{ mm}$$

In right triangle ABD:

$$\tan \gamma = \frac{AB}{BD} = \frac{150 \text{ mm}}{933.01 \text{ mm}} \qquad \gamma = 9.1333°$$

PROBLEM 4.76 (Continued)

Force Triangle



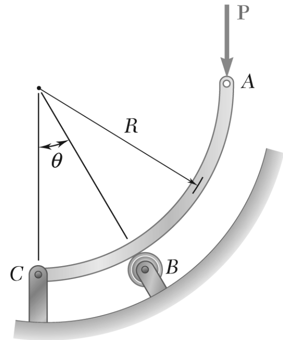
Law of sines:

$$\frac{100\text{ N}}{\sin 9.1333°} = \frac{A}{\sin 15°} = \frac{B}{\sin 155.867°}$$

A $= 163.1$ N ⦪ 55.9° ◀

B $= 258$ N ⦩ 65.0° ◀

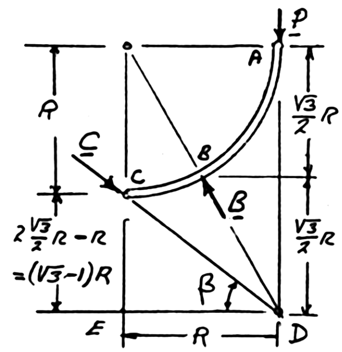
PROBLEM 4.77



Knowing that $\theta = 30°$, determine the reaction (*a*) at *B*, (*b*) at *C*.

SOLUTION

Free-Body Diagram:
(Three-force body)



Reaction at *C* must pass through *D* where force **P** and reaction at *B* intersect.

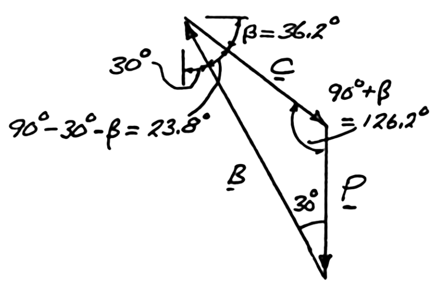
In Δ *CDE*:

$$\tan \beta = \frac{(\sqrt{3}-1)R}{R}$$

$$= \sqrt{3} - 1$$

$$\beta = 36.2°$$

Force Triangle



Law of sines:

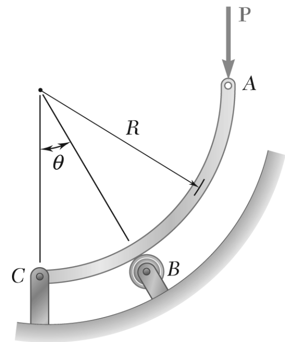
$$\frac{P}{\sin 23.8°} = \frac{B}{\sin 126.2°} = \frac{C}{\sin 30°}$$

$$B = 2.00P$$

$$C = 1.239P$$

(*a*) **B** $= 2P$ ⦨ $60.0°$ ◀

(*b*) **C** $= 1.239P$ ⦩ $36.2°$ ◀



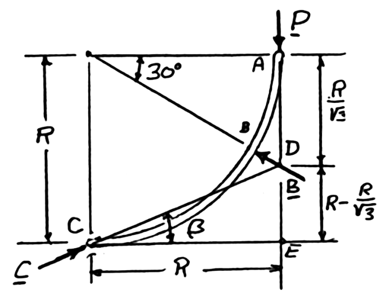
PROBLEM 4.78

Knowing that $\theta = 60°$, determine the reaction (*a*) at *B*, (*b*) at *C*.

SOLUTION

Reaction at *C* must pass through *D* where force **P** and reaction at *B* intersect.

Free-Body Diagram:
(Three-force body)



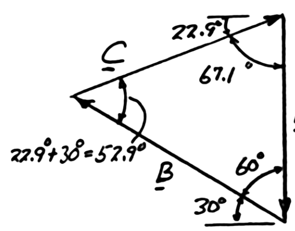
In ΔCDE:

$$\tan\beta = \frac{R - \frac{R}{\sqrt{3}}}{R}$$

$$= 1 - \frac{1}{\sqrt{3}}$$

$$\beta = 22.9°$$

Force Triangle



Law of sines:

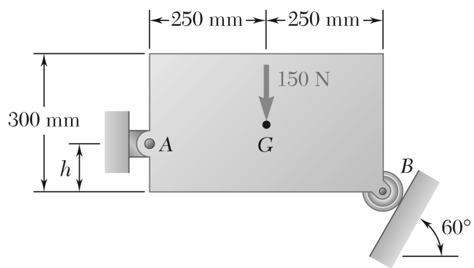
$$\frac{P}{\sin 52.9°} = \frac{B}{\sin 67.1°} = \frac{C}{\sin 60°}$$

$$B = 1.155P$$

$$C = 1.086P$$

(*a*) $\mathbf{B} = 1.155P \measuredangle 30.0°$ ◀

(*b*) $\mathbf{C} = 1.086P \measuredangle 22.9°$ ◀



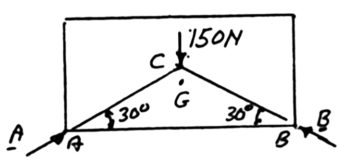
PROBLEM 4.79

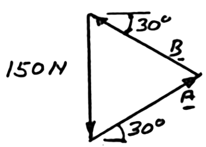
Using the method of Section 4.7, solve Problem 4.23.

PROBLEM 4.23 Determine the reactions at A and B when (*a*) $h = 0$, (*b*) $h = 200$ mm.

SOLUTION

Free-Body Diagram:





(*a*) <u>$h = 0$</u>

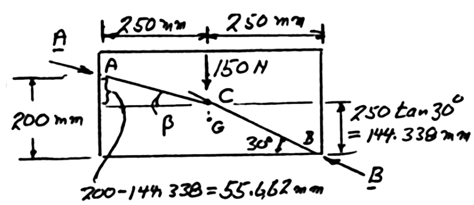
Reaction **A** must pass through C where the 150-N weight and **B** interect.

Force triangle is equilateral.

$\mathbf{A} = 150.0\text{ N} \measuredangle 30.0°$ ◀

$\mathbf{B} = 150.0\text{ N} \searrow 30.0°$ ◀

(*b*) <u>$h = 200$ mm</u>



$$\tan\beta = \frac{55.662}{250}$$

$$\beta = 12.5521°$$

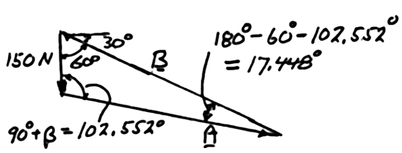
Law of sines:

$$\frac{150\text{ N}}{\sin 17.4480°} = \frac{A}{\sin 60°} = \frac{B}{\sin 102.552°}$$

$$A = 433.24\text{ N}$$

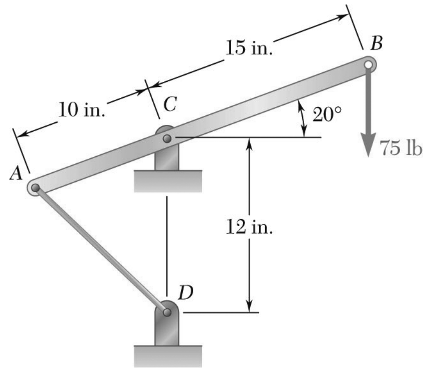
$$B = 488.31\text{ N}$$

<u>Force Triangle</u>



$\mathbf{A} = 433\text{ N} \nwarrow 12.55°$ ◀

$\mathbf{B} = 488\text{ N} \searrow 30.0°$ ◀

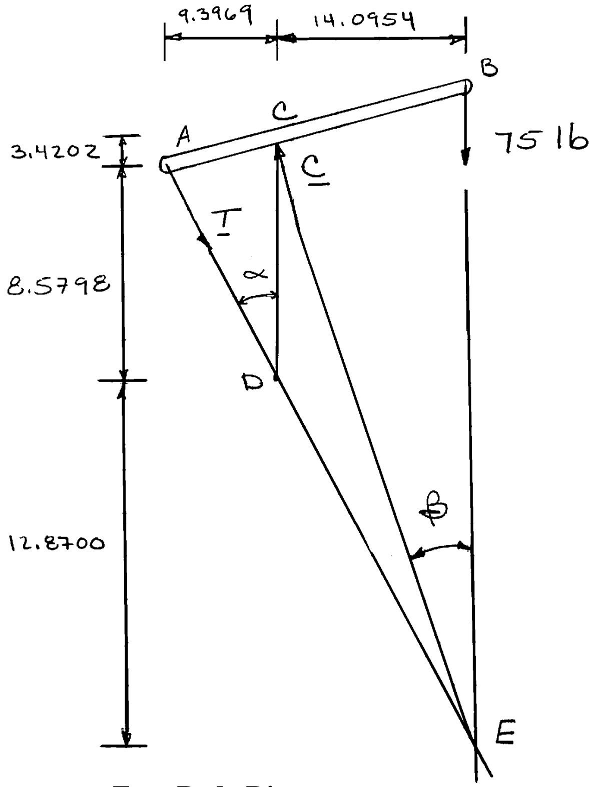


PROBLEM 4.80

Using the method of Section 4.7, solve Problem 4.24.

PROBLEM 4.24 A lever *AB* is hinged at *C* and attached to a control cable at *A*. If the lever is subjected to a 75-lb vertical force at *B*, determine (*a*) the tension in the cable, (*b*) the reaction at *C*.

SOLUTION



Free-Body Diagram:

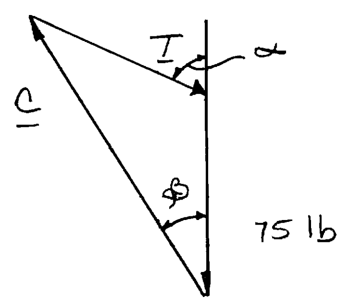
Dimensions in in.

Reaction at *C* must pass through *E*, where the 75-lb force and **T** intersect.

$$\tan\alpha = \frac{9.3969 \text{ in.}}{8.5798 \text{ in.}}$$
$$\alpha = 47.602°$$

$$\tan\beta = \frac{14.0954 \text{ in.}}{24.870 \text{ in.}}$$
$$\beta = 29.543°$$

Force Triangle

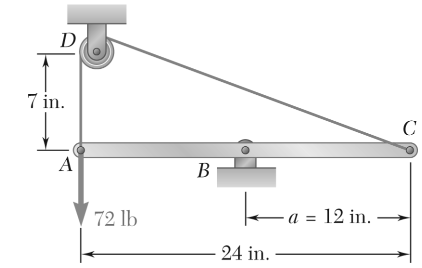


Law of sines:

$$\frac{75 \text{ lb}}{\sin 18.0590°} = \frac{T}{\sin 29.543°} = \frac{C}{\sin 132.398°}$$

(*a*) $T = 119.3$ lb ◀

(*b*) $\mathbf{C} = 178.7$ lb ⦤ 60.5° ◀



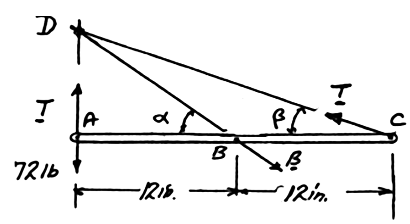
PROBLEM 4.81

Member *ABC* is supported by a pin and bracket at *B* and by an inextensible cord attached at *A* and *C* and passing over a frictionless pulley at *D*. The tension may be assumed to be the same in portions *AD* and *CD* of the cord. For the loading shown and neglecting the size of the pulley, determine the tension in the cord and the reaction at *B*.

SOLUTION

Reaction at *B* must pass through *D*.

Free-Body Diagram:



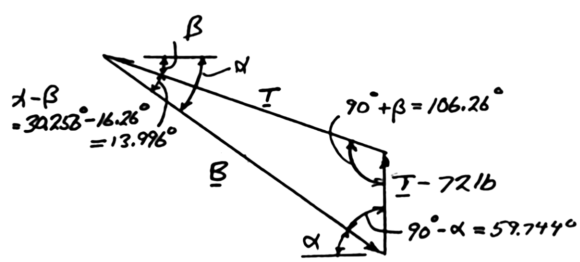
$$\tan\alpha = \frac{7\text{ in.}}{12\text{ in.}}$$

$$\alpha = 30.256°$$

$$\tan\beta = \frac{7\text{ in.}}{24\text{ in.}}$$

$$\beta = 16.26°$$

Force Triangle



Law of sines:

$$\frac{T}{\sin 59.744°} = \frac{T - 72\text{ lb}}{\sin 13.996°} = \frac{B}{\sin 106.26}$$

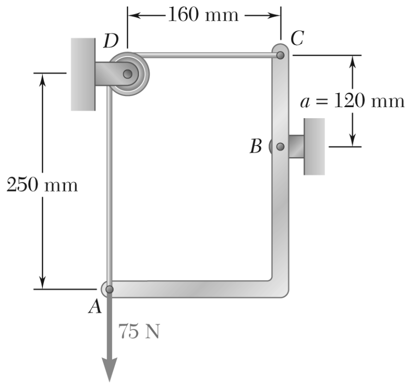
$$T(\sin 13.996°) = (T - 72\text{ lb})(\sin 59.744°)$$

$$T(0.24185) = (T - 72)(0.86378)$$

$$T = 100.00\text{ lb} \qquad T = 100.0\text{ lb} \blacktriangleleft$$

$$B = (100\text{ lb})\frac{\sin 106.26°}{\sin 59.744°}$$

$$= 111.14\text{ lb} \qquad \mathbf{B} = 111.1\text{ lb} \nwarrow 30.3° \blacktriangleleft$$

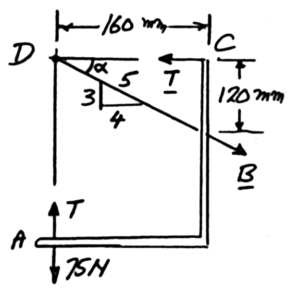


PROBLEM 4.82

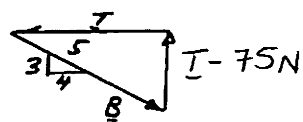
Member *ABC* is supported by a pin and bracket at *B* and by an inextensible cord attached at *A* and *C* and passing over a frictionless pulley at *D*. The tension may be assumed to be the same in portions *AD* and *CD* of the cord. For the loading shown and neglecting the size of the pulley, determine the tension in the cord and the reaction at *B*.

SOLUTION

Free-Body Diagram:



Force Triangle

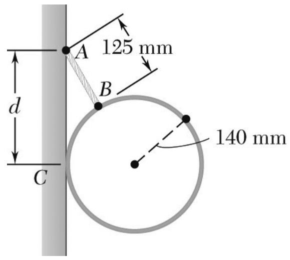


Reaction at *B* must pass through *D*.

$$\tan\alpha = \frac{120}{160}; \quad \alpha = 36.9^\circ \qquad \frac{T}{4} = \frac{T - 75\text{ N}}{3} = \frac{B}{5}$$

$$3T = 4T - 300; \quad T = 300\text{ N}$$

$$B = \frac{5}{4}T = \frac{5}{4}(300\text{ N}) = 375\text{ N} \qquad \mathbf{B} = 375\text{ N} \searrow 36.9^\circ$$



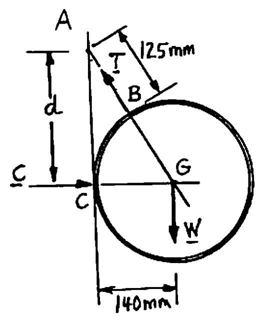
PROBLEM 4.83

A thin ring of mass 2 kg and radius $r = 140$ mm is held against a frictionless wall by a 125-mm string AB. Determine (*a*) the distance d, (*b*) the tension in the string, (*c*) the reaction at C.

SOLUTION

Free-Body Diagram:

(Three-force body)



The force T exerted at B must pass through the center G of the ring, since **C** and **W** intersect at that point.

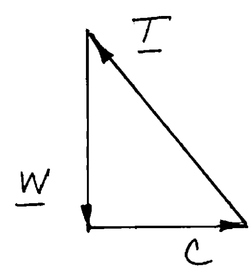
Thus, points A, B, and G are in a straight line.

(*a*) From triangle ACG:

$$d = \sqrt{(AG)^2 - (CG)^2}$$
$$= \sqrt{(265 \text{ mm})^2 - (140 \text{ mm})^2}$$
$$= 225.00 \text{ mm}$$

$d = 225$ mm ◀

Force Triangle



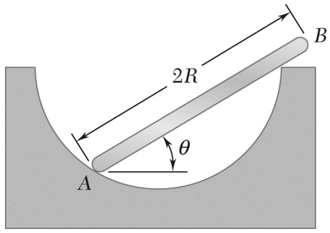
$$W = (2 \text{ kg})(9.81 \text{ m/s}^2) = 19.6200 \text{ N}$$

Law of sines:

$$\frac{T}{265 \text{ mm}} = \frac{C}{140 \text{ mm}} = \frac{19.6200 \text{ N}}{225.00 \text{ mm}}$$

(*b*) $T = 23.1$ N ◀

(*c*) $\mathbf{C} = 12.21 \text{ N} \longrightarrow$ ◀



PROBLEM 4.84

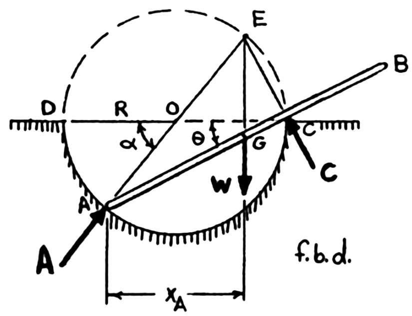
A uniform rod AB of length $2R$ rests inside a hemispherical bowl of radius R as shown. Neglecting friction, determine the angle θ corresponding to equilibrium.

SOLUTION

Based on the F.B.D., the uniform rod AB is a three-force body. Point E is the point of intersection of the three forces. Since force A passes through O, the center of the circle, and since force **C** is perpendicular to the rod, triangle ACE is a right triangle inscribed in the circle. Thus, E is a point on the circle.

Note that the angle α of triangle DOA is the central angle corresponding to the inscribed angle θ of triangle DCA.

$$\alpha = 2\theta$$



The horizontal projections of $AE, (x_{AE})$, and $AG, (x_{AG})$, are equal.

$$x_{AE} = x_{AG} = x_A$$

or $$(AE)\cos 2\theta = (AG)\cos\theta$$

and $$(2R)\cos 2\theta = R\cos\theta$$

Now $$\cos 2\theta = 2\cos^2\theta - 1$$

then $$4\cos^2\theta - 2 = \cos\theta$$

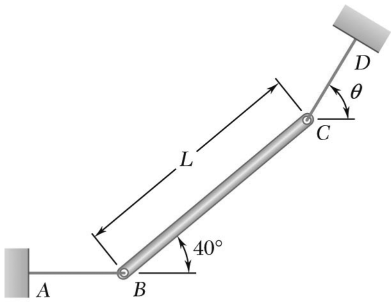
or $$4\cos^2\theta - \cos\theta - 2 = 0$$

Applying the quadratic equation,

$$\cos\theta = 0.84307 \quad \text{and} \quad \cos\theta = -0.59307$$

$$\theta = 32.534° \quad \text{and} \quad \theta = 126.375° \quad \text{(Discard)}$$

or $\theta = 32.5°$ ◀



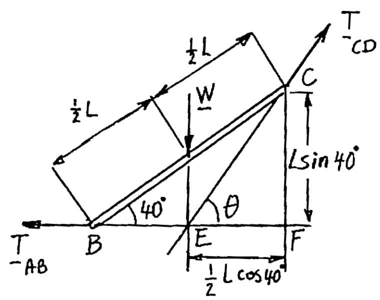
PROBLEM 4.85

A slender rod *BC* of length *L* and weight *W* is held by two cables as shown. Knowing that cable *AB* is horizontal and that the rod forms an angle of 40° with the horizontal, determine (*a*) the angle θ that cable *CD* forms with the horizontal, (*b*) the tension in each cable.

SOLUTION

Free-Body Diagram:

(Three-force body)



(*a*) The line of action of $\mathbf{T}_{CD}$ must pass through *E*, where $\mathbf{T}_{AB}$ and $\mathbf{W}$ intersect.

$$\tan\theta = \frac{CF}{EF}$$

$$= \frac{L\sin 40°}{\frac{1}{2}L\cos 40°}$$

$$= 2\tan 40°$$

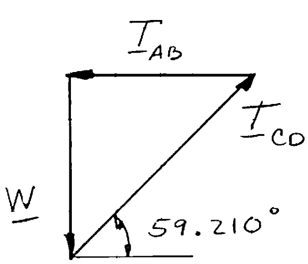
$$= 59.210°$$

$\theta = 59.2°$ ◀

(b) Force Triangle

$$T_{AB} = W\tan 30.790°$$

$$= 0.59588W$$

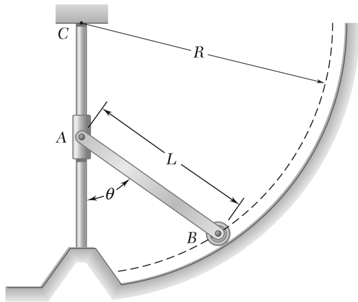


$T_{AB} = 0.596W$ ◀

$$T_{CD} = \frac{W}{\cos 30.790°}$$

$$= 1.16408W$$

$T_{CD} = 1.164W$ ◀

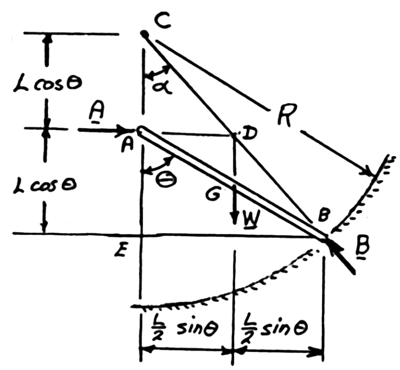


PROBLEM 4.86

A slender rod of length L and weight W is attached to a collar at A and is fitted with a small wheel at B. Knowing that the wheel rolls freely along a cylindrical surface of radius R, and neglecting friction, derive an equation in θ, L, and R that must be satisfied when the rod is in equilibrium.

SOLUTION

Free-Body Diagram (Three-force body)



Reaction **B** must pass through D where **B** and **W** intersect.

Note that ΔABC and ΔBGD are similar.

$$AC = AE = L\cos\theta$$

In ΔABC:

$$(CE)^2 + (BE)^2 = (BC)^2$$

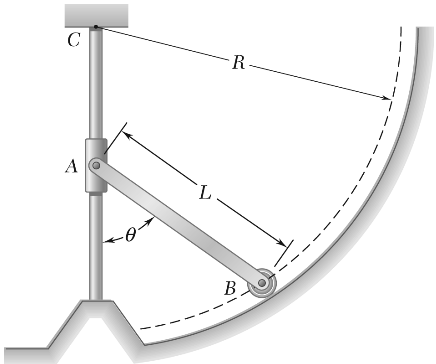
$$(2L\cos\theta)^2 + (L\sin\theta)^2 = R^2$$

$$\left(\frac{R}{L}\right)^2 = 4\cos^2\theta + \sin^2\theta$$

$$\left(\frac{R}{L}\right)^2 = 4\cos^2\theta + 1 - \cos^2\theta$$

$$\left(\frac{R}{L}\right)^2 = 3\cos^2\theta + 1$$

$$\cos^2\theta = \frac{1}{3}\left[\left(\frac{R}{L}\right)^2 - 1\right] \blacktriangleleft$$



PROBLEM 4.87

Knowing that for the rod of Problem 4.86, $L = 15$ in., $R = 20$ in., and $W = 10$ lb, determine (*a*) the angle θ corresponding to equilibrium, (*b*) the reactions at *A* and *B*.

SOLUTION

See the solution to Problem 4.86 for the free-body diagram and analysis leading to the following equation:

$$\cos^2\theta = \frac{1}{3}\left[\left(\frac{R}{L}\right)^2 - 1\right]$$

For $L = 15$ in., $R = 20$ in., and $W = 10$ lb,

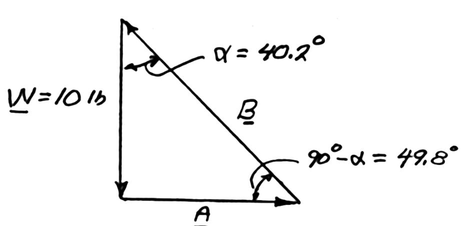
(*a*) $$\cos^2\theta = \frac{1}{3}\left[\left(\frac{20\text{ in.}}{15\text{ in.}}\right)^2 - 1\right];\quad \theta = 59.39°$$ $\theta = 59.4°$ ◀

In ΔABC: $$\tan\alpha = \frac{BE}{CE} = \frac{L\sin\theta}{2L\cos\theta} = \frac{1}{2}\tan\theta$$

$$\tan\alpha = \frac{1}{2}\tan 59.39° = 0.8452$$

$$\alpha = 40.2°$$

Force Triangle



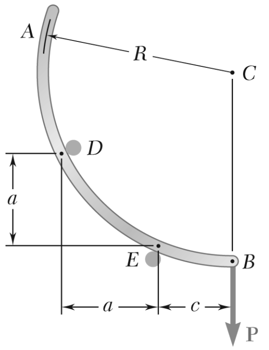
$$A = W\tan\alpha = (10\text{ lb})\tan 40.2° = 8.45\text{ lb}$$

$$B = \frac{W}{\cos\alpha} = \frac{(10\text{ lb})}{\cos 40.2°} = 13.09\text{ lb}$$

(*b*) $\mathbf{A} = 8.45$ lb ⟶ ◀

$\mathbf{B} = 13.09$ lb ↖ 49.8° ◀

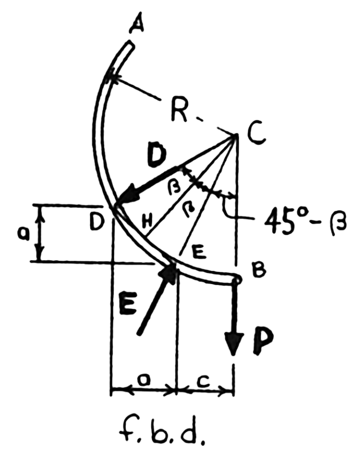
PROBLEM 4.88



Rod AB is bent into the shape of an arc of circle and is lodged between two pegs D and E. It supports a load **P** at end B. Neglecting friction and the weight of the rod, determine the distance c corresponding to equilibrium when $a = 20$ mm and $R = 100$ mm.

SOLUTION

Free-Body Diagram:



Since $y_{ED} = x_{ED} = a,$

slope of ED is $\searrow$ 45°;

slope of HC is $\measuredangle$ 45°.

Also $\qquad DE = \sqrt{2}a$

and $\qquad DH = HE = \left(\frac{1}{2}\right)DE = \frac{a}{\sqrt{2}}$

For triangles DHC and EHC, $\qquad \sin\beta = \frac{\frac{a}{\sqrt{2}}}{R} = \frac{a}{\sqrt{2}R}$

Now $\qquad c = R\sin(45° - \beta)$

For $\qquad a = 20 \text{ mm} \quad \text{and} \quad R = 100 \text{ mm}$

$$\sin\beta = \frac{20 \text{ mm}}{\sqrt{2}(100 \text{ mm})}$$

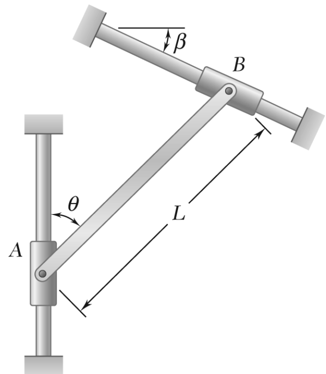
$$= 0.141421$$

$$\beta = 8.1301°$$

and $\qquad c = (100 \text{ mm})\sin(45° - 8.1301°)$

$$= 60.00 \text{ mm}$$

or $c = 60.0$ mm ◀



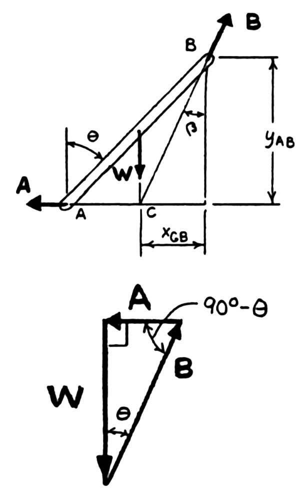
PROBLEM 4.89

A slender rod of length L is attached to collars that can slide freely along the guides shown. Knowing that the rod is in equilibrium, derive an expression for the angle θ in terms of the angle β.

SOLUTION

As shown in the free-body diagram of the slender rod AB, the three forces intersect at C. From the force geometry:

Free-Body Diagram:



$$\tan\beta = \frac{x_{GB}}{y_{AB}}$$

where

$$y_{AB} = L\cos\theta$$

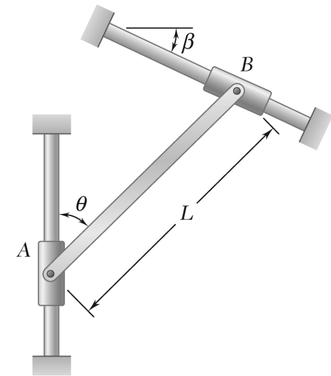
and

$$x_{GB} = \frac{1}{2}L\sin\theta$$

$$\tan\beta = \frac{\frac{1}{2}L\sin\theta}{L\cos\theta}$$

$$= \frac{1}{2}\tan\theta$$

or $\tan\theta = 2\tan\beta$ ◀



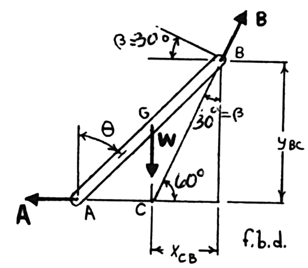
PROBLEM 4.90

An 8-kg slender rod of length L is attached to collars that can slide freely along the guides shown. Knowing that the rod is in equilibrium and that $\beta = 30°$, determine (*a*) the angle θ that the rod forms with the vertical, (*b*) the reactions at A and B.

SOLUTION

(*a*) As shown in the free-body diagram of the slender rod *AB*, the three forces intersect at *C*. From the geometry of the forces:

Free-Body Diagram:



$$\tan\beta = \frac{x_{CB}}{y_{BC}}$$

where

$$x_{CB} = \frac{1}{2}L\sin\theta$$

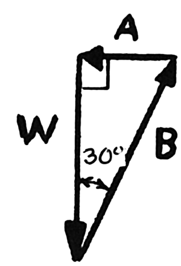
and

$$y_{BC} = L\cos\theta$$

$$\tan\beta = \frac{1}{2}\tan\theta$$

or $\tan\theta = 2\tan\beta$

For $\beta = 30°$



$$\tan\theta = 2\tan 30°$$
$$= 1.15470$$
$$\theta = 49.107°$$

or $\theta = 49.1°$ ◀

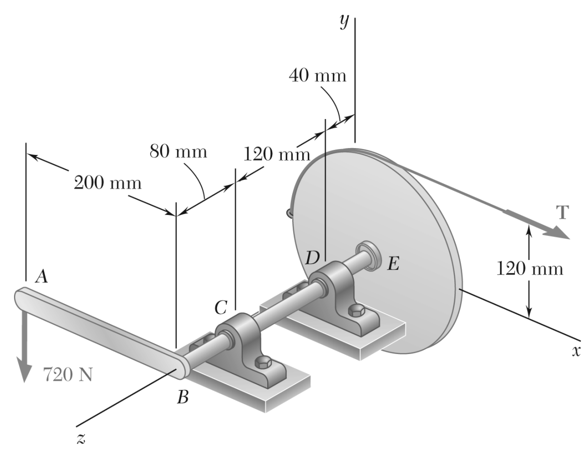
(*b*) $W = mg = (8\text{ kg})(9.81\text{ m/s}^2) = 78.480\text{ N}$

From force triangle: $A = W\tan\beta$

$$= (78.480\text{ N})\tan 30°$$
$$= 45.310\text{ N}$$

or $\mathbf{A} = 45.3\text{ N} \longleftarrow$ ◀

and $B = \dfrac{W}{\cos\beta} = \dfrac{78.480\text{ N}}{\cos 30°} = 90.621\text{ N}$ or $\mathbf{B} = 90.6\text{ N} \measuredangle\ 60.0°$ ◀

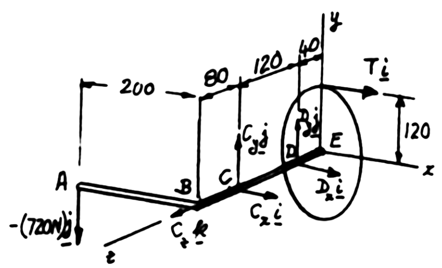


PROBLEM 4.91

A 200-mm lever and a 240-mm-diameter pulley are welded to the axle *BE* that is supported by bearings at *C* and *D*. If a 720-N vertical load is applied at *A* when the lever is horizontal, determine (*a*) the tension in the cord, (*b*) the reactions at *C* and *D*. Assume that the bearing at *D* does not exert any axial thrust.

SOLUTION

Dimensions in mm



We have six unknowns and six equations of equilibrium. —OK

$$\Sigma \mathbf{M}_C = 0: \quad (-120\mathbf{k}) \times (D_x\mathbf{i} + D_y\mathbf{j}) + (120\mathbf{j} - 160\mathbf{k}) \times T\mathbf{i} + (80\mathbf{k} - 200\mathbf{i}) \times (-720\mathbf{j}) = 0$$

$$-120D_x\mathbf{j} + 120D_y\mathbf{i} - 120T\mathbf{k} - 160T\mathbf{j} + 57.6\times10^3\mathbf{i} + 144\times10^3\mathbf{k} = 0$$

Equating to zero the coefficients of the unit vectors:

$$\mathbf{k}: \quad -120T + 144\times10^3 = 0 \qquad (a) \quad T = 1200 \text{ N} \blacktriangleleft$$

$$\mathbf{i}: \quad 120D_y + 57.6\times10^3 = 0 \qquad D_y = -480 \text{ N}$$

$$\mathbf{j}: \quad -120D_x - 160(1200 \text{ N}) = 0 \qquad D_x = -1600 \text{ N}$$

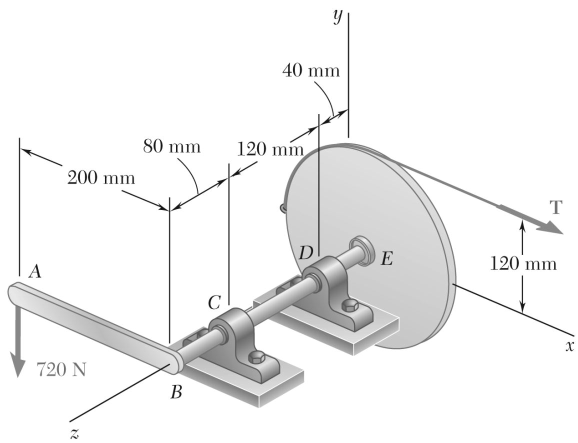
$$\Sigma F_x = 0: \quad C_x + D_x + T = 0 \qquad C_x = 1600 - 1200 = 400 \text{ N}$$

$$\Sigma F_y = 0: \quad C_y + D_y - 720 = 0 \qquad C_y = 480 + 720 = 1200 \text{ N}$$

$$\Sigma F_z = 0: \quad C_z = 0$$

(*b*) $\mathbf{C} = (400 \text{ N})\mathbf{i} + (1200 \text{ N})\mathbf{j}; \quad \mathbf{D} = -(1600 \text{ N})\mathbf{i} - (480 \text{ N})\mathbf{j} \blacktriangleleft$

PROBLEM 4.92

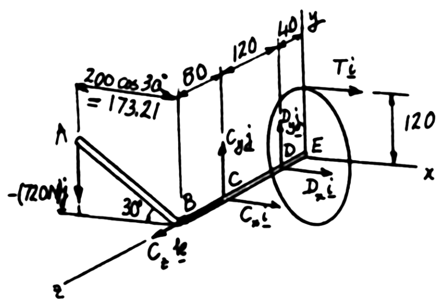


Solve Problem 4.91, assuming that the axle has been rotated clockwise in its bearings by 30° and that the 720-N load remains vertical.

