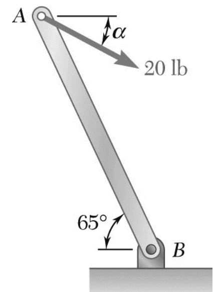
CHAPTER 3

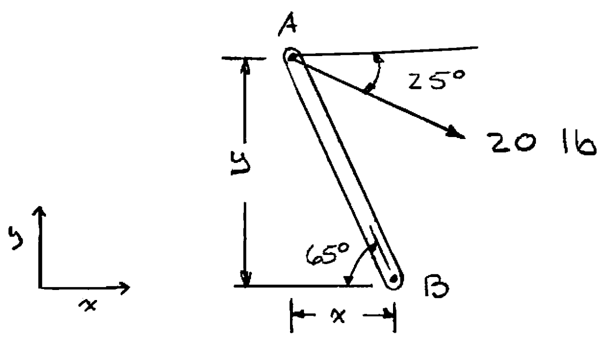


PROBLEM 3.1

A 20-lb force is applied to the control rod *AB* as shown. Knowing that the length of the rod is 9 in. and that $\alpha = 25°$, determine the moment of the force about Point *B* by resolving the force into horizontal and vertical components.

SOLUTION

Free-Body Diagram of Rod *AB*:



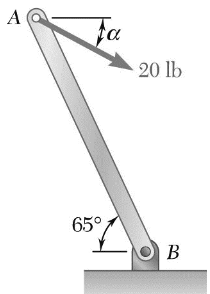
$$x = (9 \text{ in.})\cos 65°$$
$$= 3.8036 \text{ in.}$$
$$y = (9 \text{ in.})\sin 65°$$
$$= 8.1568 \text{ in.}$$

$$\mathbf{F} = F_x\mathbf{i} + F_y\mathbf{j}$$
$$= (20 \text{ lb} \cos 25°)\mathbf{i} + (-20 \text{ lb} \sin 25°)\mathbf{j}$$
$$= (18.1262 \text{ lb})\mathbf{i} - (8.4524 \text{ lb})\mathbf{j}$$

$$\mathbf{r}_{A/B} = \overrightarrow{BA} = (-3.8036 \text{ in.})\mathbf{i} + (8.1568 \text{ in.})\mathbf{j}$$

$$\mathbf{M}_B = \mathbf{r}_{A/B} \times \mathbf{F}$$
$$= (-3.8036\mathbf{i} + 8.1568\mathbf{j}) \times (18.1262\mathbf{i} - 8.4524\mathbf{j})$$
$$= 32.150\mathbf{k} - 147.852\mathbf{k}$$
$$= -115.702 \text{ lb-in.}$$

$\mathbf{M}_B = 115.7$ lb-in. ↻ ◀

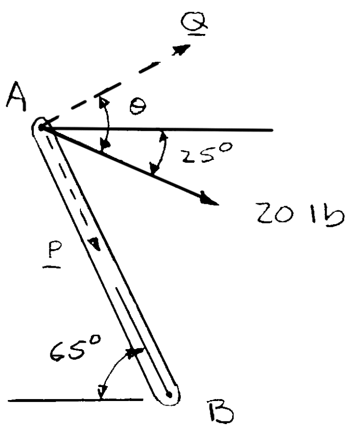


PROBLEM 3.2

A 20-lb force is applied to the control rod *AB* as shown. Knowing that the length of the rod is 9 in. and that $\alpha = 25°$, determine the moment of the force about Point *B* by resolving the force into components along *AB* and in a direction perpendicular to *AB*.

SOLUTION

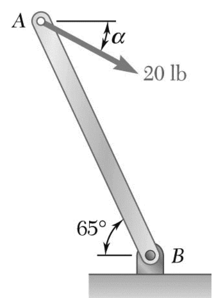
Free-Body Diagram of Rod *AB*:



$$\theta = 90° - (65° - 25°)$$
$$= 50°$$

$$Q = (20 \text{ lb})\cos 50°$$
$$= 12.8558 \text{ lb}$$
$$M_B = Q(9 \text{ in.})$$
$$= (12.8558 \text{ lb})(9 \text{ in.})$$
$$= 115.702 \text{ lb-in.}$$

$\mathbf{M}_B = 115.7$ lb-in. ↻ ◀

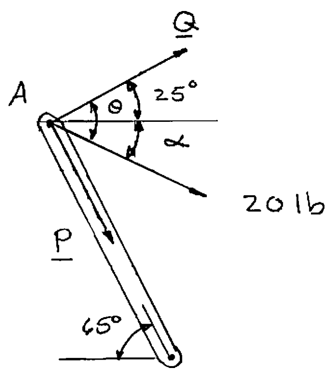


PROBLEM 3.3

A 20-lb force is applied to the control rod *AB* as shown. Knowing that the length of the rod is 9 in. and that the moment of the force about *B* is 120 lb · in. clockwise, determine the value of α.

SOLUTION

Free-Body Diagram of Rod *AB*:



$$\alpha = \theta - 25°$$

$$Q = (20\text{ lb})\cos\theta$$

and $$M_B = (Q)(9\text{ in.})$$

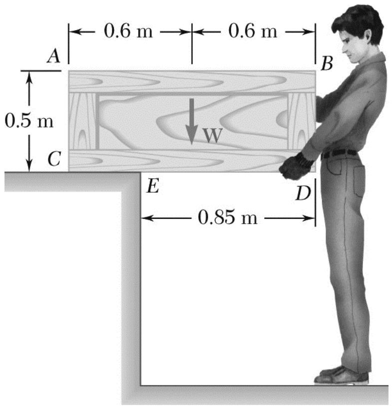
Therefore, $$120\text{ lb-in.} = (20\text{ lb})(\cos\theta)(9\text{ in.})$$

$$\cos\theta = \frac{120\text{ lb-in.}}{180\text{ lb-in.}}$$

or $$\theta = 48.190°$$

Therefore, $$\alpha = 48.190° - 25°$$

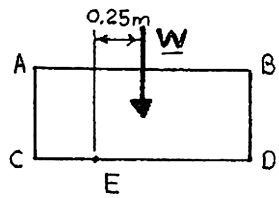
$\alpha = 23.2°$ ◀



PROBLEM 3.4

A crate of mass 80 kg is held in the position shown. Determine (*a*) the moment produced by the weight **W** of the crate about *E*, (*b*) the smallest force applied at *B* that creates a moment of equal magnitude and opposite sense about *E*.

SOLUTION

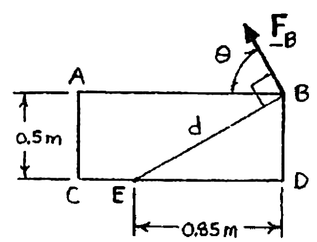


(*a*) By definition, $W = mg = 80\text{ kg}(9.81\text{ m/s}^2) = 784.8\text{ N}$

We have ΣM_E: $M_E = (784.8\text{ N})(0.25\text{ m})$

$$\mathbf{M}_E = 196.2\text{ N}\cdot\text{m} \circlearrowright \blacktriangleleft$$

(*b*) For the force at *B* to be the smallest, resulting in a moment ($\mathbf{M}_E$) about *E*, the line of action of force $\mathbf{F}_B$ must be perpendicular to the line connecting *E* to *B*. The sense of $\mathbf{F}_B$ must be such that the force produces a counterclockwise moment about *E*.



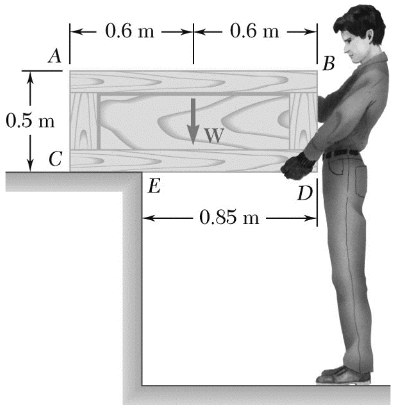
Note: $d = \sqrt{(0.85\text{ m})^2 + (0.5\text{ m})^2} = 0.98615\text{ m}$

We have ΣM_E: $196.2\text{ N}\cdot\text{m} = F_B(0.98615\text{ m})$

$$F_B = 198.954\text{ N}$$

and $\theta = \tan^{-1}\left(\dfrac{0.85\text{ m}}{0.5\text{ m}}\right) = 59.534°$

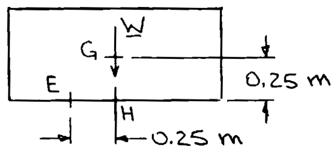
or $$\mathbf{F}_B = 199.0\text{ N} \measuredangle 59.5° \blacktriangleleft$$



PROBLEM 3.5

A crate of mass 80 kg is held in the position shown. Determine (*a*) the moment produced by the weight **W** of the crate about *E*, (*b*) the smallest force applied at *A* that creates a moment of equal magnitude and opposite sense about *E*, (*c*) the magnitude, sense, and point of application on the bottom of the crate of the smallest vertical force that creates a moment of equal magnitude and opposite sense about *E*.

SOLUTION

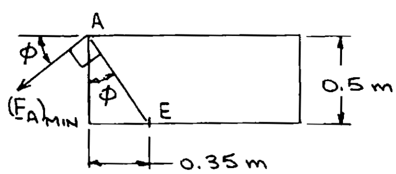


First note. . .

$$W = mg = (80\text{ kg})(9.81\text{ m/s}^2) = 784.8\text{ N}$$

(*a*) We have $M_E = r_{H/E}W = (0.25\text{ m})(784.8\text{ N}) = 196.2\text{ N}\cdot\text{m}$ or $\mathbf{M}_E = 196.2\text{ N}\cdot\text{m}$ ↻ ◀

(*b*)



For $\mathbf{F}_A$ to be minimum, it must be perpendicular to the line joining Points *A* and *E*. Then with $\mathbf{F}_A$ directed as shown, we have $(-M_E) = r_{A/E}(F_A)_{\min}$.

Where $r_{A/E} = \sqrt{(0.35\text{ m})^2 + (0.5\text{ m})^2} = 0.61033\text{ m}$

then $196.2\text{ N}\cdot\text{m} = (0.61033\text{ m})(F_A)_{\min}$

or $(F_A)_{\min} = 321\text{ N}$

Also $\tan\phi = \dfrac{0.35\text{ m}}{0.5\text{ m}}$ or $\phi = 35.0°$ $(\mathbf{F}_A)_{\min} = 321\text{ N} \measuredangle 35.0°$ ◀

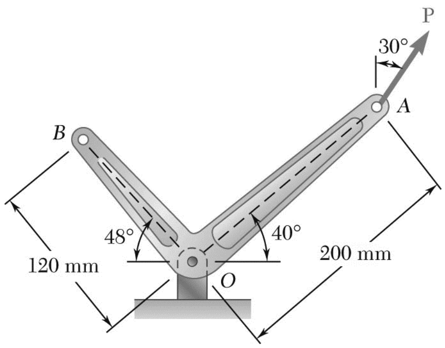
(*c*) For $\mathbf{F}_{vertical}$ to be minimum, the perpendicular distance from its line of action to Point *E* must be maximum. Thus, apply $(\mathbf{F}_{vertical})_{\min}$ at Point *D*, and then

$$(-M_E) = r_{D/E}(F_{vertical})_{\min}$$

$$196.2\text{ N}\cdot\text{m} = (0.85\text{ m})(F_{vertical})_{\min}$$

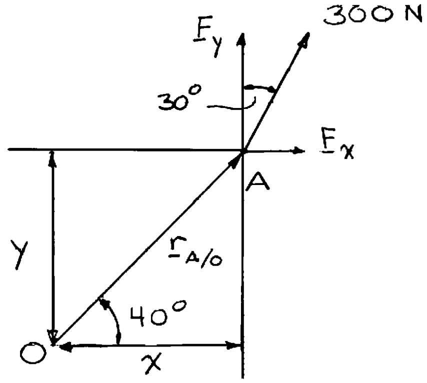
or $(\mathbf{F}_{vertical})_{\min} = 231\text{ N}\uparrow$ at Point *D* ◀

PROBLEM 3.6



A 300-N force **P** is applied at Point A of the bell crank shown. (a) Compute the moment of the force **P** about O by resolving it into horizontal and vertical components. (b) Using the result of part (a), determine the perpendicular distance from O to the line of action of **P**.

SOLUTION



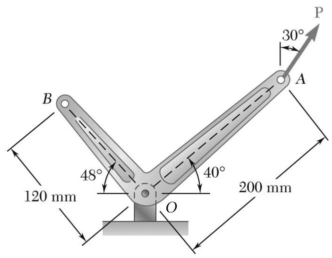
$$x = (0.2 \text{ m})\cos 40°$$
$$= 0.153209 \text{ m}$$
$$y = (0.2 \text{ m})\sin 40°$$
$$= 0.128558 \text{ m}$$
$$\therefore \quad \mathbf{r}_{A/O} = (0.153209 \text{ m})\mathbf{i} + (0.128558 \text{ m})\mathbf{j}$$

(a)
$$F_x = (300 \text{ N})\sin 30°$$
$$= 150 \text{ N}$$
$$F_y = (300 \text{ N})\cos 30°$$
$$= 259.81 \text{ N}$$
$$\mathbf{F} = (150 \text{ N})\mathbf{i} + (259.81 \text{ N})\mathbf{j}$$
$$\mathbf{M}_O = \mathbf{r}_{A/O} \times \mathbf{F}$$
$$= (0.153209\mathbf{i} + 0.128558\mathbf{j}) \text{ m} \times (150\mathbf{i} + 259.81\mathbf{j}) \text{ N}$$
$$= (39.805\mathbf{k} - 19.2837\mathbf{k}) \text{ N}\cdot\text{m}$$
$$= (20.521 \text{ N}\cdot\text{m})\mathbf{k}$$

$\mathbf{M}_O = 20.5 \text{ N}\cdot\text{m}$ ↺ ◀

(b)
$$M_O = Fd$$
$$20.521 \text{ N}\cdot\text{m} = (300 \text{ N})(d)$$
$$d = 0.068403 \text{ m}$$

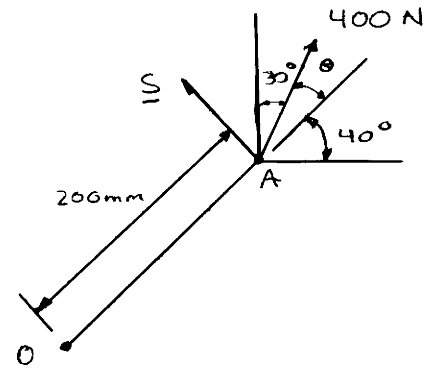
$d = 68.4 \text{ mm}$ ◀



PROBLEM 3.7

A 400-N force **P** is applied at Point *A* of the bell crank shown. (*a*) Compute the moment of the force **P** about *O* by resolving it into components along line *OA* and in a direction perpendicular to that line. (*b*) Determine the magnitude and direction of the smallest force **Q** applied at *B* that has the same moment as **P** about *O*.

SOLUTION



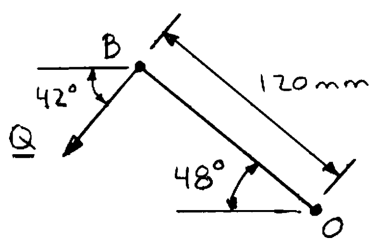
(*a*) Portion *OA* of crank:

$$\theta = 90° - 30° - 40°$$
$$\theta = 20°$$
$$S = P\sin\theta$$
$$= (400\text{ N})\sin 20°$$
$$= 136.81\text{ N}$$
$$M_O = r_{O/A}S$$
$$= (0.2\text{ m})(136.81\text{ N})$$
$$= 27.362\text{ N}\cdot\text{m}$$

$\mathbf{M}_O = 27.4\text{ N}\cdot\text{m}$ ↻ ◀

(*b*) Smallest force *Q* must be perpendicular to *OB*.

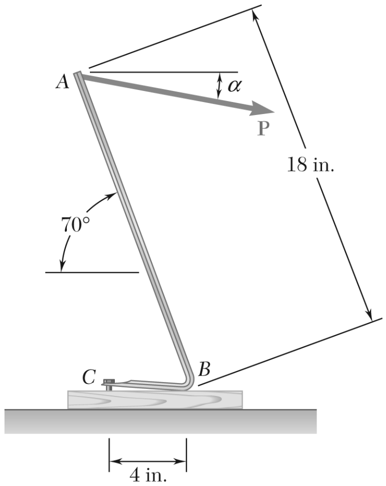
Portion *OB* of crank:



$$M_O = r_{O/B}Q$$
$$M_O = (0.120\text{ m})Q$$
$$27.362\text{ N}\cdot\text{m} = (0.120\text{ m})Q$$

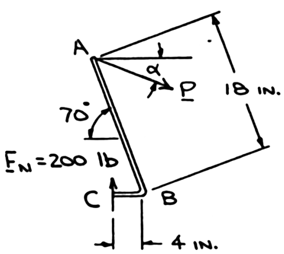
$\mathbf{Q} = 228\text{ N}$ ↗ 42.0° ◀

PROBLEM 3.8



It is known that a vertical force of 200 lb is required to remove the nail at *C* from the board. As the nail first starts moving, determine (*a*) the moment about *B* of the force exerted on the nail, (*b*) the magnitude of the force **P** that creates the same moment about *B* if $\alpha = 10°$, (*c*) the smallest force **P** that creates the same moment about *B*.

SOLUTION

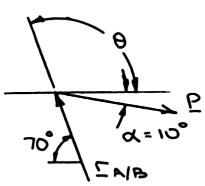


(*a*) We have $M_B = r_{C/B} F_N$

$= (4 \text{ in.})(200 \text{ lb})$

$= 800 \text{ lb} \cdot \text{in.}$

or $M_B = 800 \text{ lb} \cdot \text{in.}$ ↻ ◀



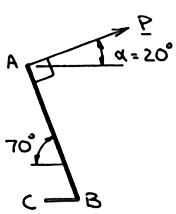
(*b*) By definition, $M_B = r_{A/B} P \sin\theta$

$\theta = 10° + (180° - 70°)$

$= 120°$

Then $800 \text{ lb} \cdot \text{in.} = (18 \text{ in.}) \times P \sin 120°$

or $P = 51.3 \text{ lb}$ ◀



(*c*) For **P** to be minimum, it must be perpendicular to the line joining Points *A* and *B*. Thus, **P** must be directed as shown.

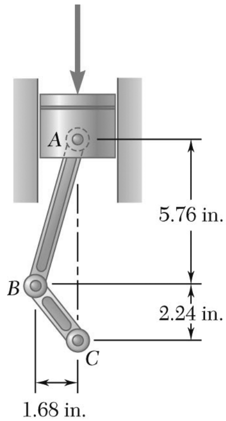
Thus $M_B = dP_{\min}$

$d = r_{A/B}$

or $800 \text{ lb} \cdot \text{in.} = (18 \text{ in.}) P_{\min}$

or $P_{\min} = 44.4 \text{ lb}$ $\mathbf{P}_{\min} = 44.4 \text{ lb} \measuredangle 20°$ ◀

PROBLEM 3.9



It is known that the connecting rod AB exerts on the crank BC a 500-lb force directed down and to the left along the centerline of AB. Determine the moment of the force about C.

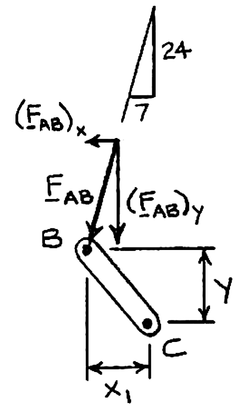
SOLUTION

Using (*a*):

$$M_C = y_1(F_{AB})_x + x_1(F_{AB})_y$$

$$= (2.24 \text{ in.})\left(\frac{7}{25}\times 500 \text{ lb}\right) + (1.68 \text{ in.})\left(\frac{24}{25}\times 500 \text{ lb}\right)$$

$$= 1120 \text{ lb}\cdot\text{in.}$$



(*a*)

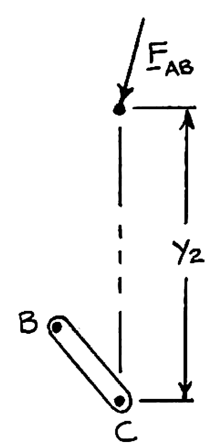
$\mathbf{M}_C = 1.120 \text{ kip}\cdot\text{in.}$ ↻ ◀

Using (*b*):

$$M_C = y_2(F_{AB})_x$$

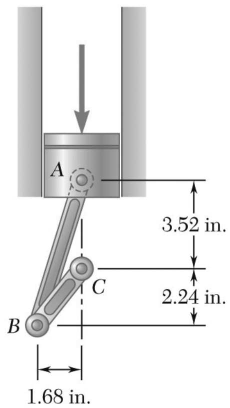
$$= (8 \text{ in.})\left(\frac{7}{25}\times 500 \text{ lb}\right)$$

$$= 1120 \text{ lb}\cdot\text{in.}$$



(*b*)

$\mathbf{M}_C = 1.120 \text{ kip}\cdot\text{in.}$ ↻ ◀



PROBLEM 3.10

It is known that the connecting rod *AB* exerts on the crank *BC* a 500-lb force directed down and to the left along the centerline of *AB*. Determine the moment of the force about *C*.

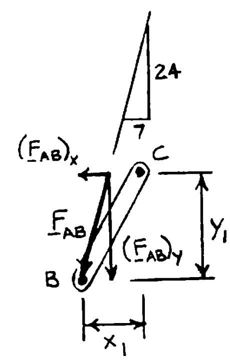
SOLUTION

Using (*a*):

$$M_C = -y_1(F_{AB})_x + x_1(F_{AB})_y$$

$$= -(2.24\text{ in.})\left(\frac{7}{25}\times 500\text{ lb}\right) + (1.68\text{ in.})\left(\frac{24}{25}\times 500\text{ lb}\right)$$

$$= +492.8\text{ lb}\cdot\text{in.}$$



(*a*)

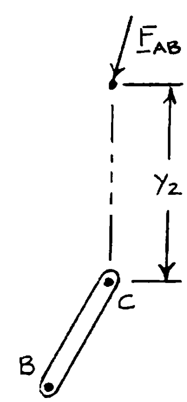
$$\mathbf{M}_C = 493\text{ lb}\cdot\text{in.} \circlearrowleft \blacktriangleleft$$

Using (*b*):

$$M_C = y_2(F_{AB})_x$$

$$= (3.52\text{ in.})\left(\frac{7}{25}\times 500\text{ lb}\right)$$

$$= +492.8\text{ lb}\cdot\text{in.}$$

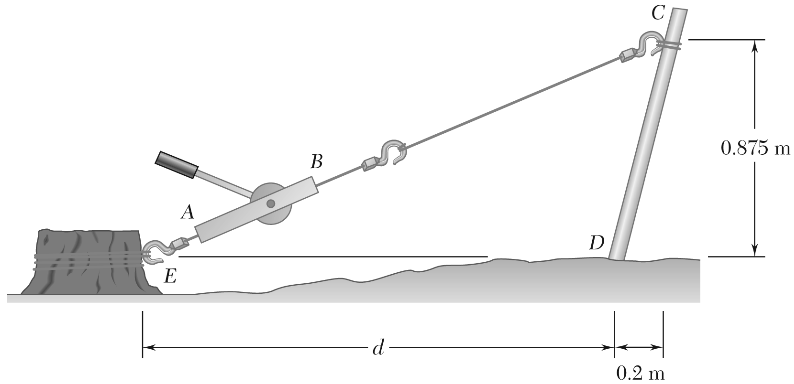


(*b*)

$$\mathbf{M}_C = 493\text{ lb}\cdot\text{in.} \circlearrowleft \blacktriangleleft$$

PROBLEM 3.11

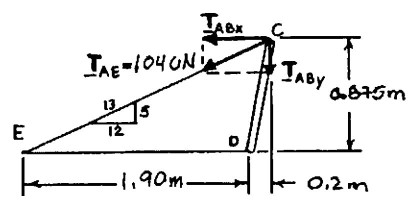
A winch puller *AB* is used to straighten a fence post. Knowing that the tension in cable *BC* is 1040 N and length *d* is 1.90 m, determine the moment about *D* of the force exerted by the cable at *C* by resolving that force into horizontal and vertical components applied (*a*) at Point *C*, (*b*) at Point *E*.



SOLUTION

(*a*) Slope of line:

$$EC = \frac{0.875\text{ m}}{1.90\text{ m} + 0.2\text{ m}} = \frac{5}{12}$$



(*a*)

Then

$$T_{ABx} = \frac{12}{13}(T_{AB})$$
$$= \frac{12}{13}(1040\text{ N})$$
$$= 960\text{ N}$$

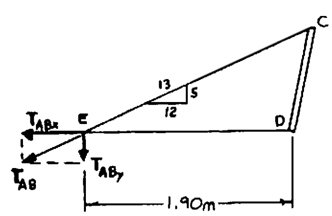
and

$$T_{ABy} = \frac{5}{13}(1040\text{ N})$$
$$= 400\text{ N}$$

Then

$$M_D = T_{ABx}(0.875\text{ m}) - T_{ABy}(0.2\text{ m})$$
$$= (960\text{ N})(0.875\text{ m}) - (400\text{ N})(0.2\text{ m})$$
$$= 760\text{ N}\cdot\text{m}$$

or $\mathbf{M}_D = 760\text{ N}\cdot\text{m}$ ↻ ◀



(*b*)

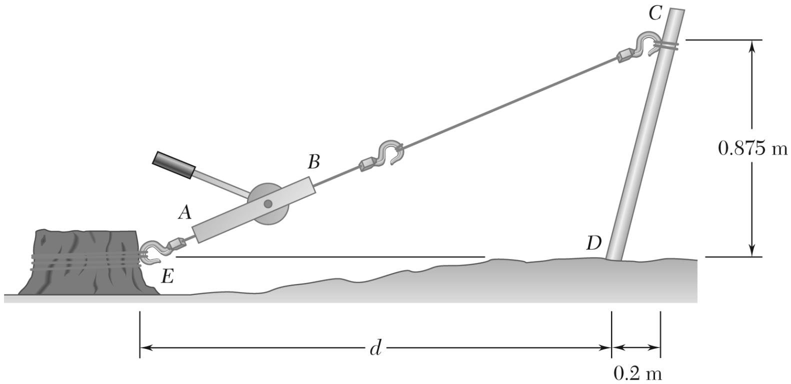
(*b*) We have

$$M_D = T_{ABx}(y) + T_{ABx}(x)$$
$$= (960\text{ N})(0) + (400\text{ N})(1.90\text{ m})$$
$$= 760\text{ N}\cdot\text{m}$$

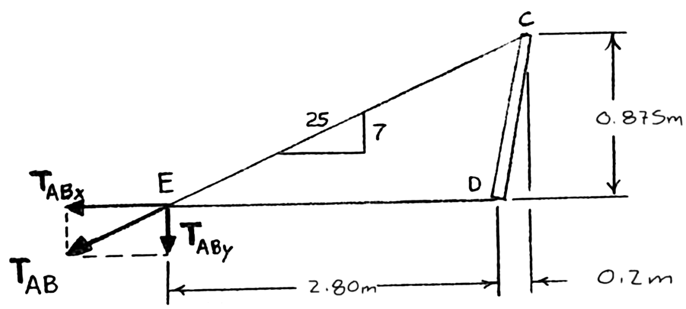
or $\mathbf{M}_D = 760\text{ N}\cdot\text{m}$ ↻ ◀

PROBLEM 3.12

It is known that a force with a moment of 960 N · m about D is required to straighten the fence post CD. If d = 2.80 m, determine the tension that must be developed in the cable of winch puller AB to create the required moment about Point D.



SOLUTION



Slope of line: $EC = \dfrac{0.875\text{ m}}{2.80\text{ m} + 0.2\text{ m}} = \dfrac{7}{24}$

Then $T_{ABx} = \dfrac{24}{25} T_{AB}$

and $T_{ABy} = \dfrac{7}{25} T_{AB}$

We have $M_D = T_{ABx}(y) + T_{ABy}(x)$

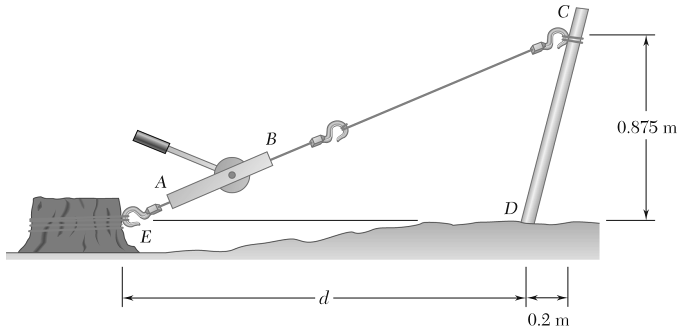
$$960\text{ N}\cdot\text{m} = \frac{24}{25}T_{AB}(0) + \frac{7}{25}T_{AB}(2.80\text{ m})$$

$$T_{AB} = 1224\text{ N}$$

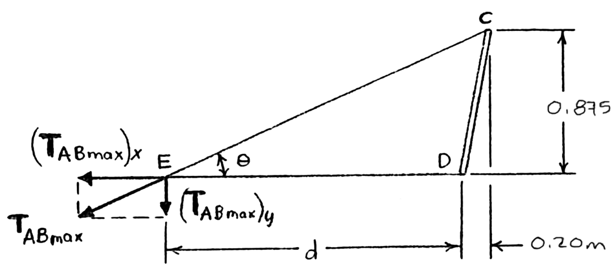
or $T_{AB} = 1224$ N ◀

PROBLEM 3.13

It is known that a force with a moment of 960 N · m about D is required to straighten the fence post CD. If the capacity of winch puller AB is 2400 N, determine the minimum value of distance d to create the specified moment about Point D.



SOLUTION



The minimum value of d can be found based on the equation relating the moment of the force $\mathbf{T}_{AB}$ about D:

$$M_D = (T_{AB\,\max})_y(d)$$

where
$$M_D = 960\ \text{N}\cdot\text{m}$$

$$(T_{AB\,\max})_y = T_{AB\,\max}\sin\theta = (2400\ \text{N})\sin\theta$$

Now
$$\sin\theta = \frac{0.875\ \text{m}}{\sqrt{(d+0.20)^2+(0.875)^2}\ \text{m}}$$

$$960\ \text{N}\cdot\text{m} = 2400\ \text{N}\left[\frac{0.875}{\sqrt{(d+0.20)^2+(0.875)^2}}\right](d)$$

or
$$\sqrt{(d+0.20)^2+(0.875)^2} = 2.1875d$$

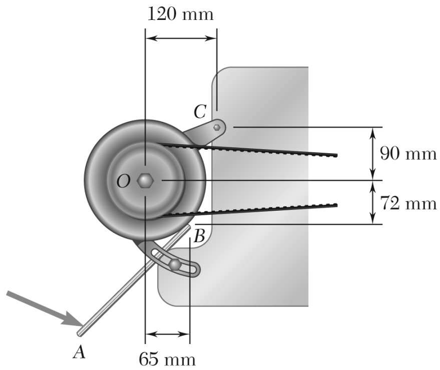
or
$$(d+0.20)^2+(0.875)^2 = 4.7852d^2$$

or
$$3.7852d^2 - 0.40d - 0.8056 = 0$$

Using the quadratic equation, the minimum values of d are 0.51719 m and -0.41151 m.

Since only the positive value applies here, $d = 0.51719$ m

or $d = 517$ mm ◀

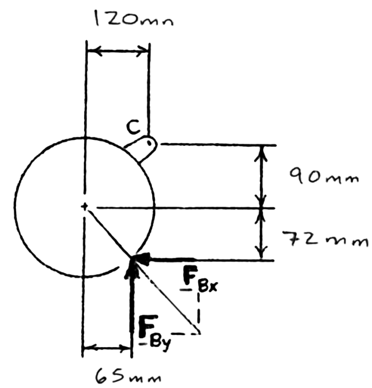


PROBLEM 3.14

A mechanic uses a piece of pipe AB as a lever when tightening an alternator belt. When he pushes down at A, a force of 485 N is exerted on the alternator at B. Determine the moment of that force about bolt C if its line of action passes through O.

SOLUTION

We have $$\mathbf{M}_C = \mathbf{r}_{B/C} \times \mathbf{F}_B$$



Noting the direction of the moment of each force component about C is clockwise,

$$M_C = xF_{By} + yF_{Bx}$$

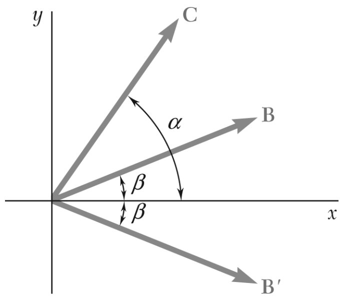
where $$x = 120\text{ mm} - 65\text{ mm} = 55\text{ mm}$$
$$y = 72\text{ mm} + 90\text{ mm} = 162\text{ mm}$$

and $$F_{Bx} = \frac{65}{\sqrt{(65)^2 + (72)^2}}(485\text{ N}) = 325\text{ N}$$

$$F_{By} = \frac{72}{\sqrt{(65)^2 + (72)^2}}(485\text{ N}) = 360\text{ N}$$

$$M_C = (55\text{ mm})(360\text{ N}) + (162)(325\text{ N})$$
$$= 72450\text{ N}\cdot\text{m}$$
$$= 72.450\text{ N}\cdot\text{m}$$

or $\mathbf{M}_C = 72.5\text{ N}\cdot\text{m}$ ↻ ◀



PROBLEM 3.15

Form the vector products **B** × **C** and **B′** × **C**, where $B = B'$, and use the results obtained to prove the identity

$$\sin\alpha\cos\beta = \frac{1}{2}\sin(\alpha+\beta) + \frac{1}{2}\sin(\alpha-\beta).$$

SOLUTION

Note:

$$\mathbf{B} = B(\cos\beta\,\mathbf{i} + \sin\beta\,\mathbf{j})$$
$$\mathbf{B}' = B(\cos\beta\,\mathbf{i} - \sin\beta\,\mathbf{j})$$
$$\mathbf{C} = C(\cos\alpha\,\mathbf{i} + \sin\alpha\,\mathbf{j})$$

By definition,

$$|\mathbf{B}\times\mathbf{C}| = BC\sin(\alpha-\beta) \quad (1)$$

$$|\mathbf{B}'\times\mathbf{C}| = BC\sin(\alpha+\beta) \quad (2)$$

Now

$$\mathbf{B}\times\mathbf{C} = B(\cos\beta\mathbf{i} + \sin\beta\mathbf{j})\times C(\cos\alpha\mathbf{i} + \sin\alpha\mathbf{j})$$

$$= BC(\cos\beta\sin\alpha - \sin\beta\cos\alpha)\mathbf{k} \quad (3)$$

and

$$\mathbf{B}'\times\mathbf{C} = B(\cos\beta\mathbf{i} - \sin\beta\mathbf{j})\times C(\cos\alpha\mathbf{i} + \sin\alpha\mathbf{j})$$

$$= BC(\cos\beta\sin\alpha + \sin\beta\cos\alpha)\mathbf{k} \quad (4)$$

Equating the magnitudes of $\mathbf{B}\times\mathbf{C}$ from Equations (1) and (3) yields:

$$BC\sin(\alpha-\beta) = BC(\cos\beta\sin\alpha - \sin\beta\cos\alpha) \quad (5)$$

Similarly, equating the magnitudes of $\mathbf{B}'\times\mathbf{C}$ from Equations (2) and (4) yields:

$$BC\sin(\alpha+\beta) = BC(\cos\beta\sin\alpha + \sin\beta\cos\alpha) \quad (6)$$

Adding Equations (5) and (6) gives:

$$\sin(\alpha-\beta) + \sin(\alpha+\beta) = 2\cos\beta\sin\alpha$$

or $\sin\alpha\cos\beta = \frac{1}{2}\sin(\alpha+\beta) + \frac{1}{2}\sin(\alpha-\beta)$ ◀

PROBLEM 3.16

The vectors **P** and **Q** are two adjacent sides of a parallelogram. Determine the area of the parallelogram when (*a*) $\mathbf{P} = -7\mathbf{i} + 3\mathbf{j} - 3\mathbf{k}$ and $\mathbf{Q} = 2\mathbf{i} + 2\mathbf{j} + 5\mathbf{k}$, (*b*) $\mathbf{P} = 6\mathbf{i} - 5\mathbf{j} - 2\mathbf{k}$ and $\mathbf{Q} = -2\mathbf{i} + 5\mathbf{j} - \mathbf{k}$.

SOLUTION

(*a*) We have $A = |\mathbf{P} \times \mathbf{Q}|$

where $\mathbf{P} = -7\mathbf{i} + 3\mathbf{j} - 3\mathbf{k}$

$\mathbf{Q} = 2\mathbf{i} + 2\mathbf{j} + 5\mathbf{k}$

Then

$$\mathbf{P} \times \mathbf{Q} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -7 & 3 & -3 \\ 2 & 2 & 5 \end{vmatrix}$$

$$= [(15+6)\mathbf{i} + (-6+35)\mathbf{j} + (-14-6)\mathbf{k}]$$

$$= (21)\mathbf{i} + (29)\mathbf{j}(-20)\mathbf{k}$$

$$A = \sqrt{(20)^2 + (29)^2 + (-20)^2}$$

or $A = 41.0$ ◀

(*b*) We have $A = |\mathbf{P} \times \mathbf{Q}|$

where $\mathbf{P} = 6\mathbf{i} - 5\mathbf{j} - 2\mathbf{k}$

$\mathbf{Q} = -2\mathbf{i} + 5\mathbf{j} - 1\mathbf{k}$

Then

$$\mathbf{P} \times \mathbf{Q} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 6 & -5 & -2 \\ -2 & 5 & -1 \end{vmatrix}$$

$$= [(5+10)\mathbf{i} + (4+6)\mathbf{j} + (30-10)\mathbf{k}]$$

$$= (15)\mathbf{i} + (10)\mathbf{j} + (20)\mathbf{k}$$

$$A = \sqrt{(15)^2 + (10)^2 + (20)^2}$$

or $A = 26.9$ ◀

PROBLEM 3.17

A plane contains the vectors **A** and **B**. Determine the unit vector normal to the plane when **A** and **B** are equal to, respectively, (*a*) **i** + 2**j** – 5**k** and 4**i** – 7**j** – 5**k**, (*b*) 3**i** – 3**j** + 2**k** and –2**i** + 6**j** – 4**k**.

SOLUTION

(*a*) We have

$$\boldsymbol{\lambda} = \frac{\mathbf{A}\times\mathbf{B}}{|\mathbf{A}\times\mathbf{B}|}$$

where

$$\mathbf{A} = 1\mathbf{i} + 2\mathbf{j} - 5\mathbf{k}$$

$$\mathbf{B} = 4\mathbf{i} - 7\mathbf{j} - 5\mathbf{k}$$

Then

$$\mathbf{A}\times\mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & +2 & -5 \\ 4 & -7 & -5 \end{vmatrix}$$

$$= (-10-35)\mathbf{i} + (20+5)\mathbf{j} + (-7-8)\mathbf{k}$$

$$= 15(3\mathbf{i} - 1\mathbf{j} - 1\mathbf{k})$$

and

$$|\mathbf{A}\times\mathbf{B}| = 15\sqrt{(-3)^2 + (-1)^2 + (-1)^2} = 15\sqrt{11}$$

$$\boldsymbol{\lambda} = \frac{15(-3\mathbf{i} - 1\mathbf{j} - 1\mathbf{k})}{15\sqrt{11}}$$

or $\boldsymbol{\lambda} = \frac{1}{\sqrt{11}}(-3\mathbf{i} - \mathbf{j} - \mathbf{k})$ ◀

(*b*) We have

$$\boldsymbol{\lambda} = \frac{\mathbf{A}\times\mathbf{B}}{|\mathbf{A}\times\mathbf{B}|}$$

where

$$\mathbf{A} = 3\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$$

$$\mathbf{B} = -2\mathbf{i} + 6\mathbf{j} - 4\mathbf{k}$$

Then

$$\mathbf{A}\times\mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & -3 & 2 \\ -2 & 6 & -4 \end{vmatrix}$$

$$= (12-12)\mathbf{i} + (-4+12)\mathbf{j} + (18-6)\mathbf{k}$$

$$= (8\mathbf{j} + 12\mathbf{k})$$

and

$$|\mathbf{A}\times\mathbf{B}| = 4\sqrt{(2)^2 + (3)^2} = 4\sqrt{13}$$

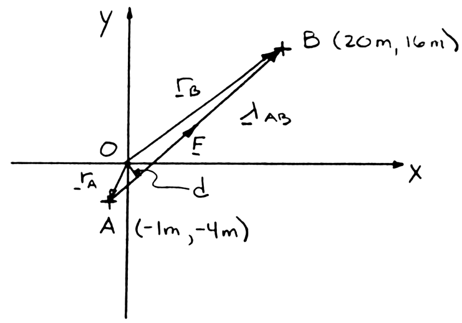
$$\boldsymbol{\lambda} = \frac{4(2\mathbf{j} + 3\mathbf{k})}{4\sqrt{13}}$$

or $\boldsymbol{\lambda} = \frac{1}{\sqrt{13}}(2\mathbf{j} + 3\mathbf{k})$ ◀

PROBLEM 3.18

A line passes through the Points (20 m, 16 m) and (−1 m, −4 m). Determine the perpendicular distance d from the line to the origin O of the system of coordinates.

SOLUTION



$$d_{AB} = \sqrt{[20\text{ m} - (-1\text{ m})]^2 + [16\text{ m} - (-4\text{ m})]^2}$$
$$= 29\text{ m}$$

Assume that a force **F**, or magnitude F(N), acts at Point A and is directed from A to B.

Then $$\mathbf{F} = F\boldsymbol{\lambda}_{AB}$$

where $$\boldsymbol{\lambda}_{AB} = \frac{\mathbf{r}_B - \mathbf{r}_A}{d_{AB}}$$
$$= \frac{1}{29}(21\mathbf{i} + 20\mathbf{j})$$

By definition, $$\mathbf{M}_O = |\mathbf{r}_A \times \mathbf{F}| = dF$$

where $$\mathbf{r}_A = -(1\text{ m})\mathbf{i} - (4\text{ m})\mathbf{j}$$

Then $$\mathbf{M}_O = [-(-1\text{ m})\mathbf{i} - (4\text{ m})\mathbf{j}] \times \frac{F}{29\text{ m}}[(21\text{ m})\mathbf{i} + (20\text{ m})\mathbf{j}]$$
$$= \frac{F}{29}[-(20)\mathbf{k} + (84)\mathbf{k}]$$
$$= \left(\frac{64}{29}F\right)\mathbf{k}\text{ N}\cdot\text{m}$$

Finally, $$\left(\frac{64}{29}F\right) = d(F)$$
$$d = \frac{64}{29}\text{ m}$$

$d = 2.21$ m ◀

PROBLEM 3.19

Determine the moment about the origin O of the force $\mathbf{F} = 4\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}$ that acts at a Point A. Assume that the position vector of A is (*a*) $\mathbf{r} = 2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}$, (*b*) $\mathbf{r} = -8\mathbf{i} + 6\mathbf{j} - 10\mathbf{k}$, (c) $\mathbf{r} = 8\mathbf{i} - 6\mathbf{j} + 5\mathbf{k}$.

SOLUTION

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F}$$

(*a*)

$$\mathbf{M}_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 3 & -4 \\ 4 & -3 & 5 \end{vmatrix}$$

$$= (15-12)\mathbf{i} + (-16-10)\mathbf{j} + (-6-12)\mathbf{k}$$

$\mathbf{M}_O = 3\mathbf{i} - 26\mathbf{j} - 18\mathbf{k}$ ◀

(*b*)

$$\mathbf{M}_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -8 & 6 & -10 \\ 4 & -3 & 5 \end{vmatrix}$$

$$= (30-30)\mathbf{i} + (-40+40)\mathbf{j} + (24-24)\mathbf{k}$$

$\mathbf{M}_O = 0$ ◀

(*c*)

$$\mathbf{M}_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 8 & -6 & 5 \\ 4 & -3 & 5 \end{vmatrix}$$

$$= (-30+15)\mathbf{i} + (20-40)\mathbf{j} + (-24+24)\mathbf{k}$$

$\mathbf{M}_O = -15\mathbf{i} - 20\mathbf{j}$ ◀

Note: The answer to Part (*b*) could have been anticipated since the elements of the last two rows of the determinant are proportional.

PROBLEM 3.20

Determine the moment about the origin O of the force $\mathbf{F} = 2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}$ that acts at a Point A. Assume that the position vector of A is (*a*) $\mathbf{r} = 3\mathbf{i} - 6\mathbf{j} + 5\mathbf{k}$, (*b*) $\mathbf{r} = \mathbf{i} - 4\mathbf{j} - 2\mathbf{k}$, (*c*) $\mathbf{r} = 4\mathbf{i} + 6\mathbf{j} - 8\mathbf{k}$.

SOLUTION

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F}$$

(*a*)

$$\mathbf{M}_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & -6 & 5 \\ 2 & 3 & -4 \end{vmatrix}$$

$$= (24 - 15)\mathbf{i} + (10 + 12)\mathbf{j} + (9 + 12)\mathbf{k}$$

$\mathbf{M}_O = 9\mathbf{i} + 22\mathbf{j} + 21\mathbf{k}$ ◀

(*b*)

$$\mathbf{M}_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -4 & -2 \\ 2 & 3 & -4 \end{vmatrix}$$

$$= (16 + 6)\mathbf{i} + (-4 + 4)\mathbf{j} + (3 + 8)\mathbf{k}$$

$\mathbf{M}_O = 22\mathbf{i} + 11\mathbf{k}$ ◀

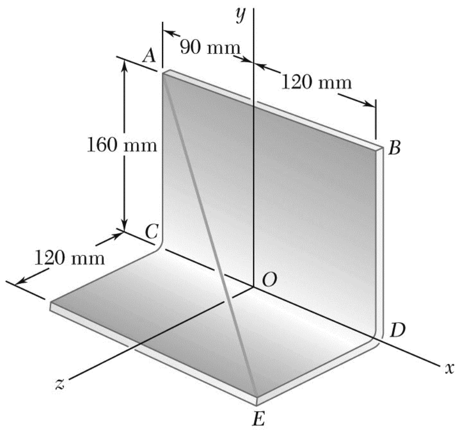
(*c*)

$$\mathbf{M}_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 6 & -8 \\ 2 & 3 & -4 \end{vmatrix}$$

$$= (-24 + 24)\mathbf{i} + (-16 + 16)\mathbf{j} + (12 - 12)\mathbf{k}$$

$\mathbf{M}_O = 0$ ◀

Note: The answer to Part (*c*) could have been anticipated since the elements of the last two rows of the determinant are proportional.



PROBLEM 3.21

The wire AE is stretched between the corners A and E of a bent plate. Knowing that the tension in the wire is 435 N, determine the moment about O of the force exerted by the wire (a) on corner A, (b) on corner E.

SOLUTION

$$\overline{AE} = (0.21\text{ m})\mathbf{i} - (0.16\text{ m})\mathbf{j} + (0.12\text{ m})\mathbf{k}$$

$$AE = \sqrt{(0.21\text{ m})^2 + (-0.16\text{ m})^2 + (0.12\text{ m})^2} = 0.29\text{ m}$$

(a)
$$\mathbf{F}_A = F_A \boldsymbol{\lambda}_{AE} = F\frac{\overline{AE}}{AE}$$

$$= (435\text{ N})\frac{0.21\mathbf{i} - 0.16\mathbf{j} + 0.12\mathbf{k}}{0.29}$$

$$= (315\text{ N})\mathbf{i} - (240\text{ N})\mathbf{j} + (180\text{ N})\mathbf{k}$$

$$\mathbf{r}_{A/O} = -(0.09\text{ m})\mathbf{i} + (0.16\text{ m})\mathbf{j}$$

$$\mathbf{M}_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.09 & 0.16 & 0 \\ 315 & -240 & 180 \end{vmatrix}$$

$$= 28.8\mathbf{i} + 16.20\mathbf{j} + (21.6 - 50.4)\mathbf{k}$$

$$\mathbf{M}_O = (28.8\text{ N}\cdot\text{m})\mathbf{i} + (16.20\text{ N}\cdot\text{m})\mathbf{j} - (28.8\text{ N}\cdot\text{m})\mathbf{k} \blacktriangleleft$$

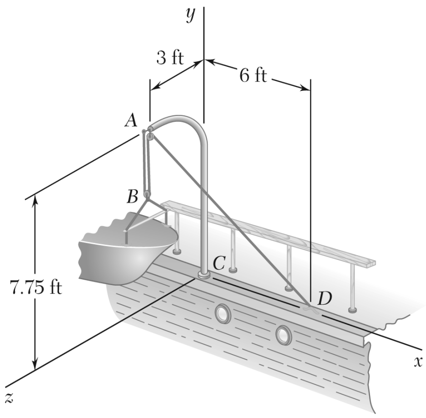
(b)
$$\mathbf{F}_E = -\mathbf{F}_A = -(315\text{ N})\mathbf{i} + (240\text{ N})\mathbf{j} - (180\text{ N})\mathbf{k}$$

$$\mathbf{r}_{E/O} = (0.12\text{ m})\mathbf{i} + (0.12\text{ m})\mathbf{k}$$

$$\mathbf{M}_O = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.12 & 0 & 0.12 \\ -315 & 240 & -180 \end{vmatrix}$$

$$= -28.8\mathbf{i} + (-37.8 + 21.6)\mathbf{j} + 28.8\mathbf{k}$$

$$\mathbf{M}_O = -(28.8\text{ N}\cdot\text{m})\mathbf{i} - (16.20\text{ N}\cdot\text{m})\mathbf{j} + (28.8\text{ N}\cdot\text{m})\mathbf{k} \blacktriangleleft$$



PROBLEM 3.22

A small boat hangs from two davits, one of which is shown in the figure. The tension in line *ABAD* is 82 lb. Determine the moment about *C* of the resultant force $\mathbf{R}_A$ exerted on the davit at *A*.

SOLUTION

We have
$$\mathbf{R}_A = 2\mathbf{F}_{AB} + \mathbf{F}_{AD}$$

where
$$\mathbf{F}_{AB} = -(82 \text{ lb})\mathbf{j}$$

and
$$\mathbf{F}_{AD} = F_{AD}\frac{\overrightarrow{AD}}{AD} = (82 \text{ lb})\frac{6\mathbf{i} - 7.75\mathbf{j} - 3\mathbf{k}}{10.25}$$
$$\mathbf{F}_{AD} = (48 \text{ lb})\mathbf{i} - (62 \text{ lb})\mathbf{j} - (24 \text{ lb})\mathbf{k}$$

Thus
$$\mathbf{R}_A = 2\mathbf{F}_{AB} + \mathbf{F}_{AD} = (48 \text{ lb})\mathbf{i} - (226 \text{ lb})\mathbf{j} - (24 \text{ lb})\mathbf{k}$$

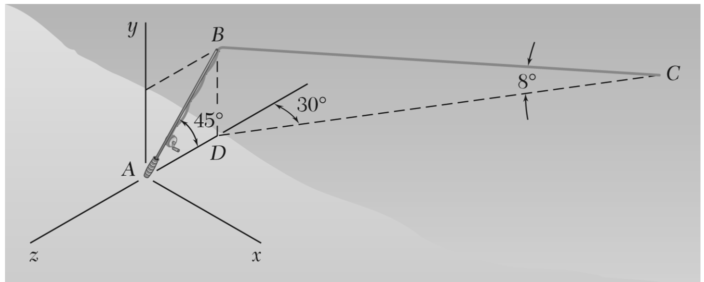
Also
$$\mathbf{r}_{A/C} = (7.75 \text{ ft})\mathbf{j} + (3 \text{ ft})\mathbf{k}$$

Using Eq. (3.21):
$$\mathbf{M}_C = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 7.75 & 3 \\ 48 & -226 & -24 \end{vmatrix}$$
$$= (492 \text{ lb}\cdot\text{ft})\mathbf{i} + (144.0 \text{ lb}\cdot\text{ft})\mathbf{j} - (372 \text{ lb}\cdot\text{ft})\mathbf{k}$$

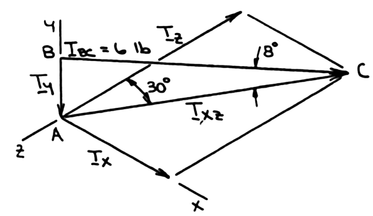
$$\mathbf{M}_C = (492 \text{ lb}\cdot\text{ft})\mathbf{i} + (144.0 \text{ lb}\cdot\text{ft})\mathbf{j} - (372 \text{ lb}\cdot\text{ft})\mathbf{k} \blacktriangleleft$$

PROBLEM 3.23

A 6-ft-long fishing rod *AB* is securely anchored in the sand of a beach. After a fish takes the bait, the resulting force in the line is 6 lb. Determine the moment about *A* of the force exerted by the line at *B*.



