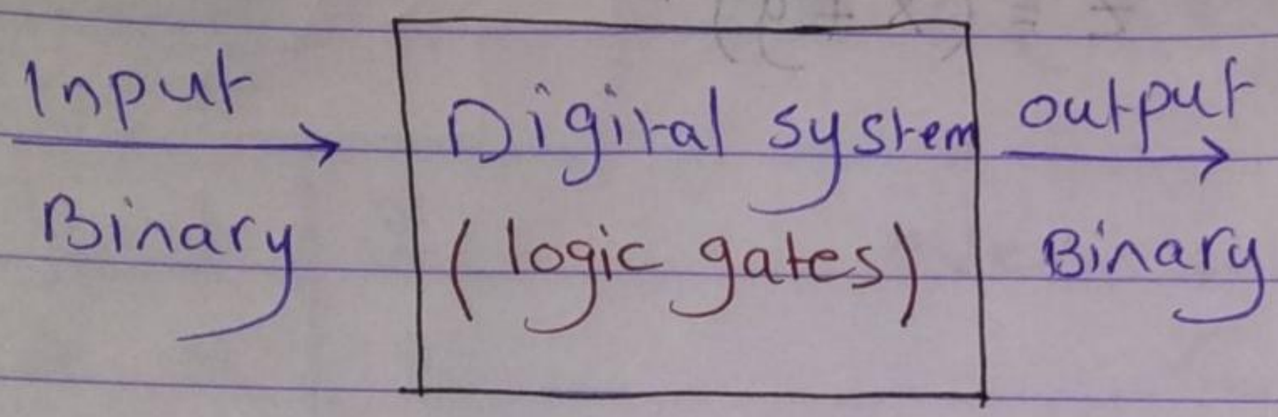
Chapter 2 Boolean algebra and logic gates.



logic gates :- is used in all of todays computers and devices, and the cost of the circits that Implement is an important factor addressed by designer

Boolean algebra :- Mathmatical system

Consist of :-

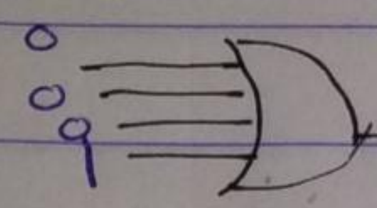
1. set of elements $B \in \{0, 1\}$.
2. set of operators ($+$, $\cdot$, not, XOR, --).
3. set of postulates مسلمات

Rules For Boolean Algebra :-

① **Closure** : The operator + and . are closed For all $x, y \in B$.

$x + y \in B$

$x \cdot y \in B$



② **Identity** :- العنصر المحايد .

0 is the Identity element for +

1 is the Identity element for .

$X + 0 = X$

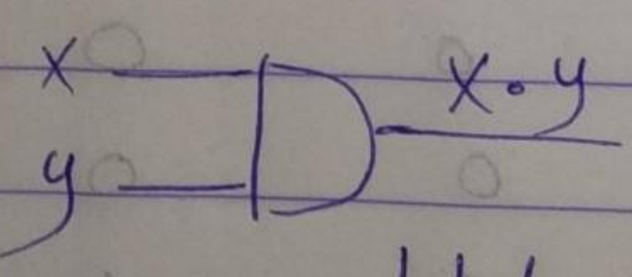
$X \begin{pmatrix} 1 \\ 0 \end{pmatrix} + 0 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} X$

$1 + 0 + 0 = 1$

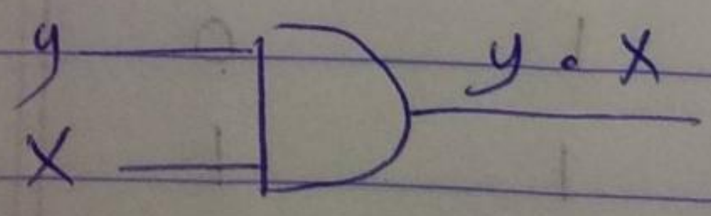
$0 + 0 = 0$

③ **Commutative** :- $x + y = y + x$

$x \cdot y = y \cdot x$



111 (the same)