PROBLEM 4.91 A 200-mm lever and a 240-mm-diameter pulley are welded to the axle *BE* that is supported by bearings at *C* and *D*. If a 720-N vertical load is applied at *A* when the lever is horizontal, determine (*a*) the tension in the cord, (*b*) the reactions at *C* and *D*. Assume that the bearing at *D* does not exert any axial thrust.

SOLUTION

Dimensions in mm



We have six unknowns and six equations of equilibrium.

$$\Sigma \mathbf{M}_C = 0: \quad (-120\mathbf{k}) \times (D_x\mathbf{i} + D_y\mathbf{j}) + (120\mathbf{j} - 160\mathbf{k}) \times T\mathbf{i} + (80\mathbf{k} - 173.21\mathbf{i}) \times (-720\mathbf{j}) = 0$$

$$-120D_x\mathbf{j} + 120D_y\mathbf{i} - 120T\,\mathbf{k} - 160T\,\mathbf{j} + 57.6\times10^3\,\mathbf{i} + 124.71\times10^3\,\mathbf{k} = 0$$

Equating to zero the coefficients of the unit vectors,

$$\mathbf{k}: \quad -120T + 124.71\times10^3 = 0 \qquad T = 1039.2 \text{ N} \qquad\qquad T = 1039 \text{ N} \blacktriangleleft$$

$$\mathbf{i}: \quad 120D_y + 57.6\times10^3 = 0 \qquad D_y = -480 \text{ N}$$

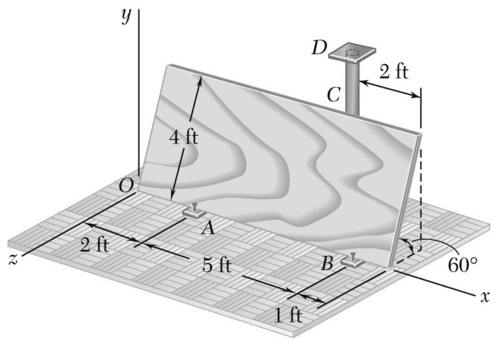
$$\mathbf{j}: \quad -120D_x - 160(1039.2) \qquad D_x = -1385.6 \text{ N}$$

$$\Sigma F_x = 0: \quad C_x + D_x + T = 0 \qquad C_x = 1385.6 - 1039.2 = 346.4$$

$$\Sigma F_y = 0: \quad C_y + D_y - 720 = 0 \qquad C_y = 480 + 720 = 1200 \text{ N}$$

$$\Sigma F_z = 0: \quad C_z = 0$$

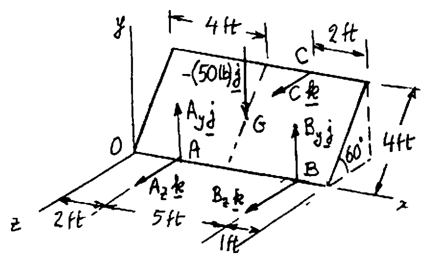
(*b*) $\mathbf{C} = (346 \text{ N})\mathbf{i} + (1200 \text{ N})\mathbf{j} \quad \mathbf{D} = -(1386 \text{ N})\mathbf{i} - (480 \text{ N})\mathbf{j}$ ◀

	**PROBLEM 4.93** A 4 × 8-ft sheet of plywood weighing 40 lb has been temporarily propped against column *CD*. It rests at *A* and *B* on small wooden blocks and against protruding nails. Neglecting friction at all surfaces of contact, determine the reactions at *A*, *B*, and *C*.

SOLUTION

Free-Body Diagram:

We have five unknowns and six equations of equilibrium. Plywood sheet is free to move in x direction, but equilibrium is maintained $(\Sigma F_x = 0)$.



$$\Sigma M_A = 0: \quad \mathbf{r}_{B/A} \times (B_y\,\mathbf{j} + B_z\mathbf{k}) + \mathbf{r}_{C/A} \times C\,\mathbf{k} + \mathbf{r}_{G/A} \times (-40 \text{ lb})\mathbf{j} = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 5 & 0 & 0 \\ 0 & B_y & B_z \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 4\sin 60^\circ & -4\cos 60^\circ \\ 0 & 0 & C \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 2\sin 60^\circ & -2\cos 60^\circ \\ 0 & -40 & 0 \end{vmatrix} = 0$$

$$(4C\sin 60^\circ - 80\cos 60^\circ)\mathbf{i} + (-5B_z - 4C)\mathbf{j} + (5B_y - 80)\mathbf{k} = 0$$

Equating the coefficients of the unit vectors to zero,

$$\mathbf{i}: \quad 4C\sin 60^\circ - 80\cos 60^\circ = 0 \qquad C = 11.5470 \text{ lb}$$

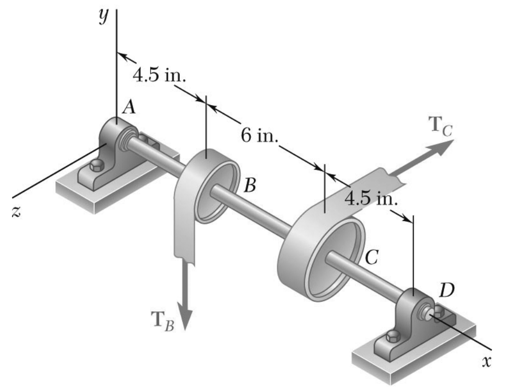
$$\mathbf{j}: \quad -5B_z - 4C = 0 \qquad B_z = 9.2376 \text{ lb}$$

$$\mathbf{k}: \quad 5B_y - 80 = 0 \qquad B_y = 16.0000 \text{ lb}$$

$$\Sigma F_y = 0: \quad A_y + B_y - 40 = 0 \qquad A_y = 40 - 16.0000 = 24.000 \text{ lb}$$

$$\Sigma F_z = 0: \quad A_z + B_z + C = 0 \qquad A_z = 9.2376 - 11.5470 = -2.3094 \text{ lb}$$

$$\mathbf{A} = (24.0 \text{ lb})\mathbf{j} - (2.31 \text{ lb})\mathbf{k}; \quad \mathbf{B} = (16.00 \text{ lb})\mathbf{j} - (9.24 \text{ lb})\mathbf{k}; \quad \mathbf{C} = (11.55 \text{ lb})\mathbf{k} \blacktriangleleft$$

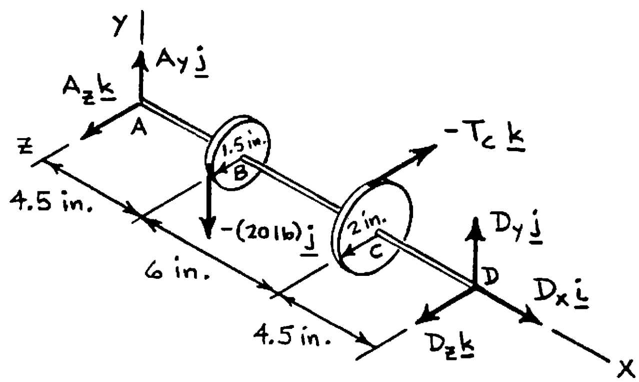


PROBLEM 4.94

Two tape spools are attached to an axle supported by bearings at A and D. The radius of spool B is 1.5 in. and the radius of spool C is 2 in. Knowing that $T_B = 20$ lb and that the system rotates at a constant rate, determine the reactions at A and D. Assume that the bearing at A does not exert any axial thrust and neglect the weights of the spools and axle.

SOLUTION

Free-Body Diagram:



We have six unknowns and six equations of equilibrium.

$$\Sigma M_A = 0: \quad (4.5\mathbf{i} + 1.5\mathbf{k}) \times (-20\mathbf{j}) + (10.5\mathbf{i} + 2\mathbf{j}) \times (-T_C\mathbf{k}) + (15\mathbf{i}) \times (D_x\mathbf{i} + D_y\mathbf{j} + D_z\mathbf{k}) = 0$$

$$-90\mathbf{k} + 30\mathbf{i} + 10.5T_C\mathbf{j} - 2T_C\mathbf{i} + 15D_y\mathbf{k} - 15D_z\mathbf{j} = 0$$

Equate coefficients of unit vectors to zero:

$$\vec{\mathbf{i}}: \quad 30 - 2T_C = 0 \qquad T_C = 15 \text{ lb}$$

$$\vec{\mathbf{j}}: \quad 10.5T_C - 15D_z = 0 \quad 10.5(15) - 15D_z = 0 \qquad D_z = 10.5 \text{ lb}$$

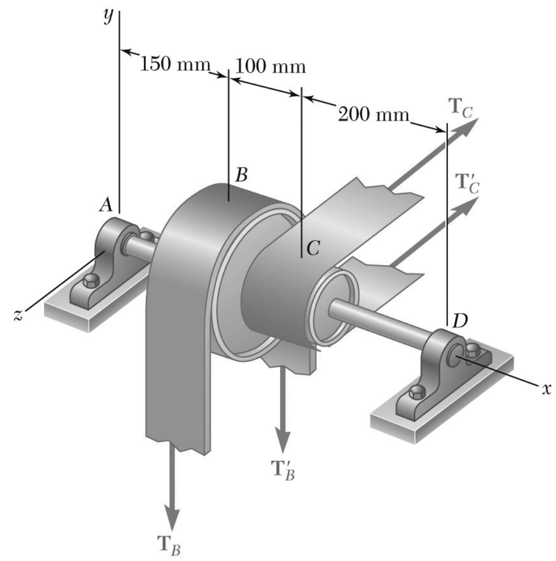
$$\vec{\mathbf{k}}: \quad -90 + 15D_y = 0 \qquad D_y = 6 \text{ lb}$$

$$\Sigma F_x = 0: \quad D_x = 0$$

$$\Sigma F_y = 0: \quad A_y + D_y - 20 \text{ lb} = 0 \qquad A_y = 20 - 6 = 14 \text{ lb}$$

$$\Sigma F_z = 0: \quad A_z + D_z - 15 \text{ lb} = 0 \qquad A_z = 15 - 10.5 = 4.5 \text{ lb}$$

$$\mathbf{A} = (14.00 \text{ lb})\mathbf{j} + (4.50 \text{ lb})\mathbf{k}; \quad \mathbf{D} = (6.00 \text{ lb})\mathbf{j} + (10.50 \text{ lb})\mathbf{k} \blacktriangleleft$$

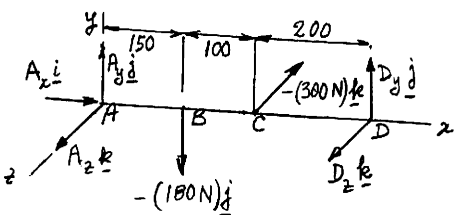


PROBLEM 4.95

Two transmission belts pass over a double-sheaved pulley that is attached to an axle supported by bearings at A and D. The radius of the inner sheave is 125 mm and the radius of the outer sheave is 250 mm. Knowing that when the system is at rest, the tension is 90 N in both portions of belt B and 150 N in both portions of belt C, determine the reactions at A and D. Assume that the bearing at D does not exert any axial thrust.

SOLUTION

We replace $\mathbf{T}_B$ and $\mathbf{T}'_B$ by their resultant $(-180\text{ N})\mathbf{j}$ and $\mathbf{T}_C$ and $\mathbf{T}'_C$ by their resultant $(-300\text{ N})\mathbf{k}$.



Dimensions in mm

We have five unknowns and six equations of equilibrium. Axle AD is free to rotate about the x-axis, but equilibrium is maintained $(\Sigma M_x = 0)$.

$$\Sigma \mathbf{M}_A = 0: \quad (150\mathbf{i})\times(-180\mathbf{j}) + (250\mathbf{i})\times(-300\mathbf{k}) + (450\mathbf{i})\times(D_y\mathbf{j} + D_z\mathbf{k}) = 0$$

$$-27\times10^3\mathbf{k} + 75\times10^3\mathbf{j} + 450D_y\mathbf{k} - 450D_z\mathbf{j} = 0$$

Equating coefficients of $\mathbf{j}$ and $\mathbf{k}$ to zero,

$$\mathbf{j}: \quad 75\times10^3 - 450D_z = 0 \qquad D_z = 166.7\text{ N}$$

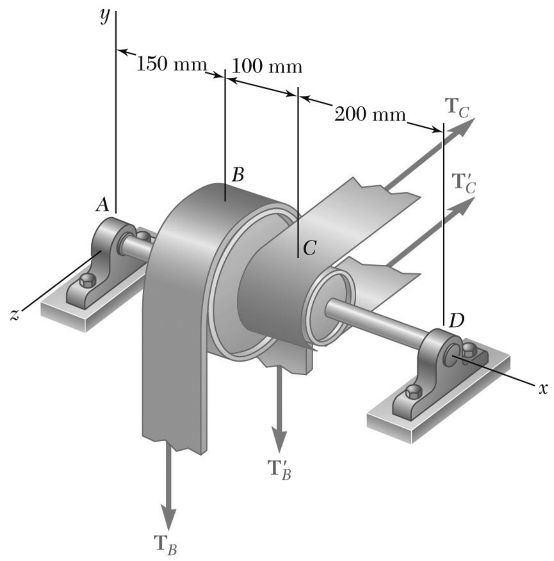
$$\mathbf{k}: \quad -27\times10^3 + 450D_y = 0 \qquad D_y = 60.0\text{ N}$$

$$\Sigma F_x = 0: \quad A_x = 0$$

$$\Sigma F_y = 0: \quad A_y + D_y - 180\text{ N} = 0 \qquad A_y = 180 - 60 = 120.0\text{ N}$$

$$\Sigma F_z = 0: \quad A_z + D_z - 300\text{ N} = 0 \qquad A_z = 300 - 166.7 = 133.3\text{ N}$$

$$\mathbf{A} = (120.0\text{ N})\mathbf{j} + (133.3\text{ N})\mathbf{k}; \quad \mathbf{D} = (60.0\text{ N})\mathbf{j} + (166.7\text{ N})\mathbf{k} \blacktriangleleft$$

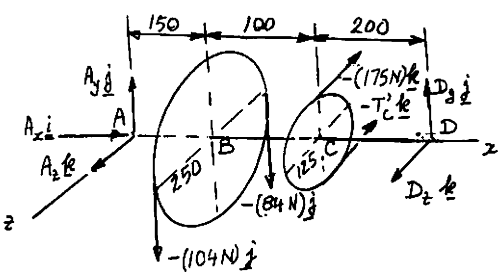


PROBLEM 4.96

Solve Problem 4.95, assuming that the pulley rotates at a constant rate and that $T_B = 104$ N, $T'_B = 84$ N, $T_C = 175$ N.

PROBLEM 4.95 Two transmission belts pass over a double-sheaved pulley that is attached to an axle supported by bearings at A and D. The radius of the inner sheave is 125 mm and the radius of the outer sheave is 250 mm. Knowing that when the system is at rest, the tension is 90 N in both portions of belt B and 150 N in both portions of belt C, determine the reactions at A and D. Assume that the bearing at D does not exert any axial thrust.

SOLUTION



Dimensions in mm

We have six unknowns and six equations of equilibrium. —OK

$$\Sigma \mathbf{M}_A = 0: \quad (150\mathbf{i} + 250\mathbf{k}) \times (-104\mathbf{j}) + (150\mathbf{i} - 250\mathbf{k}) \times (-84\mathbf{j})$$
$$+ (250\mathbf{i} + 125\mathbf{j}) \times (-175\mathbf{k}) + (250\mathbf{i} - 125\mathbf{j}) \times (-T_C)$$
$$+ 450\mathbf{i} \times (D_y\mathbf{j} + D_z\mathbf{k}) = 0$$

$$-150(104 + 84)\mathbf{k} + 250(104 - 84)\mathbf{i} + 250(175 + T'_C)\,\mathbf{j} \; - 125(175 - T'_C)$$
$$+ 450D_y\mathbf{k} - 450D_z\mathbf{j} = 0$$

Equating the coefficients of the unit vectors to zero,

$$\mathbf{i}: \quad 250(104 - 84) - 125(175 - T'_C) = 0 \qquad 175 = T'_C = 40 \qquad T'_C = 135;$$

$$\mathbf{j}: \quad 250(175 + 135) - 450D_z = 0 \qquad D_z = 172.2 \text{ N}$$

$$\mathbf{k}: \quad -150(104 + 84) + 450\,D_y = 0 \qquad D_y = 62.7 \text{ N}$$

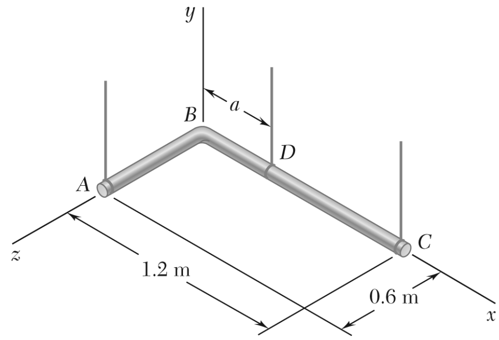
PROBLEM 4.96 (Continued)

$$\Sigma F_x = 0: \qquad A_x = 0$$

$$\Sigma F_y = 0: \qquad A_y - 104 - 84 + 62.7 = 0 \qquad A_y = 125.3 \text{ N}$$

$$\Sigma F_z = 0: \qquad A_z - 175 - 135 + 172.2 = 0 \qquad A_z = 137.8 \text{ N}$$

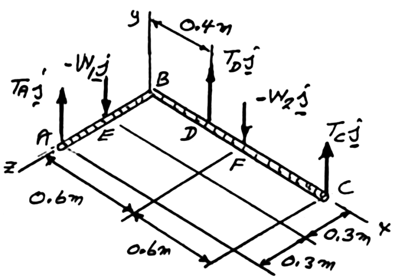
$$\mathbf{A} = (125.3 \text{ N})\mathbf{j} + (137.8 \text{ N})\mathbf{k}; \quad \mathbf{D} = (62.7 \text{ N})\mathbf{j} + (172.2 \text{ N})\mathbf{k} \blacktriangleleft$$



PROBLEM 4.97

Two steel pipes AB and BC, each having a mass per unit length of 8 kg/m, are welded together at B and supported by three wires. Knowing that $a = 0.4$ m, determine the tension in each wire.

SOLUTION



$$W_1 = 0.6m'g$$
$$W_2 = 1.2m'g$$

$$\Sigma M_D = 0: \quad \mathbf{r}_{A/D} \times T_A\mathbf{j} + \mathbf{r}_{E/D} \times (-W_1\mathbf{j}) + \mathbf{r}_{F/D} \times (-W_2\mathbf{j}) + \mathbf{r}_{C/D} \times T_C\mathbf{j} = 0$$

$$(-0.4\mathbf{i} + 0.6\mathbf{k}) \times T_A\mathbf{j} + (-0.4\mathbf{i} + 0.3\mathbf{k}) \times (-W_1\mathbf{j}) + 0.2\mathbf{i} \times (-W_2\mathbf{j}) + 0.8\mathbf{i} \times T_C\mathbf{j} = 0$$

$$-0.4T_A\mathbf{k} - 0.6T_A\mathbf{i} + 0.4W_1\mathbf{k} + 0.3W_1\mathbf{i} - 0.2W_2\mathbf{k} + 0.8T_C\mathbf{k} = 0$$

Equate coefficients of unit vectors to zero:

$$\mathbf{i}: \quad -0.6T_A + 0.3W_1 = 0; \quad T_A = \frac{1}{2}W_1 = \frac{1}{2}0.6m'g = 0.3m'g$$

$$\mathbf{k}: \quad -0.4T_A + 0.4W_1 - 0.2W_2 + 0.8T_C = 0$$

$$-0.4(0.3m'g) + 0.4(0.6m'g) - 0.2(1.2m'g) + 0.8T_C = 0$$

$$T_C = \frac{(0.12 - 0.24 - 0.24)m'g}{0.8} = 0.15m'g$$

$$\Sigma F_y = 0: \quad T_A + T_C + T_D - W_1 - W_2 = 0$$

$$0.3m'g + 0.15m'g + T_D - 0.6m'g - 1.2m'g = 0$$

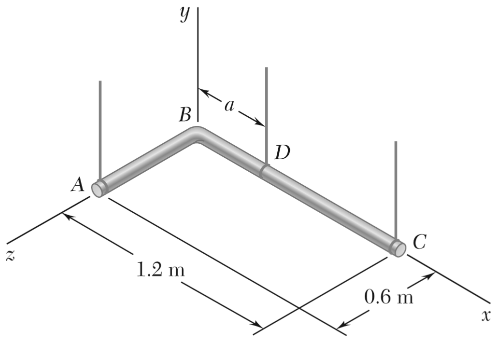
$$T_D = 1.35m'g$$

$$m'g = (8 \text{ kg/m})(9.81 \text{m/s}^2) = 78.48 \text{ N/m}$$

$T_A = 0.3m'g = 0.3 \times 78.45$ $\qquad T_A = 23.5$ N ◀

$T_B = 0.15m'g = 0.15 \times 78.45$ $\qquad T_B = 11.77$ N ◀

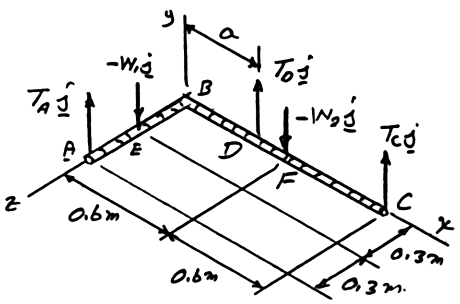
$T_C = 1.35m'g = 1.35 \times 78.45$ $\qquad T_C = 105.9$ N ◀



PROBLEM 4.98

For the pipe assembly of Problem 4.97, determine (*a*) the largest permissible value of *a* if the assembly is not to tip, (*b*) the corresponding tension in each wire.

SOLUTION



$$W_1 = 0.6m'g$$
$$W_2 = 1.2m'g$$

$$\Sigma M_D = 0: \quad \mathbf{r}_{A/D} \times T_A\mathbf{j} + \mathbf{r}_{E/D} \times (-W_1\mathbf{j}) + \mathbf{r}_{F/D} \times (-W_2\mathbf{j}) + \mathbf{r}_{C/D} \times T_C\mathbf{j} = 0$$

$$(-a\mathbf{i} + 0.6\mathbf{k}) \times T_A\mathbf{j} + (-a\mathbf{i} + 0.3\mathbf{k}) \times (-W_1\mathbf{j}) + (0.6 - a)\mathbf{i} \times (-W_2\mathbf{j}) + (1.2 - a)\mathbf{i} \times T_C\mathbf{j} = 0$$

$$-T_A a\mathbf{k} - 0.6T_A\mathbf{i} + W_1 a\mathbf{k} + 0.3W_1\mathbf{i} - W_2(0.6 - a)\mathbf{k} + T_C(1.2 - a)\mathbf{k} = 0$$

Equate coefficients of unit vectors to zero:

$$\mathbf{i}: \quad -0.6T_A + 0.3W_1 = 0; \quad T_A = \frac{1}{2}W_1 = \frac{1}{2}0.6m'g = 0.3m'g$$

$$\mathbf{k}: \quad -T_A a + W_1 a - W_2(0.6 - a) + T_C(1.2 - a) = 0$$

$$-0.3m'ga + 0.6m'ga - 1.2m'g(0.6 - a) + T_C(1.2 - a) = 0$$

$$T_C = \frac{0.3a - 0.6a + 1.2(0.6 - a)}{1.2 - a}$$ For maximum a and no tipping, $T_C = 0$.

(*a*)

$$-0.3a + 1.2(0.6 - a) = 0$$
$$-0.3a + 0.72 - 1.2a = 0$$
$$1.5a = 0.72$$

$a = 0.480\text{ m}$ ◀

PROBLEM 4.98 (Continued)

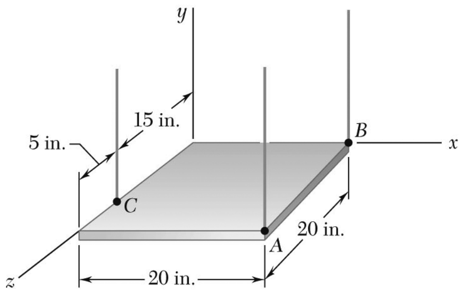
(*b*) Reactions: $m'g = (8 \text{ kg/m})\, 9.81 \text{ m/s}^2 = 78.48 \text{ N/m}$

$$T_A = 0.3m'g = 0.3 \times 78.48 = 23.544 \text{ N} \qquad T_A = 23.5 \text{ N} \blacktriangleleft$$

$$\Sigma F_y = 0: \quad T_A + T_C + T_D - W_1 - W_2 = 0$$

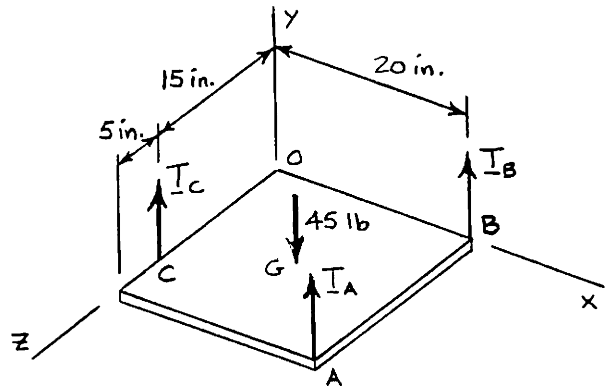
$$T_A + 0 + T_D - 0.6m'g - 1.2m'g = 0$$

$$T_D = 1.8m'g - T_A = 1.8 \times 78.48 - 23.544 = 117.72 \qquad T_D = 117.7 \text{ N} \blacktriangleleft$$

	**PROBLEM 4.99** The 45-lb square plate shown is supported by three vertical wires. Determine the tension in each wire.

SOLUTION

Free-Body Diagram:



$$\Sigma M_B = 0: \quad \mathbf{r}_{C/B} \times T_C\mathbf{j} + \mathbf{r}_{A/B} \times T_A\mathbf{j} + \mathbf{r}_{G/B} \times (-45 \text{ lb})\mathbf{j} = 0$$

$$[-(20 \text{ in.})\mathbf{i} + (15 \text{ in.})\vec{\mathbf{k}}] \times T_C\mathbf{j} + (20 \text{ in.})\mathbf{k} \times T_A\mathbf{j}$$

$$+ [-(10 \text{ in.})\mathbf{i} + (10 \text{ in.})\mathbf{k}] \times [-(45 \text{ lb})\mathbf{j}] = 0$$

$$-20T_C\mathbf{k} - 15T_C\mathbf{i} - 20T_A\mathbf{i} + 450\mathbf{k} + 450\mathbf{i} = 0$$

Equating to zero the coefficients of the unit vectors,

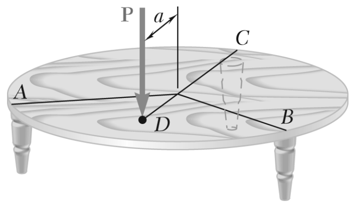
$$\mathbf{k}: \quad -20T_C + 450 = 0 \qquad T_C = 22.5 \text{ lb} \blacktriangleleft$$

$$\mathbf{i}: \quad -15(22.5) - 20T_A + 450 = 0 \qquad T_A = 5.625 \text{ lb} \blacktriangleleft$$

$$\Sigma F_y = 0: \quad T_A + T_B + T_C - 45 \text{ lb} = 0$$

$$5.625 \text{ lb} + T_B + 22.5 \text{ lb} - 45 \text{ lb} = 0 \qquad T_B = 16.875 \text{ lb} \blacktriangleleft$$

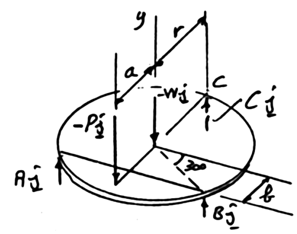
$$T_A = 5.63 \text{ lb}; \; T_B = 16.88 \text{ lb}; \; T_C = 22.5 \text{ lb} \blacktriangleleft$$



PROBLEM 4.100

The table shown weighs 30 lb and has a diameter of 4 ft. It is supported by three legs equally spaced around the edge. A vertical load **P** of magnitude 100 lb is applied to the top of the table at D. Determine the maximum value of a if the table is not to tip over. Show, on a sketch, the area of the table over which **P** can act without tipping the table.

SOLUTION



$$r = 2\text{ ft} \quad b = r\sin 30° = 1\text{ ft}$$

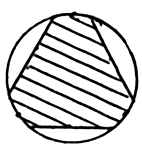
We shall sum moments about AB.

$$(b+r)C + (a-b)P - bW = 0$$

$$(1+2)C + (a-1)100 - (1)30 = 0$$

$$C = \frac{1}{3}[30 - (a-1)100]$$

If table is not to tip, $C \geq 0$.



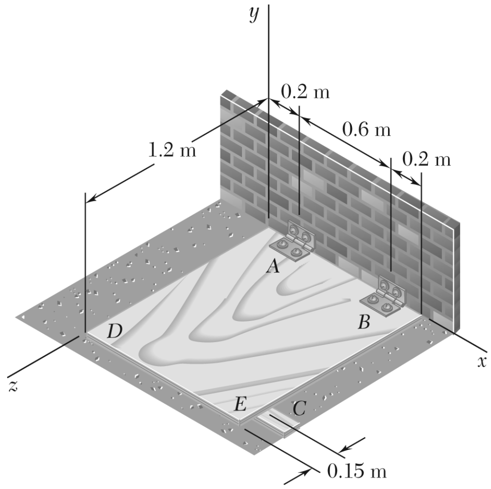
$$[30 - (a-1)100] \geq 0$$

$$30 \geq (a-1)100$$

$$a - 1 \leq 0.3 \quad a \leq 1.3\text{ ft} \quad a = 1.300\text{ ft}$$

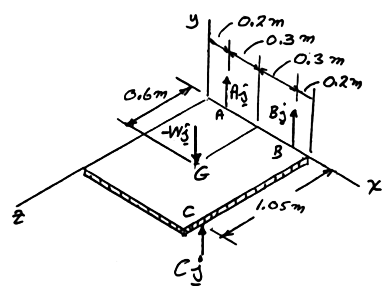
Only $\perp$ distance from P to AB matters. Same condition must be satisfied for each leg. P must be located in shaded area for no tipping.

PROBLEM 4.101



An opening in a floor is covered by a 1×1.2-m sheet of plywood of mass 18 kg. The sheet is hinged at A and B and is maintained in a position slightly above the floor by a small block C. Determine the vertical component of the reaction (*a*) at A, (*b*) at B, (*c*) at C.

SOLUTION



$$\mathbf{r}_{B/A} = 0.6\mathbf{i}$$
$$\mathbf{r}_{C/A} = 0.8\mathbf{i} + 1.05\mathbf{k}$$
$$\mathbf{r}_{G/A} = 0.3\mathbf{i} + 0.6\mathbf{k}$$

$$W = mg = (18\text{ kg})9.81$$
$$W = 176.58\text{ N}$$

$$\Sigma M_A = 0: \quad \mathbf{r}_{B/A} \times B\mathbf{j} + \mathbf{r}_{C/A} \times C\mathbf{j} + \mathbf{r}_{G/A} \times (-W\mathbf{j}) = 0$$

$$(0.6\mathbf{i}) \times B\mathbf{j} + (0.8\mathbf{i} + 1.05\mathbf{k}) \times C\mathbf{j} + (0.3\mathbf{i} + 0.6\mathbf{k}) \times (-W\mathbf{j}) = 0$$

$$0.6B\mathbf{k} + 0.8C\mathbf{k} - 1.05C\mathbf{i} - 0.3W\mathbf{k} + 0.6W\mathbf{i} = 0$$

Equate coefficients of unit vectors to zero:

$$\mathbf{i}: \quad 1.05C + 0.6W = 0 \quad C = \left(\frac{0.6}{1.05}\right)176.58\text{ N} = 100.90\text{ N}$$

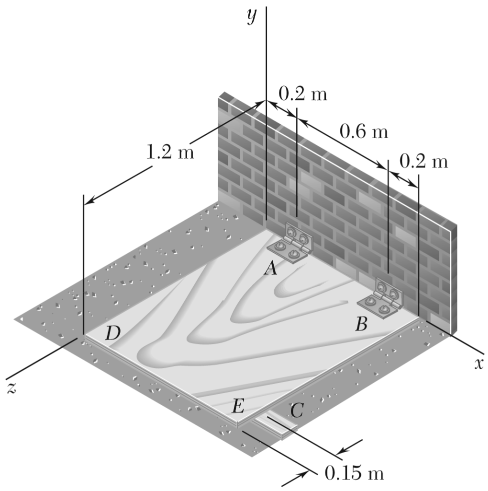
$$\mathbf{k}: \quad 0.6B + 0.8C - 0.3W = 0$$

$$0.6B + 0.8(100.90\text{ N}) - 0.3(176.58\text{ N}) = 0 \quad B = -46.24\text{ N}$$

$$\Sigma F_y = 0: \quad A + B + C - W = 0$$

$$A - 46.24\text{ N} + 100.90\text{ N} + 176.58\text{ N} = 0 \quad A = 121.92\text{ N}$$

$$(a) \quad A = 121.9\text{ N} \quad (b) \quad B = -46.2\text{ N} \quad (c) \quad C = 100.9\text{ N} \blacktriangleleft$$

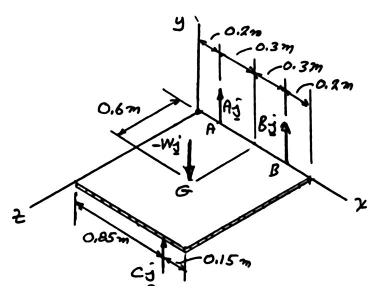


PROBLEM 4.102

Solve Problem 4.101, assuming that the small block C is moved and placed under edge DE at a point 0.15 m from corner E.

PROBLEM 4.101 An opening in a floor is covered by a 1×1.2-m sheet of plywood of mass 18 kg. The sheet is hinged at A and B and is maintained in a position slightly above the floor by a small block C. Determine the vertical component of the reaction (*a*) at A, (*b*) at B, (*c*) at C.

SOLUTION



$$\mathbf{r}_{B/A} = 0.6\mathbf{i}$$
$$\mathbf{r}_{C/A} = 0.65\mathbf{i} + 1.2\mathbf{k}$$
$$\mathbf{r}_{G/A} = 0.3\mathbf{i} + 0.6\mathbf{k}$$

$$W = mg = (18 \text{ kg})\, 9.81 \text{ m/s}^2$$
$$W = 176.58 \text{ N}$$

$$\Sigma M_A = 0: \quad \mathbf{r}_{B/A} \times B\mathbf{j} + \mathbf{r}_{C/A} \times C\mathbf{j} + \mathbf{r}_{G/A} \times (-W\mathbf{j}) = 0$$

$$0.6\mathbf{i} \times B\mathbf{j} + (0.65\mathbf{i} + 1.2\mathbf{k}) \times C\mathbf{j} + (0.3\mathbf{i} + 0.6\mathbf{k}) \times (-W\mathbf{j}) = 0$$

$$0.6B\mathbf{k} + 0.65C\mathbf{k} - 1.2C\mathbf{i} - 0.3W\mathbf{k} + 0.6W\mathbf{i} = 0$$

Equate coefficients of unit vectors to zero:

$$\mathbf{i}: \quad -1.2C + 0.6W = 0 \qquad C = \left(\frac{0.6}{1.2}\right) 176.58 \text{ N} = 88.29 \text{ N}$$

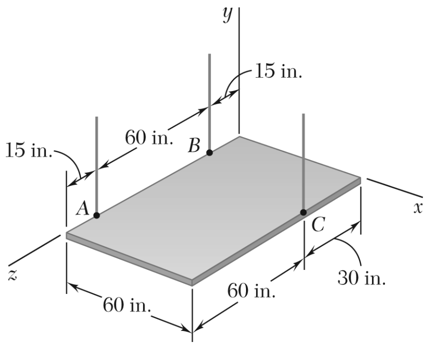
$$\mathbf{k}: \quad 0.6B + 0.65C - 0.3W = 0$$

$$0.6B + 0.65(88.29 \text{ N}) - 0.3(176.58 \text{ N}) = 0 \quad B = -7.36 \text{ N}$$

$$\Sigma F_y = 0: \quad A + B + C - W = 0$$

$$A - 7.36 \text{ N} + 88.29 \text{ N} - 176.58 \text{ N} = 0 \qquad A = 95.648 \text{ N}$$

$$(a) \quad A = 95.6 \text{ N} \quad (b) \quad B = -7.36 \text{ N} \quad (c) \quad C = 88.3 \text{ N} \blacktriangleleft$$

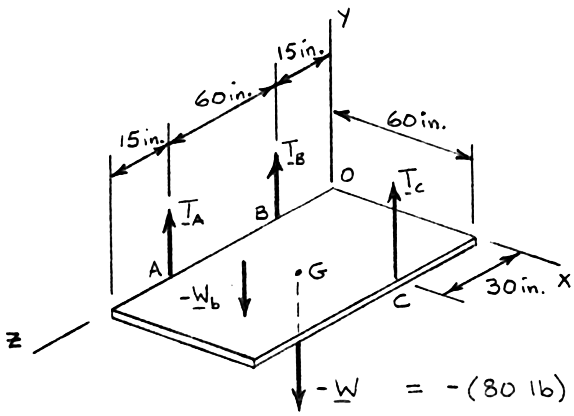


PROBLEM 4.103

The rectangular plate shown weighs 80 lb and is supported by three vertical wires. Determine the tension in each wire.

