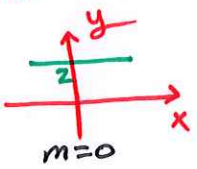
2.4 Special Functions and Their Graphs

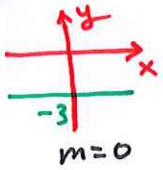
- Linear functions $f(x) = ax + b$ can be either

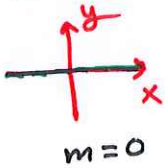
① Horizontal line when $a = 0$

slope $= m = 0$

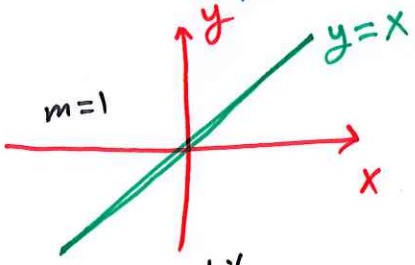
Exp $y = 2$, $y = -3$, $y = 0$, ... constant functions



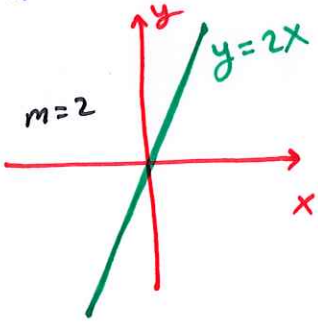


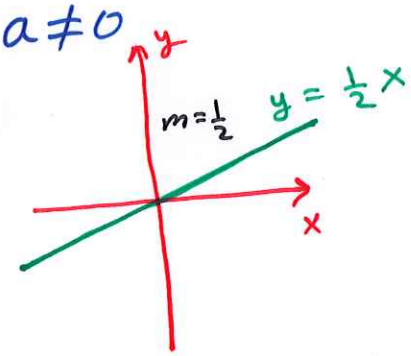


or ② Non Horizontal line when $a \neq 0$

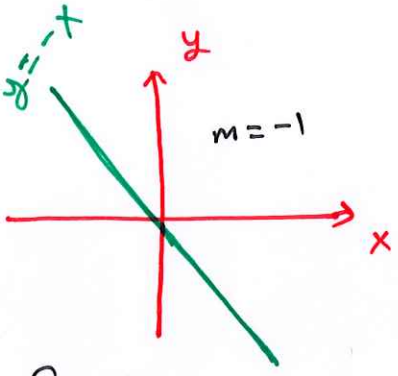


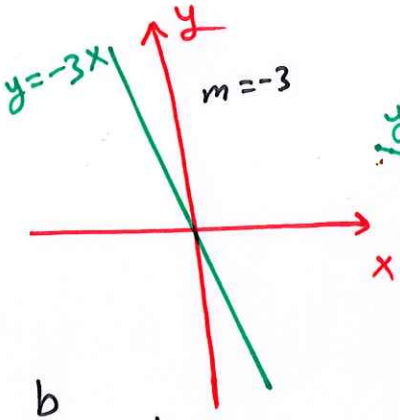
identity function since $b = 1$

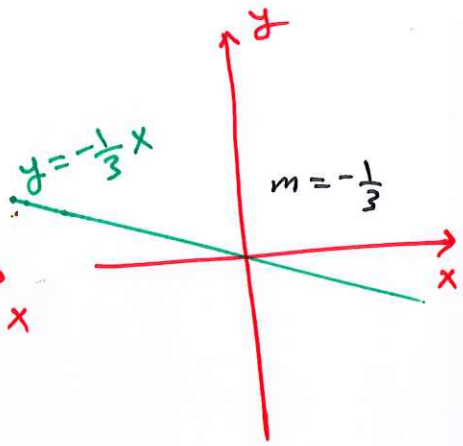




يمكن عمل جدول بسيط للتحقق







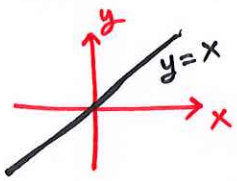
- Power functions $y = a x^b$, $b > 0$ can be either

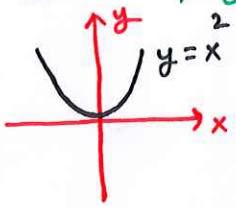
① linear functions if $b = 1$ → H. Line if $a = 0$

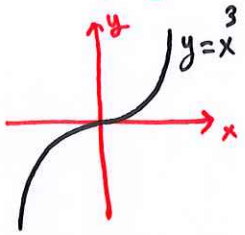
→ Non H. Line if $a \neq 0$

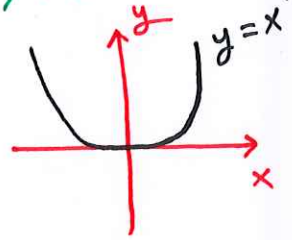
② Polynomial functions if b is positive integer

Exp $y = x$, $y = x^2$, $y = x^3$, $y = x^4$, ...