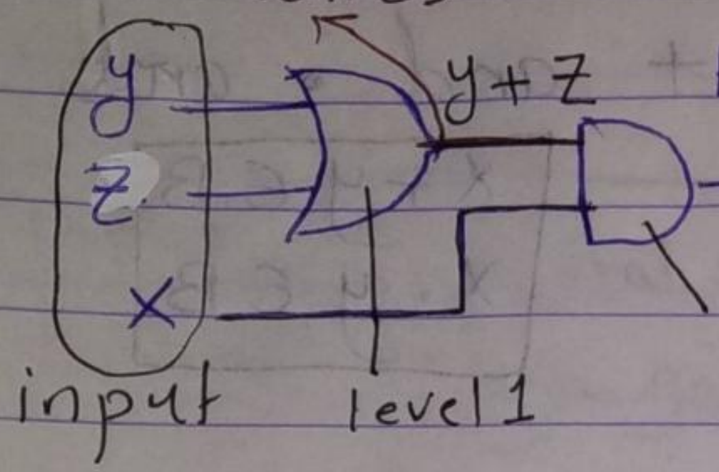


$0 + 1 \equiv 1 + 0$

$0.1 \equiv 1.0$

④ **Distributed** :- $X \cdot (y+z) \equiv X \cdot y + X \cdot z$

توزيع and على OR



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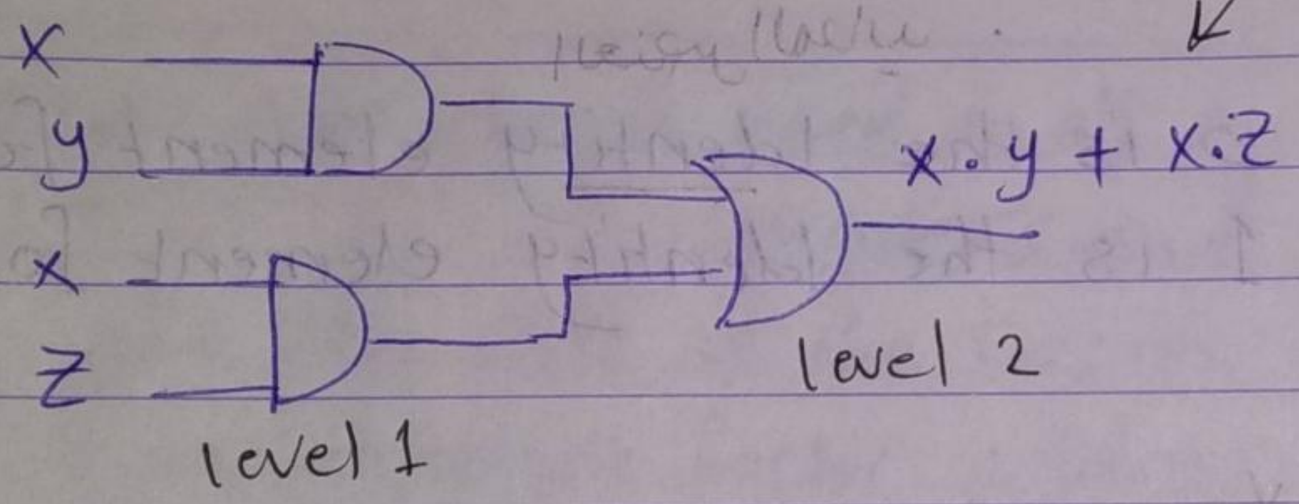
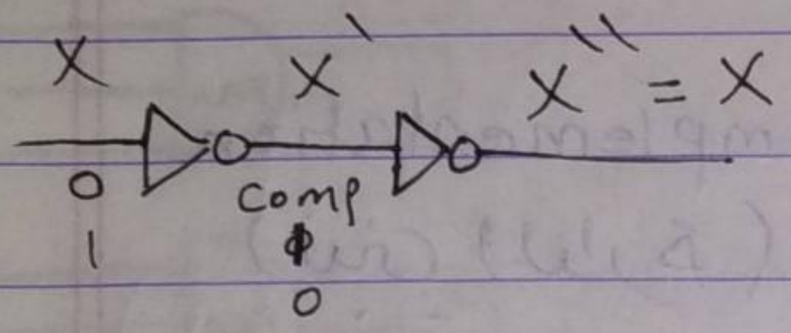


x	y	z	y+z	x . (y+z)	x . y	x . z	x . y + x . z
0	0	0	0	0	0	0	0
0	0	1	1	0	0	0	0
0	1	0	1	0	0	0	0
0	1	1	1	0	0	0	0
1	0	0	0	0	0	0	0
1	0	1	1	0	0	1	1
1	1	0	1	1	1	0	1
1	1	1	1	1	1	1	1

=

(5) The Complement.

The Complement of X is X', $\bar{X}$, $X^{\prime}$
the complement 0 is 1'
and complement 1' is 0



(6) Demorgan Law.