SOLUTION

Free-Body Diagram:



$$\Sigma \mathbf{M}_B = 0: \quad \mathbf{r}_{A/B} \times T_A \mathbf{j} + \mathbf{r}_{C/B} \times T_C \mathbf{j} + \mathbf{r}_{G/B} \times (-80 \text{ lb})\mathbf{j} = 0$$

$$(60 \text{ in.})\mathbf{k} \times T_A \mathbf{j} + [(60 \text{ in.})\mathbf{i} + (15 \text{ in.})\mathbf{k}] \times T_C \mathbf{j} + [(30 \text{ in.})\mathbf{i} + (30 \text{ in.})\mathbf{k}] \times (-80 \text{ lb})\mathbf{j} = 0$$

$$-60 T_A \mathbf{i} + 60 T_C \mathbf{k} - 15 T_C \mathbf{i} - 2400\mathbf{k} + 2400\mathbf{i} = 0$$

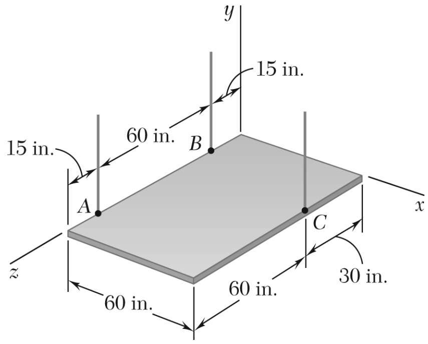
Equating to zero the coefficients of the unit vectors,

$$\mathbf{i}: \quad 60 T_A - 15(40) + 2400 = 0 \qquad T_A = 30.0 \text{ lb} \blacktriangleleft$$

$$\mathbf{k}: \quad 60 T_C - 2400 = 0 \qquad T_C = 40.0 \text{ lb} \blacktriangleleft$$

$$\Sigma F_y = 0: \quad T_A + T_B + T_C - 80 \text{ lb} = 0$$

$$30 \text{ lb} + T_B + 40 \text{ lb} - 80 \text{ lb} = 0 \qquad T_B = 10.00 \text{ lb} \blacktriangleleft$$

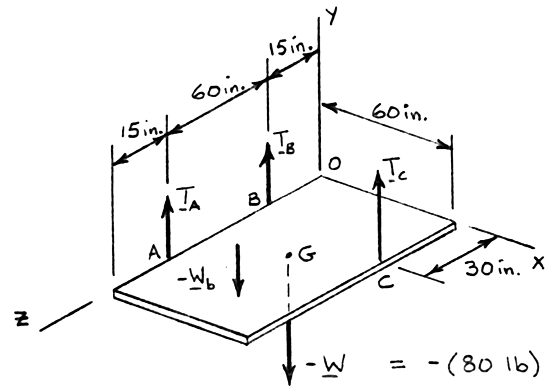


PROBLEM 4.104

The rectangular plate shown weighs 80 lb and is supported by three vertical wires. Determine the weight and location of the lightest block that should be placed on the plate if the tensions in the three wires are to be equal.

SOLUTION

Free-Body Diagram:



Let $-W_b\mathbf{j}$ be the weight of the block and x and z the block's coordinates.

Since tensions in wires are equal, let

$$T_A = T_B = T_C = T$$

$$\Sigma M_0 = 0: \quad (\mathbf{r}_A \times T\mathbf{j}) + (\mathbf{r}_B \times T\mathbf{j}) + (\mathbf{r}_C \times T\mathbf{j}) + \mathbf{r}_G \times (-W\mathbf{j}) + (x\mathbf{i} + z\mathbf{k}) \times (-W_b\mathbf{j}) = 0$$

or $\quad (75\,\mathbf{k}) \times T\mathbf{j} + (15\,\mathbf{k}) \times T\mathbf{j} + (60\mathbf{i} + 30\mathbf{k}) \times T\mathbf{j} + (30\mathbf{i} + 45\mathbf{k}) \times (-W\mathbf{j}) + (x\mathbf{i} + z\mathbf{k}) \times (-W_b\mathbf{j}) = 0$

or $\quad -75T\,\mathbf{i} - 15T\,\mathbf{i} + 60T\,\mathbf{k} - 30T\,\mathbf{i} - 30W\mathbf{k} + 45W\,\mathbf{i} - W_b \times \mathbf{k} + W_b\, z\mathbf{i} = 0$

Equate coefficients of unit vectors to zero:

$$\mathbf{i}: \quad -120T + 45W + W_b z = 0 \qquad (1)$$

$$\mathbf{k}: \quad 60T - 30W - W_b x = 0 \qquad (2)$$

Also, $\quad \Sigma F_y = 0: \quad 3T - W - W_b = 0 \qquad (3)$

Eq. (1) + 40 Eq. (3): $\quad 5W + (z - 40)W_b = 0 \qquad (4)$

Eq. (2) – 20 Eq. (3): $\quad -10W - (x - 20)W_b = 0 \qquad (5)$

PROBLEM 4.104 (Continued)

Solving Eqs. (4) and (5) for W_b/W and recalling that $0 \le x \le 60$ in., $0 \le z \le 90$ in.,

Eq. (4): $$\frac{W_b}{W} = \frac{5}{40 - z} \ge \frac{5}{40 - 0} = 0.125$$

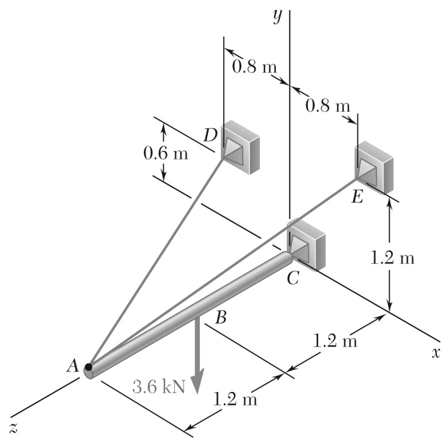
Eq. (5): $$\frac{W_b}{W} = \frac{10}{20 - x} \ge \frac{10}{20 - 0} = 0.5$$

Thus, $(W_b)_{\min} = 0.5W = 0.5(80) = 40$ lb $\qquad (W_b)_{\min} = 40.0$ lb ◀

Making $W_b = 0.5W$ in Eqs. (4) and (5):

$$5W + (z - 40)(0.5W) = 0 \qquad z = 30.0 \text{ in.} ◀$$

$$-10W - (x - 20)(0.5W) = 0 \qquad x = 0 \text{ in.} ◀$$

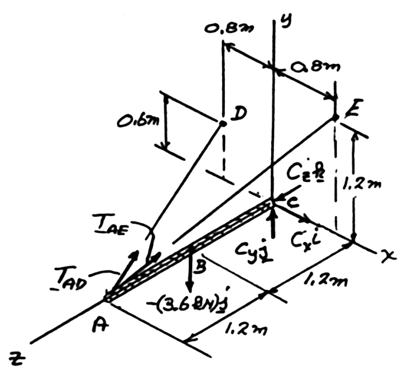


PROBLEM 4.105

A 2.4-m boom is held by a ball-and-socket joint at C and by two cables AD and AE. Determine the tension in each cable and the reaction at C.

SOLUTION

Free-Body Diagram: Five unknowns and six equations of equilibrium, but equilibrium is maintained ($\Sigma M_{AC} = 0$).



$$\mathbf{r}_B = 1.2\mathbf{k}$$
$$\mathbf{r}_A = 2.4\mathbf{k}$$

$$\overrightarrow{AD} = -0.8\mathbf{i} + 0.6\mathbf{j} - 2.4\mathbf{k} \qquad AD = 2.6 \text{ m}$$
$$\overrightarrow{AE} = 0.8\mathbf{i} + 1.2\mathbf{j} - 2.4\mathbf{k} \qquad AE = 2.8 \text{ m}$$

$$T_{AD} = \frac{\overrightarrow{AD}}{AD} = \frac{T_{AD}}{2.6}(-0.8\mathbf{i} + 0.6\mathbf{j} - 2.4\mathbf{k})$$
$$T_{AE} = \frac{\overrightarrow{AE}}{AE} = \frac{T_{AE}}{2.8}(0.8\mathbf{i} + 1.2\mathbf{j} - 2.4\mathbf{k})$$

$$\Sigma M_C = 0: \quad \mathbf{r}_A \times \mathbf{T}_{AD} + \mathbf{r}_A \times \mathbf{T}_{AE} + \mathbf{r}_B \times (-3 \text{ kN})\mathbf{j} = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 2.4 \\ -0.8 & 0.6 & -2.4 \end{vmatrix} \frac{T_{AD}}{2.6} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 2.4 \\ 0.8 & 1.2 & -2.4 \end{vmatrix} \frac{T_{AE}}{2.8} + 1.2\mathbf{k} \times (-3.6 \text{ kN})\mathbf{j} = 0$$

Equate coefficients of unit vectors to zero:

$$\mathbf{i}: \quad -0.55385T_{AD} - 1.02857T_{AE} + 4.32 = 0 \qquad (1)$$
$$\mathbf{j}: \quad -0.73846T_{AD} + 0.68671T_{AE} = 0$$

$$T_{AD} = 0.92857T_{AE} \qquad (2)$$

From Eq. (1): $\quad -0.55385(0.92857)T_{AE} - 1.02857T_{AE} + 4.32 = 0$

$$1.54286T_{AE} = 4.32$$
$$T_{AE} = 2.800 \text{ kN} \qquad T_{AE} = 2.80 \text{ kN} \blacktriangleleft$$

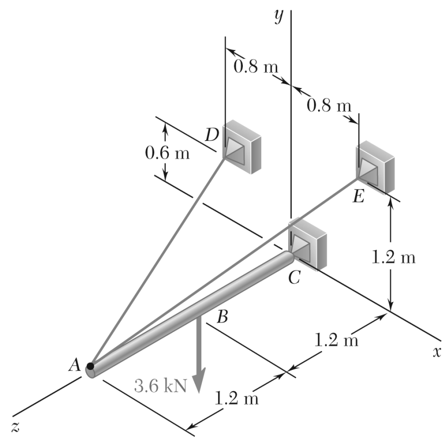
From Eq. (2): $T_{AD} = 0.92857(2.80) = 2.600\text{ kN}$ $\qquad T_{AD} = 2.60\text{ kN}$ ◀

$$\Sigma F_x = 0:\quad C_x - \frac{0.8}{2.6}(2.6\text{ kN}) + \frac{0.8}{2.8}(2.8\text{ kN}) = 0 \qquad C_x = 0$$

$$\Sigma F_y = 0:\quad C_y + \frac{0.6}{2.6}(2.6\text{ kN}) + \frac{1.2}{2.8}(2.8\text{ kN}) - (3.6\text{ kN}) = 0 \qquad C_y = 1.800\text{ kN}$$

$$\Sigma F_z = 0:\quad C_z - \frac{2.4}{2.6}(2.6\text{ kN}) - \frac{2.4}{2.8}(2.8\text{ kN}) = 0 \qquad C_z = 4.80\text{ kN}$$

$\mathbf{C} = (1.800\text{ kN})\mathbf{j} + (4.80\text{ kN})\mathbf{k}$ ◀



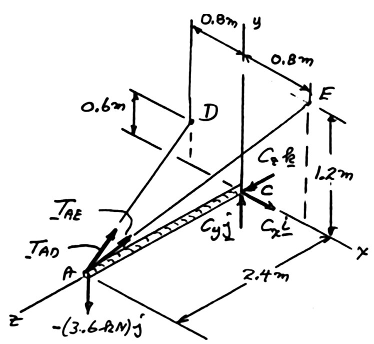
PROBLEM 4.106

Solve Problem 4.105, assuming that the 3.6-kN load is applied at Point A.

PROBLEM 4.105 A 2.4-m boom is held by a ball-and-socket joint at C and by two cables AD and AE. Determine the tension in each cable and the reaction at C.

SOLUTION

Free-Body Diagram: Five unknowns and six equations of equilibrium, but equilibrium is maintained ($\Sigma M_{AC} = 0$).



$$\overrightarrow{AD} = -0.8\mathbf{i} + 0.6\mathbf{j} - 2.4\mathbf{k} \qquad AD = 2.6 \text{ m}$$

$$\overrightarrow{AE} = 0.8\mathbf{i} + 1.2\mathbf{j} - 2.4\mathbf{k} \qquad AE = 2.8 \text{ m}$$

$$T_{AD} = \frac{\overrightarrow{AD}}{AD} = \frac{T_{AD}}{2.6}(-0.8\mathbf{i} + 0.6\mathbf{j} - 2.4\mathbf{k})$$

$$T_{AE} = \frac{\overrightarrow{AE}}{AE} = \frac{T_{AE}}{2.8}(0.8\mathbf{i} + 1.2\mathbf{j} - 2.4\mathbf{k})$$

$$\Sigma M_C = 0: \quad \mathbf{r}_A \times \mathbf{T}_{AD} + \mathbf{r}_A \times \mathbf{T}_{AE} + \mathbf{r}_A \times (-3.6 \text{ kN})\mathbf{j}$$

Factor r_A:

$$\mathbf{r}_A \times (\mathbf{T}_{AD} + \mathbf{T}_{AE} - (3.6 \text{ kN})\mathbf{j})$$

or

$$\mathbf{T}_{AD} + \mathbf{T}_{AE} - (3 \text{ kN})\mathbf{j} = 0 \quad \text{(Forces concurrent at } A\text{)}$$

<u>Coefficient of **i**</u>:

$$-\frac{T_{AD}}{2.6}(0.8) + \frac{T_{AE}}{2.8}(0.8) = 0$$

$$T_{AD} = \frac{2.6}{2.8}T_{AE} \tag{1}$$

<u>Coefficient of **j**</u>:

$$\frac{T_{AD}}{2.6}(0.6) + \frac{T_{AE}}{2.8}(1.2) - 3.6 \text{ kN} = 0$$

$$\frac{2.6}{2.8}T_{AE}\left(\frac{0.6}{2.6}\right) + \frac{1.2}{2.8}T_{AE} - 3.6 \text{ kN} = 0$$

$$T_{AE}\left(\frac{0.6 + 1.2}{2.8}\right) = 3.6 \text{ kN}$$

$$T_{AE} = 5.600 \text{ kN}$$

$T_{AE} = 5.60$ kN ◀

PROBLEM 4.106 (Continued)

From Eq. (1): $T_{AD} = \frac{2.6}{2.8}(5.6) = 5.200 \text{ kN}$ $\qquad T_{AD} = 5.20 \text{ kN}$ ◀

$$\Sigma F_x = 0: \quad C_x - \frac{0.8}{2.6}(5.2 \text{ kN}) + \frac{0.8}{2.8}(5.6 \text{ kN}) = 0 \qquad C_x = 0$$

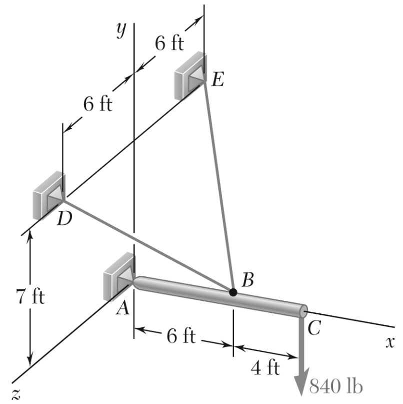
$$\Sigma F_y = 0: \quad C_y + \frac{0.6}{2.6}(5.2 \text{ kN}) + \frac{1.2}{2.8}(5.6 \text{ kN}) - 3.6 \text{ kN} = 0 \qquad C_y = 0$$

$$\Sigma F_z = 0: \quad C_z - \frac{2.4}{2.6}(5.2 \text{ kN}) - \frac{2.4}{2.8}(5.6 \text{ kN}) = 0 \qquad C_z = 9.60 \text{ kN}$$

$\mathbf{C} = (9.60 \text{ kN})\mathbf{k}$ ◀

Note: Since the forces and reaction are concurrent at *A*, we could have used the methods of Chapter 2.

PROBLEM 4.107

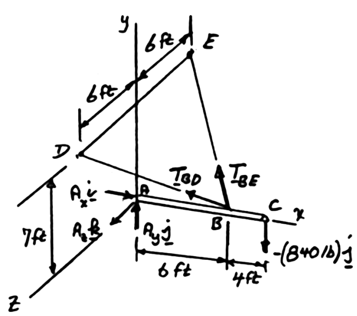


A 10-ft boom is acted upon by the 840-lb force shown. Determine the tension in each cable and the reaction at the ball-and-socket joint at A.

SOLUTION

We have five unknowns and six equations of equilibrium, but equilibrium is maintained $(\Sigma M_x = 0)$.

Free-Body Diagram:



$$\overrightarrow{BD} = (-6\text{ ft})\mathbf{i} + (7\text{ ft})\mathbf{j} + (6\text{ ft})\mathbf{k} \quad BD = 11\text{ ft}$$

$$\overrightarrow{BE} = (-6\text{ ft})\mathbf{i} + (7\text{ ft})\mathbf{j} - (6\text{ ft})\mathbf{k} \quad BE = 11\text{ ft}$$

$$T_{BD} = T_{BD}\frac{\overrightarrow{BD}}{BD} = \frac{T_{BD}}{11}(-6\mathbf{i} + 7\mathbf{j} + 6\mathbf{k})$$

$$T_{BE} = T_{BE}\frac{\overrightarrow{BE}}{BE} = \frac{T_{BE}}{11}(-6\mathbf{i} + 7\mathbf{j} - 6\mathbf{k})$$

$$\Sigma M_A = 0: \quad \mathbf{r}_B \times T_{BD} + \mathbf{r}_B \times T_{BE} + \mathbf{r}_C \times (-840\mathbf{j}) = 0$$

$$6\mathbf{i} \times \frac{T_{BD}}{11}(-6\mathbf{i} + 7\mathbf{j} + 6\mathbf{k}) + 6\mathbf{i} \times \frac{T_{BE}}{11}(-6\mathbf{i} + 7\mathbf{j} - 6\mathbf{k}) + 10\mathbf{i} \times (-840\mathbf{j}) = 0$$

$$\frac{42}{11}T_{BD}\mathbf{k} - \frac{36}{11}T_{BD}\mathbf{j} + \frac{42}{11}T_{BE}\mathbf{k} + \frac{36}{11}T_{BE}\mathbf{j} - 8400\mathbf{k} = 0$$

Equate coefficients of unit vectors to zero:

$$\mathbf{i}: \quad -\frac{36}{11}T_{BD} + \frac{36}{11}T_{BE} = 0 \quad T_{BE} = T_{BD}$$

$$\mathbf{k}: \quad \frac{42}{11}T_{BD} + \frac{42}{11}T_{BE} - 8400 = 0$$

$$2\left(\frac{42}{11}T_{BD}\right) = 8400$$

$T_{BD} = 1100 \text{ lb}$ ◀

$T_{BE} = 1100 \text{ lb}$ ◀

PROBLEM 4.107 (Continued)

$$\Sigma F_x = 0: \quad A_x - \frac{6}{11}(1100\text{ lb}) - \frac{6}{11}(1100\text{ lb}) = 0$$

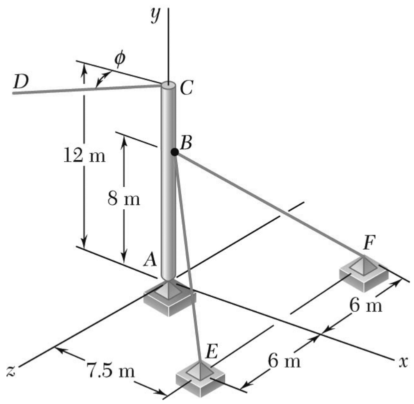
$$A_x = 1200\text{ lb}$$

$$\Sigma F_y = 0: \quad A_y + \frac{7}{11}(1100\text{ lb}) + \frac{7}{11}(1100\text{ lb}) - 840\text{ lb} = 0$$

$$A_y = -560\text{ lb}$$

$$\Sigma F_z = 0: \quad A_z + \frac{6}{11}(1100\text{ lb}) - \frac{6}{11}(1100\text{ lb}) = 0$$

$$A_z = 0 \qquad \mathbf{A} = (1200\text{ lb})\mathbf{i} - (560\text{ lb})\mathbf{j} \blacktriangleleft$$

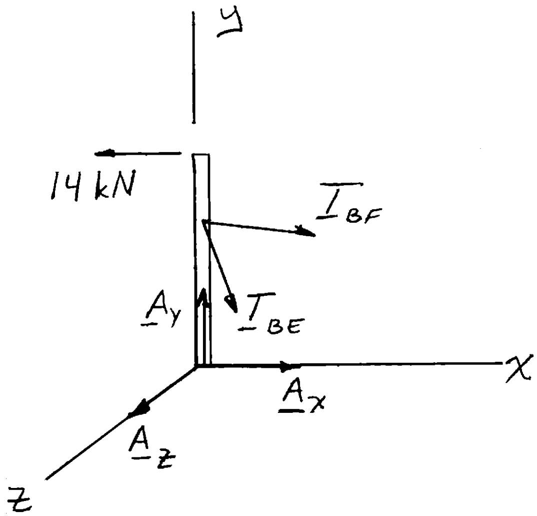


PROBLEM 4.108

A 12-m pole supports a horizontal cable *CD* and is held by a ball and socket at *A* and two cables *BE* and *BF*. Knowing that the tension in cable *CD* is 14 kN and assuming that *CD* is parallel to the *x*-axis ($\phi = 0$), determine the tension in cables *BE* and *BF* and the reaction at *A*.

SOLUTION

Free-Body Diagram:



There are five unknowns and six equations of equilibrium. The pole is free to rotate about the *y*-axis, but equilibrium is maintained under the given loading $(\Sigma M_y = 0)$.

Resolve $\overrightarrow{BE}$ and $\overrightarrow{BF}$ into components:

$$\overrightarrow{BE} = (7.5 \text{ m})\mathbf{i} - (8 \text{ m})\mathbf{j} + (6 \text{ m})\mathbf{k} \qquad BE = 12.5 \text{ m}$$

$$\overrightarrow{BF} = (7.5 \text{ m})\mathbf{i} - (8 \text{ m})\mathbf{j} - (6 \text{ m})\mathbf{k} \qquad BF = 12.5 \text{ m}$$

Express $\mathbf{T}_{BE}$ and $\mathbf{T}_{BF}$ in terms of components:

$$\mathbf{T}_{BE} = T_{BE}\frac{\overrightarrow{BE}}{BE} = T_{BE}(0.60\mathbf{i} - 0.64\mathbf{j} + 0.48\mathbf{k}) \qquad (1)$$

$$\mathbf{T}_{BF} = T_{BF}\frac{\overrightarrow{BF}}{BF} = T_{BF}(0.60\mathbf{i} - 0.64\mathbf{j} - 0.48\mathbf{k}) \qquad (2)$$

$$\Sigma M_A = 0: \quad \mathbf{r}_{B/A} \times T_{BE} + \mathbf{r}_{B/A} \times T_{BF} + \mathbf{r}_{C/A} \times (-14 \text{ kN})\mathbf{i} = 0$$

$$8\mathbf{j} \times T_{BE}(0.60\mathbf{i} - 0.64\mathbf{j} + 0.48\mathbf{k}) + 8\mathbf{j} \times T_{BF}(0.60\mathbf{i} - 0.64\mathbf{j} - 0.48\mathbf{k}) + 12\mathbf{j} \times (-14\mathbf{i}) = 0$$

$$-4.8\, T_{BE}\mathbf{k} + 3.84\, T_{BE}\mathbf{i} - 4.8 T_{BF}\mathbf{k} - 3.84 T_{BF}\mathbf{i} + 168\mathbf{k} = 0$$

Equating the coefficients of the unit vectors to zero,

$\mathbf{i}$: $\quad 3.84 T_{BE} - 3.84 T_{BF} = 0 \qquad T_{BE} = T_{BF}$

$\mathbf{k}$: $\quad -4.8 T_{BE} - 4.8 T_{BF} + 168 = 0 \qquad T_{BE} = T_{BF} = 17.50 \text{ kN}$ ◀

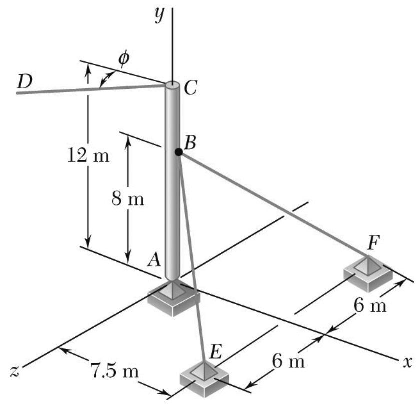
$\Sigma F_x = 0$: $\quad A_x + 2(0.60)(17.50 \text{ kN}) - 14 \text{ kN} = 0 \qquad A_x = 7.00 \text{ kN}$

$\Sigma F_y = 0$: $\quad A_y - z(0.64)(17.50 \text{ kN}) = 0 \qquad A_y = 22.4 \text{ kN}$

$\Sigma F_z = 0$: $\quad A_z + 0 = 0 \qquad A_z = 0$

$$\mathbf{A} = -(7.00 \text{ kN})\mathbf{i} + (22.4 \text{ kN})\mathbf{j}$$ ◀

Because of the symmetry, we could have noted at the outset that $T_{BF} = T_{BE}$ and eliminated one unknown.



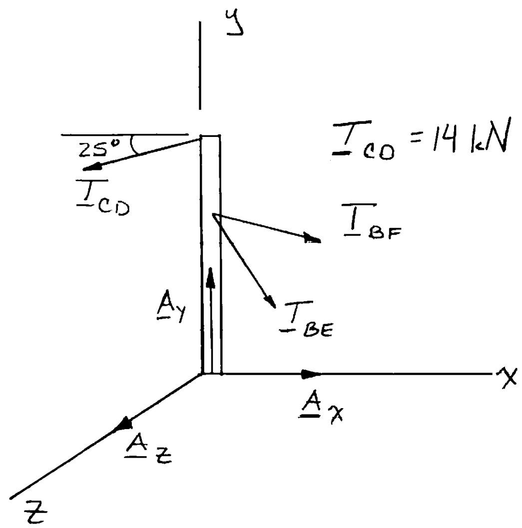
PROBLEM 4.109

Solve Problem 4.108, assuming that cable CD forms an angle $\phi = 25°$ with the vertical xy plane.

PROBLEM 4.108 A 12-m pole supports a horizontal cable CD and is held by a ball and socket at A and two cables BE and BF. Knowing that the tension in cable CD is 14 kN and assuming that CD is parallel to the x-axis ($\phi = 0$), determine the tension in cables BE and BF and the reaction at A.

SOLUTION

Free-Body Diagram:



$$\overrightarrow{BE} = (7.5 \text{ m})\mathbf{i} - (8 \text{ m})\mathbf{j} + (6 \text{ m})\mathbf{k}$$
$$BE = 12.5 \text{ m}$$
$$\overrightarrow{BF} = (7.5 \text{ m})\mathbf{i} - (8 \text{ m})\mathbf{j} - (6 \text{ m})\mathbf{k}$$
$$BF = 12.5 \text{ m}$$

$$\mathbf{T}_{BE} = T_{BE}\frac{\overrightarrow{BE}}{BE} = T_{BE}(0.60\mathbf{i} - 0.64\mathbf{j} + 0.48\mathbf{k})$$

$$\mathbf{T}_{BF} = T_{BF}\frac{\overrightarrow{BF}}{BF} = T_{BF}(0.60\mathbf{i} - 0.64\mathbf{j} - 0.48\mathbf{k})$$

$$\Sigma M_A = 0: \quad \mathbf{r}_{B/A} \times \mathbf{T}_{BE} + \mathbf{r}_{B/A} \times \mathbf{T}_{BF} + \mathbf{r}_{C/A} \times \mathbf{T}_{CD} = 0$$

$$8\mathbf{j} \times T_{BE}(0.60\mathbf{i} - 0.64\mathbf{j} + 0.48\mathbf{k}) + 8\mathbf{j} \times T_{BF}(0.60\mathbf{i} - 0.64\mathbf{j} - 0.48\mathbf{k})$$
$$+ 12\mathbf{j} \times (19 \text{ kN})(-\cos 25°\,\mathbf{i} + \sin 25°\mathbf{k}) = 0$$

$$-4.8T_{BE}\,\mathbf{k} + 3.84T_{BE}\,\mathbf{i} - 4.8T_{BF}\,\mathbf{k} - 3.84T_{BF}\,\mathbf{i} + 152.6\,\mathbf{k} - 71.00\,\mathbf{i} = 0$$

Equating the coefficients of the unit vectors to zero,

$$\mathbf{i:} \quad 3.84T_{BE} - 3.84T_{BF} + 71.00 = 0; \qquad T_{BF} - T_{BE} = 18.4896$$

$$\mathbf{k:} \quad -4.8T_{BE} - 4.8\,T_{BF} + 152.26 = 0; \qquad T_{BF} + T_{BE} = 31.721$$

Solving simultaneously, $\quad T_{BE} = 6.6157$ kN; $\qquad T_{BF} = 25.105$ kN

$$T_{BE} = 6.62 \text{ kN}; \; T_{BF} = 25.1 \text{ kN} \blacktriangleleft$$

$$\Sigma F_x = 0: \quad A_x + (0.60)(T_{BF} + T_{BE}) - 14\cos 25° = 0$$
$$A_x = 12.6883 - 0.60(31.7207)$$
$$A_x = -6.34 \text{ kN}$$

PROBLEM 4.109 (Continued)

$$\Sigma F_y = 0: \quad A_y - (0.64)(T_{BF} + T_{BE}) = 0$$

$$A_y = 0.64(31.721)$$

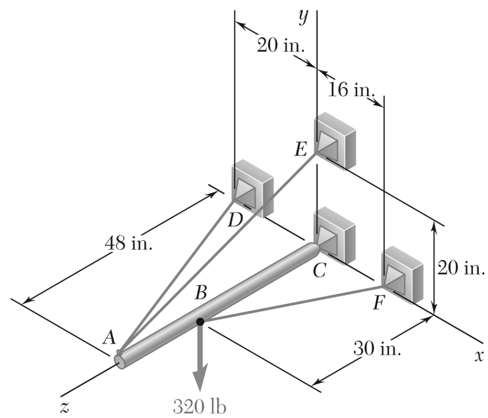
$$A_y = 20.3 \text{ kN}$$

$$\Sigma F_z = 0: \quad A_z - 0.48(T_{BF} - T_{BE}) + 14 \sin 25° = 0$$

$$A_z = 0.48(18.4893) - 5.9167$$

$$A_z = 2.96 \text{ kN}$$

$$\mathbf{A} = -(6.34 \text{ kN})\mathbf{i} + (20.3 \text{ kN})\mathbf{j} + (2.96 \text{ kN})\mathbf{k} \blacktriangleleft$$

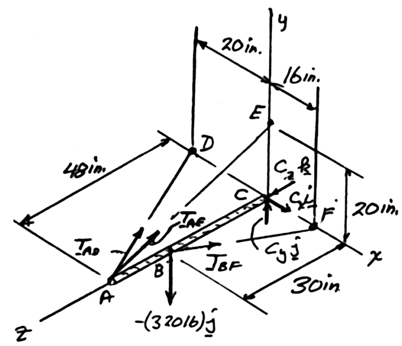


PROBLEM 4.110

A 48-in. boom is held by a ball-and-socket joint at C and by two cables BF and DAE; cable DAE passes around a frictionless pulley at A. For the loading shown, determine the tension in each cable and the reaction at C.

SOLUTION

Free-Body Diagram:



Five unknowns and six equations of equilibrium, but equilibrium is maintained $(\Sigma M_{AC} = 0)$.

$T =$ Tension in both parts of cable DAE.

$$\mathbf{r}_B = 30\mathbf{k}$$
$$\mathbf{r}_A = 48\mathbf{k}$$

$$\overrightarrow{AD} = -20\mathbf{i} - 48\mathbf{k} \qquad AD = 52 \text{ in.}$$
$$\overrightarrow{AE} = 20\mathbf{j} - 48\mathbf{k} \qquad AE = 52 \text{ in.}$$
$$\overrightarrow{BF} = 16\mathbf{i} - 30\mathbf{k} \qquad BF = 34 \text{ in.}$$

$$\mathbf{T}_{AD} = T\frac{\overrightarrow{AD}}{AD} = \frac{T}{52}(-20\mathbf{i} - 48\mathbf{k}) = \frac{T}{13}(-5\mathbf{i} - 12\mathbf{k})$$
$$\mathbf{T}_{AE} = T\frac{\overrightarrow{AE}}{AE} = \frac{T}{52}(20\mathbf{j} - 48\mathbf{k}) = \frac{T}{13}(5\mathbf{j} - 12\mathbf{k})$$
$$\mathbf{T}_{BF} = T_{BF}\frac{\overrightarrow{BF}}{BF} = \frac{T_{BF}}{34}(16\mathbf{i} - 30\mathbf{k}) = \frac{T_{BF}}{17}(8\mathbf{i} - 15\mathbf{k})$$

$$\Sigma \mathbf{M}_C = 0: \quad \mathbf{r}_A \times \mathbf{T}_{AD} + \mathbf{r}_A \times \mathbf{T}_{AE} + \mathbf{r}_B \times \mathbf{T}_{BF} + \mathbf{r}_B \times (-320 \text{ lb})\mathbf{j} = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 48 \\ -5 & 0 & -12 \end{vmatrix}\frac{T}{13} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 48 \\ 0 & 5 & -12 \end{vmatrix}\frac{T}{13} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 30 \\ 8 & 0 & -15 \end{vmatrix}\frac{T_{BF}}{17} + (30\mathbf{k}) \times (-320\mathbf{j}) = 0$$

Coefficient of **i**: $\qquad -\frac{240}{13}T + 9600 = 0 \qquad T = 520 \text{ lb}$

PROBLEM 4.110 (Continued)

Coefficient of **j**: $$-\frac{240}{13}T + \frac{240}{17}T_{BD} = 0$$

$$T_{BD} = \frac{17}{13}T = \frac{17}{13}(520) \quad T_{BD} = 680 \text{ lb}$$

$$\Sigma\mathbf{F} = 0: \quad \mathbf{T}_{AD} + \mathbf{T}_{AE} + \mathbf{T}_{BF} - 320\mathbf{j} + C = 0$$

Coefficient of **i**: $$-\frac{20}{52}(520) + \frac{8}{17}(680) + C_x = 0$$

$$-200 + 320 + C_x = 0 \qquad C_x = -120 \text{ lb}$$

Coefficient of **j**: $$\frac{20}{52}(520) - 320 + C_y = 0$$

$$200 - 320 + C_y = 0 \qquad C_y = 120 \text{ lb}$$

Coefficient of **k**: $$-\frac{48}{52}(520) - \frac{48}{52}(520) - \frac{30}{34}(680) + C_z = 0$$

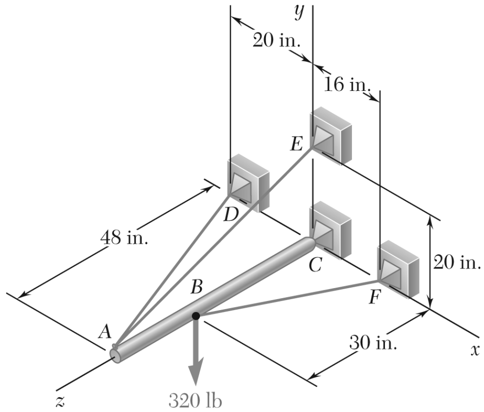
$$-480 - 480 - 600 + C_z = 0$$

$$C_z = 1560 \text{ lb}$$

Answers: $T_{DAE} = T$ $\quad T_{DAE} = 520 \text{ lb}$ ◀

$T_{BD} = 680 \text{ lb}$ ◀

$\mathbf{C} = -(120.0 \text{ lb})\mathbf{i} + (120.0 \text{ lb})\mathbf{j} + (1560 \text{ lb})\mathbf{k}$ ◀



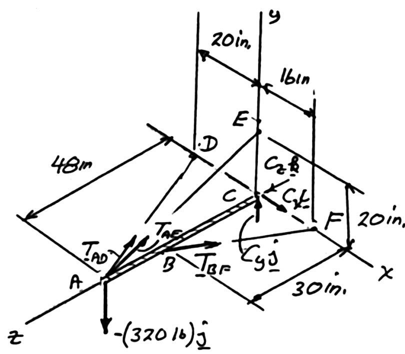
PROBLEM 4.111

Solve Problem 4.110, assuming that the 320-lb load is applied at A.

