

1.1 Solutions of linear Equations and Inequalities in one variable

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Equation is a statement of two algebraic expressions separated by equal (=)

Exp 1) $2x - 3 = 7$ is a linear equation of one variable x

equivalent equations

$$\begin{array}{l} 2x - 3 = 7 \\ +3 \quad +3 \\ \hline 2x = 10 \\ \frac{2x}{2} = \frac{10}{2} \Rightarrow x = 5 \end{array}$$

→ $x = 5$ is the solution because $x = 5$ is the only value makes the equation true or correct

→ we also write the solution set is $\{5\}$

check: $2(5) - 3 \stackrel{?}{=} 7 \Leftrightarrow 7 = 7 \checkmark$

→ x is called variable or unknown

(2) $3y - 12 = 8 - y$ is a linear equation of one variable y

equivalent equations

$$\begin{array}{l} 3y - 12 = 8 - y \\ +12 \quad +12 \\ \hline 3y = 20 - y \\ +y \quad +y \\ \hline 4y = 20 \\ \frac{4y}{4} = \frac{20}{4} \Rightarrow y = 5 \end{array}$$

unknown

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because it makes the equation true

check: $3(5) - 12 \stackrel{?}{=} 8 - (5) \Leftrightarrow 3 = 3 \checkmark$

→ Hence, the solution set is $\{5\}$

Remark Two equations are **equivalent** if they have same solution set.

Exp Equations in Exp 1 and (2) are equivalent

Def The equation is linear if the degree of the variable is one

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Exp solve the following linear equations

(1) $x + 8 = 8(x + 1)$

$$\begin{array}{r} x + 8 \\ -8 \end{array} = \begin{array}{r} 8x + 8 \\ -8 \end{array}$$

$$\Rightarrow \begin{array}{r} x \\ -x \end{array} = \begin{array}{r} 8x \\ -x \end{array} \Rightarrow 0 = 7x$$

$$\Rightarrow \frac{7x}{7} = \frac{0}{7}$$

$$\Rightarrow \boxed{x = 0}$$

check $\boxed{0} + 8 = 8(\boxed{0} + 1)$ ✓
 $8 = 8$

(2) $\frac{3x+1}{2} = \frac{x}{3} - 3$

$$\begin{array}{r} -\frac{x}{3} \\ -\frac{x}{3} \end{array}$$

or multiply both sides by LCD = 6

$$\frac{3}{3} \cdot \frac{3x+1}{2} - \frac{x}{3} \cdot \frac{2}{2} = -3$$

$$\Rightarrow \frac{3(3x+1)}{3 \cdot 2} - \frac{2x}{3 \cdot 2} = -3$$

$$\frac{9x+3}{6} - \frac{2x}{6} = -3$$

$$\Rightarrow \frac{9x+3-2x}{6} = -3$$

$$\frac{7x+3}{6} = -\frac{3}{1}$$

$$\Rightarrow (7x+3) \cdot 1 = -3 \cdot 6$$

$$\begin{array}{r} 7x+3 \\ -3 \end{array} = \begin{array}{r} -18 \\ -3 \end{array}$$

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$$\frac{7x}{7} = \frac{-21}{7}$$

$$\boxed{x = -3} \text{ is the solution}$$

check $\frac{3\boxed{-3}+1}{2} = \frac{\boxed{-3}}{3} - 3$

$$\textcircled{-4} = \frac{-8}{2} = \frac{-9+1}{2} = -1-3 = \textcircled{-4}$$

Def A fraction equation is an equation that contains a variable in a denominator.

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To solve fraction equation we multiply both sides by LCD

Exp (Solving Fractional Equations)

Solve (a) $\frac{3x}{2x+10} = 1 + \frac{1}{x+5}$

Multiply both sides by the LCD = $2x+10$

$$3x = \cancel{(2x+10)}_{-2x} + \frac{2x+10}{x+5}$$

$$x = 10 + \frac{2(x+5)}{x+5}$$

$$x = 10 + 2$$

$$\boxed{x = 12}$$

check $\frac{3 \boxed{12}}{2 \boxed{12} + 10} \stackrel{??}{=} 1 + \frac{1}{\boxed{12} + 5}$

$$\frac{36}{34} \stackrel{?}{=} 1 + \frac{1}{17} \Leftrightarrow \frac{18}{17} = \frac{18}{17} \checkmark$$

(b) $\frac{2x-1}{x-3} = 4 + \frac{5}{x-3}$

Multiply both sides by LCD = $x-3$

$$\begin{aligned} 2x-1 &= 4(x-3) + 5 \\ 2x-1 &= 4x-12+5 \\ -4x &\quad -4x \end{aligned}$$

$$\begin{aligned} -2x-1 &= -7 \\ +1 &\quad +1 \end{aligned}$$

$$\frac{-2x}{-2} = \frac{-6}{-2}$$

$$\boxed{x = 3}$$

check $\frac{2 \boxed{3}-1}{\boxed{3}-3} \stackrel{??}{=} 4 + \frac{5}{\boxed{3}-3}$

