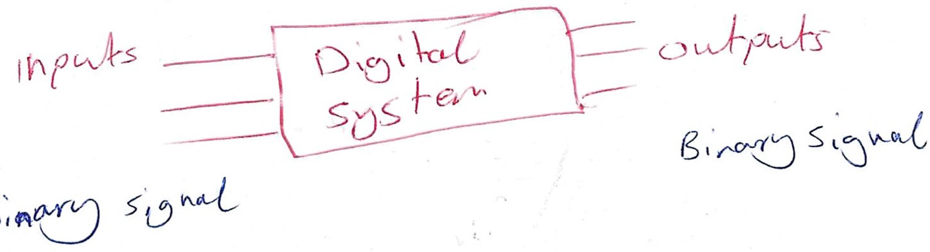
Chapter 2

Boolean Algebra and logic gates



our goal is to design the system with minimum components (logic gates) (minimum cost)

*** Boolean Algebra :-** is a branch of mathematics that deals with elements, operations and axioms with biary Variables.

elements (x, y, z) (0, 1)

operations :- $\overset{\text{and}}{\cdot}$, $\overset{\text{or}}{+}$, not - - -

Axioms or postulates :- $x + 0 = x$, - - -

$x + 0 = x$	$x \cdot 1 = x$
$x + x' = 1$	$x \cdot x' = 0$
$x + x = x$	$x \cdot x = x$
$x + 1 = 1$	$x \cdot 0 = 0$

*** Rules for boolean algebra :-**

① **cluser :-** The oberation $\overset{\text{or}}{+}$, $\overset{\text{and}}{\cdot}$ are closed for all $x, y \in B$

$x + y \in B$

$$\left.\begin{array}{l} 0 + 0 = 0 \\ 0 + 1 = 1 \\ 1 + 0 = 1 \\ 1 + 1 = 1 \end{array}\right\} \in B$$

$x \cdot y \in B$

$$\left.\begin{array}{l} 0 \cdot 0 = 0 \\ 0 \cdot 1 = 0 \\ 1 \cdot 0 = 0 \\ 1 \cdot 1 = 1 \end{array}\right\} \in B$$

② Identity:- 0 is the identity element for +

1 is the identity element for $\cdot$

$\boxed{\text{duality}}$

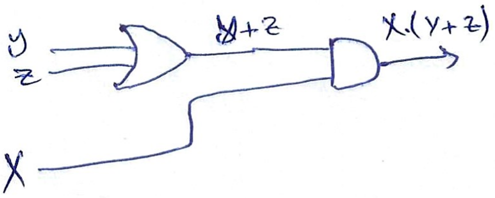
$X + 0 = X$	$X \cdot 1 = X$
$1 + 0 = 1$	$0 \cdot 1 = 0$
$0 + 0 = 0$	$1 \cdot 1 = 1$

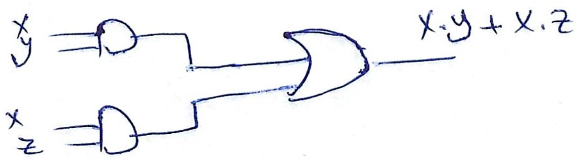
③ Commulative:-

$$X + y = y + X \qquad X \cdot y = y \cdot X$$

④ Distributive:-

$$X \cdot (y + z) = X \cdot y + X \cdot z \qquad X + (y \cdot z) = (x+y)(x+z)$$





X	Y	z
0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1

⑤ Associative :- $X + (y+z) = (x+y)+z$ $\quad$ $x\cdot(yz) = (xy)z$

⑥ The Complement

The complement of x is $\bar{x}, x', x^c$

involution :- $(x')' = x$

The complement of 0 is 1

The complement of 1 is 0

⑦ De Morgan

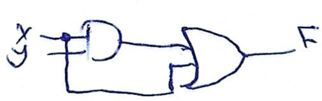
$(x+y)' = x' \cdot y'$ $\quad$ $(x \cdot y)' = x' + y'$