$(X + y)' = X' \cdot y'$ Truth table

X	y	X+y	$\overline{(X+y)}$	$\bar{X}\,\bar{y}$	$\bar{X} \cdot \bar{y}$
0	0	0	1	1 1	1
0	1	1	0	1 0	0
1	0	1	0	0 1	0
1	1	1	0	0 0	0

$(X \cdot y)' = \bar{X} + \bar{y}$

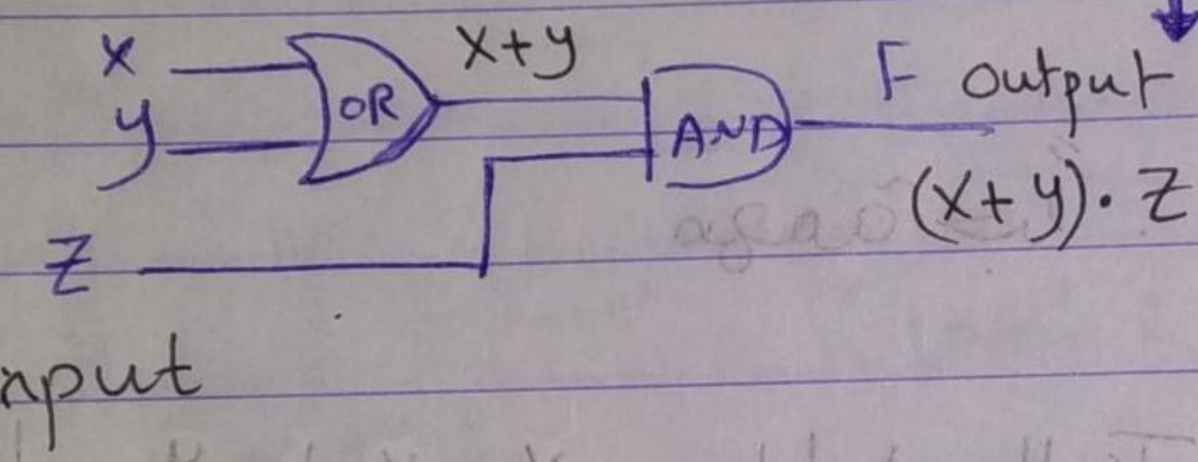
x	y	x.y	$(x.y)'$	$\bar{x}$	$\bar{y}$	$\bar{x} + \bar{y}$
0	0	0	1	1	1	1
0	1	0	1	1	0	1
1	6	0	1	0	1	1
1	1	1	0	0	0	0

Boolean Function:- Expression formed with Boolean variable, operator (+, ·, ~) and equal sign.

$F = (x + y) \cdot z$ ← Function.

variables, operator, equal.

Implementation (تبني البوابة)



$F = (x \cdot z) + (y \cdot z)$

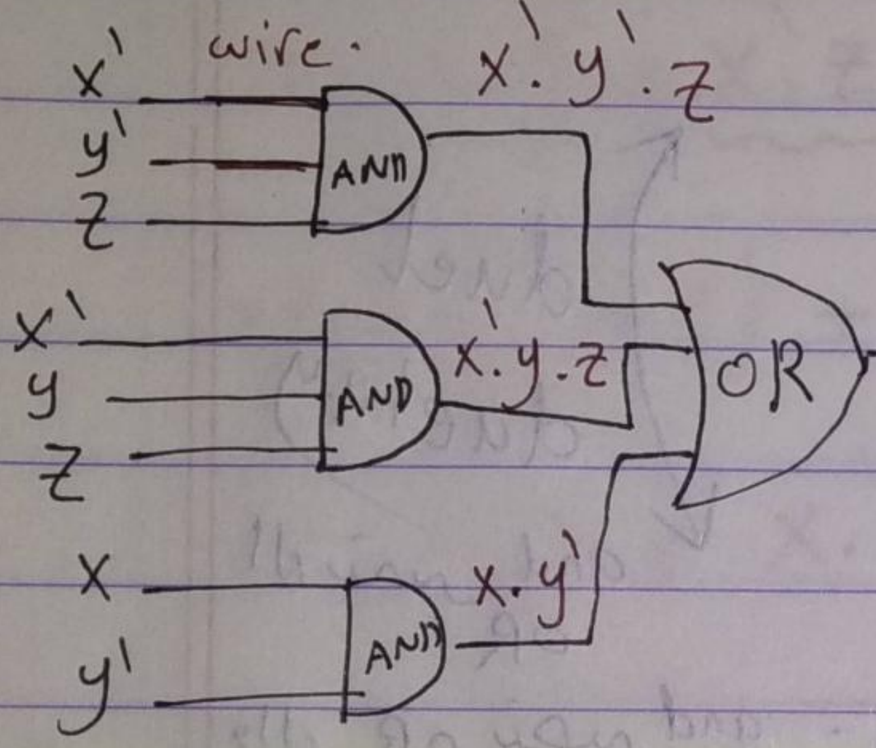
input

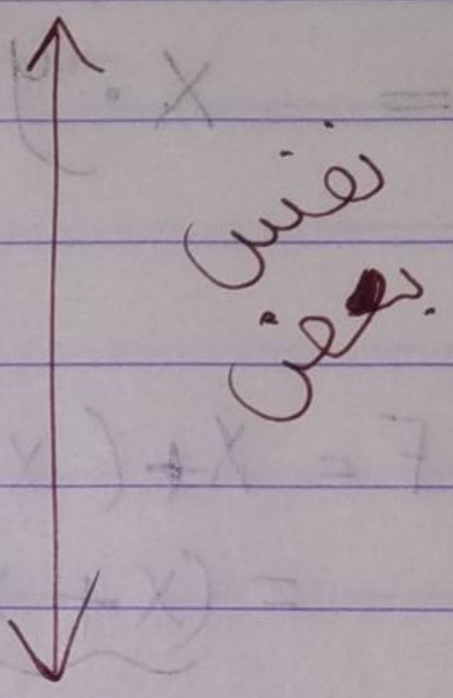
x	y	z	$F = (x+y) \cdot z$
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