SOLUTION



We have $$T_{xz} = (6 \text{ lb})\cos 8° = 5.9416 \text{ lb}$$

Then $$T_x = T_{xz} \sin 30° = 2.9708 \text{ lb}$$
$$T_y = T_{BC} \sin 8° = -0.83504 \text{ lb}$$
$$T_z = T_{xz} \cos 30° = -5.1456 \text{ lb}$$

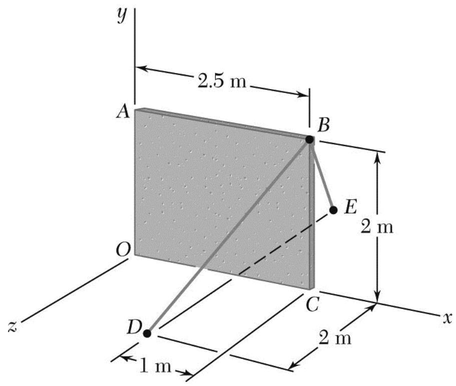
Now $$\mathbf{M}_A = \mathbf{r}_{B/A} \times \mathbf{T}_{BC}$$

where $$\mathbf{r}_{B/A} = (6\sin 45°)\mathbf{j} - (6\cos 45°)\mathbf{k}$$
$$= \frac{6 \text{ ft}}{\sqrt{2}}(\mathbf{j} - \mathbf{k})$$

Then $$\mathbf{M}_A = \frac{6}{\sqrt{2}}\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 1 & -1 \\ 2.9708 & -0.83504 & -5.1456 \end{vmatrix}$$
$$= \frac{6}{\sqrt{2}}(-5.1456 - 0.83504)\mathbf{i} - \frac{6}{\sqrt{2}}(2.9708)\mathbf{j} - \frac{6}{\sqrt{2}}(2.9708)\mathbf{k}$$

or $$\mathbf{M}_A = -(25.4 \text{ lb}\cdot\text{ft})\mathbf{i} - (12.60 \text{ lb}\cdot\text{ft})\mathbf{j} - (12.60 \text{ lb}\cdot\text{ft})\mathbf{k}$$ ◀

PROBLEM 3.24



A precast concrete wall section is temporarily held by two cables as shown. Knowing that the tension in cable BD is 900 N, determine the moment about Point O of the force exerted by the cable at B.

SOLUTION

$$\mathbf{F} = F\frac{\overrightarrow{BD}}{BD} \quad \text{where} \quad F = 900\text{ N}$$

$$\overrightarrow{BD} = -(1\text{ m})\mathbf{i} - (2\text{ m})\mathbf{j} + (2\text{ m})\mathbf{k}$$

$$BD = \sqrt{(-1\text{ m})^2 + (-2\text{ m})^2 + (2\text{ m})^2}$$

$$= 3\text{ m}$$

$$\mathbf{F} = (900\text{ N})\frac{-\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}}{3}$$

$$= -(300\text{ N})\mathbf{i} - (600\text{ N})\mathbf{j} + (600\text{ N})\mathbf{k}$$

$$\mathbf{r}_{B/O} = (2.5\text{ m})\mathbf{i} + (2\text{ m})\mathbf{j}$$

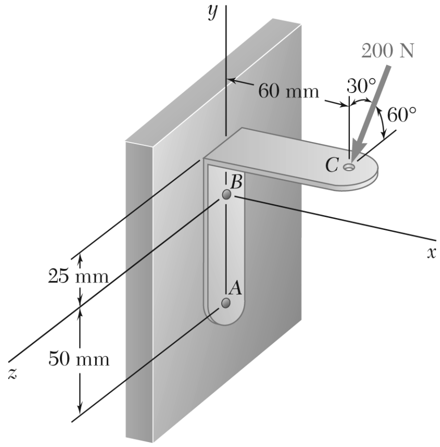
$$\mathbf{M}_O = \mathbf{r}_{B/O} \times \mathbf{F}$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2.5 & 2 & 0 \\ -300 & -600 & 600 \end{vmatrix}$$

$$= 1200\mathbf{i} - 1500\mathbf{j} + (-1500 + 600)\mathbf{k}$$

$$\mathbf{M}_O = (1200\text{ N}\cdot\text{m})\mathbf{i} - (1500\text{ N}\cdot\text{m})\mathbf{j} - (900\text{ N}\cdot\text{m})\mathbf{k} \quad \blacktriangleleft$$

PROBLEM 3.25



A 200-N force is applied as shown to the bracket *ABC*. Determine the moment of the force about *A*.

SOLUTION

We have $\mathbf{M}_A = \mathbf{r}_{C/A} \times \mathbf{F}_C$

where $\mathbf{r}_{C/A} = (0.06 \text{ m})\mathbf{i} + (0.075 \text{ m})\mathbf{j}$

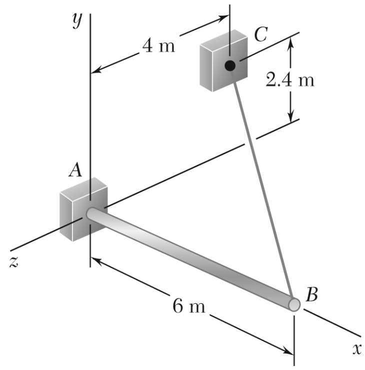
$$\mathbf{F}_C = -(200 \text{ N})\cos 30°\mathbf{j} + (200 \text{ N})\sin 30°\mathbf{k}$$

Then

$$\mathbf{M}_A = 200\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.06 & 0.075 & 0 \\ 0 & -\cos 30° & \sin 30° \end{vmatrix}$$

$$= 200[(0.075\sin 30°)\mathbf{i} - (0.06\sin 30°)\mathbf{j} - (0.06\cos 30°)\mathbf{k}]$$

or $\mathbf{M}_A = (7.50 \text{ N}\cdot\text{m})\mathbf{i} - (6.00 \text{ N}\cdot\text{m})\mathbf{j} - (10.39 \text{ N}\cdot\text{m})\mathbf{k}$ ◀



PROBLEM 3.26

The 6-m boom AB has a fixed end A. A steel cable is stretched from the free end B of the boom to a Point C located on the vertical wall. If the tension in the cable is 2.5 kN, determine the moment about A of the force exerted by the cable at B.

SOLUTION

First note
$$d_{BC} = \sqrt{(-6)^2 + (2.4)^2 + (-4)^2}$$
$$= 7.6 \text{ m}$$

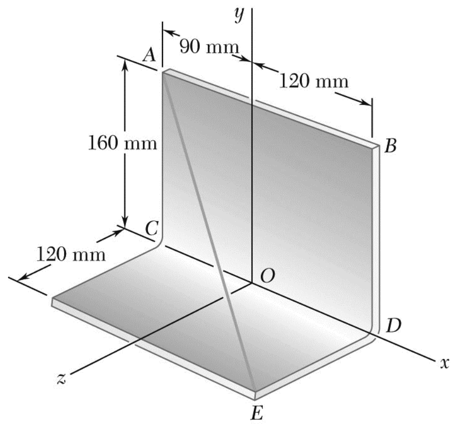
Then
$$\mathbf{T}_{BC} = \frac{2.5 \text{ kN}}{7.6}(-6\mathbf{i} + 2.4\mathbf{j} - 4\mathbf{k})$$

We have
$$\mathbf{M}_A = \mathbf{r}_{B/A} \times \mathbf{T}_{BC}$$

where
$$\mathbf{r}_{B/A} = (6 \text{ m})\mathbf{i}$$

Then
$$\mathbf{M}_A = (6 \text{ m})\mathbf{i} \times \frac{2.5 \text{ kN}}{7.6}(-6\mathbf{i} + 2.4\mathbf{j} - 4\mathbf{k})$$

or $\mathbf{M}_A = (7.89 \text{ kN} \cdot \text{m})\mathbf{j} + (4.74 \text{ kN} \cdot \text{m})\mathbf{k}$ ◀



PROBLEM 3.27

In Prob. 3.21, determine the perpendicular distance from point *O* to wire *AE*.

PROBLEM 3.21 The wire *AE* is stretched between the corners *A* and *E* of a bent plate. Knowing that the tension in the wire is 435 N, determine the moment about *O* of the force exerted by the wire (*a*) on corner *A*, (*b*) on corner *E*.

SOLUTION

From the solution to Prob. 3.21

$$\mathbf{M}_O = (28.8 \text{ N}\cdot\text{m})\mathbf{i} + (16.20 \text{ N}\cdot\text{m})\mathbf{j} - (28.8 \text{ N}\cdot\text{m})\mathbf{k}$$

$$M_O = \sqrt{(28.8)^2 + (16.20)^2 + (28.8)^2}$$

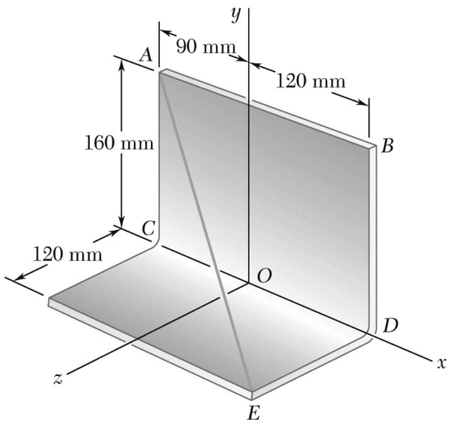
$$= 43.8329 \text{ N}\cdot\text{m}$$

But $M_O = F_A d$ or $d = \dfrac{M_O}{F_A}$

$$d = \frac{43.8329 \text{ N}\cdot\text{m}}{435 \text{ N}}$$

$$= 0.100765 \text{ m}$$

$d = 100.8 \text{ mm}$ ◀



PROBLEM 3.28

In Prob. 3.21, determine the perpendicular distance from point B to wire AE.

PROBLEM 3.21 The wire AE is stretched between the corners A and E of a bent plate. Knowing that the tension in the wire is 435 N, determine the moment about O of the force exerted by the wire (a) on corner A, (b) on corner E.

SOLUTION

From the solution to Prob. 3.21

$$\mathbf{F}_A = (315\text{ N})\mathbf{i} - (240\text{ N})\mathbf{j} + (180\text{ N})\mathbf{k}$$

$$\mathbf{r}_{A/B} = -(0.210\text{ m})\mathbf{i}$$

$$\mathbf{M}_B = \mathbf{r}_{A/B} \times \mathbf{F}_A = -0.21\mathbf{i} \times (315\mathbf{i} - 240\mathbf{j} + 180\mathbf{k})$$

$$= 50.4\mathbf{k} + 37.8\mathbf{j}$$

$$M_B = \sqrt{(50.4)^2 + (37.8)^2}$$

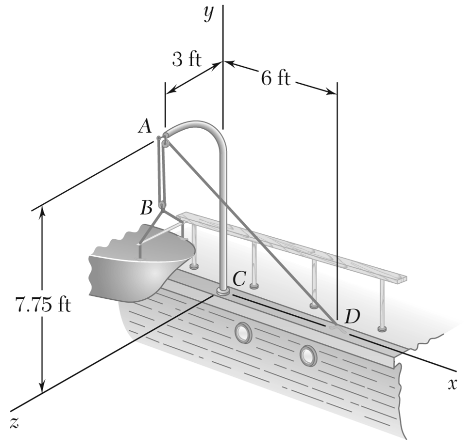
$$= 63.0\text{ N}\cdot\text{m}$$

$$M_B = F_A d \qquad \text{or} \qquad d = \frac{M_B}{F_A}$$

$$d = \frac{63.0\text{ N}\cdot\text{m}}{435\text{ N}}$$

$$= 0.144829\text{ m}$$

$$d = 144.8\text{ mm} \blacktriangleleft$$



PROBLEM 3.29

In Problem 3.22, determine the perpendicular distance from point C to portion AD of the line $ABAD$.

PROBLEM 3.22 A small boat hangs from two davits, one of which is shown in the figure. The tension in line $ABAD$ is 82 lb. Determine the moment about C of the resultant force $\mathbf{R}_A$ exerted on the davit at A.

SOLUTION

First compute the moment about C of the force $\mathbf{F}_{DA}$ exerted by the line on D:

From Problem 3.22:

$$\mathbf{F}_{DA} = -\mathbf{F}_{AD}$$
$$= -(48\text{ lb})\mathbf{i} + (62\text{ lb})\mathbf{j} + (24\text{ lb})\mathbf{k}$$
$$\mathbf{M}_C = \mathbf{r}_{D/C} \times \mathbf{F}_{DA}$$
$$= +(6\text{ ft})\mathbf{i} \times [-(48\text{ lb})\mathbf{i} + (62\text{ lb})\mathbf{j} + (24\text{ lb})\mathbf{k}]$$
$$= -(144\text{ lb}\cdot\text{ft})\mathbf{j} + (372\text{ lb}\cdot\text{ft})\mathbf{k}$$
$$\mathbf{M}_C = \sqrt{(144)^2 + (372)^2}$$
$$= 398.90\text{ lb}\cdot\text{ft}$$

Then

$$\mathbf{M}_C = \mathbf{F}_{DA} d$$

Since

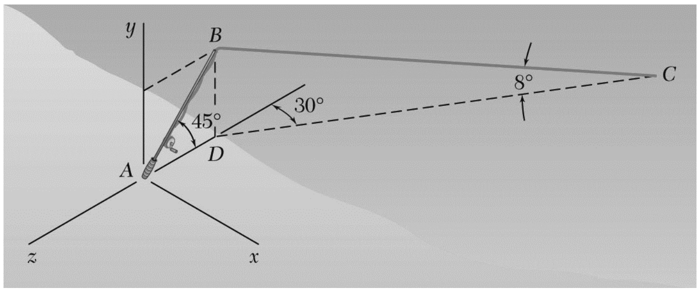
$$F_{DA} = 82\text{ lb}$$
$$d = \frac{M_C}{F_{DA}}$$
$$= \frac{398.90\text{ lb}\cdot\text{ft}}{82\text{ lb}}$$

$d = 4.86\text{ ft}$ ◀

PROBLEM 3.30

In Prob. 3.23, determine the perpendicular distance from point *A* to a line drawn through points *B* and *C*.

PROBLEM 3.23 A 6-ft-long fishing rod *AB* is securely anchored in the sand of a beach. After a fish takes the bait, the resulting force in the line is 6 lb. Determine the moment about *A* of the force exerted by the line at *B*.



SOLUTION

From the solution to Prob. 3.23:

$$\mathbf{M}_A = -(25.4 \text{ lb}\cdot\text{ft})\mathbf{i} - (12.60 \text{ lb}\cdot\text{ft})\mathbf{j} - (12.60 \text{ lb}\cdot\text{ft})\mathbf{k}$$

$$M_A = \sqrt{(-25.4)^2 + (-12.60)^2 + (-12.60)^2}$$
$$= 31.027 \text{ lb}\cdot\text{ft}$$

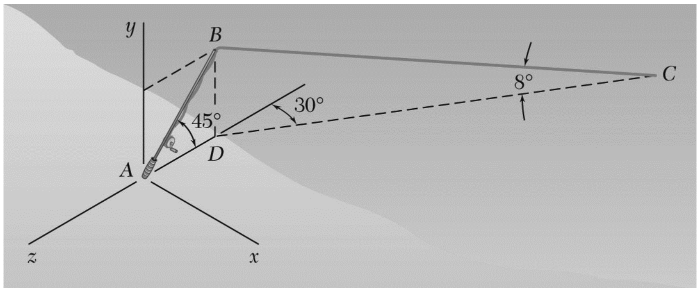
$$M_A = T_{BC}d \qquad \text{or} \qquad d = \frac{M_A}{T_{BC}}$$
$$= \frac{31.027 \text{ lb}\cdot\text{ft}}{6 \text{ lb}}$$
$$= 5.1712 \text{ ft}$$

$d = 5.17$ ft ◀

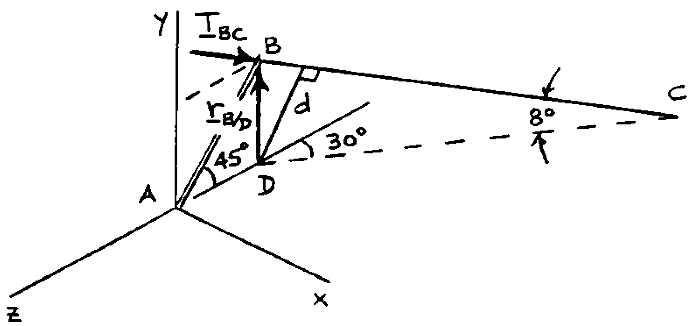
PROBLEM 3.31

In Prob. 3.23, determine the perpendicular distance from point D to a line drawn through points B and C.

PROBLEM 3.23 A 6-ft-long fishing rod AB is securely anchored in the sand of a beach. After a fish takes the bait, the resulting force in the line is 6 lb. Determine the moment about A of the force exerted by the line at B.



SOLUTION



$\overline{AB} = 6$ ft

$T_{BC} = 6$ lb

We have $\quad |\mathbf{M}_D| = T_{BC} d$

where d = perpendicular distance from D to line BC.

$$\mathbf{M}_D = \mathbf{r}_{B/D} \times \mathbf{T}_{BC} \qquad \mathbf{r}_{B/D} = (6 \sin 45° \text{ ft})\mathbf{j} = (4.2426 \text{ ft})$$

$$\mathbf{T}_{BC}: \quad (T_{BC})_x = (6 \text{ lb}) \cos 8° \sin 30° = 2.9708 \text{ lb}$$

$$(T_{BC})_y = -(6 \text{ lb}) \sin 8° = -0.83504 \text{ lb}$$

$$(T_{BC})_z = -(6 \text{ lb}) \cos 8° \cos 30° = -5.1456 \text{ lb}$$

$$\mathbf{T}_{BC} = (2.9708 \text{ lb})\mathbf{i} - (0.83504 \text{ lb})\mathbf{j} - (5.1456 \text{ lb})\mathbf{k}$$

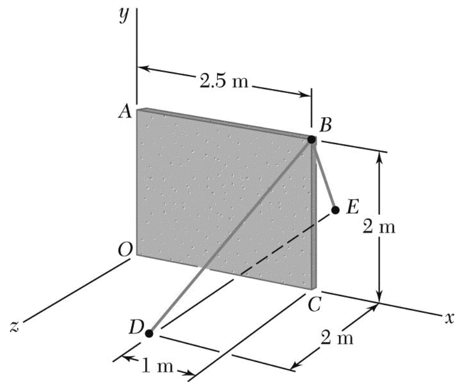
$$\mathbf{M}_D = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 4.2426 & 0 \\ 2.9708 & -0.83504 & -5.1456 \end{vmatrix}$$

$$= -(21.831 \text{ lb} \cdot \text{ft})\mathbf{i} - (12.6039 \text{ lb} \cdot \text{ft})$$

$$|M_D| = \sqrt{(-21.831)^2 + (-12.6039)^2} = 25.208 \text{ lb} \cdot \text{ft}$$

$$25.208 \text{ lb} \cdot \text{ft} = (6 \text{ lb})d$$

$d = 4.20$ ft ◀



PROBLEM 3.32

In Prob. 3.24, determine the perpendicular distance from point *O* to cable *BD*.

PROBLEM 3.24 A precast concrete wall section is temporarily held by two cables as shown. Knowing that the tension in cable *BD* is 900 N, determine the moment about Point *O* of the force exerted by the cable at *B*.

SOLUTION

From the solution to Prob. 3.24 we have

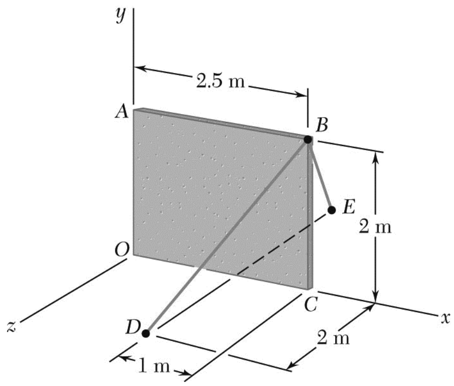
$$\mathbf{M}_O = (1200\ \text{N}\cdot\text{m})\mathbf{i} - (1500\ \text{N}\cdot\text{m})\mathbf{j} - (900\ \text{N}\cdot\text{m})\mathbf{k}$$

$$M_O = \sqrt{(1200)^2 + (-1500)^2 + (-900)^2} = 2121.3\ \text{N}\cdot\text{m}$$

$$M_O = Fd \qquad d = \frac{M_O}{F}$$

$$= \frac{2121.3\ \text{N}\cdot\text{m}}{900\ \text{N}}$$

$d = 2.36$ m ◀



PROBLEM 3.33

In Prob. 3.24, determine the perpendicular distance from point C to cable BD.

PROBLEM 3.24 A precast concrete wall section is temporarily held by two cables as shown. Knowing that the tension in cable BD is 900 N, determine the moment about Point O of the force exerted by the cable at B.

SOLUTION

From the solution to Prob. 3.24 we have

$$\mathbf{F} = -(300\text{ N})\mathbf{i} - (600\text{ N})\mathbf{j} + (600\text{ N})\mathbf{k}$$

$$\mathbf{r}_{B/C} = (2\text{ m})\mathbf{j}$$

$$\mathbf{M}_C = \mathbf{r}_{B/C} \times \mathbf{F} = (2\text{ m})\mathbf{j} \times (-300\text{ N}\mathbf{i} - 600\text{ N}\mathbf{j} + 600\text{ N}\mathbf{k})$$

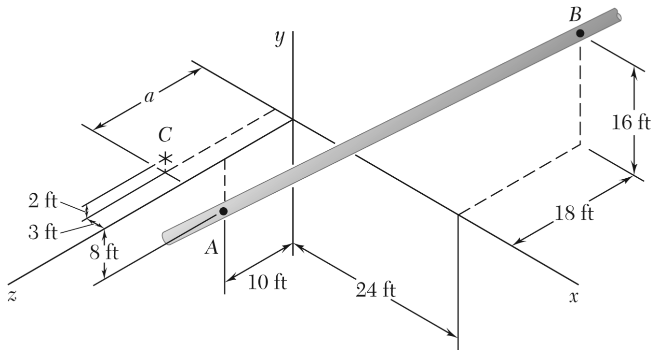
$$= (600\text{ N}\cdot\text{m})\mathbf{k} + (1200\text{ N}\cdot\text{m})\mathbf{i}$$

$$M_C = \sqrt{(600)^2 + (1200)^2} = 1341.64\text{ N}\cdot\text{m}$$

$$M_C = Fd \qquad d = \frac{M_C}{F}$$

$$= \frac{1341.64\text{ N}\cdot\text{m}}{900\text{ N}}$$

$$d = 1.491\text{ m} \blacktriangleleft$$



PROBLEM 3.34

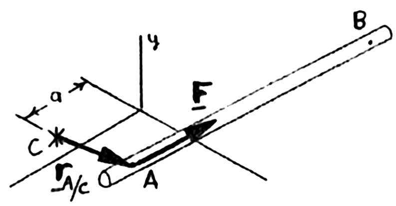
Determine the value of a that minimizes the perpendicular distance from Point C to a section of pipeline that passes through Points A and B.

SOLUTION

Assuming a force **F** acts along AB,

$$|\mathbf{M}_C| = |\mathbf{r}_{A/C} \times \mathbf{F}| = F(d)$$

where $d =$ perpendicular distance from C to line AB



$$\mathbf{F} = \boldsymbol{\lambda}_{AB} F$$

$$= \frac{(24 \text{ ft})\,\mathbf{i} + (24 \text{ ft})\,\mathbf{j} - (28 \text{ ft})\,\mathbf{k}}{\sqrt{(24)^2 + (24)^2 + (28)^2} \text{ ft}} F$$

$$= \frac{F}{11}(6)\,\mathbf{i} + (6)\,\mathbf{j} - (7)\,\mathbf{k}$$

$$\mathbf{r}_{A/C} = (3 \text{ ft})\mathbf{i} - (10 \text{ ft})\mathbf{j} - (a - 10 \text{ ft})\mathbf{k}$$

$$\mathbf{M}_C = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & -10 & 10a \\ 6 & 6 & -7 \end{vmatrix} \frac{F}{11}$$

$$= [(10 + 6a)\mathbf{i} + (81 - 6a)\mathbf{j} + 78\,\mathbf{k}]\frac{F}{11}$$

Since $\quad |\mathbf{M}_C| = \sqrt{|\mathbf{r}_{A/C} \times \mathbf{F}^2|} \quad$ or $\quad |\mathbf{r}_{A/C} \times \mathbf{F}^2| = (dF)^2$

$$\frac{1}{121}(10 + 6a)^2 + (81 - 6a)^2 + (78)^2 = d^2$$

Setting $\frac{d}{da}(d^2) = 0$ to find a to minimize d:

$$\frac{1}{121}[2(6)(10 + 6a) + 2(-6)(81 - 6a)] = 0$$

Solving $\quad a = 5.92$ ft $\qquad$ or $\quad a = 5.92$ ft ◀

PROBLEM 3.35

Given the vectors **P** = 3**i** − **j** + 2**k**, **Q** = 4**i** + 5**j** − 3**k**, and **S** = −2**i** + 3**j** − **k**, compute the scalar products **P** · **Q**, **P** · **S**, and **Q** · **S**.

SOLUTION

$$\begin{aligned}\mathbf{P}\cdot\mathbf{Q} &= (3\mathbf{i}-\mathbf{j}+2\mathbf{k})\cdot(4\mathbf{i}+5\mathbf{j}-3\mathbf{k})\\ &= (3)(4)+(-1)(5)+(2)(-3)\\ &= 12-5-6\end{aligned}$$

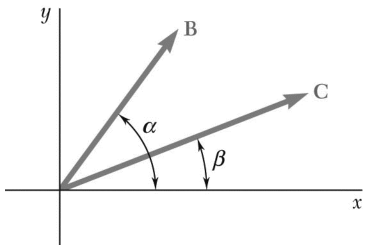
$$\mathbf{P}\cdot\mathbf{Q} = +1 \blacktriangleleft$$

$$\begin{aligned}\mathbf{P}\cdot\mathbf{S} &= (3\mathbf{i}-\mathbf{j}+2\mathbf{k})\cdot(-2\mathbf{i}+3\mathbf{j}-\mathbf{k})\\ &= (3)(-2)+(-1)(3)+(2)(-1)\\ &= -6-3-2\end{aligned}$$

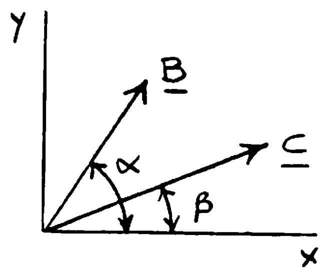
$$\mathbf{P}\cdot\mathbf{S} = -11 \blacktriangleleft$$

$$\begin{aligned}\mathbf{Q}\cdot\mathbf{S} &= (4\mathbf{i}+5\mathbf{j}-3\mathbf{k})\cdot(-2\mathbf{i}+3\mathbf{j}-\mathbf{k})\\ &= (4)(-2)+(5)(3)+(-3)(-1)\\ &= -8+15+3\end{aligned}$$

$$\mathbf{Q}\cdot\mathbf{S} = +10 \blacktriangleleft$$

	**PROBLEM 3.36** Form the scalar product **B** · **C** and use the result obtained to prove the identity $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta.$

SOLUTION



$$\mathbf{B} = B\cos\alpha\mathbf{i} + B\sin\alpha\mathbf{j} \qquad (1)$$

$$\mathbf{C} = C\cos\beta\mathbf{i} + C\sin\beta\mathbf{j} \qquad (2)$$

By definition:

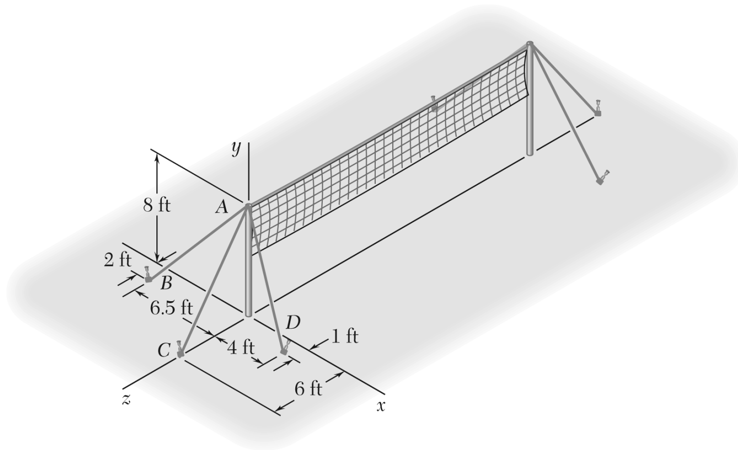
$$\mathbf{B}\cdot\mathbf{C} = BC\cos(\alpha - \beta) \qquad (3)$$

From (1) and (2):

$$\begin{aligned}\mathbf{B}\cdot\mathbf{C} &= (B\cos\alpha\mathbf{i} + B\sin\alpha\mathbf{j})\cdot(C\cos\beta\mathbf{i} + C\sin\beta\mathbf{j})\\ &= BC(\cos\alpha\cos\beta + \sin\alpha\sin\beta) \qquad (4)\end{aligned}$$

Equating the right-hand members of (3) and (4),

$$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta \blacktriangleleft$$

	**PROBLEM 3.37** Consider the volleyball net shown. Determine the angle formed by guy wires *AB* and *AC*.

SOLUTION

First note: $AB = \sqrt{(-6.5)^2 + (-8)^2 + (2)^2} = 10.5 \text{ ft}$

$$AC = \sqrt{(0)^2 + (-8)^2 + (6)^2} = 10 \text{ ft}$$

and $\overrightarrow{AB} = -(6.5 \text{ ft})\mathbf{i} - (8 \text{ ft})\mathbf{j} + (2 \text{ ft})\mathbf{k}$

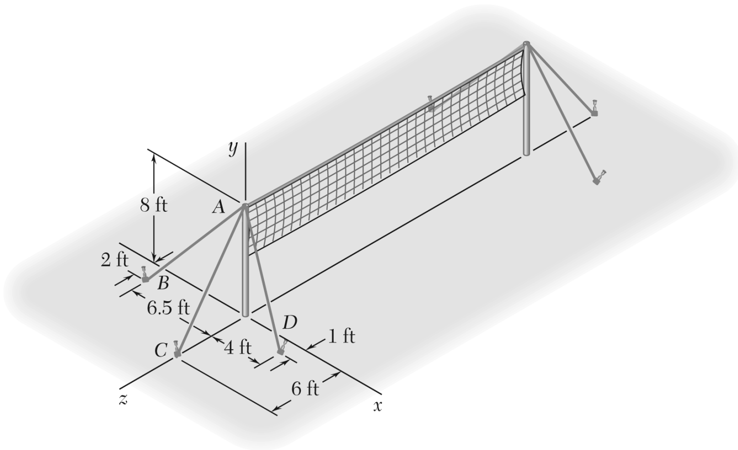
$$\overrightarrow{AC} = -(8 \text{ ft})\mathbf{j} + (6 \text{ ft})\mathbf{k}$$

By definition, $\overrightarrow{AB} \cdot \overrightarrow{AC} = (AB)(AC)\cos\theta$

or $(-6.5\mathbf{i} - 8\mathbf{j} + 2\mathbf{k}) \cdot (-8\mathbf{j} + 6\mathbf{k}) = (10.5)(10)\cos\theta$

$$(-6.5)(0) + (-8)(-8) + (2)(6) = 105\cos\theta$$

or $\cos\theta = 0.72381$ or $\theta = 43.6°$ ◀



PROBLEM 3.38

Consider the volleyball net shown. Determine the angle formed by guy wires *AC* and *AD*.

SOLUTION

First note:

$$AC = \sqrt{(0)^2 + (-8)^2 + (6)^2}$$
$$= 10 \text{ ft}$$
$$AD = \sqrt{(4)^2 + (-8)^2 + (1)^2}$$
$$= 9 \text{ ft}$$

and

$$\overrightarrow{AC} = -(8 \text{ ft})\mathbf{j} + (6 \text{ ft})\mathbf{k}$$
$$\overrightarrow{AD} = (4 \text{ ft})\mathbf{i} - (8 \text{ ft})\mathbf{j} + (1 \text{ ft})\mathbf{k}$$

By definition,

$$\overrightarrow{AC} \cdot \overrightarrow{AD} = (AC)(AD)\cos\theta$$

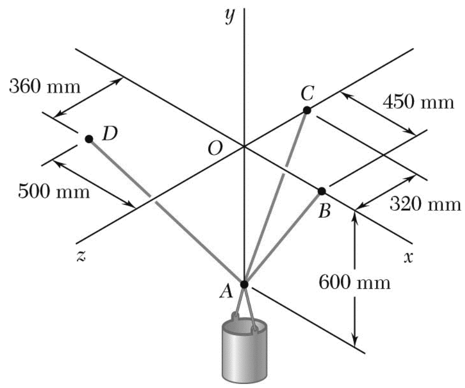
or

$$(-8\mathbf{j} + 6\mathbf{k}) \cdot (4\mathbf{i} - 8\mathbf{j} + \mathbf{k}) = (10)(9)\cos\theta$$
$$(0)(4) + (-8)(-8) + (6)(1) = 90\cos\theta$$

or

$$\cos\theta = 0.77778$$

or $\theta = 38.9°$ ◀



PROBLEM 3.39

Three cables are used to support a container as shown. Determine the angle formed by cables AB and AD.

SOLUTION

First note:

$$AB = \sqrt{(450 \text{ mm})^2 + (600 \text{ mm})^2}$$
$$= 750 \text{ mm}$$
$$AD = \sqrt{(-500 \text{ mm})^2 + (600 \text{ mm})^2 + (360 \text{ mm})^2}$$
$$= 860 \text{ mm}$$

and

$$\overrightarrow{AB} = (450 \text{ mm})\mathbf{i} + (600 \text{ mm})\mathbf{j}$$
$$\overrightarrow{AD} = (-500 \text{ mm})\mathbf{i} + (600 \text{ mm})\mathbf{j} + (360 \text{ mm})\mathbf{k}$$

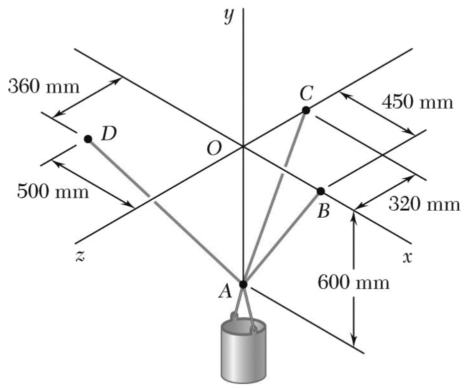
By definition,

$$\overrightarrow{AB} \cdot \overrightarrow{AD} = (AB)(AD)\cos\theta$$
$$(450\mathbf{i} + 600\mathbf{j}) \cdot (-500\mathbf{i} - 600\mathbf{j} + 360\mathbf{k}) = (750)(860)\cos\theta$$
$$(450)(-500) + (600)(600) + (0)(360) = (750)(860)\cos\theta$$

or

$$\cos\theta = 0.20930 \qquad \theta = 77.9° \blacktriangleleft$$

PROBLEM 3.40



Three cables are used to support a container as shown. Determine the angle formed by cables *AC* and *AD*.

SOLUTION

First note:

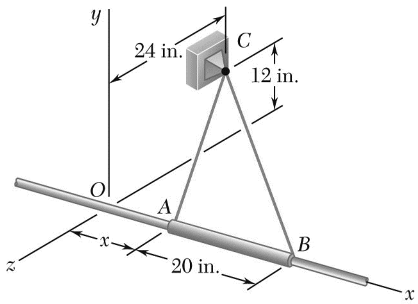
$$AC = \sqrt{(600 \text{ mm})^2 + (-320 \text{ mm})^2}$$
$$= 680 \text{ mm}$$
$$AD = \sqrt{(-500 \text{ mm})^2 + (600 \text{ mm})^2 + (360 \text{ mm})^2}$$
$$= 860 \text{ mm}$$

and

$$\overrightarrow{AC} = (600 \text{ mm})\mathbf{j} + (-320 \text{ mm})\mathbf{k}$$
$$\overrightarrow{AD} = (-500 \text{ mm})\mathbf{i} + (600 \text{ mm})\mathbf{j} + (360 \text{ mm})\mathbf{k}$$

By definition,

$$\overrightarrow{AC} \cdot \overrightarrow{AD} = (AC)(AD)\cos\theta$$
$$(600\mathbf{j} - 320\mathbf{k}) \cdot (-500\mathbf{i} + 600\mathbf{j} + 360\mathbf{k}) = (680)(860)\cos\theta$$
$$0(-500) + (600)(600) + (-320)(360) = (680)(860)\cos\theta$$
$$\cos\theta = 0.41860 \qquad \theta = 65.3° \blacktriangleleft$$



PROBLEM 3.41

The 20-in. tube AB can slide along a horizontal rod. The ends A and B of the tube are connected by elastic cords to the fixed point C. For the position corresponding to $x = 11$ in., determine the angle formed by the two cords (*a*) using Eq. (3.32), (*b*) applying the law of cosines to triangle ABC.

SOLUTION

(*a*) Using Eq. (3.32):

$$\overline{CA} = 11\mathbf{i} - 12\mathbf{j} + 24\mathbf{k}$$

$$CA = \sqrt{(11)^2 + (-12)^2 + (24)^2} = 29 \text{ in.}$$

$$\overline{CB} = 31\mathbf{i} - 12\mathbf{j} + 24\mathbf{k}$$

$$CB = \sqrt{(31)^2 + (-12)^2 + (24)^2} = 41 \text{ in.}$$

$$\cos\theta = \frac{\overline{CA} \cdot \overline{CB}}{(CA)(CB)}$$

$$= \frac{(11\mathbf{i} - 12\mathbf{j} + 24\mathbf{k}) \cdot (31\mathbf{i} - 12\mathbf{j} + 24\mathbf{k})}{(29)(41)}$$

$$= \frac{(11)(31) + (-12)(-12) + (24)(24)}{(29)(41)}$$

$$= 0.89235$$

$\theta = 26.8°$ ◀

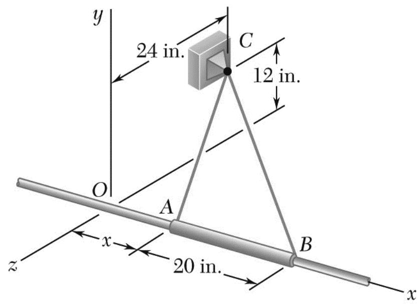
(*b*) Law of cosines:

$$(AB)^2 = (CA)^2 + (CB)^2 - 2(CA)(CB)\cos\theta$$

$$(20)^2 = (29)^2 + (41)^2 - 2(29)(41)\cos\theta$$

$$\cos\theta = 0.89235$$

$\theta = 26.8°$ ◀



PROBLEM 3.42

Solve Prob. 3.41 for the position corresponding to $x = 4$ in.

PROBLEM 3.41 The 20-in. tube *AB* can slide along a horizontal rod. The ends *A* and *B* of the tube are connected by elastic cords to the fixed point *C*. For the position corresponding to $x = 11$ in., determine the angle formed by the two cords (*a*) using Eq. (3.32), (*b*) applying the law of cosines to triangle *ABC*.

SOLUTION

(*a*) Using Eq. (3.32):

$$\overrightarrow{CA} = 4\mathbf{i} - 12\mathbf{j} + 24\mathbf{k}$$

$$CA = \sqrt{(4)^2 + (-12)^2 + (24)^2} = 27.129 \text{ in.}$$

$$\overrightarrow{CB} = 24\mathbf{i} - 12\mathbf{j} + 24\mathbf{k}$$

$$CB = \sqrt{(24)^2 + (-12)^2 + (24)^2} = 36 \text{ in.}$$

$$\cos\theta = \frac{\overrightarrow{CA} \cdot \overrightarrow{CB}}{(CA)(CB)}$$

$$= \frac{(4\mathbf{i} - 12\mathbf{j} + 24\mathbf{k}) \cdot (24\mathbf{i} - 12\mathbf{j} + 24\mathbf{k})}{(27.129)(36)}$$

$$= 0.83551$$

$\theta = 33.3°$ ◀

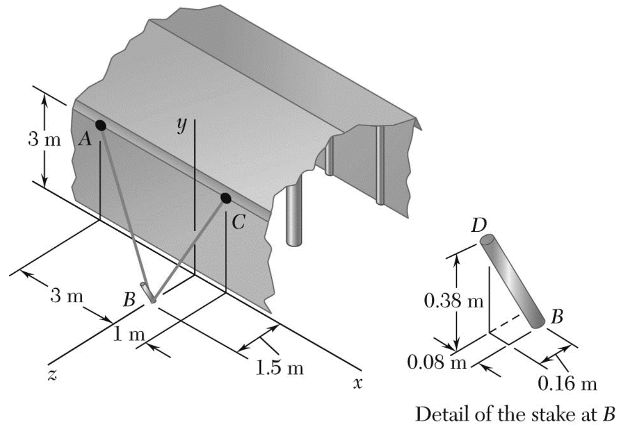
(*b*) Law of cosines:

$$(AB)^2 = (CA)^2 + (CB)^2 - 2(CA)(CB)\cos\theta$$

$$(20)^2 = (27.129)^2 + (36)^2 - 2(27.129)(36)\cos\theta$$

$$\cos\theta = 0.83551$$

$\theta = 33.3°$ ◀



PROBLEM 3.43

Ropes *AB* and *BC* are two of the ropes used to support a tent. The two ropes are attached to a stake at *B*. If the tension in rope *AB* is 540 N, determine (*a*) the angle between rope *AB* and the stake, (*b*) the projection on the stake of the force exerted by rope *AB* at Point *B*.

SOLUTION

First note:

$$BA = \sqrt{(-3)^2 + (3)^2 + (-1.5)^2} = 4.5 \text{ m}$$

$$BD = \sqrt{(-0.08)^2 + (0.38)^2 + (0.16)^2} = 0.42 \text{ m}$$

Then

$$\mathbf{T}_{BA} = \frac{T_{BA}}{4.5}(-3\mathbf{i} + 3\mathbf{j} - 1.5\mathbf{k})$$

$$= \frac{T_{BA}}{3}(-2\mathbf{i} + 2\mathbf{j} - \mathbf{k})$$

$$\boldsymbol{\lambda}_{BD} = \frac{\overrightarrow{BD}}{BD} = \frac{1}{0.42}(-0.08\mathbf{i} + 0.38\mathbf{j} + 0.16\mathbf{k})$$

$$= \frac{1}{21}(-4\mathbf{i} + 19\mathbf{j} + 8\mathbf{k})$$

(*a*) We have

$$\mathbf{T}_{BA} \cdot \boldsymbol{\lambda}_{BD} = T_{BA} \cos\theta$$

or

$$\frac{T_{BA}}{3}(-2\mathbf{i} + 2\mathbf{j} - \mathbf{k}) \cdot \frac{1}{21}(-4\mathbf{i} + 19\mathbf{j} + 8\mathbf{k}) = T_{BA} \cos\theta$$

or

$$\cos\theta = \frac{1}{63}[(-2)(-4) + (2)(19) + (-1)(8)]$$

$$= 0.60317$$

or $\theta = 52.9°$ ◀

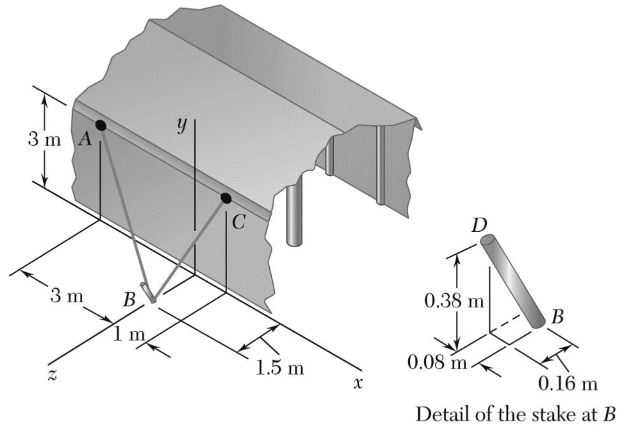
(*b*) We have

$$(T_{BA})_{BD} = \mathbf{T}_{BA} \cdot \boldsymbol{\lambda}_{BD}$$

$$= T_{BA} \cos\theta$$

$$= (540 \text{ N})(0.60317)$$

or $(T_{BA})_{BD} = 326 \text{ N}$ ◀



Detail of the stake at B

PROBLEM 3.44

Ropes AB and BC are two of the ropes used to support a tent. The two ropes are attached to a stake at B. If the tension in rope BC is 490 N, determine (*a*) the angle between rope BC and the stake, (*b*) the projection on the stake of the force exerted by rope BC at Point B.

SOLUTION

First note:

$$BC = \sqrt{(1)^2 + (3)^2 + (-1.5)^2} = 3.5 \text{ m}$$

$$BD = \sqrt{(-0.08)^2 + (0.38)^2 + (0.16)^2} = 0.42 \text{ m}$$

$$\mathbf{T}_{BC} = \frac{T_{BC}}{3.5}(\mathbf{i} + 3\mathbf{j} - 1.5\mathbf{k})$$

$$= \frac{T_{BC}}{7}(2\mathbf{i} + 6\mathbf{j} - 3\mathbf{k})$$

$$\lambda_{BD} = \frac{\overrightarrow{BD}}{BD} = \frac{1}{0.42}(-0.08\mathbf{i} + 0.38\mathbf{j} + 0.16\mathbf{k})$$

$$= \frac{1}{21}(-4\mathbf{i} + 19\mathbf{j} + 8\mathbf{k})$$

(*a*)

$$\mathbf{T}_{BC} \cdot \lambda_{BD} = T_{BC} \cos\theta$$

$$\frac{T_{BC}}{7}(2\mathbf{i} + 6\mathbf{j} - 3\mathbf{k}) \cdot \frac{1}{21}(-4\mathbf{i} + 19\mathbf{j} + 8\mathbf{k}) = T_{BC} \cos\theta$$

$$\cos\theta = \frac{1}{147}[(2)(-4) + (6)(19) + (-3)(8)]$$

$$= 0.55782$$

$$\theta = 56.1° \blacktriangleleft$$

(*b*)

$$(T_{BC})_{BD} = \mathbf{T}_{BC} \cdot \lambda_{BD}$$

$$= T_{BC} \cos\theta$$

$$= (490 \text{ N})(0.55782)$$

$$(T_{BC})_{BD} = 273 \text{ N} \blacktriangleleft$$

PROBLEM 3.45

Given the vectors $\mathbf{P} = 4\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$, $\mathbf{Q} = 2\mathbf{i} + 4\mathbf{j} - 5\mathbf{k}$, and $\mathbf{S} = S_x\mathbf{i} - \mathbf{j} + 2\mathbf{k}$, determine the value of S_x for which the three vectors are coplanar.

SOLUTION

If **P**, **Q**, and **S** are coplanar, then **P** must be perpendicular to $(\mathbf{Q} \times \mathbf{S})$.

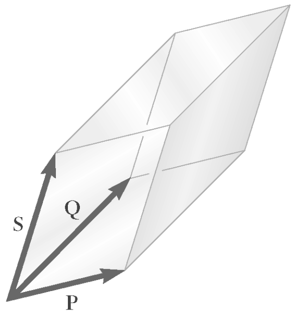
$$\mathbf{P} \cdot (\mathbf{Q} \times \mathbf{S}) = 0$$

(or, the volume of a parallelepiped defined by **P**, **Q**, and **S** is zero).

Then
$$\begin{vmatrix} 4 & -2 & 3 \\ 2 & 4 & -5 \\ S_x & -1 & 2 \end{vmatrix} = 0$$

or
$$32 + 10S_x - 6 - 20 + 8 - 12S_x = 0$$

$S_x = 7$ ◀



PROBLEM 3.46

Determine the volume of the parallelepiped of Fig. 3.25 when (*a*) $\mathbf{P} = 4\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$, $\mathbf{Q} = -2\mathbf{i} - 5\mathbf{j} + \mathbf{k}$, and $\mathbf{S} = 7\mathbf{i} + \mathbf{j} - \mathbf{k}$, (*b*) $\mathbf{P} = 5\mathbf{i} - \mathbf{j} + 6\mathbf{k}$, $\mathbf{Q} = 2\mathbf{i} + 3\mathbf{j} + \mathbf{k}$, and $\mathbf{S} = -3\mathbf{i} - 2\mathbf{j} + 4\mathbf{k}$.

SOLUTION

Volume of a parallelepiped is found using the mixed triple product.

(*a*)

$$\text{Vol.} = \mathbf{P} \cdot (\mathbf{Q} \times \mathbf{S})$$

$$= \begin{vmatrix} 4 & -3 & 2 \\ -2 & -5 & 1 \\ 7 & 1 & -1 \end{vmatrix} \text{in.}^3$$

$$= (20 - 21 - 4 + 70 + 6 - 4)$$

$$= 67$$

or $\text{Volume} = 67.0$ ◀

(*b*)

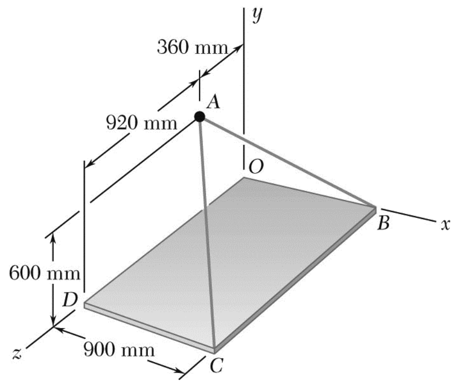
$$\text{Vol.} = \mathbf{P} \cdot (\mathbf{Q} \times \mathbf{S})$$

$$= \begin{vmatrix} 5 & -1 & 6 \\ 2 & 3 & 1 \\ -3 & -2 & 4 \end{vmatrix} \text{in.}^3$$

$$= (60 + 3 - 24 + 54 + 8 + 10)$$

$$= 111$$

or $\text{Volume} = 111.0$ ◀



PROBLEM 3.47

Knowing that the tension in cable *AB* is 570 N, determine the moment about each of the coordinate axes of the force exerted on the plate at *B*.

SOLUTION

$$\overline{BA} = (-900\text{ mm})\mathbf{i} + (600\text{ mm})\mathbf{j} + (360\text{ mm})\mathbf{k}$$

$$BA = \sqrt{(-900)^2 + (600)^2 + (360)^2} = 1140\text{ mm}$$

$$\mathbf{F}_B = F_B \frac{\overline{BA}}{BA}$$

$$= (570\text{ N})\frac{-900\mathbf{i} + 600\mathbf{j} + 360\mathbf{k}}{1140}$$

$$= -(450\text{ N})\mathbf{i} + (300\text{ N})\mathbf{j} + (180\text{ N})\mathbf{k}$$

$$\mathbf{r}_B = (0.9\text{ m})\mathbf{i}$$

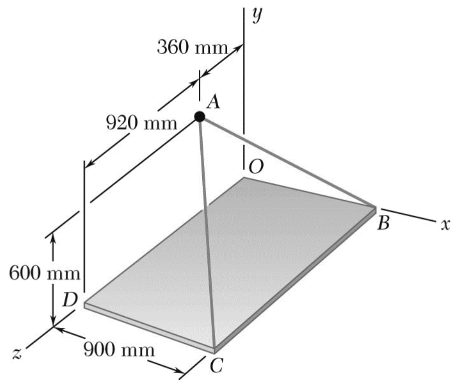
$$\mathbf{M}_O = \mathbf{r}_B \times \mathbf{F}_B = 0.9\mathbf{i} \times (-450\mathbf{i} + 300\mathbf{j} + 180\mathbf{k})$$

$$= 270\mathbf{k} - 162\mathbf{j}$$

$$\mathbf{M}_O = M_x\mathbf{i} + M_y\mathbf{j} + M_z\mathbf{k}$$

$$= -(162\text{ N}\cdot\text{m})\mathbf{j} + (270\text{ N}\cdot\text{m})\mathbf{k}$$

Therefore, $M_x = 0,\ M_y = -162.0\text{ N}\cdot\text{m},\ M_z = +270\text{ N}\cdot\text{m}$ ◀



PROBLEM 3.48

Knowing that the tension in cable *AC* is 1065 N, determine the moment about each of the coordinate axes of the force exerted on the plate at *C*.