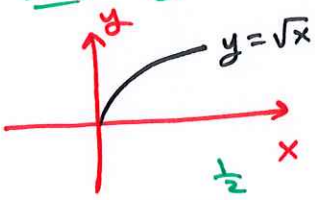


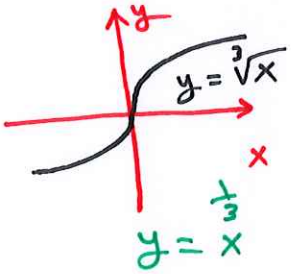


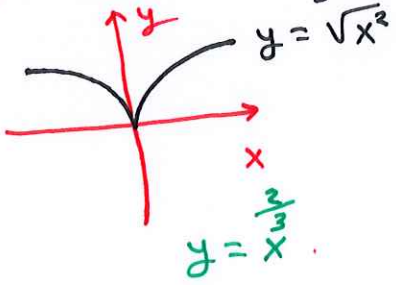
عمل جدول للتحقق

③ Root functions if $0<b<1$

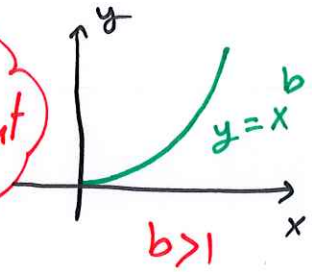
Exp $y=\sqrt{x}$, $y=\sqrt[3]{x}$, $y=\sqrt[3]{x^2}$, ...

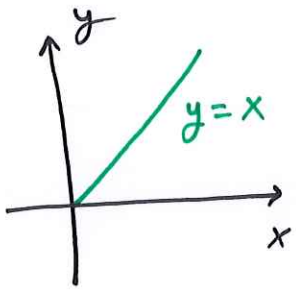


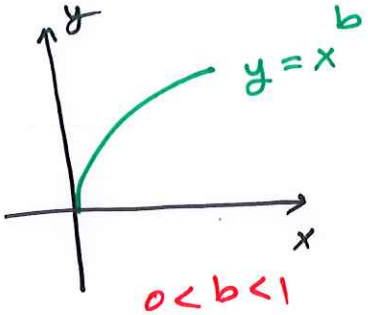




$y=x^b$ in 1st quadrant







Shifted Graph of $y=f(x)$ when $a>0$ and $b>0$

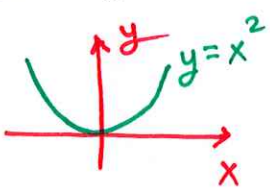
→ $y=f(x-a)$ shift of $f(x)$ by a units to the Right

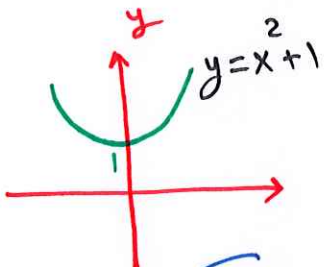
→ $y=f(x+a)$ shifts $f(x)$ by a units to the left

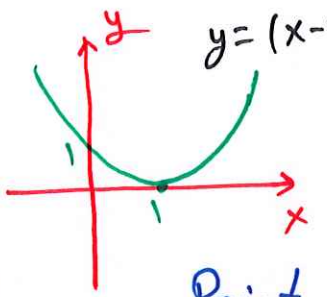
→ $y=f(x)+b$ shifts $f(x)$ by b units up

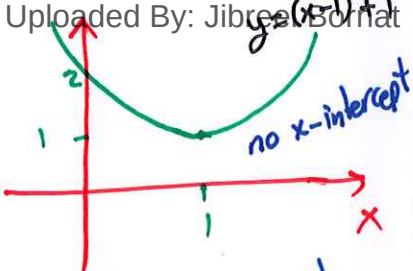
→ $y=f(x)-b$ shifts $f(x)$ by b units down