Implement the Following Function. — سؤال:

$$F = \underbrace{X' \cdot y' \cdot Z}_{\text{And}} + \underbrace{X' \cdot y \cdot Z}_{\text{And}} + \underbrace{X \cdot y'}_{\text{And}}$$

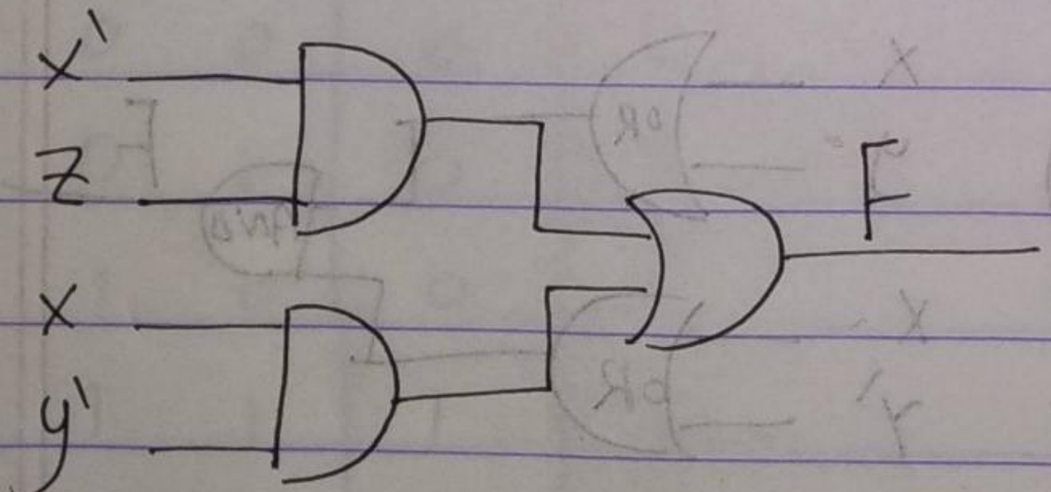
OR





$$F = X' \cdot Z \cdot \underbrace{(y' + y)}_{=1} + X \cdot y' \qquad y \cdot y' = 0$$

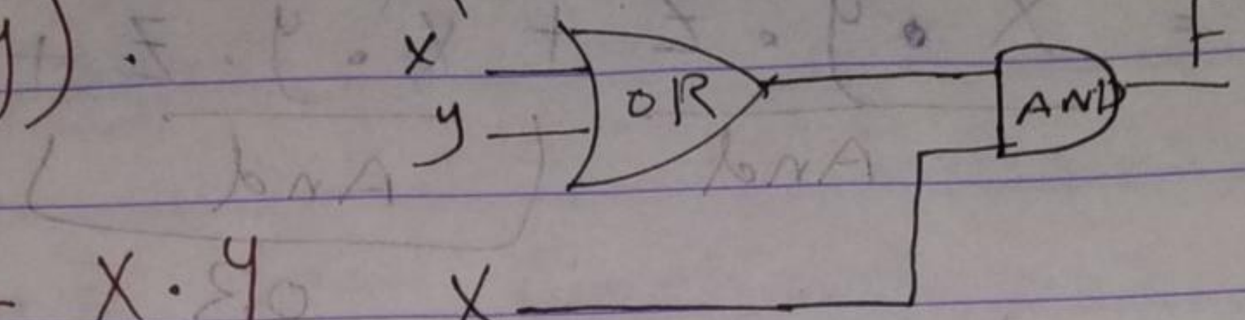
$$F = X' \cdot Z \cdot (1) + X \cdot y' \quad = \quad X' \cdot Z + X \cdot y'$$



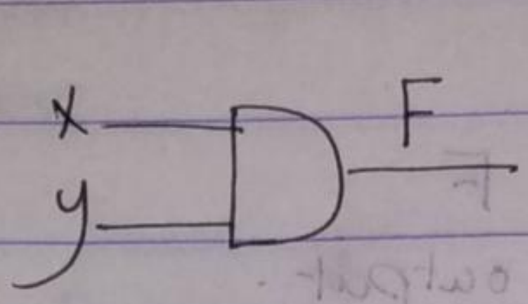
example Minimize the Following Function:

