Rotation

a Rigid body :- a body that can rotates without any change in its shape

a fixed axis :- the rotation occurs about an axis that does not move

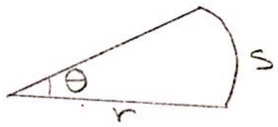
Pure rotation: angular motion

Angular Position

θ: is measured relative to the positive x-axis

$\theta = \frac{s}{r}$ → s: length of the circular arc; r: radius of the circle

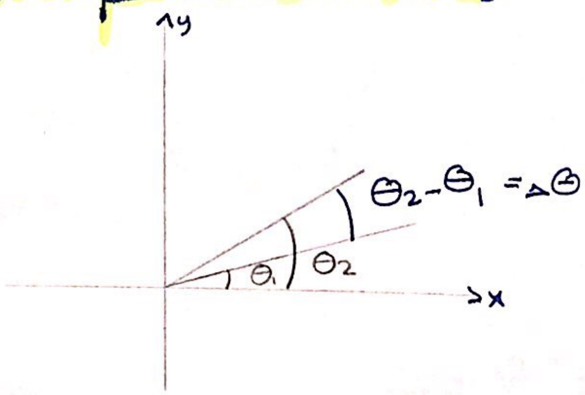
θ in radians



1 revolution $= 360^\circ = \frac{2\pi r}{r} = 2\pi$ rad

Angular displacement

$\Delta\theta = \theta_2 - \theta_1$



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- $\Delta\theta > 0 \Rightarrow$ if the movement is counterclockwise قارب الساعة
- $\Delta\theta < 0 \Rightarrow$ if the movement is clockwise ع عقارب الساعة

Angular velocity

ω average :-

$$\omega_{avg} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t}$$

unit : rad/s
or rev/s

ω instantaneous

$$\omega_{inst} = \frac{d\theta}{dt}$$

Magnitude of ω is called the angular speed

$\omega > 0 \Rightarrow$ if the movement is counterclockwise
$\omega < 0 \Rightarrow$ if the movement is clockwise

Angular acceleration

- If ω is not constant then the body han an angular acceleration

$$\alpha_{avg} = \frac{\omega_2 - \omega_1}{t_2 - t_1} = \frac{\Delta\omega}{\Delta t}$$

unit : rad/s^2
or rev/s^2

$$\alpha_{inst} = \frac{d\omega}{dt}$$

Maa Btaiw..

Are Angular Quantities vectors?

- Yes it is but we don't need to use vector notation Because we only have 2 cases: Counterclockwise and we use the plus sign (+) and for clockwise we use minus sign (−)

Rotation with Constant Angular acceleration

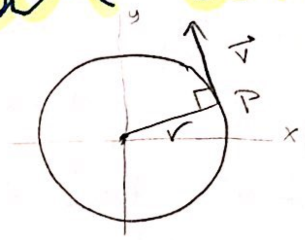
- when α is constant you can use these three equations

1- $\omega = \omega_0 + \alpha t$

2- $\Delta\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

3- $\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$

Relating the linear and Angular Variables



The position :- $S = \theta r$

The speed :- $\frac{ds}{dt} = \frac{d\theta}{dt} r$

$V = \omega r$

θ: in radians

ω in rad/s

If ω is constant $\longrightarrow$ uniform circular Motion

Then $T = \frac{2\pi r}{V} = \frac{2\pi r}{\omega r}$

$= \frac{2\pi}{\omega}$

- linear speed is always tangental to the circular Path

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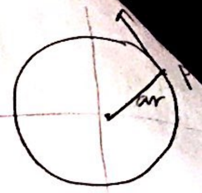