X	y
0	0
0	1
1	0
1	1

⑧ Absorption

$$x + xy = x$$
$$= x(1+y)$$
$$= x(1) = x$$

$x(x+y) = x$

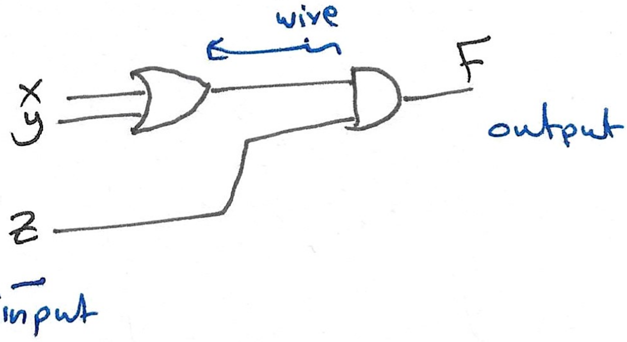


* **Boolean Function:-** is an expressions that consist of Variables, operators and equal sign

Example:- $F(x,y,z) = (x+y) \cdot z$ boolean function

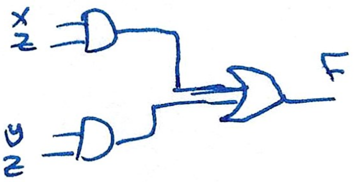
$x, y, z \in B$

Implementation:-



x	y	z
0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1

$F = x \cdot z + y \cdot z$

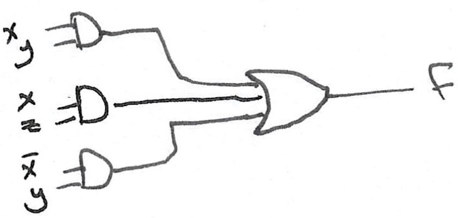


The truth table unique for both expressions

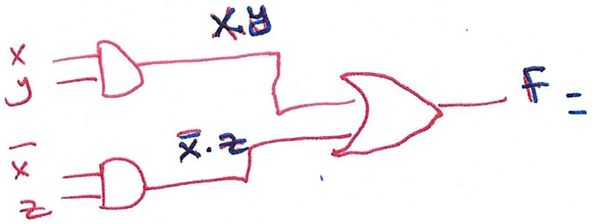
The mathematical expression is not unique

Example:- Implement the following function

$F(x,y,z) = x \cdot y + x \cdot z + x' \cdot y$



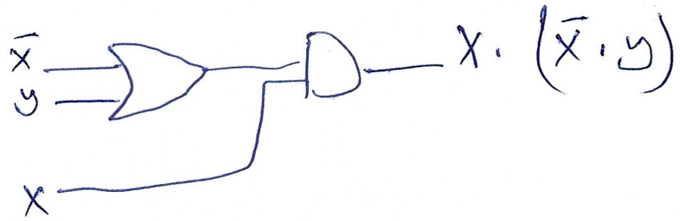
Example :-



$F = x \cdot y + \bar{x} \cdot z$

Example :- Minimize the following function :