$F = X \cdot (X' + y)$

$F = \underbrace{X \cdot X'}_{0} + X \cdot y$



$F = X \cdot y$



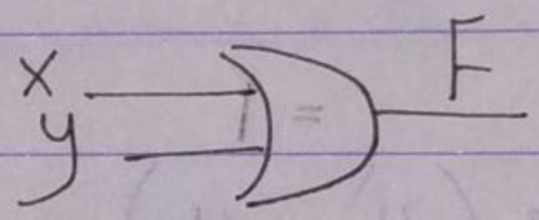
duel
duality

ال تنصير and
OR
وال OR بتصير and

ex $F = X + (X' \cdot y)$

$= \underbrace{(X + X')}_{1} \cdot (X + y)$

$F = (X' + y)$

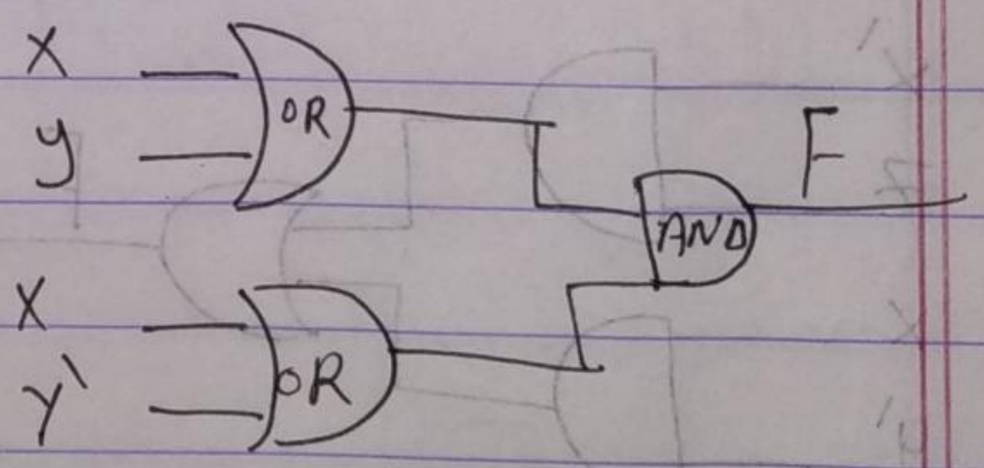


ex $F = (x + y) \cdot (x + y') \cdot$

$F = (X \cdot X) + (X \cdot y') + (X \cdot y) + \underbrace{(y \cdot y')}_{0}$

$F = X + X \cdot y' + X \cdot y$

$F = X(1 + y' + y) \cdot = X$



طريقة ثانية :-

$F = (x + y) \cdot (x + y')$

$F = x + \underbrace{(y \cdot y')}_{0} = x$

ex Minimize the Following Function.

$$F = x.y + x'.z + y.z$$

$$= x.y + x'.z + y.z.(x + x').$$

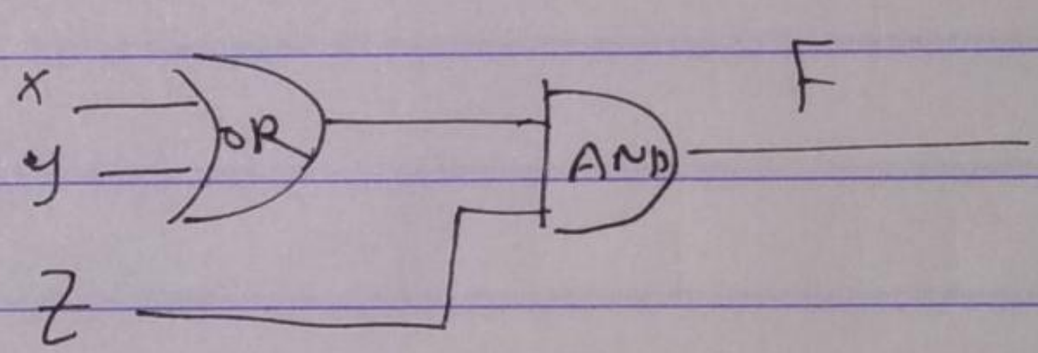
$$= \underline{x.y} + \underline{x'.z} + \underline{x.y.z} + \underline{x'.y.z}$$

$$= x.y(1+z) + x'.z(1+y).$$

$$= x.y + x'.z$$

Complement of the Function.

$$F = (x+y).z$$



x	y	z	F	F'
0	0	0	0	1
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	0	1
1	1	1	1	0

$$F' = (x+y)' + z'$$

$$= (x'.y') + z'$$

ex $F' = [x.(y'z' + yz)]'$

$F' = x' + (y'z' + yz)'$

$F' = x' + (y'z')' \cdot (yz)'$

$F = x' + (y + z) \cdot (y' + z')$.