The acceleration

$$\frac{dv}{dt} = \frac{d\omega}{dt} r$$

$a_t = \alpha r$ tangental acceleration

in rad/s^2

$a_r = \frac{v^2}{r} = \omega^2 r$ radial acceleration

Kinetic Energy of Rotation

$K = \frac{1}{2} I \omega^2$ → ω is the same for all points

↳ rotational inertia $I = \sum m_i r_i^2$

I is smaller → rotation is easier

- If we have a system of particles then we can calculate I by calculating rotational inertia for each particle in the system:

if we had a system of 2 particls Then:

$$I = I_1 + I_2$$
$$= m_1 r_1^2 + m_2 r_2^2$$

Knowing that m is the mass and r is perpendicular distance between the particle and the rotation axis

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If we have a continuous system (نظام متصل يعني انه هناك عدد لانهائي من الاجسام الصغيرة (النقاط))

, we use integration

$I = \int r^2 \, dm$ or we use **The parallel axis Theory**

This Theory is used when you have I_{com} (inertia for the center of mass) and h (which is the perpendicular distance between the given axis and the axis passing through the com)

in one condition: The given axis and the axis passing through com should be **Parallel**

Note :-

I : دائماً تتكون من ثابت x الكتلة x البعد تربيع

$\Rightarrow I = I_{com} + Mh^2$

↳ Mass of the body

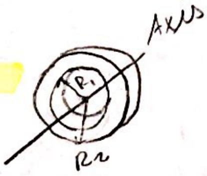
I for continuous systems :-

- **Hoop about central axis**

  $I = MR^2$

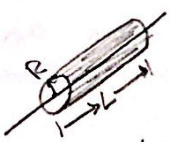
- **Annular cylinder (ring)**

  $I = \frac{1}{2} M (R_1^2 + R_2^2)$



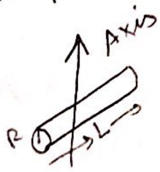
- **Solid cylinder (disk)**

  $I = \frac{1}{2} MR^2$



- **Solid cylinder (disk)**

  $I = \frac{1}{4} MR^2 + \frac{1}{12} ML^2$



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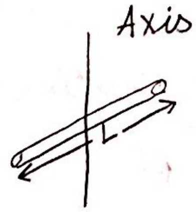
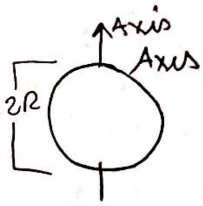
Thin rod about axis

$I = \frac{1}{12} M L^2$



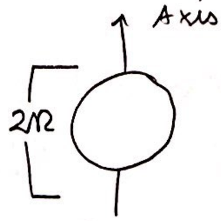
Solid sphere

$I = \frac{2}{5} M R^2$



Thin spherical shell

$I = \frac{2}{3} M R^2$

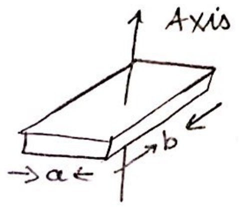


Hoop

$I = \frac{1}{2} M R^2$

Slab

$I = \frac{1}{12} M (a^2 + b^2)$



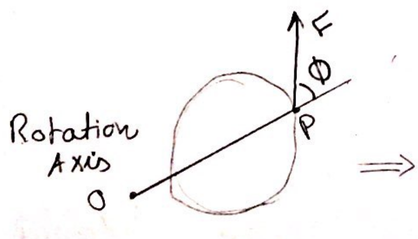
Torque (عزم الدوران)

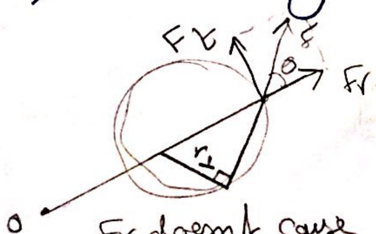
- is aproduct of 2 factors (F and r)

$\tau = F r \sin\theta$

or $= F_t r$ where F_t is the tangential Component of $\vec{F}$

or $= r_\perp F$ where $r_\perp$ is the moment ram (distance between axis of rotation and the extended line running throug $\vec{F}$ (Fig 1)