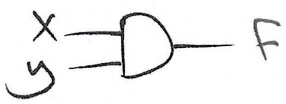
① $F = x \cdot (\bar{x} + y)$



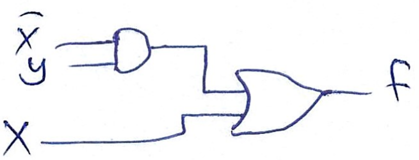
$= x \cdot \bar{x} + x \cdot y$

$= 0 + x \cdot y$

$= x \cdot y$



② $F = x + (\bar{x} \cdot y)$

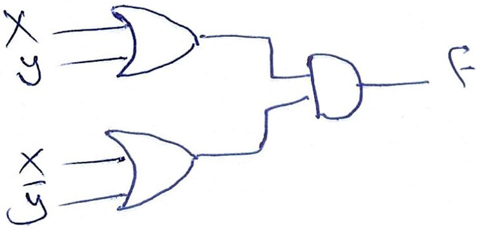


$= (x + \bar{x}) \cdot (x + y)$

$= 1 \cdot (x + y)$

$= x + y$

③ $F = (x+y)\cdot(x+\bar{y})$



$= x\cdot x + x\cdot\bar{y} + x\cdot y + y\bar{y}$

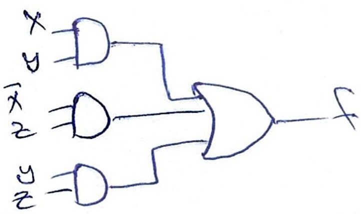
$= x + x\cdot\bar{y} + x\cdot y + 0$

$= x\cdot(1+\bar{y}+y)$

$= x\cdot 1$

$= x$

④ $F = x\cdot y + \bar{x}\cdot z + y\cdot z$



$= x\cdot y + \bar{x}\cdot z + y\cdot z\cdot 1$

$= x\cdot y + \bar{x}\cdot z + y\cdot z\cdot(x+\bar{x})$

$= x\cdot y + \bar{x}\cdot z + y\cdot z\cdot x + y\cdot z\cdot\bar{x}$

$= x\cdot y(1+z) + \bar{x}\cdot z(1+y)$

$= x\cdot y + \bar{x}\cdot z$

* Complement of the function

Example :- $F = X \cdot (y+z)$, find F'

$$F' = [X \cdot (y+z)]'$$

$$= \bar{X} + (y+z)'$$

$$= \bar{X} + \bar{y} \cdot \bar{z}$$

x	y	z
0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1

Example :- Find the complement of $F = X \cdot [\bar{y}\bar{z} + y \cdot z]$

$$F' = [X \cdot [\bar{y}\bar{z} + y \cdot z]]'$$

$$= \bar{x} + [\bar{y}\bar{z} + y \cdot z]'$$

$$= \bar{X} + (\bar{y}\bar{z})' \cdot (y \cdot z)'$$

$$= \bar{X} + (y+z) \cdot (\bar{y} + \bar{z})$$

$$= \bar{X} + [y\bar{y} + z\bar{y} + z\bar{z} + y\bar{z}]$$

$$= \bar{X} + [0 + z\bar{y} + 0 + y\bar{z}]$$

$$= \bar{x} + y\bar{z} + z\bar{y}$$

* Canonical and standard form

① Canonical form:-

Note:- Function of two variables x, y

Combinations:- $xy, \bar{x}y, x\bar{y}, \bar{x}\bar{y}$

n Variables $\Rightarrow$ 2^n combinations, each combination called min terms, or Max terms

each minterm denoted by m_i, $0 \le i \le 2^n - 1$

If I have 3 Variables $\Rightarrow$ $2^3 = 8$ combinations

$m_0, m_1, m_2, m_3, m_4, m_5, m_6, m_7$

The complement of miterm is Max term

$M_0, M_1, M_2, M_3, M_4, M_5, M_6, M_7$

Canonical form can be presented as

① Sum of minterms (SOM)

② Product of max terms (POM)

Examples:- 3 Variables systems:

$2^3 = 8$ combinations

Variables			min term		max term	
x	y	z	term	Designation	term	Designation
0	0	0	$\bar{x}.\bar{y}\,\bar{z}$	m_0	$x+y+z$	M_0
0	0	1	$\bar{x}.\bar{y}.z$	m_1	$x+y+\bar{z}$	M_1
0	1	0	$\bar{x}.y.\bar{z}$	m_2	$x+\bar{y}+z$	M_2
0	1	1	$\bar{x}.y.z$	m_3	$x+\bar{y}+\bar{z}$	M_3
1	0	0	$x.\bar{y}.\bar{z}$	m_4	$\bar{x}+y+z$	M_4
1	0	1	$x.\bar{y}.z$	m_5	$\bar{x}+y+\bar{z}$	M_5
1	1	0	$x.y.\bar{z}$	m_6	$\bar{x}+\bar{y}+z$	M_6
1	1	1	$x,y.z$	m_7	$\bar{x}+\bar{y}+\bar{z}$	M_7

① sum of minterms

Example:- $F(x,y,z) = \bar{x}\bar{y}\bar{z} + \bar{x}\bar{y}z + xyz$

This is a Canonical form, Sum of minterms

$F = m_0 + m_1 + \cancel{m_7}$

$= \Sigma(0,1,7)$

Example:- $F(A,B,C) = \Sigma(0,2,4,6)$

$F = \underset{000}{m_0} + \underset{010}{m_2} + \underset{100}{m_4} + \underset{110}{m_6}$