SOLUTION

$$\overrightarrow{CA} = (-900 \text{ mm})\mathbf{i} + (600 \text{ mm})\mathbf{j} + (-920 \text{ mm})\mathbf{k}$$

$$CA = \sqrt{(-900)^2 + (600)^2 + (-920)^2} = 1420 \text{ mm}$$

$$\mathbf{F}_C = F_C \frac{\overrightarrow{CA}}{CA}$$

$$= (1065 \text{ N})\frac{-900\mathbf{i} + 600\mathbf{j} - 920\mathbf{k}}{1420}$$

$$= -(675 \text{ N})\mathbf{i} + (450 \text{ N})\mathbf{j} - (690 \text{ N})\mathbf{k}$$

$$\mathbf{r}_C = (0.9 \text{ m})\mathbf{i} + (1.28 \text{ m})\mathbf{k}$$

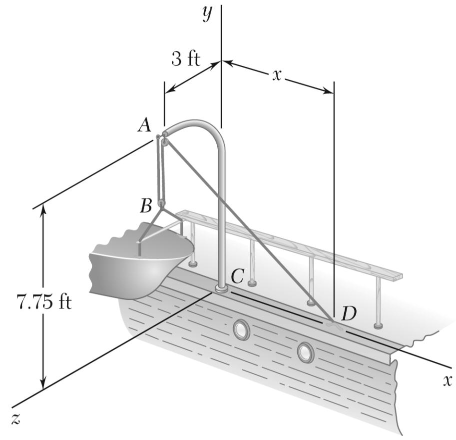
Using Eq. (3.19):

$$\mathbf{M}_O = \mathbf{r}_C \times \mathbf{F}_C = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.9 & 0 & 1.28 \\ -675 & 450 & -690 \end{vmatrix}$$

$$\mathbf{M}_O = -(576 \text{ N}\cdot\text{m})\mathbf{i} - (243 \text{ N}\cdot\text{m})\mathbf{j} + (405 \text{ N}\cdot\text{m})\mathbf{k}$$

But $\mathbf{M}_O = M_x\mathbf{i} + M_y\mathbf{j} + M_z\mathbf{k}$

Therefore, $M_x = -576 \text{ N}\cdot\text{m}, \; M_y = -243 \text{ N}\cdot\text{m}, \; M_z = +405 \text{ N}\cdot\text{m}$ ◀



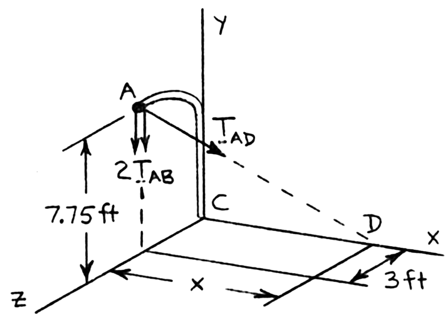
PROBLEM 3.49

A small boat hangs from two davits, one of which is shown in the figure. It is known that the moment about the z-axis of the resultant force $\mathbf{R}_A$ exerted on the davit at A must not exceed 279 lb·ft in absolute value. Determine the largest allowable tension in line $ABAD$ when $x = 6$ ft.

SOLUTION

First note: $\mathbf{R}_A = 2\mathbf{T}_{AB} + \mathbf{T}_{AD}$

Also note that only $\mathbf{T}_{AD}$ will contribute to the moment about the z-axis.



Now
$$AD = \sqrt{(6)^2 + (-7.75)^2 + (-3)^2}$$
$$= 10.25 \text{ ft}$$

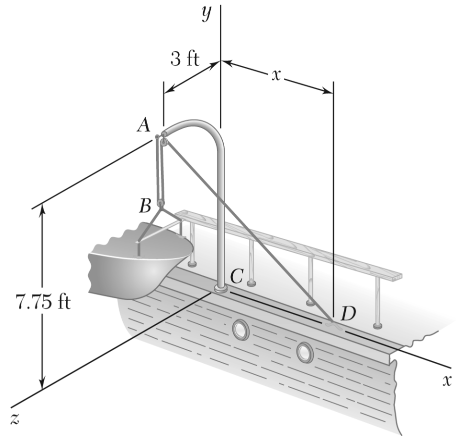
Then
$$\mathbf{T}_{AD} = T\frac{\overrightarrow{AD}}{AD}$$
$$= \frac{T}{10.25}(6\mathbf{i} - 7.75\mathbf{j} - 3\mathbf{k})$$

Now
$$M_z = \mathbf{k} \cdot (\mathbf{r}_{A/C} \times \mathbf{T}_{AD})$$

where
$$\mathbf{r}_{A/C} = (7.75 \text{ ft})\mathbf{j} + (3 \text{ ft})\mathbf{k}$$

Then for T_{max},
$$279 = \left| \frac{T_{max}}{10.25} \begin{vmatrix} 0 & 0 & 1 \\ 0 & 7.75 & 3 \\ 6 & -7.75 & -3 \end{vmatrix} \right|$$
$$= \frac{T_{max}}{10.25}|-(1)(7.75)(6)|$$

or $T_{max} = 61.5$ lb ◀



PROBLEM 3.50

For the davit of Problem 3.49, determine the largest allowable distance x when the tension in line $ABAD$ is 60 lb.

SOLUTION

From the solution of Problem 3.49, $\mathbf{T}_{AD}$ is now

$$\mathbf{T}_{AD} = T\frac{\overrightarrow{AD}}{AD}$$

$$= \frac{60 \text{ lb}}{\sqrt{x^2 + (-7.75)^2 + (-3)^2}}(x\mathbf{i} - 7.75\mathbf{j} - 3\mathbf{k})$$

Then $M_z = \mathbf{k} \cdot (\mathbf{r}_{A/C} \times \mathbf{T}_{AD})$ becomes

$$279 = \left| \frac{60}{\sqrt{x^2 + (-7.75)^2 + (-3)^2}} \begin{vmatrix} 0 & 0 & 1 \\ 0 & 7.75 & 3 \\ x & -7.75 & -3 \end{vmatrix} \right|$$

$$279 = \frac{60}{\sqrt{x^2 + 69.0625}} \left| -(1)(7.75)(x) \right|$$

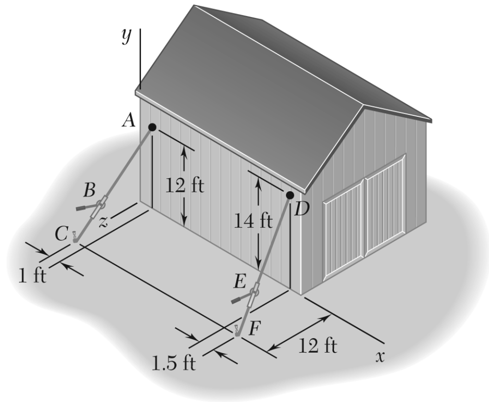
$$279\sqrt{x^2 + 69.0625} = 465x$$

$$0.6\sqrt{x^2 + 69.0625} = x$$

Squaring both sides: $$0.36x^2 + 24.8625 = x^2$$

$$x^2 = 38.848$$

$x = 6.23$ ft ◀

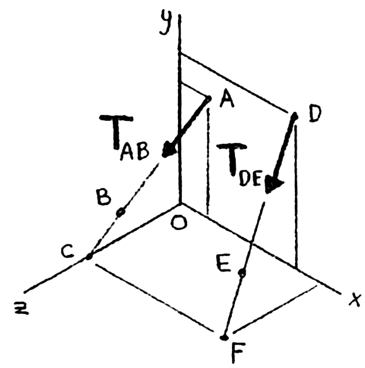


PROBLEM 3.51

A farmer uses cables and winch pullers B and E to plumb one side of a small barn. If it is known that the sum of the moments about the x-axis of the forces exerted by the cables on the barn at Points A and D is equal to 4728 lb · ft, determine the magnitude of $\mathbf{T}_{DE}$ when $T_{AB} = 255$ lb.

SOLUTION

The moment about the x-axis due to the two cable forces can be found using the z components of each force acting at their intersection with the xy plane (A and D). The x components of the forces are parallel to the x-axis, and the y components of the forces intersect the x-axis. Therefore, neither the x or y components produce a moment about the x-axis.



We have $\quad \Sigma M_x: \quad (T_{AB})_z(y_A) + (T_{DE})_z(y_D) = M_x$

where

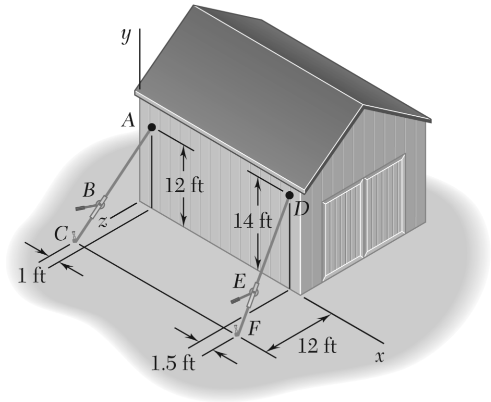
$$(T_{AB})_z = \mathbf{k} \cdot \mathbf{T}_{AB}$$
$$= \mathbf{k} \cdot (T_{AB}\lambda_{AB})$$
$$= \mathbf{k} \cdot \left[255 \text{ lb}\left(\frac{-\mathbf{i} - 12\mathbf{j} + 12\mathbf{k}}{17}\right)\right]$$
$$= 180 \text{ lb}$$

$$(T_{DE})_z = \mathbf{k} \cdot \mathbf{T}_{DE}$$
$$= \mathbf{k} \cdot (T_{DE}\lambda_{DE})$$
$$= \mathbf{k} \cdot \left[T_{DE}\left(\frac{1.5\mathbf{i} - 14\mathbf{j} + 12\mathbf{k}}{18.5}\right)\right]$$
$$= 0.64865T_{DE}$$

$$y_A = 12 \text{ ft}$$
$$y_D = 14 \text{ ft}$$
$$M_x = 4728 \text{ lb} \cdot \text{ft}$$

$$(180 \text{ lb})(12 \text{ ft}) + (0.64865T_{DE})(14 \text{ ft}) = 4728 \text{ lb} \cdot \text{ft}$$

and $\quad T_{DE} = 282.79 \text{ lb} \qquad$ or $T_{DE} = 283$ lb ◀



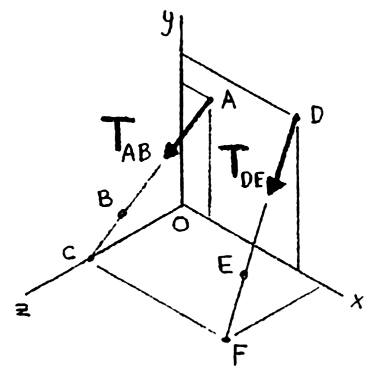
PROBLEM 3.52

Solve Problem 3.51 when the tension in cable *AB* is 306 lb.

PROBLEM 3.51 A farmer uses cables and winch pullers *B* and *E* to plumb one side of a small barn. If it is known that the sum of the moments about the *x*-axis of the forces exerted by the cables on the barn at Points *A* and *D* is equal to 4728 lb · ft, determine the magnitude of $\mathbf{T}_{DE}$ when $T_{AB} = 255$ lb.

SOLUTION

The moment about the *x*-axis due to the two cable forces can be found using the *z* components of each force acting at the intersection with the *xy* plane (*A* and *D*). The *x* components of the forces are parallel to the *x*-axis, and the *y* components of the forces intersect the *x*-axis. Therefore, neither the *x* or *y* components produce a moment about the *x*-axis.



We have ΣM_x: $(T_{AB})_z(y_A) + (T_{DE})_z(y_D) = M_x$

Where

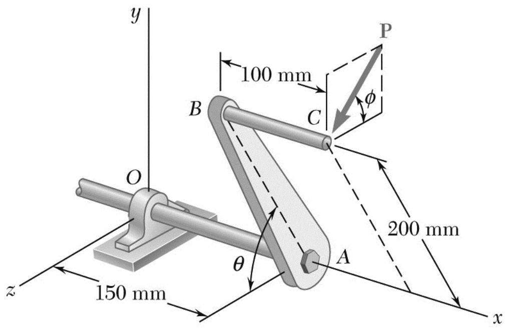
$$(T_{AB})_z = \mathbf{k} \cdot \mathbf{T}_{AB}$$
$$= \mathbf{k} \cdot (T_{AB} \boldsymbol{\lambda}_{AB})$$
$$= \mathbf{k} \cdot \left[306 \text{ lb} \left(\frac{-\mathbf{i} - 12\mathbf{j} + 12\mathbf{k}}{17} \right) \right]$$
$$= 216 \text{ lb}$$

$$(T_{DE})_z = \mathbf{k} \cdot \mathbf{T}_{DE}$$
$$= \mathbf{k} \cdot (T_{DE} \boldsymbol{\lambda}_{DE})$$
$$= \mathbf{k} \cdot \left[T_{DE} \left(\frac{1.5\mathbf{i} - 14\mathbf{j} + 12\mathbf{k}}{18.5} \right) \right]$$
$$= 0.64865 T_{DE}$$

$$y_A = 12 \text{ ft}$$
$$y_D = 14 \text{ ft}$$
$$M_x = 4728 \text{ lb} \cdot \text{ft}$$

$$(216 \text{ lb})(12 \text{ ft}) + (0.64865 T_{DE})(14 \text{ ft}) = 4728 \text{ lb} \cdot \text{ft}$$

and $T_{DE} = 235.21$ lb or $T_{DE} = 235$ lb ◀



PROBLEM 3.53

A single force **P** acts at *C* in a direction perpendicular to the handle *BC* of the crank shown. Knowing that $M_x = +20$ N · m and $M_y = -8.75$ N · m, and $M_z = -30$ N · m, determine the magnitude of **P** and the values of ϕ and θ.

SOLUTION

$$\mathbf{r}_C = (0.25 \text{ m})\mathbf{i} + (0.2 \text{ m})\sin\theta\mathbf{j} + (0.2 \text{ m})\cos\theta\mathbf{k}$$

$$\mathbf{P} = -P\sin\phi\mathbf{j} + P\cos\phi\mathbf{k}$$

$$\mathbf{M}_O = \mathbf{r}_C \times \mathbf{P} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.25 & 0.2\sin\theta & 0.2\cos\theta \\ 0 & -P\sin\phi & P\cos\phi \end{vmatrix}$$

Expanding the determinant, we find

$$M_x = (0.2)P(\sin\theta\cos\phi + \cos\theta\sin\phi)$$

$$M_x = (0.2)P\sin(\theta+\phi) \qquad (1)$$

$$M_y = -(0.25)P\cos\phi \qquad (2)$$

$$M_z = -(0.25)P\sin\phi \qquad (3)$$

Dividing Eq. (3) by Eq. (2) gives: $\tan\phi = \dfrac{M_z}{M_y}$ (4)

$$\tan\phi = \frac{-30 \text{ N}\cdot\text{m}}{-8.75 \text{ N}\cdot\text{m}}$$

$$\phi = 73.740 \qquad \phi = 73.7° \blacktriangleleft$$

Squaring Eqs. (2) and (3) and adding gives:

$$M_y^2 + M_z^2 = (0.25)^2 P^2 \quad \text{or} \quad P = 4\sqrt{M_y^2 + M_z^2} \qquad (5)$$

$$P = 4\sqrt{(8.75)^2 + (30)^2}$$

$$= 125.0 \text{ N} \qquad P = 125.0 \text{ N} \blacktriangleleft$$

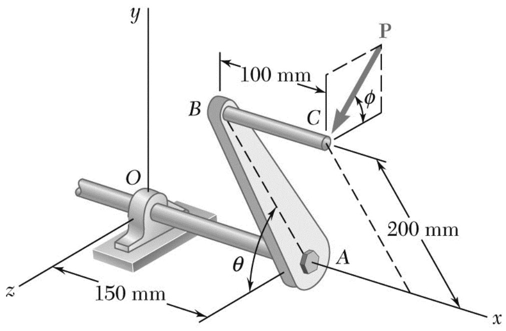
Substituting data into Eq. (1):

$$(+20 \text{ N}\cdot\text{m}) = 0.2 \text{ m}(125.0 \text{ N})\sin(\theta+\phi)$$

$$(\theta+\phi) = 53.130° \quad \text{and} \quad (\theta+\phi) = 126.87°$$

$$\theta = -20.6° \quad \text{and} \quad \theta = 53.1°$$

$$Q = 53.1° \blacktriangleleft$$



PROBLEM 3.54

A single force **P** acts at C in a direction perpendicular to the handle BC of the crank shown. Determine the moment M_x of **P** about the x-axis when $\theta = 65°$, knowing that $M_y = -15\text{ N}\cdot\text{m}$ and $M_z = -36\text{ N}\cdot\text{m}$.

SOLUTION

See the solution to Prob. 3.53 for the derivation of the following equations:

$$M_x = (0.2)P\sin(\theta + \phi) \qquad (1)$$

$$\tan\phi = \frac{M_z}{M_y} \qquad (4)$$

$$P = 4\sqrt{M_y^2 + M_z^2} \qquad (5)$$

Substituting for known data gives:

$$\tan\phi = \frac{-36\text{ N}\cdot\text{m}}{-15\text{ N}\cdot\text{m}}$$

$$\phi = 67.380°$$

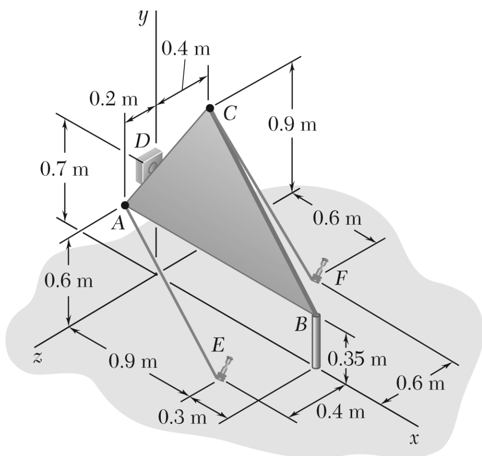
$$P = 4\sqrt{(-15)^2 + (-36)^2}$$

$$P = 156.0\text{ N}$$

$$M_x = 0.2\text{ m}(156.0\text{ N})\sin(65° + 67.380°)$$

$$= 23.047\text{ N}\cdot\text{m}$$

$$M_x = 23.0\text{ N}\cdot\text{m} \blacktriangleleft$$



PROBLEM 3.55

The triangular plate *ABC* is supported by ball-and-socket joints at *B* and *D* and is held in the position shown by cables *AE* and *CF*. If the force exerted by cable *AE* at *A* is 55 N, determine the moment of that force about the line joining Points *D* and *B*.

SOLUTION

First note:

$$\mathbf{T}_{AE} = T_{AE}\frac{\overrightarrow{AE}}{AE}$$

$$AE = \sqrt{(0.9)^2 + (-0.6)^2 + (0.2)^2} = 1.1\text{ m}$$

Then

$$\mathbf{T}_{AE} = \frac{55\text{ N}}{1.1}(0.9\mathbf{i} - 0.6\mathbf{j} + 0.2\mathbf{k})$$

$$= 5[(9\text{ N})\mathbf{i} - (6\text{ N})\mathbf{j} + (2\text{ N})\mathbf{k}]$$

Also,

$$DB = \sqrt{(1.2)^2 + (-0.35)^2 + (0)^2}$$

$$= 1.25\text{ m}$$

Then

$$\boldsymbol{\lambda}_{DB} = \frac{\overrightarrow{DB}}{DB}$$

$$= \frac{1}{1.25}(1.2\mathbf{i} - 0.35\mathbf{j})$$

$$= \frac{1}{25}(24\mathbf{i} - 7\mathbf{j})$$

Now

$$M_{DB} = \boldsymbol{\lambda}_{DB} \cdot (\mathbf{r}_{A/D} \times \mathbf{T}_{AE})$$

where

$$\mathbf{r}_{A/D} = -(0.1\text{ m})\mathbf{j} + (0.2\text{ m})\mathbf{k}$$

Then

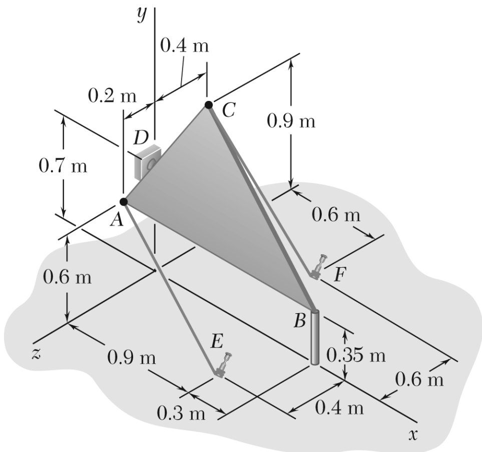
$$M_{DB} = \frac{1}{25}(5)\begin{vmatrix} 24 & -7 & 0 \\ 0 & -0.1 & 0.2 \\ 9 & -6 & 2 \end{vmatrix}$$

$$= \frac{1}{5}(-4.8 - 12.6 + 28.8)$$

or $M_{DB} = 2.28\text{ N}\cdot\text{m}$ ◀

PROBLEM 3.56

The triangular plate *ABC* is supported by ball-and-socket joints at *B* and *D* and is held in the position shown by cables *AE* and *CF*. If the force exerted by cable *CF* at *C* is 33 N, determine the moment of that force about the line joining Points *D* and *B*.



SOLUTION

First note:
$$\mathbf{T}_{CF} = T_{CF}\frac{\overrightarrow{CF}}{CF}$$

$$CF = \sqrt{(0.6)^2 + (-0.9)^2 + (-0.2)^2} = 1.1\text{ m}$$

Then
$$\mathbf{T}_{CF} = \frac{33\text{ N}}{1.1}(0.6\mathbf{i} - 0.9\mathbf{j} + 0.2\mathbf{k})$$
$$= 3[(6\text{ N})\mathbf{i} - (9\text{ N})\mathbf{j} - (2\text{ N})\mathbf{k}]$$

Also,
$$DB = \sqrt{(1.2)^2 + (-0.35)^2 + (0)^2}$$
$$= 1.25\text{ m}$$

Then
$$\boldsymbol{\lambda}_{DB} = \frac{\overrightarrow{DB}}{DB}$$
$$= \frac{1}{1.25}(1.2\mathbf{i} - 0.35\mathbf{j})$$
$$= \frac{1}{25}(24\mathbf{i} - 7\mathbf{j})$$

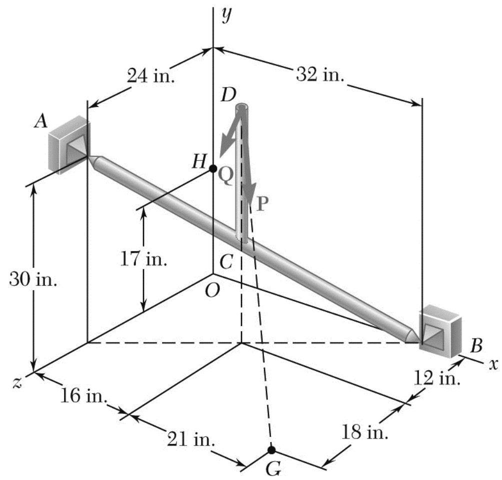
Now
$$M_{DB} = \boldsymbol{\lambda}_{DB}\cdot(\mathbf{r}_{C/D} \times \mathbf{T}_{CF})$$

where
$$\mathbf{r}_{C/D} = (0.2\text{ m})\mathbf{j} - (0.4\text{ m})\mathbf{k}$$

Then
$$M_{DB} = \frac{1}{25}(3)\begin{vmatrix} 24 & -7 & 0 \\ 0 & 0.2 & -0.4 \\ 6 & -9 & -2 \end{vmatrix}$$
$$= \frac{3}{25}(-9.6 + 16.8 - 86.4)$$

or $M_{DB} = -9.50\text{ N}\cdot\text{m}$ ◀

PROBLEM 3.57



The 23-in. vertical rod *CD* is welded to the midpoint *C* of the 50-in. rod *AB*. Determine the moment about *AB* of the 235-lb force **P**.

SOLUTION

$$\overline{AB} = (32\text{ in.})\mathbf{i} - (30\text{ in.})\mathbf{j} - (24\text{ in.})\mathbf{k}$$

$$AB = \sqrt{(32)^2 + (-30)^2 + (-24)^2} = 50\text{ in.}$$

$$\boldsymbol{\lambda}_{AB} = \frac{\overline{AB}}{AB} = 0.64\mathbf{i} - 0.60 - 0.48\mathbf{k}$$

We shall apply the force *P* at Point *G*:

$$\mathbf{r}_{G/B} = (5\text{ in.})\mathbf{i} + (30\text{ in.})\mathbf{k}$$

$$\overrightarrow{DG} = (21\text{ in.})\mathbf{i} - (38\text{ in.})\mathbf{j} + (18\text{ in.})\mathbf{k}$$

$$DG = \sqrt{(21)^2 + (-38)^2 + (18)^2} = 47\text{ in.}$$

$$\mathbf{P} = P\frac{\overrightarrow{DG}}{DG} = (235\text{ lb})\frac{21\mathbf{i} - 38\mathbf{j} + 18\mathbf{k}}{47}$$

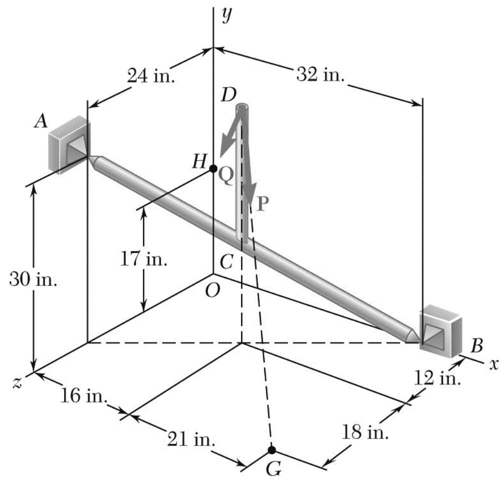
$$\mathbf{P} = (105\text{ lb})\mathbf{i} - (190\text{ lb})\mathbf{j} + (90\text{ lb})\mathbf{k}$$

The moment of **P** about *AB* is given by Eq. (3.46):

$$\mathbf{M}_{AB} = \boldsymbol{\lambda}_{AB} \cdot (\mathbf{r}_{G/B} \times P) = \begin{vmatrix} 0.64 & -0.60 & -0.48 \\ 5\text{ in.} & 0 & 30\text{ in.} \\ 105\text{ lb} & -190\text{ lb} & 90\text{ lb} \end{vmatrix}$$

$$\begin{aligned} \mathbf{M}_{AB} &= 0.64[0 - (30\text{ in.})(-190\text{ lb})] \\ &\quad - 0.60[(30\text{ in.})(105\text{ lb}) - (5\text{ in.})(90\text{ lb})] \\ &\quad - 0.48[(5\text{ in.})(-190\text{ lb}) - 0] \\ &= +2484\text{ lb}\cdot\text{in.} \end{aligned}$$

$$\mathbf{M}_{AB} = +207\text{ lb}\cdot\text{ft}$$ ◀



PROBLEM 3.58

The 23-in. vertical rod *CD* is welded to the midpoint *C* of the 50-in. rod *AB*. Determine the moment about *AB* of the 174-lb force **Q**.

SOLUTION

$$\overline{AB} = (32\text{ in.})\mathbf{i} - (30\text{ in.})\mathbf{j} - (24\text{ in.})\mathbf{k}$$

$$AB = \sqrt{(32)^2 + (-30)^2 + (-24)^2} = 50\text{ in.}$$

$$\boldsymbol{\lambda}_{AB} = \frac{\overline{AB}}{AB} = 0.64\mathbf{i} - 0.60\mathbf{j} - 0.48\mathbf{k}$$

We shall apply the force **Q** at Point *H*:

$$\mathbf{r}_{H/B} = -(32\text{ in.})\mathbf{i} + (17\text{ in.})\mathbf{j}$$

$$\overline{DH} = -(16\text{ in.})\mathbf{i} - (21\text{ in.})\mathbf{j} - (12\text{ in.})\mathbf{k}$$

$$DH = \sqrt{(16)^2 + (-21)^2 + (-12)^2} = 29\text{ in.}$$

$$\mathbf{Q} = \frac{\overline{DH}}{DH} = (174\text{ lb})\frac{-16\mathbf{i} - 21\mathbf{j} - 12\mathbf{k}}{29}$$

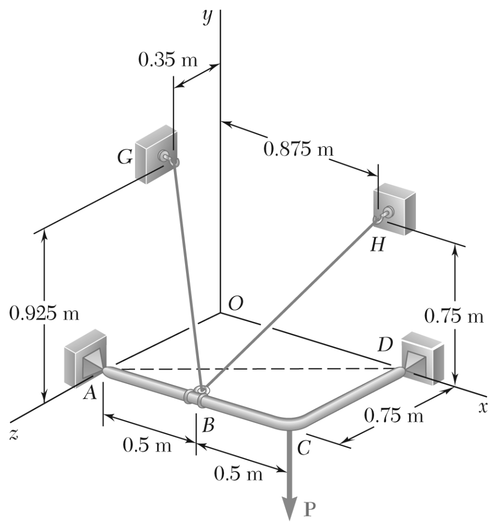
$$Q = -(96\text{ lb})\mathbf{i} - (126\text{ lb})\mathbf{j} - (72\text{ lb})\mathbf{k}$$

The moment of **Q** about *AB* is given by Eq. (3.46):

$$\mathbf{M}_{AB} = \boldsymbol{\lambda}_{AB} \cdot (\mathbf{r}_{H/B} \times \mathbf{Q}) = \begin{vmatrix} 0.64 & -0.60 & -0.48 \\ -32\text{ in.} & 17\text{ in.} & 0 \\ -96\text{ lb} & -126\text{ lb} & -72\text{ lb} \end{vmatrix}$$

$$\begin{aligned} \mathbf{M}_{AB} &= 0.64[(17\text{ in.})(-72\text{ lb}) - 0] \\ &\quad - 0.60[(0 - (-32\text{ in.})(-72\text{ lb})] \\ &\quad - 0.48[(-32\text{ in.})(-126\text{ lb}) - (17\text{ in.})(-96\text{ lb})] \\ &= -2119.7\text{ lb}\cdot\text{in.} \end{aligned}$$

$\mathbf{M}_{AB} = 176.6\text{ lb}\cdot\text{ft}$ ◀



PROBLEM 3.59

The frame *ACD* is hinged at *A* and *D* and is supported by a cable that passes through a ring at *B* and is attached to hooks at *G* and *H*. Knowing that the tension in the cable is 450 N, determine the moment about the diagonal *AD* of the force exerted on the frame by portion *BH* of the cable.

SOLUTION

$$M_{AD} = \boldsymbol{\lambda}_{AD} \cdot (\mathbf{r}_{B/A} \times \mathbf{T}_{BH})$$

Where

$$\boldsymbol{\lambda}_{AD} = \frac{1}{5}(4\mathbf{i} - 3\mathbf{k})$$

$$\mathbf{r}_{B/A} = (0.5 \text{ m})\mathbf{i}$$

and

$$d_{BH} = \sqrt{(0.375)^2 + (0.75)^2 + (-0.75)^2}$$

$$= 1.125 \text{ m}$$

Then

$$\mathbf{T}_{BH} = \frac{450 \text{ N}}{1.125}(0.375\mathbf{i} + 0.75\mathbf{j} - 0.75\mathbf{k})$$

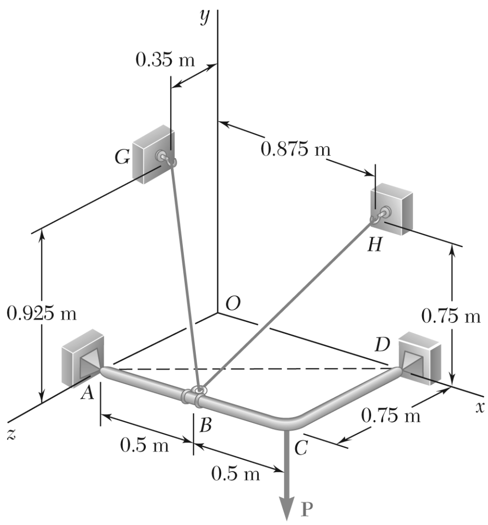
$$= (150 \text{ N})\mathbf{i} + (300 \text{ N})\mathbf{j} - (300 \text{ N})\mathbf{k}$$

Finally,

$$M_{AD} = \frac{1}{5}\begin{vmatrix} 4 & 0 & -3 \\ 0.5 & 0 & 0 \\ 150 & 300 & -300 \end{vmatrix}$$

$$= \frac{1}{5}[(-3)(0.5)(300)]$$

or $M_{AD} = -90.0 \text{ N}\cdot\text{m}$ ◀



PROBLEM 3.60

In Problem 3.59, determine the moment about the diagonal *AD* of the force exerted on the frame by portion *BG* of the cable.

PROBLEM 3.59 The frame *ACD* is hinged at *A* and *D* and is supported by a cable that passes through a ring at *B* and is attached to hooks at *G* and *H*. Knowing that the tension in the cable is 450 N, determine the moment about the diagonal *AD* of the force exerted on the frame by portion *BH* of the cable.

SOLUTION

$$M_{AD} = \boldsymbol{\lambda}_{AD} \cdot (\mathbf{r}_{B/A} \times \mathbf{T}_{BG})$$

Where

$$\boldsymbol{\lambda}_{AD} = \frac{1}{5}(4\mathbf{i} - 3\mathbf{k})$$

$$\mathbf{r}_{B/A} = (0.5 \text{ m})\mathbf{j}$$

and

$$BG = \sqrt{(-0.5)^2 + (0.925)^2 + (-0.4)^2}$$
$$= 1.125 \text{ m}$$

Then

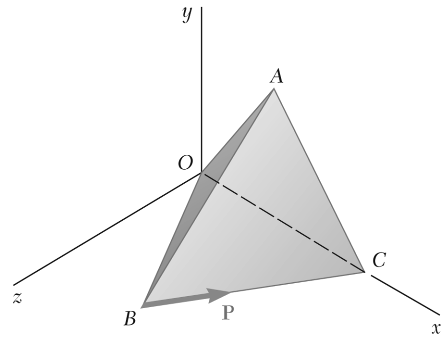
$$\mathbf{T}_{BG} = \frac{450 \text{ N}}{1.125}(-0.5\mathbf{i} + 0.925\mathbf{j} - 0.4\mathbf{k})$$
$$= -(200 \text{ N})\mathbf{i} + (370 \text{ N})\mathbf{j} - (160 \text{ N})\mathbf{k}$$

Finally,

$$M_{AD} = \frac{1}{5}\begin{vmatrix} 4 & 0 & -3 \\ 0.5 & 0 & 0 \\ -200 & 370 & -160 \end{vmatrix}$$

$$= \frac{1}{5}[(-3)(0.5)(370)]$$

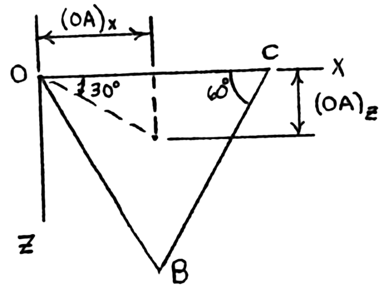
$M_{AD} = -111.0 \text{ N}\cdot\text{m}$ ◀



PROBLEM 3.61

A regular tetrahedron has six edges of length a. A force **P** is directed as shown along edge BC. Determine the moment of **P** about edge OA.

SOLUTION



We have $$M_{OA} = \boldsymbol{\lambda}_{OA} \cdot (\mathbf{r}_{C/O} \times \mathbf{P})$$

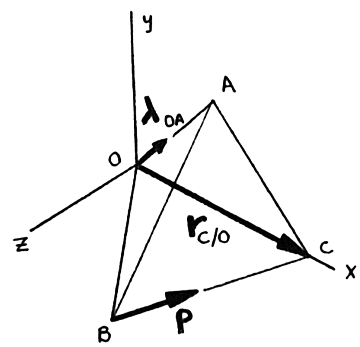
From triangle OBC: $$(OA)_x = \frac{a}{2}$$

$$(OA)_z = (OA)_x \tan 30° = \frac{a}{2}\left(\frac{1}{\sqrt{3}}\right) = \frac{a}{2\sqrt{3}}$$

Since $$(OA)^2 = (OA)_x^2 + (OA)_y^2 + (OA_z)^2$$

or $$a^2 = \left(\frac{a}{2}\right)^2 + (OA)_y^2 + \left(\frac{a}{2\sqrt{3}}\right)^2$$

$$(OA)_y = \sqrt{a^2 - \frac{a^2}{4} - \frac{a^2}{12}} = a\sqrt{\frac{2}{3}}$$



Then $$\mathbf{r}_{A/O} = \frac{a}{2}\mathbf{i} + a\sqrt{\frac{2}{3}}\mathbf{j} + \frac{a}{2\sqrt{3}}\mathbf{k}$$

and $$\boldsymbol{\lambda}_{OA} = \frac{1}{2}\mathbf{i} + \sqrt{\frac{2}{3}}\mathbf{j} + \frac{1}{2\sqrt{3}}\mathbf{k}$$

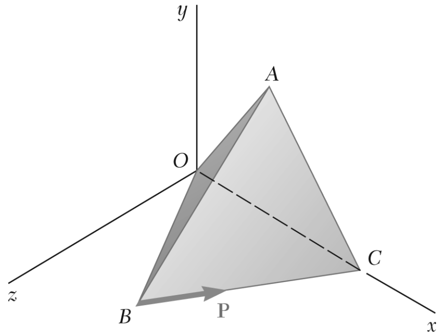
$$\mathbf{P} = \boldsymbol{\lambda}_{BC}P = \frac{(a\sin 30°)\mathbf{i} - (a\cos 30°)\mathbf{k}}{a}(P) = \frac{P}{2}(\mathbf{i} - \sqrt{3}\mathbf{k})$$

$$\mathbf{r}_{C/O} = a\mathbf{i}$$

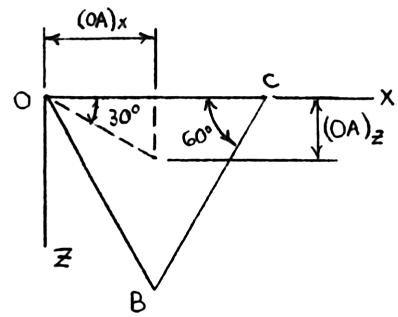
$$M_{OA} = \begin{vmatrix} \frac{1}{2} & \sqrt{\frac{2}{3}} & \frac{1}{2\sqrt{3}} \\ 1 & 0 & 0 \\ 1 & 0 & -\sqrt{3} \end{vmatrix}(a)\left(\frac{P}{2}\right)$$

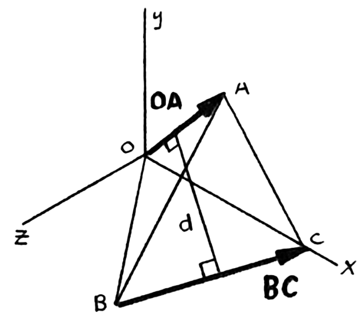
$$= \frac{aP}{2}\left(-\sqrt{\frac{2}{3}}\right)(1)(-\sqrt{3}) = \frac{aP}{\sqrt{2}}$$

$$M_{OA} = \frac{aP}{\sqrt{2}} \blacktriangleleft$$

	**PROBLEM 3.62** A regular tetrahedron has six edges of length a. (a) Show that two opposite edges, such as OA and BC, are perpendicular to each other. (b) Use this property and the result obtained in Problem 3.61 to determine the perpendicular distance between edges OA and BC.

SOLUTION





(a) For edge OA to be perpendicular to edge BC,

$$\overrightarrow{OA}\cdot\overrightarrow{BC}=0$$

From triangle OBC: $\quad (OA)_x=\dfrac{a}{2}$

$$(OA)_z=(OA)_x\tan 30°=\frac{a}{2}\left(\frac{1}{\sqrt{3}}\right)=\frac{a}{2\sqrt{3}}$$

$$\overrightarrow{OA}=\left(\frac{a}{2}\right)\mathbf{i}+(OA)_y\,\mathbf{j}+\left(\frac{a}{2\sqrt{3}}\right)\mathbf{k}$$

and

$$\overrightarrow{BC}=(a\sin 30°)\mathbf{i}-(a\cos 30°)\mathbf{k}$$

$$=\frac{a}{2}\mathbf{i}-\frac{a\sqrt{3}}{2}\mathbf{k}=\frac{a}{2}(\mathbf{i}-\sqrt{3}\,\mathbf{k})$$

Then

$$\left[\frac{a}{2}\mathbf{i}+(OA)_y\mathbf{j}+\left(\frac{a}{2\sqrt{3}}\right)\mathbf{k}\right]\cdot(\mathbf{i}-\sqrt{3}\mathbf{k})\frac{a}{2}=0$$

or

$$\frac{a^2}{4}+(OA)_y(0)-\frac{a^2}{4}=0$$

$$\overrightarrow{OA}\cdot\overrightarrow{BC}=0$$

so that $\qquad \overrightarrow{OA}$ is perpendicular to $\overrightarrow{BC}$. ◀

(b) We have $M_{OA}=Pd$, with P acting along BC and d the perpendicular distance from $\overrightarrow{OA}$ to $\overrightarrow{BC}$.

From the results of Problem 3.57,

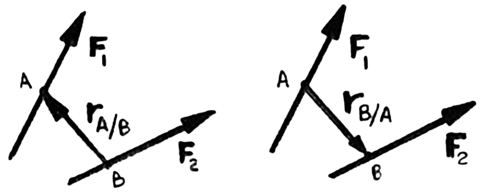
$$M_{OA}=\frac{Pa}{\sqrt{2}}$$

$$\frac{Pa}{\sqrt{2}}=Pd \qquad \text{or}\quad d=\frac{a}{\sqrt{2}}\ ◀$$

PROBLEM 3.63

Two forces $\mathbf{F}_1$ and $\mathbf{F}_2$ in space have the same magnitude F. Prove that the moment of $\mathbf{F}_1$ about the line of action of $\mathbf{F}_2$ is equal to the moment of $\mathbf{F}_2$ about the line of action of $\mathbf{F}_1$.

SOLUTION



First note that $\mathbf{F}_1 = F_1\boldsymbol{\lambda}_1$ and $\mathbf{F}_2 = F_2\boldsymbol{\lambda}_2$

Let $M_1 =$ moment of $\mathbf{F}_2$ about the line of action of $\mathbf{F}_1$ and $M_2 =$ moment of $\mathbf{F}_1$ about the line of action of $\mathbf{F}_2$.

Now, by definition,

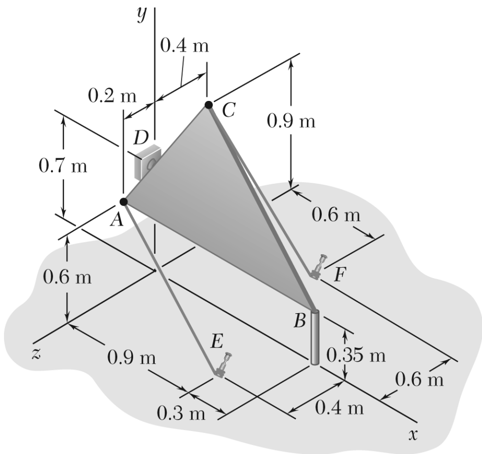
$$M_1 = \boldsymbol{\lambda}_1 \cdot (\mathbf{r}_{B/A} \times \mathbf{F}_2)$$
$$= \boldsymbol{\lambda}_1 \cdot (\mathbf{r}_{B/A} \times \boldsymbol{\lambda}_2)F_2$$
$$M_2 = \boldsymbol{\lambda}_2 \cdot (\mathbf{r}_{A/B} \times \mathbf{F}_1)$$
$$= \boldsymbol{\lambda}_2 \cdot (\mathbf{r}_{A/B} \times \boldsymbol{\lambda}_1)F_1$$

Since $F_1 = F_2 = F$ and $\mathbf{r}_{A/B} = -\mathbf{r}_{B/A}$

$$M_1 = \boldsymbol{\lambda}_1 \cdot (\mathbf{r}_{B/A} \times \boldsymbol{\lambda}_2)F$$
$$M_2 = \boldsymbol{\lambda}_2 \cdot (-\mathbf{r}_{B/A} \times \boldsymbol{\lambda}_1)F$$

Using Equation (3.39): $\boldsymbol{\lambda}_1 \cdot (\mathbf{r}_{B/A} \times \boldsymbol{\lambda}_2) = \boldsymbol{\lambda}_2 \cdot (-\mathbf{r}_{B/A} \times \boldsymbol{\lambda}_1)$

so that $M_2 = \boldsymbol{\lambda}_1 \cdot (\mathbf{r}_{B/A} \times \boldsymbol{\lambda}_2)F$ $\quad M_{12} = M_{21}$ ◀



PROBLEM 3.64

In Problem 3.55, determine the perpendicular distance between cable *AE* and the line joining Points *D* and *B*.

PROBLEM 3.55 The triangular plate *ABC* is supported by ball-and-socket joints at *B* and *D* and is held in the position shown by cables *AE* and *CF*. If the force exerted by cable *AE* at *A* is 55 N, determine the moment of that force about the line joining Points *D* and *B*.

SOLUTION

From the solution to Problem 3.55:

$$T_{AE} = 55 \text{ N}$$
$$\mathbf{T}_{AE} = 5[(9 \text{ N})\mathbf{i} - (6 \text{ N})\mathbf{j} + (2 \text{ N})\mathbf{k}]$$

$$|M_{DB}| = 2.28 \text{ N}\cdot\text{m}$$

$$\boldsymbol{\lambda}_{DB} = \frac{1}{25}(24\mathbf{i} - 7\mathbf{j})$$

Based on the discussion of Section 3.11, it follows that only the perpendicular component of $\mathbf{T}_{AE}$ will contribute to the moment of $\mathbf{T}_{AE}$ about line $\overrightarrow{DB}$.

Now
$$(T_{AE})_{\text{parallel}} = \mathbf{T}_{AE} \cdot \boldsymbol{\lambda}_{DB}$$
$$= 5(9\mathbf{i} - 6\mathbf{j} + 2\mathbf{k}) \cdot \frac{1}{25}(24\mathbf{i} - 7\mathbf{j})$$
$$= \frac{1}{5}[(9)(24) + (-6)(-7)]$$
$$= 51.6 \text{ N}$$

Also,
$$\mathbf{T}_{AE} = (\mathbf{T}_{AE})_{\text{parallel}} + (\mathbf{T}_{AE})_{\text{perpendicular}}$$

so that
$$(\mathbf{T}_{AE})_{\text{perpendicular}} = \sqrt{(55)^2 + (51.6)^2} = 19.0379 \text{ N}$$

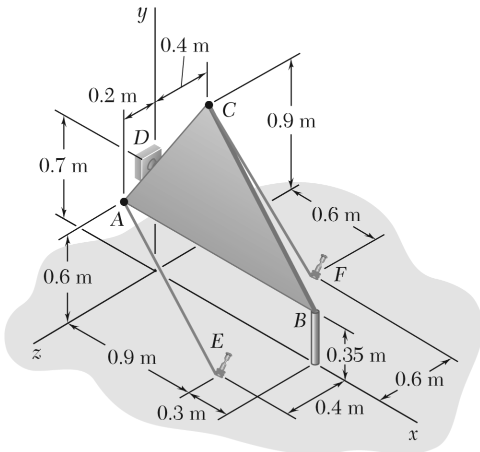
Since $\boldsymbol{\lambda}_{DB}$ and $(\mathbf{T}_{AE})_{\text{perpendicular}}$ are perpendicular, it follows that

$$M_{DB} = d(T_{AE})_{\text{perpendicular}}$$

or
$$2.28 \text{ N}\cdot\text{m} = d(19.0379 \text{ N})$$

$$d = 0.119761$$

$d = 0.1198$ m ◀



PROBLEM 3.65

In Problem 3.56, determine the perpendicular distance between cable *CF* and the line joining Points *D* and *B*.

PROBLEM 3.56 The triangular plate *ABC* is supported by ball-and-socket joints at *B* and *D* and is held in the position shown by cables *AE* and *CF*. If the force exerted by cable *CF* at *C* is 33 N, determine the moment of that force about the line joining Points *D* and *B*.

SOLUTION

From the solution to Problem 3.56:

$$T_{CF} = 33 \text{ N}$$

$$\mathbf{T}_{CF} = 3[(6 \text{ N})\mathbf{i} - (9 \text{ N})\mathbf{j} - (2 \text{ N})\mathbf{k}]$$

$$|M_{DB}| = 9.50 \text{ N}\cdot\text{m}$$

$$\boldsymbol{\lambda}_{DB} = \frac{1}{25}(24\mathbf{i} - 7\mathbf{j})$$

Based on the discussion of Section 3.11, it follows that only the perpendicular component of $\mathbf{T}_{CF}$ will contribute to the moment of $\mathbf{T}_{CF}$ about line $\overrightarrow{DB}$.

Now

$$(\mathbf{T}_{CF})_{\text{parallel}} = \mathbf{T}_{CF} \cdot \boldsymbol{\lambda}_{DB}$$

$$= 3(6\mathbf{i} - 9\mathbf{j} - 2\mathbf{k}) \cdot \frac{1}{25}(24\mathbf{i} - 7\mathbf{j})$$

$$= \frac{3}{25}[(6)(24) + (-9)(-7)]$$

$$= 24.84 \text{ N}$$

Also,

$$\mathbf{T}_{CF} = (\mathbf{T}_{CF})_{\text{parallel}} + (\mathbf{T}_{CF})_{\text{perpendicular}}$$

so that

$$(\mathbf{T}_{CF})_{\text{perpendicular}} = \sqrt{(33)^2 - (24.84)^2}$$

$$= 21.725 \text{ N}$$

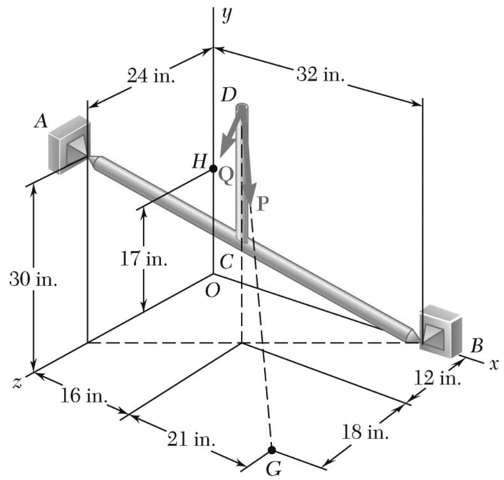
Since $\boldsymbol{\lambda}_{DB}$ and $(\mathbf{T}_{CF})_{\text{perpendicular}}$ are perpendicular, it follows that

$$|M_{DB}| = d(T_{CF})_{\text{perpendicular}}$$

or

$$9.50 \text{ N}\cdot\text{m} = d \times 21.725 \text{ N}$$

or $d = 0.437$ m ◀



PROBLEM 3.66

In Prob. 3.57, determine the perpendicular distance between rod *AB* and the line of action of **P**.

PROBLEM 3.57 The 23-in. vertical rod *CD* is welded to the midpoint *C* of the 50-in. rod *AB*. Determine the moment about *AB* of the 235-lb force **P**.