$F = x' + (\cancel{yy'}^{0} + yz' + y'z + \cancel{zz}^{0})$.

$F = x' + yz' + y'z$.

Canonical and Standard Forms :-

$F = \underbrace{(x.y)}_{\text{term1}} + \underbrace{z}_{\text{term 2}}$

By Distance.

$$F = x'.y.z' + x'y' + xy' + xyz + xy'z$$

palestine ⟶ Texas. (Implemetet).

← هيك صعب نتعامل معها صرمكان لهكا بدنا نتبع طريقة canonical form. ال

Canonical form :- . لغة يستخدموها ال designer في كل مكان

Note : IF we have 2 variables x, y

- F(x, y) = 4 combinations.
  output input.

$x.y$, $x'.y$, $x.y'$, $x'.y'$

✱ n variables $\Longrightarrow 2^n$ Combinations.

$2 \Rightarrow 2^2$, $3 \Rightarrow 2^3$

= each combination called minterm, denoted by m_i $0 \leq i \leq 2^n - 1$.

ex Function Consist of 3 variables.

8 Combinations. $\rightarrow$ $2^3 - 1$. $8 - 1 = 7$.

m_0 → minterm0
m_1 → minterm1
m_2
m_3
m_4
m_5
m_6
m_7

8.

Canonical forms :-

1. Sum of minterms (SOM).
2. Product of Maxterm (POM).

ex Function 3 variables. $\equiv 2^3 = 8$ combination.

$F(x, y, z)$.

. And دائماً ال term جوابها 1 وجودها

x	y	z	Minterm term	Minterm Designed	Maxterm term	Maxterm Designed
0	0	0	$x'y'z'$	m_0	$x+y+z$	M_0
0	0	1	$x'y'z$	m_1	$x+y+z'$	M_1
0	1	0	$x'yz'$	m_2	$x+y'+z$	M_2
0	1	1	$x'yz$	m_3	$x+y'+z$	M_3
1	0	0	$xy'z'$	m_4	$x'+y+z$	M_4
1	0	1	$xy'z$	m_5	$x'+y+z'$	M_5
1	1	0	xyz'	m_6	$x'+y'+z$	M_6
1	1	1	xyz	m_7	$x'+y'+z'$	M_7

ex $F(x,y,z) = x'y'z' + x.y.z$

term1 $0'.0'.0'$ + term2 $1.1.1$ minterems

$F(x,y,z) = m_0 + m_7 = \sum(0,7)$

ex $F(x,y,z) = x'yz + xy'z + xyz.$

minterms $= \underline{0'.1.1} + \underline{1.0'.1} + \underline{1.1.1}$

$= m_3 + m_5 + m_7.$

$= \sum(3,5,7).$

ex what is boolean function.

$F(x,y,z) = \sum 0,3,5,7.$

$= m_0 + m_3 + m_5 + m_7$

000 011 101 111

$= x'y'z' + x'yz + xy'z, xyz$

ex $F(A,B,C) = \sum(0,2,7)$ Truth Table.

$= m_0 + m_2 + m_7$

$= 000 \quad 010 \quad 111$

$= A'.B'.C' + A'BC' + ABC$

$0'.0'0' + 0'.0.0' + 0.0.0$

$1 + 0 + 0$

A	B	C	F	
0	0	0	1	0
0	0	1	0	
0	1	0	1	2
0	1	1	0	
1	0	0	0	
1	0	1	0	
1	1	0	0	
1	1	1	1	7

ex Find the truth table and mathmetical expression for the following.

$F(A,B,C,D) = \sum 0, 1, 15$

0000 0001 1111

$m_0 + m_1 + m_{15}$

A	B	C	D	F	
0	0	0	0	1	0
0	0	0	1	1	1
0	0	1	0	0	
0	0	1	1	0	
0	1	0	0	0	
0	1	0	1	0	
0	1	1	0	0	
0	1	1	1	0	
1	0	0	0	0	
1	0	0	1	0	
1	0	1	0	0	
1	0	1	1	0	
1	1	0	0	0	
1	1	0	1	0	
1	1	1	0	0	
1	1	1	1	1	15

ex $F(x,y,z) = (x+y+z) \cdot (x'+y'+z')$

$0+0+0 \qquad 1'+1'+1'$

$= M_0 \cdot M_2$

$= \Pi(0,7)$.

ex $F(A,B,C,D) = (A+B+C+D') \cdot (A+B'+C+D)$.

$= (0+0+0+1') \cdot (0+1'+0+0)$.

