Chap 6 :-

6.1 • Volumes using Cross sections

Volumes Rules

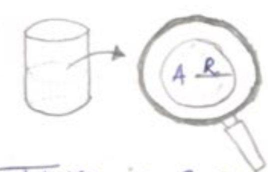
1- cylindrical solid:-
$V = (\text{Base Area})\,\text{Height}$

2-

There is 3 methods used to find a volume of a given solid

Disk Method

• it's a special case of Washer Method. $[r(x)=0]$



$A = \pi R^2$

• There is 2 cases :-

$CS \perp x$-axis
$A(x) = \pi R^2(x)$

$CS \perp y$-axis
$A(y) = \pi R^2(y)$

Washer Method

• 2 cases
we use it when the solid does not border on or cross the axis of revolution

→ If the $CS \perp x$-axis which results by Rotation about x-axis with outer Radius $R(x)$ and Inner Radius $r(x)$ then

$$V = \int_a^b A(x)\,dx = \int_a^b \pi\,[R^2(x) - r^2(x)]\,dx$$

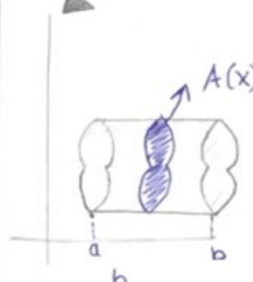
→ if the $CS \perp y$-axis which results by Rotation about y-axis with outer Radius $R(x)$ and Inner Radius $r(x)$ then

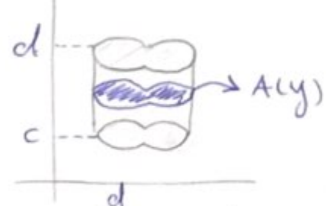
$$V = \int_c^d \pi\,[R^2(y) - r^2(y)]\,dy$$

Shell Method

Slicing by parallel planes:-

* How to find a volume given
  - Graph the solid
  - Determine the cross section
  - Then if $CS \perp x$-axis : 1st case
    if $CS \perp y$-axis : 2nd case





• $V = \int_a^b A(x)\,dx$

• $V = \int_c^d A(y)\,dy$

• Rotation about x-axis or any line parallel to it

• Rotatation about y-axis or any line parallel to it

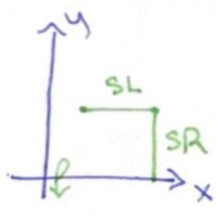
6.2 Shell method

we use this method to find the volume of a solid generated by revolving a given region about :-

X-axis d → c, d على محور الصادات

$$V = 2\pi \int_c^d \binom{\text{shell}}{\text{Radius}} \binom{\text{shell}}{\text{length}} \, dy$$

- distance between $\boxed{\text{shell length}}$ and the axis of Revolution (x)
  SR
- the segment's length That is parallel to the axis of revolution (x)
  SL

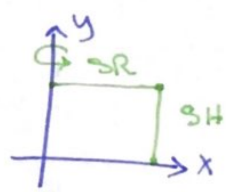


y-axis

$$V = 2\pi \int_a^b \binom{\text{shell}}{\text{Radius}} \binom{\text{shell}}{\text{hight}} \, dx$$

a, b على محور السينات

- distance between the shell's hight and the axis of Revolution (y)
- sigment's hight that is parallel to the axis of Revolution (y)



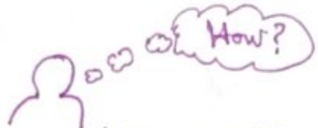
 Written by Alaa Etaiwi

6.3 Arc length

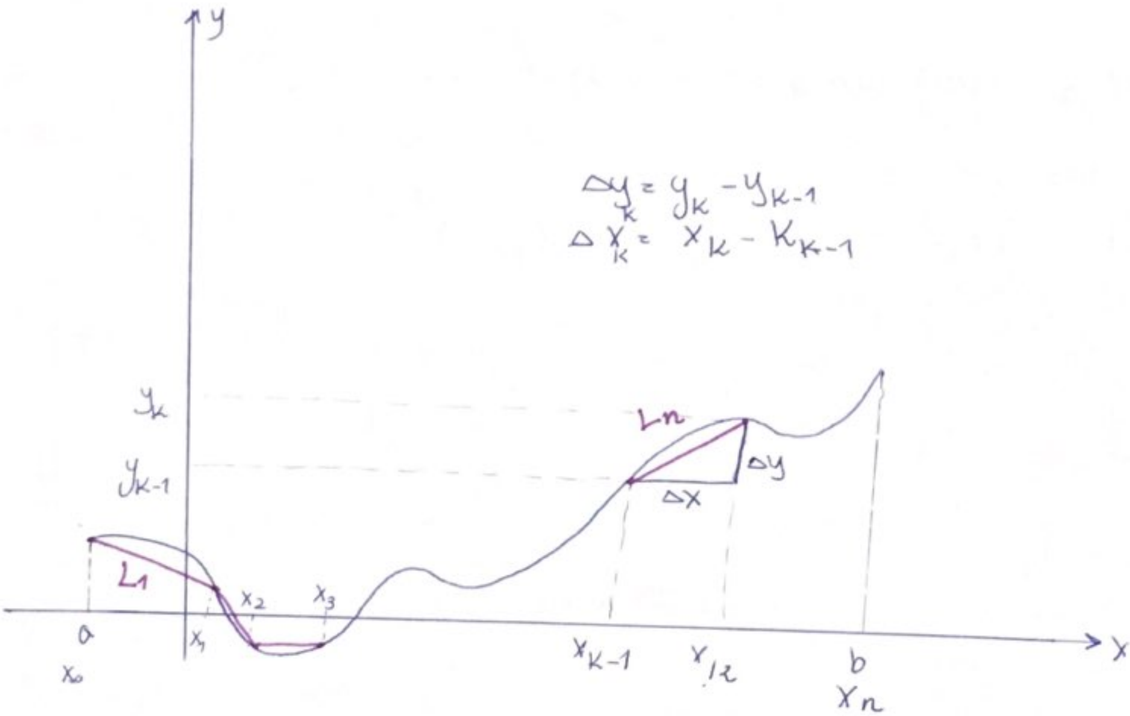
If $f'(x)$ is cont on $[a,b]$ then the Arc length of the curve $y=f(x)$ is given by:-