PROBLEM 4.110 A 48-in. boom is held by a ball-and-socket joint at C and by two cables BF and DAE; cable DAE passes around a frictionless pulley at A. For the loading shown, determine the tension in each cable and the reaction at C.

SOLUTION

Free-Body Diagram:



Five unknowns and six equations of equilibrium, but equilibrium is maintained $(\Sigma M_{AC} = 0)$.

T = tension in both parts of cable DAE.

$$\mathbf{r}_B = 30\mathbf{k}$$
$$\mathbf{r}_A = 48\mathbf{k}$$

$$\overrightarrow{AD} = -20\mathbf{i} - 48\mathbf{k} \qquad AD = 52 \text{ in.}$$
$$\overrightarrow{AE} = 20\mathbf{j} - 48\mathbf{k} \qquad AE = 52 \text{ in.}$$
$$\overrightarrow{BF} = 16\mathbf{i} - 30\mathbf{k} \qquad BF = 34 \text{ in.}$$

$$\mathbf{T}_{AD} = T\frac{\overrightarrow{AD}}{AD} = \frac{T}{52}(-20\mathbf{i} - 48\mathbf{k}) = \frac{T}{13}(-5\mathbf{i} - 12\mathbf{k})$$
$$\mathbf{T}_{AE} = T\frac{\overrightarrow{AE}}{AE} = \frac{T}{52}(20\mathbf{j} - 48\mathbf{k}) = \frac{T}{13}(5\mathbf{j} - 12\mathbf{k})$$
$$\mathbf{T}_{BF} = T_{BF}\frac{\overrightarrow{BF}}{BF} = \frac{T_{BF}}{34}(16\mathbf{i} - 30\mathbf{k}) = \frac{T_{BF}}{17}(8\mathbf{i} - 15\mathbf{k})$$

$$\Sigma M_C = 0: \quad \mathbf{r}_A \times \mathbf{T}_{AD} + \mathbf{r}_A \times \mathbf{T}_{AE} + \mathbf{r}_B \times \mathbf{T}_{BF} + \mathbf{r}_A \times (-320 \text{ lb})\mathbf{j} = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 48 \\ -5 & 0 & -12 \end{vmatrix}\frac{T}{13} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 48 \\ 0 & 5 & -12 \end{vmatrix}\frac{T}{13} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 30 \\ 8 & 0 & -15 \end{vmatrix}\frac{T_{BF}}{17} + 48\mathbf{k} \times (-320\mathbf{j}) = 0$$

Coefficient of $\mathbf{i}$: $$-\frac{240}{13}T + 15{,}360 = 0 \qquad T = 832 \text{ lb}$$

Coefficient of **j**: $-\frac{240}{13}T + \frac{240}{17}T_{BD} = 0$

$$T_{BD} = \frac{17}{13}T = \frac{17}{13}(832) \qquad T_{BD} = 1088 \text{ lb}$$

$$\Sigma \mathbf{F} = 0: \quad \mathbf{T}_{AD} + \mathbf{T}_{AE} + \mathbf{T}_{BF} - 320\mathbf{j} + \mathbf{C} = 0$$

Coefficient of **i**: $-\frac{20}{52}(832) + \frac{8}{17}(1088) + C_x = 0$

$$-320 + 512 + C_x = 0 \qquad C_x = -192 \text{ lb}$$

Coefficient of **j**: $\frac{20}{52}(832) - 320 + C_y = 0$

$$320 - 320 + C_y = 0 \qquad C_y = 0$$

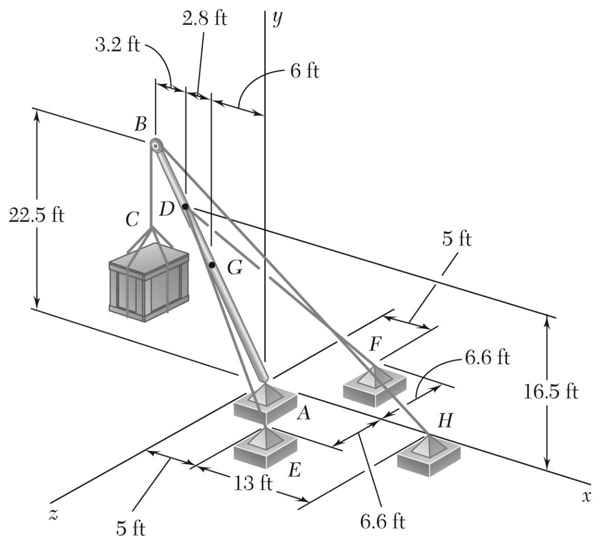
Coefficient of **k**: $-\frac{48}{52}(832) - \frac{48}{52}(852) - \frac{30}{34}(1088) + C_z = 0$

$$-768 - 768 - 960 + C_z = 0 \qquad C_z = 2496 \text{ lb}$$

Answers: $T_{DAE} = T$ $\qquad T_{DAE} = 832 \text{ lb}$ ◀

$T_{BD} = 1088 \text{ lb}$ ◀

$\mathbf{C} = -(192.0 \text{ lb})\mathbf{i} + (2496 \text{ lb})\mathbf{k}$ ◀

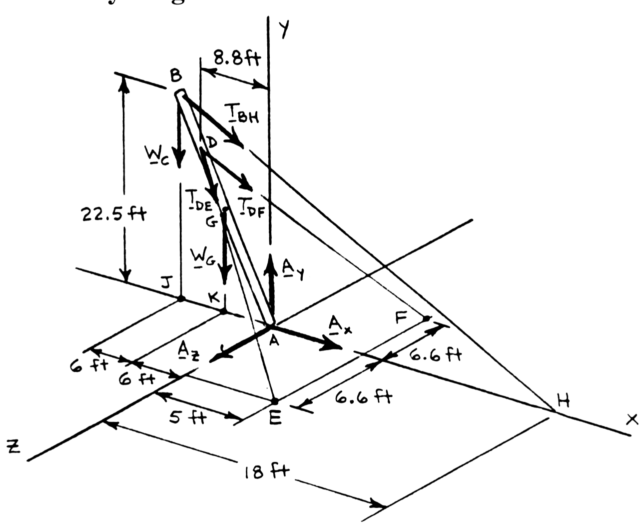


PROBLEM 4.112

A 600-lb crate hangs from a cable that passes over a pulley B and is attached to a support at H. The 200-lb boom AB is supported by a ball-and-socket joint at A and by two cables DE and DF. The center of gravity of the boom is located at G. Determine (*a*) the tension in cables DE and DF, (*b*) the reaction at A.

SOLUTION

Free-Body Diagram:



$W_C = 600 \text{ lb}$

$W_G = 200 \text{ lb}$

We have five unknowns $(T_{DE}, T_{DF}, A_x, A_y, A_z)$ and five equilibrium equations. The boom is free to spin about the AB axis, but equilibrium is maintained, since $\Sigma M_{AB} = 0$.

We have

$$\overrightarrow{BH} = (30 \text{ ft})\mathbf{i} - (22.5 \text{ ft})\mathbf{j} \qquad BH = 37.5 \text{ ft}$$

$$\overrightarrow{DE} = (13.8 \text{ ft})\mathbf{i} - \frac{8.8}{12}(22.5 \text{ ft})\mathbf{j} + (6.6 \text{ ft})\mathbf{k}$$

$$= (13.8 \text{ ft})\mathbf{i} - (16.5 \text{ ft})\mathbf{j} + (6.6 \text{ ft})\mathbf{k} \qquad DE = 22.5 \text{ ft}$$

$$\overrightarrow{DF} = (13.8 \text{ ft})\mathbf{i} - (16.5 \text{ ft})\mathbf{j} - (6.6 \text{ ft})\mathbf{k} \qquad DF = 22.5 \text{ ft}$$

Thus:
$$\mathbf{T}_{BH} = \mathbf{T}_{BH}\frac{\overrightarrow{BH}}{BH} = (600\text{ lb})\frac{30\mathbf{i} - 22.5\mathbf{j}}{37.5} = (480\text{ lb})\mathbf{i} - (360\text{ lb})\mathbf{j}$$
$$\mathbf{T}_{DE} = \mathbf{T}_{DE}\frac{\overrightarrow{DE}}{DE} = \frac{T_{DE}}{22.5}(13.8\mathbf{i} - 16.5\mathbf{j} + 6.6\mathbf{k})$$
$$\mathbf{T}_{DF} = \mathbf{T}_{DF}\frac{\overrightarrow{DF}}{DF} = \frac{T_{DE}}{22.5}(13.8\mathbf{i} - 16.5\mathbf{j} - 6.6\mathbf{k})$$

(*a*) $\Sigma\mathbf{M}_A = 0$: $(\mathbf{r}_J \times \mathbf{W}_C) + (\mathbf{r}_K \times \mathbf{W}_G) + (\mathbf{r}_H \times \mathbf{T}_{BH}) + (\mathbf{r}_E \times \mathbf{T}_{DE}) + (\mathbf{r}_F \times \mathbf{T}_{DF}) = 0$
$$-(12\mathbf{i}) \times (-600\mathbf{j}) - (6\mathbf{i}) \times (-200\mathbf{j}) + (18\mathbf{i}) \times (480\mathbf{i} - 360\mathbf{j})$$
$$+\frac{T_{DE}}{22.5}\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 5 & 0 & 6.6 \\ 13.8 & -16.5 & 6.6 \end{vmatrix} + \frac{T_{DF}}{22.5}\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 5 & 0 & -6.6 \\ 13.8 & -16.5 & -6.6 \end{vmatrix} = 0$$

or
$$7200\mathbf{k} + 1200\mathbf{k} - 6480\mathbf{k} + 4.84(T_{DE} - T_{DF})\mathbf{i}$$
$$+\frac{58.08}{22.5}(T_{DE} - T_{DF})\mathbf{j} - \frac{82.5}{22.5}(T_{DE} + T_{DF})\mathbf{k} = 0$$

Equating to zero the coefficients of the unit vectors,

i or **j**: $T_{DE} - T_{DF} = 0 \qquad T_{DE} = T_{DF}$*

k: $7200 + 1200 - 6480 - \dfrac{82.5}{22.5}(2T_{DE}) = 0 \qquad T_{DE} = 261.82\text{ lb}$

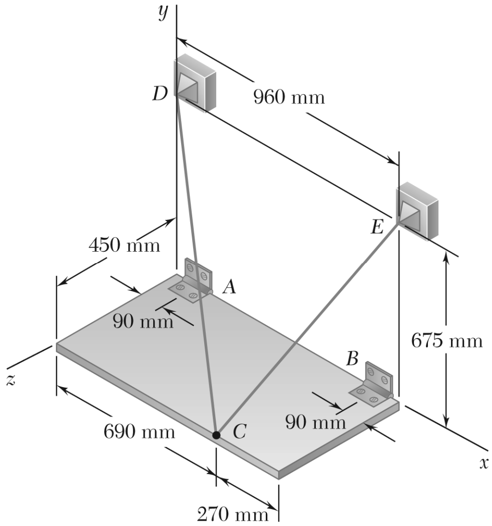
$$T_{DE} = T_{DF} = 262\text{ lb} \blacktriangleleft$$

(*b*) $\Sigma F_x = 0$: $A_x + 480 + 2\left(\dfrac{13.8}{22.5}\right)(261.82) = 0 \qquad A_x = -801.17\text{ lb}$

$\Sigma F_y = 0$: $A_y - 600 - 200 - 360 - 2\left(\dfrac{16.5}{22.5}\right)(261.82) = 0 \qquad A_y = 1544.00\text{ lb}$

$\Sigma F_z = 0$: $A_z = 0$
$$\mathbf{A} = -(801\text{ lb})\mathbf{i} + (1544\text{ lb})\mathbf{j} \blacktriangleleft$$

*Remark: The fact is that $T_{DE} = T_{DF}$ could have been noted at the outset from the symmetry of structure with respect to *xy* plane.

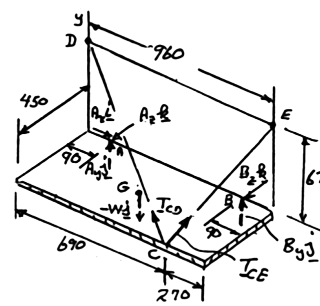


PROBLEM 4.113

A 100-kg uniform rectangular plate is supported in the position shown by hinges A and B and by cable DCE that passes over a frictionless hook at C. Assuming that the tension is the same in both parts of the cable, determine (*a*) the tension in the cable, (*b*) the reactions at A and B. Assume that the hinge at B does not exert any axial thrust.

SOLUTION

Dimensions in mm



$$\mathbf{r}_{B/A}(960-180)\mathbf{i} = 780\mathbf{i}$$

$$\mathbf{r}_{G/A} = \left(\frac{960}{2} - 90\right)\mathbf{i} + \frac{450}{2}\mathbf{k}$$

$$= 390\mathbf{i} + 225\mathbf{k}$$

$$\mathbf{r}_{C/A} = 600\mathbf{i} + 450\mathbf{k}$$

T = Tension in cable DCE

$$\overrightarrow{CD} = -690\mathbf{i} + 675\mathbf{j} - 450\mathbf{k} \qquad CD = 1065 \text{ mm}$$

$$\overrightarrow{CE} = 270\mathbf{i} + 675\mathbf{j} - 450\mathbf{k} \qquad CE = 855 \text{ mm}$$

$$\mathbf{T}_{CD} = \frac{T}{1065}(-690\mathbf{i} + 675\mathbf{j} - 450\mathbf{k})$$

$$\mathbf{T}_{CE} = \frac{T}{855}(270\mathbf{i} + 675\mathbf{j} - 450\mathbf{k})$$

$$\mathbf{W} = -mg\mathbf{i} = -(100 \text{ kg})(9.81 \text{ m/s}^2)\mathbf{j} = -(981 \text{ N})\mathbf{j}$$

$$\Sigma\mathbf{M}_A = 0: \quad \mathbf{r}_{C/A} \times \mathbf{T}_{CD} + \mathbf{r}_{C/A} \times \mathbf{T}_{CE} + \mathbf{r}_{G/A} \times (-W\mathbf{j}) + \mathbf{r}_{B/A} \times \mathbf{B} = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 600 & 0 & 450 \\ -690 & 675 & -450 \end{vmatrix} \frac{T}{1065} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 600 & 0 & 450 \\ 270 & 675 & -450 \end{vmatrix} \frac{T}{855}$$

$$+ \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 390 & 0 & 225 \\ 0 & -981 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 780 & 0 & 0 \\ 0 & B_y & B_z \end{vmatrix} = 0$$

PROBLEM 4.113 (Continued)

Coefficient of **i**: $-(450)(675)\dfrac{T}{1065}-(450)(675)\dfrac{T}{855}+220.73\times 10^3=0$

$T = 344.64$ N $\qquad T = 345$ N ◀

Coefficient of **j**: $(-690\times 450+600\times 450)\dfrac{344.64}{1065}+(270\times 450+600\times 450)\dfrac{344.64}{855}-780B_z=0$

$B_z = 185.516$ N

Coefficient of **k**: $(600)(675)\dfrac{344.64}{1065}+(600)(675)\dfrac{344.64}{855}-382.59\times 10^3+780B_y=0 \quad B_y=113.178$ N

$\mathbf{B} = (113.2 \text{ N})\mathbf{j} + (185.5 \text{ N})\mathbf{k}$ ◀

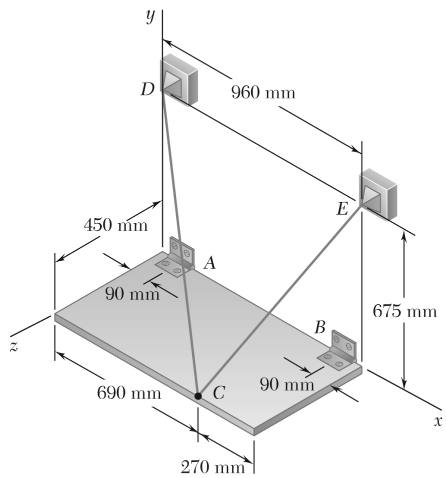
$\Sigma\mathbf{F}=0: \quad \mathbf{A}+\mathbf{B}+\mathbf{T}_{CD}+\mathbf{T}_{CE}+\mathbf{W}=0$

Coefficient of **i**: $A_x-\dfrac{690}{1065}(344.64)+\dfrac{270}{855}(344.64)=0 \qquad A_x=114.5$ N

Coefficient of **j**: $A_y+113.178+\dfrac{675}{1065}(344.64)+\dfrac{675}{855}(344.64)-981=0 \qquad A_y=377$ N

Coefficient of **k**: $A_z+185.516-\dfrac{450}{1065}(344.64)-\dfrac{450}{855}(344.64)=0 \qquad A_z=141.5$ N

$\mathbf{A} = (114.5 \text{ N})\mathbf{i} + (377 \text{ N})\mathbf{j} + (144.5 \text{ N})\mathbf{k}$ ◀



PROBLEM 4.114

Solve Problem 4.113, assuming that cable *DCE* is replaced by a cable attached to Point *E* and hook *C*.

PROBLEM 4.113 A 100-kg uniform rectangular plate is supported in the position shown by hinges *A* and *B* and by cable *DCE* that passes over a frictionless hook at *C*. Assuming that the tension is the same in both parts of the cable, determine (*a*) the tension in the cable, (*b*) the reactions at *A* and *B*. Assume that the hinge at *B* does not exert any axial thrust.

SOLUTION

See solution to Problem 4.113 for free-body diagram and analysis leading to the following:

$$CD = 1065 \text{ mm}$$

$$CE = 855 \text{ mm}$$

Now,

$$\mathbf{T}_{CD} = \frac{T}{1065}(-690\mathbf{i} + 675\mathbf{j} - 450\mathbf{k})$$

$$\mathbf{T}_{CE} = \frac{T}{855}(270\mathbf{i} + 675\mathbf{j} - 450\mathbf{k})$$

$$\mathbf{W} = -mg\mathbf{i} = -(100 \text{ kg})(9.81 \text{ m/s}^2)\mathbf{j} = -(981 \text{ N})\mathbf{j}$$

$$\Sigma\mathbf{M}_A = 0: \quad \mathbf{r}_{C/A} \times T_{CE} + \mathbf{r}_{G/A} \times (-W\mathbf{j}) + \mathbf{r}_{B/A} \times B = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 600 & 0 & 450 \\ 270 & 675 & -450 \end{vmatrix} \frac{T}{855} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 390 & 0 & 225 \\ 0 & -981 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 780 & 0 & 0 \\ 0 & B_y & B_z \end{vmatrix} = 0$$

Coefficient of **i**:

$$-(450)(675)\frac{T}{855} + 220.73 \times 10^3 = 0$$

$$T = 621.31 \text{ N} \qquad\qquad T = 621 \text{ N} \blacktriangleleft$$

Coefficient of **j**:

$$(270 \times 450 + 600 \times 450)\frac{621.31}{855} - 780B_z = 0 \quad B_z = 364.74 \text{ N}$$

Coefficient of **k**:

$$(600)(675)\frac{621.31}{855} - 382.59 \times 10^3 + 780B_y = 0 \quad B_y = 113.186 \text{ N}$$

$$\mathbf{B} = (113.2 \text{ N})\mathbf{j} + (365 \text{ N})\mathbf{k} \blacktriangleleft$$

PROBLEM 4.114 (Continued)

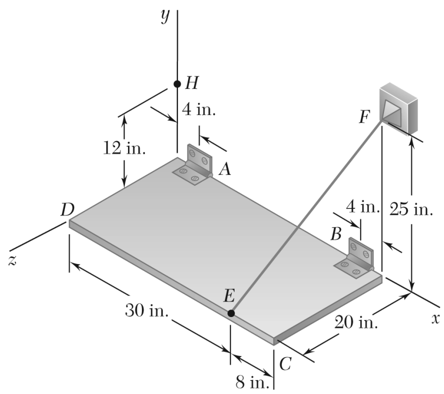
$$\Sigma \mathbf{F} = 0: \quad \mathbf{A} + \mathbf{B} + \mathbf{T}_{CE} + \mathbf{W} = 0$$

Coefficient of **i**: $\quad A_x + \frac{270}{855}(621.31) = 0 \qquad A_x = -196.2 \text{ N}$

Coefficient of **j**: $\quad A_y + 113.186 + \frac{675}{855}(621.31) - 981 = 0 \qquad A_y = 377.3 \text{ N}$

Coefficient of **k**: $\quad A_z + 364.74 - \frac{450}{855}(621.31) = 0 \qquad A_z = -37.7 \text{ N}$

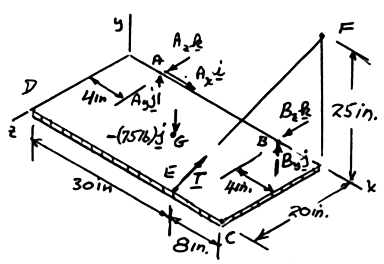
$$\mathbf{A} = -(196.2 \text{ N})\mathbf{i} + (377 \text{ N})\mathbf{j} - (37.7 \text{ N})\mathbf{k} \blacktriangleleft$$



PROBLEM 4.115

The rectangular plate shown weighs 75 lb and is held in the position shown by hinges at A and B and by cable EF. Assuming that the hinge at B does not exert any axial thrust, determine (*a*) the tension in the cable, (*b*) the reactions at A and B.

SOLUTION



$$\mathbf{r}_{B/A} = (38-8)\mathbf{i} = 30\mathbf{i}$$

$$\mathbf{r}_{E/A} = (30-4)\mathbf{i} + 20\mathbf{k}$$
$$= 26\mathbf{i} + 20\mathbf{k}$$

$$\mathbf{r}_{G/A} = \frac{38}{2}\mathbf{i} + 10\mathbf{k}$$
$$= 19\mathbf{i} + 10\mathbf{k}$$

$$\overrightarrow{EF} = 8\mathbf{i} + 25\mathbf{j} - 20\mathbf{k}$$

$$EF = 33 \text{ in.}$$

$$\mathbf{T} = T\frac{\overrightarrow{AE}}{AE} = \frac{T}{33}(8\mathbf{i} + 25\mathbf{j} - 20\mathbf{k})$$

$$\Sigma\mathbf{M}_A = 0: \quad \mathbf{r}_{E/A} \times T + \mathbf{r}_{G/A} \times (-75\mathbf{j}) + \mathbf{r}_{B/A} \times B = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 26 & 0 & 20 \\ 8 & 25 & -20 \end{vmatrix}\frac{T}{33} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 19 & 0 & 10 \\ 0 & -75 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 30 & 0 & 0 \\ 0 & B_y & B_z \end{vmatrix} = 0$$

Coefficient of $\mathbf{i}$: $\quad -(25)(20)\dfrac{T}{33} + 750 = 0$: $\quad T = 49.5 \text{ lb}$ ◀

Coefficient of $\mathbf{j}$: $\quad (160+520)\dfrac{49.5}{33} - 30B_z = 0$: $\quad B_z = 34 \text{ lb}$

Coefficient of $\mathbf{k}$: $\quad (26)(25)\dfrac{49.5}{33} - 1425 + 30B_y = 0$: $\quad B_y = 15 \text{ lb}$ $\quad \mathbf{B} = (15.00 \text{ lb})\mathbf{j} + (34.0 \text{ lb})\mathbf{k}$ ◀

PROBLEM 4.115 (Continued)

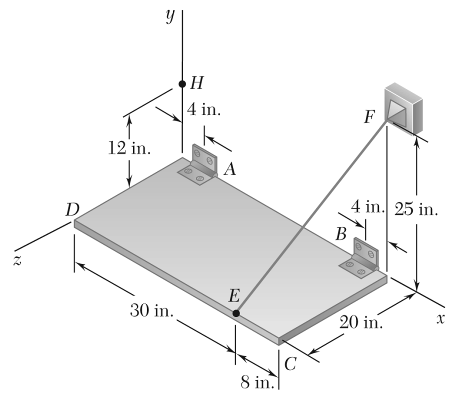
$$\Sigma \mathbf{F} = 0: \quad \mathbf{A} + \mathbf{B} + \mathbf{T} - (75 \text{ lb})\mathbf{j} = 0$$

Coefficient of **i**: $A_x + \dfrac{8}{33}(49.5) = 0 \quad A_x = -12.00 \text{ lb}$

Coefficient of **j**: $A_y + 15 + \dfrac{25}{33}(49.5) - 75 = 0 \quad A_y = 22.5 \text{ lb}$

Coefficient of **k**: $A_z + 34 - \dfrac{20}{33}(49.5) = 0 \quad A_z = -4.00 \text{ lb}$

$$\mathbf{A} = -(12.00 \text{ lb})\mathbf{i} + (22.5 \text{ lb})\mathbf{j} - (4.00 \text{ lb})\mathbf{k} \blacktriangleleft$$

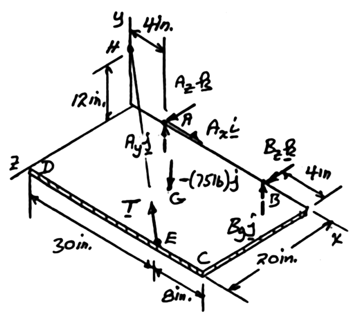


PROBLEM 4.116

Solve Problem 4.115, assuming that cable EF is replaced by a cable attached at points E and H.

PROBLEM 4.115 The rectangular plate shown weighs 75 lb and is held in the position shown by hinges at A and B and by cable EF. Assuming that the hinge at B does not exert any axial thrust, determine (*a*) the tension in the cable, (*b*) the reactions at A and B.

SOLUTION



$$\mathbf{r}_{B/A} = (38-8)\mathbf{i} = 30\mathbf{i}$$

$$\mathbf{r}_{E/A} = (30-4)\mathbf{i} + 20\mathbf{k}$$
$$= 26\mathbf{i} + 20\mathbf{k}$$

$$\mathbf{r}_{G/A} = \frac{38}{2}\mathbf{i} + 10\mathbf{k}$$
$$= 19\mathbf{i} + 10\mathbf{k}$$

$$\overrightarrow{EH} = -30\mathbf{i} + 12\mathbf{j} - 20\mathbf{k}$$
$$EH = 38 \text{ in.}$$

$$\mathbf{T} = T\frac{\overrightarrow{EH}}{EH} = \frac{T}{38}(-30\mathbf{i} + 12\mathbf{j} - 20\mathbf{k})$$

$$\Sigma\mathbf{M}_A = 0: \quad \mathbf{r}_{E/A} \times \mathbf{T} + \mathbf{r}_{G/A} \times (-75\mathbf{j}) + \mathbf{r}_{B/A} \times \mathbf{B} = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 26 & 0 & 20 \\ -30 & 12 & -20 \end{vmatrix}\frac{T}{38} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 19 & 0 & 10 \\ 0 & -75 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 30 & 0 & 0 \\ 0 & B_y & B_z \end{vmatrix} = 0$$

Coefficient of **i**: $\quad -(12)(20)\dfrac{T}{38} + 750 = 0 \qquad T = 118.75$ $\qquad T = 118.8\,\text{lb}$ ◀

Coefficient of **j**: $\quad (-600+520)\dfrac{118.75}{38} - 30B_z = 0 \qquad B_z = -8.33\,\text{lb}$

Coefficient of **k**: $\quad (26)(12)\dfrac{118.75}{38} - 1425 + 30B_y = 0 \qquad B_y = 15.00\text{ lb}$ $\qquad \mathbf{B} = (15.00\text{ lb})\mathbf{j} - (8.33\text{ lb})\mathbf{k}$ ◀

PROBLEM 4.116 (Continued)

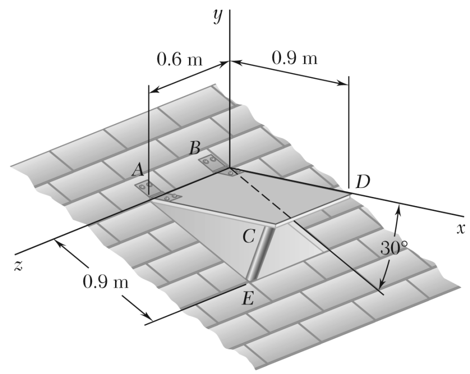
$$\Sigma \mathbf{F} = 0: \quad \mathbf{A} + \mathbf{B} + \mathbf{T} - (75 \text{ lb})\mathbf{j} = 0$$

Coefficient of **i**: $A_x - \frac{30}{38}(118.75) = 0 \quad A_x = 93.75 \text{ lb}$

Coefficient of **j**: $A_y + 15 + \frac{12}{38}(118.75) - 75 = 0 \quad A_y = 22.5 \text{ lb}$

Coefficient of **k**: $A_z - 8.33 - \frac{20}{38}(118.75) = 0 \quad A_z = 70.83 \text{ lb}$

$$\mathbf{A} = (93.8 \text{ lb})\mathbf{i} + (22.5 \text{ lb})\mathbf{j} + (70.8 \text{ lb})\mathbf{k} \blacktriangleleft$$

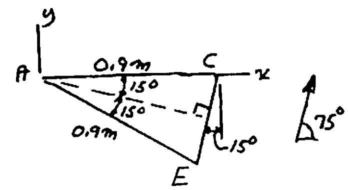


PROBLEM 4.117

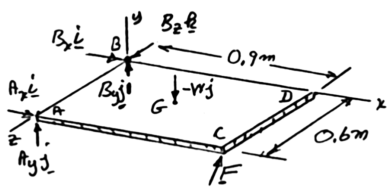
A 20-kg cover for a roof opening is hinged at corners *A* and *B*. The roof forms an angle of 30° with the horizontal, and the cover is maintained in a horizontal position by the brace *CE*. Determine (*a*) the magnitude of the force exerted by the brace, (*b*) the reactions at the hinges. Assume that the hinge at *A* does not exert any axial thrust.

SOLUTION

Force exerted by *CE*:



$$\mathbf{F} = F(\cos 75°)\mathbf{i} + F(\sin 75°)\mathbf{j}$$
$$\mathbf{F} = F(0.25882\mathbf{i} + 0.96593\mathbf{j})$$
$$W = mg = 20\text{ kg}(9.81\text{ m/s}^2) = 196.2\text{ N}$$



$$\mathbf{r}_{A/B} = 0.6\mathbf{k}$$
$$\mathbf{r}_{C/B} = 0.9\mathbf{i} + 0.6\mathbf{k}$$
$$\mathbf{r}_{G/B} = 0.45\mathbf{i} + 0.3\mathbf{k}$$
$$\mathbf{F} = F(0.25882\mathbf{i} + 0.96593\mathbf{j})$$

$$\Sigma\mathbf{M}_B = 0: \quad \mathbf{r}_{G/B} \times (-196.2\mathbf{j}) + \mathbf{r}_{C/B} \times \mathbf{F} + \mathbf{r}_{A/B} \times \mathbf{A} = 0$$

(*a*)

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.45 & 0 & 0.3 \\ 0 & -196.2 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.9 & 0 & 0.6 \\ 0.25882 & +0.96593 & 0 \end{vmatrix} F + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 0.6 \\ A_x & A_y & 0 \end{vmatrix} = 0$$

Coefficient of **i** : $\quad +58.86 - 0.57956F - 0.6A_y = 0 \quad (1)$

Coefficient of **j**: $\quad +0.155292F + 0.6A_x = 0 \quad (2)$

Coefficient of **k**: $\quad -88.29 + 0.86934F = 0: \quad F = 101.56\text{ N}$

From Eq. (2): $\quad +58.86 - 0.57956(101.56) - 0.6A_y = 0 \quad A_y = 0$

From Eq. (3): $\quad +0.155292(101.56) + 0.6A_x = 0 \quad A_x = -26.286\text{ N}$

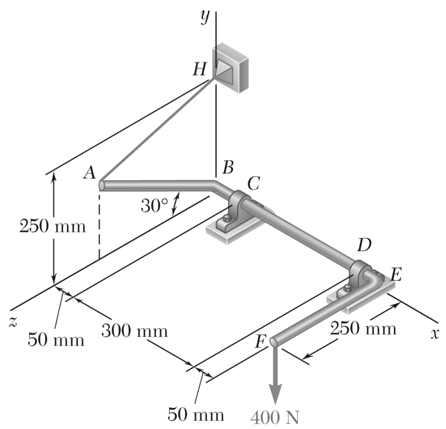
$F = (101.6\text{ N})$ ◀

(*b*) $\quad \Sigma\mathbf{F}: \quad \mathbf{A} + \mathbf{B} + \mathbf{F} - W_{\mathbf{j}} = 0$

Coefficient of **i**: $\quad 26.286 + B_x + 0.25882(101.56) = 0 \quad B_x = 0$

Coefficient of **j**: $\quad B_y + 0.96593(101.56) - 196.2 = 0 \quad B_y = 98.1\text{ N}$

Coefficient of **k**: $\quad B_z = 0 \quad$ **A** $= -(26.3\text{ N})\mathbf{i}$; **B** $= (98.1\text{ N})\mathbf{j}$ ◀



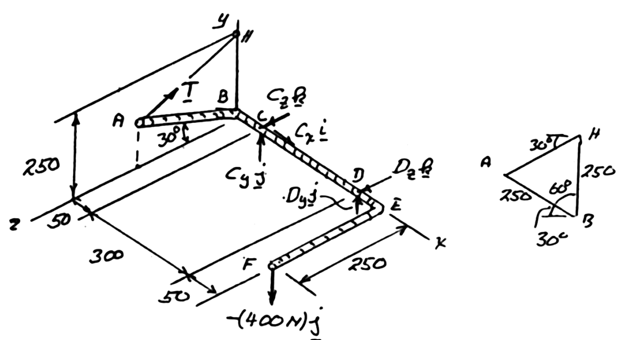
PROBLEM 4.118

The bent rod *ABEF* is supported by bearings at *C* and *D* and by wire *AH*. Knowing that portion *AB* of the rod is 250 mm long, determine (*a*) the tension in wire *AH*, (*b*) the reactions at *C* and *D*. Assume that the bearing at *D* does not exert any axial thrust.

SOLUTION

Free-Body Diagram: ΔABH is equilateral.