$= \bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + A\bar{B}\bar{C} + AB\bar{C}$

Example: $F(A,B,C,D) = \Sigma(0,2,10,15)$

$$F = m_0 + m_2 + m_{10} + m_{15}$$

$0000 \quad 0010 \quad 1010 \quad 1111$

$$F = \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}C\bar{D} + A\bar{B}C\bar{D} + ABCD$$

② Product of Maxterms

Example: $F(x,y,z) = (x+y+z)\cdot(x+\bar{y}+z)\cdot(\bar{x}+\bar{y}+\bar{z})$

$000 \quad 010 \quad 111$

$$F = M_0 \cdot M_2 \cdot M_7 = \Pi(0,2,7)$$

Example: $F(A,B,C) = \Pi(0,2,5)$

$$= M_0 \cdot M_2 \cdot M_5$$

$000 \quad 010 \quad 101$

$$= (A+B+C)\cdot(A+\bar{B}+C)\cdot(\bar{A}+B+\bar{C})$$

Example: $F(A,B,C,D) = (A+B+C+D)\cdot(\bar{A}+\bar{B}+C+D)\cdot(\bar{A}+B+\bar{C}+D)$

$0000 \quad 1100 \quad 1010$

$$= M_0 \cdot M_{12} \cdot M_{10}$$

$$= \Pi(0,10,12)$$

Example :- $F(A,B,C) = \Sigma(0,7)$

$$= m_0 + m_7$$

$$\begin{array}{cc} 000 & 111 \end{array}$$

$$= \bar{A}\bar{B}\bar{C} + ABC$$

Truth table:-

A	B	C	F
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

Example:- $F(A,B,C) = (A+B+C) \cdot (\bar{A}+\bar{B}+\bar{C})$

$$\begin{array}{cc} 0\;0\;0 & 1\;1\;1 \end{array}$$

$$F = M_0 \cdot M_7$$

A	B	C	F
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

Example :- $F(A,B,C) = \Sigma(0,2,4)$ write

① Mathematical expression

② Truth table

③ POM

①

$$F = \underset{000}{m_0} + \underset{010}{m_2} + \underset{100}{m_4}$$

$$= \bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + A\bar{B}\bar{C}$$

②

A	B	C	F
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	0

③ $F(A,B,C) = \Pi(1,3,5,6,7)$

$$= \underset{001}{M_1} \cdot \underset{011}{M_3} \cdot \underset{101}{M_5} \cdot \underset{110}{M_6} \cdot \underset{111}{M_7}$$

$$= (A+B+\bar{C}) \cdot (A+\bar{B}+\bar{C}) \cdot (\bar{A}+B+\bar{C}) \cdot (\bar{A}+\bar{B}+C) \cdot (\bar{A}+\bar{B}+\bar{C})$$

Example: $F(A,B,C) = (A+B+C) \cdot (\bar{A}+\bar{B}+C)$

$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad 0 \quad 0 \quad 0 \quad\quad 1 \quad 1 \quad 0$

$= M_0 \cdot M_6$

$= \Pi(0,6)$

Truth table

A	B	C	F
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1

SOM

$F = \Sigma(1,2,3,4,5,7)$

* Complement of the Function

Example:- $F(A,B,C) = \Sigma(0,2,4,6)$

Find the complement as a product of maxterm.

A	B	C	F	$\bar{F}$
0	0	0	1	0 ←
0	0	1	0	1
0	1	0	1	0 ←
0	1	1	0	1
1	0	0	1	0 ←
1	0	1	0	1
1	1	0	1	0 ←
1	1	1	0	1

$\bar{F}(A,B,C) = \Pi(0,2,4,6)$

Example:- $\bar{F}(A,B,C) = \Pi(0,2,5)$ Find F as SOM

A	B	C	$\bar{F}$	F
0	0	0	0	1 ←
0	0	1	1	0
0	1	0	0	1 ←
0	1	1	1	0
1	0	0	1	0
1	0	1	0	1 ←
1	1	0	1	0
1	1	1	1	0