SOLUTION

$$\overline{AB} = (32\text{ in.})\mathbf{i} - (30\text{ in.})\mathbf{j} - (24\text{ in.})\mathbf{k}$$

$$AB = \sqrt{(32)^2 + (-30)^2 + (-24)^2} = 50\text{ in.}$$

$$\boldsymbol{\lambda}_{AB} = \frac{\overline{AB}}{AB} = 0.64\mathbf{i} - 0.60\mathbf{j} - 0.48\mathbf{k}$$

$$\boldsymbol{\lambda}_P = \frac{\mathbf{P}}{P} = \frac{105\mathbf{i} - 190\mathbf{j} + 90\mathbf{k}}{235}$$

Angle θ between *AB* and **P**:

$$\cos\theta = \boldsymbol{\lambda}_{AB} \cdot \boldsymbol{\lambda}_P$$

$$= (0.64\mathbf{i} - 0.60\mathbf{j} - 0.48\mathbf{k}) \cdot \frac{105\mathbf{i} - 190\mathbf{j} + 90\mathbf{k}}{235}$$

$$= 0.58723$$

$$\therefore \quad \theta = 54.039°$$

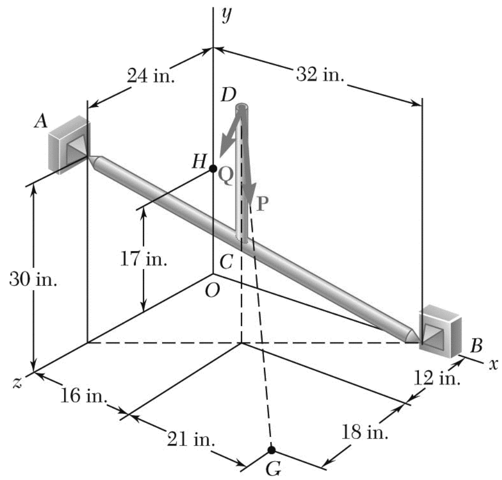
The moment of **P** about *AB* may be obtained by multiplying the projection of **P** on a plane perpendicular to *AB* by the perpendicular distance *d* from *AB* to **P**:

$$\mathbf{M}_{AB} = (P\sin\theta)d$$

From the solution to Prob. 3.57: $\mathbf{M}_{AB} = 207\text{ lb}\cdot\text{ft} = 2484\text{ lb}\cdot\text{in.}$

We have $\quad 2484\text{ lb}\cdot\text{in.} = (235\text{ lb})(\sin 54.039)d$

$d = 13.06$ in. ◀



PROBLEM 3.67

In Prob. 3.58, determine the perpendicular distance between rod *AB* and the line of action of **Q**.

PROBLEM 3.58 The 23-in. vertical rod *CD* is welded to the midpoint *C* of the 50-in. rod *AB*. Determine the moment about *AB* of the 174-lb force **Q**.

SOLUTION

$$\overline{AB} = (32 \text{ in.})\mathbf{i} - (30 \text{ in.})\mathbf{j} - (24 \text{ in.})\mathbf{k}$$

$$AB = \sqrt{(32)^2 + (-30)^2 + (-24)^2} = 50 \text{ in.}$$

$$\boldsymbol{\lambda}_{AB} = \frac{\overline{AB}}{AB} = 0.64\mathbf{i} - 0.60\mathbf{j} - 0.48\mathbf{k}$$

$$\boldsymbol{\lambda}_Q = \frac{\mathbf{Q}}{Q} = \frac{-96\mathbf{i} - 126\mathbf{j} - 72\mathbf{k}}{174}$$

Angle θ between *AB* and **Q**:

$$\cos\theta = \boldsymbol{\lambda}_{AB} \cdot \boldsymbol{\lambda}_Q$$

$$= (0.64\mathbf{i} - 0.60\mathbf{j} - 0.48\mathbf{k}) \cdot \frac{(-96\mathbf{i} - 126\mathbf{j} - 72\mathbf{k})}{174}$$

$$= 0.28000$$

$$\therefore \quad \theta = 73.740°$$

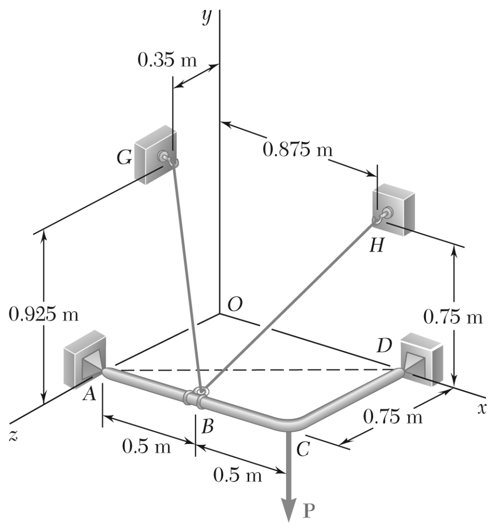
The moment of **Q** about *AB* may be obtained by multiplying the projection of **Q** on a plane perpendicular to *AB* by the perpendicular distance *d* from *AB* to **Q**:

$$\mathbf{M}_{AB} = (Q\sin\theta)d$$

From the solution to Prob. 3.58: $\mathbf{M}_{AB} = 176.6 \text{ lb}\cdot\text{ft} = 2119.2 \text{ lb}\cdot\text{in.}$

$$2119.2 \text{ lb}\cdot\text{in.} = (174 \text{ lb})(\sin 73.740°)d$$

$d = 12.69$ in. ◀



PROBLEM 3.68

In Problem 3.59, determine the perpendicular distance between portion *BH* of the cable and the diagonal *AD*.

PROBLEM 3.59 The frame *ACD* is hinged at *A* and *D* and is supported by a cable that passes through a ring at *B* and is attached to hooks at *G* and *H*. Knowing that the tension in the cable is 450 N, determine the moment about the diagonal *AD* of the force exerted on the frame by portion *BH* of the cable.

SOLUTION

From the solution to Problem 3.59:
$$T_{BH} = 450 \text{ N}$$
$$\mathbf{T}_{BH} = (150 \text{ N})\mathbf{i} + (300 \text{ N})\mathbf{j} - (300 \text{ N})\mathbf{k}$$
$$|M_{AD}| = 90.0 \text{ N}\cdot\text{m}$$
$$\boldsymbol{\lambda}_{AD} = \frac{1}{5}(4\mathbf{i} - 3\mathbf{k})$$

Based on the discussion of Section 3.11, it follows that only the perpendicular component of $\mathbf{T}_{BH}$ will contribute to the moment of $\mathbf{T}_{BH}$ about line $\overrightarrow{AD}$.

Now
$$(T_{BH})_{\text{parallel}} = \mathbf{T}_{BH}\cdot\boldsymbol{\lambda}_{AD}$$
$$= (150\mathbf{i} + 300\mathbf{j} - 300\mathbf{k})\cdot\frac{1}{5}(4\mathbf{i} - 3\mathbf{k})$$
$$= \frac{1}{5}[(150)(4) + (-300)(-3)]$$
$$= 300 \text{ N}$$

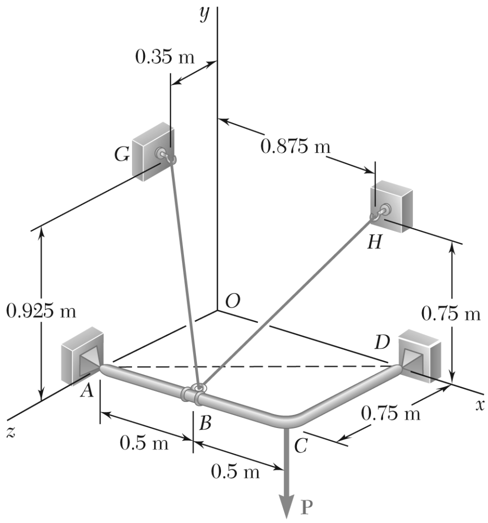
Also,
$$\mathbf{T}_{BH} = (\mathbf{T}_{BH})_{\text{parallel}} + (\mathbf{T}_{BH})_{\text{perpendicular}}$$

so that
$$(T_{BH})_{\text{perpendicular}} = \sqrt{(450)^2 - (300)^2} = 335.41 \text{ N}$$

Since $\boldsymbol{\lambda}_{AD}$ and $(\mathbf{T}_{BH})_{\text{perpendicular}}$ are perpendicular, it follows that
$$M_{AD} = d(T_{BH})_{\text{perpendicular}}$$

or
$$90.0 \text{ N}\cdot\text{m} = d(335.41 \text{ N})$$
$$d = 0.26833 \text{ m}$$

$d = 0.268$ m ◀



PROBLEM 3.69

In Problem 3.60, determine the perpendicular distance between portion *BG* of the cable and the diagonal *AD*.

PROBLEM 3.60 In Problem 3.59, determine the moment about the diagonal *AD* of the force exerted on the frame by portion *BG* of the cable.

SOLUTION

From the solution to Problem 3.60: $T_{BG} = 450$ N

$$\mathbf{T}_{BG} = -(200\text{ N})\mathbf{i} + (370\text{ N})\mathbf{j} - (160\text{ N})\mathbf{k}$$

$$|M_{AD}| = 111\text{ N}\cdot\text{m}$$

$$\boldsymbol{\lambda}_{AD} = \frac{1}{5}(4\mathbf{i} - 3\mathbf{k})$$

Based on the discussion of Section 3.11, it follows that only the perpendicular component of $\mathbf{T}_{BG}$ will contribute to the moment of $\mathbf{T}_{BG}$ about line $\overline{AD}$.

Now
$$(T_{BG})_{\text{parallel}} = \mathbf{T}_{BG} \cdot \boldsymbol{\lambda}_{AD}$$
$$= (-200\mathbf{i} + 370\mathbf{j} - 160\mathbf{k}) \cdot \frac{1}{5}(4\mathbf{i} - 3\mathbf{k})$$
$$= \frac{1}{5}[(-200)(4) + (-160)(-3)]$$
$$= -64\text{ N}$$

Also,
$$\bar{\mathbf{T}}_{BG} = (\mathbf{T}_{BG})_{\text{parallel}} + (\mathbf{T}_{BG})_{\text{perpendicular}}$$

so that
$$(\mathbf{T}_{BG})_{\text{perpendicular}} = \sqrt{(450)^2 - (-64)^2} = 445.43\text{ N}$$

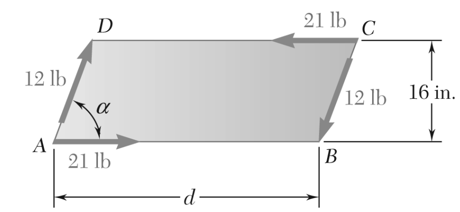
Since $\boldsymbol{\lambda}_{AD}$ and $(\mathbf{T}_{BG})_{\text{perpendicular}}$ are perpendicular, it follows that

$$M_{AD} = d(T_{BG})_{\text{perpendicular}}$$

or
$$111\text{ N}\cdot\text{m} = d(445.43\text{ N})$$

$$d = 0.24920\text{ m}$$

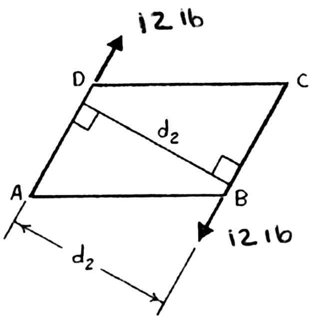
$d = 0.249$ m ◀

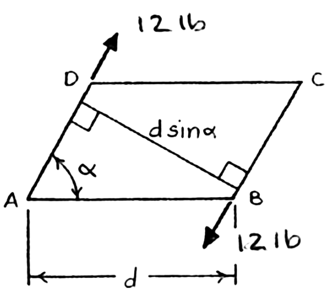


PROBLEM 3.70

A plate in the shape of a parallelogram is acted upon by two couples. Determine (*a*) the moment of the couple formed by the two 21-lb forces, (*b*) the perpendicular distance between the 12-lb forces if the resultant of the two couples is zero, (*c*) the value of α if the resultant couple is $72\text{ lb}\cdot\text{in.}$ clockwise and d is 42 in.

SOLUTION





(*a*) We have $M_1 = d_1 F_1$

where $d_1 = 16\text{ in.}$

$F_1 = 21\text{ lb}$

$$M_1 = (16\text{ in.})(21\text{ lb})$$
$$= 336\text{ lb}\cdot\text{in.}$$

or $\mathbf{M}_1 = 336\text{ lb}\cdot\text{in.}$ ↻ ◀

(*b*) We have $\mathbf{M}_1 + \mathbf{M}_2 = 0$

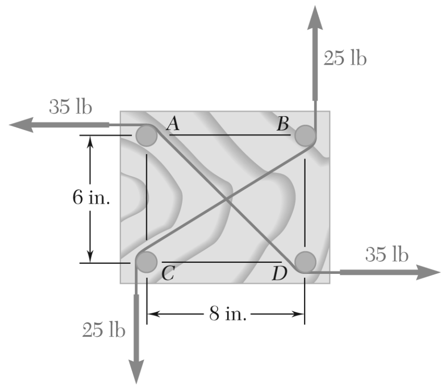
or $336\text{ lb}\cdot\text{in.} - d_2(12\text{ lb}) = 0$ $\qquad d_2 = 28.0\text{ in.}$ ◀

(*c*) We have $\mathbf{M}_{\text{total}} = \mathbf{M}_1 + \mathbf{M}_2$

or $-72\text{ lb}\cdot\text{in.} = 336\text{ lb}\cdot\text{in.} - (42\text{ in.})(\sin\alpha)(12\text{ lb})$

$$\sin\alpha = 0.80952$$

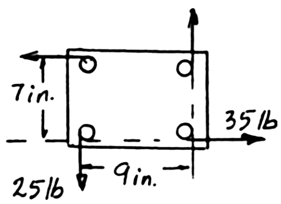
and $\alpha = 54.049°$ $\qquad$ or $\alpha = 54.0°$ ◀



PROBLEM 3.71

Four 1-in.-diameter pegs are attached to a board as shown. Two strings are passed around the pegs and pulled with the forces indicated. (*a*) Determine the resultant couple acting on the board. (*b*) If only one string is used, around which pegs should it pass and in what directions should it be pulled to create the same couple with the minimum tension in the string? (*c*) What is the value of that minimum tension?

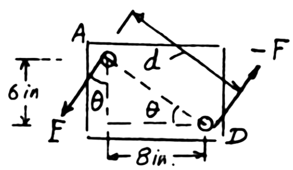
SOLUTION



(*a*) $$+\circlearrowleft M = (35 \text{ lb})(7 \text{ in.}) + (25 \text{ lb})(9 \text{ in.})$$

$$= 245 \text{ lb}\cdot\text{in.} + 225 \text{ lb}\cdot\text{in.}$$

$$M = 470 \text{ lb}\cdot\text{in.} \circlearrowleft \blacktriangleleft$$



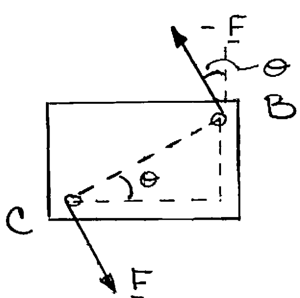
(*b*) With only one string, pegs *A* and *D*, or *B* and *C* should be used. We have

$$\tan\theta = \frac{6}{8} \qquad \theta = 36.9° \qquad 90° - \theta = 53.1°$$

Direction of forces:

With pegs *A* and *D*: $\theta = 53.1°$ ◀

With pegs *B* and *C*: $\theta = 53.1°$ ◀



(*c*) The distance between the centers of the two pegs is

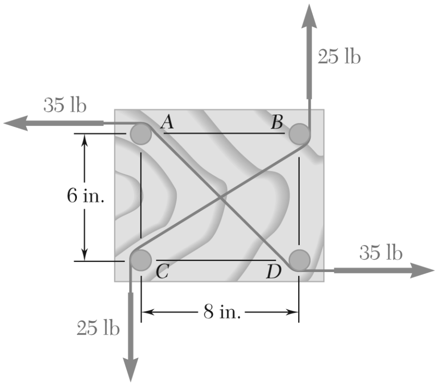
$$\sqrt{8^2 + 6^2} = 10 \text{ in.}$$

Therefore, the perpendicular distance *d* between the forces is

$$d = 10 \text{ in.} + 2\left(\frac{1}{2} \text{ in.}\right)$$

$$= 11 \text{ in.}$$

We must have $M = Fd \quad 470 \text{ lb}\cdot\text{in.} = F(11 \text{ in.}) \qquad F = 42.7 \text{ lb}$ ◀



PROBLEM 3.72

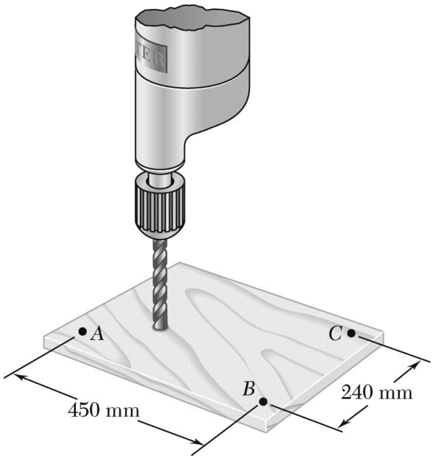
Four pegs of the same diameter are attached to a board as shown. Two strings are passed around the pegs and pulled with the forces indicated. Determine the diameter of the pegs knowing that the resultant couple applied to the board is 485 lb·in. counterclockwise.

SOLUTION

$$M = d_{AD}F_{AD} + d_{BC}F_{BC}$$

$$485 \text{ lb}\cdot\text{in.} = [(6+d)\text{ in.}](35\text{ lb}) + [(8+d)\text{ in.}](25\text{ lb})$$

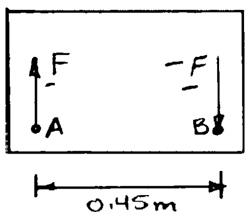
$d = 1.250$ in. ◀



PROBLEM 3.73

A piece of plywood in which several holes are being drilled successively has been secured to a workbench by means of two nails. Knowing that the drill exerts a 12-N·m couple on the piece of plywood, determine the magnitude of the resulting forces applied to the nails if they are located (*a*) at *A* and *B*, (*b*) at *B* and *C*, (*c*) at *A* and *C*.

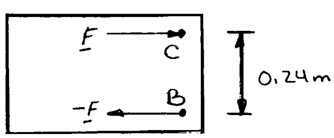
SOLUTION



(*a*) $M = Fd$

$12\text{ N}\cdot\text{m} = F(0.45\text{ m})$

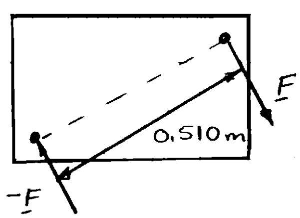
$F = 26.7\text{ N}$ ◀



(*b*) $M = Fd$

$12\text{ N}\cdot\text{m} = F(0.24\text{ m})$

$F = 50.0\text{ N}$ ◀

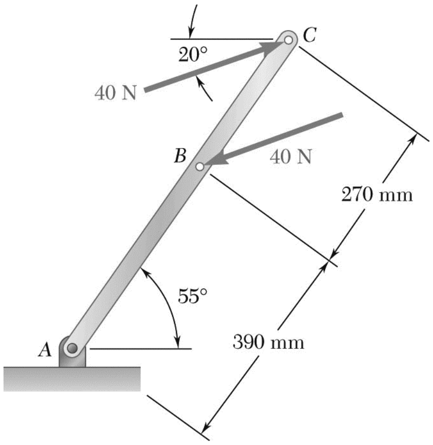


(*c*) $M = Fd \qquad d = \sqrt{(0.45\text{ m})^2 + (0.24\text{ m})^2}$

$= 0.510\text{ m}$

$12\text{ N}\cdot\text{m} = F(0.510\text{ m})$

$F = 23.5\text{ N}$ ◀

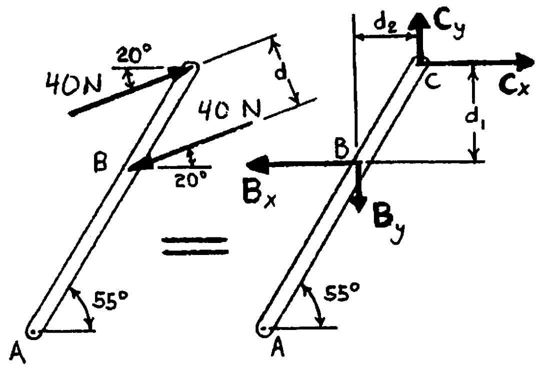


PROBLEM 3.74

Two parallel 40-N forces are applied to a lever as shown. Determine the moment of the couple formed by the two forces (*a*) by resolving each force into horizontal and vertical components and adding the moments of the two resulting couples, (*b*) by using the perpendicular distance between the two forces, (*c*) by summing the moments of the two forces about Point *A*.

SOLUTION

(*a*) We have $\Sigma \mathbf{M}_B: \; -d_1 C_x + d_2 C_y = \mathbf{M}$



where

$$d_1 = (0.270 \text{ m})\sin 55° = 0.22117 \text{ m}$$

$$d_2 = (0.270 \text{ m})\cos 55° = 0.154866 \text{ m}$$

$$C_x = (40 \text{ N})\cos 20° = 37.588 \text{ N}$$

$$C_y = (40 \text{ N})\sin 20° = 13.6808 \text{ N}$$

$$\mathbf{M} = -(0.22117 \text{ m})(37.588 \text{ N})\mathbf{k} + (0.154866 \text{ m})(13.6808 \text{ N})\mathbf{k}$$

$$= -(6.1946 \text{ N}\cdot\text{m})\mathbf{k}$$

or $\mathbf{M} = 6.19 \text{ N}\cdot\text{m}$ ↻ ◀

(*b*) We have

$$\mathbf{M} = Fd(-\mathbf{k}) = 40 \text{ N}[(0.270 \text{ m})\sin(55° - 20°)](-\mathbf{k}) = -(6.1946 \text{ N}\cdot\text{m})\mathbf{k}$$

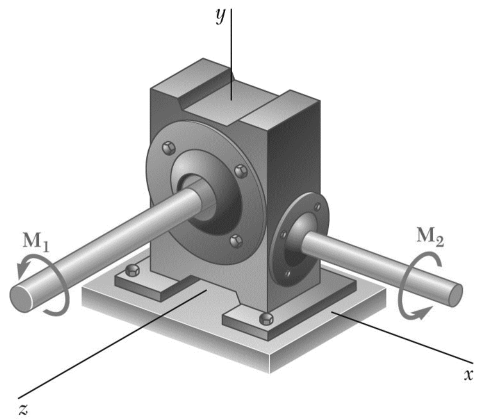
or $\mathbf{M} = 6.19 \text{ N}\cdot\text{m}$ ↻ ◀

(*c*) We have $\Sigma \mathbf{M}_A: \; \Sigma(\mathbf{r}_A \times \mathbf{F}) = \mathbf{r}_{B/A} \times \mathbf{F}_B + \mathbf{r}_{C/A} \times \mathbf{F}_C = \mathbf{M}$

$$M = (0.390 \text{ m})(40 \text{ N})\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \cos 55° & \sin 55° & 0 \\ -\cos 20° & -\sin 20° & 0 \end{vmatrix} + (0.660 \text{ m})(40 \text{ N})\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \cos 55° & \sin 55° & 0 \\ \cos 20° & \sin 20° & 0 \end{vmatrix}$$

$$= (8.9478 \text{ N}\cdot\text{m} - 15.1424 \text{ N}\cdot\text{m})\mathbf{k} = -(6.1946 \text{ N}\cdot\text{m})\mathbf{k}$$

or $\mathbf{M} = 6.19 \text{ N}\cdot\text{m}$ ↻ ◀



PROBLEM 3.75

The two shafts of a speed-reducer unit are subjected to couples of magnitude M_1 = 15 lb·ft and M_2 = 3 lb·ft, respectively. Replace the two couples with a single equivalent couple, specifying its magnitude and the direction of its axis.

SOLUTION

$$M_1 = (15\ \text{lb}\cdot\text{ft})\mathbf{k}$$

$$M_2 = (3\ \text{lb}\cdot\text{ft})\mathbf{i}$$

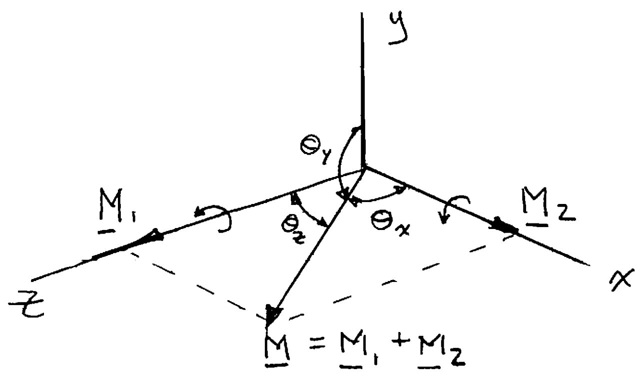
$$M = \sqrt{M_1^2 + M_2^2}$$
$$= \sqrt{(15)^2 + (3)^2}$$
$$= 15.30\ \text{lb}\cdot\text{ft}$$

$$\tan\theta_x = \frac{15}{3} = 5$$

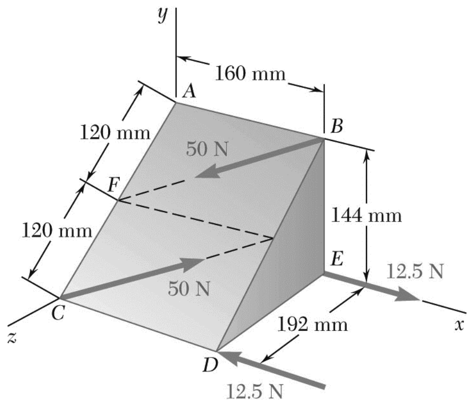
$$\theta_x = 78.7°$$

$$\theta_y = 90°$$

$$\theta_z = 90° - 78.7°$$
$$= 11.30°$$



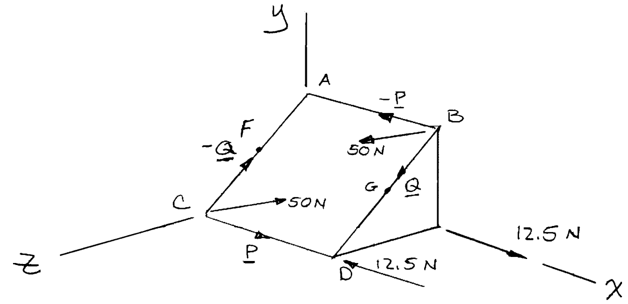
$M = 15.30\ \text{lb}\cdot\text{ft};\ \theta_x = 78.7°,\ \theta_y = 90.0°,\ \theta_z = 11.30°$ ◀



PROBLEM 3.76

Replace the two couples shown with a single equivalent couple, specifying its magnitude and the direction of its axis.

SOLUTION



Replace the couple in the *ABCD* plane with two couples *P* and *Q* shown:

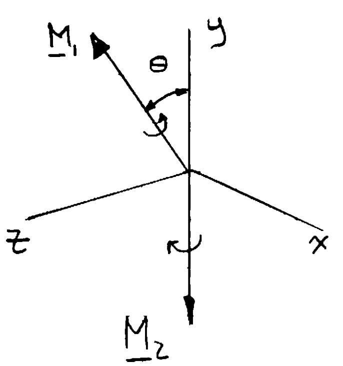
$$P = (50\text{ N})\frac{CD}{CG} = (50\text{ N})\left(\frac{160\text{ mm}}{200\text{ mm}}\right) = 40\text{ N}$$

$$Q = (50\text{ N})\frac{CF}{CG} = (50\text{ N})\left(\frac{120\text{ mm}}{200\text{ mm}}\right) = 30\text{ N}$$

Couple vector $\mathbf{M}_1$ perpendicular to plane *ABCD*:

$$+\circlearrowleft M_1 = (40\text{ N})(0.24\text{ m}) - (30\text{ N})(0.16\text{ m}) = 4.80\text{ N}\cdot\text{m}$$

Couple vector $\mathbf{M}_2$ in the *xy* plane:



$$+\circlearrowleft M_2 = -(12.5\text{ N})(0.192\text{ m}) = -2.40\text{ N}\cdot\text{m}$$

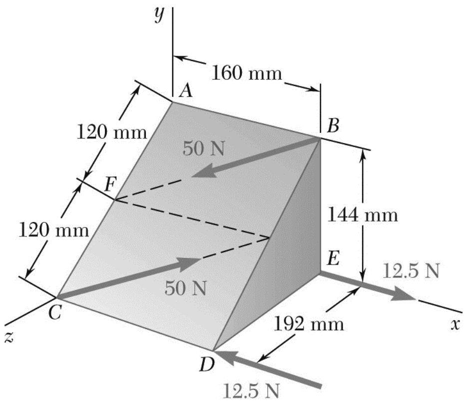
$$\tan\theta = \frac{144\text{ mm}}{192\text{ mm}} \qquad \theta = 36.870°$$

$$\mathbf{M}_1 = (4.80\cos 36.870°)\mathbf{j} + (4.80\sin 36.870°)\mathbf{k}$$
$$= 3.84\mathbf{j} + 2.88\mathbf{k}$$

$$\mathbf{M}_2 = -2.40\mathbf{j}$$

$$\mathbf{M} = \mathbf{M}_1 + \mathbf{M}_2 = 1.44\mathbf{j} + 2.88\mathbf{k}$$

$$\mathbf{M} = 3.22\text{ N}\cdot\text{m}; \ \theta_x = 90.0°, \ \theta_y = 53.1°, \ \theta_z = 36.9° \ ◀$$

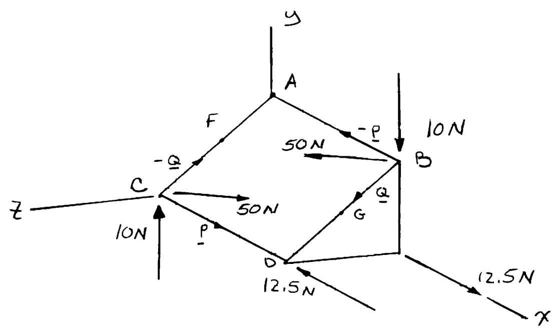


PROBLEM 3.77

Solve Prob. 3.76, assuming that two 10-N vertical forces have been added, one acting upward at *C* and the other downward at *B*.

PROBLEM 3.76 Replace the two couples shown with a single equivalent couple, specifying its magnitude and the direction of its axis.

SOLUTION



Replace the couple in the *ABCD* plane with two couples *P* and *Q* shown.

$$P = (50 \text{ N})\frac{CD}{CG} = (50 \text{ N})\left(\frac{160 \text{ mm}}{200 \text{ mm}}\right) = 40 \text{ N}$$

$$Q = (50 \text{ N})\frac{CF}{CG} = (50 \text{ N})\left(\frac{120 \text{ mm}}{200 \text{ mm}}\right) = 30 \text{ N}$$

Couple vector $\mathbf{M}_1$ perpendicular to plane *ABCD*.

$$+\circlearrowleft M_1 = (40 \text{ N})(0.24 \text{ m}) - (30 \text{ N})(0.16 \text{ m}) = 4.80 \text{ N}\cdot\text{m}$$

$$\tan\theta = \frac{144 \text{ mm}}{192 \text{ mm}} \qquad \theta = 36.870°$$

$$\mathbf{M}_1 = (4.80\cos 36.870°)\mathbf{j} + (4.80\sin 36.870°)\mathbf{k}$$
$$= 3.84\mathbf{j} + 2.88\mathbf{k}$$

$$+\circlearrowleft M_2 = -(12.5 \text{ N})(0.192 \text{ m}) = -2.40 \text{ N}\cdot\text{m}$$
$$= -2.40\mathbf{j}$$

$$\mathbf{M}_3 = \mathbf{r}_{B/C} \times M_3;\ \mathbf{r}_{B/C} = (0.16 \text{ m})\mathbf{i} + (0.144 \text{ m})\mathbf{j} - (0.192 \text{ m})\mathbf{k}$$
$$= (0.16 \text{ m})\mathbf{i} + (0.144 \text{ m})\mathbf{j} - (0.192 \text{ m})\mathbf{k} \times (-10 \text{ N})\mathbf{j}$$
$$= -1.92\mathbf{i} - 1.6\mathbf{k}$$

PROBLEM 3.77 (Continued)

$$M = M_1 + M_2 + M_3 = (3.84\mathbf{j} + 2.88\mathbf{k}) - 2.40\mathbf{j} + (-1.92\mathbf{i} - 1.6\mathbf{k})$$
$$= -(1.92\text{ N}\cdot\text{m})\mathbf{i} + (1.44\text{ N}\cdot\text{m})\mathbf{j} + (1.28\text{ N}\cdot\text{m})\mathbf{k}$$

$$M = \sqrt{(-1.92)^2 + (1.44)^2 + (1.28)^2} = 2.72\text{ N}\cdot\text{m}$$

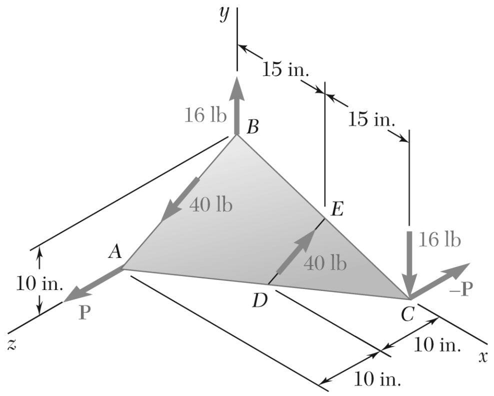
$M = 2.72\text{ N}\cdot\text{m}$ ◀

$\cos\theta_x = -1.92/2.72$

$\cos\theta_y = 1.44/2.72$

$\cos\theta_z = 1.28/2.72$

$\theta_x = 134.9°\quad \theta_y = 58.0°\quad \theta_z = 61.9°$ ◀



PROBLEM 3.78

If $P = 0$, replace the two remaining couples with a single equivalent couple, specifying its magnitude and the direction of its axis.

SOLUTION

$$\mathbf{M} = \mathbf{M}_1 + \mathbf{M}_2; \quad F_1 = 16 \text{ lb}, \quad F_2 = 40 \text{ lb}$$

$$\mathbf{M_1} = \mathbf{r}_C \times \mathbf{F}_1 = (30 \text{ in.})\mathbf{i} \times [-(16 \text{ lb})\mathbf{j}] = -(480 \text{ lb}\cdot\text{in.})\mathbf{k}$$

$$\mathbf{M_2} = \mathbf{r}_{E/B} \times \mathbf{F}_2; \quad \mathbf{r}_{E/B} = (15 \text{ in.})\mathbf{i} - (5 \text{ in.})\mathbf{j}$$

$$d_{DE} = \sqrt{(0)^2 + (5)^2 + (10)^2} = 5\sqrt{5} \text{ in.}$$

$$F_2 = \frac{40 \text{ lb}}{5\sqrt{5}}(5\mathbf{j} - 10\mathbf{k})$$

$$= 8\sqrt{5}[(1 \text{ lb})\mathbf{j} - (2 \text{ lb})\mathbf{k}]$$

$$\mathbf{M}_2 = 8\sqrt{5}\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 15 & -5 & 0 \\ 0 & 1 & -2 \end{vmatrix}$$

$$= 8\sqrt{5}[(10 \text{ lb}\cdot\text{in.})\mathbf{i} + (30 \text{ lb}\cdot\text{in.})\mathbf{j} + (15 \text{ lb}\cdot\text{in.})\mathbf{k}]$$

$$\mathbf{M} = -(480 \text{ lb}\cdot\text{in.})\mathbf{k} + 8\sqrt{5}[(10 \text{ lb}\cdot\text{in.})\mathbf{i} + (30 \text{ lb}\cdot\text{in.})\mathbf{j} + (15 \text{ lb}\cdot\text{in.})\mathbf{k}]$$

$$= (178.885 \text{ lb}\cdot\text{in.})\mathbf{i} + (536.66 \text{ lb}\cdot\text{in.})\mathbf{j} - (211.67 \text{ lb}\cdot\text{in.})\mathbf{k}$$

$$M = \sqrt{(178.885)^2 + (536.66)^2 + (-211.67)^2}$$

$$= 603.99 \text{ lb}\cdot\text{in}$$

$M = 604 \text{ lb}\cdot\text{in.}$ ◀

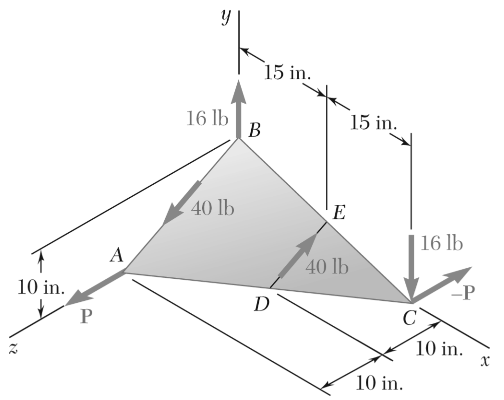
$$\lambda_{\text{axis}} = \frac{\mathbf{M}}{M} = 0.29617\mathbf{i} + 0.88852\mathbf{j} - 0.35045\mathbf{k}$$

$$\cos\theta_x = 0.29617$$

$$\cos\theta_y = 0.88852$$

$$\cos\theta_z = -0.35045$$

$\theta_x = 72.8°$ $\theta_y = 27.3°$ $\theta_z = 110.5°$ ◀



PROBLEM 3.79

If $P = 20$ lb, replace the three couples with a single equivalent couple, specifying its magnitude and the direction of its axis.

SOLUTION

From the solution to Problem. 3.78:

16-lb force: $M_1 = -(480 \text{ lb}\cdot\text{in.})\mathbf{k}$

40-lb force: $M_2 = 8\sqrt{5}[(10 \text{ lb}\cdot\text{in.})\mathbf{i} + (30 \text{ lb}\cdot\text{in.})\mathbf{j} + (15 \text{ lb}\cdot\text{in.})\mathbf{k}]$

$P = 20$ lb

$$M_3 = \mathbf{r}_C \times P$$
$$= (30 \text{ in.})\mathbf{i} \times (20 \text{ lb})\mathbf{k}$$
$$= (600 \text{ lb}\cdot\text{in.})\mathbf{j}$$

$$\mathbf{M} = \mathbf{M}_1 + \mathbf{M}_2 + \mathbf{M}_3$$
$$= -(480)\mathbf{k} + 8\sqrt{5}(10\mathbf{i} + 30\mathbf{j} + 15\mathbf{k}) + 600\mathbf{j}$$
$$= (178.885 \text{ lb}\cdot\text{in.})\mathbf{i} + (1136.66 \text{ lb}\cdot\text{in.})\mathbf{j} - (211.67 \text{ lb}\cdot\text{in.})\mathbf{k}$$

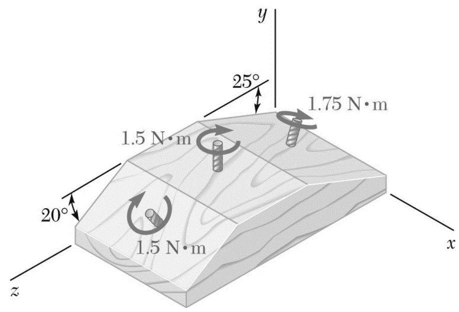
$$M = \sqrt{(178.885)^2 + (113.66)^2 + (211.67)^2}$$
$$= 1169.96 \text{ lb}\cdot\text{in.}$$

$M = 1170 \text{ lb}\cdot\text{in.}$ ◀

$$\lambda_{\text{axis}} = \frac{\mathbf{M}}{M} = 0.152898\mathbf{i} + 0.97154\mathbf{j} - 0.180921\mathbf{k}$$

$$\cos\theta_x = 0.152898$$
$$\cos\theta_y = 0.97154$$
$$\cos\theta_z = -0.180921$$

$\theta_x = 81.2°$ $\theta_y = 13.70°$ $\theta_z = 100.4°$ ◀



PROBLEM 3.80

In a manufacturing operation, three holes are drilled simultaneously in a workpiece. If the holes are perpendicular to the surfaces of the workpiece, replace the couples applied to the drills with a single equivalent couple, specifying its magnitude and the direction of its axis.

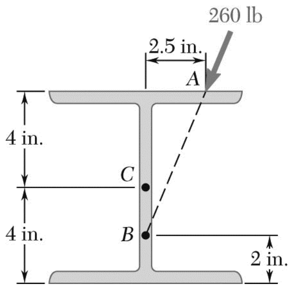
SOLUTION

$$\mathbf{M} = \mathbf{M}_1 + \mathbf{M}_2 + \mathbf{M}_3$$
$$= (1.5 \text{ N}\cdot\text{m})(-\cos 20°\mathbf{j} + \sin 20°\mathbf{k}) - (1.5 \text{ N}\cdot\text{m})\mathbf{j}$$
$$+ (1.75 \text{ N}\cdot\text{m})(-\cos 25°\mathbf{j} + \sin 25°\mathbf{k})$$
$$= -(4.4956 \text{ N}\cdot\text{m})\mathbf{j} + (0.22655 \text{ N}\cdot\text{m})\mathbf{k}$$
$$M = \sqrt{(0)^2 + (-4.4956)^2 + (0.22655)^2}$$
$$= 4.5013 \text{ N}\cdot\text{m}$$

$M = 4.50$ N·m ◀

$$\boldsymbol{\lambda}_{\text{axis}} = \frac{\mathbf{M}}{M} = -(0.99873\mathbf{j} + 0.050330\mathbf{k})$$
$$\cos\theta_x = 0$$
$$\cos\theta_y = -0.99873$$
$$\cos\theta_z = 0.050330$$

$\theta_x = 90.0°$, $\theta_y = 177.1°$, $\theta_z = 87.1°$ ◀



PROBLEM 3.81

A 260-lb force is applied at A to the rolled-steel section shown. Replace that force with an equivalent force-couple system at the center C of the section.

SOLUTION

$$AB = \sqrt{(2.5 \text{ in.})^2 + (6.0 \text{ in.})^2} = 6.50 \text{ in.}$$

$$\sin\alpha = \frac{2.5 \text{ in.}}{6.5 \text{ in.}} = \frac{5}{13}$$

$$\cos\alpha = \frac{6.0 \text{ in.}}{6.5 \text{ in.}} = \frac{12}{13} \qquad \alpha = 22.6°$$

$$\mathbf{F} = -F\sin\alpha\mathbf{i} - F\cos\alpha\mathbf{j}$$

$$= -(260 \text{ lb})\frac{5}{13}\mathbf{i} - (260 \text{ lb})\frac{12}{13}\mathbf{j}$$

$$= -(100.0 \text{ lb})\mathbf{i} - (240 \text{ lb})\mathbf{j}$$

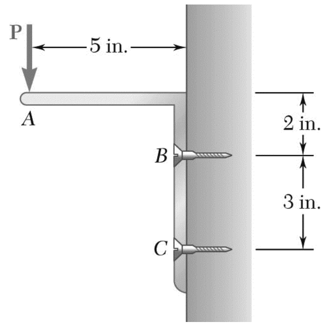
$$\mathbf{M}_C = \mathbf{r}_{A/C} \times \mathbf{F}$$

$$= (2.5\mathbf{i} + 4.0\mathbf{j}) \times (-100.0\mathbf{i} - 240\mathbf{j})$$

$$= 400\mathbf{k} - 600\mathbf{k}$$

$$= -(200 \text{ lb} \cdot \text{in.})\mathbf{k}$$

$\mathbf{F} = 260 \text{ lb} \measuredangle 67.4°$; $\mathbf{M}_C = 200 \text{ lb} \cdot \text{in.} \circlearrowright$ ◀



PROBLEM 3.82

A 30-lb vertical force **P** is applied at *A* to the bracket shown, which is held by screws at *B* and *C*. (*a*) Replace **P** with an equivalent force-couple system at *B*. (*b*) Find the two horizontal forces at *B* and *C* that are equivalent to the couple obtained in part *a*.

SOLUTION

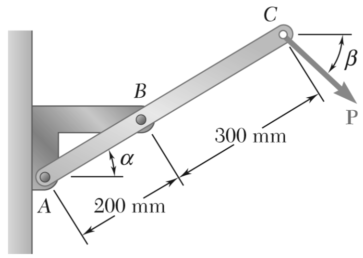
(*a*) $M_B = (30 \text{ lb})(5 \text{ in.})$

$= 150.0 \text{ lb} \cdot \text{in.}$

$$\mathbf{F} = 30.0 \text{ lb} \downarrow, \quad \mathbf{M}_B = 150.0 \text{ lb} \cdot \text{in.} \circlearrowright \blacktriangleleft$$

(*b*) $B = C = \dfrac{150 \text{ lb} \cdot \text{in.}}{3.0 \text{ in.}} = 50.0 \text{ lb}$

$$\mathbf{B} = 50.0 \text{ lb} \longleftarrow; \quad \mathbf{C} = 50.0 \text{ lb} \longrightarrow \blacktriangleleft$$



PROBLEM 3.83

The force **P** has a magnitude of 250 N and is applied at the end C of a 500-mm rod AC attached to a bracket at A and B. Assuming $\alpha = 30°$ and $\beta = 60°$, replace **P** with (*a*) an equivalent force-couple system at B, (*b*) an equivalent system formed by two parallel forces applied at A and B.

SOLUTION

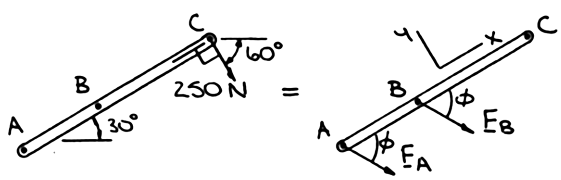
(*a*) Equivalence requires $\quad \Sigma\mathbf{F}: \quad \mathbf{F} = \mathbf{P} \quad$ or $\quad \mathbf{F} = 250\text{ N}$ ↘ 60°

$$\Sigma\mathbf{M}_B: \quad M = -(0.3\text{ m})(250\text{ N}) = -75\text{ N}\cdot\text{m}$$

The equivalent force-couple system at B is

$\mathbf{F}_B = 250\text{ N}$ ↘ 60° $\qquad \mathbf{M}_B = 75.0\text{ N}\cdot\text{m}$ ↻ ◀

(*b*) We require



Equivalence then requires

$$\Sigma F_x: \quad 0 = F_A \cos\phi + F_B \cos\phi$$

$$F_A = -F_B \quad \text{or} \quad \cos\phi = 0$$

$$\Sigma F_y: \quad -250 = -F_A \sin\phi - F_B \sin\phi$$

Now if $\quad F_A = -F_B \Rightarrow -250 = 0$, reject.

$$\cos\phi = 0$$

or $\quad \phi = 90°$

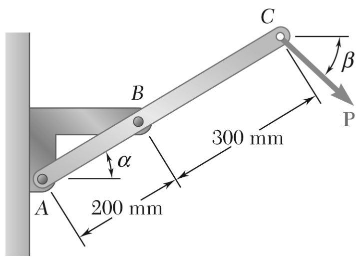
and $\quad F_A + F_B = 250$

Also, $\quad \Sigma M_B: \quad -(0.3\text{ m})(250\text{ N}) = (0.2\text{m})F_A$

or $\quad F_A = -375\text{ N}$

and $\quad F_B = 625\text{ N}$

$\mathbf{F}_A = 375\text{ N}$ ↖ 60° $\qquad \mathbf{F}_B = 625\text{ N}$ ↘ 60.0° ◀

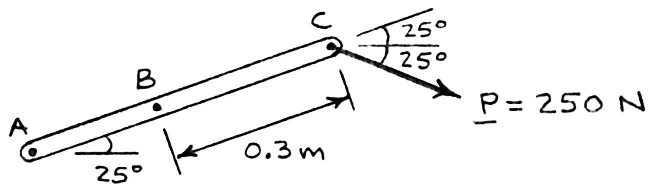


PROBLEM 3.84

Solve Problem 3.83, assuming $\alpha = \beta = 25°$.

PROBLEM 3.83 The force **P** has a magnitude of 250 N and is applied at the end *C* of a 500-mm rod *AC* attached to a bracket at *A* and *B*. Assuming $\alpha = 30°$ and $\beta = 60°$, replace **P** with (*a*) an equivalent force-couple system at *B*, (*b*) an equivalent system formed by two parallel forces applied at A and *B*.

SOLUTION



(*a*) Equivalence requires

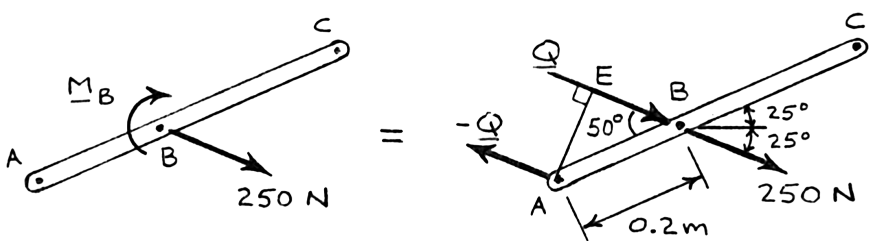
$$\Sigma \mathbf{F}: \quad \mathbf{F}_B = \mathbf{P} \quad \text{or} \quad \mathbf{F}_B = 250 \text{ N} \searrow 25.0°$$

$$\Sigma \mathbf{M}_B: \quad M_B = -(0.3 \text{ m})[(250 \text{ N})\sin 50°] = -57.453 \text{ N}\cdot\text{m}$$

The equivalent force-couple system at *B* is

$$\mathbf{F}_B = 250 \text{ N} \searrow 25.0° \qquad \mathbf{M}_B = 57.5 \text{ N}\cdot\text{m} \circlearrowright$$ ◀

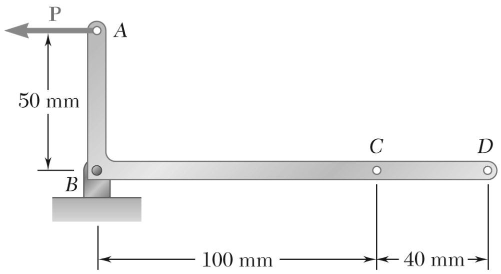
(*b*) We require



Equivalence requires

$$M_B = d_{AE}Q \quad (0.3 \text{ m})[(250 \text{ N})\sin 50°]$$
$$= [(0.2 \text{ m})\sin 50°]Q$$
$$Q = 375 \text{ N}$$

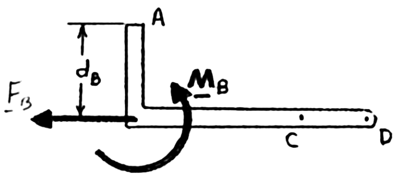
Adding the forces at *B*: $\mathbf{F}_A = 375 \text{ N} \nwarrow 25.0°$ $\qquad \mathbf{F}_B = 625 \text{ N} \searrow 25.0°$ ◀



PROBLEM 3.85

The 80-N horizontal force **P** acts on a bell crank as shown. (*a*) Replace **P** with an equivalent force-couple system at *B*. (*b*) Find the two vertical forces at *C* and *D* that are equivalent to the couple found in part *a*.

SOLUTION



(*a*) Based on $\Sigma F: \; F_B = F = 80 \text{ N}$ or $\mathbf{F}_B = 80.0 \text{ N} \leftarrow$ ◀

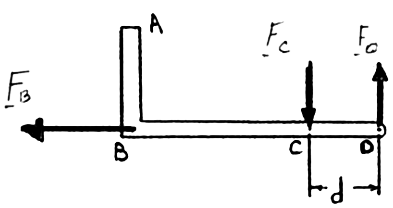
$$\Sigma M: \; M_B = Fd_B$$

$$= 80 \text{ N } (0.05 \text{ m})$$

$$= 4.0000 \text{ N} \cdot \text{m}$$

or $\mathbf{M}_B = 4.00 \text{ N} \cdot \text{m} \circlearrowright$ ◀

(*b*) If the two vertical forces are to be equivalent to $\mathbf{M}_B$, they must be a couple. Further, the sense of the moment of this couple must be counterclockwise.



