6.4 Exponential Probability Distribution "Continuous"

The exponential Prob. distribution is continuous prob. distribution used for random variables s.t:

- the time between arrivals at a car wash.
- the time between the arrival customers at a bank.
- the time required to load تحميل a truck
- the distance between defects تلفيات/عيوب in a highway, and so on...

} service time

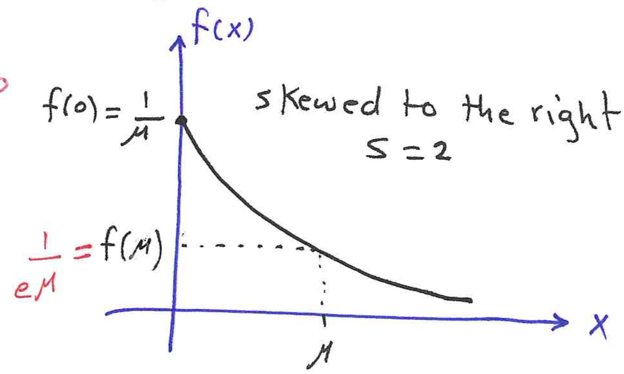
* The Exponential Prob. Density Function is

$$f(x) = \frac{e^{-\frac{x}{\mu}}}{\mu} \quad \text{for } x \geq 0 \text{ and } \mu > 0$$

where μ = expected value or Mean

μ = standard deviation σ

σ^2 = Variance $= \mu^2$



* The cumulative Probabilities for the exponential distribution is

$$P(x \leq x_0) = 1 - e^{-\frac{x_0}{\mu}}$$

Example (Q33 page 248) Consider the following exponential prob. density function $f(x) = \frac{1}{3} e^{-x/3}$ for $x \geq 0$

[a] write the formula for the cumulative probabilities?

$$P(X \leq X_0) = 1 - e^{-X_0/3}$$

[b] Find $P(X \leq 2) = 1 - e^{-\frac{2}{3}} = 1 - 0.5134 = 0.4866$

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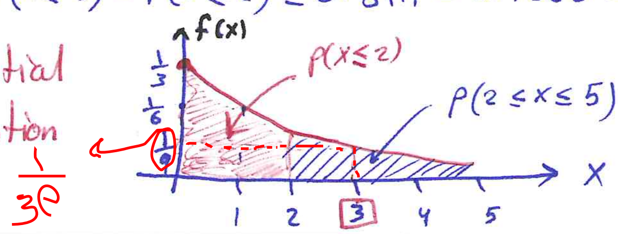
[c] Find $P(X \geq 3) = 1 - P(X < 3) = 1 - [1 - e^{-3/3}] = e^{-1} = 0.3679$

[d] Find $P(X \leq 5) = 1 - e^{-5/3} = 1 - 0.1889 = 0.8111$

[e] Find $P(2 \leq x \leq 5) = P(x \leq 5) - P(x \leq 2) = 0.8111 - 0.4866 = 0.3245$

[f] sketch this exponential probability distribution



[g] Find $\sigma = \mu = 3$

[h] Find $\sigma^2 = \mu^2 = 9$

Relationship Between the Poisson and Exponential Distributions:

* Recall that the Poisson distribution is a discrete prob. distribution used to examine the number of occurrence of an event over a specified interval of time : $f(x) = \frac{\mu^x e^{-\mu}}{x!}$ where

$\sigma^2 = Var(x) = \mu$ = expected value or mean number of occurrence over specified interval.

* The continuous exponential prob. distribution describes the length of the interval between occurrence.

* That is, if arrivals follow a Poisson distribution, then the time between arrivals must follow an exponential distribution.

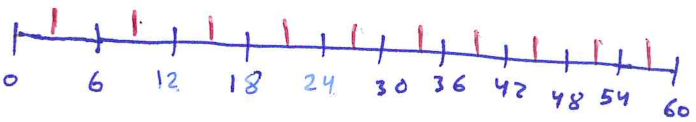
Example: Suppose the number of cars arrive at a car wash during one hour is described by a Poisson prob. distribution with mean 10 cars per hour.

Hence, the Poisson prob. function that gives the prob. of x arrivals car per hour is $f(x) = \frac{10^x e^{-10}}{x!}$ $\mu_p = 10$

. Now the average time between cars arriving is $= \frac{1 \text{ hour}}{10 \text{ cars}}$

Hence, the corresponding exponential distribution that describes the time between the arrivals has mean

$\mu_e = 0.1$ hour/car $= 6$ mint/car



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$\mu_e = 0.1$ hour/car. Thus, the exponential prob. density function is

$$f(x) = \frac{1}{0.1} e^{-x/0.1} = 10 e^{-10x}$$