

Exercises:

Q36: if X has a Gamma distribution with $\alpha=3$ and $\beta=4$. Find

$$Pr(3.28 < X < 25.2)$$

$$X \sim \text{Gamma}(3, 4)$$

$$Y = \frac{2X}{\beta} = \frac{2X}{4} = \frac{X}{2}$$

$$\Rightarrow Y \sim \chi^2(r) \quad \text{where } r = 2\alpha = 2(3) = 6$$

$$\text{So } Pr(3.28 < X < 25.2) = Pr(1.64 < Y < 12.6) \quad , \quad r=6$$

$$\text{By table} \quad = 0.95 - 0.05$$

$$= 0.9$$

Q40: let X have a Gamma distribution with p.d.f

$$f(x) = \begin{cases} \frac{1}{\beta^\alpha} x e^{-\frac{x}{\beta}} & , \quad 0 < x < \infty \\ 0 & , \quad \text{elsewhere} \end{cases}$$

if $\underline{x=2}$ is the unique mode of the distribution find the parameter β and $Pr(X < 9.49)$.

$$\text{from } f(x) \rightarrow \alpha=2$$

$$\text{Mode} = (\alpha-1)\beta$$

$$2 = (2-1)\beta$$

$$\Rightarrow \boxed{\beta=2}$$

$$\text{So } X \sim \text{Gamma}(2, 2)$$

$$\Rightarrow Y = \frac{2X}{\beta} = \frac{2X}{2} = X$$

$$Y \sim \chi^2(r) \quad \text{where } r = 2\alpha = 2(2) = 4$$

$$\text{So } \Pr(X < 9.49) = \Pr(Y < 9.49)$$

$$= 0.95$$

By chi-square table

Q42: let X have a gamma distribution with parameters α and β . show that

$$\Pr(X \geq 2\alpha\beta) \leq \left(\frac{2}{e}\right)^\alpha$$