Fr doesnt cause rotation But Ft does

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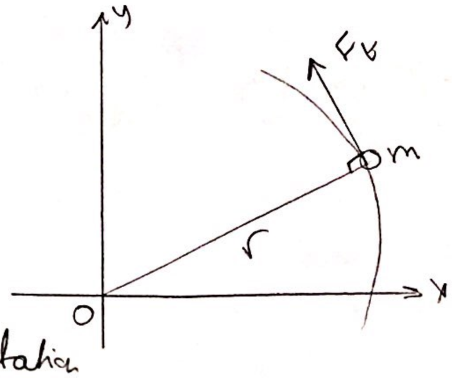
Newton's second law for Rotation

$\tau_{net} = I\alpha$



Axis of Rotation

Work & Rotational Kinetic Energy

$\rightarrow$ Work

$W = \Delta K$

$= \frac{1}{2} I \omega_f^2 - \frac{1}{2} I \omega_i^2$

$\rightarrow$ Angular velocity

or $W = \tau(\theta_f - \theta_i)$

and P (Power) $= \tau\omega$

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How to solve Problems

1- If you had $\theta(t)$ you can find angular velocity (ω)s and α By differentiation

2- If the question gives you number of revolutions it equals θ and you ~~can~~ can turn it into radians by multiplying it $(\times 2\pi)$

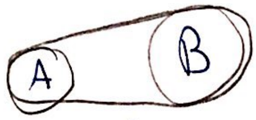
3- If the question asks for "average" acceleration or velocity you use $\alpha_{avg} = \frac{\Delta\omega}{\Delta t}$, $\omega_{avg} = \frac{\Delta\theta}{\Delta t}$

4- always pay attention to units because it helps you ~~to~~ to know if you're steps are right or wrong

5- θ_{max} can be found using $\Delta\theta = \theta_{max} - \theta_0 = \omega_0 t + \frac{1}{2}\alpha t^2$ if α is constant

6- If you're given V (speed) you can turn it into angular velocity using $\omega = \frac{V}{r}$

7- $V_A = V_B$ (careful V not ω)



(7)

if the question tells you that the belt does not slip

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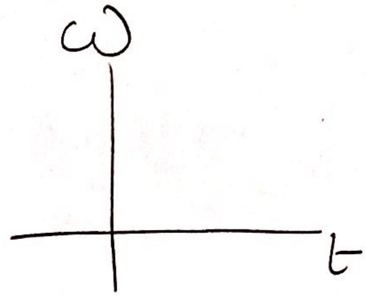
8- In these graphs
α is the Slope



9 - ~~In~~ questions that has particles and rods with mass

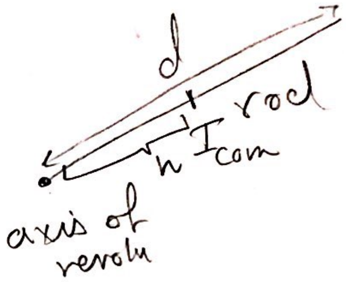
$$I = I_{particl_1} + I_{particle_2} + I_{rod_1} + I_{rod_2}$$

$$I_{particle} = md^2$$

I_{rod} : (you have to use The parallel axis Theory)

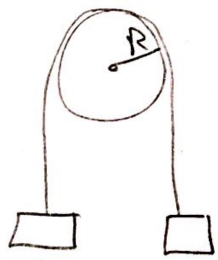
meaning :- $I_{rod} = I_{com} + Mh^2$

$$= \frac{1}{12}Md^2 + M\left(\frac{d}{2}\right)^2$$



10 - When the question asks for the Torque pay attention to θ (It's not always the given angle)

11- In such Questions
at for the pulley = a of the box



12- If you're given F as a function you can find τ and then α using

$$\tau = Fr$$

and $\alpha = \frac{\tau}{I}$

13- you can use the ~~Theory~~ Conservation of Energy Theory In a lot of questions Just pay attention !

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