Exp $y=x^2$

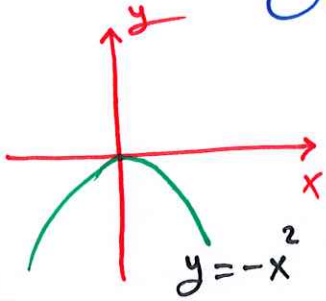


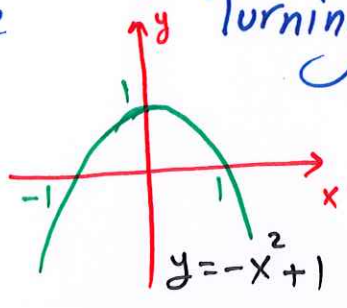


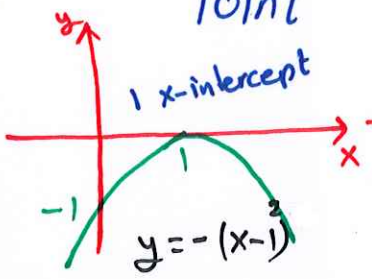


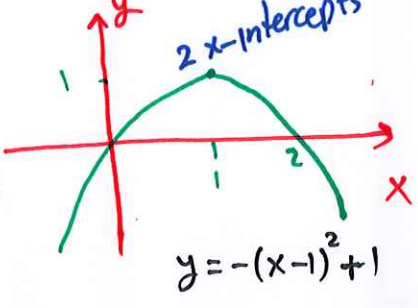


One Turning Point

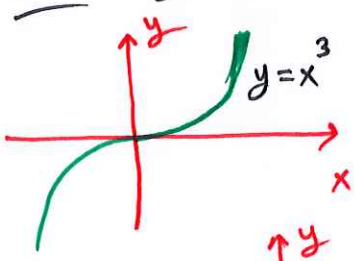




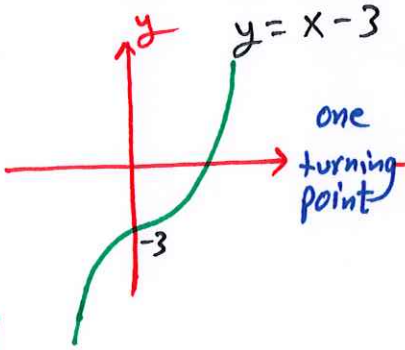




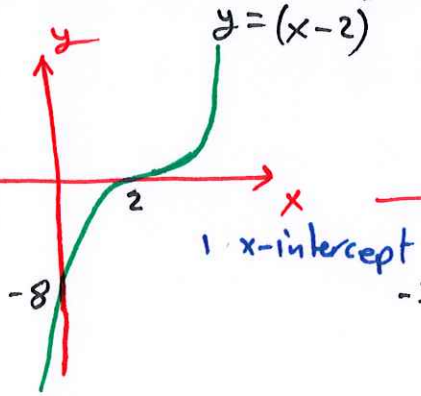
Exp $y = x^3$

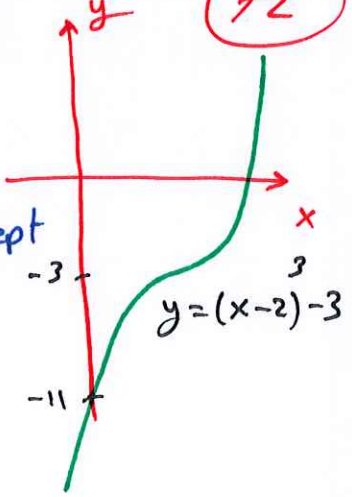


$y = x^3 - 3$

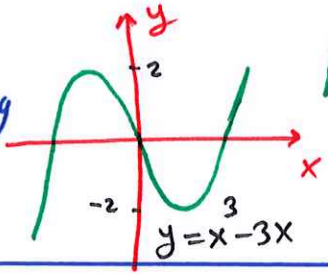


$y = (x-2)^3$



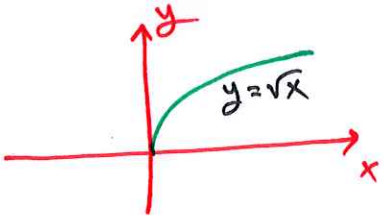


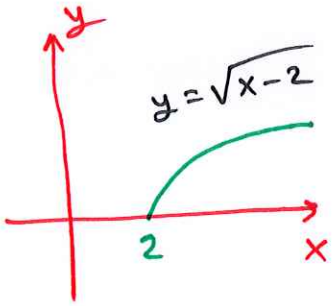
2 turning points

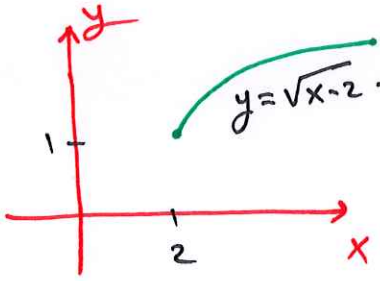


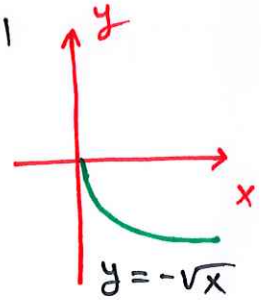
3 x-intercept

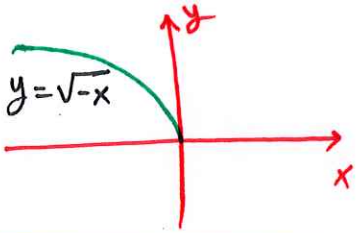
Exp $y = \sqrt{x}$

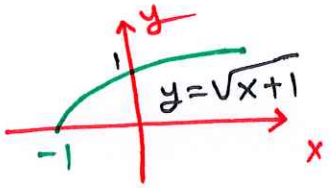


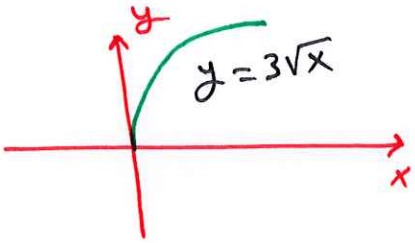


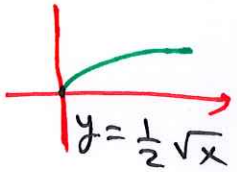