Dimensions in mm



$$\mathbf{r}_{H/C} = -50\mathbf{i} + 250\mathbf{j}$$
$$\mathbf{r}_{D/C} = 300\mathbf{i}$$
$$\mathbf{r}_{F/C} = 350\mathbf{i} + 250\mathbf{k}$$
$$\mathbf{T} = T(\sin 30°)\mathbf{j} - T(\cos 30°)\mathbf{k} = T(0.5\mathbf{j} - 0.866\mathbf{k})$$

$$\Sigma\mathbf{M}_C = 0: \quad \mathbf{r}_{H/C} \times \mathbf{T} + \mathbf{r}_D \times \mathbf{D} + \mathbf{r}_{F/C} \times (-400\mathbf{j}) = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -50 & 250 & 0 \\ 0 & 0.5 & -0.866 \end{vmatrix} T + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 300 & 0 & 0 \\ 0 & D_y & D_z \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 350 & 0 & 250 \\ 0 & -400 & 0 \end{vmatrix} = 0$$

Coefficient **i**: $-216.5T + 100 \times 10^3 = 0$

$T = 461.9 \text{ N}$ $\qquad T = 462 \text{ N}$ ◀

Coefficient of **j**: $-43.3T - 300D_z = 0$

$-43.3(461.9) - 300D_z = 0 \qquad D_z = -66.67 \text{ N}$

PROBLEM 4.118 (Continued)

Coefficient of **k**: $-25T + 300D_y - 140\times10^3 = 0$

$-25(461.9) + 300D_y - 140\times10^3 = 0 \qquad D_y = 505.1\text{ N}$

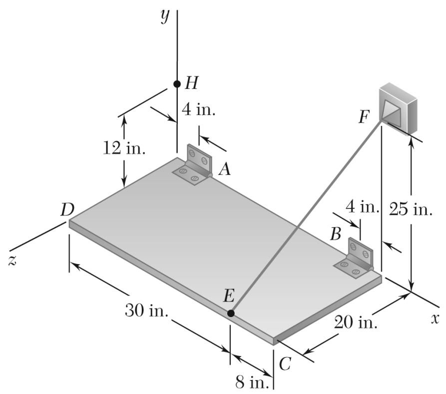
$\mathbf{D} = (505\text{ N})\mathbf{j} - (66.7\text{ N})\mathbf{k}$ ◀

$\Sigma\mathbf{F} = 0: \quad \mathbf{C} + \mathbf{D} + \mathbf{T} - 400\mathbf{j} = 0$

Coefficient **i**: $C_x = 0 \qquad C_x = 0$

Coefficient **j**: $C_y + (461.9)0.5 + 505.1 - 400 = 0 \qquad C_y = -336\text{ N}$

Coefficient **k**: $C_z - (461.9)0.866 - 66.67 = 0 \qquad C_z = 467\text{ N} \qquad \mathbf{C} = -(336\text{ N})\mathbf{j} + (467\text{ N})\mathbf{k}$ ◀

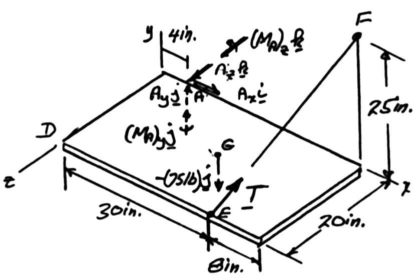


PROBLEM 4.119

Solve Problem 4.115, assuming that the hinge at *B* is removed and that the hinge at *A* can exert couples about axes parallel to the *y* and *z* axes.

PROBLEM 4.115 The rectangular plate shown weighs 75 lb and is held in the position shown by hinges at *A* and *B* and by cable *EF*. Assuming that the hinge at *B* does not exert any axial thrust, determine (*a*) the tension in the cable, (*b*) the reactions at *A* and *B*.

SOLUTION



$$\mathbf{r}_{E/A} = (30-4)\mathbf{i} + 20\mathbf{k} = 26\mathbf{i} + 20\mathbf{k}$$

$$\mathbf{r}_{G/A} = (0.5\times 38)\mathbf{i} + 10\mathbf{k} = 19\mathbf{i} + 10\mathbf{k}$$

$$\overrightarrow{AE} = 8\mathbf{i} + 25\mathbf{j} - 20\mathbf{k}$$

$$AE = 33 \text{ in.}$$

$$T = T\frac{\overrightarrow{AE}}{AE} = \frac{T}{33}(8\mathbf{i} + 25\mathbf{j} - 20\mathbf{k})$$

$$\Sigma\mathbf{M}_A = 0: \quad \mathbf{r}_{E/A}\times\mathbf{T} + \mathbf{r}_{G/A}\times(-75\mathbf{j}) + (M_A)_y\mathbf{j} + (M_A)_z\mathbf{k} = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 26 & 0 & 20 \\ 8 & 25 & -20 \end{vmatrix}\frac{T}{33} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 19 & 0 & 10 \\ 0 & -75 & 0 \end{vmatrix} + (M_A)_y\mathbf{j} + (M_A)_z\mathbf{k} = 0$$

Coefficient of **i**: $\quad -(20)(25)\dfrac{T}{33} + 750 = 0 \qquad T = 49.5 \text{ lb}$ ◀

Coefficient of **j**: $\quad (160+520)\dfrac{49.5}{33} + (M_A)_y = 0 \quad (M_A)_y = -1020 \text{ lb}\cdot\text{in.}$

Coefficient of **k**: $\quad (26)(25)\dfrac{49.5}{33} - 1425 + (M_A)_z = 0 \quad (M_A)_z = 450 \text{ lb}\cdot\text{in.}$

$$\Sigma F = 0: \quad A + T - 75\mathbf{j} = 0$$

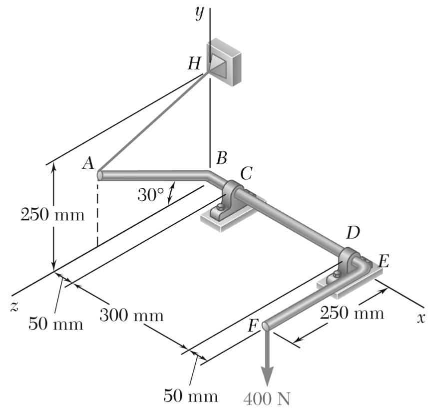
$\mathbf{M}_A = -(1020 \text{ lb}\cdot\text{in.})\mathbf{j} + (450 \text{ lb}\cdot\text{in.})\mathbf{k}$ ◀

Coefficient of **i**: $\quad A_x + \dfrac{8}{33}(49.5) = 0 \quad A_x = 12.00 \text{ lb}$

Coefficient of **j**: $\quad A_y + \dfrac{25}{33}(49.5) - 75 = 0 \quad A_y = 37.5 \text{ lb}$

Coefficient of **k**: $\quad A_z - \dfrac{20}{33}(49.5) = 0 \quad A_z = 30.0 \text{ lb}$

$\mathbf{A} = -(12.00 \text{ lb})\mathbf{i} + (37.5 \text{ lb})\mathbf{j} + (30.0 \text{ lb})\mathbf{k}$ ◀



PROBLEM 4.120

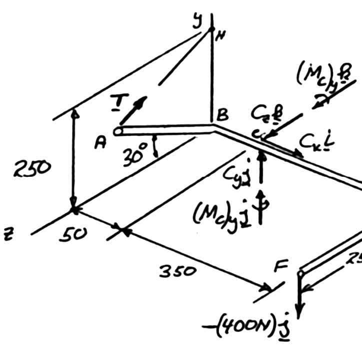
Solve Problem 4.118, assuming that the bearing at D is removed and that the bearing at C can exert couples about axes parallel to the y and z axes.

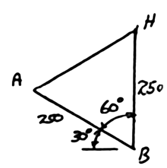
PROBLEM 4.118 The bent rod $ABEF$ is supported by bearings at C and D and by wire AH. Knowing that portion AB of the rod is 250 mm long, determine (*a*) the tension in wire AH, (*b*) the reactions at C and D. Assume that the bearing at D does not exert any axial thrust.

SOLUTION

Free-Body Diagram: ΔABH is equilateral.

Dimensions in mm





$$\mathbf{r}_{H/C} = -50\mathbf{i} + 250\mathbf{j}$$
$$\mathbf{r}_{F/C} = 350\mathbf{i} + 250\mathbf{k}$$
$$\mathbf{T} = T(\sin 30°)\mathbf{j} - T(\cos 30°)\mathbf{k} = T(0.5\mathbf{j} - 0.866\mathbf{k})$$

$$\Sigma \mathbf{M}_C = 0: \quad \mathbf{r}_{F/C} \times (-400\mathbf{j}) + \mathbf{r}_{H/C} \times T + (M_C)_y \mathbf{j} + (M_C)_z \mathbf{k} = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 350 & 0 & 250 \\ 0 & -400 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -50 & 250 & 0 \\ 0 & 0.5 & -0.866 \end{vmatrix} T + (M_C)_y \mathbf{j} + (M_C)_z \mathbf{k} = 0$$

Coefficient of **i**: $+100 \times 10^3 - 216.5T = 0 \quad T = 461.9 \text{ N}$ $\qquad T = 462 \text{ N}$ ◀

Coefficient of **j**: $-43.3(461.9) + (M_C)_y = 0$

$$(M_C)_y = 20 \times 10^3 \text{ N} \cdot \text{mm}$$
$$(M_C)_y = 20.0 \text{ N} \cdot \text{m}$$

PROBLEM 4.120 (Continued)

Coefficient of **k**: $-140\times 10^3 - 25(461.9) + (M_C)_z = 0$

$$(M_C)_z = 151.54\times 10^3 \text{ N}\cdot\text{mm}$$
$$(M_C)_z = 151.5 \text{ N}\cdot\text{m}$$

$$\Sigma F = 0: \quad C + T - 400\mathbf{j} = 0$$

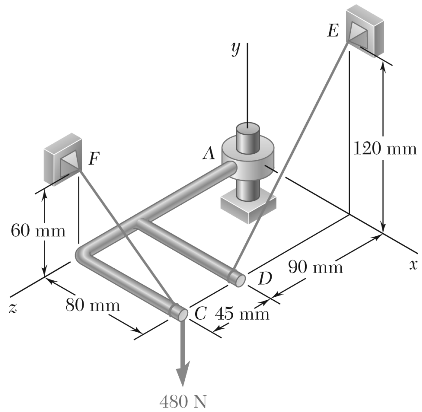
$$\mathbf{M}_C = (20.0 \text{ N}\cdot\text{m})\mathbf{j} + (151.5 \text{ N}\cdot\text{m})\mathbf{k} \blacktriangleleft$$

Coefficient of **i**: $C_x = 0$

Coefficient of **j**: $C_y + 0.5(461.9) - 400 = 0 \quad C_y = 169.1 \text{ N}$

Coefficient of **k**: $C_z - 0.866(461.9) = 0 \quad C_z = 400 \text{ N}$

$$\mathbf{C} = (169.1 \text{ N})\mathbf{j} + (400 \text{ N})\mathbf{k} \blacktriangleleft$$

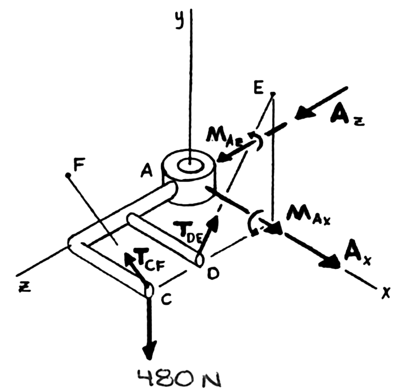


PROBLEM 4.121

The assembly shown is welded to collar A that fits on the vertical pin shown. The pin can exert couples about the x and z axes but does not prevent motion about or along the y-axis. For the loading shown, determine the tension in each cable and the reaction at A.

SOLUTION

Free-Body Diagram:



First note:

$$\mathbf{T}_{CF} = \boldsymbol{\lambda}_{CF} T_{CF} = \frac{-(0.08 \text{ m})\mathbf{i} + (0.06 \text{ m})\mathbf{j}}{\sqrt{(0.08)^2 + (0.06)^2} \text{ m}} T_{CF}$$

$$= T_{CF}(-0.8\mathbf{i} + 0.6\mathbf{j})$$

$$\mathbf{T}_{DE} = \boldsymbol{\lambda}_{DE} T_{DE} = \frac{(0.12 \text{ m})\mathbf{j} - (0.09 \text{ m})\mathbf{k}}{\sqrt{(0.12)^2 + (0.09)^2} \text{ m}} T_{DE}$$

$$= T_{DE}(0.8\mathbf{j} - 0.6\mathbf{k})$$

(*a*) From F.B.D. of assembly:

$$\Sigma F_y = 0: \quad 0.6T_{CF} + 0.8T_{DE} - 480 \text{ N} = 0$$

or $$0.6T_{CF} + 0.8T_{DE} = 480 \text{ N} \qquad (1)$$

$$\Sigma M_y = 0: \quad -(0.8T_{CF})(0.135 \text{ m}) + (0.6T_{DE})(0.08 \text{ m}) = 0$$

or $$T_{DE} = 2.25T_{CF} \qquad (2)$$

Substituting Equation (2) into Equation (1),

$$0.6T_{CF} + 0.8[(2.25)T_{CF}] = 480 \text{ N}$$

$$T_{CF} = 200.00 \text{ N}$$

or $$T_{CF} = 200 \text{ N} \blacktriangleleft$$

and from Equation (2): $$T_{DE} = 2.25(200.00 \text{ N}) = 450.00$$

or $$T_{DE} = 450 \text{ N} \blacktriangleleft$$

PROBLEM 4.121 (Continued)

(*b*) From F.B.D. of assembly:

$$\Sigma F_z = 0: \quad A_z - (0.6)(450.00\text{ N}) = 0 \quad A_z = 270.00\text{ N}$$

$$\Sigma F_x = 0: \quad A_x - (0.8)(200.00\text{ N}) = 0 \quad A_x = 160.000\text{ N}$$

or $\mathbf{A} = (160.0\text{ N})\mathbf{i} + (270\text{ N})\mathbf{k}$ ◀

$$\Sigma M_x = 0: \quad M_{A_x} + (480\text{ N})(0.135\text{ m}) - [(200.00\text{ N})(0.6)](0.135\text{ m}) - [(450\text{ N})(0.8)](0.09\text{ m}) = 0$$

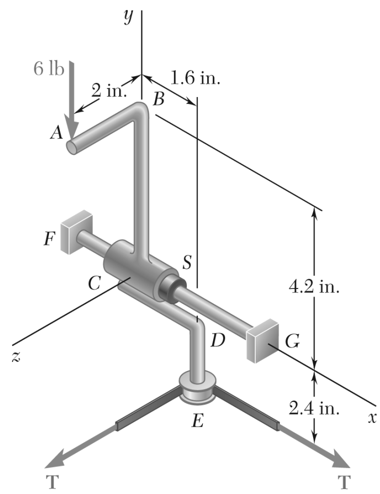
$$M_{A_x} = -16.2000\text{ N}\cdot\text{m}$$

$$\Sigma M_z = 0: \quad M_{A_z} - (480\text{ N})(0.08\text{ m}) + [(200.00\text{ N})(0.6)](0.08\text{ m}) + [(450\text{ N})(0.8)](0.08\text{ m}) = 0$$

$$M_{A_z} = 0$$

or $\mathbf{M}_A = -(16.20\text{ N}\cdot\text{m})\mathbf{i}$ ◀

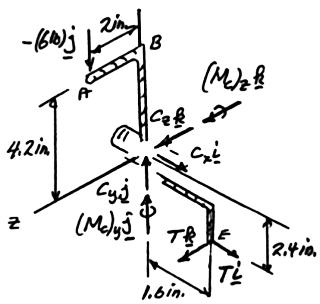
PROBLEM 4.122



The assembly shown is used to control the tension T in a tape that passes around a frictionless spool at E. Collar C is welded to rods ABC and CDE. It can rotate about shaft FG but its motion along the shaft is prevented by a washer S. For the loading shown, determine (*a*) the tension T in the tape, (*b*) the reaction at C.

SOLUTION

Free-Body Diagram:



$$\mathbf{r}_{A/C} = 4.2\mathbf{j} + 2\mathbf{k}$$

$$\mathbf{r}_{E/C} = 1.6\mathbf{i} - 2.4\mathbf{j}$$

$$\Sigma M_C = 0: \quad \mathbf{r}_{A/C} \times (-6\mathbf{j}) + \mathbf{r}_{E/C} \times T(\mathbf{i}+\mathbf{k}) + (M_C)_y\mathbf{j} + (M_C)_z\mathbf{k} = 0$$

$$(4.2\,\mathbf{j} + 2\mathbf{k}) \times (-6\mathbf{j}) + (1.6\mathbf{i} - 2.4\mathbf{j}) \times T(\mathbf{i}+\mathbf{k}) + (M_C)_y\mathbf{j} + (M_C)_z\mathbf{k} = 0$$

Coefficient of **i**: $\quad 12 - 2.4T = 0 \qquad T = 5.00 \text{ lb}$ ◀

Coefficient of **j**: $\quad -1.6(5 \text{ lb}) + (M_C)_y = 0 \quad (M_C)_y = 8 \text{ lb}\cdot\text{in.}$

Coefficient of **k**: $\quad 2.4(5 \text{ lb}) + (M_C)_z = 0 \quad (M_C)_z = -12 \text{ lb}\cdot\text{in.}$

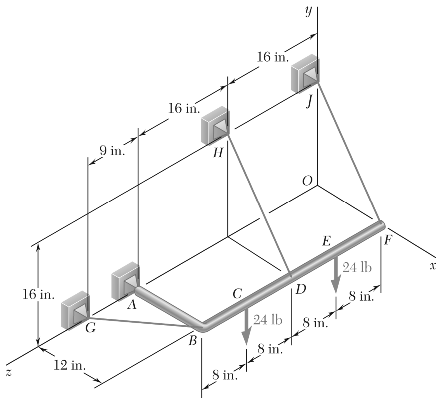
$$\mathbf{M}_C = (8.00 \text{ lb}\cdot\text{in.})\mathbf{j} - (12.00 \text{ lb}\cdot\text{in.})\mathbf{k} \ ◀$$

$$\Sigma F = 0: \quad C_x\mathbf{i} + C_y\mathbf{j} + C_z\mathbf{k} - (6 \text{ lb})\mathbf{j} + (5 \text{ lb})\mathbf{i} + (5 \text{ lb})\mathbf{k} = 0$$

Equate coefficients of unit vectors to zero.

$$C_x = -5 \text{ lb} \quad C_y = 6 \text{ lb} \quad C_z = -5 \text{ lb} \ ◀$$

$$\mathbf{C} = -(5.00 \text{ lb})\mathbf{i} + (6.00 \text{ lb})\mathbf{j} - (5.00 \text{ lb})\mathbf{k} \ ◀$$

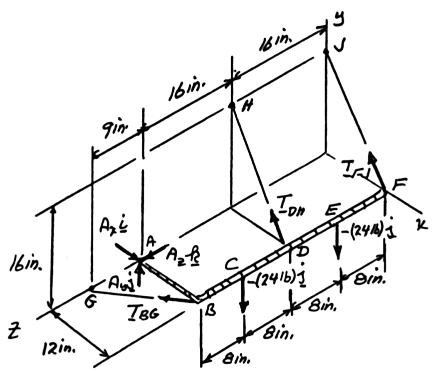


PROBLEM 4.123

The rigid L-shaped member *ABF* is supported by a ball-and-socket joint at *A* and by three cables. For the loading shown, determine the tension in each cable and the reaction at *A*.

SOLUTION

Free-Body Diagram:



$$\mathbf{r}_{B/A} = 12\mathbf{i}$$
$$\mathbf{r}_{F/A} = 12\mathbf{j} - 8\mathbf{k}$$
$$\mathbf{r}_{D/A} = 12\mathbf{i} - 16\mathbf{k}$$
$$\mathbf{r}_{E/A} = 12\mathbf{i} - 24\mathbf{k}$$
$$\mathbf{r}_{F/A} = 12\mathbf{i} - 32\mathbf{k}$$

$$\overrightarrow{BG} = -12\mathbf{i} + 9\mathbf{k}$$
$$BG = 15 \text{ in.}$$
$$\boldsymbol{\lambda}_{BG} = -0.8\mathbf{i} + 0.6\mathbf{k}$$

$$\overrightarrow{DH} = -12\mathbf{i} + 16\mathbf{j}; \quad DH = 20 \text{ in.}; \quad \lambda_{DH} = -0.6\mathbf{i} + 0.8\mathbf{j}$$
$$\overrightarrow{FJ} = -12\mathbf{i} + 16\mathbf{j}; \quad FJ = 20 \text{ in.}; \quad \lambda_{FJ} = -0.6\mathbf{i} + 0.8\mathbf{j}$$

$$\Sigma\mathbf{M}_A = 0\text{:} \quad \mathbf{r}_{B/A} \times \mathbf{T}_{BG}\lambda_{BG} + \mathbf{r}_{DH} \times \mathbf{T}_{DH}\lambda_{DH} + \mathbf{r}_{F/A} \times \mathbf{T}_{FJ}\lambda_{FJ}$$
$$+\mathbf{r}_{F/A} \times (-24\mathbf{j}) + \mathbf{r}_{E/A} \times (-24\mathbf{j}) = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 12 & 0 & 0 \\ -0.8 & 0 & 0.6 \end{vmatrix} T_{BG} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 12 & 0 & -16 \\ -0.6 & 0.8 & 0 \end{vmatrix} T_{DH} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 12 & 0 & -32 \\ -0.6 & 0.8 & 0 \end{vmatrix} T_{FJ}$$

$$+ \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 12 & 0 & -8 \\ 0 & -24 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 12 & 0 & -24 \\ 0 & -24 & 0 \end{vmatrix} = 0$$

Coefficient of **i**: $\quad +12.8T_{DH} + 25.6T_{FJ} - 192 - 576 = 0 \quad (1)$

Coefficient of **k**: $\quad +9.6T_{DH} + 9.6T_{FJ} - 288 - 288 = 0 \quad (2)$

$\frac{3}{4}$ Eq. (1) – Eq. (2): $\quad 9.6T_{FJ} = 0 \qquad T_{FJ} = 0$ ◀

PROBLEM 4.123 (Continued)

From Eq. (1): $12.8T_{DH} - 268 = 0$ $\quad T_{DH} = 60 \text{ lb}$ ◀

Coefficient of **j:** $-7.2T_{BG} + (16 \times 0.6)(60.0 \text{ lb}) = 0$ $\quad T_{BG} = 80.0 \text{ lb}$ ◀

$\Sigma\mathbf{F} = 0$: $\quad \mathbf{A} + T_{BG}\boldsymbol{\lambda}_{BG} + T_{DH}\boldsymbol{\lambda}_{DH} + T_{FJ} - 24\mathbf{j} - 24\mathbf{j} = 0$

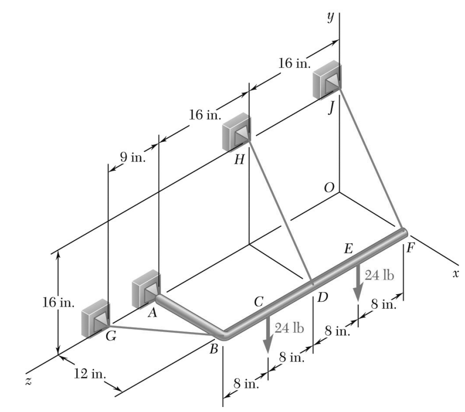
Coefficient of **i**: $A_x + (80)(-0.8) + (60.0)(-0.6) = 0$ $\quad A_x = 100.0 \text{ lb}$

Coefficient of **j**: $A_y + (60.0)(0.8) - 24 - 24 = 0$ $\quad A_y = 0$

Coefficient of **k:** $A_z + (80.0)(+0.6) = 0$ $\quad A_z = -48.0 \text{ lb}$

$\mathbf{A} = (100.0 \text{ lb})\mathbf{i} - (48.0 \text{ lb})\mathbf{j}$ ◀

Note: The value $A_y = 0$ can be confirmed by considering $\Sigma M_{BF} = 0$.



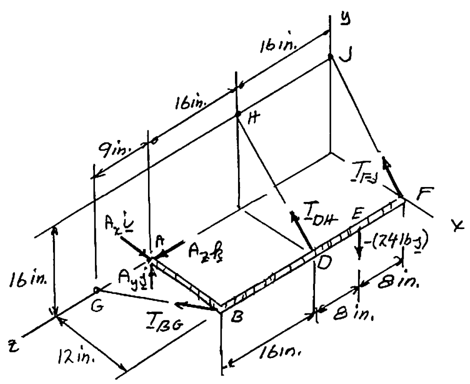
PROBLEM 4.124

Solve Problem 4.123, assuming that the load at *C* has been removed.

PROBLEM 4.123 The rigid L-shaped member *ABF* is supported by a ball-and-socket joint at *A* and by three cables. For the loading shown, determine the tension in each cable and the reaction at *A*.

SOLUTION

Free-Body Diagram:



$$\mathbf{r}_{B/A} = 12\mathbf{i}$$
$$\mathbf{r}_{B/A} = 12\mathbf{i} - 16\mathbf{k}$$
$$\mathbf{r}_{E/A} = 12\mathbf{i} - 24\mathbf{k}$$
$$\mathbf{r}_{F/A} = 12\mathbf{i} - 32\mathbf{k}$$

$$\overrightarrow{BG} = -12\mathbf{i} + 9\mathbf{k}; \quad BG = 15 \text{ in.}; \quad \boldsymbol{\lambda}_{BG} = -0.8\mathbf{i} + 0.6\mathbf{k}$$
$$\overrightarrow{DH} = -12\mathbf{i} + 16\mathbf{j}; \quad DH = 20 \text{ in.}; \quad \boldsymbol{\lambda}_{DH} = -0.6\mathbf{i} + 0.8\mathbf{j}$$
$$\overrightarrow{FJ} = -12\mathbf{i} + 16\mathbf{j}; \quad FJ = 20 \text{ in.}; \quad \boldsymbol{\lambda}_{FJ} = -0.6\mathbf{i} + 0.8\mathbf{j}$$

$$\Sigma\mathbf{M}_A = 0: \quad \mathbf{r}_{B/A} \times T_{BG}\,\boldsymbol{\lambda}_{BG} + \mathbf{r}_{D/A} \times T_{DH}\,\boldsymbol{\lambda}_{DH} + \mathbf{r}_{F/A} \times T_{FJ}\boldsymbol{\lambda}_{FJ} + \mathbf{r}_{E/A} \times (-24\mathbf{j}) = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 12 & 0 & 0 \\ -0.8 & 0 & 0.6 \end{vmatrix} T_{BG} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 12 & 0 & -16 \\ -0.6 & 0.8 & 0 \end{vmatrix} T_{DH} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 12 & 0 & -32 \\ -0.6 & 0.8 & 0 \end{vmatrix} T_{FJ} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 12 & 0 & -24 \\ 0 & -24 & 0 \end{vmatrix} = 0$$

$$\mathbf{i}: \quad +12.8T_{DH} + 25.6T_{FJ} - 576 = 0 \qquad (1)$$

$$\mathbf{k}: \quad +9.6T_{DH} + 9.6T_{FJ} - 288 = 0 \qquad (2)$$

PROBLEM 4.124 (Continued)

Multiply Eq. (1) by $\frac{3}{4}$ and subtract Eq. (2):

$$9.6T_{FJ} - 144 = 0 \qquad T_{FJ} = 15.00 \text{ lb} \blacktriangleleft$$

From Eq. (1):

$$12.8T_{DH} + 25.6(15.00) - 576 = 0 \qquad T_{DH} = 15.00 \text{ lb} \blacktriangleleft$$

$$\mathbf{j}: \quad -7.2T_{BG} + (16)(0.6)(15) + (32)(0.6)(15) = 0$$

$$-7.2T_{BG} + 432 = 0 \qquad T_{BG} = 60.0 \text{ lb} \blacktriangleleft$$

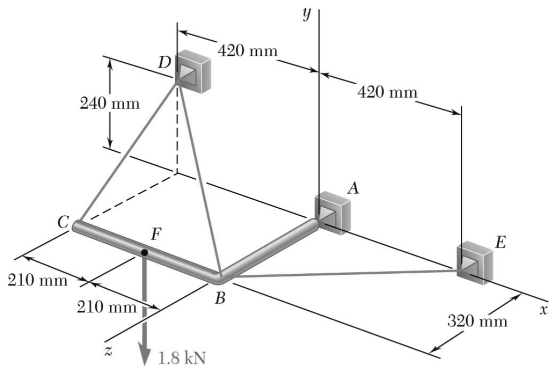
$$\Sigma F = 0: \quad \mathbf{A} + T_{BG}\boldsymbol{\lambda}_{BG} + T_{DA}\boldsymbol{\lambda}_{DH} + T_{FJ}\boldsymbol{\lambda}_{FJ} - 24\mathbf{j} = 0$$

$$\mathbf{i}: \quad A_x + (60)(-0.8) + (15)(-0.6) + (15)(-0.6) = 0 \qquad A_x = 66.0 \text{ lb}$$

$$\mathbf{j}: \quad A_y + (15)(0.8) + (15)(0.8) - 24 = 0 \qquad A_y = 0$$

$$\mathbf{k}: \quad A_z + (60)(0.6) = 0 \qquad A_z = -36.0 \text{ lb}$$

$$\mathbf{A} = (66.0 \text{ lb})\mathbf{i} - (36.0 \text{ lb})\mathbf{k} \blacktriangleleft$$



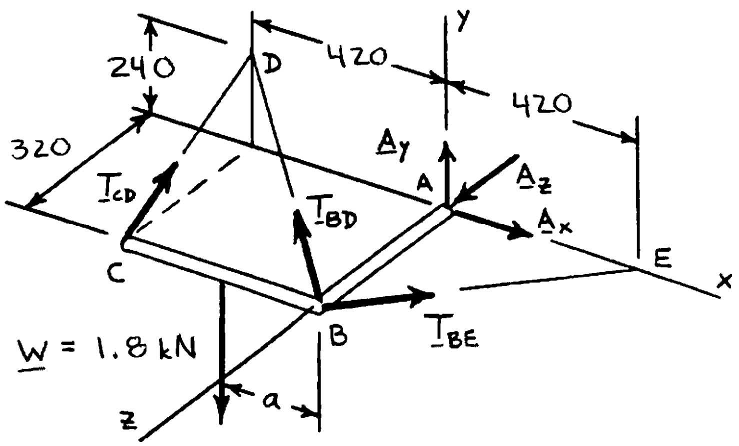
PROBLEM 4.125

The rigid L-shaped member *ABC* is supported by a ball-and-socket joint at *A* and by three cables. If a 1.8-kN load is applied at *F*, determine the tension in each cable.

SOLUTION

Free-Body Diagram:

Dimensions in mm



In this problem: $a = 210$ mm

We have
$$\overrightarrow{CD} = (240 \text{ mm})\mathbf{j} - (320 \text{ mm})\mathbf{k} \quad CD = 400 \text{ mm}$$
$$\overrightarrow{BD} = -(420 \text{ mm})\mathbf{i} + (240 \text{ mm})\mathbf{j} - (320 \text{ mm})\mathbf{k} \quad BD = 580 \text{ mm}$$
$$\overrightarrow{BE} = (420 \text{ mm})\mathbf{i} - (320 \text{ mm})\mathbf{k} \quad BE = 528.02 \text{ mm}$$

Thus,
$$\mathbf{T}_{CD} = T_{CD}\frac{\overrightarrow{CD}}{CD} = T_{CD}(0.6\mathbf{j} - 0.8\mathbf{k})$$
$$\mathbf{T}_{BD} = T_{BD}\frac{\overrightarrow{BD}}{BD} = T_{BD}(-0.72414\mathbf{i} + 0.41379\mathbf{j} - 0.55172\mathbf{k})$$
$$\mathbf{T}_{BE} = T_{BE}\frac{\overrightarrow{BE}}{BE} = T_{BE}(0.79542\mathbf{i} - 0.60604\mathbf{k})$$

$$\Sigma\mathbf{M}_A = 0\text{:} \quad (\mathbf{r}_C \times \mathbf{T}_{CD}) + (\mathbf{r}_B \times \mathbf{T}_{BD}) + (\mathbf{r}_B \times \mathbf{T}_{BE}) + (\mathbf{r}_W \times \mathbf{W}) = 0$$

PROBLEM 4.125 (Continued)

Noting that
$$\mathbf{r}_C = -(420\text{ mm})\mathbf{i} + (320\text{ mm})\mathbf{k}$$
$$\mathbf{r}_B = (320\text{ mm})\mathbf{k}$$
$$\mathbf{r}_W = -a\mathbf{i} + (320\text{ mm})\mathbf{k}$$

and using determinants, we write

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -420 & 0 & 320 \\ 0 & 0.6 & -0.8 \end{vmatrix} T_{CD} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 320 \\ -0.72414 & 0.41379 & -0.55172 \end{vmatrix} T_{BD}$$
$$+ \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 320 \\ 0.79542 & 0 & -0.60604 \end{vmatrix} T_{BE} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -a & 0 & 320 \\ 0 & -1.8 & 0 \end{vmatrix} = 0$$

Equating to zero the coefficients of the unit vectors,

$$\mathbf{i}: \quad -192T_{CD} - 132.413T_{BD} + 576 = 0 \qquad (1)$$

$$\mathbf{j}: \quad -336T_{CD} - 231.72T_{BD} + 254.53T_{BE} = 0 \qquad (2)$$

$$\mathbf{k}: \quad -252T_{CD} + 1.8a = 0 \qquad (3)$$

Recalling that $a = 210$ mm, Eq. (3) yields

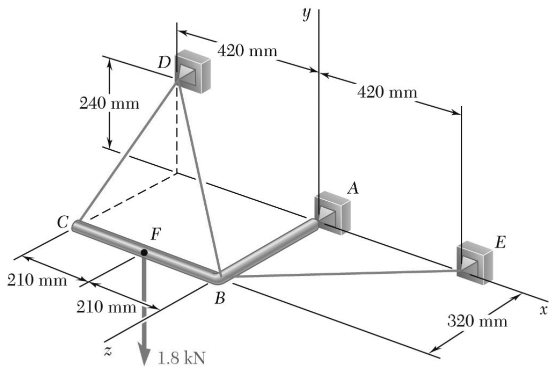
$$T_{CD} = \frac{1.8(210)}{252} = 1.500\text{ kN} \qquad T_{CD} = 1.500\text{ kN} \blacktriangleleft$$

From Eq. (1):
$$-192(1.5) - 132.413T_{BD} + 576 = 0$$

$$T_{BD} = 2.1751\text{ kN} \qquad T_{BD} = 2.18\text{ kN} \blacktriangleleft$$

From Eq. (2):
$$-336(1.5) - 231.72(2.1751) + 254.53T_{BE} = 0$$

$$T_{BE} = 3.9603\text{ kN} \qquad T_{BE} = 3.96\text{ kN} \blacktriangleleft$$



PROBLEM 4.126

Solve Problem 4.125, assuming that the 1.8-kN load is applied at *C*.

PROBLEM 4.125 The rigid L-shaped member *ABC* is supported by a ball-and-socket joint at *A* and by three cables. If a 1.8-kN load is applied at *F*, determine the tension in each cable.

SOLUTION

See solution of Problem 4.125 for free-body diagram and derivation of Eqs. (1), (2), and (3):

$$-192T_{CD} - 132.413T_{BD} + 576 = 0 \qquad (1)$$

$$-336T_{CD} - 231.72T_{BD} + 254.53T_{BE} = 0 \qquad (2)$$

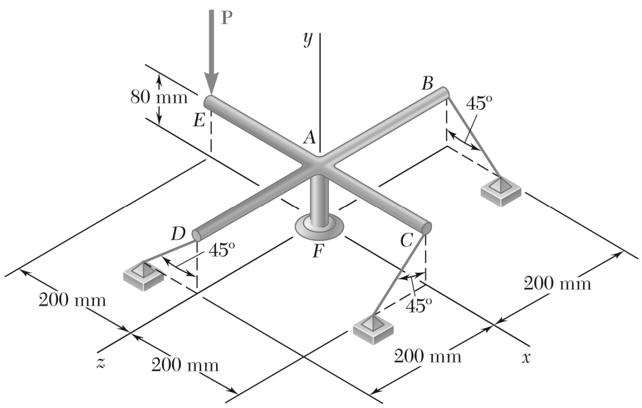
$$-252T_{CD} + 1.8a = 0 \qquad (3)$$

In this problem, the 1.8-kN load is applied at *C* and we have $a = 420$ mm. Carrying into Eq. (3) and solving for T_{CD},

$$T_{CD} = 3.00 \qquad T_{CD} = 3.00 \text{ kN} \blacktriangleleft$$

From Eq. (1): $-(192)(3) - 132.413T_{BD} + 576 = 0$ $\qquad T_{BD} = 0 \blacktriangleleft$

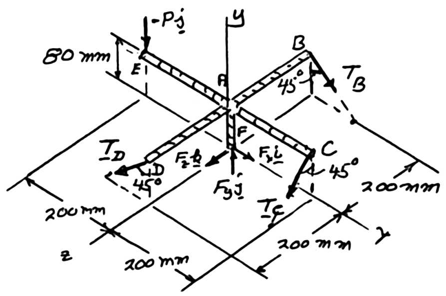
From Eq. (2): $-336(3) - 0 + 254.53T_{BE} = 0$ $\qquad T_{BE} = 3.96 \text{ kN} \blacktriangleleft$



PROBLEM 4.127

The assembly shown consists of an 80-mm rod AF that is welded to a cross consisting of four 200-mm arms. The assembly is supported by a ball-and-socket joint at F and by three short links, each of which forms an angle of 45° with the vertical. For the loading shown, determine (*a*) the tension in each link, (*b*) the reaction at F.