$F = \Sigma(0,2,5)$

$F \cdot \bar{F} = 0$

Example: write the following function as sum of minterms

$$F(A,B,C) = AB + AB\bar{C}$$

$$F(A,B,C) = AB(C+\bar{C}) + AB\bar{C}$$

$$= ABC + AB\bar{C} + AB\bar{C}$$

$$X + X = X$$

$$F(A,B,C) = ABC + AB\bar{C}$$

Example: write the following function as sum of minterms

$$F(A,B,C) = A + \bar{A}C$$

A	B	C	F
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

$$F = m_1 + m_3 + m_4 + m_5 + m_6 + m_7$$

$$= \Sigma(1,3,4,5,6,7)$$

or

$$F = A(B+\bar{B}) + \bar{A}C(B+\bar{B})$$

$$= AB + A\bar{B} + \bar{A}CB + \bar{A}C\bar{B}$$

$$= AB(C+\bar{C}) + A\bar{B}(C+\bar{C}) + \bar{A}BC + \bar{A}\bar{B}C$$

$$= ABC + AB\bar{C} + A\bar{B}C + A\bar{B}\bar{C} + \bar{A}BC + \bar{A}\bar{B}C$$

$$m_{111} + m_{110} + m_{101} + m_{100} + m_{011} + m_{001}$$

$$= m_1 + m_3 + m_4 + m_5 + m_6 + m_7$$

Example: write the following function as product of maxterms

$F(x,y,z) = xy + \bar{x}z$

x	y	z	F
0	0	0	0 ←
0	0	1	1
0	1	0	0 ←
0	1	1	1
1	0	0	0 ←
1	0	1	0 ←
1	1	0	1
1	1	1	1

$$F = \Pi(0,2,4,5)$$

$$= (x+y+z)\cdot(x+\bar{y}+z)\cdot(\bar{x}+y+z)\cdot(\bar{x}+y+\bar{z})$$

or

$$F = xy + \bar{x}z$$

$$= (xy+\bar{x})\cdot(xy+z)$$

$$= (\cancel{x+\bar{x}})^{1}\cdot(y+\bar{x})\cdot(x+z)\cdot(y+z$$

$$= (\bar{x}+y)\cdot(x+z)\cdot(y+z)$$

$$= (\bar{x}+y+z\cdot\bar{z})\cdot(x+z+y\cdot\bar{y})\cdot(y+z+x\cdot\bar{x}$$

$$= (\bar{x}+y+z)\cdot(\bar{x}+y+\bar{z})\cdot(x+y+z)\cdot$$

$$(x+\bar{y}+z)\cdot(x+y+z)+(\bar{x}+y+z)$$

$$= M_{100}\cdot M_{101}\cdot M_{000}\cdot M_{010}\cdot M_{000}\cdot M_{100}$$

$$= M_4\cdot M_5\cdot M_0\cdot M_2$$

② standard form

① Sum of products

② product of sums

Example: The Function

$F_1 = x \cdot y \cdot z + x \cdot y \cdot \bar{z}$ is sum of minterms

but

$F_2 = \underbrace{x \cdot y}_{\text{term but not minterm}} + x y \bar{z}$ is not sum of minterms

but we can call F_2 as sum of products

Example: $F_1 = \bar{x} \cdot y \cdot z + x \cdot y \cdot z$

F_1 is sum of minterms

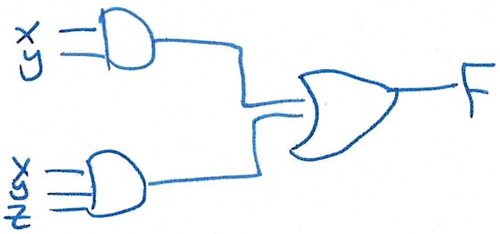
and sum of products

$F_2 = \bar{x} \cdot y + x \cdot y \cdot z$

F_2 is sum of products

Example:- $F = xy + xyz$

F is sum of products



AND

Level 1

OR

Level 2

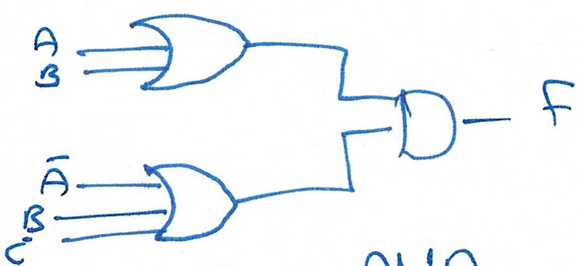
Example:- $F_1 = (A+B+C) \cdot (\bar{A}+B+\bar{C})$