Polynomial function of degree n has the form

$$y = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \cdots + a_1 x^1 + a_0 \quad , \quad a_n \neq 0$$

$n = 0, 1, 2, 3, \cdots$

Exp

- The linear function $y = ax + b$ is poly. of degree 1
- The quadratic function $y = ax^2 + bx + c$ is poly. of degree 2
- $y = 3x^3 - 2x$ is poly. of degree 3
- $y = 5x^{10}$ is poly. of degree 10
- Any poly. of degree 2 has only one turning point

Exp which of the following are poly.?

① $f(x) = x^4 - x^2 + 2x$ Yes Poly. of degree 4

② $g(x) = \frac{x^2-1}{x^3} = x^{-1} - x^{-3}$ No since n is negative

③ $h(x) = 3 - \sqrt{x}$ No since $n = \frac{1}{2}$

④ $r(x) = 5x - \frac{3}{2}$ Yes Poly. of degree 1

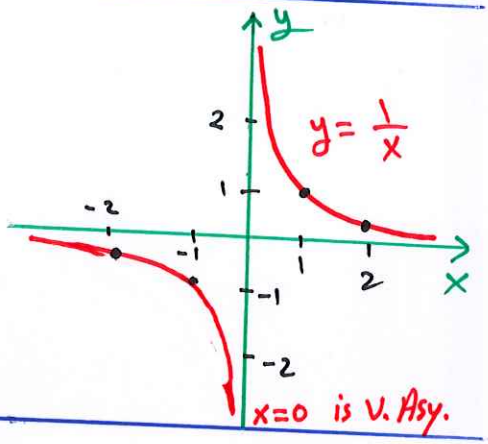
⑤ $g(x) = -\sqrt{7}$ Yes Poly. of degree 0 "constant function"

Rational function is $y = \frac{f(x)}{g(x)}$, $g(x) \neq 0$, f and g are poly.