SOLUTION



$$\mathbf{r}_{E/F} = -200\,\mathbf{i} + 80\mathbf{j}$$

$$\mathbf{T}_B = T_B(\mathbf{i} - \mathbf{j})/\sqrt{2} \qquad \mathbf{r}_{B/F} = 80\mathbf{j} - 200\mathbf{k}$$

$$\mathbf{T}_C = T_C(-\mathbf{j} + \mathbf{k})/\sqrt{2} \qquad \mathbf{r}_{C/F} = 200\mathbf{i} + 80\mathbf{j}$$

$$\mathbf{T}_D = T_D(-\mathbf{i} + \mathbf{j})/\sqrt{2} \qquad \mathbf{r}_{D/E} = 80\mathbf{j} + 200\mathbf{k}$$

$$\Sigma M_F = 0\text{:} \quad \mathbf{r}_{B/F} \times \mathbf{T}_B + \mathbf{r}_{C/F} \times \mathbf{T}_C + \mathbf{r}_{D/F} \times T_D + \mathbf{r}_{E/F} \times (-P\mathbf{j}) = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 80 & -200 \\ 1 & -1 & 0 \end{vmatrix} \frac{T_B}{\sqrt{2}} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 200 & 80 & 0 \\ 0 & -1 & 1 \end{vmatrix} \frac{T_C}{\sqrt{2}} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 80 & 200 \\ -1 & -1 & 0 \end{vmatrix} \frac{T_D}{\sqrt{2}} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -200 & 80 & 0 \\ 0 & -P & 0 \end{vmatrix} = 0$$

Equate coefficients of unit vectors to zero and multiply each equation by $\sqrt{2}$.

$$\mathbf{i}\text{:} \qquad -200T_B + 80T_C + 200\,T_D = 0 \tag{1}$$

$$\mathbf{j}\text{:} \qquad -200T_B - 200T_C - 200T_D = 0 \tag{2}$$

$$\mathbf{k}\text{:} \qquad -80T_B - 200T_C + 80T_D + 200\sqrt{2}P = 0 \tag{3}$$

$$\frac{80}{200}(2)\text{:} \qquad -80T_B - 80T_C - 80T_D = 0 \tag{4}$$

$$\text{Eqs. } (3) + (4)\text{:} \qquad -160T_B - 280T_C + 200\sqrt{2}P = 0 \tag{5}$$

$$\text{Eqs. } (1) + (2)\text{:} \qquad -400T_B - 120T_C = 0$$

$$T_B = -\frac{120}{400}T_C - 0.3T_C \tag{6}$$

PROBLEM 4.127 (Continued)

Eqs. (6) $\longrightarrow$ (5): $-160(-0.3T_C) - 280T_C + 200\sqrt{2}P = 0$

$$-232T_C + 200\sqrt{2}P = 0$$

$$T_C = 1.2191P \qquad T_C = 1.219P \blacktriangleleft$$

From Eq. (6): $T_B = -0.3(1.2191P) = -0.36574 = P$ $\qquad T_B = -0.366P \blacktriangleleft$

From Eq. (2): $-200(-0.3657P) - 200(1.2191P) - 200T_{\theta D} = 0$

$$T_D = -0.8534P \qquad T_D = -0.853P \blacktriangleleft$$

$\Sigma\mathbf{F} = 0$: $\quad \mathbf{F} + \mathbf{T}_B + \mathbf{T}_C + \mathbf{T}_D - P\mathbf{j} = 0$

$\mathbf{i}$: $\quad F_x + \dfrac{(-0.36574P)}{\sqrt{2}} - \dfrac{(-0.8534P)}{\sqrt{2}} = 0$

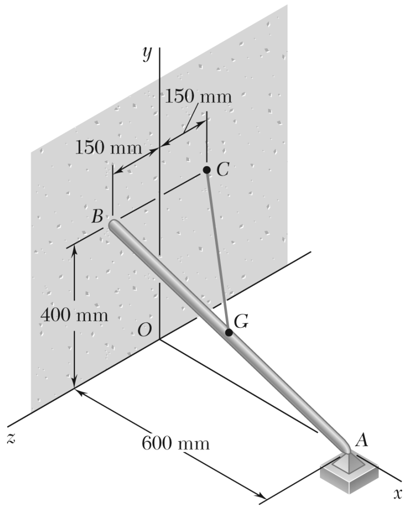
$F_x = -0.3448P \qquad F_x = -0.345P$

$\mathbf{j}$: $\quad F_y - \dfrac{(-0.36574P)}{\sqrt{2}} - \dfrac{(1.2191P)}{\sqrt{2}} - \dfrac{(-0.8534P)}{\sqrt{2}} - 200 = 0$

$F_y = P \qquad F_y = P$

$\mathbf{k}$: $\quad F_z + \dfrac{(1.2191P)}{\sqrt{2}} = 0$

$F_z = -0.8620P \quad F_z = -0.862P$ $\qquad \mathbf{F} = -0.345P\mathbf{i} + P\mathbf{j} - 0.862P\mathbf{k} \blacktriangleleft$

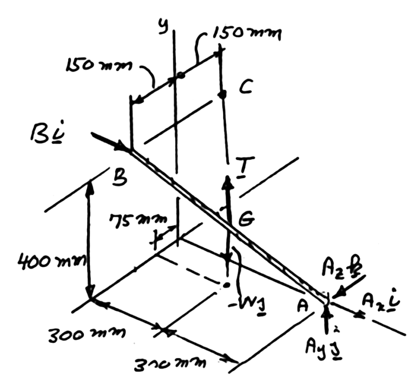


PROBLEM 4.128

The uniform 10-kg rod AB is supported by a ball-and-socket joint at A and by the cord CG that is attached to the midpoint G of the rod. Knowing that the rod leans against a frictionless vertical wall at B, determine (*a*) the tension in the cord, (*b*) the reactions at A and B.

SOLUTION

Free-Body Diagram:



Five unknowns and six equations of equilibrium, but equilibrium is maintained $(\Sigma M_{AB} = 0)$.

$$W = mg$$
$$= (10 \text{ kg})\, 9.81 \text{ m/s}^2$$
$$W = 98.1 \text{ N}$$

$$\overrightarrow{GC} = -300\mathbf{i} + 200\mathbf{j} - 225\mathbf{k} \qquad GC = 425 \text{ mm}$$

$$\mathbf{T} = T\frac{\overrightarrow{GC}}{GC} = \frac{T}{425}(-300\mathbf{i} + 200\mathbf{j} - 225\mathbf{k})$$

$$\mathbf{r}_{B/A} = -600\mathbf{i} + 400\mathbf{j} + 150 \text{ mm}$$

$$\mathbf{r}_{G/A} = -300\mathbf{i} + 200\mathbf{j} + 75 \text{ mm}$$

$$\Sigma M_A = 0\text{:} \quad \mathbf{r}_{B/A} \times \mathbf{B} + \mathbf{r}_{G/A} \times \mathbf{T} + \mathbf{r}_{G/A} \times (-W\mathbf{j}) = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -600 & 400 & 150 \\ B & 0 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -300 & 200 & 75 \\ -300 & 200 & -225 \end{vmatrix} \frac{T}{425} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -300 & 200 & 75 \\ 0 & -98.1 & 0 \end{vmatrix}$$

Coefficient of $\mathbf{i}$: $\quad (-105.88 - 35.29)T + 7357.5 = 0$

$$T = 52.12 \text{ N}$$

$T = 52.1 \text{ N}$ ◀

PROBLEM 4.128 (Continued)

Coefficient of $\mathbf{j}$: $150B-(300\times75+300\times225)\dfrac{52.12}{425}=0$

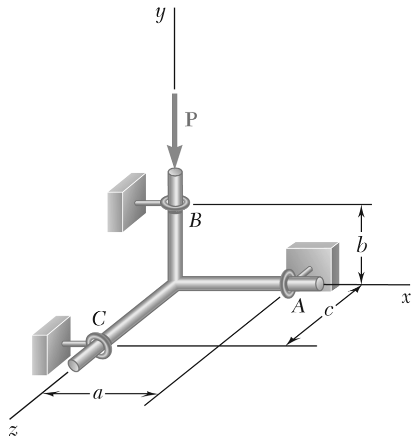
$B=73.58\text{ N}$ $\qquad\qquad \mathbf{B}=(73.6\text{ N})\mathbf{i}$ ◀

$\Sigma\mathbf{F}=0$: $\mathbf{A}+\mathbf{B}+\mathbf{T}-W\mathbf{j}=0$

Coefficient of $\mathbf{i}$: $A_x+73.58-52.15\dfrac{300}{425}=0$ $\qquad\qquad A_x=-36.8\text{ N}$ ◀

Coefficient of $\mathbf{j}$: $A_y+52.15\dfrac{200}{425}-98.1=0$ $\qquad\qquad A_y=73.6\text{ N}$ ◀

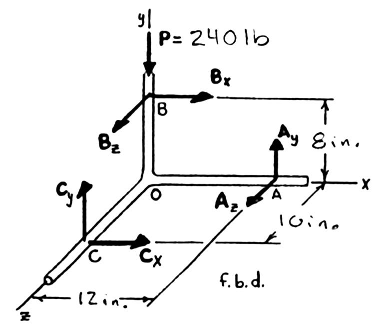
Coefficient of $\mathbf{k}$: $A_z-52.15\dfrac{225}{425}=0$ $\qquad\qquad A_z=27.6\text{ N}$ ◀



PROBLEM 4.129

Three rods are welded together to form a "corner" that is supported by three eyebolts. Neglecting friction, determine the reactions at A, B, and C when $P = 240$ lb, $a = 12$ in., $b = 8$ in., and $c = 10$ in.

SOLUTION



From F.B.D. of weldment:

$$\Sigma \mathbf{M}_O = 0: \quad \mathbf{r}_{A/O} \times \mathbf{A} + \mathbf{r}_{B/O} \times \mathbf{B} + \mathbf{r}_{C/O} \times \mathbf{C} = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 12 & 0 & 0 \\ 0 & A_y & A_z \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 8 & 0 \\ B_x & 0 & B_z \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 10 \\ C_x & C_y & 0 \end{vmatrix} = 0$$

$$(-12A_z\mathbf{j} + 12A_y\mathbf{k}) + (8B_z\mathbf{i} - 8B_x\mathbf{k}) + (-10C_y\mathbf{i} + 10C_x\mathbf{j}) = 0$$

From **i**-coefficient: $\quad 8B_z - 10C_y = 0$

or $\quad B_z = 1.25C_y \quad (1)$

j-coefficient: $\quad -12A_z + 10C_x = 0$

or $\quad C_x = 1.2A_z \quad (2)$

k-coefficient: $\quad 12A_y - 8B_x = 0$

or $\quad B_x = 1.5A_y \quad (3)$

$$\Sigma \mathbf{F} = 0: \quad \mathbf{A} + \mathbf{B} + \mathbf{C} - \mathbf{P} = 0$$

or $\quad (B_x + C_x)\mathbf{i} + (A_y + C_y - 240 \text{ lb})\mathbf{j} + (A_z + B_z)\mathbf{k} = 0$

From **i**-coefficient: $\quad B_x + C_x = 0$

or $\quad C_x = -B_x \quad (4)$

j-coefficient: $\quad A_y + C_y - 240 \text{ lb} = 0$

or $\quad A_y + C_y = 240 \text{ lb} \quad (5)$

PROBLEM 4.129 (Continued)

k-coefficient: $A_z + B_z = 0$

or $$A_z = -B_z \tag{6}$$

Substituting C_x from Equation (4) into Equation (2),

$$-B_z = 1.2A_z \tag{7}$$

Using Equations (1), (6), and (7),

$$C_y = \frac{B_z}{1.25} = \frac{-A_z}{1.25} = \frac{1}{1.25}\left(\frac{B_x}{1.2}\right) = \frac{B_x}{1.5} \tag{8}$$

From Equations (3) and (8):

$$C_y = \frac{1.5A_y}{1.5} \quad \text{or} \quad C_y = A_y$$

and substituting into Equation (5),

$$2A_y = 240 \text{ lb}$$

$$A_y = C_y = 120 \text{ lb} \tag{9}$$

Using Equation (1) and Equation (9),

$$B_z = 1.25(120 \text{ lb}) = 150.0 \text{ lb}$$

Using Equation (3) and Equation (9),

$$B_x = 1.5(120 \text{ lb}) = 180.0 \text{ lb}$$

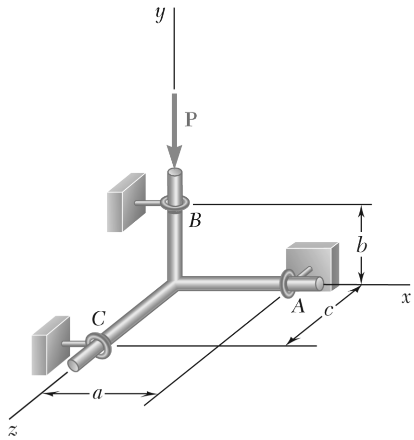
From Equation (4): $C_x = -180.0 \text{ lb}$

From Equation (6): $A_z = -150.0 \text{ lb}$

Therefore, $\mathbf{A} = (120.0 \text{ lb})\mathbf{j} - (150.0 \text{ lb})\mathbf{k}$ ◀

$\mathbf{B} = (180.0 \text{ lb})\mathbf{i} + (150.0 \text{ lb})\mathbf{k}$ ◀

$\mathbf{C} = -(180.0 \text{ lb})\mathbf{i} + (120.0 \text{ lb})\mathbf{j}$ ◀

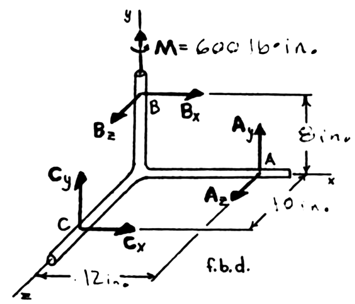


PROBLEM 4.130

Solve Problem 4.129, assuming that the force **P** is removed and is replaced by a couple $\mathbf{M} = +(600 \text{ lb} \cdot \text{in.})\mathbf{j}$ acting at B.

PROBLEM 4.129 Three rods are welded together to form a "corner" that is supported by three eyebolts. Neglecting friction, determine the reactions at A, B, and C when $P = 240$ lb, $a = 12$ in., $b = 8$ in., and $c = 10$ in.

SOLUTION



From F.B.D. of weldment:

$$\Sigma \mathbf{M}_O = 0: \quad \mathbf{r}_{A/O} \times \mathbf{A} + \mathbf{r}_{B/O} \times \mathbf{B} + \mathbf{r}_{C/O} \times \mathbf{C} + \mathbf{M} = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 12 & 0 & 0 \\ 0 & A_y & A_z \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 8 & 0 \\ B_x & 0 & B_z \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 10 \\ C_x & C_y & 0 \end{vmatrix} + (600 \text{ lb} \cdot \text{in.})\mathbf{j} = 0$$

$$(-12A_z\mathbf{j} + 12A_y\mathbf{k}) + (8B_z\mathbf{j} - 8B_x\mathbf{k}) + (-10C_y\mathbf{i} + 10C_x\mathbf{j}) + (600 \text{ lb} \cdot \text{in.})\mathbf{j} = 0$$

From **i**-coefficient: $\quad 8B_z - 10C_y = 0$

or $\quad C_y = 0.8B_z \quad (1)$

j-coefficient: $\quad -12A_z + 10C_x + 600 = 0$

or $\quad C_x = 1.2A_z - 60 \quad (2)$

k-coefficient: $\quad 12A_y - 8B_x = 0$

or $\quad B_x = 1.5A_y \quad (3)$

$$\Sigma \mathbf{F} = 0: \quad \mathbf{A} + \mathbf{B} + \mathbf{C} = 0$$

$$(B_x + C_x)\mathbf{i} + (A_y + C_y)\mathbf{j} + (A_z + B_z)\mathbf{k} = 0$$

From **i**-coefficient: $\quad C_x = -B_x \quad (4)$

j-coefficient: $\quad C_y = -A_y \quad (5)$

k-coefficient: $\quad A_z = -B_z \quad (6)$

PROBLEM 4.130 (Continued)

Substituting C_x from Equation (4) into Equation (2),

$$A_z = 50 - \left(\frac{B_x}{1.2}\right) \tag{7}$$

Using Equations (1), (6), and (7),

$$C_y = 0.8B_z = -0.8A_z = \left(\frac{2}{3}\right)B_x - 40 \tag{8}$$

From Equations (3) and (8):

$$C_y = A_y - 40$$

Substituting into Equation (5), $2A_y = 40$

$$A_y = 20.0 \text{ lb}$$

From Equation (5): $C_y = -20.0 \text{ lb}$

Equation (1): $B_z = -25.0 \text{ lb}$

Equation (3): $B_x = 30.0 \text{ lb}$

Equation (4): $C_x = -30.0 \text{ lb}$

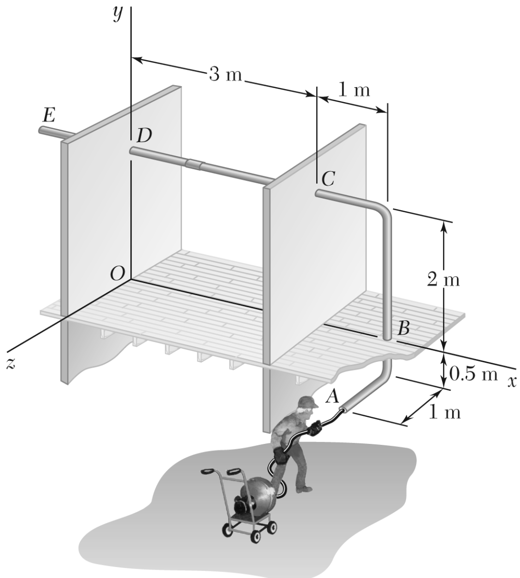
Equation (6): $A_z = 25.0 \text{ lb}$

Therefore,

$\mathbf{A} = (20.0 \text{ lb})\mathbf{j} + (25.0 \text{ lb})\mathbf{k}$ ◀

$\mathbf{B} = (30.0 \text{ lb})\mathbf{i} - (25.0 \text{ lb})\mathbf{k}$ ◀

$\mathbf{C} = -(30.0 \text{ lb})\mathbf{i} - (20.0 \text{ lb})\mathbf{j}$ ◀

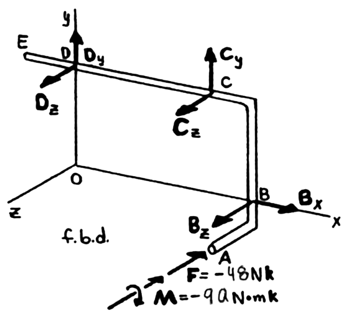


PROBLEM 4.131

In order to clean the clogged drainpipe *AE*, a plumber has disconnected both ends of the pipe and inserted a power snake through the opening at *A*. The cutting head of the snake is connected by a heavy cable to an electric motor that rotates at a constant speed as the plumber forces the cable into the pipe. The forces exerted by the plumber and the motor on the end of the cable can be represented by the wrench $\mathbf{F} = -(48\text{ N})\mathbf{k}$, $\mathbf{M} = -(90\text{ N}\cdot\text{m})\mathbf{k}$. Determine the additional reactions at *B*, *C*, and *D* caused by the cleaning operation. Assume that the reaction at each support consists of two force components perpendicular to the pipe.

SOLUTION

From F.B.D. of pipe assembly *ABCD*:



$$\Sigma F_x = 0: \quad B_x = 0$$

$$\Sigma M_{D(x\text{-axis})} = 0: \quad (48\text{ N})(2.5\text{ m}) - B_z(2\text{ m}) = 0$$

$$B_z = 60.0\text{ N}$$

and $\mathbf{B} = (60.0\text{ N})\mathbf{k}$ ◀

$$\Sigma M_{D(z\text{-axis})} = 0: \quad C_y(3\text{ m}) - 90\text{ N}\cdot\text{m} = 0$$

$$C_y = 30.0\text{ N}$$

$$\Sigma M_{D(y\text{-axis})} = 0: \quad -C_z(3\text{ m}) - (60.0\text{ N})(4\text{ m}) + (48\text{ N})(4\text{ m}) = 0$$

$$C_z = -16.00\text{ N}$$

and $\mathbf{C} = (30.0\text{ N})\mathbf{j} - (16.00\text{ N})\mathbf{k}$ ◀

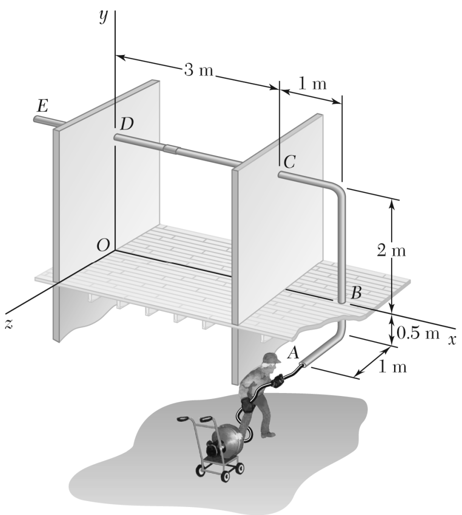
$$\Sigma F_y = 0: \quad D_y + 30.0 = 0$$

$$D_y = -30.0\text{ N}$$

$$\Sigma F_z = 0: \quad D_z - 16.00\text{ N} + 60.0\text{ N} - 48\text{ N} = 0$$

$$D_z = 4.00\text{ N}$$

and $\mathbf{D} = -(30.0\text{ N})\mathbf{j} + (4.00\text{ N})\mathbf{k}$ ◀



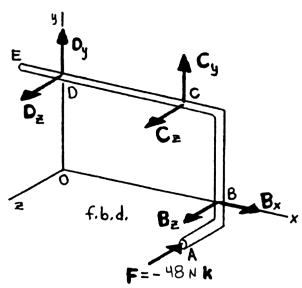
PROBLEM 4.132

Solve Problem 4.131, assuming that the plumber exerts a force $\mathbf{F} = -(48\text{ N})\mathbf{k}$ and that the motor is turned off $(\mathbf{M} = 0)$.

PROBLEM 4.131 In order to clean the clogged drainpipe *AE*, a plumber has disconnected both ends of the pipe and inserted a power snake through the opening at *A*. The cutting head of the snake is connected by a heavy cable to an electric motor that rotates at a constant speed as the plumber forces the cable into the pipe. The forces exerted by the plumber and the motor on the end of the cable can be represented by the wrench $\mathbf{F} = -(48\text{ N})\mathbf{k}, \mathbf{M} = -(90\text{ N}\cdot\text{m})\mathbf{k}$. Determine the additional reactions at *B*, *C*, and *D* caused by the cleaning operation. Assume that the reaction at each support consists of two force components perpendicular to the pipe.

SOLUTION

From F.B.D. of pipe assembly *ABCD*:



$$\Sigma F_x = 0: \quad B_x = 0$$

$$\Sigma M_{D(x\text{-axis})} = 0: \quad (48\text{ N})(2.5\text{ m}) - B_z(2\text{ m}) = 0$$

$$B_z = 60.0\text{ N} \qquad \text{and } \mathbf{B} = (60.0\text{ N})\mathbf{k} \blacktriangleleft$$

$$\Sigma M_{D(z\text{-axis})} = 0: \quad C_y(3\text{ m}) - B_x(2\text{ m}) = 0$$

$$C_y = 0$$

$$\Sigma M_{D(y\text{-axis})} = 0: \quad C_z(3\text{ m}) - (60.0\text{ N})(4\text{ m}) + (48\text{ N})(4\text{ m}) = 0$$

$$C_z = -16.00\text{ N} \qquad \text{and } \mathbf{C} = -(16.00\text{ N})\mathbf{k} \blacktriangleleft$$

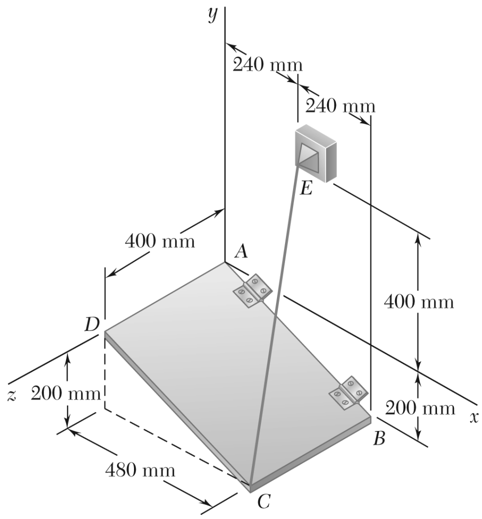
$$\Sigma F_y = 0: \quad D_y + C_y = 0$$

$$D_y = 0$$

$$\Sigma F_z = 0: \quad D_z + B_z + C_z - F = 0$$

$$D_z + 60.0\text{ N} - 16.00\text{ N} - 48\text{ N} = 0$$

$$D_z = 4.00\text{ N} \qquad \text{and } \mathbf{D} = (4.00\text{ N})\mathbf{k} \blacktriangleleft$$

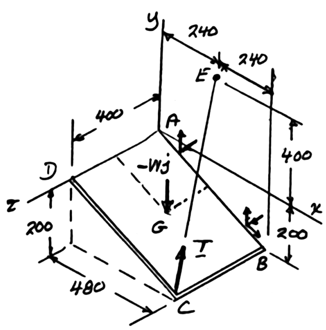


PROBLEM 4.133

The 50-kg plate *ABCD* is supported by hinges along edge *AB* and by wire *CE*. Knowing that the plate is uniform, determine the tension in the wire.

SOLUTION

Free-Body Diagram:



$$W = mg = (50\text{ kg})(9.81\text{ m/s}^2)$$

$$W = 490.50\text{ N}$$

$$\overrightarrow{CE} = -240\mathbf{i} + 600\mathbf{j} - 400\mathbf{k}$$

$$CE = 760\text{ mm}$$

$$\mathbf{T} = T\frac{\overrightarrow{CE}}{CE} = \frac{T}{760}(-240\mathbf{i} + 600\mathbf{j} - 400\mathbf{k})$$

$$\boldsymbol{\lambda}_{AB} = \frac{\overrightarrow{AB}}{AB} = \frac{480\mathbf{i} - 200\mathbf{j}}{520} = \frac{1}{13}(12\mathbf{i} - 5\mathbf{j})$$

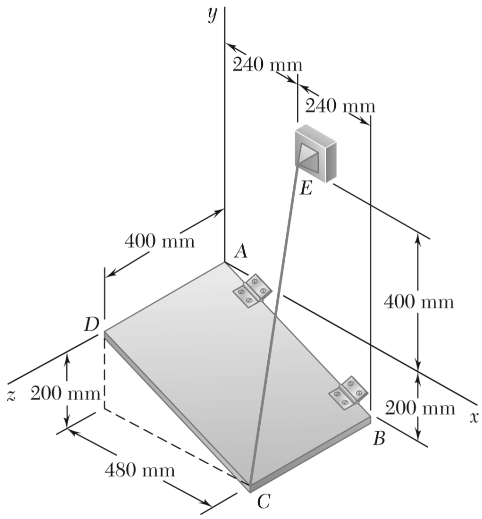
$$\Sigma \mathbf{M}_{AB} = 0: \quad \boldsymbol{\lambda}_{AB} \cdot (\mathbf{r}_{E/A} \times T) + \boldsymbol{\lambda}_{AB} \cdot (\mathbf{r}_{G/A} \times -W\mathbf{j}) = 0$$

$$\mathbf{r}_{E/A} = 240\mathbf{i} + 400\mathbf{j}; \quad \mathbf{r}_{G/A} = 240\mathbf{i} - 100\mathbf{j} + 200\mathbf{k}$$

$$\begin{vmatrix} 12 & -5 & 0 \\ 240 & 400 & 0 \\ -240 & 600 & -400 \end{vmatrix} \frac{T}{13 \times 20} + \begin{vmatrix} 12 & -5 & 0 \\ 240 & -100 & 200 \\ 0 & -W & 0 \end{vmatrix} \frac{1}{13} = 0$$

$$(-12 \times 400 \times 400 - 5 \times 240 \times 400)\frac{T}{760} + 12 \times 200W = 0$$

$$T = 0.76W = 0.76(490.50\text{ N}) \qquad T = 373\text{ N} \blacktriangleleft$$



PROBLEM 4.134

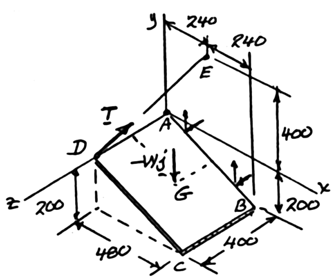
Solve Problem 4.133, assuming that wire CE is replaced by a wire connecting E and D.

PROBLEM 4.133 The 50-kg plate $ABCD$ is supported by hinges along edge AB and by wire CE. Knowing that the plate is uniform, determine the tension in the wire.

SOLUTION

Free-Body Diagram:

Dimensions in mm



$$W = mg = (50\text{ kg})(9.81\text{ m/s}^2)$$

$$W = 490.50\text{ N}$$

$$\overrightarrow{DE} = -240\mathbf{i} + 400\mathbf{j} - 400\mathbf{k}$$

$$DE = 614.5\text{ mm}$$

$$\mathbf{T} = T\frac{\overrightarrow{DE}}{DE} = \frac{T}{614.5}(240\mathbf{i} + 400\mathbf{j} - 400\mathbf{k})$$

$$\boldsymbol{\lambda}_{AB} = \frac{\overrightarrow{AB}}{AB} = \frac{480\mathbf{i} - 200\mathbf{j}}{520} = \frac{1}{13}(12\mathbf{i} - 5\mathbf{j})$$

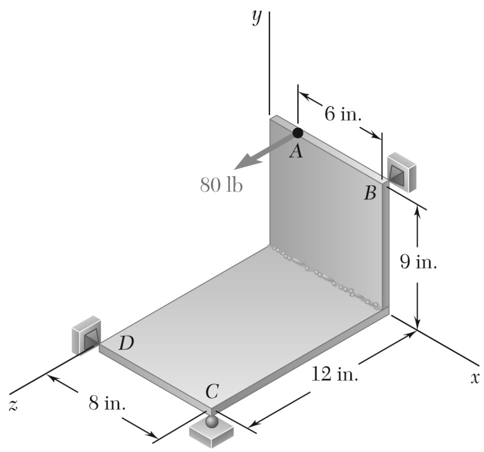
$$\mathbf{r}_{E/A} = 240\mathbf{i} + 400\mathbf{j}; \quad \mathbf{r}_{G/A} = 240\mathbf{i} - 100\mathbf{j} + 200\mathbf{k}$$

$$\begin{vmatrix} 12 & -5 & 0 \\ 240 & 400 & 0 \\ 240 & 400 & -400 \end{vmatrix} \frac{T}{13 \times 614.5} + \begin{vmatrix} 12 & 5 & 0 \\ 240 & -100 & 200 \\ 0 & -W & 0 \end{vmatrix} \frac{1}{13} = 0$$

$$(-12 \times 400 \times 400 - 5 \times 240 \times 400)\frac{T}{614.5} + 12 \times 200 \times W = 0$$

$$T = 0.6145W = 0.6145(490.50\text{ N})$$

$$T = 301\text{ N} \blacktriangleleft$$

	**PROBLEM 4.135** Two rectangular plates are welded together to form the assembly shown. The assembly is supported by ball-and-socket joints at *B* and *D* and by a ball on a horizontal surface at *C*. For the loading shown, determine the reaction at *C*.

SOLUTION

First note:

$$\lambda_{BD} = \frac{-(6\text{ in.})\mathbf{i} - (9\text{ in.})\mathbf{j} + (12\text{ in.})\mathbf{k}}{\sqrt{(6)^2 + (9)^2 + (12)^2}\text{ in.}}$$

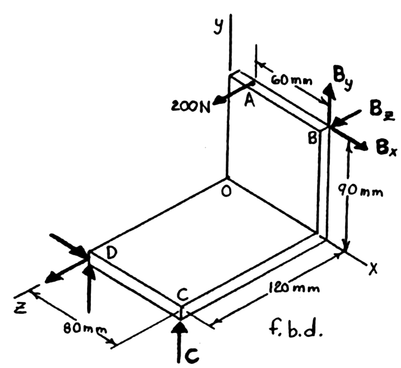
$$= \frac{1}{16.1555}(-6\mathbf{i} - 9\mathbf{j} + 12\mathbf{k})$$

$$\mathbf{r}_{A/B} = -(6\text{ in.})\mathbf{i}$$

$$\mathbf{P} = (80\text{ lb})\mathbf{k}$$

$$\mathbf{r}_{C/D} = (8\text{ in.})\mathbf{i}$$

$$\mathbf{C} = (C)\mathbf{j}$$



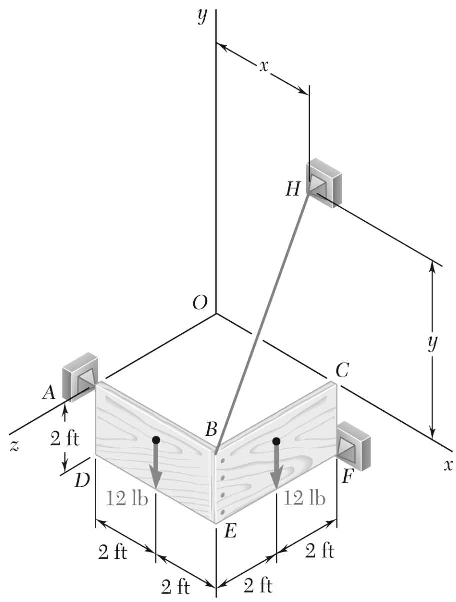
From the F.B.D. of the plates:

$$\Sigma M_{BD} = 0: \quad \lambda_{BD} \cdot (\mathbf{r}_{A/B} \times \mathrm{P}) + \lambda_{BD} \cdot (\mathbf{r}_{C/D} \times \mathrm{C}) = 0$$

$$\begin{vmatrix} -6 & -9 & 12 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{vmatrix} \left[\frac{6(80)}{16.1555}\right] + \begin{vmatrix} -6 & -9 & 12 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix} \left[\frac{C(8)}{16.1555}\right] = 0$$

$$(-9)(6)(80) + (12)(8)C = 0$$

$$C = 45.0\text{ lb} \qquad \text{or } \mathbf{C} = (45.0\text{ lb})\mathbf{j} \blacktriangleleft$$

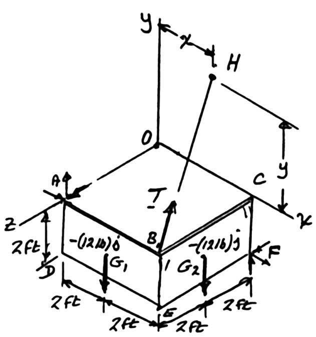


PROBLEM 4.136

Two 2×4-ft plywood panels, each of weight 12 lb, are nailed together as shown. The panels are supported by ball-and-socket joints at A and F and by the wire BH. Determine (*a*) the location of H in the xy plane if the tension in the wire is to be minimum, (*b*) the corresponding minimum tension.