Then with F_C and F_D acting as shown,

$$\Sigma M: \; M_D = F_C d$$

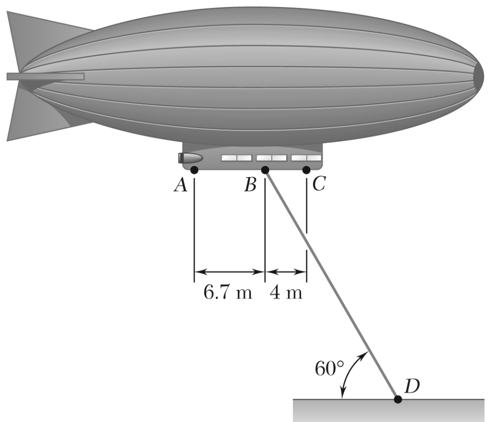
$$4.0000 \text{ N} \cdot \text{m} = F_C (0.04 \text{ m})$$

$F_C = 100.000 \text{ N}$ or $\mathbf{F}_C = 100.0 \text{ N} \downarrow$ ◀

$$\Sigma F_y: \; 0 = F_D - F_C$$

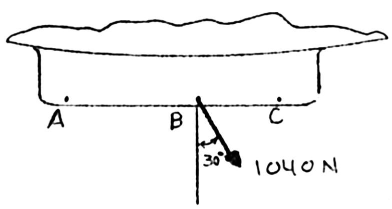
$F_D = 100.000 \text{ N}$ or $\mathbf{F}_D = 100.0 \text{ N} \uparrow$ ◀

PROBLEM 3.86



A dirigible is tethered by a cable attached to its cabin at B. If the tension in the cable is 1040 N, replace the force exerted by the cable at B with an equivalent system formed by two parallel forces applied at A and C.

SOLUTION

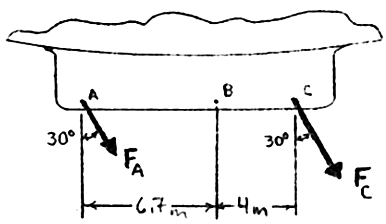


Require the equivalent forces acting at A and C be parallel and at an angle of α with the vertical.

Then for equivalence,

$$\Sigma F_x: \quad (1040 \text{ N})\sin 30° = F_A \sin\alpha + F_B \sin\alpha \qquad (1)$$

$$\Sigma F_y: \quad -(1040 \text{ N})\cos 30° = -F_A \cos\alpha - F_B \cos\alpha \qquad (2)$$



Dividing Equation (1) by Equation (2),

$$\frac{(1040 \text{ N})\sin 30°}{-(1040 \text{ N})\cos 30°} = \frac{(F_A + F_B)\sin\alpha}{-(F_A + F_B)\cos\alpha}$$

Simplifying yields $\alpha = 30°$.

Based on

$$\Sigma M_C: \quad [(1040 \text{ N})\cos 30°](4 \text{ m}) = (F_A \cos 30°)(10.7 \text{ m})$$

$$F_A = 388.79 \text{ N}$$

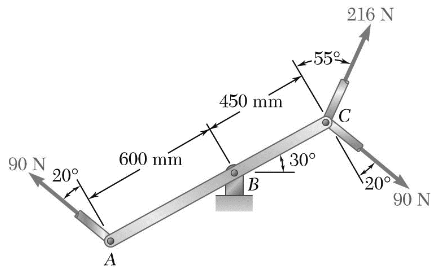
or $\mathbf{F}_A = 389 \text{ N} \; \nwarrow 60.0°$ ◀

Based on

$$\Sigma M_A: \quad -[(1040 \text{ N})\cos 30°](6.7 \text{ m}) = (F_C \cos 30°)(10.7 \text{ m})$$

$$F_C = 651.21 \text{ N}$$

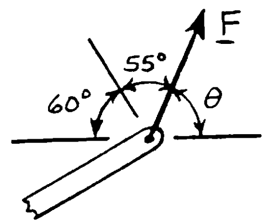
or $\mathbf{F}_C = 651 \text{ N} \; \nwarrow 60.0°$ ◀



PROBLEM 3.87

Three control rods attached to a lever *ABC* exert on it the forces shown. (*a*) Replace the three forces with an equivalent force-couple system at *B*. (*b*) Determine the single force that is equivalent to the force-couple system obtained in part *a*, and specify its point of application on the lever.

SOLUTION



(*a*) First note that the two 90-N forces form a couple. Then

$$\mathbf{F} = 216 \text{ N} \measuredangle \theta$$

where $\theta = 180° - (60° + 55°) = 65°$

and

$$M = \Sigma M_B$$
$$= (0.450 \text{ m})(216 \text{ N})\cos 55° - (1.050 \text{ m})(90 \text{ N})\cos 20°$$
$$= -33.049 \text{ N}\cdot\text{m}$$

The equivalent force-couple system at *B* is

$$\mathbf{F} = 216 \text{ N} \measuredangle 65.0°; \quad \mathbf{M} = 33.0 \text{ N}\cdot\text{m} \circlearrowright \blacktriangleleft$$

(*b*) The single equivalent force **F**′ is equal to **F**. Further, since the sense of **M** is clockwise, **F**′ must be applied between *A* and *B*. For equivalence,

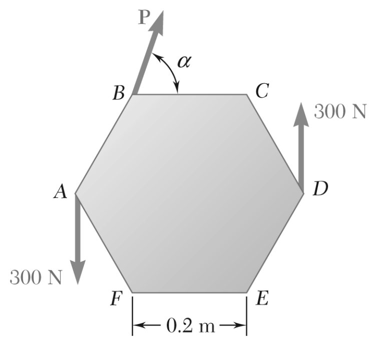
$$\Sigma M_B: \quad M = aF' \cos 55°$$

where *a* is the distance from *B* to the point of application of **F**′. Then

$$-33.049 \text{ N}\cdot\text{m} = -a(216 \text{ N})\cos 55°$$

$$a = 0.26676 \text{ m}$$

or $\mathbf{F}' = 216 \text{ N} \measuredangle 65.0°$ applied to the lever 267 mm to the left of *B* ◀

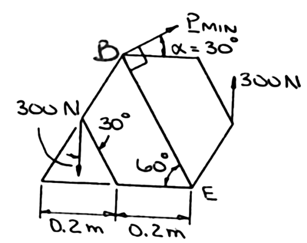


PROBLEM 3.88

A hexagonal plate is acted upon by the force **P** and the couple shown. Determine the magnitude and the direction of the smallest force **P** for which this system can be replaced with a single force at *E*.

SOLUTION

From the statement of the problem, it follows that $\Sigma M_E = 0$ for the given force-couple system. Further, for $\mathbf{P}_{\text{min}}$, we must require that **P** be perpendicular to $\mathbf{r}_{B/E}$. Then



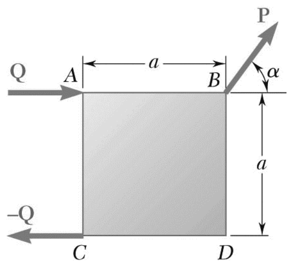
$$\Sigma M_E: \quad (0.2 \sin 30° + 0.2)\text{m} \times 300 \text{ N} + (0.2 \text{ m})\sin 30° \times 300 \text{ N} - (0.4 \text{ m})P_{\text{min}} = 0$$

or

$$P_{\text{min}} = 300 \text{ N}$$

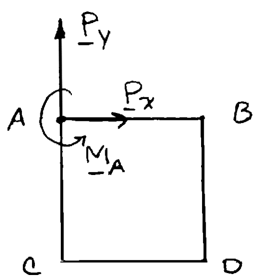
$\mathbf{P}_{\text{min}} = 300$ N ⦣ 30.0° ◀

PROBLEM 3.89



A force and couple act as shown on a square plate of side $a = 25$ in. Knowing that $P = 60$ lb, $Q = 40$ lb, and $\alpha = 50°$, replace the given force and couple by a single force applied at a point located (*a*) on line *AB*, (*b*) on line *AC*. In each case determine the distance from *A* to the point of application of the force.

SOLUTION

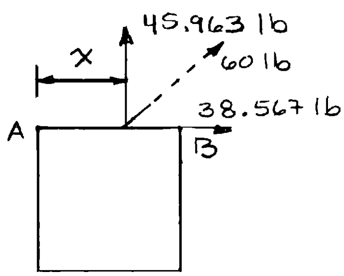


Replace the given force-couple system with an equivalent force-couple system at *A*.

$$P_x = (60 \text{ lb})(\cos 50°) = 38.567 \text{ lb}$$

$$P_y = (60 \text{ lb})(\sin 50°) = 45.963 \text{ lb}$$

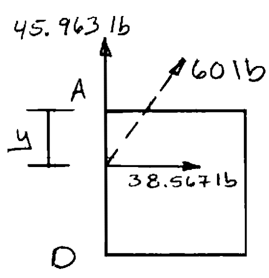
$$+\circlearrowleft M_A = P_y a - Qa$$
$$= (45.963 \text{ lb})(25 \text{ in.}) - (40 \text{ lb})(25 \text{ in.})$$
$$= 149.075 \text{ lb} \cdot \text{in.} \circlearrowleft$$



(*a*) Equating moments about *A* gives:

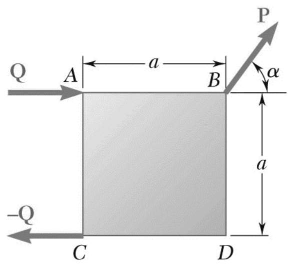
$$149.075 \text{ lb} \cdot \text{in.} = (45.963 \text{ lb})x$$
$$x = 3.24 \text{ in.}$$

P = 60.0 lb ∠ 50.0°; 3.24 in. from *A* ◀



(*b*) $149.075 \text{ lb} \cdot \text{in.} = (38.567 \text{ lb})y$
$$y = 3.87 \text{ in.}$$

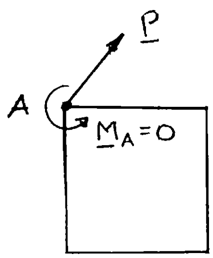
P = 60.0 lb ∠ 50.0°; 3.87 in. below *A* ◀



PROBLEM 3.90

The force and couple shown are to be replaced by an equivalent single force. Knowing that $P = 2Q$, determine the required value of α if the line of action of the single equivalent force is to pass through (*a*) Point *A*, (*b*) Point *C*.

SOLUTION

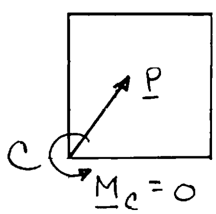


(*a*) We must have $M_A = 0$

$$(P\sin\alpha)a - Q(a) = 0$$

$$\sin\alpha = \frac{Q}{P} = \frac{Q}{2Q} = \frac{1}{2}$$

$$\alpha = 30.0° \blacktriangleleft$$



(*b*) We must have $M_C = 0$

$$(P\sin\alpha)a - (P\cos\alpha)a - Q(a) = 0$$

$$\sin\alpha - \cos\alpha = \frac{Q}{P} = \frac{Q}{2Q} = \frac{1}{2}$$

$$\sin\alpha = \cos\alpha + \frac{1}{2} \qquad (1)$$

$$\sin^2\alpha = \cos^2\alpha + \cos\alpha + \frac{1}{4}$$

$$1 - \cos^2\alpha = \cos^2\alpha + \cos\alpha + \frac{1}{4}$$

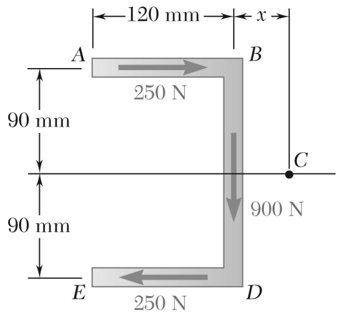
$$2\cos^2\alpha + \cos\alpha - 0.75 = 0 \qquad (2)$$

Solving the quadratic in $\cos\alpha$:

$$\cos\alpha = \frac{-1 \pm \sqrt{7}}{4} \qquad \alpha = 65.7° \text{ or } 155.7°$$

Only the first value of α satisfies Eq. (1),

therefore $\alpha = 65.7° \blacktriangleleft$

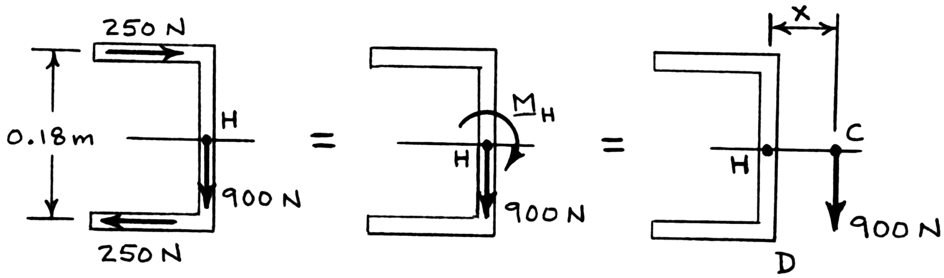


PROBLEM 3.91

The shearing forces exerted on the cross section of a steel channel can be represented by a 900-N vertical force and two 250-N horizontal forces as shown. Replace this force and couple with a single force **F** applied at Point *C*, and determine the distance *x* from *C* to line *BD*. (Point *C* is defined as the *shear center* of the section.)

SOLUTION

Replace the 250-N forces with a couple and move the 900-N force to Point *C* such that its moment about *H* is equal to the moment of the couple

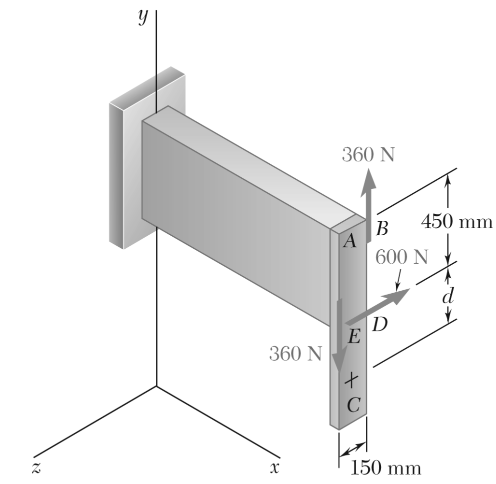


$$M_H = (0.18)(250\text{ N})$$
$$= 45\text{ N}\cdot\text{m}$$

Then
$$M_H = x(900\text{ N})$$

or
$$45\text{ N}\cdot\text{m} = x(900\text{ N})$$
$$x = 0.05\text{ m}$$

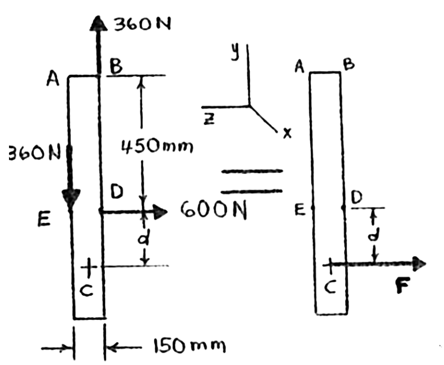
$\mathbf{F} = 900\text{ N}\downarrow \quad x = 50.0\text{ mm}$ ◀



PROBLEM 3.92

A force and a couple are applied as shown to the end of a cantilever beam. (*a*) Replace this system with a single force **F** applied at Point *C*, and determine the distance *d* from *C* to a line drawn through Points *D* and *E*. (*b*) Solve part *a* if the directions of the two 360-N forces are reversed.

SOLUTION



(*a*) We have $\Sigma \mathbf{F}$: $\mathbf{F} = (360 \text{ N})\mathbf{j} - (360 \text{ N})\mathbf{j} - (600 \text{ N})\mathbf{k}$

or $\mathbf{F} = -(600 \text{ N})\mathbf{k}$ ◀

and ΣM_D: $(360 \text{ N})(0.15 \text{ m}) = (600 \text{ N})(d)$

$d = 0.09 \text{ m}$

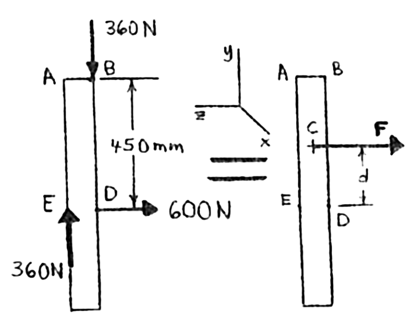
or $d = 90.0$ mm below *ED* ◀

(*b*) We have from part *a*: $\mathbf{F} = -(600 \text{ N})\mathbf{k}$ ◀

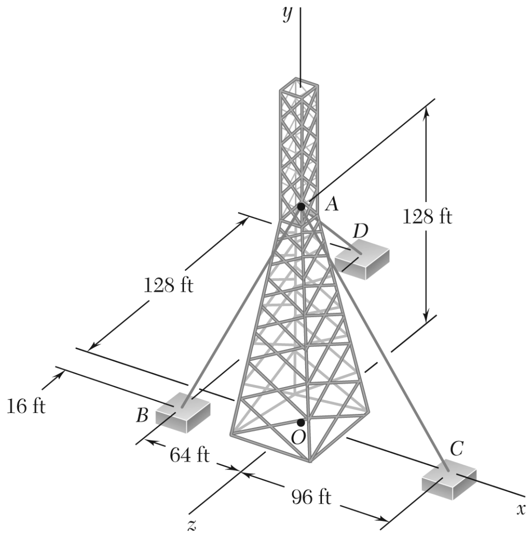
and ΣM_D: $-(360 \text{ N})(0.15 \text{ m}) = -(600 \text{ N})(d)$

$d = 0.09 \text{ m}$

or $d = 90.0$ mm above *ED* ◀



PROBLEM 3.93



An antenna is guyed by three cables as shown. Knowing that the tension in cable *AB* is 288 lb, replace the force exerted at *A* by cable *AB* with an equivalent force-couple system at the center *O* of the base of the antenna.

SOLUTION

We have $$d_{AB} = \sqrt{(-64)^2 + (-128)^2 + (16)^2} = 144 \text{ ft}$$

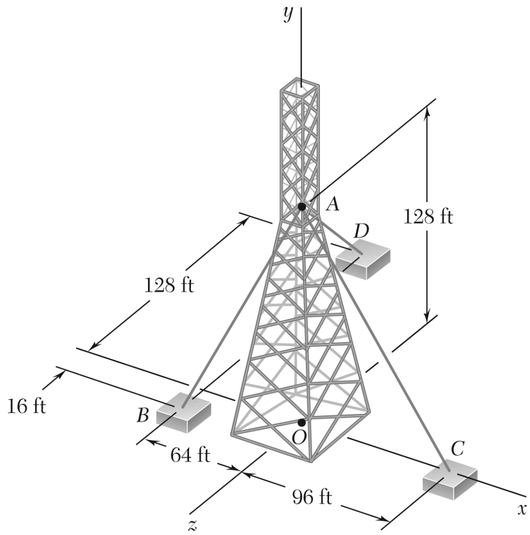
Then $$\mathbf{T}_{AB} = \frac{288 \text{ lb}}{144}(-64\mathbf{i} - 128\mathbf{j} + 16\mathbf{k})$$
$$= (32 \text{ lb})(-4\mathbf{i} - 8\mathbf{j} + \mathbf{k})$$

Now $$\mathbf{M} = \mathbf{M}_O = \mathbf{r}_{A/O} \times \mathbf{T}_{AB}$$
$$= 128\mathbf{j} \times 32(-4\mathbf{i} - 8\mathbf{j} + \mathbf{k})$$
$$= (4096 \text{ lb} \cdot \text{ft})\mathbf{i} + (16,384 \text{ lb} \cdot \text{ft})\mathbf{k}$$

The equivalent force-couple system at *O* is

$$\mathbf{F} = -(128.0 \text{ lb})\mathbf{i} - (256 \text{ lb})\mathbf{j} + (32.0 \text{ lb})\mathbf{k} \quad \blacktriangleleft$$

$$\mathbf{M} = (4.10 \text{ kip} \cdot \text{ft})\mathbf{i} + (16.38 \text{ kip} \cdot \text{ft})\mathbf{k} \quad \blacktriangleleft$$



PROBLEM 3.94

An antenna is guyed by three cables as shown. Knowing that the tension in cable AD is 270 lb, replace the force exerted at A by cable AD with an equivalent force-couple system at the center O of the base of the antenna.

SOLUTION

We have
$$d_{AD} = \sqrt{(-64)^2 + (-128)^2 + (-128)^2}$$
$$= 192 \text{ ft}$$

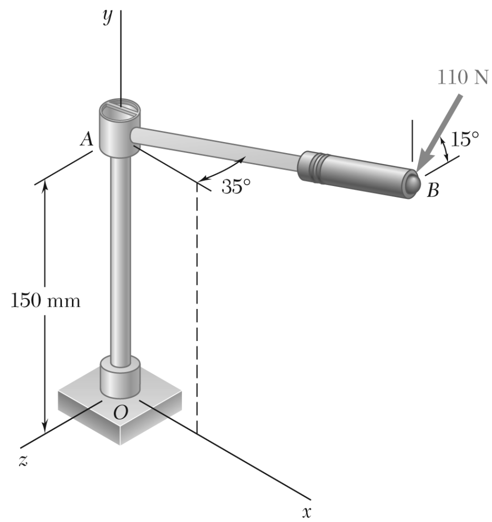
Then
$$\mathbf{T}_{AD} = \frac{270 \text{ lb}}{192}(-64\mathbf{i} - 128\mathbf{j} + 128\mathbf{k})$$
$$= (90 \text{ lb})(-\mathbf{i} - 2\mathbf{j} - 2\mathbf{k})$$

Now
$$\mathbf{M} = \mathbf{M}_O = \mathbf{r}_{A/O} \times \mathbf{T}_{AD}$$
$$= 128\mathbf{j} \times 90(-\mathbf{i} - 2\mathbf{j} - 2\mathbf{k})$$
$$= -(23{,}040 \text{ lb} \cdot \text{ft})\mathbf{i} + (11{,}520 \text{ lb} \cdot \text{ft})\mathbf{k}$$

The equivalent force-couple system at O is

$$\mathbf{F} = -(90.0 \text{ lb})\mathbf{i} - (180.0 \text{ lb})\mathbf{j} - (180.0 \text{ lb})\mathbf{k} \blacktriangleleft$$

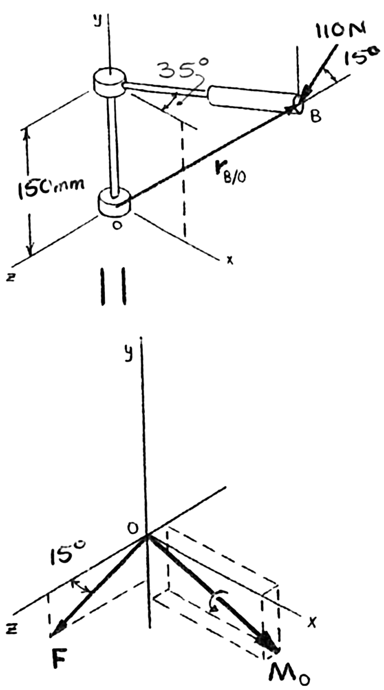
$$\mathbf{M} = -(23.0 \text{ kip} \cdot \text{ft})\mathbf{i} + (11.52 \text{ kip} \cdot \text{ft})\mathbf{k} \blacktriangleleft$$



PROBLEM 3.95

A 110-N force acting in a vertical plane parallel to the yz-plane is applied to the 220-mm-long horizontal handle AB of a socket wrench. Replace the force with an equivalent force-couple system at the origin O of the coordinate system.

SOLUTION



We have $\Sigma \mathbf{F}: \quad \mathbf{P}_B = \mathbf{F}$

where
$$\mathbf{P}_B = 110 \text{ N}[-(\sin 15°)\mathbf{j} + (\cos 15°)\mathbf{k}]$$
$$= -(28.470 \text{ N})\mathbf{j} + (106.252 \text{ N})\mathbf{k}$$

$$\text{or } \mathbf{F} = -(28.5 \text{ N})\mathbf{j} + (106.3 \text{ N})\mathbf{k} \blacktriangleleft$$

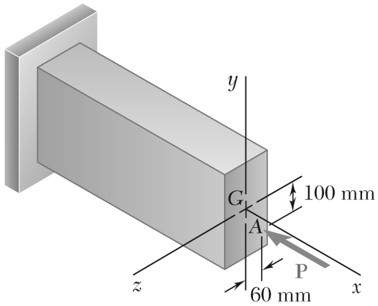
We have $\Sigma M_O: \quad \mathbf{r}_{B/O} \times \mathbf{P}_B = \mathbf{M}_O$

where
$$\mathbf{r}_{B/O} = [(0.22\cos 35°)\mathbf{i} + (0.15)\mathbf{j} - (0.22\sin 35°)\mathbf{k}] \text{ m}$$
$$= (0.180213 \text{ m})\mathbf{i} + (0.15 \text{ m})\mathbf{j} - (0.126187 \text{ m})\mathbf{k}$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.180213 & 0.15 & 0.126187 \\ 0 & -28.5 & 106.3 \end{vmatrix} \text{N} \cdot \text{m} = \mathbf{M}_O$$

$$\mathbf{M}_O = [(12.3487)\mathbf{i} - (19.1566)\mathbf{j} - (5.1361)\mathbf{k}] \text{ N} \cdot \text{m}$$

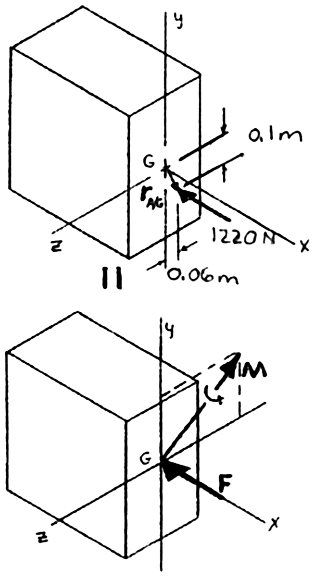
$$\text{or } \mathbf{M}_O = (12.35 \text{ N} \cdot \text{m})\mathbf{i} - (19.16 \text{ N} \cdot \text{m})\mathbf{j} - (5.13 \text{ N} \cdot \text{m})\mathbf{k} \blacktriangleleft$$



PROBLEM 3.96

An eccentric, compressive 1220-N force **P** is applied to the end of a cantilever beam. Replace **P** with an equivalent force-couple system at G.

SOLUTION



We have

$$\Sigma\mathbf{F}: \quad -(1220\text{ N})\mathbf{i} = \mathbf{F}$$

$$\mathbf{F} = -(1220\text{ N})\mathbf{i} \quad \blacktriangleleft$$

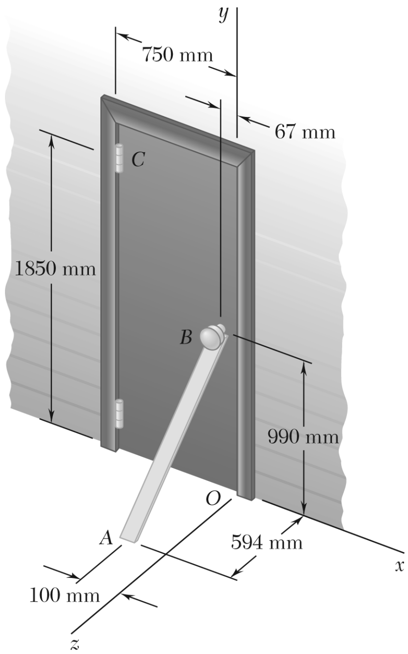
Also, we have

$$\Sigma\mathbf{M}_G: \quad \mathbf{r}_{A/G} \times \mathbf{P} = \mathbf{M}$$

$$1220\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -0.1 & -0.06 \\ -1 & 0 & 0 \end{vmatrix} \text{N}\cdot\text{m} = \mathbf{M}$$

$$\mathbf{M} = (1220\text{ N}\cdot\text{m})[(-0.06)(-1)\mathbf{j} - (-0.1)(-1)\mathbf{k}]$$

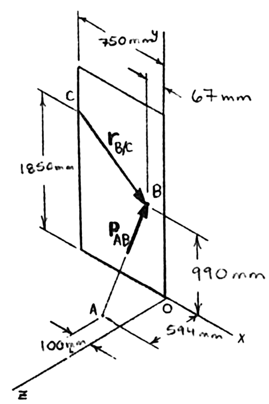
$$\text{or} \quad \mathbf{M} = (73.2\text{ N}\cdot\text{m})\mathbf{j} - (122\text{ N}\cdot\text{m})\mathbf{k} \quad \blacktriangleleft$$



PROBLEM 3.97

To keep a door closed, a wooden stick is wedged between the floor and the doorknob. The stick exerts at B a 175-N force directed along line AB. Replace that force with an equivalent force-couple system at C.

SOLUTION



We have

$$\Sigma \mathbf{F}: \quad \mathbf{P}_{AB} = \mathbf{F}_C$$

where

$$\mathbf{P}_{AB} = \boldsymbol{\lambda}_{AB} P_{AB}$$

$$= \frac{(33 \text{ mm})\mathbf{i} + (990 \text{ mm})\mathbf{j} - (594 \text{ mm})\mathbf{k}}{1155.00 \text{ mm}}(175 \text{ N})$$

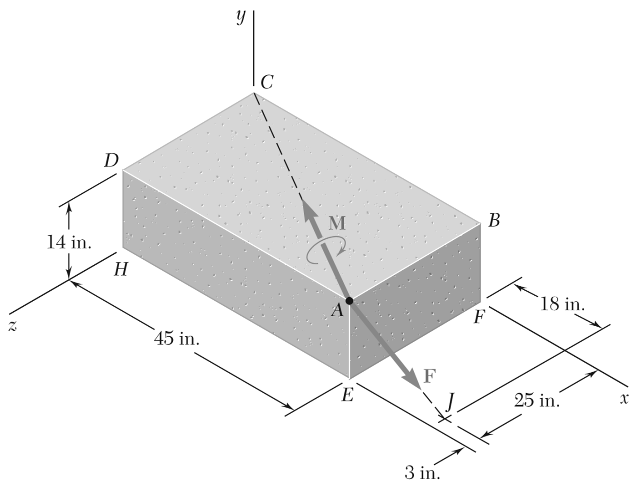
or $\mathbf{F}_C = (5.00 \text{ N})\mathbf{i} + (150.0 \text{ N})\mathbf{j} - (90.0 \text{ N})\mathbf{k}$ ◀

We have $\quad \Sigma \mathbf{M}_C: \quad \mathbf{r}_{B/C} \times \mathbf{P}_{AB} = \mathbf{M}_C$

$$\mathbf{M}_C = 5\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.683 & -0.860 & 0 \\ 1 & 30 & -18 \end{vmatrix} \text{N} \cdot \text{m}$$

$$= (5)\{(-0.860)(-18)\mathbf{i} - (0.683)(-18)\mathbf{j} + [(0.683)(30) - (0.860)(1)]\mathbf{k}\}$$

or $\mathbf{M}_C = (77.4 \text{ N} \cdot \text{m})\mathbf{i} + (61.5 \text{ N} \cdot \text{m})\mathbf{j} + (106.8 \text{ N} \cdot \text{m})\mathbf{k}$ ◀



PROBLEM 3.98

A 46-lb force **F** and a 2120-lb·in. couple **M** are applied to corner *A* of the block shown. Replace the given force-couple system with an equivalent force-couple system at corner *H*.

SOLUTION

We have $$d_{AJ} = \sqrt{(18)^2 + (-14)^2 + (-3)^2} = 23 \text{ in.}$$

Then $$\mathbf{F} = \frac{46 \text{ lb}}{23}(18\mathbf{i} - 14\mathbf{j} - 3\mathbf{k})$$
$$= (36 \text{ lb})\mathbf{i} - (28 \text{ lb})\mathbf{j} - (6 \text{ lb})\mathbf{k}$$

Also $$d_{AC} = \sqrt{(-45)^2 + (0)^2 + (-28)^2} = 53 \text{ in.}$$

Then $$\mathbf{M} = \frac{2120 \text{ lb} \cdot \text{in.}}{53}(-45\mathbf{i} - 28\mathbf{k})$$
$$= -(1800 \text{ lb} \cdot \text{in.})\mathbf{i} - (1120 \text{ lb} \cdot \text{in.})\mathbf{k}$$

Now $$\mathbf{M}' = \mathbf{M} + \mathbf{r}_{A/H} \times \mathbf{F}$$

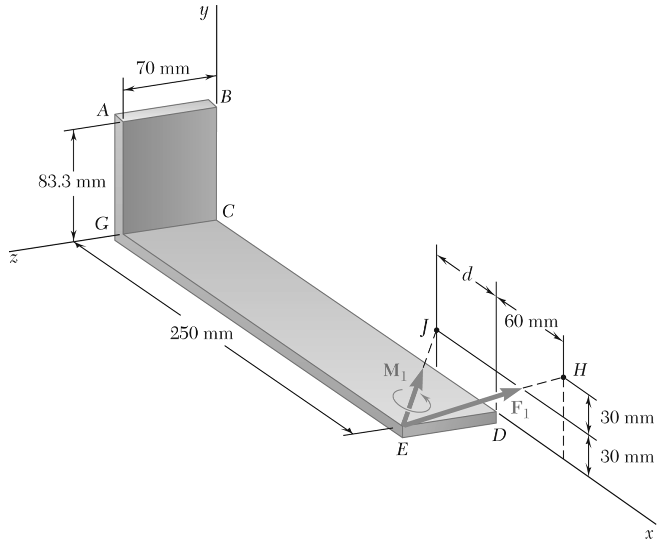
where $$\mathbf{r}_{A/H} = (45 \text{ in.})\mathbf{i} + (14 \text{ in.})\mathbf{j}$$

Then $$\mathbf{M}' = (-1800\mathbf{i} - 1120\mathbf{k}) + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 45 & 14 & 0 \\ 36 & -28 & -6 \end{vmatrix}$$

$$= (-1800\mathbf{i} - 1120\mathbf{k}) + \{[(14)(-6)]\mathbf{i} + [-(45)(-6)]\mathbf{j} + [(45)(-28) - (14)(36)]\mathbf{k}\}$$
$$= (-1800 - 84)\mathbf{i} + (270)\mathbf{j} + (-1120 - 1764)\mathbf{k}$$
$$= -(1884 \text{ lb} \cdot \text{in.})\mathbf{i} + (270 \text{ lb} \cdot \text{in.})\mathbf{j} - (2884 \text{ lb} \cdot \text{in.})\mathbf{k}$$
$$= -(157 \text{ lb} \cdot \text{ft})\mathbf{i} + (22.5 \text{ lb} \cdot \text{ft})\mathbf{j} - (240 \text{ lb} \cdot \text{ft})\mathbf{k}$$

The equivalent force-couple system at *H* is $\mathbf{F}' = (36.0 \text{ lb})\mathbf{i} - (28.0 \text{ lb})\mathbf{j} - (6.00 \text{ lb})\mathbf{k}$ ◀

$\mathbf{M}' = -(157.0 \text{ lb} \cdot \text{ft})\mathbf{i} + (22.5 \text{ lb} \cdot \text{ft})\mathbf{j} - (240 \text{ lb} \cdot \text{ft})\mathbf{k}$ ◀



PROBLEM 3.99

A 77-N force $\mathbf{F}_1$ and a 31-N · m couple $\mathbf{M}_1$ are applied to corner E of the bent plate shown. If $\mathbf{F}_1$ and $\mathbf{M}_1$ are to be replaced with an equivalent force-couple system ($\mathbf{F}_2$, $\mathbf{M}_2$) at corner B and if $(M_2)_z = 0$, determine (*a*) the distance d, (*b*) $\mathbf{F}_2$ and $\mathbf{M}_2$.

SOLUTION

(*a*) We have $\Sigma M_{Bz}: \quad M_{2z} = 0$

$$\mathbf{k} \cdot (\mathbf{r}_{H/B} \times \mathbf{F}_1) + M_{1z} = 0 \quad (1)$$

where $\mathbf{r}_{H/B} = (0.31\text{ m})\mathbf{i} - (0.0233)\mathbf{j}$

$$\mathbf{F}_1 = \boldsymbol{\lambda}_{EH} F_1$$

$$= \frac{(0.06\text{ m})\mathbf{i} + (0.06\text{ m})\mathbf{j} - (0.07\text{ m})\mathbf{k}}{0.11\text{ m}}(77\text{ N})$$

$$= (42\text{ N})\mathbf{i} + (42\text{ N})\mathbf{j} - (49\text{ N})\mathbf{k}$$

$$M_{1z} = \mathbf{k} \cdot \mathbf{M}_1$$

$$\mathbf{M}_1 = \boldsymbol{\lambda}_{EJ} M_1$$

$$= \frac{-d\mathbf{i} + (0.03\text{ m})\mathbf{j} - (0.07\text{ m})\mathbf{k}}{\sqrt{d^2 + 0.0058}\text{ m}}(31\text{ N}\cdot\text{m})$$

Then from Equation (1),

$$\begin{vmatrix} 0 & 0 & 1 \\ 0.31 & -0.0233 & 0 \\ 42 & 42 & -49 \end{vmatrix} + \frac{(-0.07\text{ m})(31\text{ N}\cdot\text{m})}{\sqrt{d^2 + 0.0058}} = 0$$

Solving for d, Equation (1) reduces to

$$(13.0200 + 0.9786) - \frac{2.17\text{ N}\cdot\text{m}}{\sqrt{d^2 + 0.0058}} = 0$$

from which $d = 0.1350\text{ m}$ or $d = 135.0\text{ mm}$ ◀

PROBLEM 3.99 (Continued)

(*b*) $\mathbf{F}_2 = \mathbf{F}_1 = (42\mathbf{i} + 42\mathbf{j} - 49\mathbf{k})\text{ N}$ or $\mathbf{F}_2 = (42.0\text{ N})\mathbf{i} + (42.0\text{ N})\mathbf{j} - (49.0\text{ N})\mathbf{k}$ ◀

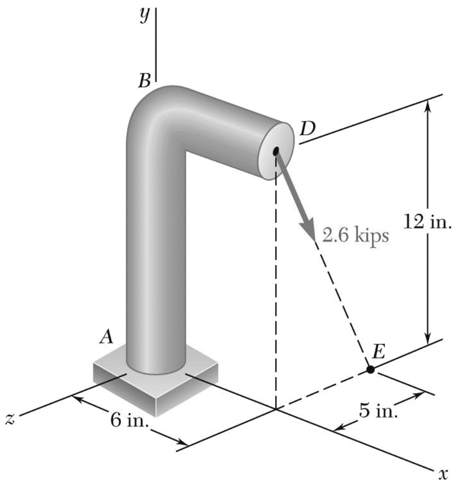
$$\mathbf{M}_2 = \mathbf{r}_{H/B} \times \mathbf{F}_1 + \mathbf{M}_1$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.31 & -0.0233 & 0 \\ 42 & 42 & -49 \end{vmatrix} + \frac{(0.1350)\mathbf{i} + 0.03\mathbf{j} - 0.07\mathbf{k}}{0.155000}(31\text{ N}\cdot\text{m})$$

$$= (1.14170\mathbf{i} + 15.1900\mathbf{j} + 13.9986\mathbf{k})\text{ N}\cdot\text{m}$$
$$+ (-27.000\mathbf{i} + 6.0000\mathbf{j} - 14.0000\mathbf{k})\text{ N}\cdot\text{m}$$

$$\mathbf{M}_2 = -(25.858\text{ N}\cdot\text{m})\mathbf{i} + (21.190\text{ N}\cdot\text{m})\mathbf{j}$$

or $\mathbf{M}_2 = -(25.9\text{ N}\cdot\text{m})\mathbf{i} + (21.2\text{ N}\cdot\text{m})\mathbf{j}$ ◀



PROBLEM 3.100

A 2.6-kip force is applied at Point *D* of the cast iron post shown. Replace that force with an equivalent force-couple system at the center *A* of the base section.

SOLUTION

$$\overrightarrow{DE} = -(12 \text{ in.})\mathbf{j} - (5 \text{ in.})\mathbf{k}; \quad DE = 13.00 \text{ in.}$$

$$\mathbf{F} = (2.6 \text{ kips})\frac{\overrightarrow{DE}}{DE}$$

$$\mathbf{F} = (2.6 \text{ kips})\frac{-12\mathbf{j} - 5\mathbf{k}}{13}$$

$$\mathbf{F} = -(2.40 \text{ kips})\mathbf{j} - (1.000 \text{ kip})\mathbf{k} \blacktriangleleft$$

$$\mathbf{M}_A = \mathbf{r}_{D/A} \times \mathbf{F}$$

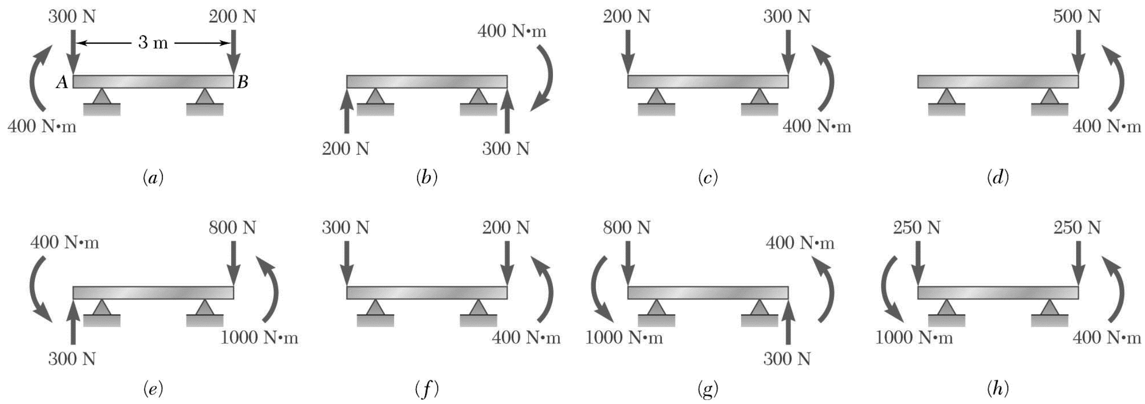
where $\mathbf{r}_{D/A} = (6 \text{ in.})\mathbf{i} + (12 \text{ in.})\mathbf{j}$

$$\mathbf{M}_A = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 6 \text{ in.} & 12 \text{ in.} & 0 \\ 0 & -2.4 \text{ kips} & -1.0 \text{ kips} \end{vmatrix}$$

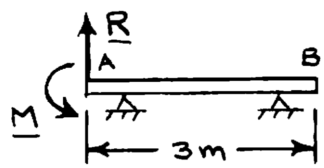
$$\mathbf{M}_A = -(12.00 \text{ kip} \cdot \text{in.})\mathbf{i} + (6.00 \text{ kip} \cdot \text{in.})\mathbf{j} - (14.40 \text{ kip} \cdot \text{in.})\mathbf{k} \blacktriangleleft$$

PROBLEM 3.101

A 3-m-long beam is subjected to a variety of loadings. (*a*) Replace each loading with an equivalent force-couple system at end *A* of the beam. (*b*) Which of the loadings are equivalent?



SOLUTION



(*a*) (*a*) We have $\Sigma F_Y: \quad -300 \text{ N} - 200 \text{ N} = R_a$

or $\mathbf{R}_a = 500 \text{ N} \downarrow$ ◀

and $\Sigma M_A: \quad -400 \text{ N}\cdot\text{m} - (200 \text{ N})(3 \text{ m}) = M_a$

or $\mathbf{M}_a = 1000 \text{ N}\cdot\text{m}$ ↻ ◀

(*b*) We have $\Sigma F_Y: \quad 200 \text{ N} + 300 \text{ N} = R_b$

or $\mathbf{R}_b = 500 \text{ N} \uparrow$ ◀

and $\Sigma M_A: \quad -400 \text{ N}\cdot\text{m} + (300 \text{ N})(3 \text{ m}) = M_b$

or $\mathbf{M}_b = 500 \text{ N}\cdot\text{m}$ ↺ ◀

(*c*) We have $\Sigma F_Y: \quad -200 \text{ N} - 300 \text{ N} = R_c$

or $\mathbf{R}_c = 500 \text{ N} \downarrow$ ◀

and $\Sigma M_A: \quad 400 \text{ N}\cdot\text{m} - (300 \text{ N})(3 \text{ m}) = M_c$

or $\mathbf{M}_c = 500 \text{ N}\cdot\text{m}$ ↻ ◀

(*d*) We have $\Sigma F_Y: \; -500 \text{ N} = R_d$

or $\mathbf{R}_d = 500 \text{ N}\downarrow$ ◀

and $\Sigma M_A: \; 400 \text{ N}\cdot\text{m} - (500 \text{ N})(3 \text{ m}) = M_d$

or $\mathbf{M}_d = 1100 \text{ N}\cdot\text{m}$ ↻ ◀

(*e*) We have $\Sigma F_Y: \; 300 \text{ N} - 800 \text{ N} = R_e$

or $\mathbf{R}_e = 500 \text{ N}\downarrow$ ◀

and $\Sigma M_A: \; 400 \text{ N}\cdot\text{m} + 1000 \text{ N}\cdot\text{m} - (800 \text{ N})(3 \text{ m}) = M_e$

or $\mathbf{M}_e = 1000 \text{ N}\cdot\text{m}$ ↻ ◀

(*f*) We have $\Sigma F_Y: \; -300 \text{ N} - 200 \text{ N} = R_f$

or $\mathbf{R}_f = 500 \text{ N}\downarrow$ ◀

and $\Sigma M_A: \; 400 \text{ N}\cdot\text{m} - (200 \text{ N})(3 \text{ m}) = M_f$

or $\mathbf{M}_f = 200 \text{ N}\cdot\text{m}$ ↻ ◀

(*g*) We have $\Sigma F_Y: \; -800 \text{ N} + 300 \text{ N} = R_g$

or $\mathbf{R}_g = 500 \text{ N}\downarrow$ ◀

and $\Sigma M_A: \; 1000 \text{ N}\cdot\text{m} + 400 \text{ N}\cdot\text{m} + (300 \text{ N})(3 \text{ m}) = M_g$

or $\mathbf{M}_g = 2300 \text{ N}\cdot\text{m}$ ↺ ◀

(*h*) We have $\Sigma F_Y: \; -250 \text{ N} - 250 \text{ N} = R_h$

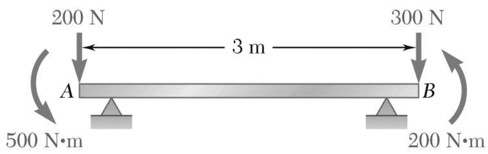
or $\mathbf{R}_h = 500 \text{ N}\downarrow$ ◀

and $\Sigma M_A: \; 1000 \text{ N}\cdot\text{m} + 400 \text{ N}\cdot\text{m} - (250 \text{ N})(3 \text{ m}) = M_h$

or $\mathbf{M}_h = 650 \text{ N}\cdot\text{m}$ ↺ ◀

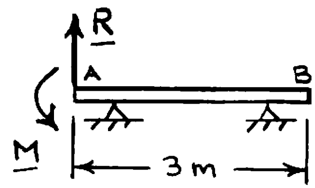
(*b*) Therefore, loadings (*a*) and (*e*) are equivalent.

PROBLEM 3.102



A 3-m-long beam is loaded as shown. Determine the loading of Prob. 3.101 that is equivalent to this loading.

SOLUTION



We have ΣF_Y: $-200\text{ N} - 300\text{ N} = R$

or $\mathbf{R} = 500\text{ N}\downarrow$

and ΣM_A: $500\text{ N}\cdot\text{m} + 200\text{ N}\cdot\text{m} - (300\text{ N})(3\text{ m}) = M$

or $\mathbf{M} = 200\text{ N}\cdot\text{m}$ ↻

Problem 3.101 equivalent force-couples at A:

Case	$\overrightarrow{\mathbf{R}}$	$\overrightarrow{\mathbf{M}}$
(*a*)	500 N ↓	1000 N·m ↻
(*b*)	500 N ↑	500 N·m ↺
(*c*)	500 N ↓	500 N·m ↻
(*d*)	500 N ↓	1100 N·m ↻
(*e*)	500 N ↓	1000 N·m ↻
(*f*)	500 N ↓	200 N·m ↻ ←
(*g*)	500 N ↓	2300 N·m ↺
(*h*)	500 N ↓	650 N·m ↺

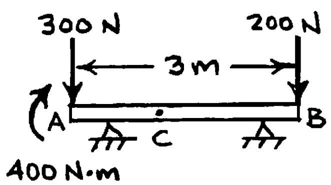
Equivalent to case (*f*) of Problem 3.101 ◀

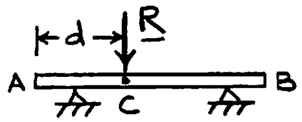
PROBLEM 3.103

Determine the single equivalent force and the distance from Point A to its line of action for the beam and loading of (*a*) Prob. 3.101*a*, (*b*) Prob. 3.101*b*, (*c*) Prob. 3.102.

SOLUTION

For equivalent single force at distance d from A:



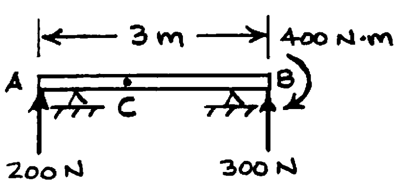


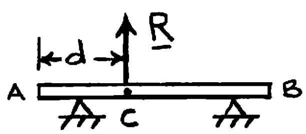
(*a*) We have $\Sigma F_Y\!: \; -300\text{ N} - 200\text{ N} = R$

or $\mathbf{R} = 500\text{ N}\downarrow$ ◀

and $\Sigma M_C\!: \; -400\text{ N}\cdot\text{m} + (300\text{ N})(d) - (200\text{ N})(3-d) = 0$

or $d = 2.00\text{ m}$ ◀



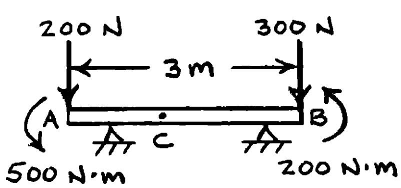


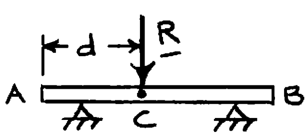
(*b*) We have $\Sigma F_Y\!: \; 200\text{ N} + 300\text{ N} = R$

or $\mathbf{R} = 500\text{ N}\uparrow$ ◀

and $\Sigma M_C\!: \; -400\text{ N}\cdot\text{m} - (200\text{ N})(d) + (300\text{ N})(3-d) = 0$

or $d = 1.000\text{ m}$ ◀





(*c*) We have $\Sigma F_Y\!: \; -200\text{ N} - 300\text{ N} = R$

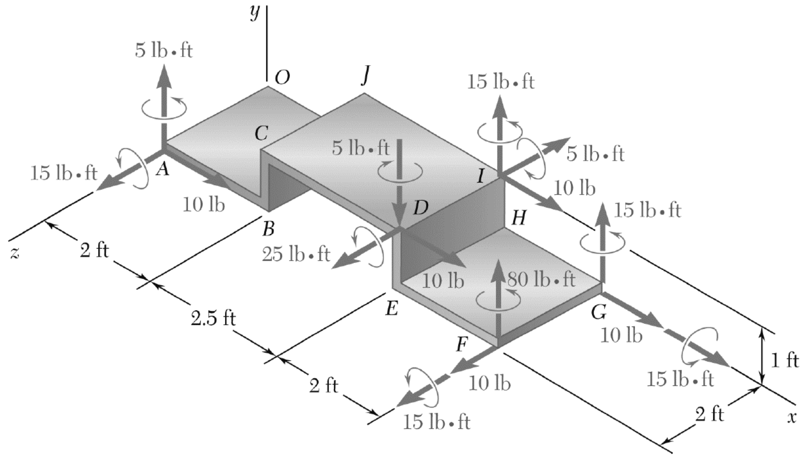
or $\mathbf{R} = 500\text{ N}\downarrow$ ◀

and $\Sigma M_C\!: \; 500\text{ N}\cdot\text{m} + 200\text{ N}\cdot\text{m} + (200\text{ N})(d) - (300\text{ N})(3-d) = 0$

or $d = 0.400\text{ m}$ ◀

PROBLEM 3.104

Five separate force-couple systems act at the corners of a piece of sheet metal, which has been bent into the shape shown. Determine which of these systems is equivalent to a force $\mathbf{F} = (10 \text{ lb})\mathbf{i}$ and a couple of moment $\mathbf{M} = (15 \text{ lb} \cdot \text{ft})\mathbf{j} + (15 \text{ lb} \cdot \text{ft})\mathbf{k}$ located at the origin.



SOLUTION

First note that the force-couple system at F cannot be equivalent because of the direction of the force [The force of the other four systems is $(10 \text{ lb})\mathbf{i}$]. Next, move each of the systems to the origin O; the forces remain unchanged.

$$A:\quad \mathbf{M}_A = \Sigma\mathbf{M}_O = (5 \text{ lb}\cdot\text{ft})\mathbf{j} + (15 \text{ lb}\cdot\text{ft})\mathbf{k} + (2 \text{ ft})\mathbf{k}\times(10 \text{ lb})\mathbf{i}$$

$$= (25 \text{ lb}\cdot\text{ft})\mathbf{j} + (15 \text{ lb}\cdot\text{ft})\mathbf{k}$$

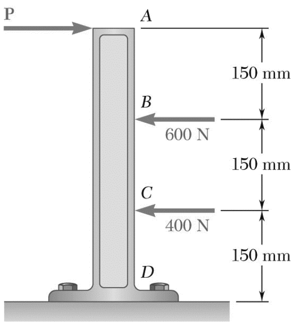
$$D:\quad \mathbf{M}_D = \Sigma\mathbf{M}_O = -(5 \text{ lb}\cdot\text{ft})\mathbf{j} + (25 \text{ lb}\cdot\text{ft})\mathbf{k}$$
$$+[(4.5 \text{ ft})\mathbf{i} + (1 \text{ ft})\mathbf{j} + (2 \text{ ft})\mathbf{k}]\times 10 \text{ lb})\mathbf{i}$$
$$= (15 \text{ lb}\cdot\text{ft})\mathbf{i} + (15 \text{ lb}\cdot\text{ft})\mathbf{k}$$

$$G:\quad \mathbf{M}_G = \Sigma\mathbf{M}_O = (15 \text{ lb}\cdot\text{ft})\mathbf{i} + (15 \text{ lb}\cdot\text{ft})\mathbf{j}$$

$$I:\quad \mathbf{M}_I = \Sigma\mathbf{M}_I = (15 \text{ lb}\cdot\text{ft})\mathbf{j} - (5 \text{ lb}\cdot\text{ft})\mathbf{k}$$
$$+[(4.5 \text{ ft})\mathbf{i} + (1 \text{ ft})\mathbf{j}]\times(10 \text{ lb})\mathbf{j}$$

$$= (15 \text{ lb}\cdot\text{ft})\mathbf{j} - (15 \text{ lb}\cdot\text{ft})\mathbf{k}$$

The equivalent force-couple system is the system at corner D. ◀

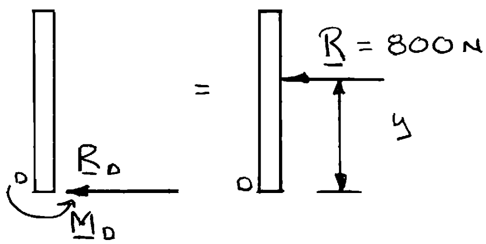


PROBLEM 3.105

Three horizontal forces are applied as shown to a vertical cast iron arm. Determine the resultant of the forces and the distance from the ground to its line of action when (*a*) $P = 200$ N, (*b*) $P = 2400$ N, (*c*) $P = 1000$ N.

SOLUTION

(*a*)



$$\xrightarrow{+} R_D = +200 \text{ N} - 600 \text{ N} - 400 \text{ N} = -800 \text{ N}$$

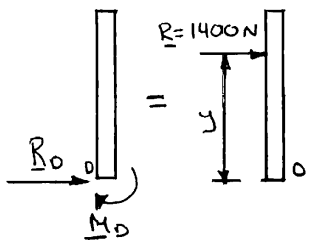
$$+\circlearrowleft M_D = -(200 \text{ N})(0.450 \text{ m}) + (600 \text{ N})(0.300 \text{ m}) + (400 \text{ N})(0.1500 \text{ m})$$

$$= +150.0 \text{ N}\cdot\text{m}$$

$$y = \frac{M_D}{R} = \frac{150 \text{ N}\cdot\text{m}}{800 \text{ N}} = 0.1875 \text{ m}$$

$\mathbf{R} = 800 \text{ N} \longleftarrow; \; y = 187.5 \text{ mm}$ ◀

(*b*)



$$\xrightarrow{+} R_D = +2400 \text{ N} - 600 \text{ N} - 400 \text{ N} = +1400 \text{ N}$$

$$+\circlearrowleft M_D = -(2400 \text{ N})(0.450 \text{ m}) + (600 \text{ N})(0.300 \text{ m}) + (400 \text{ N})(0.1500 \text{ m})$$

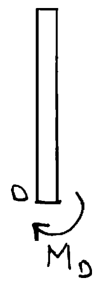
$$= -840 \text{ N}\cdot\text{m}$$

$$y = \frac{M_D}{R} = \frac{840 \text{ N}\cdot\text{m}}{1400 \text{ N}} = 0.600 \text{ m}$$

$\mathbf{R} = 1400 \text{ N} \longrightarrow; \; y = 600 \text{ mm}$ ◀

PROBLEM 3.105 (Continued)

(*c*)

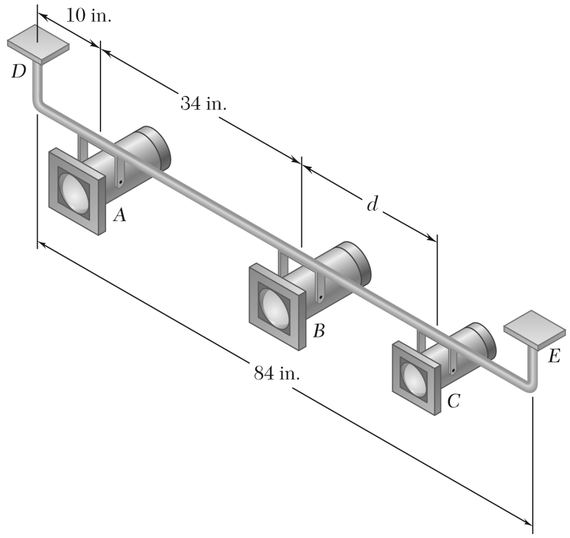


$$\xrightarrow{+} R_D = +1000 - 600 - 400 = 0$$

$$+\circlearrowright M_D = -(1000\text{ N})(0.450\text{ m}) + (600\text{ N})(0.300\text{ m}) + (400\text{ N})(0.1500\text{ m})$$
$$= -210\text{ N}\cdot\text{m}$$

$\therefore \quad y = \infty$ System reduces to a couple.

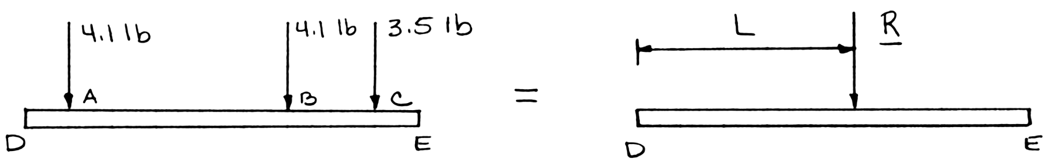
$\mathbf{M}_D = 210\text{ N}\cdot\text{m} \circlearrowright$ ◀



PROBLEM 3.106

Three stage lights are mounted on a pipe as shown. The lights at A and B each weigh 4.1 lb, while the one at C weighs 3.5 lb. (a) If d = 25 in., determine the distance from D to the line of action of the resultant of the weights of the three lights. (b) Determine the value of d so that the resultant of the weights passes through the midpoint of the pipe.

SOLUTION



For equivalence,

$$\Sigma F_y: \quad -4.1-4.1-3.5=-R \quad \text{or} \quad \mathbf{R}=11.7 \text{ lb}\downarrow$$

$$\Sigma F_D: \quad -(10 \text{ in.})(4.1 \text{ lb})-(44 \text{ in.})(4.1 \text{ lb})$$
$$-[(4.4+d) \text{ in.}](3.5 \text{ lb})=-(L \text{ in.})(11.7 \text{ lb})$$

or
$$375.4+3.5d=11.7L \quad (d, L \text{ in in.})$$

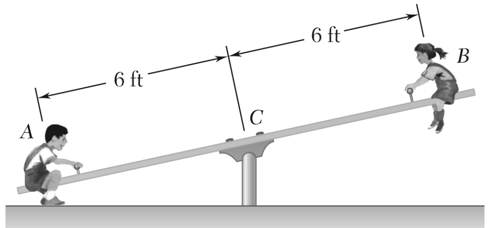
(a)
$$d=25 \text{ in.}$$

We have $375.4+3.5(25)=11.7L$ or $L=39.6$ in.

The resultant passes through a point 39.6 in. to the right of D. ◀

(b)
$$L=42 \text{ in.}$$

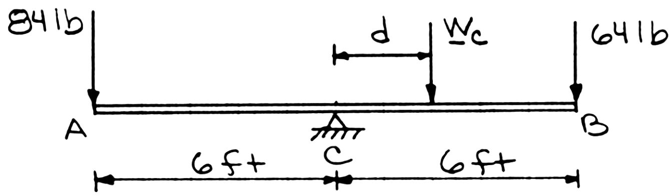
We have $375.4+3.5d=11.7(42)$ or $d=33.1$ in. ◀



PROBLEM 3.107

The weights of two children sitting at ends A and B of a seesaw are 84 lb and 64 lb, respectively. Where should a third child sit so that the resultant of the weights of the three children will pass through C if she weighs (a) 60 lb, (b) 52 lb.

SOLUTION



(a) For the resultant weight to act at C, $\Sigma M_C = 0 \quad W_C = 60 \text{ lb}$

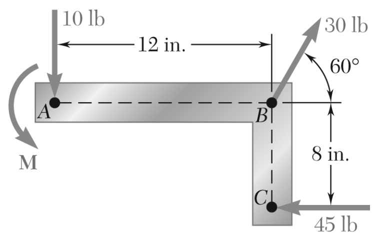
Then $(84 \text{ lb})(6 \text{ ft}) - 60 \text{ lb}(d) - 64 \text{ lb}(6 \text{ ft}) = 0$

$d = 2.00$ ft to the right of C ◀

(b) For the resultant weight to act at C, $\Sigma M_C = 0 \quad W_C = 52 \text{ lb}$

Then $(84 \text{ lb})(6 \text{ ft}) - 52 \text{ lb}(d) - 64 \text{ lb}(6 \text{ ft}) = 0$

$d = 2.31$ ft to the right of C ◀



PROBLEM 3.108

A couple of magnitude $M = 54$ lb · in. and the three forces shown are applied to an angle bracket. (*a*) Find the resultant of this system of forces. (*b*) Locate the points where the line of action of the resultant intersects line *AB* and line *BC*.

SOLUTION

(*a*) We have

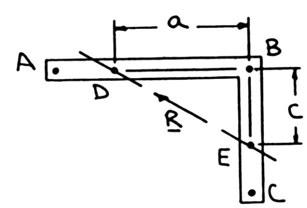
$$\Sigma \mathbf{F}: \quad \mathbf{R} = (-10\mathbf{j}) + (30\cos 60°)\mathbf{i} + 30\sin 60°\mathbf{j} + (-45\mathbf{i})$$
$$= -(30 \text{ lb})\mathbf{i} + (15.9808 \text{ lb})\mathbf{j}$$

or $\mathbf{R} = 34.0 \text{ lb} \;\angle\; 28.0°$ ◀

(*b*) First reduce the given forces and couple to an equivalent force-couple system $(\mathbf{R}, \mathbf{M}_B)$ at *B*.

We have $\Sigma M_B: \quad M_B = (54 \text{ lb}\cdot\text{in}) + (12 \text{ in.})(10 \text{ lb}) - (8 \text{ in.})(45 \text{ lb})$

$$= -186 \text{ lb}\cdot\text{in.}$$



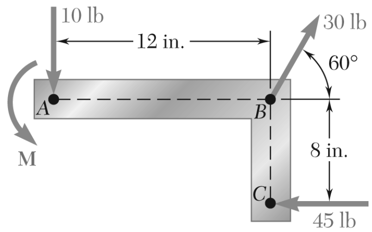
Then with **R** at *D*, $\Sigma M_B: \quad -186 \text{ lb}\cdot\text{in} = a(15.9808 \text{ lb})$

or $a = 11.64$ in.

and with **R** at *E*, $\Sigma M_B: \quad -186 \text{ lb}\cdot\text{in} = C(30 \text{ lb})$

or $C = 6.2$ in.

The line of action of **R** intersects line *AB* 11.64 in. to the left of *B* and intersects line *BC* 6.20 in. below *B*. ◀



PROBLEM 3.109

A couple **M** and the three forces shown are applied to an angle bracket. Find the moment of the couple if the line of action of the resultant of the force system is to pass through (*a*) Point *A*, (*b*) Point *B*, (*c*) Point *C*.

SOLUTION

In each case, we must have $\mathbf{M}_1^R = 0$

(*a*) $+\circlearrowleft M_A^B = \Sigma M_A = M + (12 \text{ in.})[(30 \text{ lb})\sin 60°] - (8 \text{ in.})(45 \text{ lb}) = 0$

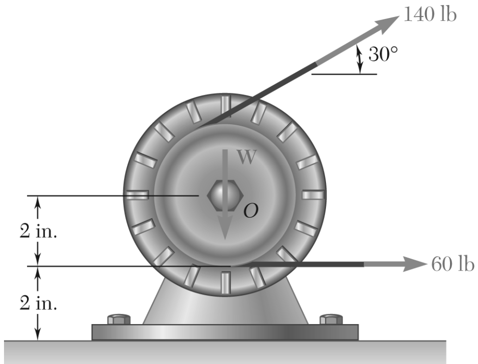
$M = +48.231 \text{ lb} \cdot \text{in.}$ $\qquad \mathbf{M} = 48.2 \text{ lb} \cdot \text{in.} \circlearrowleft$ ◀

(*b*) $+\circlearrowleft M_B^R = \Sigma M_B = M + (12 \text{ in.})(10 \text{ lb}) - (8 \text{ in.})(45 \text{ lb}) = 0$

$M = +240 \text{ lb} \cdot \text{in.}$ $\qquad \mathbf{M} = 240 \text{ lb} \cdot \text{in.} \circlearrowleft$ ◀

(*c*) $+\circlearrowleft M_C^R = \Sigma M_C = M + (12 \text{ in.})(10 \text{ lb}) - (8 \text{ in.})[(30 \text{ lb})\cos 60°] = 0$

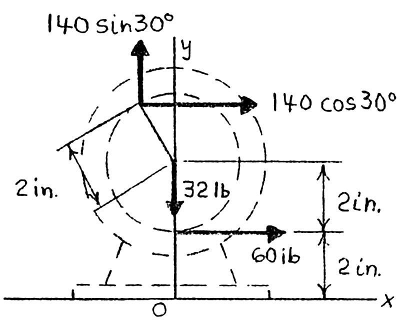
$M = 0$ $\qquad \mathbf{M} = 0$ ◀



PROBLEM 3.110

A 32-lb motor is mounted on the floor. Find the resultant of the weight and the forces exerted on the belt, and determine where the line of action of the resultant intersects the floor.

SOLUTION



We have

$$\Sigma \mathbf{F}: \quad (60\text{ lb})\mathbf{i} - (32\text{ lb})\mathbf{j} + (140\text{ lb})(\cos 30°\mathbf{i} + \sin 30°\mathbf{j}) = \mathbf{R}$$

$$\mathbf{R} = (181.244\text{ lb})\mathbf{i} + (38.0\text{ lb})\mathbf{j}$$

or $\mathbf{R} = 185.2\text{ lb} \measuredangle 11.84°$ ◀

We have $\quad \Sigma M_O: \quad \Sigma M_O = xR_y$

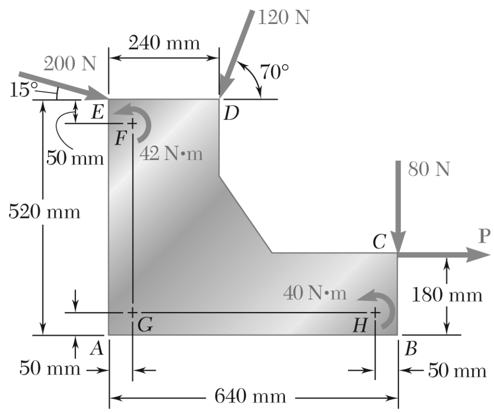
$$-[(140\text{ lb})\cos 30°][(4 + 2\cos 30°)\text{in.}] - [(140\text{ lb})\sin 30°][(2\text{ in.})\sin 30°]$$

$$-(60\text{ lb})(2\text{ in.}) = x(38.0\text{ lb})$$

$$x = \frac{1}{38.0}(-694.97 - 70.0 - 120)\text{ in.}$$

and $\quad x = -23.289\text{ in.}$

Or resultant intersects the base (x-axis) 23.3 in. to the left of the vertical centerline (y-axis) of the motor. ◀

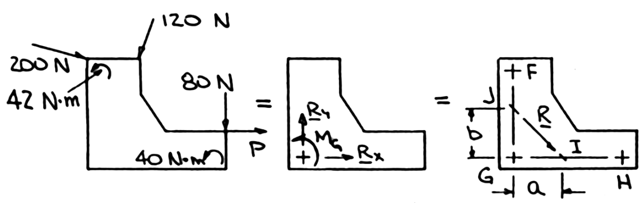


PROBLEM 3.111

A machine component is subjected to the forces and couples shown. The component is to be held in place by a single rivet that can resist a force but not a couple. For $P = 0$, determine the location of the rivet hole if it is to be located (*a*) on line *FG*, (*b*) on line *GH*.

SOLUTION

We have



First replace the applied forces and couples with an equivalent force-couple system at *G*.

Thus, ΣF_x: $200\cos 15° - 120\cos 70° + P = R_x$

or $R_x = (152.142 + P)$ N

ΣF_y: $-200\sin 15° - 120\sin 70° - 80 = R_y$

or $R_y = -244.53$ N

$$\Sigma M_G: \; -(0.47 \text{ m})(200 \text{ N})\cos 15° + (0.05 \text{ m})(200 \text{ N})\sin 15° + (0.47 \text{ m})(120 \text{ N})\cos 70° - (0.19 \text{ m})(120 \text{ N})\sin 70° - (0.13 \text{ m})(P \text{ N}) - (0.59 \text{ m})(80 \text{ N}) + 42 \text{ N}\cdot\text{m} + 40 \text{ N}\cdot\text{m} = M_G$$

or $M_G = -(55.544 + 0.13P)$ N · m (1)

Setting $P = 0$ in Eq. (1):

Now with **R** at *I*, ΣM_G: $-55.544 \text{ N}\cdot\text{m} = -a(244.53 \text{ N})$

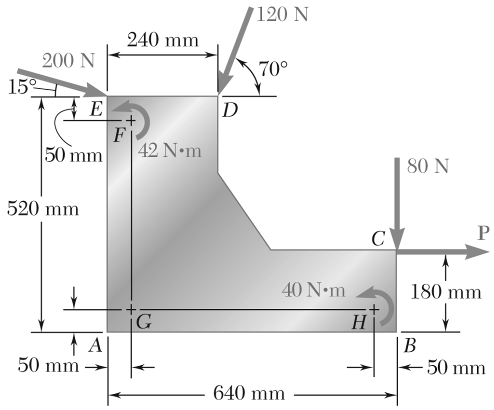
or $a = 0.227$ m

and with **R** at *J*, ΣM_G: $-55.544 \text{ N}\cdot\text{m} = -b(152.142 \text{ N})$

or $b = 0.365$ m

(*a*) The rivet hole is 0.365 m above *G*. ◀

(*b*) The rivet hole is 0.227 m to the right of *G*. ◀



PROBLEM 3.112

Solve Problem 3.111, assuming that $P = 60$ N.

PROBLEM 3.111 A machine component is subjected to the forces and couples shown. The component is to be held in place by a single rivet that can resist a force but not a couple. For $P = 0$, determine the location of the rivet hole if it is to be located (*a*) on line *FG*, (*b*) on line *GH*.

SOLUTION

See the solution to Problem 3.111 leading to the development of Equation (1):

$$M_G = -(55.544 + 0.13P) \text{ N} \cdot \text{m}$$

and $$R_x = (152.142 + P) \text{ N}$$

For $$P = 60 \text{ N}$$

we have $$R_x = (152.142 + 60) = 212.14 \text{ N}$$

$$M_G = -[55.544 + 0.13(60)] = -63.344 \text{ N} \cdot \text{m}$$

Then with **R** at *I*, $$\Sigma M_G: \quad -63.344 \text{ N} \cdot \text{m} = -a(244.53 \text{ N})$$

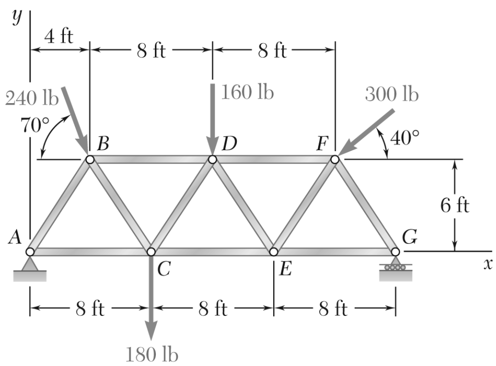
or $$a = 0.259 \text{ m}$$

and with **R** at *J*, $$\Sigma M_G: \quad -63.344 \text{ N} \cdot \text{m} = -b(212.14 \text{ N})$$

or $$b = 0.299 \text{ m}$$

(*a*) The rivet hole is 0.299 m above *G*. ◀

(*b*) The rivet hole is 0.259 m to the right of *G*. ◀



PROBLEM 3.113

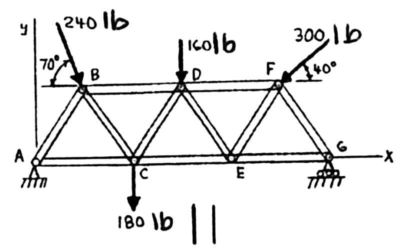
A truss supports the loading shown. Determine the equivalent force acting on the truss and the point of intersection of its line of action with a line drawn through Points A and G.

SOLUTION

We have $\mathbf{R} = \Sigma \mathbf{F}$

$$\mathbf{R} = (240\text{ lb})(\cos 70°\mathbf{i} - \sin 70°\mathbf{j}) - (160\text{ lb})\mathbf{j} + (300\text{ lb})(-\cos 40°\mathbf{i} - \sin 40°\mathbf{j}) - (180\text{ lb})\mathbf{j}$$

$$\mathbf{R} = -(147.728\text{ lb})\mathbf{i} - (758.36\text{ lb})\mathbf{j}$$



$$R = \sqrt{R_x^2 + R_y^2}$$

$$= \sqrt{(147.728)^2 + (758.36)^2}$$

$$= 772.62\text{ lb}$$

$$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right)$$

$$= \tan^{-1}\left(\frac{-758.36}{-147.728}\right)$$

$$= 78.977°$$

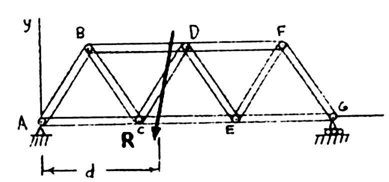
or $\mathbf{R} = 773\text{ lb}$ ↙ $79.0°$ ◀

We have $\Sigma M_A = dR_y$

where

$$\Sigma M_A = -[240\text{ lb}\cos 70°](6\text{ ft}) - [240\text{ lb}\sin 70°](4\text{ ft}) - (160\text{ lb})(12\text{ ft}) + [300\text{ lb}\cos 40°](6\text{ ft}) - [300\text{ lb}\sin 40°](20\text{ ft}) - (180\text{ lb})(8\text{ ft})$$

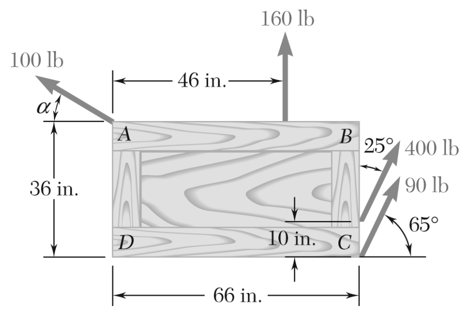
$$= -7232.5\text{ lb}\cdot\text{ft}$$



$$d = \frac{-7232.5\text{ lb}\cdot\text{ft}}{-758.36\text{ lb}}$$

$$= 9.5370\text{ ft}$$

or $d = 9.54$ ft to the right of A ◀

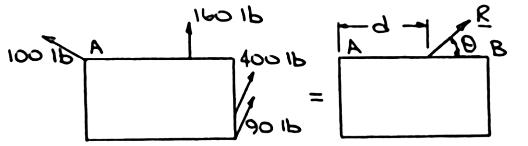


PROBLEM 3.114

Four ropes are attached to a crate and exert the forces shown. If the forces are to be replaced with a single equivalent force applied at a point on line AB, determine (*a*) the equivalent force and the distance from A to the point of application of the force when $\alpha = 30°$, (*b*) the value of α so that the single equivalent force is applied at Point B.

SOLUTION

We have



(*a*) For equivalence, ΣF_x: $-100\cos 30° + 400\cos 65° + 90\cos 65° = R_x$

or $R_x = 120.480$ lb

ΣF_y: $100\sin\alpha + 160 + 400\sin 65° + 90\sin 65° = R_y$

or $R_y = (604.09 + 100\sin\alpha)$ lb (1)

With $\alpha = 30°$, $R_y = 654.09$ lb

Then $R = \sqrt{(120.480)^2 + (654.09)^2}$ $\tan\theta = \dfrac{654.09}{120.480}$

$= 665$ lb or $\theta = 79.6°$

Also ΣM_A: $(46\text{ in.})(160\text{ lb}) + (66\text{ in.})(400\text{ lb})\sin 65°$
$+ (26\text{ in.})(400\text{ lb})\cos 65° + (66\text{ in.})(90\text{ lb})\sin 65°$
$+ (36\text{ in.})(90\text{ lb})\cos 65° = d(654.09\text{ lb})$

or $\Sigma M_A = 42{,}435\text{ lb}\cdot\text{in.}$ and $d = 64.9$ in. $R = 665\text{ lb} \measuredangle 79.6°$ ◀

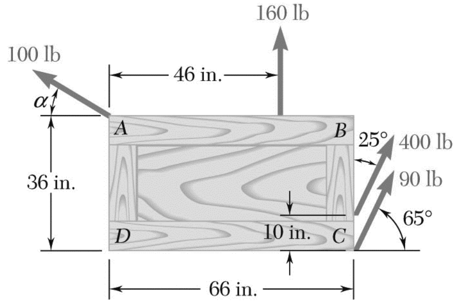
and **R** is applied 64.9 in. to the right of A. ◀

(*b*) We have $d = 66$ in.

Then ΣM_A: $42{,}435\text{ lb}\cdot\text{in} = (66\text{ in.})R_y$

or $R_y = 642.95$ lb

Using Eq. (1): $642.95 = 604.09 + 100\sin\alpha$ or $\alpha = 22.9°$ ◀

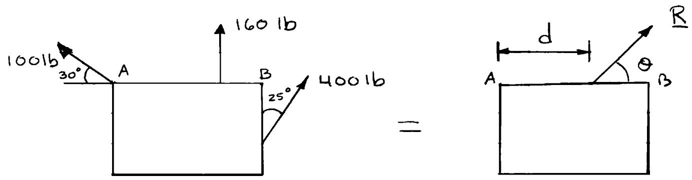


PROBLEM 3.115

Solve Prob. 3.114, assuming that the 90-lb force is removed.

PROBLEM 3.114 Four ropes are attached to a crate and exert the forces shown. If the forces are to be replaced with a single equivalent force applied at a point on line *AB*, determine (*a*) the equivalent force and the distance from *A* to the point of application of the force when $\alpha = 30°$, (*b*) the value of α so that the single equivalent force is applied at Point *B*.

SOLUTION



(*a*) For equivalence, $\Sigma F_x: \quad -(100 \text{ lb})\cos 30° + (400 \text{ lb})\sin 25° = R_x$

or $R_x = 82.445 \text{ lb}$

$\Sigma F_y: \quad 160 \text{ lb} + (100 \text{ lb})\sin 30° + (400 \text{ lb})\cos 25° = R_y$

or $R_y = 572.52 \text{ lb}$

$$R = \sqrt{(82.445)^2 + (572.52)} = 578.43 \text{ lb}$$

$$\tan\theta = \frac{572.52}{82.445} \quad \text{or} \quad \theta = 81.806°$$

$$\Sigma M_A: \quad (46 \text{ in.})(160 \text{ lb}) + (66 \text{ in.})(400 \text{ lb})\cos 25° + (26 \text{ in.})(400 \text{ lb})\sin 25°$$
$$= d(527.52 \text{ lb})$$
$$d = 62.3 \text{ in.}$$

R = 578 lb ∠ 81.8° and is applied 62.3 in. to the right of *A*. ◀

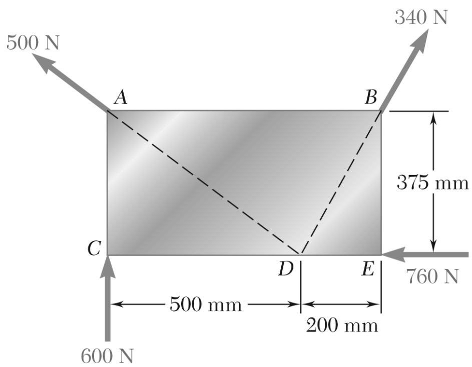
(*b*) We have $d = 66.0$ in. For R applied at B,

$$\Sigma M_A: \quad R_y(66 \text{ in.}) = (160 \text{ lb})(46 \text{ in.}) + (66 \text{ in.})(400 \text{ lb})\cos 25° + (26 \text{ in.})(400 \text{ lb})\sin 25°$$

$$R_y = 540.64 \text{ lb}$$

$$\Sigma F_Y: \quad 160 \text{ lb} + (100 \text{ lb})\sin\alpha + (400 \text{ lb})\cos 25° = 540.64 \text{ lb}$$

$\alpha = 10.44°$ ◀



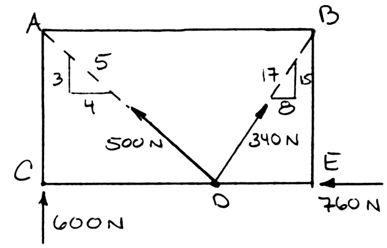
PROBLEM 3.116

Four forces act on a 700 × 375-mm plate as shown. (*a*) Find the resultant of these forces. (*b*) Locate the two points where the line of action of the resultant intersects the edge of the plate.

SOLUTION

(*a*)

$$\mathbf{R} = \Sigma\mathbf{F}$$
$$= (-400\text{ N} + 160\text{ N} - 760\text{ N})\mathbf{i} + (600\text{ N} + 300\text{ N} + 300\text{ N})\mathbf{j}$$
$$= -(1000\text{ N})\mathbf{i} + (1200\text{ N})\mathbf{j}$$



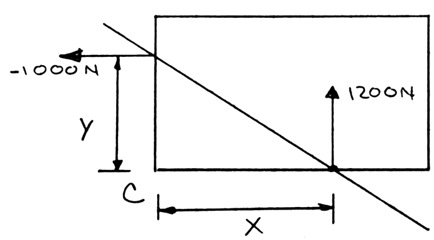
$$R = \sqrt{(1000\text{ N})^2 + (1200\text{ N})^2}$$
$$= 1562.09\text{ N}$$
$$\tan\theta = \left(-\frac{1200\text{ N}}{1000\text{ N}}\right)$$
$$= -1.20000$$
$$\theta = -50.194°$$

$\mathbf{R} = 1562\text{ N} \measuredangle 50.2°$ ◀

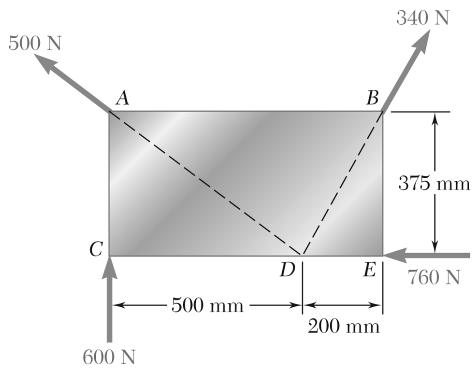
(*b*)

$$\mathbf{M}_C^R = \Sigma\mathbf{r}\times\mathbf{F}$$
$$= (0.5\text{ m})\mathbf{i}\times(300\text{ N} + 300\text{ N})\mathbf{j}$$
$$= (300\text{ N}\cdot\text{m})\mathbf{k}$$

$$(300\text{ N}\cdot\text{m})\mathbf{k} = x\mathbf{i}\times(1200\text{ N})\mathbf{j}$$
$$x = 0.25000\text{ m}$$
$$x = 250\text{ mm}$$
$$(300\text{ N}\cdot\text{m}) = y\mathbf{j}\times(-1000\text{ N})\mathbf{i}$$
$$y = 0.30000\text{ m}$$
$$y = 300\text{ mm}$$



Intersection 250 mm to right of *C* and 300 mm above *C* ◀

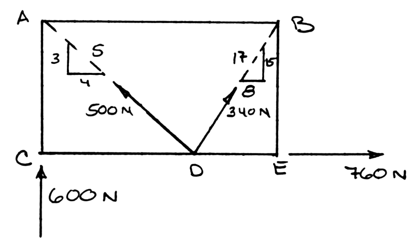


PROBLEM 3.117

Solve Problem 3.116, assuming that the 760-N force is directed to the right.

PROBLEM 3.116 Four forces act on a 700 × 375-mm plate as shown. (*a*) Find the resultant of these forces. (*b*) Locate the two points where the line of action of the resultant intersects the edge of the plate.

SOLUTION



(*a*)

$$\mathbf{R} = \Sigma\mathbf{F}$$
$$= (-400\text{N} + 160\text{ N} + 760\text{ N})\mathbf{i}$$
$$+ (600\text{ N} + 300\text{ N} + 300\text{ N})\mathbf{j}$$
$$= (520\text{ N})\mathbf{i} + (1200\text{ N})\mathbf{j}$$

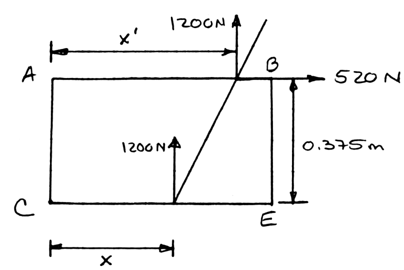
$$R = \sqrt{(520\text{ N})^2 + (1200\text{ N})^2} = 1307.82\text{ N}$$

$$\tan\theta = \left(\frac{1200\text{ N}}{520\text{ N}}\right) = 2.3077$$

$$\theta = 66.5714°$$

$\mathbf{R} = 1308\text{ N} \measuredangle 66.6°$ ◀

(*b*)



$$\mathbf{M}_C^R = \Sigma\mathbf{r}\times\mathbf{F}$$
$$= (0.5\text{ m})\mathbf{i}\times(300\text{ N} + 300\text{ N})\mathbf{j}$$
$$= (300\text{ N}\cdot\text{m})\mathbf{k}$$

$$(300\text{ N}\cdot\text{m})\mathbf{k} = x\mathbf{i}\times(1200\text{ N})\mathbf{j}$$
$$x = 0.25000\text{ m}$$

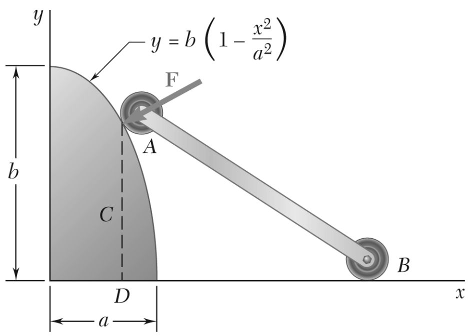
or $\quad x = 0.250\text{ mm}$

$$(300\text{ N}\cdot\text{m})\mathbf{k} = [x'\mathbf{i} + (0.375\text{ m})\mathbf{j}]\times[(520\text{ N})\mathbf{i} + (1200\text{ N})\mathbf{j}]$$
$$= (1200x' - 195)\mathbf{k}$$

$$x' = 0.41250\text{ m}$$

or $\quad x' = 412.5\text{ mm}$

Intersection 412 mm to the right of *A* and 250 mm to the right of *C* ◀



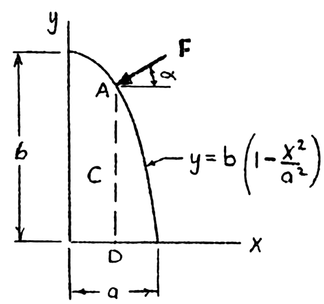
PROBLEM 3.118

As follower AB rolls along the surface of member C, it exerts a constant force **F** perpendicular to the surface. (*a*) Replace **F** with an equivalent force-couple system at Point D obtained by drawing the perpendicular from the point of contact to the x-axis. (*b*) For $a = 1$ m and $b = 2$ m, determine the value of x for which the moment of the equivalent force-couple system at D is maximum.

SOLUTION

(*a*) The slope of any tangent to the surface of member C is

$$\frac{dy}{dx} = \frac{d}{dx}\left[b\left(1 - \frac{x^2}{a^2}\right)\right] = \frac{-2b}{a^2}x$$



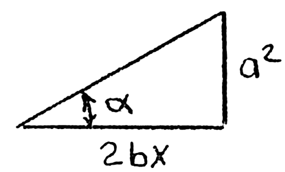
Since the force **F** is perpendicular to the surface,

$$\tan\alpha = -\left(\frac{dy}{dx}\right)^{-1} = \frac{a^2}{2b}\left(\frac{1}{x}\right)$$

For equivalence,

$$\Sigma F: \quad \mathbf{F} = \mathbf{R}$$

$$\Sigma M_D: \quad (F\cos\alpha)(y_A) = M_D$$



where

$$\cos\alpha = \frac{2bx}{\sqrt{(a^2)^2 + (2bx)^2}}$$

$$y_A = b\left(1 - \frac{x^2}{a^2}\right)$$

$$M_D = \frac{2Fb^2\left(x - \frac{x^3}{a^2}\right)}{\sqrt{a^4 + 4b^2x^2}}$$

Therefore, the equivalent force-couple system at D is

$$\mathbf{R} = F \measuredangle \tan^{-1}\left(\frac{a^2}{2bx}\right) \blacktriangleleft$$

$$\mathbf{M} = \frac{2Fb^2\left(x - \frac{x^3}{a^2}\right)}{\sqrt{a^4 + 4b^2x^2}} \blacktriangleleft$$

PROBLEM 3.118 (Continued)

(*b*) To maximize M, the value of x must satisfy $\frac{dM}{dx} = 0$

where for $a = 1 \text{ m}, \quad b = 2 \text{ m}$

$$M = \frac{8F(x - x^3)}{\sqrt{1 + 16x^2}}$$

$$\frac{dM}{dx} = 8F \frac{\sqrt{1+16x^2}(1-3x^2) - (x - x^3)\left[\frac{1}{2}(32x)(1+16x^2)^{-1/2}\right]}{(1+16x^2)} = 0$$

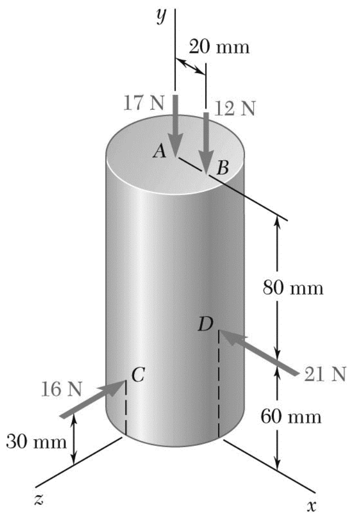
$$(1 + 16x^2)(1 - 3x^2) - 16x(x - x^3) = 0$$

or

$$32x^4 + 3x^2 - 1 = 0$$

$$x^2 = \frac{-3 \pm \sqrt{9 - 4(32)(-1)}}{2(32)} = 0.136011 \text{ m}^2 \quad \text{and} \quad -0.22976 \text{ m}^2$$

Using the positive value of x^2: $x = 0.36880 \text{ m}$ or $x = 369 \text{ mm}$ ◀



PROBLEM 3.119

As plastic bushings are inserted into a 60-mm-diameter cylindrical sheet metal enclosure, the insertion tools exert the forces shown on the enclosure. Each of the forces is parallel to one of the coordinate axes. Replace these forces with an equivalent force-couple system at C.

SOLUTION

For equivalence,

$$\Sigma \mathbf{F}: \quad \mathbf{R} = \mathbf{F}_A + \mathbf{F}_B + \mathbf{F}_C + \mathbf{F}_D$$

$$= -(17\text{ N})\mathbf{j} - (12\text{ N})\mathbf{j} - (16\text{ N})\mathbf{k} - (21\text{ N})\mathbf{i}$$

$$= -(21\text{ N})\mathbf{i} - (29\text{ N})\mathbf{j} - (16\text{ N})\mathbf{k}$$

$$\Sigma M_C: \quad \mathbf{M} = \mathbf{r}_{A/C} \times \mathbf{F}_A + \mathbf{r}_{B/C} \times \mathbf{F}_B + \mathbf{r}_{D/C} \times \mathbf{F}_D$$

$$M = [(0.11\text{ m})\mathbf{j} - (0.03\text{ m})\mathbf{k}] \times [-(17\text{ N})]\mathbf{j}$$

$$+ [(0.02\text{ m})\mathbf{i} + (0.11\text{ m})\mathbf{j} - (0.03\text{ m})\mathbf{k}] \times [-(12\text{ N})]\mathbf{j}$$

$$+ [(0.03\text{ m})\mathbf{i} + (0.03\text{ m})\mathbf{j} - (0.03\text{ m})\mathbf{k}] \times [-(21\text{ N})]\mathbf{i}$$

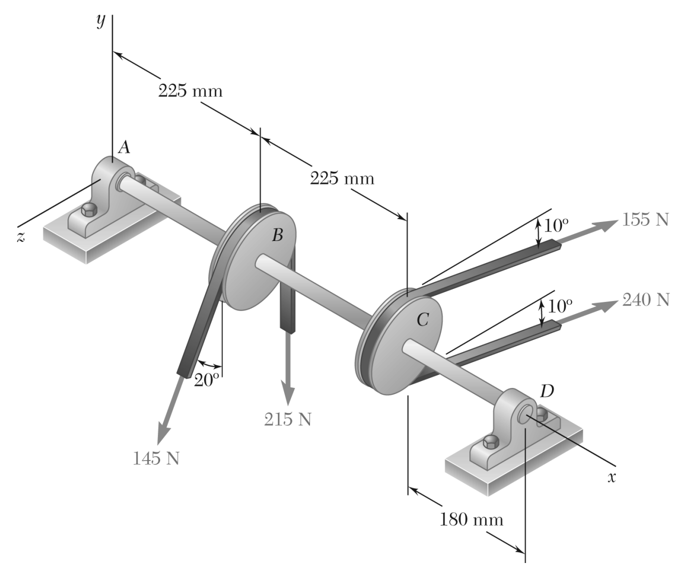
$$= -(0.51\text{ N}\cdot\text{m})\mathbf{i} + [-(0.24\text{ N}\cdot\text{m})\mathbf{k} - (0.36\text{ N}\cdot\text{m})\mathbf{i}]$$

$$+ [(0.63\text{ N}\cdot\text{m})\mathbf{k} + (0.63\text{ N}\cdot\text{m})\mathbf{j}]$$

∴ The equivalent force-couple system at C is

$$\mathbf{R} = -(21.0\text{ N})\mathbf{i} - (29.0\text{ N})\mathbf{j} - (16.00\text{ N})\mathbf{k} \blacktriangleleft$$

$$\mathbf{M} = -(0.870\text{ N}\cdot\text{m})\mathbf{i} + (0.630\text{ N}\cdot\text{m})\mathbf{j} + (0.390\text{ N}\cdot\text{m})\mathbf{k} \blacktriangleleft$$

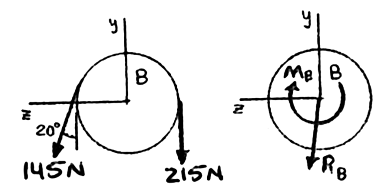


PROBLEM 3.120

Two 150-mm-diameter pulleys are mounted on line shaft *AD*. The belts at *B* and *C* lie in vertical planes parallel to the *yz*-plane. Replace the belt forces shown with an equivalent force-couple system at *A*.

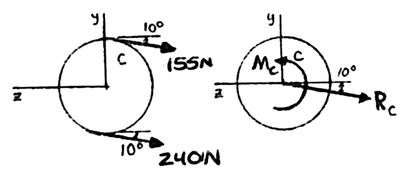
SOLUTION

Equivalent force-couple at each pulley:



Pulley *B*:

$$\mathbf{R}_B = (145\text{ N})(-\cos 20°\mathbf{j} + \sin 20°\mathbf{k}) - 215\text{ N}\mathbf{j}$$
$$= -(351.26\text{ N})\mathbf{j} + (49.593\text{ N})\mathbf{k}$$
$$\mathbf{M}_B = -(215\text{ N} - 145\text{ N})(0.075\text{ m})\mathbf{i}$$
$$= -(5.25\text{ N}\cdot\text{m})\mathbf{i}$$



Pulley *C*:

$$\mathbf{R}_C = (155\text{ N} + 240\text{ N})(-\sin 10°\mathbf{j} - \cos 10°\mathbf{k})$$
$$= -(68.591\text{ N})\mathbf{j} - (389.00\text{ N})\mathbf{k}$$
$$\mathbf{M}_C = (240\text{ N} - 155\text{ N})(0.075\text{ m})\mathbf{i}$$
$$= (6.3750\text{ N}\cdot\text{m})\mathbf{i}$$

Then

$$\mathbf{R} = \mathbf{R}_B + \mathbf{R}_C = -(419.85\text{ N})\mathbf{j} - (339.41)\mathbf{k}$$

or $\mathbf{R} = (420\text{ N})\mathbf{j} - (339\text{ N})\mathbf{k}$ ◀

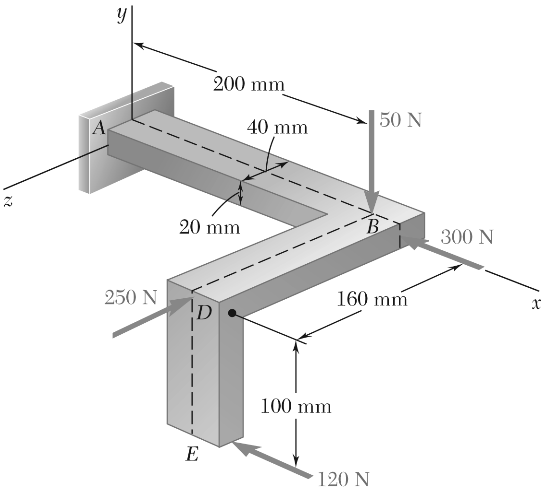
$$\mathbf{M}_A = \mathbf{M}_B + \mathbf{M}_C + \mathbf{r}_{B/A} \times \mathbf{R}_B + \mathbf{r}_{C/A} \times \mathbf{R}_C$$

$$= -(5.25\text{ N}\cdot\text{m})\mathbf{i} + (6.3750\text{ N}\cdot\text{m})\mathbf{i} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.225 & 0 & 0 \\ 0 & -351.26 & 49.593 \end{vmatrix} \text{N}\cdot\text{m}$$

$$+ \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.45 & 0 & 0 \\ 0 & -68.591 & -389.00 \end{vmatrix} \text{N}\cdot\text{m}$$

$$= (1.12500\text{ N}\cdot\text{m})\mathbf{i} + (163.892\text{ N}\cdot\text{m})\mathbf{j} - (109.899\text{ N}\cdot\text{m})\mathbf{k}$$

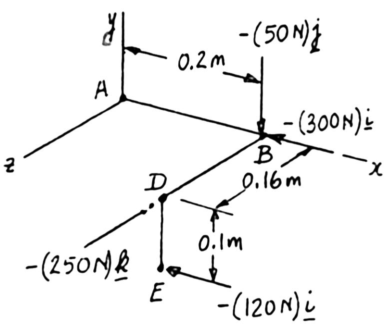
or $\mathbf{M}_A = (1.125\text{ N}\cdot\text{m})\mathbf{i} + (163.9\text{ N}\cdot\text{m})\mathbf{j} - (109.9\text{ N}\cdot\text{m})\mathbf{k}$ ◀



PROBLEM 3.121

Four forces are applied to the machine component *ABDE* as shown. Replace these forces with an equivalent force-couple system at *A*.

SOLUTION



$$\mathbf{R} = -(50\text{ N})\mathbf{j} - (300\text{ N})\mathbf{i} - (120\text{ N})\mathbf{i} - (250\text{ N})\mathbf{k}$$

$$\mathbf{R} = -(420\text{ N})\mathbf{i} - (50\text{ N})\mathbf{j} - (250\text{ N})\mathbf{k}$$

$$\mathbf{r}_B = (0.2\text{ m})\mathbf{i}$$

$$\mathbf{r}_D = (0.2\text{ m})\mathbf{i} + (0.16\text{ m})\mathbf{k}$$

$$\mathbf{r}_E = (0.2\text{ m})\mathbf{i} - (0.1\text{ m})\mathbf{j} + (0.16\text{ m})\mathbf{k}$$

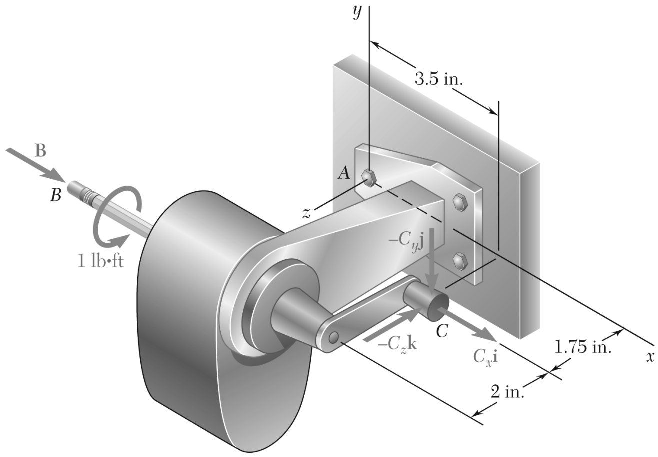
$$\mathbf{M}_A^R = \mathbf{r}_B \times [-(300\text{ N})\mathbf{i} - (50\text{ N})\mathbf{j}] + \mathbf{r}_D \times (-250\text{ N})\mathbf{k} + \mathbf{r} \times (-120\text{ N})\mathbf{i}$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.2\text{ m} & 0 & 0 \\ -300\text{ N} & -50\text{ N} & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.2\text{ m} & 0 & 0.16\text{ m} \\ 0 & 0 & -250\text{ N} \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.2\text{ m} & -0.1\text{ m} & 0.16\text{ m} \\ -120\text{ N} & 0 & 0 \end{vmatrix}$$

$$= -(10\text{ N}\cdot\text{m})\mathbf{k} + (50\text{ N}\cdot\text{m})\mathbf{j} - (19.2\text{ N}\cdot\text{m})\mathbf{j} - (12\text{ N}\cdot\text{m})\mathbf{k}$$

Force-couple system at *A* is

$$\mathbf{R} = -(420\text{ N})\mathbf{i} - (50\text{ N})\mathbf{j} - (250\text{ N})\mathbf{k} \quad \mathbf{M}_A^R = (30.8\text{ N}\cdot\text{m})\mathbf{j} - (220\text{ N}\cdot\text{m})\mathbf{k} \blacktriangleleft$$



PROBLEM 3.122

While using a pencil sharpener, a student applies the forces and couple shown. (*a*) Determine the forces exerted at *B* and *C* knowing that these forces and the couple are equivalent to a force-couple system at *A* consisting of the force $\mathbf{R} = (2.6\text{ lb})\mathbf{i} + R_y\mathbf{j} - (0.7\text{ lb})\mathbf{k}$ and the couple $\mathbf{M}_A^R = M_x\mathbf{i} + (1.0\text{ lb}\cdot\text{ft})\mathbf{j} - (0.72\text{ lb}\cdot\text{ft})\mathbf{k}$. (*b*) Find the corresponding values of R_y and M_x.

SOLUTION

(*a*) From the statement of the problem, equivalence requires

$$\Sigma\mathbf{F}:\quad \mathbf{B} + \mathbf{C} = \mathbf{R}$$

or

$$\Sigma F_x:\quad B_x + C_x = 2.6\text{ lb} \tag{1}$$

$$\Sigma F_y:\quad -C_y = R_y \tag{2}$$

$$\Sigma F_z:\quad -C_z = -0.7\text{ lb} \quad\text{or}\quad C_z = 0.7\text{ lb}$$

and

$$\Sigma\mathbf{M}_A:\quad (\mathbf{r}_{B/A}\times\mathbf{B} + \mathbf{M}_B) + \mathbf{r}_{C/A}\times\mathbf{C} = M_A^R$$

or

$$\Sigma\mathbf{M}_x:\quad (1\text{ lb}\cdot\text{ft}) + \left(\frac{1.75}{12}\text{ ft}\right)(C_y) = M_x \tag{3}$$

$$\Sigma M_y:\quad \left(\frac{3.75}{12}\text{ ft}\right)(B_x) + \left(\frac{1.75}{12}\text{ ft}\right)(C_x) + \left(\frac{3.5}{12}\text{ ft}\right)(0.7\text{ lb}) = 1\text{ lb}\cdot\text{ft}$$

or

$$3.75B_x + 1.75C_x = 9.55$$

Using Eq. (1):

$$3.75B_x + 1.75(2.6B_x) = 9.55$$

or

$$B_x = 2.5\text{ lb}$$

and

$$C_x = 0.1\text{ lb}$$

$$\Sigma M_z:\quad -\left(\frac{3.5}{12}\text{ ft}\right)(C_y) = -0.72\text{ lb}\cdot\text{ft}$$

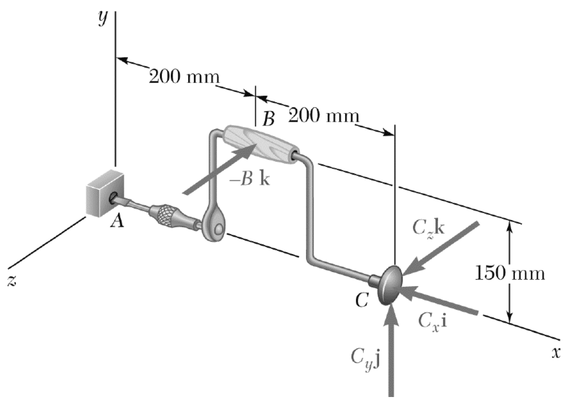
or

$$C_y = 2.4686\text{ lb}$$

$$\mathbf{B} = (2.50\text{ lb})\mathbf{i} \quad \mathbf{C} = (0.1000\text{ lb})\mathbf{i} - (2.47\text{ lb})\mathbf{j} - (0.700\text{ lb})\mathbf{k}$$ ◀

(*b*) Eq. (2) ⇒ $R_y = -2.47\text{ lb}$ ◀

Using Eq. (3): $1 + \left(\frac{1.75}{12}\right)(2.4686) = M_x$ or $M_x = 1.360\text{ lb}\cdot\text{ft}$ ◀



PROBLEM 3.123

A blade held in a brace is used to tighten a screw at A. (a) Determine the forces exerted at B and C, knowing that these forces are equivalent to a force-couple system at A consisting of $\mathbf{R} = -(30\text{ N})\mathbf{i} + R_y\mathbf{j} + R_z\mathbf{k}$ and $\mathbf{M}_A^R = -(12\text{ N}\cdot\text{m})\mathbf{i}$. ($b$) Find the corresponding values of R_y and R_z. (c) What is the orientation of the slot in the head of the screw for which the blade is least likely to slip when the brace is in the position shown?

SOLUTION

(a) Equivalence requires $\Sigma\mathbf{F}$: $\mathbf{R} = \mathbf{B} + \mathbf{C}$

or

$$-(30\text{ N})\mathbf{i} + R_y\mathbf{j} + R_z\mathbf{k} = -B\mathbf{k} + (-C_x\mathbf{i} + C_y\mathbf{j} + C_z\mathbf{k})$$

Equating the **i** coefficients: $\mathbf{i}$: $-30\text{ N} = -C_x$ or $C_x = 30\text{ N}$

Also, $\Sigma\mathbf{M}_A$: $\mathbf{M}_A^R = \mathbf{r}_{B/A} \times \mathbf{B} + \mathbf{r}_{C/A} \times \mathbf{C}$

or

$$-(12\text{ N}\cdot\text{m})\mathbf{i} = [(0.2\text{ m})\mathbf{i} + (0.15\text{ m})\mathbf{j}] \times (-B)\mathbf{k}$$
$$+(0.4\text{ m})\mathbf{i} \times [-(30\text{ N})\mathbf{i} + C_y\mathbf{j} + C_z\mathbf{k}]$$

Equating coefficients:

$\mathbf{i}$: $-12\text{ N}\cdot\text{m} = -(0.15\text{ m})B$ or $B = 80\text{ N}$

$\mathbf{k}$: $0 = (0.4\text{ m})C_y$ or $C_y = 0$

$\mathbf{j}$: $0 = (0.2\text{ m})(80\text{ N}) - (0.4\text{ m})C_z$ or $C_z = 40\text{ N}$

$$\mathbf{B} = -(80.0\text{ N})\mathbf{k} \quad \mathbf{C} = -(30.0\text{ N})\mathbf{i} + (40.0\text{ N})\mathbf{k}$$ ◀

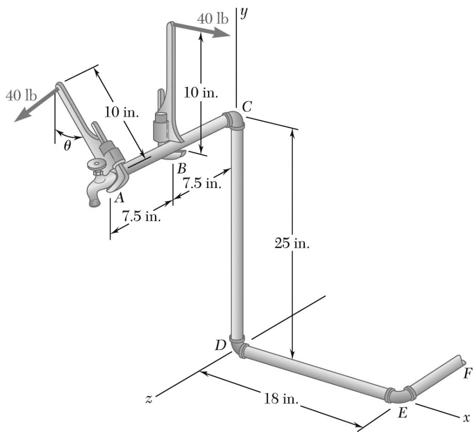
(b) Now we have for the equivalence of forces

$$-(30\text{ N})\mathbf{i} + R_y\mathbf{j} + R_z\mathbf{k} = -(80\text{ N})\mathbf{k} + [(-30\text{ N})\mathbf{i} + (40\text{ N})\mathbf{k}]$$

Equating coefficients: $\mathbf{j}$: $R_y = 0$ $\qquad R_y = 0$ ◀

$\mathbf{k}$: $R_z = -80 + 40$ $\qquad$ or $R_z = -40.0\text{ N}$ ◀

(c) First note that $\mathbf{R} = -(30\text{ N})\mathbf{i} - (40\text{ N})\mathbf{k}$. Thus, the screw is best able to resist the lateral force R_z when the slot in the head of the screw is vertical. ◀



PROBLEM 3.124

In order to unscrew the tapped faucet A, a plumber uses two pipe wrenches as shown. By exerting a 40-lb force on each wrench, at a distance of 10 in. from the axis of the pipe and in a direction perpendicular to the pipe and to the wrench, he prevents the pipe from rotating, and thus avoids loosening or further tightening the joint between the pipe and the tapped elbow C. Determine (*a*) the angle θ that the wrench at A should form with the vertical if elbow C is not to rotate about the vertical, (*b*) the force-couple system at C equivalent to the two 40-lb forces when this condition is satisfied.

SOLUTION

We first reduce the given forces to force-couple systems at A and B, noting that

$$|\mathbf{M}_A| = |\mathbf{M}_B| = (40 \text{ lb})(10 \text{ in.})$$
$$= 400 \text{ lb} \cdot \text{in.}$$

We now determine the equivalent force-couple system at C.

$$\mathbf{R} = (40 \text{ lb})(1-\cos\theta)\mathbf{i} - (40 \text{ lb})\sin\theta\mathbf{j} \qquad (1)$$

$$\mathbf{M}_C^R = \mathbf{M}_A + \mathbf{M}_B + (15 \text{ in.})\mathbf{k} \times [-(40 \text{ lb})\cos\theta\mathbf{i} - (40 \text{ lb})\sin\theta\mathbf{j}]$$
$$+ (7.5 \text{ in.})\mathbf{k} \times (40 \text{ lb})\mathbf{i}$$
$$= +400 - 400 - 600\cos\theta\mathbf{j} + 600\sin\theta\mathbf{i} + 300\mathbf{j}$$
$$= (600 \text{ lb} \cdot \text{in.})\sin\theta\mathbf{i} + (300 \text{ lb} \cdot \text{in.})(1 - 2\cos\theta)\mathbf{j} \qquad (2)$$

(*a*) For no rotation about vertical, y component of $\mathbf{M}_C^R$ must be zero.

$$1 - 2\cos\theta = 0$$
$$\cos\theta = 1/2$$

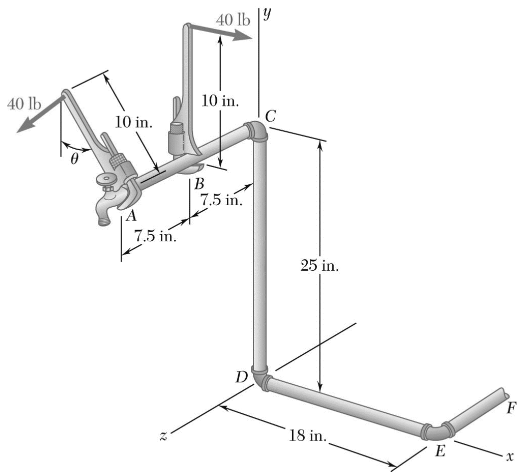
$\theta = 60.0°$ ◀

(*b*) For $\theta = 60.0°$ in Eqs. (1) and (2),

$$\mathbf{R} = (20.0 \text{ lb})\mathbf{i} - (34.641 \text{ lb})\mathbf{j}; \quad \mathbf{M}_C^R = (519.62 \text{ lb} \cdot \text{in.})\mathbf{i}$$

$\mathbf{R} = (20.0 \text{ lb})\mathbf{i} - (34.6 \text{ lb})\mathbf{j}$; $\mathbf{M}_C^R = (520 \text{ lb} \cdot \text{in.})\mathbf{i}$ ◀

PROBLEM 3.125



Assuming $\theta = 60°$ in Prob. 3.124, replace the two 40-lb forces with an equivalent force-couple system at *D* and determine whether the plumber's action tends to tighten or loosen the joint between (*a*) pipe *CD* and elbow *D*, (*b*) elbow *D* and pipe *DE*. Assume all threads to be right-handed.

PROBLEM 3.124 In order to unscrew the tapped faucet *A*, a plumber uses two pipe wrenches as shown. By exerting a 40-lb force on each wrench, at a distance of 10 in. from the axis of the pipe and in a direction perpendicular to the pipe and to the wrench, he prevents the pipe from rotating, and thus avoids loosening or further tightening the joint between the pipe and the tapped elbow *C*. Determine (*a*) the angle θ that the wrench at *A* should form with the vertical if elbow *C* is not to rotate about the vertical, (*b*) the force-couple system at *C* equivalent to the two 40-lb forces when this condition is satisfied.

SOLUTION

The equivalent force-couple system at *C* for $\theta = 60°$ was obtained in the solution to Prob. 3.124:

$$\mathbf{R} = (20.0 \text{ lb})\mathbf{i} - (34.641 \text{ lb})\mathbf{j}$$

$$\mathbf{M}_C^R = (519.62 \text{ lb}\cdot\text{in.})\mathbf{i}$$

The equivalent force-couple system at *D* is made of **R** and $\mathbf{M}_D^R$ where

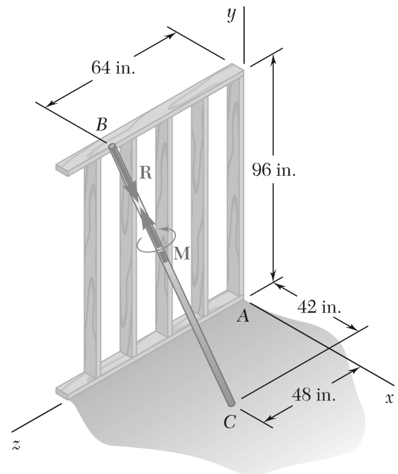
$$\begin{aligned}\mathbf{M}_D^R &= \mathbf{M}_C^R + \mathbf{r}_{C/D} \times \mathbf{R} \\ &= (519.62 \text{ lb}\cdot\text{in.})\mathbf{i} + (25.0 \text{ in.})\mathbf{j} \times [(20.0 \text{ lb})\mathbf{i} - (34.641 \text{ lb})\mathbf{j}] \\ &= (519.62 \text{ lb}\cdot\text{in.})\mathbf{i} - (500 \text{ lb}\cdot\text{in.})\mathbf{k}\end{aligned}$$

Equivalent force-couple at *D*:

$$\mathbf{R} = (20.0 \text{ lb})\mathbf{i} - (34.6 \text{ lb})\mathbf{j}; \;\; \mathbf{M}_C^R = (520 \text{ lb}\cdot\text{in.})\mathbf{i} - (500 \text{ lb}\cdot\text{in.})\mathbf{k} \;\; \blacktriangleleft$$

(*a*) Since $\mathbf{M}_D^R$ has no component along the *y*-axis, the plumber's action will neither loosen nor tighten the joint between pipe *CD* and elbow. ◀

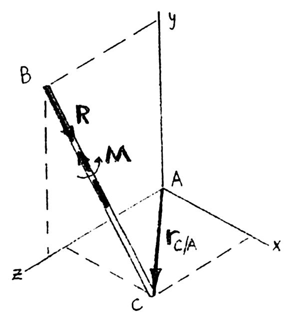
(*b*) Since the *x* component of $\mathbf{M}_D^R$ is , the plumber's action will tend to tighten the joint between elbow and pipe *DE*. ◀



PROBLEM 3.126

As an adjustable brace *BC* is used to bring a wall into plumb, the force-couple system shown is exerted on the wall. Replace this force-couple system with an equivalent force-couple system at *A* if $R = 21.2$ lb and $M = 13.25$ lb · ft.

SOLUTION



We have $\Sigma\mathbf{F}: \quad \mathbf{R} = \mathbf{R}_A = \mathbf{R}\lambda_{BC}$

where
$$\lambda_{BC} = \frac{(42 \text{ in.})\mathbf{i} - (96 \text{ in.})\mathbf{j} - (16 \text{ in.})\mathbf{k}}{106 \text{ in.}}$$

$$\mathbf{R}_A = \frac{21.2 \text{ lb}}{106}(42\mathbf{i} - 96\mathbf{j} - 16\mathbf{k})$$

or
$$\mathbf{R}_A = (8.40 \text{ lb})\mathbf{i} - (19.20 \text{ lb})\mathbf{j} - (3.20 \text{ lb})\mathbf{k} \quad \blacktriangleleft$$

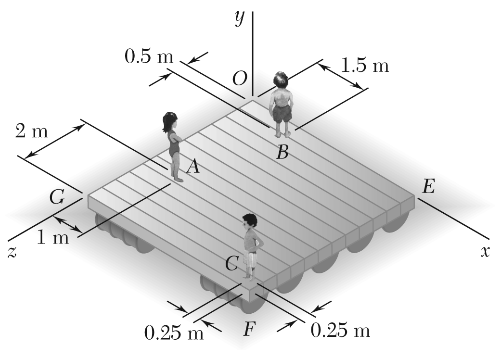
We have $\Sigma\mathbf{M}_A: \quad \mathbf{r}_{C/A} \times \mathbf{R} + \mathbf{M} = \mathbf{M}_A$

where
$$\mathbf{r}_{C/A} = (42 \text{ in.})\mathbf{i} + (48 \text{ in.})\mathbf{k} = \frac{1}{12}(42\mathbf{i} + 48\mathbf{k})\text{ft}$$
$$= (3.5 \text{ ft})\mathbf{i} + (4.0 \text{ ft})\mathbf{k}$$
$$\mathbf{R} = (8.40 \text{ lb})\mathbf{i} - (19.50 \text{ lb})\mathbf{j} - (3.20 \text{ lb})\mathbf{k}$$
$$\mathbf{M} = -\lambda_{BC}M$$
$$= \frac{-42\mathbf{i} + 96\mathbf{j} + 16\mathbf{k}}{106}(13.25 \text{ lb} \cdot \text{ft})$$
$$= -(5.25 \text{ lb} \cdot \text{ft})\mathbf{i} + (12 \text{ lb} \cdot \text{ft})\mathbf{j} + (2 \text{ lb} \cdot \text{ft})\mathbf{k}$$

Then
$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3.5 & 0 & 4.0 \\ 8.40 & -19.20 & -3.20 \end{vmatrix} \text{lb} \cdot \text{ft} + (-5.25\mathbf{i} + 12\mathbf{j} + 2\mathbf{k}) \text{ lb} \cdot \text{ft} = \mathbf{M}_A$$

$$\mathbf{M}_A = (71.55 \text{ lb} \cdot \text{ft})\mathbf{i} + (56.80 \text{ lb} \cdot \text{ft})\mathbf{j} - (65.20 \text{ lb} \cdot \text{ft})\mathbf{k}$$

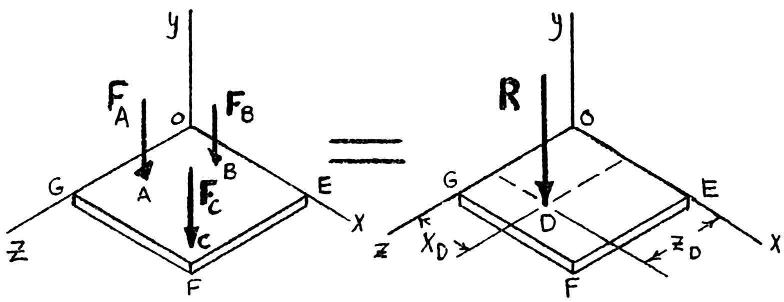
or $\mathbf{M}_A = (71.6 \text{ lb} \cdot \text{ft})\mathbf{i} + (56.8 \text{ lb} \cdot \text{ft})\mathbf{j} - (65.2 \text{ lb} \cdot \text{ft})\mathbf{k} \quad \blacktriangleleft$



PROBLEM 3.127

Three children are standing on a 5×5-m raft. If the weights of the children at Points *A*, *B*, and *C* are 375 N, 260 N, and 400 N, respectively, determine the magnitude and the point of application of the resultant of the three weights.

SOLUTION



We have $\Sigma\mathbf{F}: \quad \mathbf{F}_A + \mathbf{F}_B + \mathbf{F}_C = \mathbf{R}$

$$-(375\text{ N})\mathbf{j} - (260\text{ N})\mathbf{j} - (400\text{ N})\mathbf{j} = \mathbf{R}$$

$$-(1035\text{ N})\mathbf{j} = \mathbf{R}$$

or $R = 1035\text{ N}$ ◀

We have $\Sigma M_x: \quad F_A(z_A) + F_B(z_B) + F_C(z_C) = R(z_D)$

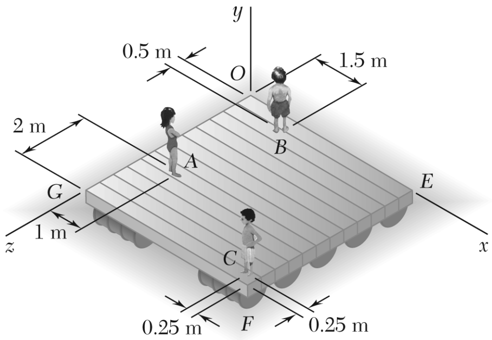
$$(375\text{ N})(3\text{ m}) + (260\text{ N})(0.5\text{ m}) + (400\text{ N})(4.75\text{ m}) = (1035\text{ N})(z_D)$$

$z_D = 3.0483\text{ m}$ or $z_D = 3.05\text{ m}$ ◀

We have $\Sigma M_z: \quad F_A(x_A) + F_B(x_B) + F_C(x_C) = R(x_D)$

$$375\text{ N}(1\text{ m}) + (260\text{ N})(1.5\text{ m}) + (400\text{ N})(4.75\text{ m}) = (1035\text{ N})(x_D)$$

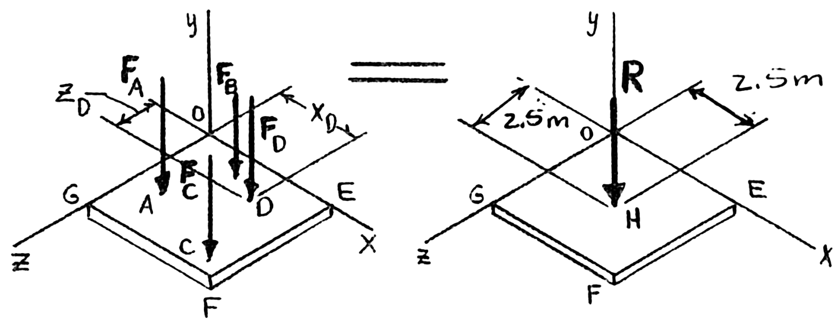
$x_D = 2.5749\text{ m}$ or $x_D = 2.57\text{ m}$ ◀



PROBLEM 3.128

Three children are standing on a 5×5-m raft. The weights of the children at Points *A*, *B*, and *C* are 375 N, 260 N, and 400 N, respectively. If a fourth child of weight 425 N climbs onto the raft, determine where she should stand if the other children remain in the positions shown and the line of action of the resultant of the four weights is to pass through the center of the raft.

SOLUTION



We have $\Sigma \mathbf{F}$: $\mathbf{F}_A + \mathbf{F}_B + \mathbf{F}_C = \mathbf{R}$

$$-(375 \text{ N})\mathbf{j} - (260 \text{ N})\mathbf{j} - (400 \text{ N})\mathbf{j} - (425 \text{ N})\mathbf{j} = \mathbf{R}$$

$$\mathbf{R} = -(1460 \text{ N})\mathbf{j}$$

We have ΣM_x: $F_A(z_A) + F_B(z_B) + F_C(z_C) + F_D(z_D) = R(z_H)$

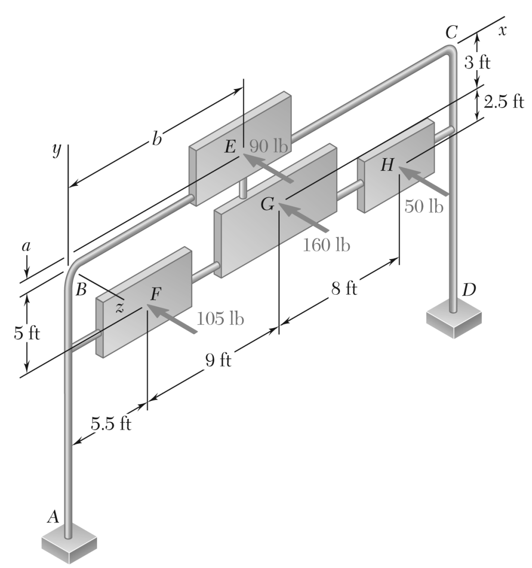
$$(375 \text{ N})(3 \text{ m}) + (260 \text{ N})(0.5 \text{ m}) + (400 \text{ N})(4.75 \text{ m})$$
$$+ (425 \text{ N})(z_D) = (1460 \text{ N})(2.5 \text{ m})$$

$$z_D = 1.16471 \text{ m} \qquad \text{or} \quad z_D = 1.165 \text{ m} \blacktriangleleft$$

We have ΣM_z: $F_A(x_A) + F_B(x_B) + F_C(x_C) + F_D(x_D) = R(x_H)$

$$(375 \text{ N})(1 \text{ m}) + (260 \text{ N})(1.5 \text{ m}) + (400 \text{ N})(4.75 \text{ m})$$
$$+ (425 \text{ N})(x_D) = (1460 \text{ N})(2.5 \text{ m})$$

$$x_D = 2.3235 \text{ m} \qquad \text{or} \quad x_D = 2.32 \text{ m} \blacktriangleleft$$

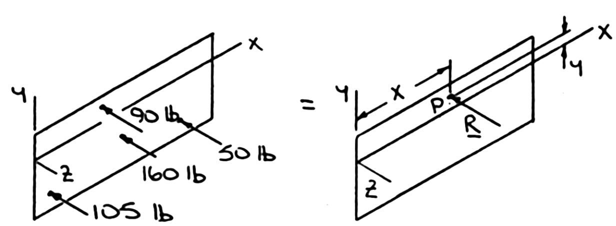


PROBLEM 3.129

Four signs are mounted on a frame spanning a highway, and the magnitudes of the horizontal wind forces acting on the signs are as shown. Determine the magnitude and the point of application of the resultant of the four wind forces when $a = 1$ ft and $b = 12$ ft.

SOLUTION

We have



Assume that the resultant **R** is applied at Point *P* whose coordinates are (*x*, *y*, 0).

Equivalence then requires

$$\Sigma F_z: \quad -105 - 90 - 160 - 50 = -R$$

or $R = 405$ lb ◀

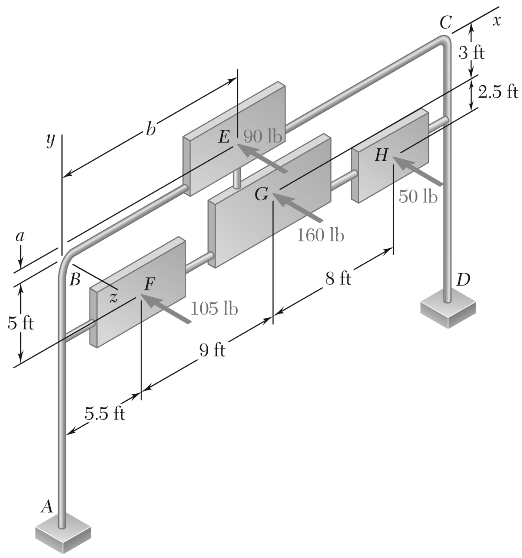
$$\Sigma M_x: \quad (5 \text{ ft})(105 \text{ lb}) - (1 \text{ ft})(90 \text{ lb}) + (3 \text{ ft})(160 \text{ lb}) + (5.5 \text{ ft})(50 \text{ lb}) = -y(405 \text{ lb})$$

or $y = -2.94$ ft

$$\Sigma M_y: \quad (5.5 \text{ ft})(105 \text{ lb}) + (12 \text{ ft})(90 \text{ lb}) + (14.5 \text{ ft})(160 \text{ lb}) + (22.5 \text{ ft})(50 \text{ lb}) = -x(405 \text{ lb})$$

or $x = 12.60$ ft

R acts 12.60 ft to the right of member *AB* and 2.94 ft below member *BC*. ◀



PROBLEM 3.130

Four signs are mounted on a frame spanning a highway, and the magnitudes of the horizontal wind forces acting on the signs are as shown. Determine a and b so that the point of application of the resultant of the four forces is at G.

SOLUTION

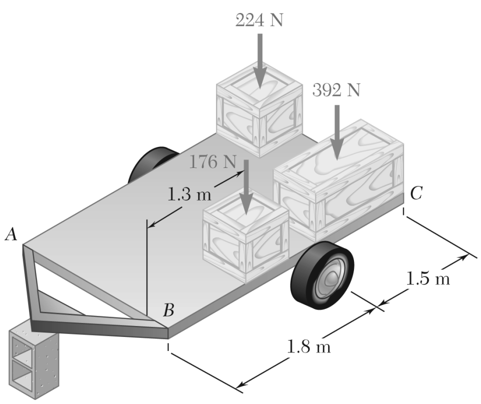
Since **R** acts at G, equivalence then requires that $\Sigma \mathbf{M}_G$ of the applied system of forces also be zero. Then at

$$G: \Sigma M_x: \quad -(a+3) \text{ ft} \times (90 \text{ lb}) + (2 \text{ ft})(105 \text{ lb}) + (2.5 \text{ ft})(50 \text{ lb}) = 0$$

or $a = 0.722$ ft ◀

$$\Sigma M_y: \quad -(9 \text{ ft})(105 \text{ ft}) - (14.5 - b) \text{ ft} \times (90 \text{ lb}) + (8 \text{ ft})(50 \text{ lb}) = 0$$

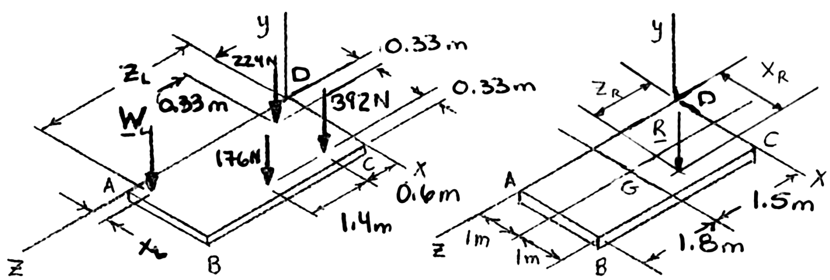
or $b = 20.6$ ft ◀



PROBLEM 3.131*

A group of students loads a 2×3.3-m flatbed trailer with two $0.66 \times 0.66 \times 0.66$-m boxes and one $0.66 \times 0.66 \times 1.2$-m box. Each of the boxes at the rear of the trailer is positioned so that it is aligned with both the back and a side of the trailer. Determine the smallest load the students should place in a second $0.66 \times 0.66 \times 1.2$-m box and where on the trailer they should secure it, without any part of the box overhanging the sides of the trailer, if each box is uniformly loaded and the line of action of the resultant of the weights of the four boxes is to pass through the point of intersection of the centerlines of the trailer and the axle. (*Hint:* Keep in mind that the box may be placed either on its side or on its end.)

SOLUTION



For the smallest weight on the trailer so that the resultant force of the four weights acts over the axle at the intersection with the center line of the trailer, the added $0.66 \times 0.66 \times 1.2$-m box should be placed adjacent to one of the edges of the trailer with the 0.66×0.66-m side on the bottom. The edges to be considered are based on the location of the resultant for the three given weights.

We have $\quad \Sigma \mathbf{F}: \quad -(224 \text{ N})\mathbf{j} - (392 \text{ N})\mathbf{j} - (176 \text{ N})\mathbf{j} = \mathbf{R}$

$$\mathbf{R} = -(792 \text{ N})\mathbf{j}$$

We have $\quad \Sigma M_z: \quad -(224 \text{ N})(0.33 \text{ m}) - (392 \text{ N})(1.67 \text{ m}) - (176 \text{ N})(1.67 \text{ m}) = (-792 \text{ N})(x)$

$$x_R = 1.29101 \text{ m}$$

We have $\quad \Sigma M_x: \quad (224 \text{ N})(0.33 \text{ m}) + (392 \text{ N})(0.6 \text{ m}) + (176 \text{ N})(2.0 \text{ m}) = (792 \text{ N})(z)$

$$z_R = 0.83475 \text{ m}$$

From the statement of the problem, it is known that the resultant of **R** from the original loading and the lightest load **W** passes through G, the point of intersection of the two center lines. Thus, $\Sigma \mathbf{M}_G = 0$.

Further, since the lightest load **W** is to be as small as possible, the fourth box should be placed as far from G as possible without the box overhanging the trailer. These two requirements imply

$$(0.33 \text{ m} \le x \le 1 \text{ m}) \; (1.5 \text{ m} \le z \le 2.97 \text{ m})$$

PROBLEM 3.131* (Continued)

With $x_L = 0.33$ m

at G: ΣM_z: $(1-0.33)\text{ m}\times W_L - (1.29101-1)\text{ m}\times(792\text{ N}) = 0$

or $W_L = 344.00$ N

Now we must check if this is physically possible,

at G: ΣM_x: $(z_L - 1.5)\text{ m}\times 344\text{ N}) - (1.5-0.83475)\text{ m}\times(792\text{ N}) = 0$

or $z_L = 3.032$ m

which is **not** acceptable.

With $z_L = 2.97$ m:

at G: ΣM_x: $(2.97-1.5)\text{ m}\times W_L - (1.5-0.83475)\text{ m}\times(792\text{ N}) = 0$

or $W_L = 358.42$ N

Now check if this is physically possible,

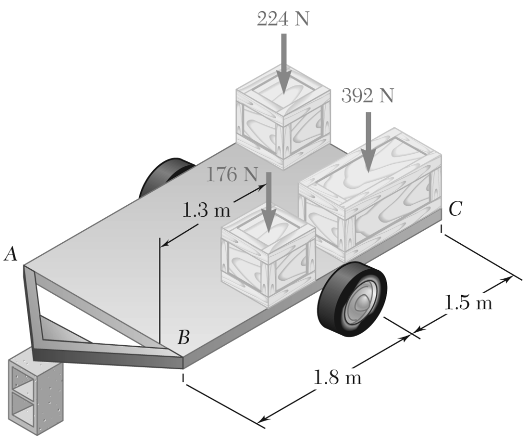
at G: ΣM_z: $(1-x_L)\text{ m}\times(358.42\text{ N}) - (1.29101-1)\text{ m}\times(792\text{ N}) = 0$

or $x_L = 0.357$ m ok!

The minimum weight of the fourth box is $W_L = 358$ N ◀

And it is placed on end A (0.66×0.66-m side down) along side *AB* with the center of the box 0.357 m from side *AD*. ◀

PROBLEM 3.132*



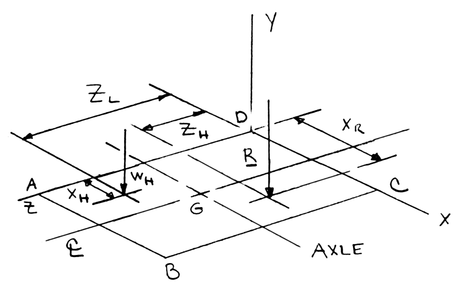
Solve Problem 3.131 if the students want to place as much weight as possible in the fourth box and at least one side of the box must coincide with a side of the trailer.

PROBLEM 3.131* A group of students loads a 2×3.3-m flatbed trailer with two $0.66\times0.66\times0.66$-m boxes and one $0.66\times0.66\times1.2$-m box. Each of the boxes at the rear of the trailer is positioned so that it is aligned with both the back and a side of the trailer. Determine the smallest load the students should place in a second $0.66\times0.66\times1.2$-m box and where on the trailer they should secure it, without any part of the box overhanging the sides of the trailer, if each box is uniformly loaded and the line of action of the resultant of the weights of the four boxes is to pass through the point of intersection of the centerlines of the trailer and the axle. (*Hint:* Keep in mind that the box may be placed either on its side or on its end.)

SOLUTION

First replace the three known loads with a single equivalent force **R** applied at coordinate $(x_R,\ 0,\ z_R)$.

Equivalence requires



$$\Sigma F_y:\quad -224-392-176=-R$$

or $$\mathbf{R}=792\ \text{N}\downarrow$$

$$\Sigma M_x:\quad (0.33\text{ m})(224\text{ N})+(0.6\text{ m})(392\text{ N})+(2\text{ m})(176\text{ N})=z_R(792\text{ N})$$

or $$z_R=0.83475\text{ m}$$

$$\Sigma M_z:\quad -(0.33\text{ m})(224\text{ N})-(1.67\text{ m})(392\text{ N})-(1.67\text{ m})(176\text{ N})=x_R(792\text{ N})$$

or $$x_R=1.29101\text{ m}$$

From the statement of the problem, it is known that the resultant of **R** and the heaviest loads $\mathbf{W}_H$ passes through G, the point of intersection of the two center lines. Thus,

$$\Sigma\mathbf{M}_G=0$$

Further, since $\mathbf{W}_H$ is to be as large as possible, the fourth box should be placed as close to G as possible while keeping one of the sides of the box coincident with a side of the trailer. Thus, the two limiting cases are

$$x_H=0.6\text{ m}\quad\text{or}\quad z_H=2.7\text{ m}$$

PROBLEM 3.132* (Continued)

Now consider these two possibilities.

With $x_H = 0.6$ m

at G: ΣM_z: $(1-0.6)\text{ m}\times W_H - (1.29101-1)\text{ m}\times(792\text{ N}) = 0$

or $W_H = 576.20$ N

Checking if this is physically possible

at G: ΣM_x: $(z_H - 1.5)\text{ m}\times(576.20\text{ N}) - (1.5-0.83475)\text{ m}\times(792\text{ N}) = 0$

or $z_H = 2.414$ m

which is acceptable.

With $z_H = 2.7$ m

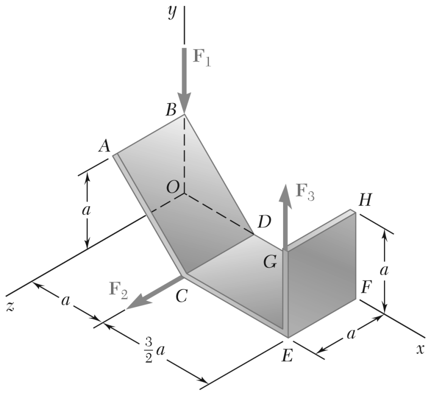
at G: ΣM_x: $(2.7-1.5)\text{ m}\times W_H - (1.5-0.83475)\text{ m}\times(792\text{ N}) = 0$

or $W_H = 439$ N

Since this is less than the first case, the maximum weight of the fourth box is

$W_H = 576$ N ◀

and it is placed with a 0.66×1.2-m side down, a 0.66-m edge along side *AD*, and the center 2.41 m from side *DC*. ◀



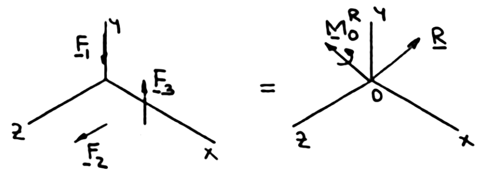
PROBLEM 3.133*

A piece of sheet metal is bent into the shape shown and is acted upon by three forces. If the forces have the same magnitude P, replace them with an equivalent wrench and determine (*a*) the magnitude and the direction of the resultant force $\mathbf{R}$, (*b*) the pitch of the wrench, (*c*) the axis of the wrench.

SOLUTION

First reduce the given forces to an equivalent force-couple system $\left(\mathbf{R},\ \mathbf{M}_O^R\right)$ at the origin.

We have



$$\Sigma\mathbf{F}:\quad -P\mathbf{j}+P\mathbf{j}+P\mathbf{k}=R$$

or

$$\mathbf{R}=P\mathbf{k}$$

$$\Sigma\mathbf{M}_O:\quad -(aP)\mathbf{j}+\left[-(aP)\mathbf{i}+\left(\frac{5}{2}aP\right)\mathbf{k}\right]=M_O^R$$

or

$$\mathbf{M}_O^R=aP\left(-\mathbf{i}-\mathbf{j}+\frac{5}{2}\mathbf{k}\right)$$

(*a*) Then for the wrench,

$$R=P \blacktriangleleft$$

and

$$\boldsymbol{\lambda}_{\text{axis}}=\frac{\mathbf{R}}{R}=\mathbf{k}$$

$$\cos\theta_x=0\quad \cos\theta_y=0\quad \cos\theta_z=1$$

or

$$\theta_x=90°\quad \theta_y=90°\quad \theta_z=0° \blacktriangleleft$$

(*b*) Now

$$M_1=\boldsymbol{\lambda}_{\text{axis}}\cdot\mathbf{M}_O^R$$

$$=\mathbf{k}\cdot aP\left(-\mathbf{i}-\mathbf{j}+\frac{5}{2}\mathbf{k}\right)$$

$$=\frac{5}{2}aP$$

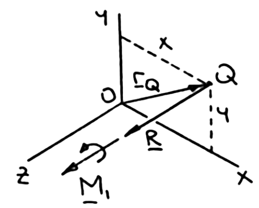
Then

$$P=\frac{\mathbf{M}_1}{R}=\frac{\frac{5}{2}aP}{P}\qquad \text{or}\quad P=\frac{5}{2}a \blacktriangleleft$$

PROBLEM 3.133* (Continued)

(*c*) The components of the wrench are $(\mathbf{R}, \mathbf{M}_1)$, where $\mathbf{M}_1 = M_1\boldsymbol{\lambda}_{\text{axis}}$, and the axis of the wrench is assumed to intersect the *xy*-plane at Point *Q*, whose coordinates are (*x*, *y*, 0). Thus, we require

$$\mathbf{M}_z = \mathbf{r}_Q \times \mathbf{R}_R$$



where

$$\mathbf{M}_z = \mathbf{M}_O \times \mathbf{M}_1$$

Then

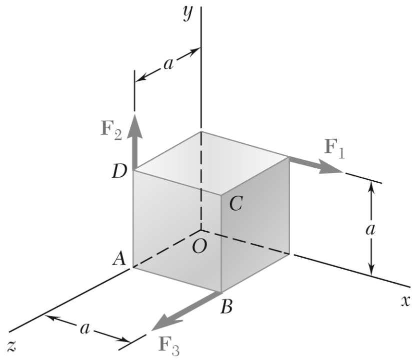
$$aP\left(-\mathbf{i} - \mathbf{j} + \frac{5}{2}\mathbf{k}\right) - \frac{5}{2}aP\mathbf{k} = (x\mathbf{i} + y\mathbf{j}) + P\mathbf{k}$$

Equating coefficients:

$$\mathbf{i}: \quad -aP = yP \quad \text{or} \quad y = -a$$

$$\mathbf{j}: \quad -aP = -xP \quad \text{or} \quad x = a$$

The axis of the wrench is parallel to the *z*-axis and intersects the *xy*-plane at $x = a,\ y = -a$. ◀

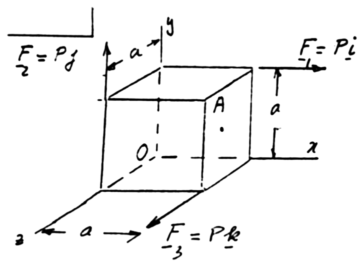


PROBLEM 3.134*

Three forces of the same magnitude P act on a cube of side a as shown. Replace the three forces by an equivalent wrench and determine (a) the magnitude and direction of the resultant force $\mathbf{R}$, (b) the pitch of the wrench, (c) the axis of the wrench.

SOLUTION

Force-couple system at O:

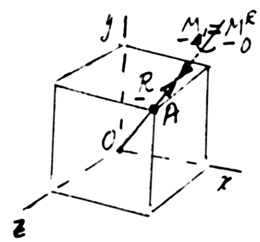


$$\mathbf{R} = P\mathbf{i} + P\mathbf{j} + P\mathbf{k} = P(\mathbf{i} + \mathbf{j} + \mathbf{k})$$

$$\mathbf{M}_O^R = a\mathbf{j} \times P\mathbf{i} + a\mathbf{k} \times P\mathbf{j} + a\mathbf{i} \times P\mathbf{k}$$
$$= -Pa\mathbf{k} - Pa\mathbf{i} - Pa\mathbf{j}$$

$$\mathbf{M}_O^R = -Pa(\mathbf{i} + \mathbf{j} + \mathbf{k})$$

Since $\mathbf{R}$ and $\mathbf{M}_O^R$ have the same direction, they form a wrench with $\mathbf{M}_1 = \mathbf{M}_O^R$. Thus, the axis of the wrench is the diagonal OA. We note that



$$\cos\theta_x = \cos\theta_y = \cos\theta_z = \frac{a}{a\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$R = P\sqrt{3} \quad \theta_x = \theta_y = \theta_z = 54.7°$$

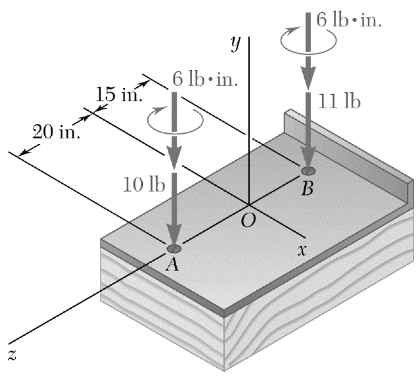
$$M_1 = M_O^R = -Pa\sqrt{3}$$

$$\text{Pitch} = p = \frac{M_1}{R} = \frac{-Pa\sqrt{3}}{P\sqrt{3}} = -a$$

(a) $R = P\sqrt{3} \quad \theta_x = \theta_y = \theta_z = 54.7°$ ◀

(b) $-a$ ◀

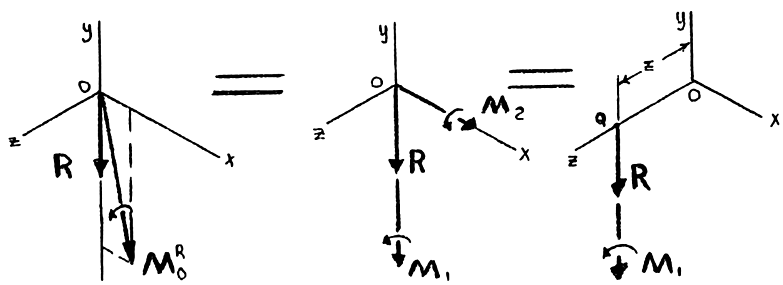
(c) Axis of the wrench is diagonal OA. ◀



PROBLEM 3.135*

The forces and couples shown are applied to two screws as a piece of sheet metal is fastened to a block of wood. Reduce the forces and the couples to an equivalent wrench and determine (*a*) the resultant force **R**, (*b*) the pitch of the wrench, (*c*) the point where the axis of the wrench intersects the *xz*-plane.

SOLUTION



First, reduce the given force system to a force-couple at the origin.

We have $\Sigma\mathbf{F}: \quad -(10\text{ lb})\mathbf{j}-(11\text{ lb})\mathbf{j}=\mathbf{R}$

$$\mathbf{R}=-(21\text{ lb})\mathbf{j}$$

We have $\Sigma\mathbf{M}_O: \quad \Sigma(\mathbf{r}_O\times\mathbf{F})+\Sigma\mathbf{M}_C=\mathbf{M}_O^R$

$$\mathbf{M}_O^R=\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 0 & 0 & 20\\ 0 & -10 & 0\end{vmatrix}\text{ lb}\cdot\text{in.}+\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 0 & 0 & -15\\ 0 & -11 & 0\end{vmatrix}\text{ lb}\cdot\text{in.}-(12\text{ lb}\cdot\text{in})\mathbf{j}$$

$$=(35\text{ lb}\cdot\text{in.})\mathbf{i}-(12\text{ lb}\cdot\text{in.})\mathbf{j}$$

(*a*) $\mathbf{R}=-(21\text{ lb})\mathbf{j}$ or $\mathbf{R}=-(21.0\text{ lb})\mathbf{j}$ ◀

(*b*) We have $M_1=\boldsymbol{\lambda}_R\cdot\mathbf{M}_O^R \quad \boldsymbol{\lambda}_R=\dfrac{\mathbf{R}}{R}$

$$=(-\mathbf{j})\cdot[(35\text{ lb}\cdot\text{in.})\mathbf{i}-(12\text{ lb}\cdot\text{in.})\mathbf{j}]$$

$$=12\text{ lb}\cdot\text{in.}\quad\text{and}\quad\mathbf{M}_1=-(12\text{ lb}\cdot\text{in.})\mathbf{j}$$

and pitch $p=\dfrac{M_1}{R}=\dfrac{12\text{ lb}\cdot\text{in.}}{21\text{ lb}}=0.57143\text{ in.}$ or $p=0.571\text{ in.}$ ◀

(*c*) We have

$$\mathbf{M}_O^R = \mathbf{M}_1 + \mathbf{M}_2$$

$$\mathbf{M}_2 = \mathbf{M}_O^R - \mathbf{M}_1 = (35 \text{ lb} \cdot \text{in.})\mathbf{i}$$

We require

$$\mathbf{M}_2 = \mathbf{r}_{Q/O} \times \mathbf{R}$$

$$(35 \text{ lb} \cdot \text{in.})\mathbf{i} = (x\mathbf{i} + z\mathbf{k}) \times [-(21 \text{ lb})\mathbf{j}]$$

$$35\mathbf{i} = -(21x)\mathbf{k} + (21z)\mathbf{i}$$

From **i**:

$$35 = 21z$$

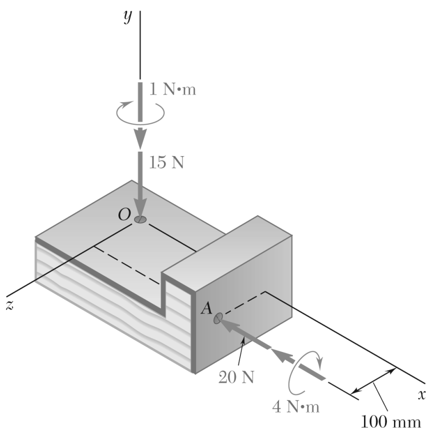
$$z = 1.66667 \text{ in.}$$

From **k**:

$$0 = -21x$$

$$z = 0$$

The axis of the wrench is parallel to the *y*-axis and intersects the *xz*-plane at $x = 0$, $z = 1.667$ in. ◀



PROBLEM 3.136*

The forces and couples shown are applied to two screws as a piece of sheet metal is fastened to a block of wood. Reduce the forces and the couples to an equivalent wrench and determine (*a*) the resultant force **R**, (*b*) the pitch of the wrench, (*c*) the point where the axis of the wrench intersects the *xz*-plane.

SOLUTION

First, reduce the given force system to a force-couple system.

We have $\Sigma\mathbf{F}: \quad -(20\text{ N})\mathbf{i} - (15\text{ N})\mathbf{j} = \mathbf{R} \quad R = 25\text{ N}$

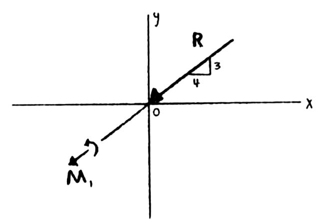
We have $\Sigma\mathbf{M}_O: \quad \Sigma(\mathbf{r}_O \times \mathbf{F}) + \Sigma\mathbf{M}_C = \mathbf{M}_O^R$

$$\mathbf{M}_O^R = -20\text{ N}(0.1\text{ m})\mathbf{j} - (4\text{ N}\cdot\text{m})\mathbf{i} - (1\text{ N}\cdot\text{m})\mathbf{j}$$
$$= -(4\text{ N}\cdot\text{m})\mathbf{i} - (3\text{ N}\cdot\text{m})\mathbf{j}$$

(*a*) $\mathbf{R} = -(20.0\text{ N})\mathbf{i} - (15.00\text{ N})\mathbf{j}$ ◀

(*b*) We have $M_1 = \boldsymbol{\lambda}_R \cdot \mathbf{M}_O^R \quad \boldsymbol{\lambda} = \dfrac{\mathbf{R}}{R}$

$$= (-0.8\mathbf{i} - 0.6\mathbf{j})\cdot[-(4\text{ N}\cdot\text{m})\mathbf{i} - (3\text{ N}\cdot\text{m})\mathbf{j}]$$
$$= 5\text{ N}\cdot\text{m}$$



Pitch: $p = \dfrac{M_1}{R} = \dfrac{5\text{ N}\cdot\text{m}}{25\text{ N}} = 0.200\text{ m}$

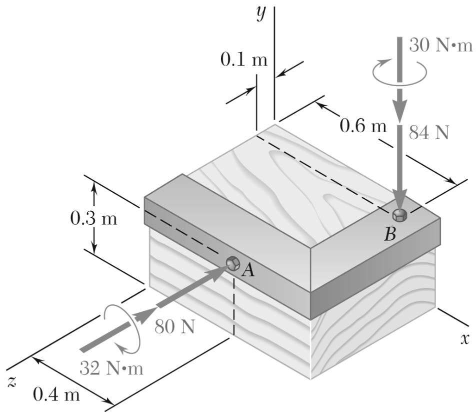
or $p = 0.200\text{ m}$ ◀

(*c*) From above, note that

$$\mathbf{M}_1 = \mathbf{M}_O^R$$

Therefore, the axis of the wrench goes through the origin. The line of action of the wrench lies in the *xy*-plane with a slope of

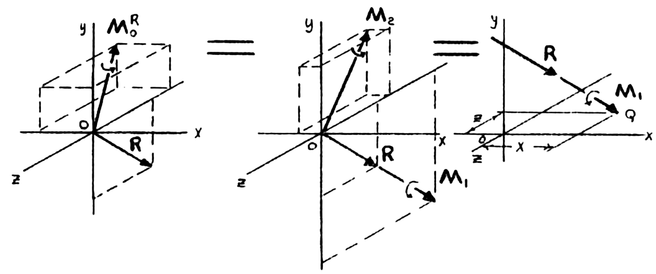
$$y = \frac{3}{4}x$$ ◀



PROBLEM 3.137*

Two bolts at A and B are tightened by applying the forces and couples shown. Replace the two wrenches with a single equivalent wrench and determine (a) the resultant $\mathbf{R}$, (b) the pitch of the single equivalent wrench, (c) the point where the axis of the wrench intersects the xz-plane.