Exp Reciprocal function $f(x) = \frac{1}{x}$

x	-2	-1	1	2	3
y	$-\frac{1}{2}$	-1	1	$\frac{1}{2}$	$\frac{1}{3}$

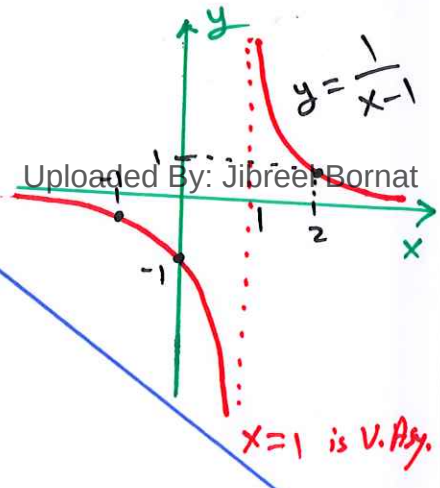
$D = \mathbb{R} \setminus \{0\}$



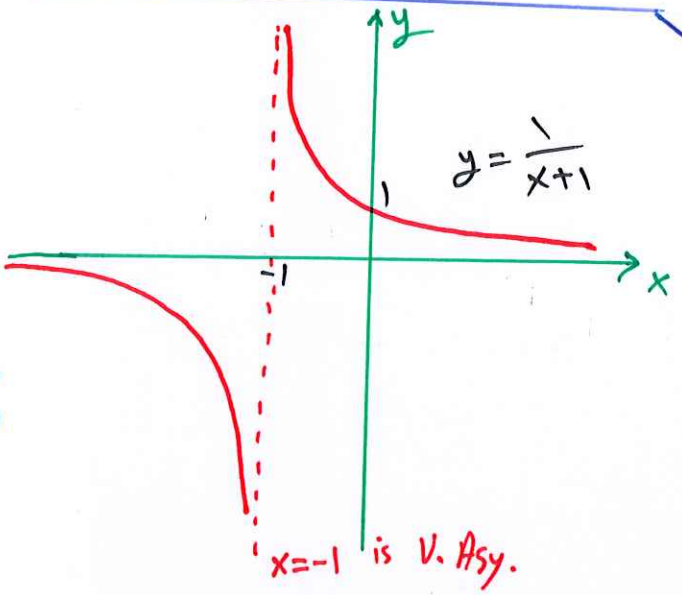
Exp $f(x) = \frac{1}{x-1}$

x	-1	0	2	3	4	-2	-3
y	$-\frac{1}{2}$	-1	1	$\frac{1}{2}$	$\frac{1}{4}$	$-\frac{1}{3}$	$-\frac{1}{4}$

$D = \mathbb{R} \setminus \{1\}$



Exp $y = \frac{1}{x+1}$



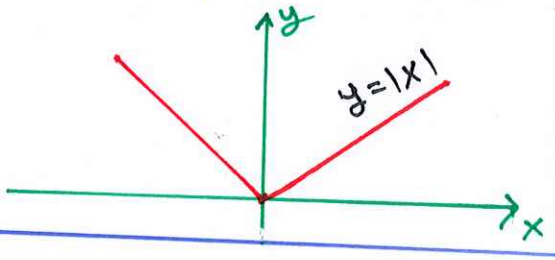
$D = \mathbb{R} \setminus \{-1\}$

V. Asy. means vertical Asymptotes

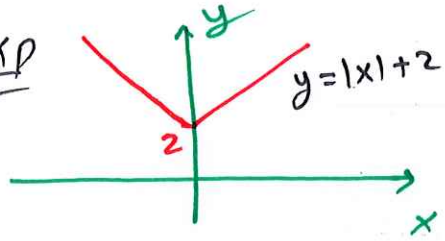
Piecewise Defined functions

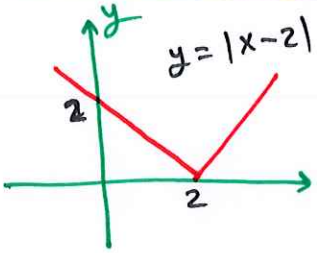
Exp The absolute value function $f(x) = |x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$

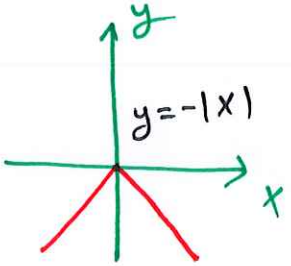
x	-3	-2	-1	0	1	2	3
y	3	2	1	0	1	2	3

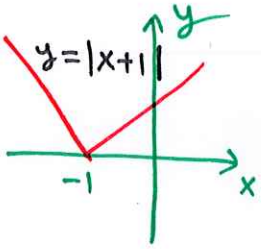


Exp









Exp If $f(x) = \begin{cases} 5 & , x \leq 0 \\ 2x & , 0 < x < 5 \\ x+6 & , 5 \leq x \end{cases}$ then find

① $f(-5) = 5$ ② $f(4) = 2(4) = 8$ ③ $f(20) = 20+6 = 26$

Exp Sketch $f(x) = \begin{cases} x^2 & \text{if } x \leq 0 \\ 4x & \text{if } 0 < x \leq 1 \\ 4 & \text{if } 1 < x \leq 2 \end{cases}$