SOLUTION

Free-Body Diagram:



$$\overrightarrow{AF} = 4\mathbf{i} - 2\mathbf{j} - 4\mathbf{k} \quad AF = 6\text{ ft}$$

$$\boldsymbol{\lambda}_{AF} = \frac{1}{3}(2\mathbf{i} - \mathbf{j} - 2\mathbf{k})$$

$$\mathbf{r}_{G_1/A} = 2\mathbf{i} - \mathbf{j}$$

$$\mathbf{r}_{G_2/A} = 4\mathbf{i} - \mathbf{j} - 2\mathbf{k}$$

$$\mathbf{r}_{B/A} = 4\mathbf{i}$$

$$\Sigma M_{AF} = 0: \quad \boldsymbol{\lambda}_{AF} \cdot (\mathbf{r}_{G_1/A} \times (-12\mathbf{j}) + \boldsymbol{\lambda}_{AF} \cdot (\mathbf{r}_{G_2/A} \times (-12\mathbf{j})) + \boldsymbol{\lambda}_{AF} \cdot (\mathbf{r}_{B/A} \times T) = 0$$

$$\begin{vmatrix} 2 & -1 & -2 \\ 2 & -1 & 0 \\ 0 & -12 & 0 \end{vmatrix} \frac{1}{3} + \begin{vmatrix} 2 & -1 & -2 \\ 4 & -1 & -2 \\ 0 & -12 & 0 \end{vmatrix} \frac{1}{3} + \boldsymbol{\lambda}_{AF} \cdot (\mathbf{r}_{B/A} \times \mathbf{T}) = 0$$

$$(2 \times 2 \times 12)\frac{1}{3} + (-2 \times 2 \times 12 + 2 \times 4 \times 12)\frac{1}{3} + \boldsymbol{\lambda}_{AF} \cdot (\mathbf{r}_{B/A} \times \mathbf{T}) = 0$$

$$\boldsymbol{\lambda}_{AF} \cdot (\mathbf{r}_{B/A} \times \mathbf{T}) = -32 \quad \text{or} \quad \mathbf{T} \cdot (\boldsymbol{\lambda}_{A/F} \times \mathbf{r}_{B/A}) = -32 \qquad (1)$$

PROBLEM 4.136 (Continued)

Projection of **T** on $(\boldsymbol{\lambda}_{AF} \times \mathbf{r}_{B/A})$ is constant. Thus, $T_{\min}$ is parallel to

$$\boldsymbol{\lambda}_{AF} \times \mathbf{r}_{B/A} = \frac{1}{3}(2\mathbf{i} - \mathbf{j} - 2\mathbf{k}) \times 4\mathbf{i} = \frac{1}{3}(-8\mathbf{j} + 4\mathbf{k})$$

Corresponding unit vector is $\frac{1}{\sqrt{5}}(-2\mathbf{j} + \mathbf{k})$.

$$T_{\min} = T(-2\mathbf{j} + \mathbf{k})\frac{1}{\sqrt{5}} \tag{2}$$

From Eq. (1):

$$\frac{T}{\sqrt{5}}(-2\mathbf{j} + \mathbf{k}) \cdot \left[\frac{1}{3}(2\mathbf{i} - \mathbf{j} - 2\mathbf{k}) \times 4\mathbf{i}\right] = -32$$

$$\frac{T}{\sqrt{5}}(-2\mathbf{j} + \mathbf{k}) \cdot \frac{1}{3}(-8\mathbf{j} + 4\mathbf{k}) = -32$$

$$\frac{T}{3\sqrt{5}}(16 + 4) = -32 \qquad T = -\frac{3\sqrt{5}(32)}{20} = 4.8\sqrt{5}$$

$$T = 10.7331 \text{ lb}$$

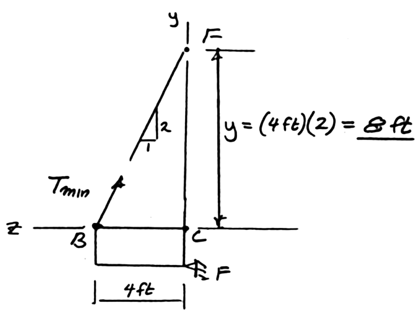
From Eq. (2):

$$T_{\min} = T(-2\mathbf{j} + \mathbf{k})\frac{1}{\sqrt{5}}$$

$$= 4.8\sqrt{5}(-2\mathbf{j} + \mathbf{k})\frac{1}{\sqrt{5}}$$

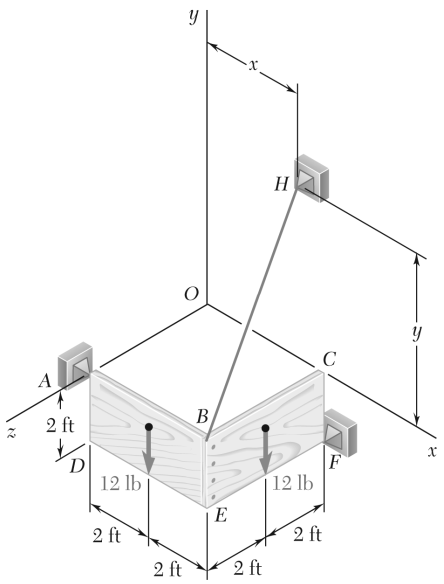
$$\mathbf{T}_{\min} = -(9.6 \text{ lb})\mathbf{j} + (4.8 \text{ lb } \mathbf{k})$$

Since $T_{\min}$ has no **i** component, wire *BH* is parallel to the *yz* plane, and $x = 4$ ft.



(*a*) $x = 4.00$ ft; $y = 8.00$ ft ◀

(*b*) $T_{\min} = 10.73$ lb ◀



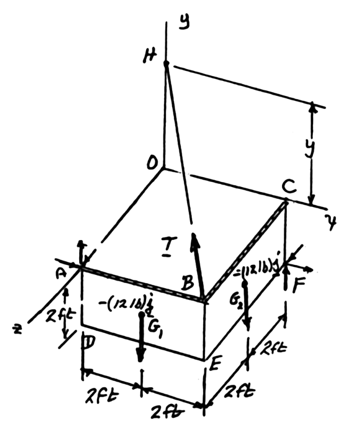
PROBLEM 4.137

Solve Problem 4.136, subject to the restriction that H must lie on the y-axis.

PROBLEM 4.136 Two 2×4-ft plywood panels, each of weight 12 lb, are nailed together as shown. The panels are supported by ball-and-socket joints at A and F and by the wire BH. Determine (a) the location of H in the xy plane if the tension in the wire is to be minimum, (b) the corresponding minimum tension.

SOLUTION

Free-Body Diagram:



$$\overrightarrow{AF} = 4\mathbf{i} - 2\mathbf{j} - 4\mathbf{k}$$

$$\boldsymbol{\lambda}_{AF} = \frac{1}{3}(2\mathbf{i} - \mathbf{j} - 2\mathbf{k})$$

$$\mathbf{r}_{G_1/A} = 2\mathbf{i} - \mathbf{j}$$

$$\mathbf{r}_{G_2/A} = 4\mathbf{i} - \mathbf{j} - 2\mathbf{k}$$

$$\mathbf{r}_{B/A} = 4\mathbf{i}$$

$$\Sigma M_{AF} = 0: \quad \boldsymbol{\lambda}_{AF}\cdot(\mathbf{r}_{G/A}\times(-12\mathbf{j}) + \boldsymbol{\lambda}_{AF}\cdot(\mathbf{r}_{G_2/A}\times(-12\mathbf{j})) + \boldsymbol{\lambda}_{AF}\cdot(\mathbf{r}_{B/A}\times T) = 0$$

$$\begin{vmatrix} 2 & -1 & 2 \\ 2 & -1 & 0 \\ 0 & -12 & 0 \end{vmatrix}\frac{1}{3} + \begin{vmatrix} 2 & -1 & -2 \\ 4 & -1 & -2 \\ 0 & -12 & 0 \end{vmatrix}\frac{1}{3} + \boldsymbol{\lambda}_{AF}\cdot(\mathbf{r}_{B/A}\times\mathbf{T}) = 0$$

$$(2\times 2\times 12)\frac{1}{3} + (-2\times 2\times 12 + 2\times 4\times 12)\frac{1}{3} + \boldsymbol{\lambda}_{AF}\cdot(\mathbf{r}_{B/A}\times\mathbf{T}) = 0$$

$$\boldsymbol{\lambda}_{AF}\cdot(\mathbf{r}_{B/A}\times\mathbf{T}) = -32 \qquad (1)$$

$$\overrightarrow{BH} = -4\mathbf{i} + y\mathbf{j} - 4\mathbf{k} \qquad BH = (32 + y^2)^{1/2}$$

$$\mathbf{T} = T\frac{\overrightarrow{BH}}{BH} = T\frac{-4\mathbf{i} + y\mathbf{j} - 4\mathbf{k}}{(32 + y^2)^{1/2}}$$

PROBLEM 4.137 (Continued)

From Eq. (1):

$$\boldsymbol{\lambda}_{AF} \cdot (\mathbf{r}_{B/A} \times T) = \begin{vmatrix} 2 & -1 & -2 \\ 4 & 0 & 0 \\ -4 & y & -4 \end{vmatrix} \frac{T}{3(32+y^2)^{1/2}} = -32$$

$$(-16-8y)T = -3\times 32(32+y^2)^{1/2} \qquad T = 96\frac{(32+y^2)^{1/2}}{8y+16} \tag{2}$$

$$\frac{dT}{dy} = 0: \quad 96\frac{(8y+16)\frac{1}{2}(32+y^2)^{-1/2}(2y)+(32+y^2)^{1/2}(8)}{(8y+16)^2}$$

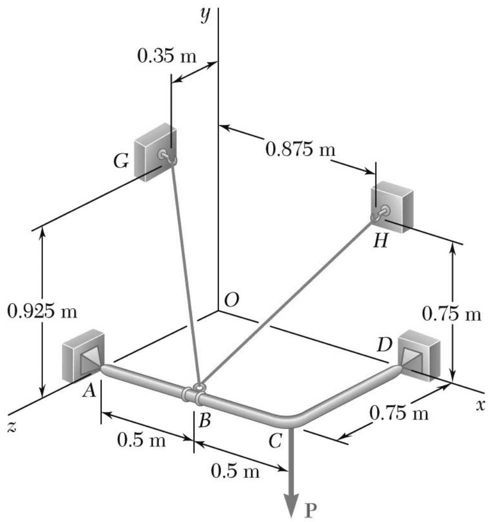
Numerator = 0: $\quad (8y+16)y = (32+y^2)8$

$$8y^2+16y = 32\times 8+8y^2$$

$x = 0$ ft; $y = 16.00$ ft ◀

From Eq. (2): $\quad T = 96\dfrac{(32+16^2)^{1/2}}{8\times 16+16} = 11.3137$ lb

$T_{\min} = 11.31$ lb ◀

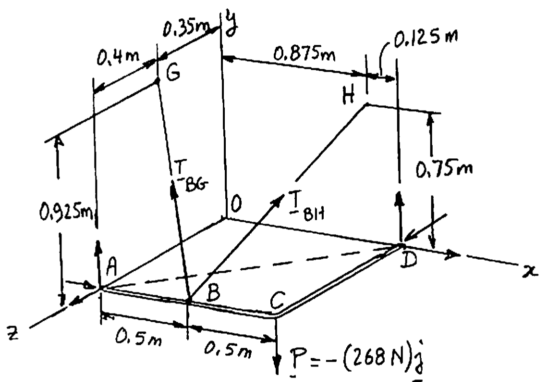


PROBLEM 4.138

The frame ACD is supported by ball-and-socket joints at A and D and by a cable that passes through a ring at B and is attached to hooks at G and H. Knowing that the frame supports at Point C a load of magnitude $P = 268$ N, determine the tension in the cable.

SOLUTION

Free-Body Diagram:



$$\boldsymbol{\lambda}_{AD} = \frac{\overrightarrow{AD}}{AD} = \frac{(1\text{ m})\mathbf{i} - (0.75\text{ m})\mathbf{k}}{1.25\text{ m}}$$

$$\boldsymbol{\lambda}_{AD} = 0.8\mathbf{i} - 0.6\mathbf{k}$$

$$\mathbf{T}_{BG} = T_{BG}\frac{\overrightarrow{BG}}{BG}$$

$$= T_{BG}\frac{-0.5\mathbf{i} + 0.925\mathbf{j} - 0.4\mathbf{k}}{1.125}$$

$$T_{BH} = T_{BH}\frac{\overrightarrow{BH}}{BH}$$

$$= T_{BH}\frac{0.375\mathbf{i} + 0.75\mathbf{j} - 0.75\mathbf{k}}{1.125}$$

PROBLEM 4.138 (Continued)

$$\mathbf{r}_{B/A} = (0.5\text{ m})\mathbf{i}; \quad \mathbf{r}_{C/A} = (1\text{ m})\mathbf{i}; \quad \mathbf{P} = -(268\text{ N})\mathbf{j}$$

To eliminate the reactions at A and D, we shall write

$$\Sigma\mathbf{M}_{AD} = 0: \quad \boldsymbol{\lambda}_{AD}\cdot(\mathbf{r}_{B/A}\times\mathbf{T}_{BG}) + \boldsymbol{\lambda}_{AD}\cdot(\mathbf{r}_{B/A}\times\mathbf{T}_{BH}) + \boldsymbol{\lambda}_{AD}\cdot(\mathbf{r}_{C/A}\times\mathbf{P}) = 0 \qquad (1)$$

Substituting for terms in Eq. (1) and using determinants,

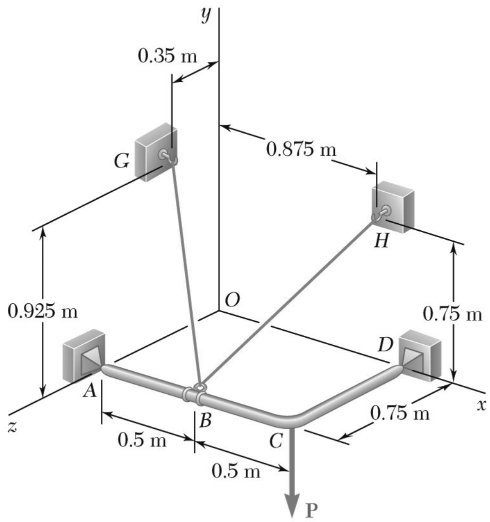
$$\begin{vmatrix} 0.8 & 0 & -0.6 \\ 0.5 & 0 & 0 \\ -0.5 & 0.925 & -0.4 \end{vmatrix}\frac{T_{BG}}{1.125} + \begin{vmatrix} 0.8 & 0 & -0.6 \\ 0.5 & 0 & 0 \\ 0.375 & 0.75 & -0.75 \end{vmatrix}\frac{T_{BH}}{1.125} + \begin{vmatrix} 0.8 & 0 & -0.6 \\ 1 & 0 & 0 \\ 0 & -268 & 0 \end{vmatrix} = 0$$

Multiplying all terms by (–1.125),

$$0.27750T_{BG} + 0.22500T_{BH} = 180.900 \qquad (2)$$

For this problem, $\quad T_{BG} = T_{BH} = T$

$$(0.27750 + 0.22500)T = 180.900 \qquad T = 360\text{ N} \blacktriangleleft$$



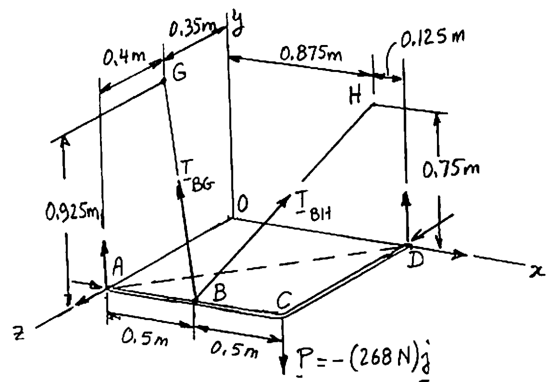
PROBLEM 4.139

Solve Prob. 4.138, assuming that cable *GBH* is replaced by a cable *GB* attached at *G* and *B*.

PROBLEM 4.138 The frame *ACD* is supported by ball-and-socket joints at *A* and *D* and by a cable that passes through a ring at *B* and is attached to hooks at *G* and *H*. Knowing that the frame supports at Point *C* a load of magnitude $P = 268$ N, determine the tension in the cable.

SOLUTION

Free-Body Diagram:



$$\lambda_{AD} = \frac{\overrightarrow{AD}}{AD} = \frac{(1 \text{ m})\mathbf{i} - (0.75 \text{ m})\mathbf{k}}{1.25 \text{ m}}$$

$$\lambda_{AD} = 0.8\mathbf{i} - 0.6\mathbf{k}$$

$$\mathbf{T}_{BG} = T_{BG}\frac{\overrightarrow{BG}}{BG}$$

$$= T_{BG}\frac{-0.5\mathbf{i} + 0.925\mathbf{j} - 0.4\mathbf{k}}{1.125}$$

$$\mathbf{T}_{BH} = T_{BH}\frac{\overrightarrow{BH}}{BH}$$

$$= T_{BH}\frac{0.375\mathbf{i} + 0.75\mathbf{j} - 0.75\mathbf{k}}{1.125}$$

PROBLEM 4.139 (Continued)

$$\mathbf{r}_{B/A} = (0.5\text{ m})\mathbf{i}; \quad \mathbf{r}_{C/A} = (1\text{ m})\mathbf{i}; \quad \mathbf{P} = -(268\text{ N})\mathbf{j}$$

To eliminate the reactions at A and D, we shall write

$$\Sigma\mathbf{M}_{AD} = 0: \quad \boldsymbol{\lambda}_{AD}\cdot(\mathbf{r}_{B/A}\times\mathbf{T}_{BG}) + \boldsymbol{\lambda}_{AD}\cdot(\mathbf{r}_{B/A}\times\mathbf{T}_{BH}) + \boldsymbol{\lambda}_{AD}\cdot(\mathbf{r}_{C/A}\times\mathbf{P}) = 0 \qquad (1)$$

Substituting for terms in Eq. (1) and using determinants,

$$\begin{vmatrix} 0.8 & 0 & -0.6 \\ 0.5 & 0 & 0 \\ -0.5 & 0.925 & -0.4 \end{vmatrix}\frac{T_{BG}}{1.125} + \begin{vmatrix} 0.8 & 0 & -0.6 \\ 0.5 & 0 & 0 \\ 0.375 & 0.75 & -0.75 \end{vmatrix}\frac{T_{BH}}{1.125} + \begin{vmatrix} 0.8 & 0 & -0.6 \\ 1 & 0 & 0 \\ 0 & -268 & 0 \end{vmatrix} = 0$$

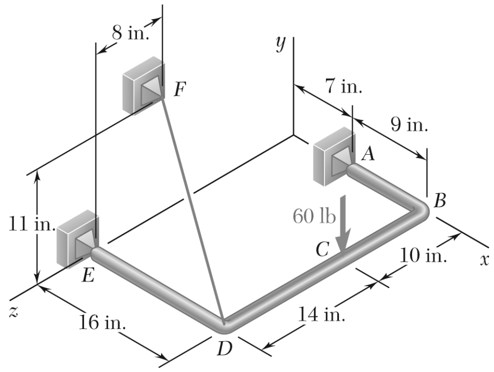
Multiplying all terms by (–1.125),

$$0.27750T_{BG} + 0.22500T_{BH} = 180.900 \qquad (2)$$

For this problem, $T_{BH} = 0$.

Thus, Eq. (2) reduces to

$$0.27750T_{BG} = 180.900 \qquad T_{BG} = 652\text{ N} \blacktriangleleft$$

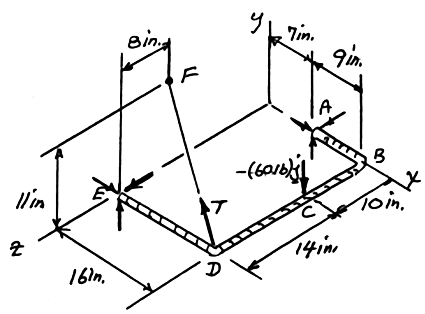


PROBLEM 4.140

The bent rod *ABDE* is supported by ball-and-socket joints at *A* and *E* and by the cable *DF*. If a 60-lb load is applied at *C* as shown, determine the tension in the cable.

SOLUTION

Free-Body Diagram:



$$\overrightarrow{DF} = -16\mathbf{i} + 11\mathbf{j} - 8\mathbf{k} \qquad DF = 21\text{ in.}$$

$$\mathbf{T} = T\frac{\overrightarrow{DE}}{DF} = \frac{T}{21}(-16\mathbf{i} + 11\mathbf{j} - 8\mathbf{k})$$

$$\mathbf{r}_{D/E} = 16\mathbf{i}$$

$$\mathbf{r}_{C/E} = 16\mathbf{i} - 14\mathbf{k}$$

$$\boldsymbol{\lambda}_{EA} = \frac{\overrightarrow{EA}}{EA} = \frac{7\mathbf{i} - 24\mathbf{k}}{25}$$

$$\Sigma M_{EA} = 0: \quad \boldsymbol{\lambda}_{EA} \cdot (\mathbf{r}_{B/E} \times \mathbf{T}) + \boldsymbol{\lambda}_{EA} \cdot (\mathbf{r}_{C/E} \cdot (-60\mathbf{j})) = 0$$

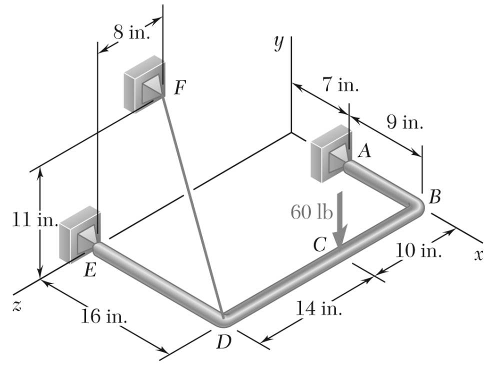
$$\begin{vmatrix} 7 & 0 & -24 \\ 16 & 0 & 0 \\ -16 & 11 & -8 \end{vmatrix} \frac{T}{21 \times 25} + \begin{vmatrix} 7 & 0 & -24 \\ 16 & 0 & -14 \\ 0 & -60 & 0 \end{vmatrix} \frac{1}{25} = 0$$

$$-\frac{24 \times 16 \times 11}{21 \times 25}T + \frac{-7 \times 14 \times 60 + 24 \times 16 \times 60}{25} = 0$$

$$201.14T + 17{,}160 = 0$$

$$T = 85.314\text{ lb}$$

$T = 85.3$ lb ◀

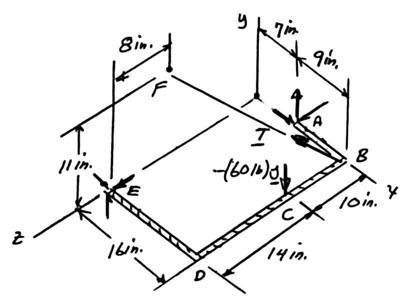


PROBLEM 4.141

Solve Problem 4.140, assuming that cable *DF* is replaced by a cable connecting *B* and *F*.

SOLUTION

Free-Body Diagram:



$$\mathbf{r}_{B/A} = 9\mathbf{i}$$

$$\mathbf{r}_{C/A} = 9\mathbf{i} + 10\mathbf{k}$$

$$\overrightarrow{BF} = -16\mathbf{i} + 11\mathbf{j} + 16\mathbf{k} \qquad BF = 25.16 \text{ in.}$$

$$\mathbf{T} = T\frac{\overrightarrow{BF}}{BF} = \frac{T}{25.16}(-16\mathbf{i} + 11\mathbf{j} + 16\mathbf{k})$$

$$\boldsymbol{\lambda}_{AE} = \frac{\overrightarrow{AE}}{AE} = \frac{7\mathbf{i} - 24\mathbf{k}}{\mathbf{25}}$$

$$\Sigma M_{AE} = 0: \quad \boldsymbol{\lambda}_{AF} \cdot (\mathbf{r}_{B/A} \times \mathbf{T}) + \boldsymbol{\lambda}_{AE} \cdot (\mathbf{r}_{C/A} \cdot (-60\mathbf{j})) = 0$$

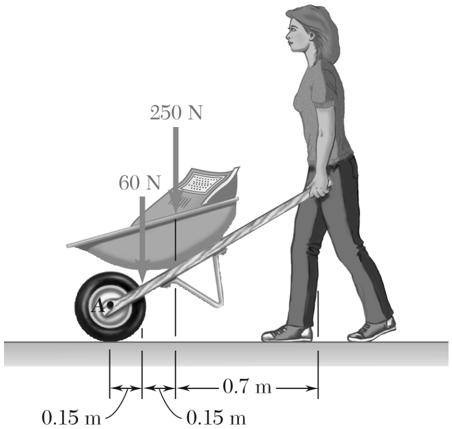
$$\begin{vmatrix} 7 & 0 & -24 \\ 9 & 0 & 0 \\ -16 & 11 & 16 \end{vmatrix} \frac{T}{25 \times 25.16} + \begin{vmatrix} 7 & 0 & -24 \\ 9 & 0 & 10 \\ 0 & -60 & 0 \end{vmatrix} \frac{1}{25} = 0$$

$$-\frac{24 \times 9 \times 11}{25 \times 25.16}T + \frac{24 \times 9 \times 60 + 7 \times 10 \times 60}{25} = 0$$

$$94.436T - 17{,}160 = 0$$

$T = 181.7$ lb ◀

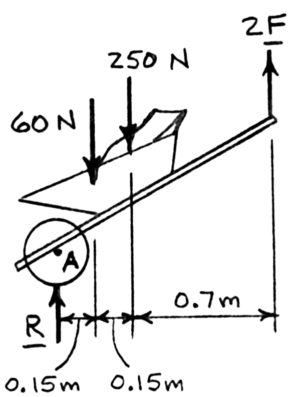
PROBLEM 4.142



A gardener uses a 60-N wheelbarrow to transport a 250-N bag of fertilizer. What force must she exert on each handle?

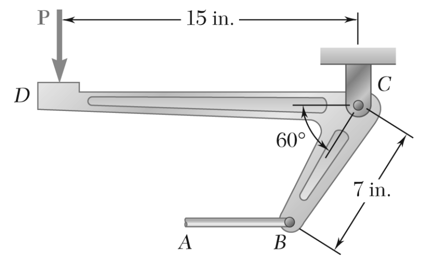
SOLUTION

Free-Body Diagram:



$$+\circlearrowleft \Sigma M_A = 0: \quad (2F)(1\text{ m}) - (60\text{ N})(0.15\text{ m}) - (250\text{ N})(0.3\text{ m}) = 0$$

$$\mathbf{F} = 42.0\text{ N}\uparrow \blacktriangleleft$$

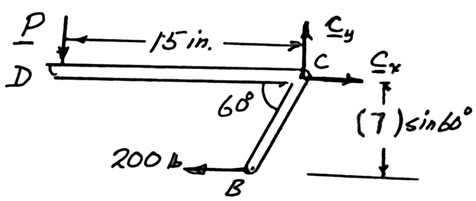


PROBLEM 4.143

The required tension in cable *AB* is 200 lb. Determine (*a*) the vertical force **P** that must be applied to the pedal, (*b*) the corresponding reaction at *C*.

SOLUTION

Free-Body Diagram:



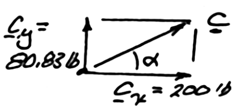
$BC = 7$ in.

(*a*) $+\circlearrowright \Sigma M_C = 0\text{:} \quad P(15 \text{ in.}) - (200 \text{ lb})(6.062 \text{ in.}) = 0$

$P = 80.83$ lb

$\mathbf{P} = 80.8 \text{ lb} \downarrow$ ◀

(*b*) $\xrightarrow{+} \Sigma F_y = 0\text{:} \quad C_x - 200 \text{ lb} = 0$ $\qquad \mathbf{C}_x = 200 \text{ lb} \longrightarrow$

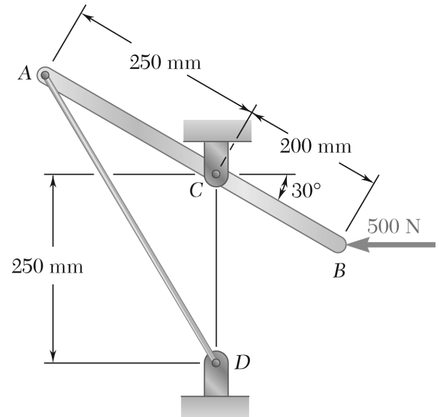


$+\uparrow \Sigma F_y = 0\text{:} \quad C_y - P = 0 \quad C_y - 80.83 \text{ lb} = 0$ $\qquad \mathbf{C}_y = 80.83 \text{ lb} \uparrow$

$\alpha = 22.0°$

$C = 215.7$ lb

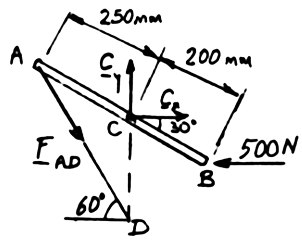
$\mathbf{C} = 216 \text{ lb} \measuredangle 22.0°$ ◀



PROBLEM 4.144

A lever AB is hinged at C and attached to a control cable at A. If the lever is subjected to a 500-N horizontal force at B, determine (*a*) the tension in the cable, (*b*) the reaction at C.

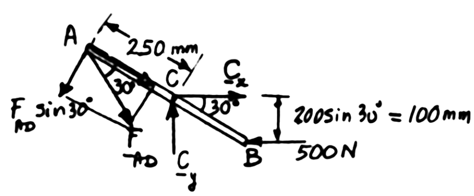
SOLUTION



Triangle ACD is isosceles with $\measuredangle C = 90° + 30° = 120°$ $\measuredangle A = \measuredangle D = \frac{1}{2}(180° - 120°) = 30°.$

Thus, DA forms angle of 60° with the horizontal axis.

(*a*) We resolve $\mathbf{F}_{AD}$ into components along AB and perpendicular to AB.

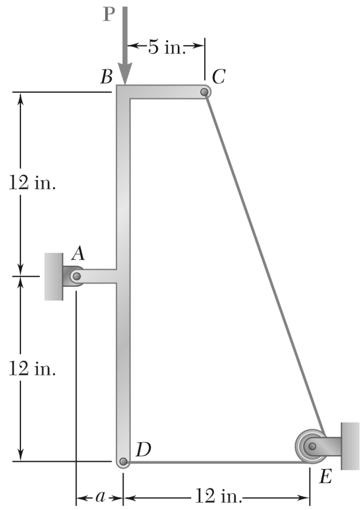


$+\circlearrowright \Sigma M_C = 0$: $(F_{AD} \sin 30°)(250 \text{ mm}) - (500 \text{ N})(100 \text{ mm}) = 0$ $\qquad F_{AD} = 400 \text{ N}$ ◀

(*b*) $\xrightarrow{+} \Sigma F_x = 0$: $-(400 \text{ N})\cos 60° + C_x - 500 \text{ N} = 0$ $\qquad C_x = +300 \text{ N}$

$+\uparrow \Sigma F_y = 0$: $-(400 \text{ N})\sin 60° + C_y = 0$ $\qquad C_y = +346.4 \text{ N}$

$\mathbf{C} = 458 \text{ N} \measuredangle 49.1°$ ◀

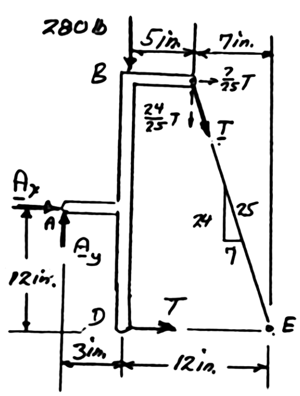


PROBLEM 4.145

A force **P** of magnitude 280 lb is applied to member *ABCD*, which is supported by a frictionless pin at *A* and by the cable *CED*. Since the cable passes over a small pulley at *E*, the tension may be assumed to be the same in portions *CE* and *ED* of the cable. For the case when $a = 3$ in., determine (*a*) the tension in the cable, (*b*) the reaction at *A*.

SOLUTION

Free-Body Diagram:



(*a*) $+\circlearrowleft \Sigma M_A = 0: \quad -(280 \text{ lb})(8 \text{ in.})$

$$T(12 \text{ in.}) - \frac{7}{25}T(12 \text{ in.})$$

$$-\frac{24}{25}T(8 \text{ in.}) = 0$$

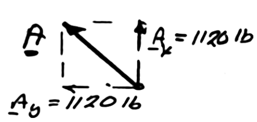
$$(12 - 11.04)T = 840 \qquad T = 875 \text{ lb} \blacktriangleleft$$

(*b*) $\xrightarrow{+} \Sigma F_x = 0: \quad \frac{7}{25}(875 \text{ lb}) + 875 \text{ lb} + A_x = 0$

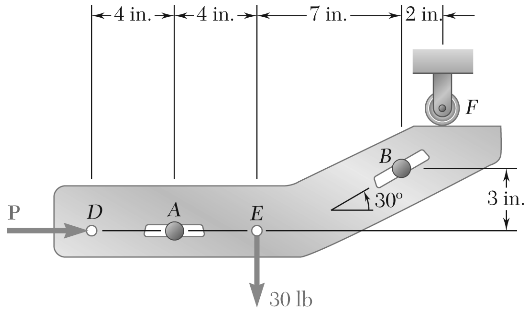
$$A_x = -1120 \qquad \mathbf{A}_x = 1120 \text{ lb} \longleftarrow$$

$+\uparrow \Sigma F_y = 0: \quad A_y - 280 \text{ lb} - \frac{24}{25}(875 \text{ lb}) = 0$

$$A_y = +1120 \qquad \mathbf{A}_y = 1120 \text{ lb}\uparrow$$



$$\mathbf{A} = 1584 \text{ lb} \nwarrow 45.0° \blacktriangleleft$$

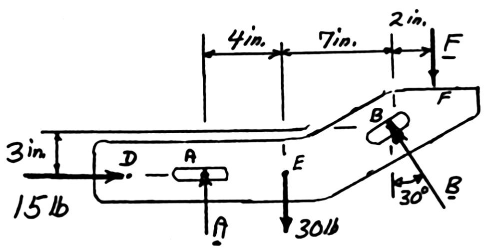


PROBLEM 4.146

Two slots have been cut in plate *DEF*, and the plate has been placed so that the slots fit two fixed, frictionless pins *A* and *B*. Knowing that $P = 15$ lb, determine (*a*) the force each pin exerts on the plate, (*b*) the reaction at *F*.

SOLUTION

Free-Body Diagram:



(*a*) $\xrightarrow{+} \Sigma F_x = 0$: $15 \text{ lb} - B \sin 30° = 0$ **B** $= 30.0$ lb $\searrow$ 60.0° ◀

(*b*) $+\circlearrowleft \Sigma M_A = 0$: $-(30 \text{ lb})(4 \text{ in.}) + B \sin 30°(3 \text{ in.}) + B \cos 30°(11 \text{ in.}) - F(13 \text{ in.}) = 0$

$$-120 \text{ lb} \cdot \text{in.} + (30 \text{ lb}) \sin 30°(3 \text{ in.}) + (30 \text{ lb}) \cos 30°(11 \text{ in.}) - F(13 \text{ in.}) = 0$$

$$F = +16.2145 \text{ lb}$$

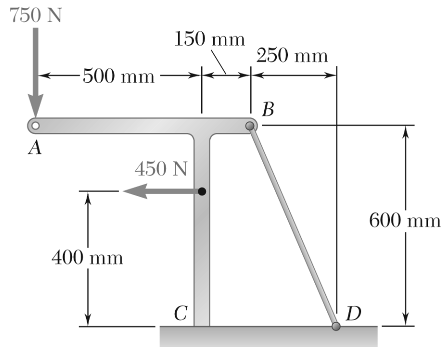
F $= 16.21$ lb $\downarrow$ ◀

(*a*) $+\uparrow \Sigma F_y = 0$: $A - 30 \text{ lb} + B \cos 30° - F = 0$

$$A - 30 \text{ lb} + (30 \text{ lb}) \cos 30° - 16.2145 \text{ lb} = 0$$

$$A = +20.23 \text{ lb}$$

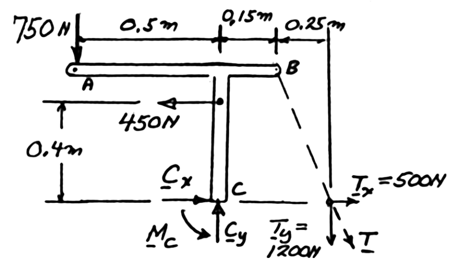
A $= 20.2$ lb $\uparrow$ ◀



PROBLEM 4.147

Knowing that the tension in wire *BD* is 1300 N, determine the reaction at the fixed support *C* of the frame shown.