$x=3$ gives 0 in the denominator $\Rightarrow$ this equation has no solution

Exp (Solving for one variable)

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① solve $y - 4 = -4(x + 2)$ for y

$$\begin{array}{r} y - 4 = -4x - 8 \\ +4 \quad \quad +4 \end{array}$$

$$y = -4x - 4$$

② solve $9x + \frac{3}{2}y = 11$ for y

$$\begin{array}{r} -9x \quad \quad -9x \end{array}$$

$$\frac{3}{2}y = 11 - 9x \quad \text{multiply by 2}$$

$$3y = 22 - 18x \quad \text{divid by 3}$$

$$y = \frac{22}{3} - 6x$$

Exp (Solving Inequalities)

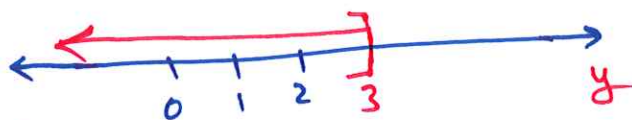
solve ① $3y - 7 \leq 5 - y$

$$\begin{array}{r} 3y - 7 \leq 5 - y \\ +7 \quad +7 \end{array}$$

$$\begin{array}{r} 3y \leq 12 - y \\ +y \quad \quad +y \end{array}$$

$$4y \leq 12 \quad \text{divide by 4}$$

$$y \leq 3$$



$$\text{solution set} = \{y: y \leq 3\}$$

check take $y = 0 \Rightarrow$

$$3(0) - 7 \stackrel{?}{\leq} 5 - (0)$$

$$-7 \leq 5 \quad \checkmark$$

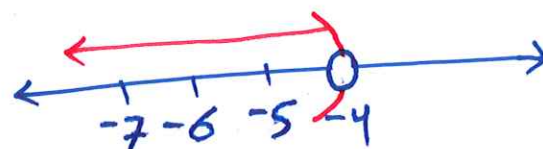
② $1 - 2x > 9$

$$\begin{array}{r} -1 \quad \quad -1 \end{array}$$

$$-2x > 8 \quad \text{divide by -2}$$

$$x < -4$$

$$\text{solution set} = \{x: x < -4\}$$



check take $x = -5$

$$1 - 2(-5) \stackrel{?}{>} 9$$

$$1 + 10 > 9$$

$$11 > 9 \quad \checkmark$$

(3) $-\frac{2x}{5} \leq -30 - x$ multiply by 5

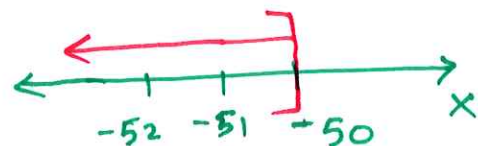
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$$\begin{array}{r} -2x \leq -150 - 5x \\ +5x \qquad \qquad +5x \end{array}$$

$$3x \leq -150 \quad \text{divide by 3}$$

$$x \leq -50$$

$$\text{solution set} = \{x : x \leq -50\}$$



check take $x = -60$

$$\frac{-2(-60)}{5} \stackrel{?}{\leq} -30 - (-60)$$

$$\frac{120}{5} \stackrel{?}{\leq} -30 + 60$$

$$24 \leq 30 \quad \checkmark$$

(4) $\frac{3x}{4} - \frac{1}{6} < x - \frac{2(x-1)}{3}$ multiply by LCD = 12

$$\frac{3x}{4} - \frac{1}{6} < 12x - \frac{2(x-1)}{3}$$

$$9x - 2 < 12x - 8(x-1)$$

$$9x - 2 < 12x - 8x + 8$$

$$9x + 2 < 4x + 10$$

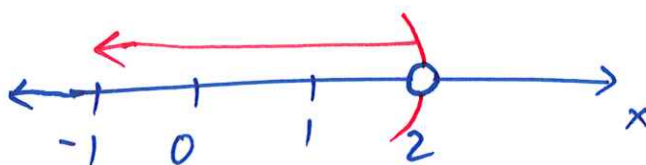
$$\begin{array}{r} 9x < 4x + 10 \\ -4x \qquad -4x \end{array}$$

$$5x < 10$$

divide by 5

$$x < 2$$

$$\text{solution set} = \{x : x < 2\}$$



check

take $x = 0$

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$$\frac{3(0)}{4} - \frac{1}{6} < (0) - \frac{2(0-1)}{3}$$

$$0 - \frac{1}{6} < \frac{2}{3}$$

$$-\frac{1}{6} < \frac{2}{3} \quad \checkmark$$