$= M_1 + M_4$

$= \Pi(1,4)$.

ex $F(A,B,C) = \Pi(0,2,6)$. Truth table

$M_0 \cdot M_2 \cdot M_6$

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A	B	C	F	
0	0	0	0	0
0	0	1	1	
0	1	0	0	2
0	1	1	1	
1	0	0	1	
1	0	1	1	
1	1	0	0	6
1	1	1	1	

$(A+B+C) \cdot (A+B'+C) \cdot (A'+B'+C)$

$(0+0+0) \cdot (0+0'+0) \cdot (0'+0'+0)$

$0 \cdot 1 \cdot 1$

$(0+0+1) \cdot (0+0'+1) \cdot (0'+0'+1)$

$1 \cdot 1 \cdot 1$

$F_1(A,B,C) = \sum 0,2,6$ SOM

$= m_0 + m_2 + m_6$

A	B	C	F
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	0

عكس بعض

$F_2(A,B,C) = \prod 0,2,6$ POM

A	B	C	F
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	1

ex Express the following Function in product of maxterm?

$F(A,B,C) = \sum 0,2,5$

Sum of minterms.

$111 = m_0 + m_2 + m_5$.

$F(A,B,C) = \Pi(1,3,4,6,7)$

$= M_1 \cdot M_3 \cdot M_4 \cdot M_6 \cdot M_7$

Express complement of F in product of maxterm.

$F(A,B,C) = \Pi(0,2,5)$

A	B	C	F	F`
0	0	0	1	0
0	0	1	0	1
0	1	0	1	0
0	1	1	0	1
1	0	0	0	1
1	0	1	1	0
1	1	0	0	1
1	1	1	1	0

ex Express the Following Function in ① sum of minterm ② complement function in SOM, POM.

A	B	C	F	F'
0	0	0	0	1
0	0	1	0	1
0	1	0	1	0
0	1	1	1	0
1	0	0	1	0
1	0	1	1	0
1	1	0	0	1
1	1	1	1	0

$F(A,B,C) = \Pi(0,1,6)$.

$F(A,B,C) = \Sigma(2,3,4,5,7)$.

$F'(A,B,C) = \sum 0,1,6$.

$F'(A,B,C) = \Pi(2,3,4,5,7)$.

ex Express the Following Function in Sum of minterm $F(A,B,C) = A + A'C$

Sum of minterm = ☐ + ☐ + ☐

term
variable.

Solution #1

$F = A + A'C$.

$F = A.(\underbrace{B' + B}_{1}) + A'C(\underbrace{B + B'}_{1})$.

$= A.B' + AB + A'BC + A'B'C$.

$= A.B'(C + C') + AB(C + C') + A'BC + A'B'C$.

$= AB'C + AB'C' + ABC + ABC' + A'BC + A'B'C$.
$\quad 1\,0\,1 \quad 1\,0\,0 \quad 1\,1\,1 \quad 1\,1\,0 \quad 0\,1\,1 \quad 0\,0\,1$

$= \Sigma 1, 3, 4, 5, 6, 7$

Solution #2

A	B	C	$F = A + A'C$
0	0	0	$0 \quad 0 + 0'.0$
0	0	1	$1 \quad 0 + 1.0$
0	1	0	$0 \quad 0 + 0.1$
0	1	1	$1 \quad 1 + -$
1	0	0	$1 \quad 1 + -$
1	0	1	$1 \quad 1 + -$
1	1	0	$1 \quad 1 + -$
1	1	1	$1 \quad 1 + -$

$= \Sigma 1, 3, 4, 5, 6, 7.$

ex $F(x,y,z) = x\cdot y + x'\cdot z$ [POM].

Solution # 1

x	y	z	$F = x\cdot y + x'\cdot z$
0	0	0	$0\cdot 0 + 0'\cdot 0$
0	0	1	$0\cdot 0 + 0'\cdot 1$
0	1	0	$0\cdot 1 + 0'\cdot 0$
0	1	1	$0\cdot 1 + 0'\cdot 1$
1	0	0	$1\cdot 0 + 1'\cdot 0$
1	0	1	$1\cdot 0 + 1'\cdot 1$
1	1	0	$1\cdot 1 + -$
1	1	1	$1\cdot 1 + -$

$F(A,B,C) = \Pi(0,2,4,5)$.

Solution #2.

$F = x\cdot y + x'\cdot z$

$= x\cdot y(z+z') + x'\cdot z(y+y')$

$= x\cdot y\cdot z + x\cdot y\cdot z' + x'\cdot y\cdot z + x'\cdot y'\cdot z$

$= m7 + m6 + m3 + m1$

$= \Sigma 1,3,6,7$

$F = \Pi(0,2,4,5)$.

