

Problem

Define $G: J_5 \times J_5 \rightarrow J_5 \times J_5$ as follows: For all $(a, b) \in J_5 \times J_5$,

$$G(a, b) = ((2a + 1) \bmod 5, (3b - 2) \bmod 5).$$

Find the following:

a. $G(4, 4)$

b. $G(2, 1)$

c. $G(3, 2)$

d. $G(1, 5)$

Step-by-step solution

Step 1 of 4

Consider the function $G: J_5 \times J_5 \rightarrow J_5 \times J_5, \forall (a, b) \in J_5 \times J_5$.

Let, $G(a, b) = ((2a + 1) \bmod 5, (3b - 2) \bmod 5)$

(a)

The objective is to determine the value of $G(4, 4)$.

$$\begin{aligned} G(4, 4) &= ((2 \cdot 4 + 1) \bmod 5, (3 \cdot 4 - 2) \bmod 5) \\ &= (9 \bmod 5, 10 \bmod 5) \\ &= (4, 0) \end{aligned}$$

Step 2 of 4

(b)

The objective is to determine the value of $G(2, 1)$.

$$\begin{aligned} G(2, 1) &= ((2 \cdot 2 + 1) \bmod 5, (3 \cdot 1 - 2) \bmod 5) \\ &= (5 \bmod 5, 1 \bmod 5) \\ &= (0, 1) \end{aligned}$$

Step 3 of 4

(c)

The objective is to determine the value of $G(3, 2)$.

$$\begin{aligned} G(3, 2) &= ((2 \cdot 3 + 1) \bmod 5, (3 \cdot 2 - 2) \bmod 5) \\ &= (7 \bmod 5, 4 \bmod 5) \\ &= (2, 4) \end{aligned}$$

Step 4 of 4

(d)

The objective is to determine the value of $G(1,5)$.

$$\begin{aligned} G(1,5) &= ((2 \cdot 1 + 1) \bmod 5, (3 \cdot 5 - 2) \bmod 5) \\ &= (3 \bmod 5, 13 \bmod 5) \\ &= (3, 3) \end{aligned}$$