F_1 is Product of maxterms

and product of sums

$F_2(A,B,C) = (A+B) \cdot (\bar{A}+B+C)$

F_2 is product of sums



OR

AND

Example: Find the product of sums of the following expression:

$$F = \bar{A}\bar{B}C + A\bar{B}\bar{C} + A\bar{B}C + ABC$$

To find the product of sums from sum of minterms

$$F(POS) = \bar{\bar{F}}(SOM)$$

① Find the Complement of sum of minterms

② simplify the sum of minterms to sum of product

③ Find the complement of the sum of product

①

$$F = \underset{001}{\bar{A}\bar{B}C} + \underset{100}{A\bar{B}\bar{C}} + \underset{101}{A\bar{B}C} + \underset{111}{ABC}$$

$$= m_1 + m_4 + m_5 + m_7$$

$$\therefore \bar{F} = \underset{000}{m_0} + \underset{010}{m_2} + \underset{011}{m_3} + \underset{110}{m_6}$$

$$= \bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + \bar{A}BC + AB\bar{C}$$

② Simplify $\bar{F}$ to sum of product

$$\bar{F} = \underbrace{\bar{A}\bar{B}\bar{C} + \overbrace{\bar{A}B\bar{C}}_{\bar{A}\bar{C}(\bar{B}+B)}^{B\bar{C}(A+\bar{A})} + \underbrace{\bar{A}BC}_{\bar{A}B(C+\bar{C})}} + AB\bar{C}$$

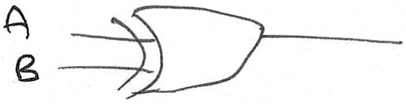
$$= \bar{A}\bar{C} + \bar{A}B + B\bar{C}$$

③ $F = \bar{\bar{F}} = \overline{(\bar{A}\bar{C} + \bar{A}B + B\bar{C})} = (A+C)(A+\bar{B})(\bar{B}+C)$

Note:-

XOR gate

odd

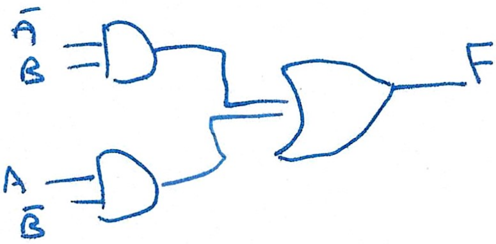


$F = A \oplus B$

A	B	F
0	0	0
0	1	1
1	0	1
1	1	0

$F = m_1 + m_2$

$= \bar{A}B + A\bar{B}$



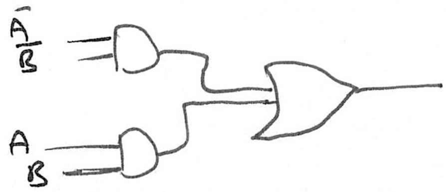
XNOR

$F = (A \oplus B)'$

A	B	F
0	0	1
0	1	0
1	0	0
1	1	1

$F = m_0 + m_3$

$= \bar{A}\bar{B} + AB$



Examples:- If $F = A \oplus B$, what is $\bar{F}$?

$F = \bar{A}B + A\bar{B}$

$\bar{F} = (\bar{A}B + A\bar{B})'$

$= \overline{(\bar{A}B)} \cdot \overline{(A\bar{B})}$

$= (A + \bar{B}) \cdot (\bar{A} + B)$

$= A\bar{A} + AB + \bar{A}\bar{B} + B\bar{B}$ $\quad = AB + \bar{A}\bar{B}$ $\quad$ XNOR

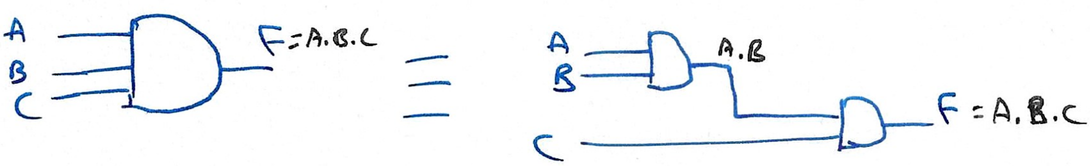
Notes:-

All gates are associative except NAND / NOR

AND

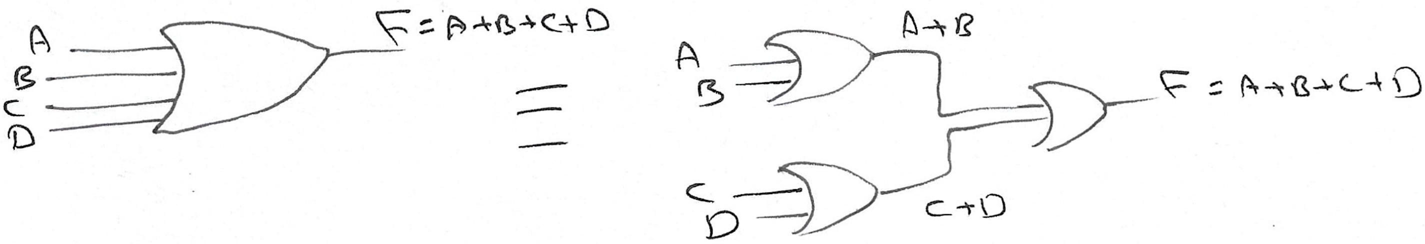
$F = A \cdot B \cdot C$

$F = (A \cdot B) \cdot C = A \cdot (B \cdot C)$

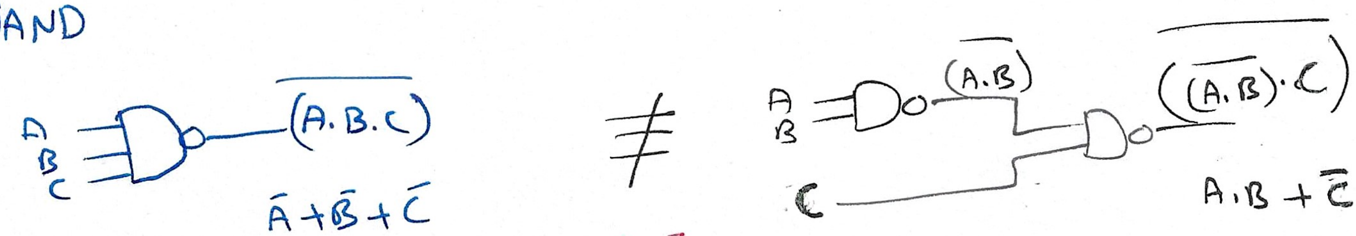


OR

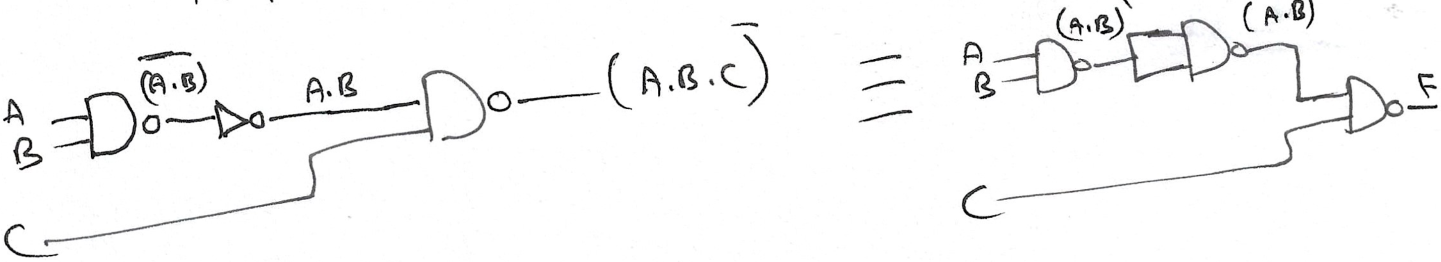
$F = (A+B)+C = A+(B+C)$



NAND



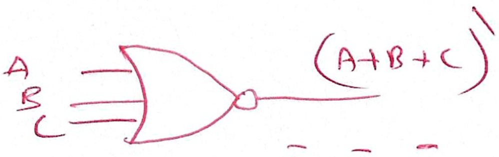
$\equiv$



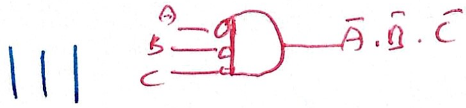
$F = (A.A)'$

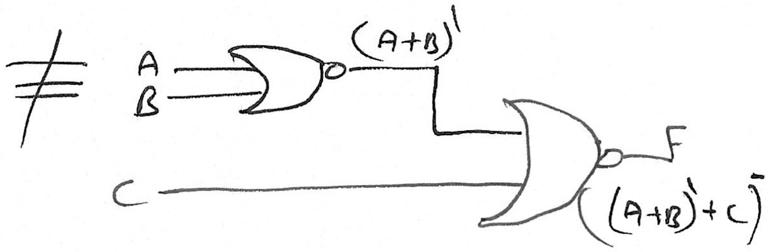
$= \bar{A}$

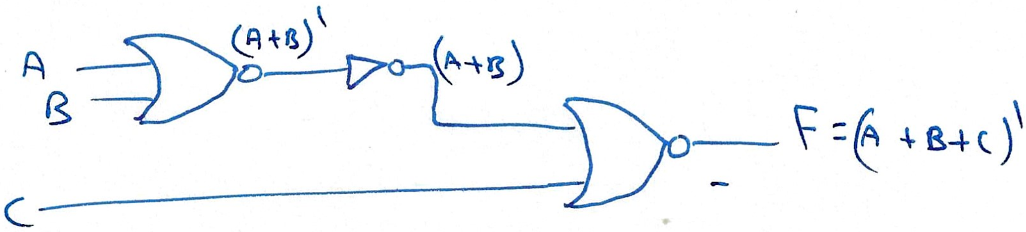
NOR

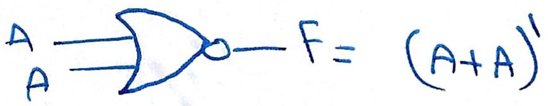


$\bar{A} \cdot \bar{B} \cdot \bar{C}$

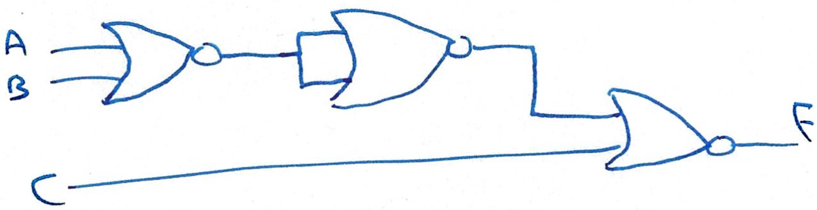




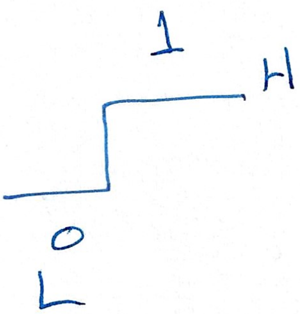


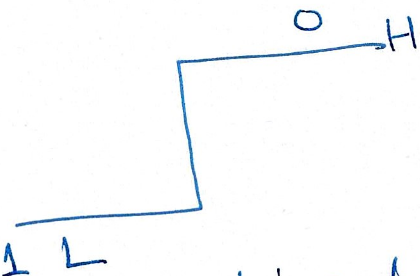


$= \bar{A}$



* positive and Nigative logic





positive logic

AND

A	B	F
L 0	L 0	0 L
L 0	H 1	0 L
H 1	L 0	0 L
H 1	H 1	1 H

negative logic

A	B	F
L 1	L 1	L 1
L 1	H 0	1 L
H 0	L 1	1 L
H 0	H 0	0 H