Solution # 3.

$$F(x, y, z) = x.y + x'.z \quad [POM]$$

$$= \omega + (x'.z)$$

$$= (\omega + x').(\omega + z)$$

$$= [(x.y) + x'] \bullet [(x.y + z]$$

$$= (x' + x).(x' + y).(x + z).(y + z)$$

$$= (x' + y).(x + z).(y + z).$$

$$= (\underbrace{x' + y}_{\omega_1} + z.z').(\underbrace{x + z}_{\omega_2} + y.y').(\underbrace{y + z}_{\omega_3} + x.x').$$

$$= (\omega_1 + z).(\omega_1 + z').(\omega_2 + y).(\omega_2 + y').(\omega_3 + x) + (\omega_3 + x').$$

$$= (x' + y + z).(x' + y + z).(x + y + z).(x + y' + z).(x + y + z).(x' + y + z).$$

$$= \Pi(0, 2, 4, 5).$$

✳ Standard Form :-

① Sum of product.

(فيش في C)

$F(A,B,C) = \underbrace{A.B}_{\text{prod}} + \underbrace{A.B.C'}_{\text{prod}}$ (sum)

Sum of minterm ✗

Sum of product ✓

ex $F(A,B,C) = A.B.C' + A'.B.C$ sum of minterm ✓

sum of product ✓

② product of sum.

$F(A,B,C) = (A+B).(A'+B+C)$.

product of maxterm ✗

product of sum ✓

ex $F(A,B,C) = (A+B).(A+BC)$ (none)

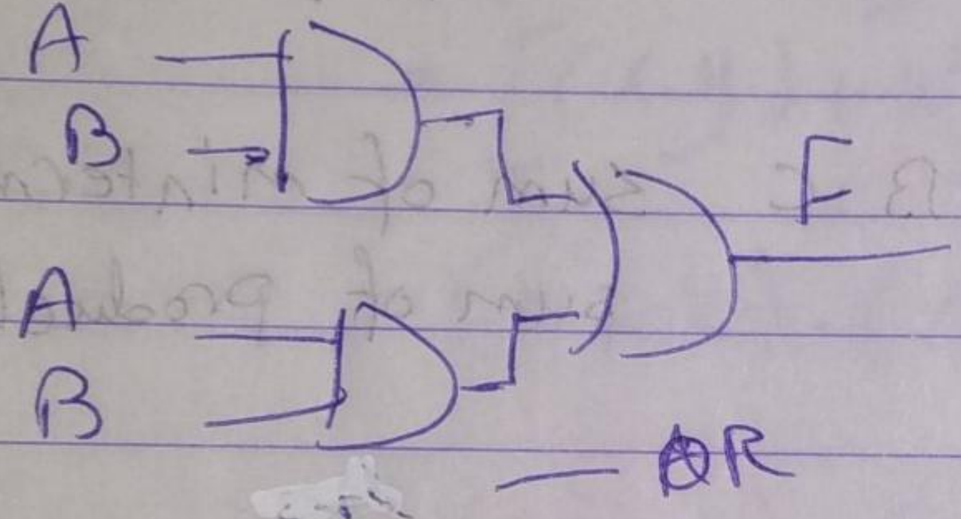
ex $F(A,B,C) = (A+B).(A'+B'+C')$.

product of sum ✓.

Note :- ① Sum of product

$$F(A,B,C) = (A \cdot B) + A' \cdot B' \cdot C'$$

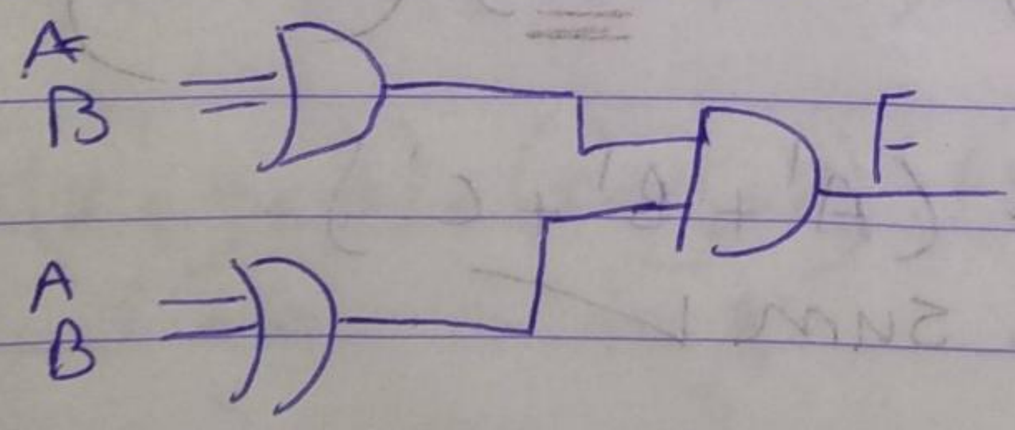
دلالة ← SoP, SoM ← and الليفل الأول
والليفل الثاني OR



② Product of Sum

$$F(A,B,C) = (A+B) \cdot (A'+B'+C')$$

POS, POM ← OR الليفل الأول
والثاني AND →



OR — AND.