SOLUTION



$$T = 1300 \text{ N}$$

$$T_x = \frac{5}{13}T$$

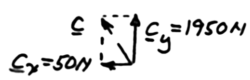
$$= 500 \text{ N}$$

$$T_y = \frac{12}{13}T$$

$$= 1200 \text{ N}$$

$\xrightarrow{+} \Sigma M_x = 0{:}\quad C_x - 450 \text{ N} + 500 \text{ N} = 0 \qquad C_x = -50 \text{ N} \qquad\qquad \mathbf{C}_x = 50 \text{ N} \longleftarrow$

$+\!\uparrow \Sigma F_y = 0{:}\quad C_y - 750 \text{ N} - 1200 \text{ N} = 0 \qquad C_y = +1950 \text{ N} \qquad\qquad \mathbf{C}_y = 1950 \text{ N} \uparrow$

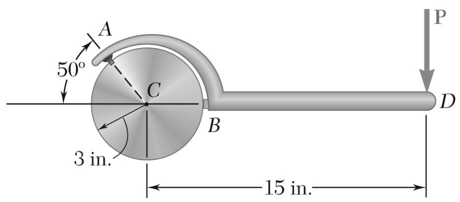


$\mathbf{C} = 1951 \text{ N} \measuredangle 88.5° \blacktriangleleft$

$+\circlearrowleft \Sigma M_C = 0{:}\quad M_C + (750 \text{ N})(0.5 \text{ m}) + (4.50 \text{ N})(0.4 \text{ m})$

$-(1200 \text{ N})(0.4 \text{ m}) = 0$

$M_C = -75.0 \text{ N} \cdot \text{m} \qquad\qquad \mathbf{M}_C = 75.0 \text{ N} \cdot \text{m} \circlearrowright \blacktriangleleft$



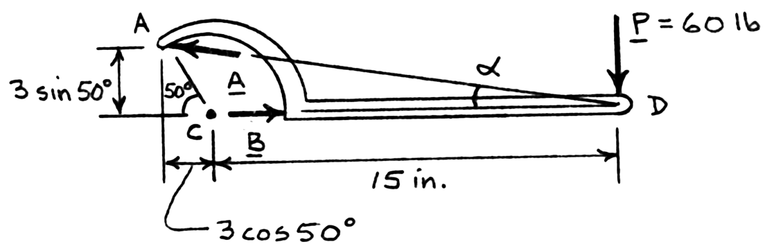
PROBLEM 4.148

The spanner shown is used to rotate a shaft. A pin fits in a hole at *A*, while a flat, frictionless surface rests against the shaft at *B*. If a 60-lb force **P** is exerted on the spanner at *D*, find the reactions at *A* and *B*.

SOLUTION

Free-Body Diagram:

(Three-force body)



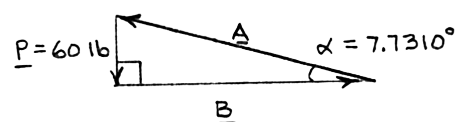
The line of action of **A** must pass through *D*, where **B** and **P** intersect.

$$\tan\alpha = \frac{3\sin 50°}{3\cos 50° + 15}$$
$$= 0.135756$$
$$\alpha = 7.7310°$$

$$A = \frac{60 \text{ lb}}{\sin 7.7310°}$$
$$= 446.02 \text{ lb}$$

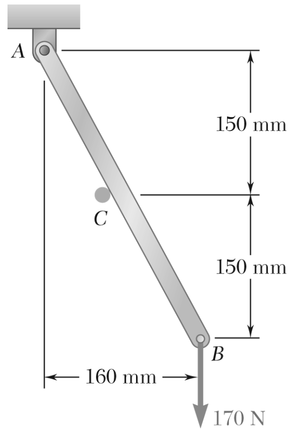
$$B = \frac{60 \text{ lb}}{\tan 7.7310°}$$
$$= 441.97 \text{ lb}$$

Force triangle



A = 446 lb ⦩ 7.73° ◀

B = 442 lb ⟶ ◀

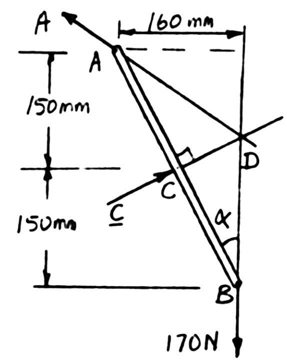


PROBLEM 4.149

Rod AB is supported by a pin and bracket at A and rests against a frictionless peg at C. Determine the reactions at A and C when a 170-N vertical force is applied at B.

SOLUTION

Free-Body Diagram:
(Three-force body)



The reaction at A must pass through D where **C** and the 170-N force intersect.

$$\tan\alpha = \frac{160 \text{ mm}}{300 \text{ mm}}$$

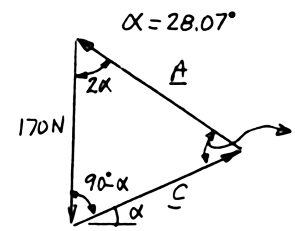
$$\alpha = 28.07°$$

We note that triangle ABD is isosceles (since $AC = BC$) and, therefore,

$$\sphericalangle CAD = \alpha = 28.07°$$

Also, since $CD \perp CB$, reaction **C** forms angle $\alpha = 28.07°$ with the horizontal axis.

Force triangle



We note that **A** forms angle 2α with the vertical axis. Thus, **A** and **C** form angle

$$180° - (90° - \alpha) - 2\alpha = 90° - \alpha$$

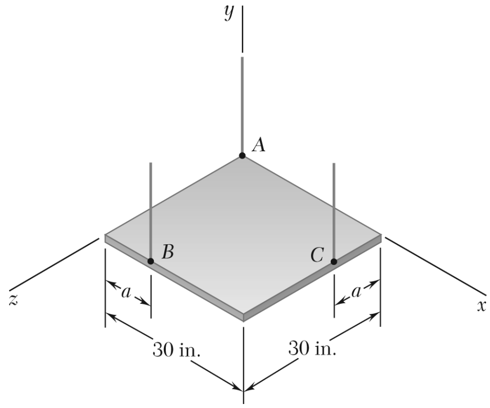
Force triangle is isosceles, and we have

$$A = 170 \text{ N}$$

$$C = 2(170 \text{ N})\sin\alpha$$

$$= 160.0 \text{ N}$$

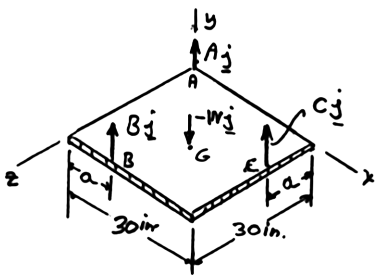
$\mathbf{A} = 170.0 \text{ N} \searrow 33.9°$; $\mathbf{C} = 160.0 \text{ N} \angle 28.1°$ ◀



PROBLEM 4.150

The 24-lb square plate shown is supported by three vertical wires. Determine (a) the tension in each wire when $a = 10$ in., (b) the value of a for which the tension in each wire is 8 lb.

SOLUTION



$$\mathbf{r}_{B/A} = a\mathbf{i} + 30\mathbf{k}$$
$$\mathbf{r}_{C/A} = 30\mathbf{i} + a\mathbf{k}$$
$$\mathbf{r}_{G/A} = 15\mathbf{i} + 15\mathbf{k}$$

By symmetry, $B = C$.

$$\Sigma M_A = 0: \quad \mathbf{r}_{B/A} \times B\mathbf{j} + \mathbf{r}_C \times C\mathbf{j} + \mathbf{r}_{G/A} \times (-W\mathbf{j}) = 0$$

$$(a\mathbf{i} + 30\mathbf{k}) \times B\mathbf{j} + (30\mathbf{i} + a\mathbf{k}) \times B\mathbf{j} + (15\mathbf{i} + 15\mathbf{k}) \times (-W\mathbf{j}) = 0$$

$$Ba\mathbf{k} - 30B\mathbf{i} + 30B\mathbf{k} - Ba\mathbf{i} - 15W\mathbf{k} + 15W\mathbf{i} = 0$$

Equate coefficient of unit vector $\mathbf{i}$ to zero:

$$\mathbf{i}: \quad -30B - Ba + 15W = 0$$

$$B = \frac{15W}{30+a} \qquad C = B = \frac{15W}{30+a} \tag{1}$$

$$\Sigma F_y = 0: \quad A + B + C - W = 0$$

$$A + 2\left[\frac{15W}{30+a}\right] - W = 0; \quad A = \frac{aW}{30+a} \tag{2}$$

(a) For $a = 10$ in.

From Eq. (1): $C = B = \dfrac{15(24 \text{ lb})}{30+10} = 9.00 \text{ lb}$

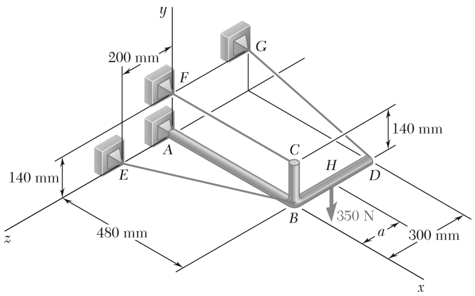
From Eq. (2): $A = \dfrac{10(24 \text{ lb})}{30+10} = 6.00 \text{ lb}$ $\qquad A = 6.00 \text{ lb}; \quad B = C = 9.00 \text{ lb}$ ◀

PROBLEM 4.150 (Continued)

(*b*) For tension in each wire = 8 lb,

From Eq. (1):
$$8\text{ lb} = \frac{15(24\text{ lb})}{30 + a}$$

$$30\text{ in.} + a = 45 \qquad a = 15.00\text{ in.} \blacktriangleleft$$



PROBLEM 4.151

Frame *ABCD* is supported by a ball-and-socket joint at *A* and by three cables. For $a = 150$ mm, determine the tension in each cable and the reaction at *A*.

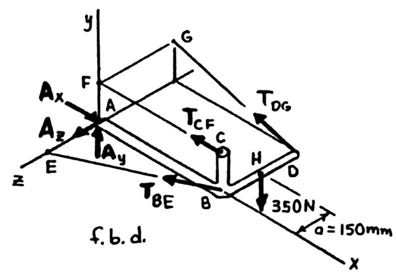
SOLUTION

First note:

$$\mathbf{T}_{DG} = \boldsymbol{\lambda}_{DG} T_{DG} = \frac{-(0.48\text{ m})\mathbf{i} + (0.14\text{ m})\mathbf{j}}{\sqrt{(0.48)^2 + (0.14)^2}\text{ m}} T_{DG}$$

$$= \frac{-0.48\mathbf{i} + 0.14\mathbf{j}}{0.50} T_{DG}$$

$$= \frac{T_{DG}}{25}(24\mathbf{i} + 7\mathbf{j})$$



$$\mathbf{T}_{BE} = \boldsymbol{\lambda}_{BE} T_{BE} = \frac{-(0.48\text{ m})\mathbf{i} + (0.2\text{ m})\mathbf{k}}{\sqrt{(0.48)^2 + (0.2)^2}\text{ m}} T_{BE}$$

$$= \frac{-0.48\mathbf{i} + 0.2\mathbf{k}}{0.52} T_{BE}$$

$$= \frac{T_{BE}}{13}(-12\mathbf{j} + 5\mathbf{k})$$

From F.B.D. of frame *ABCD*:

$$\Sigma M_x = 0: \quad \left(\frac{7}{25} T_{DG}\right)(0.3\text{ m}) - (350\text{ N})(0.15\text{ m}) = 0$$

or $\qquad T_{DG} = 625\text{ N}$ ◀

$$\Sigma M_y = 0: \quad \left(\frac{24}{25} \times 625\text{ N}\right)(0.3\text{ m}) - \left(\frac{5}{13} T_{BE}\right)(0.48\text{ m}) = 0$$

or $\qquad T_{BE} = 975\text{ N}$ ◀

$$\Sigma M_z = 0: \quad T_{CF}(0.14\text{ m}) + \left(\frac{7}{25} \times 625\text{ N}\right)(0.48\text{ m}) - (350\text{ N})(0.48\text{ m}) = 0$$

or $\qquad T_{CF} = 600\text{ N}$ ◀

PROBLEM 4.151 (Continued)

$$\Sigma F_x = 0: \quad A_x + T_{CF} + (T_{BE})_x + (T_{DG})_x = 0$$

$$A_x - 600 \text{ N} - \left(\frac{12}{13} \times 975 \text{ N}\right) - \left(\frac{24}{25} \times 625 \text{ N}\right) = 0$$

$$A_x = 2100 \text{ N}$$

$$\Sigma F_y = 0: \quad A_y + (T_{DG})_y - 350 \text{ N} = 0$$

$$A_y + \left(\frac{7}{25} \times 625 \text{ N}\right) - 350 \text{ N} = 0$$

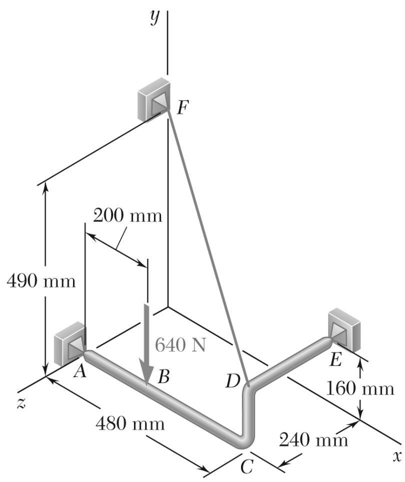
$$A_y = 175.0 \text{ N}$$

$$\Sigma F_z = 0: \quad A_z + (T_{BE})_z = 0$$

$$A_z + \left(\frac{5}{13} \times 975 \text{ N}\right) = 0$$

$$A_z = -375 \text{ N}$$

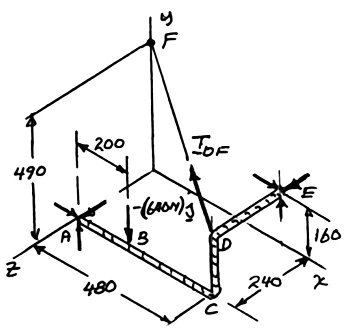
Therefore, $\mathbf{A} = (2100 \text{ N})\mathbf{i} + (175.0 \text{ N})\mathbf{j} - (375 \text{ N})\mathbf{k}$ ◀

	**PROBLEM 4.152** The pipe *ACDE* is supported by ball-and-socket joints at *A* and *E* and by the wire *DF*. Determine the tension in the wire when a 640-N load is applied at *B* as shown.

SOLUTION

Free-Body Diagram:

Dimensions in mm



$$\overrightarrow{AE} = 480\mathbf{i} + 160\mathbf{j} - 240\mathbf{k}$$

$$AE = 560 \text{ mm}$$

$$\boldsymbol{\lambda}_{AE} = \frac{\overrightarrow{AE}}{AE} = \frac{480\mathbf{i} + 160\mathbf{j} - 240\mathbf{k}}{560}$$

$$\boldsymbol{\lambda}_{AE} = \frac{6\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}}{7}$$

$$\mathbf{r}_{B/A} = 200\mathbf{i}$$

$$\mathbf{r}_{D/A} = 480\mathbf{i} + 160\mathbf{j}$$

$$\overrightarrow{DF} = -480\mathbf{i} + 330\mathbf{j} - 240\mathbf{k}; \qquad DF = 630 \text{ mm}$$

$$\mathbf{T}_{DF} = T_{DF}\frac{\overrightarrow{DF}}{DF} = T_{DF}\frac{-480\mathbf{i} + 330\mathbf{j} - 240\mathbf{k}}{630} = T_{DF}\frac{-16\mathbf{i} + 11\mathbf{j} - 8\mathbf{k}}{21}$$

$$\Sigma M_{AE} = \boldsymbol{\lambda}_{AE} \cdot (\mathbf{r}_{D/A} \times \mathbf{T}_{DF}) + \boldsymbol{\lambda}_{AE} \cdot (\mathbf{r}_{B/A} \times (-600\mathbf{j})) = 0$$

$$\begin{vmatrix} 6 & 2 & -3 \\ 480 & 160 & 0 \\ -16 & 11 & -8 \end{vmatrix} \frac{T_{DF}}{21 \times 7} + \begin{vmatrix} 6 & 2 & -3 \\ 200 & 0 & 0 \\ 0 & -640 & 0 \end{vmatrix} \frac{1}{7} = 0$$

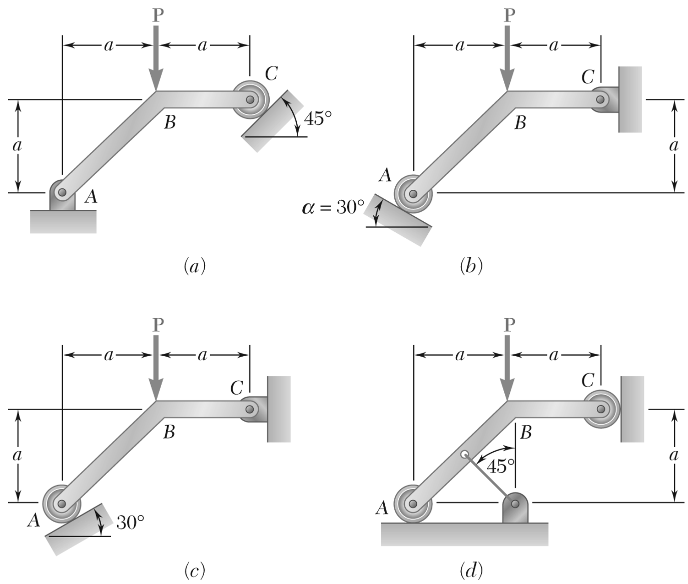
$$\frac{-6 \times 160 \times 8 + 2 \times 480 \times 8 - 3 \times 480 \times 11 - 3 \times 160 \times 16}{21 \times 7} T_{DF} + \frac{3 \times 200 \times 640}{7} = 0$$

$$-1120T_{DF} + 384 \times 10^3 = 0$$

$$T_{DF} = 342.86 \text{ N} \qquad\qquad T_{DF} = 343 \text{ N} ◀$$

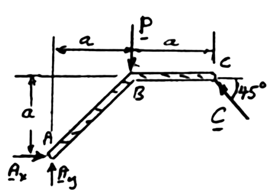
PROBLEM 4.153

A force **P** is applied to a bent rod *ABC*, which may be supported in four different ways as shown. In each case, if possible, determine the reactions at the supports.



SOLUTION

(*a*)



$$+\circlearrowleft \Sigma M_A = 0: \quad -P_a + (C\sin 45°)2a + (\cos 45°)a = 0$$

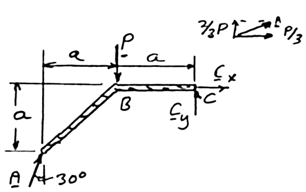
$$3\frac{C}{\sqrt{2}} = P \qquad C = \frac{\sqrt{2}}{3}P \qquad \mathbf{C} = 0.471P \measuredangle 45° \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x - \left(\frac{\sqrt{2}}{3}P\right)\frac{1}{\sqrt{2}} \qquad A_x = \frac{P}{3} \longrightarrow$$

$$+\uparrow \Sigma F_y = 0: \quad A_y - P + \left(\frac{\sqrt{2}}{3}P\right)\frac{1}{\sqrt{2}} \qquad A_y = \frac{2P}{3}\uparrow$$

$$\mathbf{A} = 0.745P \measuredangle 63.4° \blacktriangleleft$$

(*b*)



$$+\circlearrowleft \Sigma M_C = 0: \quad +Pa - (A\cos 30°)2a + (A\sin 30°)a = 0$$

$$A(1.732 - 0.5) = P \quad A = 0.812P$$

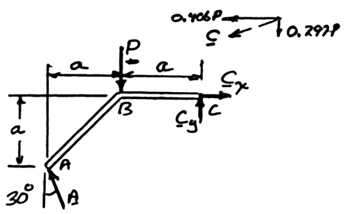
$$\mathbf{A} = 0.812P \measuredangle 60.0° \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad (0.812P)\sin 30° + C_x = 0 \qquad C_x = -0.406P$$

$$+\uparrow \Sigma F_y = 0: \quad (0.812P)\cos 30° - P + C_y = 0 \quad C_y = -0.297P$$

$$\mathbf{C} = 0.503P \measuredangle 36.2° \blacktriangleleft$$

(*c*)



$$+\circlearrowleft \Sigma M_C = 0: \quad +Pa - (A\cos 30°)2a + (A\sin 30°)a = 0$$

$$A(1.732 + 0.5) = P \quad A = 0.448P$$

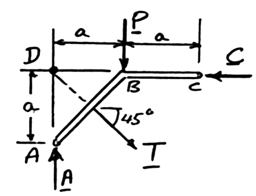
$$A = 0.448P \searrow 60.0° \blacktriangleleft$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad -(0.448P)\sin 30° + C_x = 0 \qquad C_x = 0.224P \longrightarrow$$

$$+\uparrow \Sigma F_y = 0: \quad (0.448P)\cos 30° - P + C_y = 0 \quad C_y = 0.612P \uparrow$$

$$\mathbf{C} = 0.652P \nearrow 69.9° \blacktriangleleft$$

(*d*) Force **T** exerted by wire and reactions **A** and **C** all intersect at Point *D*.

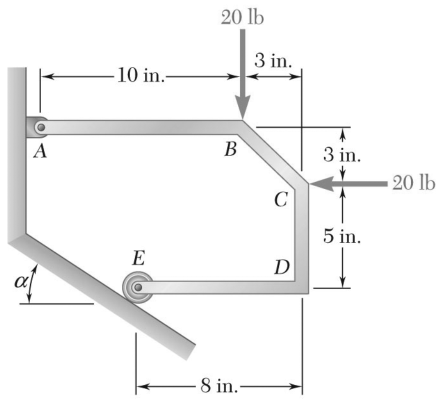


$$+\circlearrowleft \Sigma M_D = 0: \quad P_a = 0$$

Equilibrium is not maintained.

Rod is improperly constrained. ◀

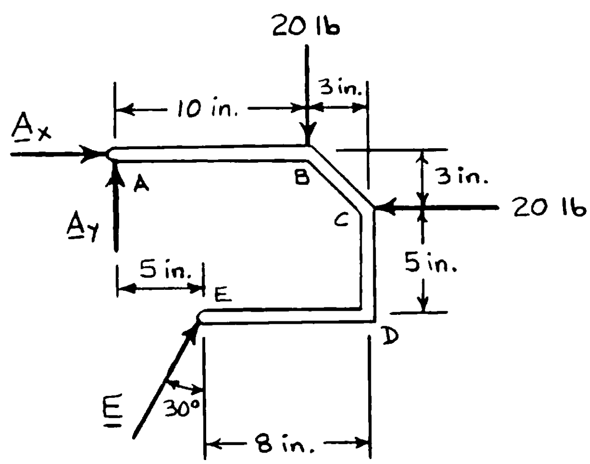
PROBLEM 4.F1

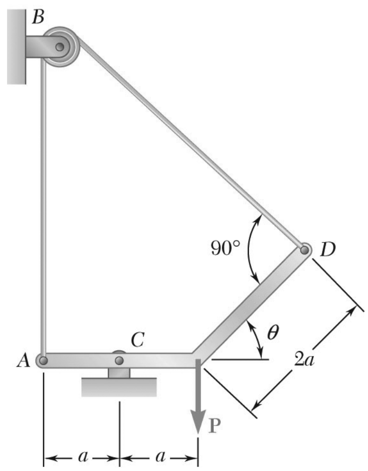


For the frame and loading shown, draw the free-body diagram needed to determine the reactions at A and E when $\alpha = 30°$.

SOLUTION

Free-Body Diagram of Frame:



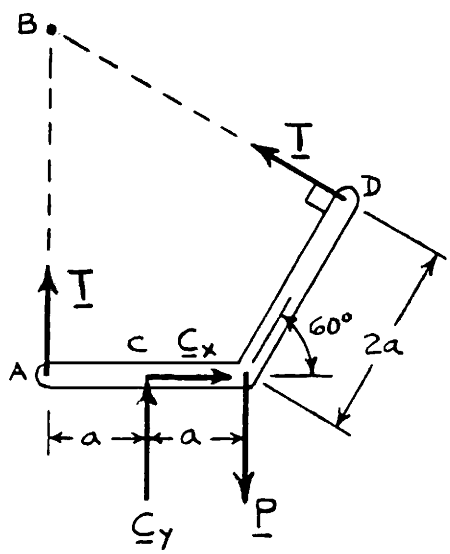


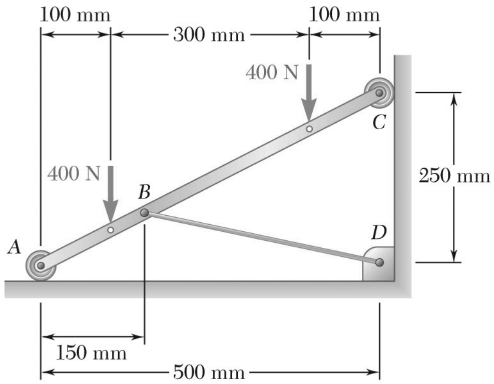
PROBLEM 4.F2

Neglecting friction, draw the free-body diagram needed to determine the tension in cable *ABD* and the reaction at *C* when $\theta = 60°$.

SOLUTION

Free-Body Diagram of Member *ACD*:



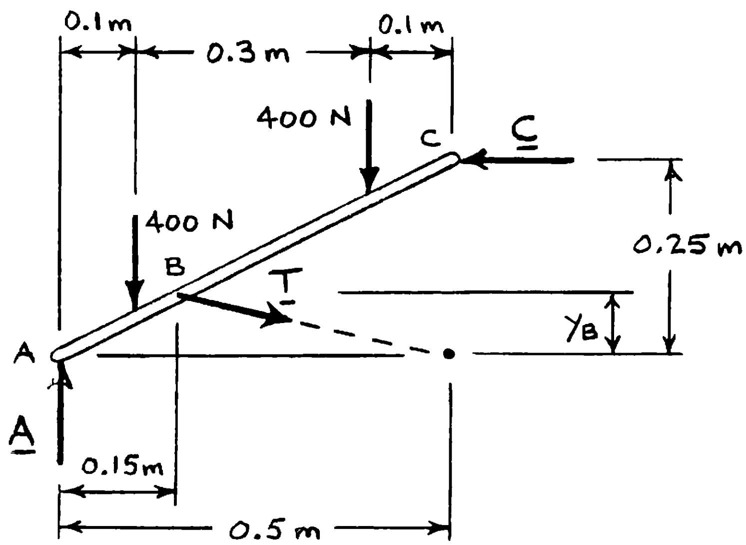


PROBLEM 4.F3

Bar AC supports two 400-N loads as shown. Rollers at A and C rest against frictionless surfaces and a cable BD is attached at B. Draw the free-body diagram needed to determine the tension in cable BD and the reactions at A and C.

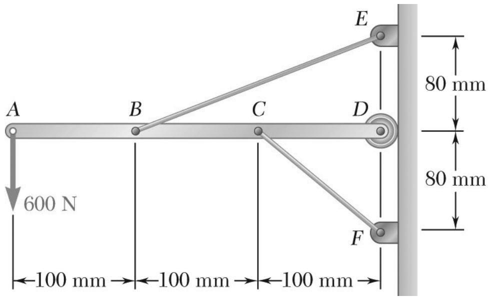
SOLUTION

Free-Body Diagram of Bar *AC*:



Note: By similar triangles

$$\frac{y_B}{0.25\text{ m}} = \frac{0.15\text{ m}}{0.5\text{ m}} \qquad y_B = 0.075\text{ m}$$

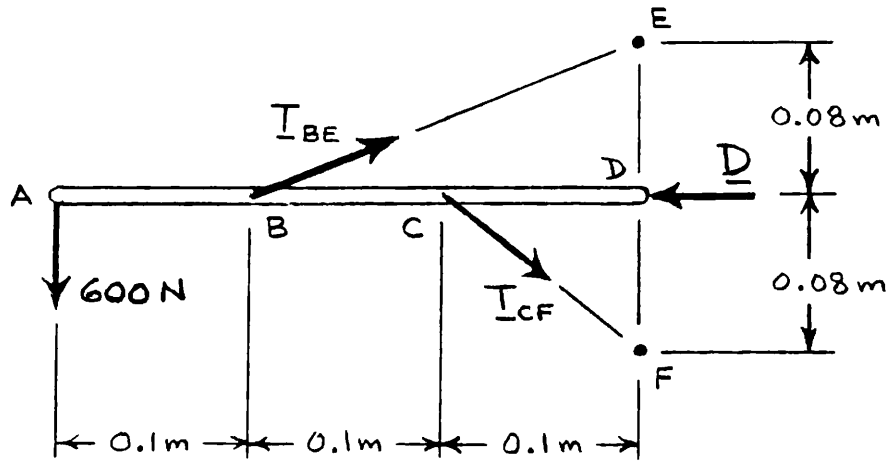


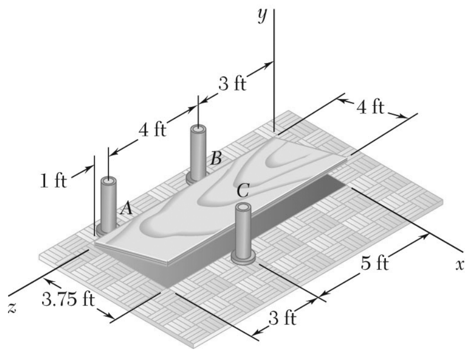
PROBLEM 4.F4

Draw the free-body diagram needed to determine the tension in each cable and the reaction at *D*.

SOLUTION

Free-Body Diagram of Member *ABCD*:



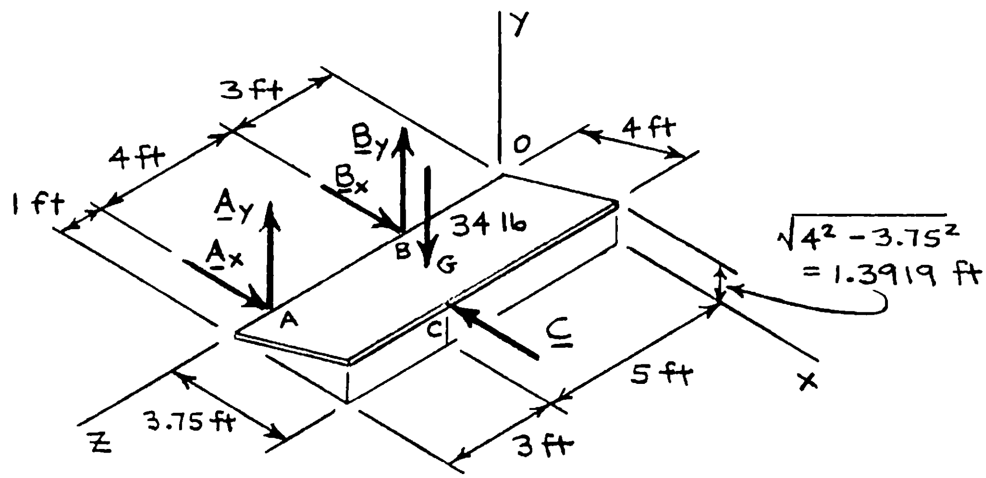


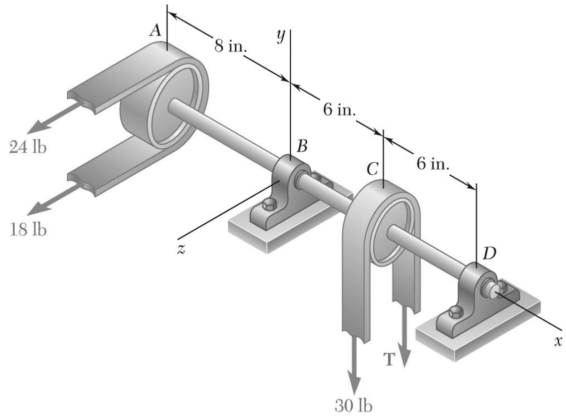
PROBLEM 4.F5

A 4 × 8-ft sheet of plywood weighing 34 lb has been temporarily placed among three pipe supports. The lower edge of the sheet rests on small collars at *A* and *B* and its upper edge leans against pipe *C*. Neglecting friction on all surfaces, draw the free-body diagram needed to determine the reactions at *A*, *B*, and *C*.

SOLUTION

Free-Body Diagram of Plywood sheet:



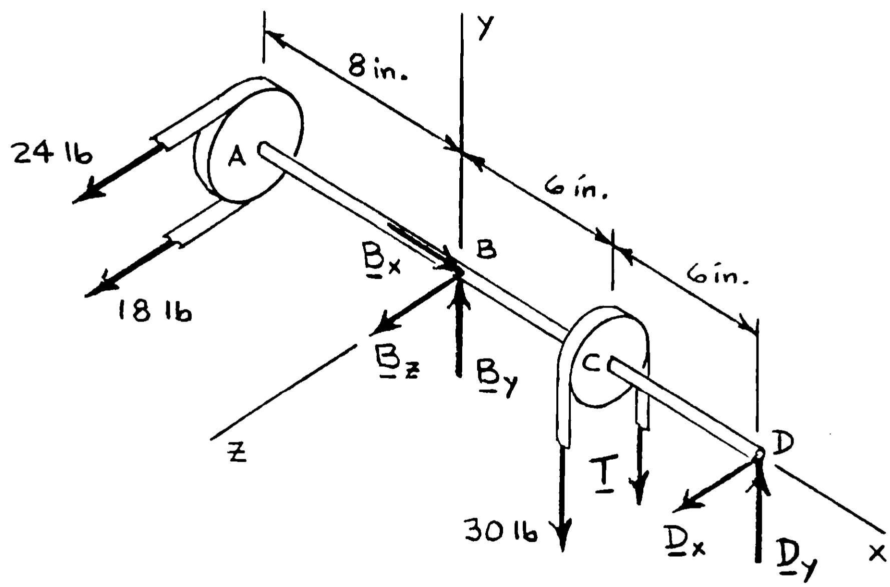


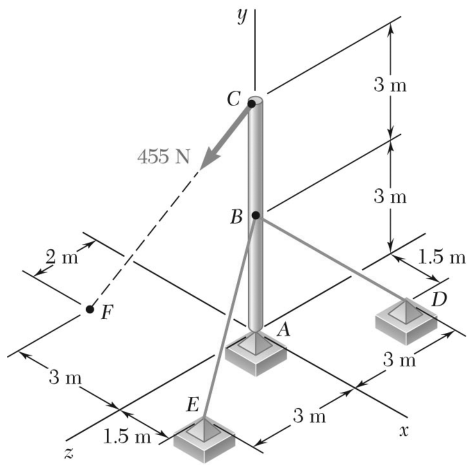
PROBLEM 4.F6

Two transmission belts pass over sheaves welded to an axle supported by bearings at B and D. The sheave at A has a radius of 2.5 in. and the sheave at C has a radius of 2 in. Knowing that the system rotates at a constant rate, draw the free-body diagram needed to determine the tension T and the reactions at B and D. Assume that the bearing at D does not exert any axial thrust and neglect the weights of the sheaves and axle.

SOLUTION

Free-Body Diagram of axle-sheave system:





PROBLEM 4.F7

The 6-m pole *ABC* is acted upon by a 455-N force as shown. The pole is held by a ball-and-socket joint at *A* and by two cables *BD* and *BE*. Draw the free-body diagram needed to determine the tension in each cable and the reaction at *A*.

SOLUTION

Free-Body Diagram of Pole: