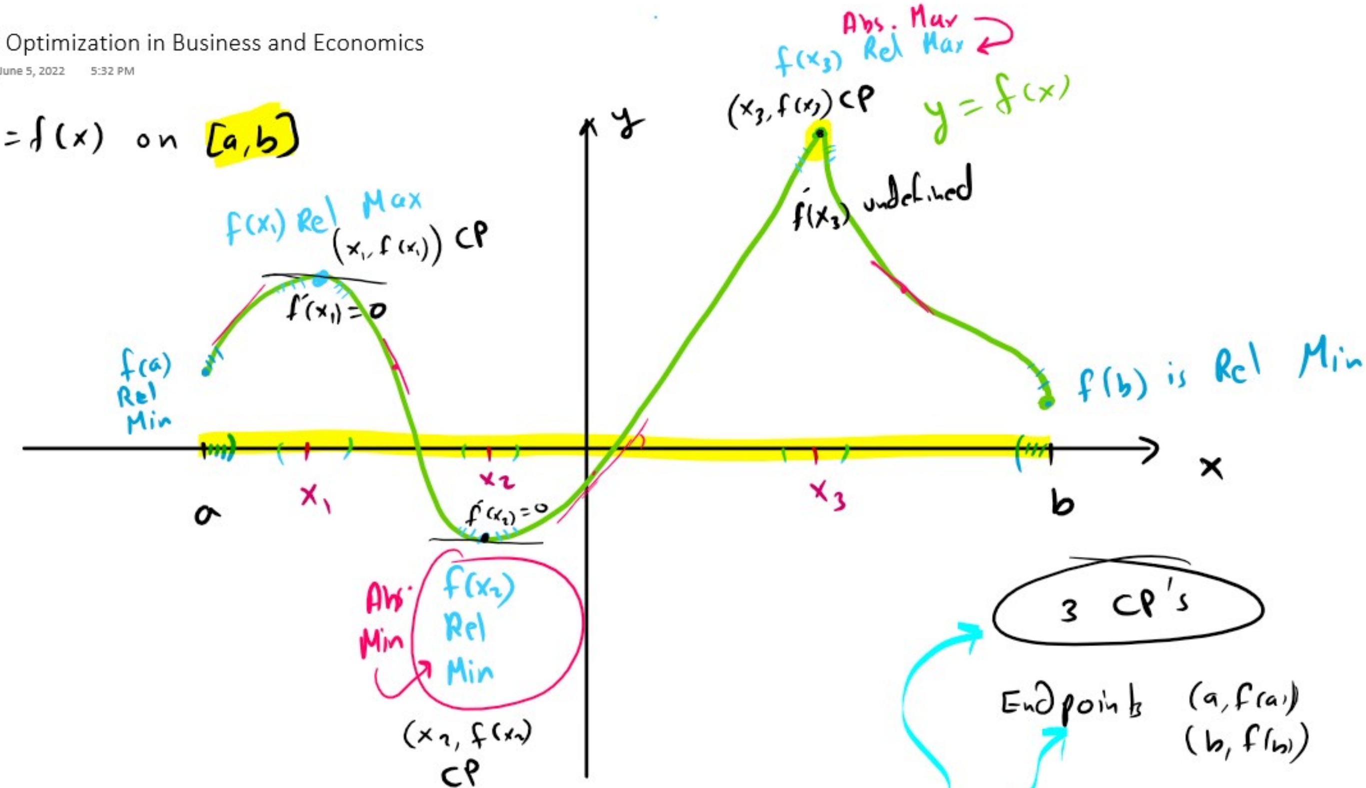
$y = f(x)$ on $\boxed{[a,b]}$



Extrem Values

Max → Abs. → Local (Rel)

Min → Abs → Local Rel

To find Extrem Values (Rel Max, Rel Min, Abs. Max, Abs. Min)

We need to check $\boxed{1}$ Critical Points

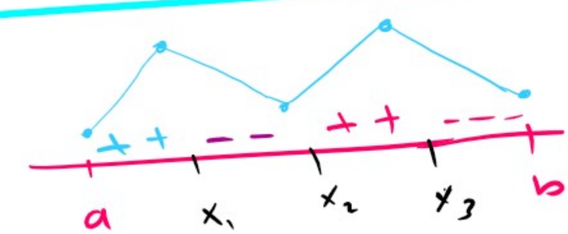
We need to check (1) Critical Points

$f'(x)=0$ $\quad$ f' undefined

(2) End points

$f(a)$, $f(b)$

1^{st} Derivative Test



2^{nd} Derivative Test

$\underline{\underline{CP's}}$: occurs at $\boxed{x_1, x_2, x_3}$ Critical Values

Find $f''(x) \Rightarrow f''(x_1) < 0 \Rightarrow f(x_1)$ is Rel Max

$f''(x_2) > 0 \Rightarrow f(x_2)$ is Rel Min

$f''(x_3)$ undefined $\Rightarrow$ Test fail

use 1^{st} Derivative Test

$\underline{\underline{Exp}}$ (Revenue)

Total Revenue $\quad R(x) = 8000x - 40x^2 - x^3$

where x is the number of units sold per day.

where x is the number of units sold per day.

If only 50 units can be sold per day.

(1) Find the number of units that must be sold to maximize revenue.

$$0 \le x \le 50$$

$$R'(x) = 8000 - 80x - 3x^2 = 0$$

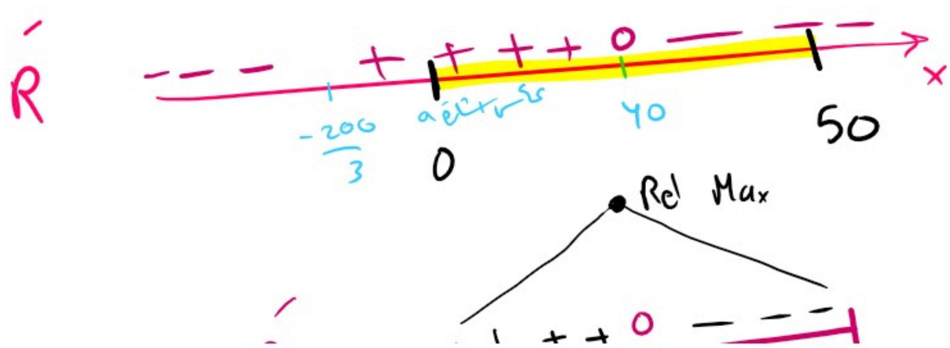
$$a = -3, \quad b = -80, \quad c = 8000$$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-80) \pm \sqrt{(-80)^2 - 4(-3)(8000)}}{2(-3)}$$

$$x_1 = 40, \quad \cancel{x_2 = -\frac{200}{3}}$$

مرفوضة



R' + + + + 0 − − − −

0   40   50

↑ gives Max Revenue

Check ① CP

② End points

② Find Max Revenue

$R(x) = 8000x - 40x^2 - x^3$

The revenue is max at $x = 40$ → # of units must be sold to maximize revenue

① $R(40) = 8000(40) - 40(40)^2 - (40)^3$

$= 320000 - 40(1600) - (32,000)$

$= \boxed{\$\,192,000}$

② $R(0) = 0 - 0 - 0 = 0$

$R(50) = 8000\boxed{50} - 40\boxed{50}^2 - \boxed{50}^3 = \$\,175,000$

The Max Revenue is \$ 192,000 occurs when # of units is $x = 40$

We can also use 2^{nd} derivative Test

$R'(x) = 8000 - 80x - 3x^2$ with critical values

$x_1 = 40$ ✓

$x_2 = -\frac{200}{3}$ ✗

$R''(x) = -80 - 6x$

$R''(x) = -80 - 6x$ $\quad x_2 = -\frac{200}{3}$ X

$R''(x_1) = R''(40) = -80 - 6\boxed{40}$

$= -80 - 240 < 0$

$R(40)$ is Max $\Rightarrow R(40) = \$\ 192,000$

Def Total Cost $C(x)$

Per unit average cost is $\bar{C} = \frac{C(x)}{x}$

$C(x) = x\bar{C}$

Exp $C(50) = \$\ 1000 \Rightarrow \bar{C} = \frac{C(50)}{50} = \frac{1000}{50} = 20$

$\downarrow$

$x = 50$

Remark: Average cost per unit is undefined if no units are produced → $x = 0$

Exp (Average Cost)

Total cost for commodity $C(x) = \frac{x^2}{4} + 4x + 100$

where x # of units produced

where x # of units produced

① How many units must be produced to **minimize** the average cost

av. cost $\bar{C} = \dfrac{C(x)}{x} = \dfrac{\frac{x^2}{4} + 4x + 100}{x}$

$$= \frac{x}{4} + 4 + \frac{100}{x}$$

$$\boxed{\bar{C} = \frac{x}{4} + 4 + 100x^{-1}}$$

$$(\bar{C})' = \frac{1}{4} + 0 + 100(-1)x^{-2} = 0$$

$$(\bar{C})' = \frac{1}{4} - \frac{100}{x^2} = 0 \Rightarrow \frac{1}{4} = \frac{100}{x^2}$$

$$x^2 = 400$$

$$x = \pm 20$$

$x = 20$, $\cancel{x = -20}$

$(\bar{C})'$ — — — 0 + + +

20

$$(\bar{C})'(10) = \frac{1}{4} - \frac{100}{100} = \frac{1}{4} - 1 < 0$$

$$(\bar{C})'(30) = \frac{1}{4} - \frac{100}{900} = \frac{1}{4} - \frac{1}{9} > 0$$

$(\bar{C})'(30) = \frac{1}{4} - \frac{100}{900} = \frac{1}{4} - \frac{1}{9} > 0$

we must produce $x = 20$ units to minimize the av. cost

② Find the minimum av. cost

Av. cost $\bar{C}(x) = \frac{x}{4} + 4 + \frac{100}{x}$

$\bar{C}(20) = \frac{20}{4} + 4 + \frac{10\cancel{0}}{2\cancel{0}}$

$= 5 + 4 + 5$

$= \$\ 14$

\$ 14 is min av. cost per unit if we produce $x = 20$ units.

Exp (Profit) Total Profit for a commodity

$P(x) = 4x^3 - 210x^2 + 3600x - 200$

where x can't exceed 30 units sold. $\boxed{0 \le x \le 30}$

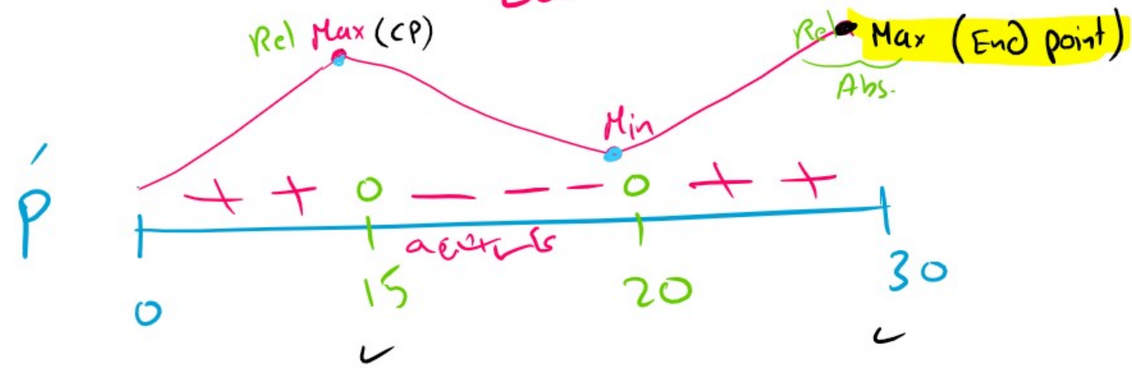
① Find # of units x that maximize the $P(x)$

$P'(x) = 12x^2 - 420x + 3600 = 0$ Find critical values

$P'(x) = 12x^2 - 420x + 3600 = 0$

$a = 12, \; b = -420, \; c = 3600$

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow x_1 = 15, \quad x_2 = 20$$



$P(15) = 4(15)^3 - 210(15)^2 + 3600(15) - 200 = \$\ 20{,}050$

$P(30) = 4(30)^3 - 210(30)^2 + 3600(30) - 200 = \$\ 26{,}800$

$x = 30$ units maximizes the profit.

② Find Max profit.

Max profit is $P(30) = \$\ 26{,}800$

We can use 2nd Derivative Test

$P'(x) = 12x^2 - 420x + 3600$, $\quad x_1 = 15, \; x_2 = 20$ critical values

$P''(x) = 24x - 420$

$$P''(x_1) = P''(15) = 24\boxed{15} - 420 = 360 - 420 < 0$$

$\Rightarrow P(15)$ is Rel Max

$$P''(x_2) = P''(20) = 24(20) - 420 = 480 - 420 > 0$$

$\Rightarrow P(20)$ is Rel Min

Exp • Demand for commodity x is given by

$$p = 168 - 0.2x$$

p is price of this commodity

• Average cost for this commodity is

$$\bar{C} = \frac{C(x)}{x}$$

$$\bar{C} = 120 + x$$

$$C(x) = x\,\bar{C}$$

① How many units must be sold to maximize profit.

Total Profit = Total Revenue − Total Cost

$$= R(x) - C(x)$$

$$= px - x\bar{C}$$

$$= (168 - 0.2x)x - x(120 + x)$$

$$= 168x - 0.2x^2 - 120x - x^2$$

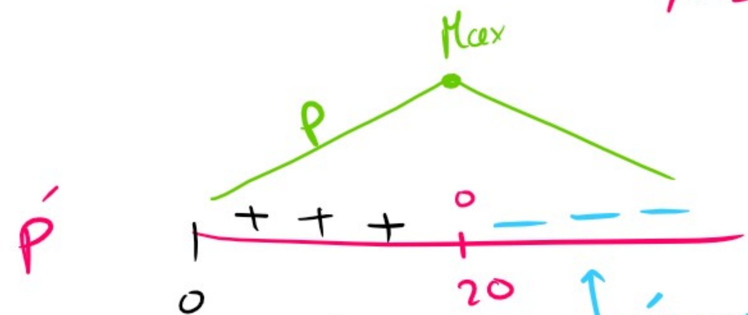
$= 168x - 0.2x^2 - 120x - x^2$

$$\boxed{P(x) = 48x - 1.2x^2}$$

$P'(x) = 48 - 2.4x = 0$

$\frac{48}{2.4} = \frac{2.4x}{2.4}$

$x = \frac{48}{2.4} = 20$



$P'(100) = 48 - 2.4(100) = 48 - 240$ negative

$P'(10) = 48 - 2.4\boxed{10} = 48 - 24 > 0$

we must produce $x = 20$ to sell and max profit.

② Find Max Profit.

$P(x) = 48x - 1.2x^2$

$P(20) = 48\boxed{20} - 1.2\boxed{20}^2$

$= \$480$

\$480 is the Max possible profit occurs when $x = 20$ units are produced and sold.

$x = 20$ units are produced and sold.

③ Find the optimal selling price.

$$P = 168 - 0.2x$$

$$P(20) = 168 - 0.2\,(\cancel{20})$$

$$= 168 - 4$$

$$= \$164$$ optimal selling price