$$L = \int_a^b \sqrt{1+(f'(x))^2}\, dx = \int_a^b \sqrt{1+(\ \ \ )^2}\, dx$$

Explanation:-



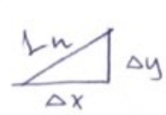
- Let's say you have the curve $f(x)$, wich happenels to be continuous and diff at $[a,b]$ WoW!



L: is the true length
→ $\tilde{L}$: Approximated length

$\tilde{L} = L_1 + L_2 + \text{——} + L_n$

$= \sum_{k=1}^{n} L_k$



$= \sum_{k=1}^{n} \sqrt{(\Delta x)^2+(\Delta y)^2}$

Remember MVT
Mean Value Theorm
$f'(c) = \dfrac{f(b)-f(a)}{b-a} = \dfrac{\Delta y}{\Delta x}$

$\Rightarrow$ By MVT there is $C_k \in (x_{k-1}, x_k)$

$f'(c_k) = \dfrac{\Delta y_k}{\Delta x_k} \Rightarrow \Delta y_k = f'(c_k)\, \Delta x_k$

But $\tilde{L} = \sum_{K=0}^{n} \sqrt{(\Delta X_K)^2 + (f'(C_K)\ \Delta X_K)^2}$

$\tilde{L} = \sum_{K=0}^{n} \sqrt{1 + f'(C_K)}\ \Delta X_K$

→ to ~~mo~~ improve $\tilde{L}$ $\Rightarrow$ we use n numbers of sub intervals
as n gets large

$L = \lim_{n \to \infty} \tilde{L}$

$= \lim_{n \to \infty} \sum_{K=1}^{n} \sqrt{1 + f'(C_K)^2}\ \Delta X_K$

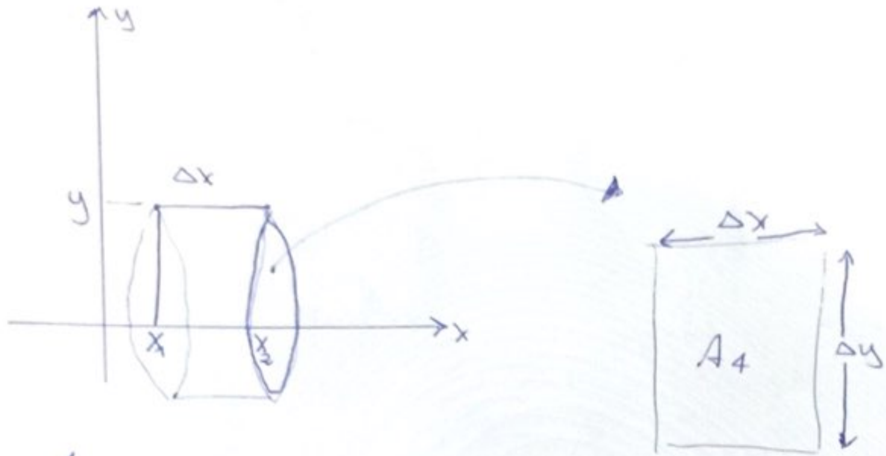
$= \int_{b}^{a} \sqrt{1 + f'(x)^2}\ dx$ Finally

Remark: very important and Tricky ⚠

- f' has to be Continuous on $[a,b]$

If not :- Try $\Rightarrow L = \int_{c}^{d} \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\ dy$

6.4 Area of surface using Revolution



Area : (Δx)(2yπ

Area = (Δx)(Δy)

⊛ X - axis :-

The surface area of the region ~~bounded~~ generated by Revolving Δx about X - axis is :-

$2\pi y \, \Delta x$

Definitions :-

X - axis

If $y = f(x) \geq 0$ is continuously differentiable on interval $[a,b]$ then the surface Area of the region generated by revolving the curve $y = f(x)$ about X-axis is

$$\boxed{S = \int_a^b 2\pi y \sqrt{1 + \underbrace{f'(x)^2}_{\left(\frac{dy}{dx}\right)^2}} \, dx}$$

y - axis

If $x = g(y) \geq 0$ is cont diff on $I = [c,d]$ then the surface Area of the region Generated by Revolving the curve $x = g(y)$ about y-axis is:-

$$\boxed{S = \int_c^d 2\pi \, g(y) \sqrt{1 + \underbrace{(g'(y))^2}_{\left(\frac{dx}{dy}\right)^2}} \, dy}$$