SOLUTION



First, reduce the given force system to a force-couple at the origin.

We have $\quad \Sigma\mathbf{F}: \quad -(84\text{ N})\mathbf{j} - (80\text{ N})\mathbf{k} = \mathbf{R} \quad R = 116\text{ N}$

and $\quad \Sigma\mathbf{M}_O: \quad \Sigma(\mathbf{r}_O \times \mathbf{F}) + \Sigma\mathbf{M}_C = \mathbf{M}_O^R$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.6 & 0 & 0.1 \\ 0 & 84 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.4 & 0.3 & 0 \\ 0 & 0 & 80 \end{vmatrix} + (-30\mathbf{j} - 32\mathbf{k})\text{ N}\cdot\text{m} = \mathbf{M}_O^R$$

$$\mathbf{M}_O^R = -(15.6\text{ N}\cdot\text{m})\mathbf{i} + (2\text{ N}\cdot\text{m})\mathbf{j} - (82.4\text{ N}\cdot\text{m})\mathbf{k}$$

(a) $\quad \mathbf{R} = -(84.0\text{ N})\mathbf{j} - (80.0\text{ N})\mathbf{k}$ ◀

(b) We have

$$M_1 = \boldsymbol{\lambda}_R \cdot \mathbf{M}_O^R \qquad \boldsymbol{\lambda}_R = \frac{\mathbf{R}}{R}$$

$$= -\frac{-84\mathbf{j} - 80\mathbf{k}}{116} \cdot [-(15.6\text{ N}\cdot\text{m})\mathbf{i} + (2\text{ N}\cdot\text{m})\mathbf{j} - (82.4\text{ N}\cdot\text{m})\mathbf{k}]$$

$$= 55.379\text{ N}\cdot\text{m}$$

and

$$\mathbf{M}_1 = M_1\lambda_R = -(40.102\text{ N}\cdot\text{m})\mathbf{j} - (38.192\text{ N}\cdot\text{m})\mathbf{k}$$

Then pitch

$$p = \frac{M_1}{R} = \frac{55.379\text{ N}\cdot\text{m}}{116\text{ N}} = 0.47741\text{ m}$$

or $p = 0.477$ m ◀

PROBLEM 3.137* (Continued)

(*c*) We have $\mathbf{M}_O^R = \mathbf{M}_1 + \mathbf{M}_2$

$$\mathbf{M}_2 = \mathbf{M}_O^R - \mathbf{M}_1 = [(-15.6\mathbf{i} + 2\mathbf{j} - 82.4\mathbf{k}) - (40.102\mathbf{j} - 38.192\mathbf{k})] \text{ N}\cdot\text{m}$$

$$= -(15.6 \text{ N}\cdot\text{m})\mathbf{i} + (42.102 \text{ N}\cdot\text{m})\mathbf{j} - (44.208 \text{ N}\cdot\text{m})\mathbf{k}$$

We require $\mathbf{M}_2 = \mathbf{r}_{Q/O} \times \mathbf{R}$

$$(-15.6\mathbf{i} + 42.102\mathbf{j} - 44.208\mathbf{k}) = (x\mathbf{i} + z\mathbf{k}) \times (84\mathbf{j} - 80\mathbf{k})$$

$$= (84z)\mathbf{i} + (80x)\mathbf{j} - (84x)\mathbf{k}$$

From **i**: $-15.6 = 84z$

$$z = -0.185714 \text{ m}$$

or $z = -0.1857$ m

From **k**: $-44.208 = -84x$

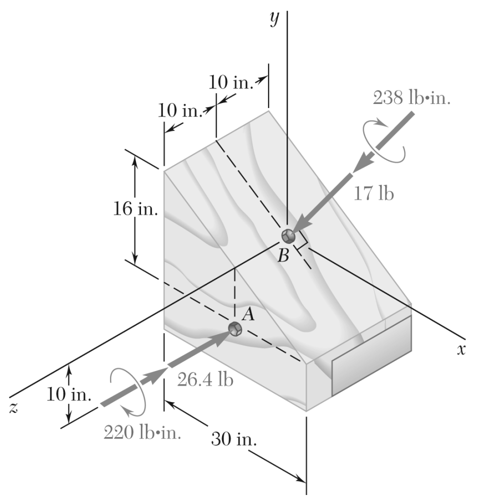
$$x = 0.52629 \text{ m}$$

or $x = 0.526$ m

The axis of the wrench intersects the *xz*-plane at

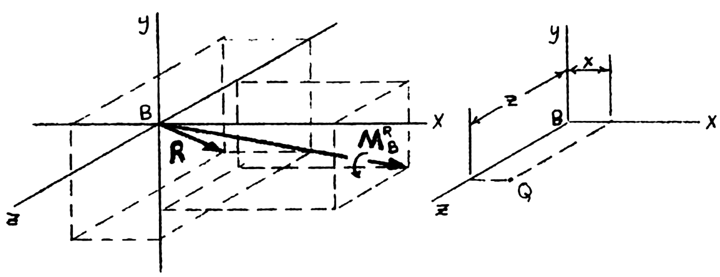
$x = 0.526$ m $\quad y = 0 \quad z = -0.1857$ m ◀

PROBLEM 3.138*



Two bolts at A and B are tightened by applying the forces and couples shown. Replace the two wrenches with a single equivalent wrench and determine (*a*) the resultant **R**, (*b*) the pitch of the single equivalent wrench, (*c*) the point where the axis of the wrench intersects the xz-plane.

SOLUTION



First, reduce the given force system to a force-couple at the origin at B.

(*a*) We have $\Sigma\mathbf{F}:\quad -(26.4\text{ lb})\mathbf{k} - (17\text{ lb})\left(\dfrac{8}{17}\mathbf{i} + \dfrac{15}{17}\mathbf{j}\right) = \mathbf{R}$

$$\mathbf{R} = -(8.00\text{ lb})\mathbf{i} - (15.00\text{ lb})\mathbf{j} - (26.4\text{ lb})\mathbf{k} \quad \blacktriangleleft$$

and $R = 31.4\text{ lb}$

We have $\Sigma\mathbf{M}_B:\quad \mathbf{r}_{A/B} \times \mathbf{F}_A + \mathbf{M}_A + \mathbf{M}_B = \mathbf{M}_B^R$

$$\mathbf{M}_B^R = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -10 & 0 \\ 0 & 0 & -26.4 \end{vmatrix} - 220\mathbf{k} - 238\left(\frac{8}{17}\mathbf{i} + \frac{15}{17}\mathbf{j}\right) = 264\mathbf{i} - 220\mathbf{k} - 14(8\mathbf{i} + 15\mathbf{j})$$

$$\mathbf{M}_B^R = (152\text{ lb}\cdot\text{in.})\mathbf{i} - (210\text{ lb}\cdot\text{in.})\mathbf{j} - (220\text{ lb}\cdot\text{in.})\mathbf{k}$$

(*b*) We have $M_1 = \boldsymbol{\lambda}_R \cdot \mathbf{M}_O^R \qquad \boldsymbol{\lambda}_R = \dfrac{\mathbf{R}}{R}$

$$= \frac{-8.00\mathbf{i} - 15.00\mathbf{j} - 26.4\mathbf{k}}{31.4} \cdot [(152\text{ lb}\cdot\text{in.})\mathbf{i} - (210\text{ lb}\cdot\text{in.})\mathbf{j} - (220\text{ lb}\cdot\text{in.})\mathbf{k}]$$

$$= 246.56\text{ lb}\cdot\text{in.}$$

PROBLEM 3.138* (Continued)

and $\mathbf{M}_1 = M_1\lambda_R = -(62.818\text{ lb}\cdot\text{in.})\mathbf{i} - (117.783\text{ lb}\cdot\text{in.})\mathbf{j} - (207.30\text{ lb}\cdot\text{in.})\mathbf{k}$

Then pitch $p = \dfrac{M_1}{R} = \dfrac{246.56\text{ lb}\cdot\text{in.}}{31.4\text{ lb}} = 7.8522\text{ in.}$ or $p = 7.85\text{ in.}$ ◀

(*c*) We have $\mathbf{M}_B^R = \mathbf{M}_1 + \mathbf{M}_2$

$$\mathbf{M}_2 = \mathbf{M}_B^R - \mathbf{M}_1 = (152\mathbf{i} - 210\mathbf{j} - 220\mathbf{k}) - (-62.818\mathbf{i} - 117.783\mathbf{j} - 207.30\mathbf{k})$$
$$= (214.82\text{ lb}\cdot\text{in.})\mathbf{i} - (92.217\text{ lb}\cdot\text{in.})\mathbf{j} - (12.7000\text{ lb}\cdot\text{in.})\mathbf{k}$$

We require $\mathbf{M}_2 = \mathbf{r}_{Q/B} \times \mathbf{R}$

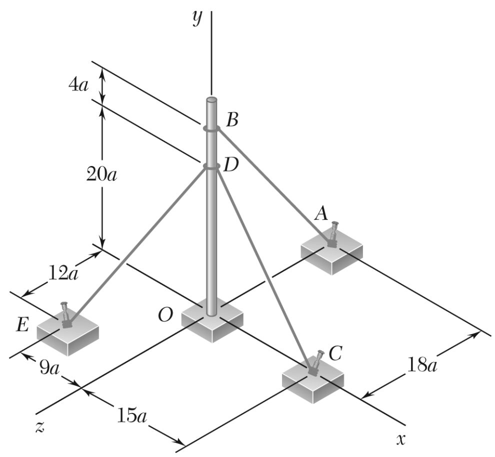
$$214.82\mathbf{i} - 92.217\mathbf{j} - 12.7000\mathbf{k} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x & 0 & z \\ -8 & -15 & -26.4 \end{vmatrix}$$
$$= (15z)\mathbf{i} - (8z)\mathbf{j} + (26.4x)\mathbf{j} - (15x)\mathbf{k}$$

From **i**: $214.82 = 15z \qquad z = 14.3213\text{ in.}$

From **k**: $-12.7000 = -15x \qquad x = 0.84667\text{ in.}$

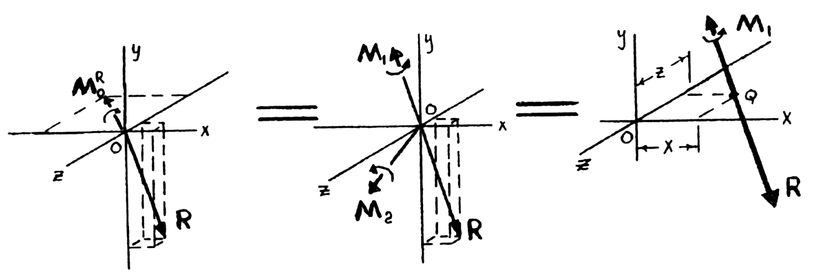
The axis of the wrench intersects the *xz*-plane at $x = 0.847\text{ in.} \quad y = 0 \quad z = 14.32\text{ in.}$ ◀

PROBLEM 3.139*



A flagpole is guyed by three cables. If the tensions in the cables have the same magnitude P, replace the forces exerted on the pole with an equivalent wrench and determine (*a*) the resultant force **R**, (*b*) the pitch of the wrench, (*c*) the point where the axis of the wrench intersects the *xz*-plane.

SOLUTION



(*a*) First reduce the given force system to a force-couple at the origin.

We have $\Sigma\mathbf{F}: \quad P\boldsymbol{\lambda}_{BA} + P\boldsymbol{\lambda}_{DC} + P\boldsymbol{\lambda}_{DE} = \mathbf{R}$

$$\mathbf{R} = P\left[\left(\frac{4}{5}\mathbf{j} - \frac{3}{5}\mathbf{k}\right) + \left(\frac{3}{5}\mathbf{i} - \frac{4}{5}\mathbf{j}\right) + \left(\frac{-9}{25}\mathbf{i} - \frac{4}{5}\mathbf{j} + \frac{12}{25}\mathbf{k}\right)\right]$$

$$\mathbf{R} = \frac{3P}{25}(2\mathbf{i} - 20\mathbf{j} - \mathbf{k}) \blacktriangleleft$$

$$R = \frac{3P}{25}\sqrt{(2)^2 + (20)^2 + (1)^2} = \frac{27\sqrt{5}}{25}P$$

We have $\Sigma\mathbf{M}: \quad \Sigma(\mathbf{r}_O \times P) = \mathbf{M}_O^R$

$$(24a)\mathbf{j} \times \left(\frac{-4P}{5}\mathbf{j} - \frac{3P}{5}\mathbf{k}\right) + (20a)\mathbf{j} \times \left(\frac{3P}{5}\mathbf{i} - \frac{4P}{5}\mathbf{j}\right) + (20a)\mathbf{j} \times \left(\frac{-9P}{25}\mathbf{i} - \frac{4P}{5}\mathbf{j} + \frac{12P}{25}\mathbf{k}\right) = \mathbf{M}_O^R$$

$$\mathbf{M}_O^R = \frac{24Pa}{5}(-\mathbf{i} - \mathbf{k})$$

PROBLEM 3.139* (Continued)

(*b*) We have $M_1 = \boldsymbol{\lambda}_R \cdot \mathbf{M}_O^R$

where $\boldsymbol{\lambda}_R = \dfrac{\mathbf{R}}{R} = \dfrac{3P}{25}(2\mathbf{i} - 20\mathbf{j} - \mathbf{k})\dfrac{25}{27\sqrt{5}P} = \dfrac{1}{9\sqrt{5}}(2\mathbf{i} - 20\mathbf{j} - \mathbf{k})$

Then $M_1 = \dfrac{1}{9\sqrt{5}}(2\mathbf{i} - 20\mathbf{j} - \mathbf{k}) \cdot \dfrac{24Pa}{5}(-\mathbf{i} - \mathbf{k}) = \dfrac{-8Pa}{15\sqrt{5}}$

and pitch $p = \dfrac{M_1}{R} = \dfrac{-8Pa}{15\sqrt{5}}\left(\dfrac{25}{27\sqrt{5}P}\right) = \dfrac{-8a}{81}$ or $p = -0.0988a$ ◀

(*c*) $\mathbf{M}_1 = M_1 \boldsymbol{\lambda}_R = \dfrac{-8Pa}{15\sqrt{5}}\left(\dfrac{1}{9\sqrt{5}}\right)(2\mathbf{i} - 20\mathbf{j} - \mathbf{k}) = \dfrac{8Pa}{675}(-2\mathbf{i} + 20\mathbf{j} + \mathbf{k})$

Then $\mathbf{M}_2 = \mathbf{M}_O^R - \mathbf{M}_1 = \dfrac{24Pa}{5}(-\mathbf{i} - \mathbf{k}) - \dfrac{8Pa}{675}(-2\mathbf{i} + 20\mathbf{j} + \mathbf{k}) = \dfrac{8Pa}{675}(-430\mathbf{i} - 20\mathbf{j} - 406\mathbf{k})$

We require $\mathbf{M}_2 = \mathbf{r}_{Q/O} \times \mathbf{R}$

$$\left(\frac{8Pa}{675}\right)(-403\mathbf{i} - 20\mathbf{j} - 406\mathbf{k}) = (x\mathbf{i} + z\mathbf{k}) \times \left(\frac{3P}{25}\right)(2\mathbf{i} - 20\mathbf{j} - \mathbf{k})$$

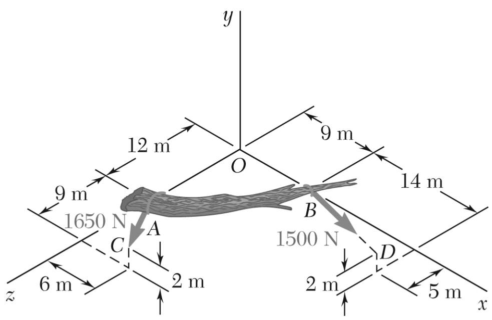
$$= \left(\frac{3P}{25}\right)[20z\mathbf{i} + (x + 2z)\mathbf{j} - 20x\mathbf{k}]$$

From **i**: $8(-403)\dfrac{Pa}{675} = 20z\left(\dfrac{3P}{25}\right)$ $z = -1.99012a$

From **k**: $8(-406)\dfrac{Pa}{675} = -20x\left(\dfrac{3P}{25}\right)$ $x = 2.0049a$

The axis of the wrench intersects the *xz*-plane at

$x = 2.00a, \ z = -1.990a$ ◀



PROBLEM 3.140*

Two ropes attached at *A* and *B* are used to move the trunk of a fallen tree. Replace the forces exerted by the ropes with an equivalent wrench and determine (*a*) the resultant force **R**, (*b*) the pitch of the wrench, (*c*) the point where the axis of the wrench intersects the *yz*-plane.

SOLUTION

(*a*) First replace the given forces with an equivalent force-couple system $\left(\mathbf{R},\ \mathbf{M}_O^R\right)$ at the origin.

We have

$$d_{AC} = \sqrt{(6)^2 + (2)^2 + (9)^2} = 11 \text{ m}$$

$$d_{BD} = \sqrt{(14)^2 + (2)^2 + (5)^2} = 15 \text{ m}$$

Then

$$T_{AC} = \frac{1650 \text{ N}}{11} = (6\mathbf{i} + 2\mathbf{j} + 9\mathbf{k})$$

$$= (900 \text{ N})\mathbf{i} + (300 \text{ N})\mathbf{j} + (1350 \text{ N})\mathbf{k}$$

and

$$T_{BD} = \frac{1500 \text{ N}}{15} = (14\mathbf{i} + 2\mathbf{j} + 5\mathbf{k})$$

$$= (1400 \text{ N})\mathbf{i} + (200 \text{ N})\mathbf{j} + (500 \text{ N})\mathbf{k}$$

Equivalence then requires

$$\Sigma\mathbf{F}: \quad \mathbf{R} = \mathbf{T}_{AC} + \mathbf{T}_{BD}$$

$$= (900\mathbf{i} + 300\mathbf{j} + 1350\mathbf{k})$$

$$+ (1400\mathbf{i} + 200\mathbf{j} + 500\mathbf{k})$$

$$= (2300 \text{ N})\mathbf{i} + (500 \text{ N})\mathbf{j} + (1850 \text{ N})\mathbf{k}$$

$$\Sigma\mathbf{M}_O: \quad \mathbf{M}_O^R = \mathbf{r}_A \times \mathbf{T}_{AC} + \mathbf{r}_B \times \mathbf{T}_{BD}$$

$$= (12 \text{ m})\mathbf{k} \times [(900 \text{ N})\mathbf{i} + (300 \text{ N})\mathbf{j} + (1350 \text{ N})\mathbf{k}]$$

$$+ (9 \text{ m})\mathbf{i} \times [(1400 \text{ N})\mathbf{i} + (200 \text{ N})\mathbf{j} + (500 \text{ N})\mathbf{k}]$$

$$= -(3600)\mathbf{i} + (10{,}800 - 4500)\mathbf{j} + (1800)\mathbf{k}$$

$$= -(3600 \text{ N}\cdot\text{m})\mathbf{i} + (6300 \text{ N}\cdot\text{m})\mathbf{j} + (1800 \text{ N}\cdot\text{m})\mathbf{k}$$

The components of the wrench are $(\mathbf{R},\ \mathbf{M}_1)$, where

$$\mathbf{R} = (2300 \text{ N})\mathbf{i} + (500 \text{ N})\mathbf{j} + (1850 \text{ N})\mathbf{k}$$ ◀

PROBLEM 3.140* (Continued)

(*b*) We have

$$R = 100\sqrt{(23)^2 + (5)^2 + (18.5)^2} = 2993.7 \text{ N}$$

Let

$$\boldsymbol{\lambda}_{\text{axis}} = \frac{\mathbf{R}}{R} = \frac{1}{29.937}(23\mathbf{i} + 5\mathbf{j} + 18.5\mathbf{k})$$

Then

$$M_1 = \boldsymbol{\lambda}_{\text{axis}} \cdot \mathbf{M}_O^R$$
$$= \frac{1}{29.937}(23\mathbf{i} + 5\mathbf{j} + 18.5\mathbf{k}) \cdot (-3600\mathbf{i} + 6300\mathbf{j} + 1800\mathbf{k})$$
$$= \frac{1}{0.29937}[(23)(-36) + (5)(63) + (18.5)(18)]$$
$$= -601.26 \text{ N} \cdot \text{m}$$

Finally,

$$P = \frac{M_1}{R} = \frac{-601.26 \text{ N} \cdot \text{m}}{2993.7 \text{ N}}$$

or $P = -0.201$ m ◀

(*c*) We have

$$\mathbf{M}_1 = M_1 \boldsymbol{\lambda}_{\text{axis}}$$
$$= (-601.26 \text{ N} \cdot \text{m}) \times \frac{1}{29.937}(23\mathbf{i} + 5\mathbf{j} + 18.5\mathbf{k})$$

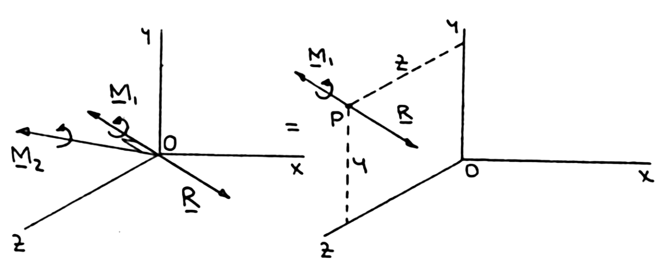
or

$$\mathbf{M}_1 = -(461.93 \text{ N} \cdot \text{m})\mathbf{i} - (100.421 \text{ N} \cdot \text{m})\mathbf{j} - (371.56 \text{ N} \cdot \text{m})\mathbf{k}$$

Now

$$\mathbf{M}_2 = \mathbf{M}_O^R - \mathbf{M}_1$$
$$= (-3600\mathbf{i} + 6300\mathbf{j} + 1800\mathbf{k})$$
$$-(-461.93\mathbf{i} - 100.421\mathbf{j} - 371.56\mathbf{k})$$
$$= -(3138.1 \text{ N} \cdot \text{m})\mathbf{i} + (6400.4 \text{ N} \cdot \text{m})\mathbf{j} + (2171.6 \text{ N} \cdot \text{m})\mathbf{k}$$

For equivalence:



PROBLEM 3.140* (Continued)

Thus, we require $\mathbf{M}_2 = \mathbf{r}_P \times \mathbf{R}$ $\quad \mathbf{r} = (y\mathbf{j} + z\mathbf{k})$

Substituting:

$$-3138.1\mathbf{i} + 6400.4\mathbf{j} + 2171.6\mathbf{k} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & y & z \\ 2300 & 500 & 1850 \end{vmatrix}$$

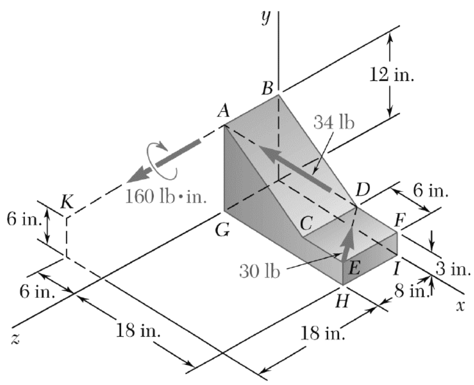
Equating coefficients:

$$\mathbf{j}: \quad 6400.4 = 2300z \quad \text{or} \quad z = 2.78 \text{ m}$$

$$\mathbf{k}: \quad 2171.6 = -2300y \quad \text{or} \quad y = -0.944 \text{ m}$$

The axis of the wrench intersects the yz-plane at $y = -0.944$ m $\quad z = 2.78$ m ◀

PROBLEM 3.141*



Determine whether the force-and-couple system shown can be reduced to a single equivalent force **R**. If it can, determine **R** and the point where the line of action of **R** intersects the *yz*-plane. If it cannot be so reduced, replace the given system with an equivalent wrench and determine its resultant, its pitch, and the point where its axis intersects the *yz*-plane.

SOLUTION

First determine the resultant of the forces at D. We have

$$d_{DA} = \sqrt{(-12)^2 + (9)^2 + (8)^2} = 17 \text{ in.}$$

$$d_{ED} = \sqrt{(-6)^2 + (0)^2 + (-8)^2} = 10 \text{ in.}$$

Then

$$\mathbf{F}_{DA} = \frac{34 \text{ lb}}{17} = (-12\mathbf{i} + 9\mathbf{j} + 8\mathbf{k})$$

$$= -(24 \text{ lb})\mathbf{i} + (18 \text{ lb})\mathbf{j} + (16 \text{ lb})\mathbf{k}$$

and

$$\mathbf{F}_{ED} = \frac{30 \text{ lb}}{10} = (-6\mathbf{i} - 8\mathbf{k})$$

$$= -(18 \text{ lb})\mathbf{i} - (24 \text{ lb})\mathbf{k}$$

Then

$$\Sigma\mathbf{F}: \quad \mathbf{R} = \mathbf{F}_{DA} + \mathbf{F}_{ED}$$

$$= (-24\mathbf{i} + 18\mathbf{j} + 16\mathbf{k} + (-18\mathbf{i} - 24\mathbf{k})$$

$$= -(42 \text{ lb})\mathbf{i} + (18 \text{ lb})\mathbf{j} - (8 \text{ lb})\mathbf{k}$$

For the applied couple

$$d_{AK} = \sqrt{(-6)^2 + (-6)^2 + (18)^2} = 6\sqrt{11} \text{ in.}$$

Then

$$\mathbf{M} = \frac{160 \text{ lb} \cdot \text{in.}}{6\sqrt{11}}(-6\mathbf{i} - 6\mathbf{j} + 18\mathbf{k})$$

$$= \frac{160}{\sqrt{11}}[-(1 \text{ lb} \cdot \text{in.})\mathbf{i} - (1 \text{ lb} \cdot \text{in.})\mathbf{j} + (3 \text{ lb} \cdot \text{in.})\mathbf{k}]$$

To be able to reduce the original forces and couple to a single equivalent force, **R** and **M** must be perpendicular. Thus

$$\mathbf{R} \cdot \mathbf{M} \stackrel{?}{=} 0$$

Substituting

$$(-42\mathbf{i}+18\mathbf{j}-8\mathbf{k})\cdot\frac{160}{\sqrt{11}}(-\mathbf{i}-\mathbf{j}+3\mathbf{k})\overset{?}{=}0$$

or

$$\frac{160}{\sqrt{11}}[(-42)(-1)+(18)(-1)+(-8)(3)]\overset{?}{=}0$$

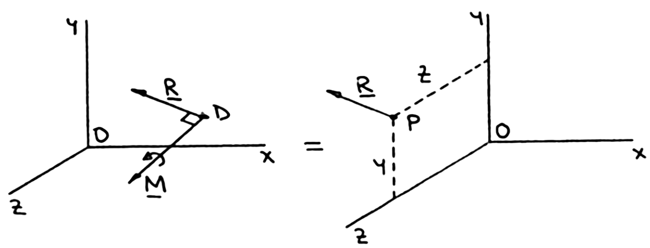
or

$$0\overset{\checkmark}{=}0$$

R and **M** are perpendicular so that the given system can be reduced to the single equivalent force.

$$\mathbf{R}=-(42.0\text{ lb})\mathbf{i}+(18.00\text{ lb})\mathbf{j}-(8.00\text{ lb})\mathbf{k}$$ ◀

Then for equivalence,



Thus, we require $\mathbf{M}=\mathbf{r}_{P/D}\times\mathbf{R}$

where $\mathbf{r}_{P/D}=-(12\text{ in.})\mathbf{i}+[(y-3)\text{in.}]\mathbf{j}+(z\text{ in.})\mathbf{k}$

Substituting:

$$\frac{160}{\sqrt{11}}(-\mathbf{i}-\mathbf{j}+3\mathbf{k})=\begin{vmatrix}\mathbf{i}&\mathbf{j}&\mathbf{k}\\-12&(y-3)&z\\-42&18&-8\end{vmatrix}$$

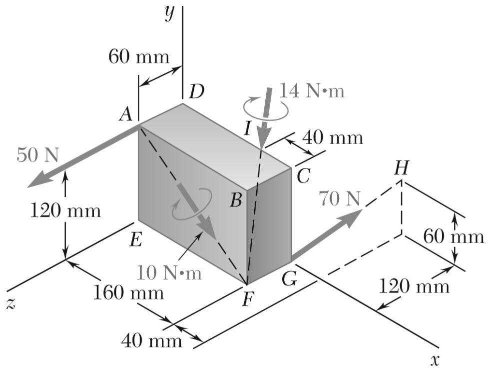
$$=[(y-3)(-8)-(z)(18)]\mathbf{i}+[(z)(-42)-(-12)(-8)]\mathbf{j}+[(-12)(18)-(y-3)(-42)]\mathbf{k}$$

Equating coefficients:

$$\mathbf{j}:\quad -\frac{160}{\sqrt{11}}=-42z-96 \qquad \text{or} \qquad z=-1.137\text{ in.}$$

$$\mathbf{k}:\quad \frac{480}{\sqrt{11}}=-216+42(y-3) \qquad \text{or} \qquad y=11.59\text{ in.}$$

The line of action of **R** intersects the yz-plane at $x=0 \quad y=11.59\text{ in.} \quad z=-1.137\text{ in.}$ ◀



PROBLEM 3.142*

Determine whether the force-and-couple system shown can be reduced to a single equivalent force **R**. If it can, determine **R** and the point where the line of action of **R** intersects the *yz*-plane. If it cannot be so reduced, replace the given system with an equivalent wrench and determine its resultant, its pitch, and the point where its axis intersects the *yz*-plane.

SOLUTION

First, reduce the given force system to a force-couple at the origin.

We have $\Sigma\mathbf{F}: \quad \mathbf{F}_A + \mathbf{F}_G = \mathbf{R}$

$$\mathbf{R} = (50\text{ N})\mathbf{k} + 70\text{ N}\left[\frac{(40\text{ mm})\mathbf{i} + (60\text{ mm})\mathbf{j} - (120\text{ mm})\mathbf{k}}{140\text{ mm}}\right]$$

$$= (20\text{ N})\mathbf{i} + (30\text{ N})\mathbf{j} - (10\text{ N})\mathbf{k}$$

and $R = 37.417\text{ N}$

We have $\Sigma\mathbf{M}_O: \quad \Sigma(\mathbf{r}_O \times \mathbf{F}) + \Sigma\mathbf{M}_C = \mathbf{M}_O^R$

$$\mathbf{M}_O^R = [(0.12\text{ m})\mathbf{j} \times (50\text{ N})\mathbf{k}] + \{(0.16\text{ m})\mathbf{i} \times [(20\text{ N})\mathbf{i} + (30\text{ N})\mathbf{j} - (60\text{ N})\mathbf{k}]\}$$

$$+ (10\text{ N}\cdot\text{m})\left[\frac{(160\text{ mm})\mathbf{i} - (120\text{ mm})\mathbf{j}}{200\text{ mm}}\right]$$

$$+ (14\text{ N}\cdot\text{m})\left[\frac{(40\text{ mm})\mathbf{i} - (120\text{ mm})\mathbf{j} + (60\text{ mm})\mathbf{k}}{140\text{ mm}}\right]$$

$$\mathbf{M}_0^R = (18\text{ N}\cdot\text{m})\mathbf{i} - (8.4\text{ N}\cdot\text{m})\mathbf{j} + (10.8\text{ N}\cdot\text{m})\mathbf{k}$$

To be able to reduce the original forces and couples to a single equivalent force, **R** and **M** must be perpendicular. Thus, $\mathbf{R}\cdot\mathbf{M} = 0$.

Substituting

$$(20\mathbf{i} + 30\mathbf{j} - 10\mathbf{k}) \cdot (18\mathbf{i} - 8.4\mathbf{j} + 10.8\mathbf{k}) \stackrel{?}{=} 0$$

or $$(20)(18) + (30)(-8.4) + (-10)(10.8) \stackrel{?}{=} 0$$

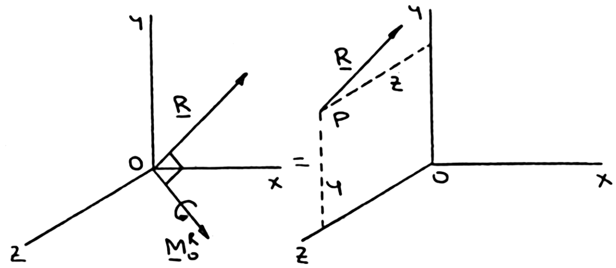
or $$0 \stackrel{\checkmark}{=} 0$$

R and **M** are perpendicular so that the given system can be reduced to the single equivalent force.

$$\mathbf{R} = (20.0\text{ N})\mathbf{i} + (30.0\text{ N})\mathbf{j} - (10.00\text{ N})\mathbf{k} \blacktriangleleft$$

PROBLEM 3.142* (Continued)

Then for equivalence,



Thus, we require $\mathbf{M}_O^R = \mathbf{r}_p \times \mathbf{R} \quad \mathbf{r}_p = y\mathbf{j} + z\mathbf{k}$

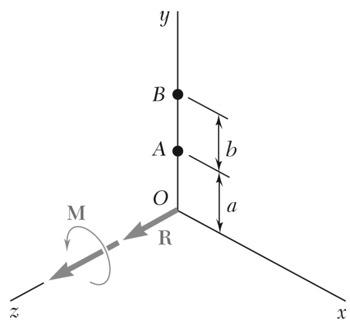
Substituting:

$$18\mathbf{i} - 8.4\mathbf{j} + 10.8\mathbf{k} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & y & z \\ 20 & 30 & -10 \end{vmatrix}$$

Equating coefficients:

$$\mathbf{j}: \quad -8.4 = 20z \quad \text{or} \quad z = -0.42 \text{ m}$$
$$\mathbf{k}: \quad 10.8 = -20y \quad \text{or} \quad y = -0.54 \text{ m}$$

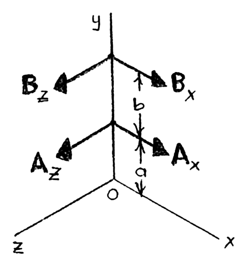
The line of action of **R** intersects the *yz*-plane at $x = 0 \quad y = -0.540 \text{ m} \quad z = -0.420 \text{ m}$ ◀



PROBLEM 3.143*

Replace the wrench shown with an equivalent system consisting of two forces perpendicular to the y-axis and applied respectively at A and B.

SOLUTION



Express the forces at A and B as

$$\mathbf{A} = A_x\mathbf{i} + A_z\mathbf{k}$$
$$\mathbf{B} = B_x\mathbf{i} + B_z\mathbf{k}$$

Then, for equivalence to the given force system,

$$\Sigma F_x: \quad A_x + B_x = 0 \qquad (1)$$
$$\Sigma F_z: \quad A_z + B_z = R \qquad (2)$$
$$\Sigma M_x: \quad A_z(a) + B_z(a+b) = 0 \qquad (3)$$
$$\Sigma M_z: \quad -A_x(a) - B_x(a+b) = M \qquad (4)$$

From Equation (1), $\quad B_x = -A_x$

Substitute into Equation (4):

$$-A_x(a) + A_x(a+b) = M$$
$$A_x = \frac{M}{b} \quad \text{and} \quad B_x = -\frac{M}{b}$$

From Equation (2), $\quad B_z = R - A_z$

and Equation (3), $\quad A_z a + (R - A_z)(a+b) = 0$

$$A_z = R\left(1 + \frac{a}{b}\right)$$

and $\quad B_z = R - R\left(1 + \frac{a}{b}\right)$

$$B_z = -\frac{a}{b}R$$

Then

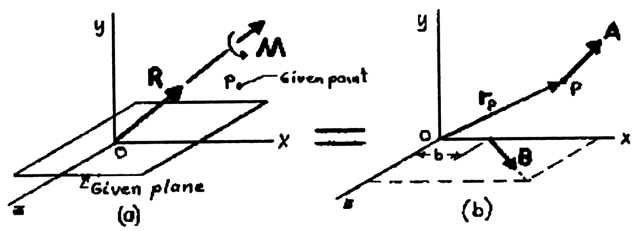
$$\mathbf{A} = \left(\frac{M}{b}\right)\mathbf{i} + R\left(1 + \frac{a}{b}\right)\mathbf{k} \blacktriangleleft$$

$$\mathbf{B} = -\left(\frac{M}{b}\right)\mathbf{i} - \left(\frac{a}{b}R\right)\mathbf{k} \blacktriangleleft$$

PROBLEM 3.144*

Show that, in general, a wrench can be replaced with two forces chosen in such a way that one force passes through a given point while the other force lies in a given plane.

SOLUTION



First, choose a coordinate system so that the *xy*-plane coincides with the given plane. Also, position the coordinate system so that the line of action of the wrench passes through the origin as shown in Figure *a*. Since the orientation of the plane and the components (**R**, **M**) of the wrench are known, it follows that the scalar components of **R** and **M** are known relative to the shown coordinate system.

A force system to be shown as equivalent is illustrated in Figure *b*. Let **A** be the force passing through the given Point *P* and **B** be the force that lies in the given plane. Let *b* be the *x*-axis intercept of **B**.

The known components of the wrench can be expressed as

$$\mathbf{R} = R_x\mathbf{i} + R_y\mathbf{j} + R_z\mathbf{k} \quad \text{and} \quad M = M_x\mathbf{i} + M_y\mathbf{j} + M_z\mathbf{k}$$

while the unknown forces **A** and **B** can be expressed as

$$\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k} \quad \text{and} \quad \mathbf{B} = B_x\mathbf{i} + B_z\mathbf{k}$$

Since the position vector of Point *P* is given, it follows that the scalar components (*x*, *y*, *z*) of the position vector $\mathbf{r}_P$ are also known.

Then, for equivalence of the two systems,

$$\Sigma F_x: \quad R_x = A_x + B_x \qquad (1)$$

$$\Sigma F_y: \quad R_y = A_y \qquad (2)$$

$$\Sigma F_z: \quad R_z = A_z + B_z \qquad (3)$$

$$\Sigma M_x: \quad M_x = yA_z - zA_y \qquad (4)$$

$$\Sigma M_y: \quad M_y = zA_x - xA_z - bB_z \qquad (5)$$

$$\Sigma M_z: \quad M_z = xA_y - yA_x \qquad (6)$$

Based on the above six independent equations for the six unknowns $(A_x, A_y, A_z, B_x, B_z, b)$, there exists a unique solution for **A** and **B**.

From Equation (2), $A_y = R_y$ ◀

PROBLEM 3.144* (Continued)

Equation (6): $$A_x = \left(\frac{1}{y}\right)(xR_y - M_z) \blacktriangleleft$$

Equation (1): $$B_x = R_x - \left(\frac{1}{y}\right)(xR_y - M_z) \blacktriangleleft$$

Equation (4): $$A_z = \left(\frac{1}{y}\right)(M_x + zR_y) \blacktriangleleft$$

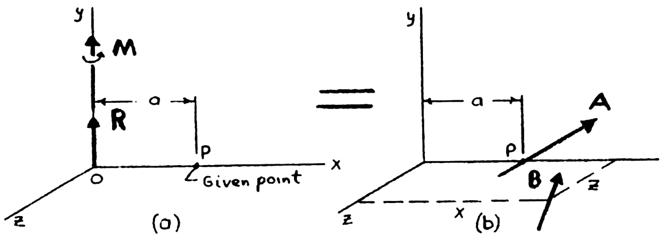
Equation (3): $$B_z = R_z - \left(\frac{1}{y}\right)(M_x + zR_y) \blacktriangleleft$$

Equation (5): $$b = \frac{(xM_x + yM_y + zM_z)}{(M_x - yR_z + zR_y)} \blacktriangleleft$$

PROBLEM 3.145*

Show that a wrench can be replaced with two perpendicular forces, one of which is applied at a given point.

SOLUTION



First, observe that it is always possible to construct a line perpendicular to a given line so that the constructed line also passes through a given point. Thus, it is possible to align one of the coordinate axes of a rectangular coordinate system with the axis of the wrench while one of the other axes passes through the given point.

See Figures *a* and *b*.

We have $\mathbf{R} = R\mathbf{j}$ and $\mathbf{M} = M\mathbf{j}$ and are known.

The unknown forces **A** and **B** can be expressed as

$$\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k} \quad \text{and} \quad \mathbf{B} = B_x\mathbf{i} + B_y\mathbf{j} + B_z\mathbf{k}$$

The distance *a* is known. It is assumed that force **B** intersects the *xz*-plane at (*x*, 0, *z*). Then for equivalence,

$$\Sigma F_x: \quad 0 = A_x + B_x \quad (1)$$

$$\Sigma F_y: \quad R = A_y + B_y \quad (2)$$

$$\Sigma F_z: \quad 0 = A_z + B_z \quad (3)$$

$$\Sigma M_x: \quad 0 = -zB_y \quad (4)$$

$$\Sigma M_y: \quad M = -aA_z - xB_z + zB_x \quad (5)$$

$$\Sigma M_z: \quad 0 = aA_y + xB_y \quad (6)$$

Since **A** and **B** are made perpendicular,

$$\mathbf{A} \cdot \mathbf{B} = 0 \quad \text{or} \quad A_xB_x + A_yB_y + A_zB_z = 0 \quad (7)$$

There are eight unknowns: $A_x, A_y, A_z, B_x, B_y, B_z, x, z$

But only seven independent equations. Therefore, *there exists an infinite number of solutions.*

Next, consider Equation (4): $0 = -zB_y$

If $B_y = 0$, Equation (7) becomes $A_xB_x + A_zB_z = 0$

Using Equations (1) and (3), this equation becomes $A_x^2 + A_z^2 = 0$

Since the components of **A** must be real, a nontrivial solution is not possible. Thus, it is required that $B_y \neq 0$, so that from Equation (4), $z = 0$.

To obtain one possible solution, arbitrarily let $A_x = 0$.

(*Note:* Setting A_y, A_z, or B_z equal to zero results in unacceptable solutions.)

The defining equations then become

$$0 = B_x \quad (1)'$$

$$R = A_y + B_y \quad (2)$$

$$0 = A_z + B_z \quad (3)$$

$$M = -aA_z - xB_z \quad (5)'$$

$$0 = aA_y + xB_y \quad (6)$$

$$A_yB_y + A_zB_z = 0 \quad (7)'$$

Then Equation (2) can be written $A_y = R - B_y$

Equation (3) can be written $B_z = -A_z$

Equation (6) can be written $x = -\dfrac{aA_y}{B_y}$

Substituting into Equation (5)′,

$$M = -aA_z - \left(-a\frac{R - B_y}{B_y}\right)(-A_z)$$

or

$$A_z = -\frac{M}{aR}B_y \quad (8)$$

Substituting into Equation (7)′,

$$(R - B_y)B_y + \left(-\frac{M}{aR}B_y\right)\left(\frac{M}{aR}B_y\right) = 0$$

or

$$B_y = \frac{a^2R^3}{a^2R^2 + M^2}$$

Then from Equations (2), (8), and (3),

$$A_y = R - \frac{a^2R^2}{a^2R^2 + M^2} = \frac{RM^2}{a^2R^2 + M^2}$$

$$A_z = -\frac{M}{aR}\left(\frac{a^2R^3}{a^2R^2 + M^2}\right) = -\frac{aR^2M}{a^2R^2 + M^2}$$

$$B_z = \frac{aR^2M}{a^2R^2 + M^2}$$

In summary,

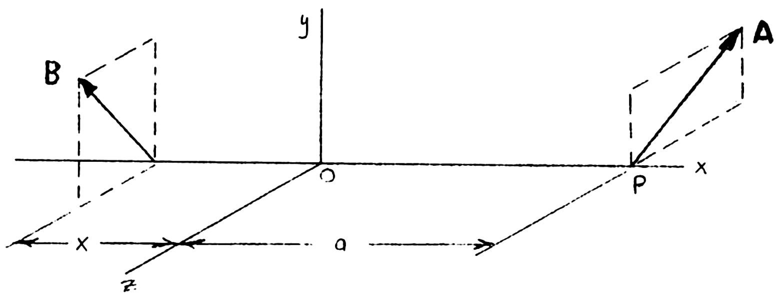
$$\mathbf{A} = \frac{RM}{a^2R^2 + M^2}(M\mathbf{j} - aR\mathbf{k}) \quad \blacktriangleleft$$

$$\mathbf{B} = \frac{aR^2}{a^2R^2 + M^2}(aR\mathbf{j} + M\mathbf{k}) \quad \blacktriangleleft$$

Which shows that it is possible to replace a wrench with two perpendicular forces, one of which is applied at a given point.

Lastly, if $R > 0$ and $M > 0$, it follows from the equations found for **A** and **B** that $A_y > 0$ and $B_y > 0$.

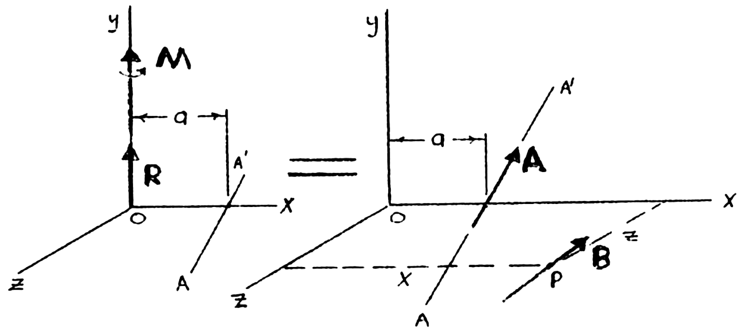
From Equation (6), $x < 0$ (assuming $a > 0$). Then, as a consequence of letting $A_x = 0$, force **A** lies in a plane parallel to the *yz*-plane and to the right of the origin, while force **B** lies in a plane parallel to the *yz*-plane but to the left to the origin, as shown in the figure below.



PROBLEM 3.146*

Show that a wrench can be replaced with two forces, one of which has a prescribed line of action.

SOLUTION



First, choose a rectangular coordinate system where one axis coincides with the axis of the wrench and another axis intersects the prescribed line of action (AA'). Note that it has been assumed that the line of action of force **B** intersects the *xz*-plane at Point $P(x, 0, z)$. Denoting the known direction of line AA' by

$$\lambda_A = \lambda_x \mathbf{i} + \lambda_y \mathbf{j} + \lambda_z \mathbf{k}$$

it follows that force **A** can be expressed as

$$\mathbf{A} = A\lambda_A = A(\lambda_x \mathbf{i} + \lambda_y \mathbf{j} + \lambda_z \mathbf{k})$$

Force **B** can be expressed as

$$\mathbf{B} = B_x \mathbf{i} + B_y \mathbf{j} + B_z \mathbf{k}$$

Next, observe that since the axis of the wrench and the prescribed line of action AA' are known, it follows that the distance a can be determined. In the following solution, it is assumed that a is known.

Then for equivalence,

$$\Sigma F_x: \quad 0 = A\lambda_x + B_x \tag{1}$$

$$\Sigma F_y: \quad R = A\lambda_y + B_y \tag{2}$$

$$\Sigma F_z: \quad 0 = A\lambda_z + B_z \tag{3}$$

$$\Sigma M_x: \quad 0 = -zB_y \tag{4}$$

$$\Sigma M_y: \quad M = -aA\lambda_z + zB_x - xB_z \tag{5}$$

$$\Sigma M_x: \quad 0 = -aA\lambda_y + xB_y \tag{6}$$

Since there are six unknowns (A, B_x, B_y, B_z, x, z) and six independent equations, it will be possible to obtain a solution.

PROBLEM 3.146* (Continued)

Case 1: Let $z = 0$ to satisfy Equation (4).

Now Equation (2): $A\lambda_y = R - B_y$

Equation (3): $B_z = -A\lambda_z$

Equation (6): $x = -\dfrac{aA\lambda_y}{B_y} = -\left(\dfrac{a}{B_y}\right)(R - B_y)$

Substitution into Equation (5):

$$M = -aA\lambda_z - \left[-\left(\frac{a}{B_y}\right)(R - B_y)(-A\lambda_z)\right]$$

$$A = -\frac{1}{\lambda_z}\left(\frac{M}{aR}\right)B_y$$

Substitution into Equation (2):

$$R = -\frac{1}{\lambda_z}\left(\frac{M}{aR}\right)B_y\lambda_y + B_y$$

$$B_y = \frac{\lambda_z aR^2}{\lambda_z aR - \lambda_y M}$$

Then $$A = -\frac{MR}{\lambda_z aR - \lambda_y M} = \frac{R}{\lambda_y - \frac{aR}{M}\lambda_z}$$

$$B_x = -A\lambda_x = \frac{\lambda_x MR}{\lambda_z aR - \lambda_y M}$$

$$B_z = -A\lambda_z = \frac{\lambda_z MR}{\lambda_z aR - \lambda_y M}$$

In summary,

$$\mathbf{A} = \frac{P}{\lambda_y - \frac{aR}{M}\lambda_z}\lambda_A \blacktriangleleft$$

$$\mathbf{B} = \frac{R}{\lambda_z aR - \lambda_y M}(\lambda_x M\,\mathbf{i} + \lambda_z aR\mathbf{j} + \lambda_z M\,\mathbf{k}) \blacktriangleleft$$

and $$x = a\left(1 - \frac{R}{B_y}\right)$$

$$= a\left[1 - R\left(\frac{\lambda_z aR - \lambda_y M}{\lambda_z aR^2}\right)\right]$$

or $$x = \frac{\lambda_y M}{\lambda_z R} \blacktriangleleft$$

Note that for this case, the lines of action of both **A** and **B** intersect the *x*-axis.

PROBLEM 3.146* (Continued)

Case 2: Let $B_y = 0$ to satisfy Equation (4).

Now Equation (2): $$A = \frac{R}{\lambda_y}$$

Equation (1): $$B_x = -R\left(\frac{\lambda_x}{\lambda_y}\right)$$

Equation (3): $$B_z = -R\left(\frac{\lambda_z}{\lambda_y}\right)$$

Equation (6): $aA\lambda_y = 0$ which requires $a = 0$

Substitution into Equation (5):

$$M = z\left[-R\left(\frac{\lambda_x}{\lambda_y}\right)\right] - x\left[-R\left(\frac{\lambda_z}{\lambda_y}\right)\right] \quad \text{or} \quad \lambda_z x - \lambda_x z = \left(\frac{M}{R}\right)\lambda_y$$

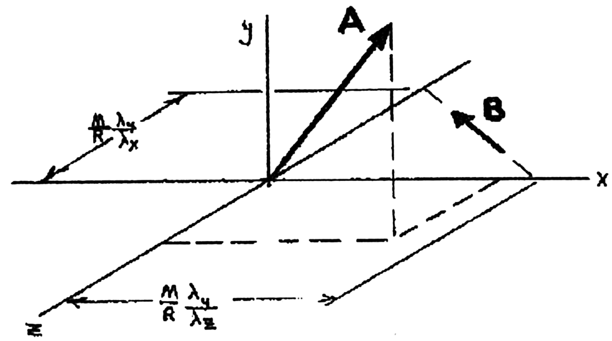
This last expression is the equation for the line of action of force **B**.

In summary,

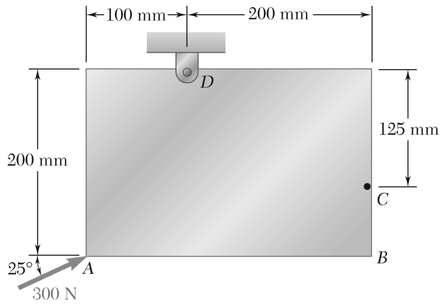
$$\mathbf{A} = \left(\frac{R}{\lambda_y}\right)\lambda_A$$ ◀

$$\mathbf{B} = \left(\frac{R}{\lambda_y}\right)(-\lambda_x\mathbf{i} - \lambda_x\mathbf{k})$$ ◀

Assuming that $\lambda_x, \lambda_y, \lambda_z > 0$, the equivalent force system is as shown below.



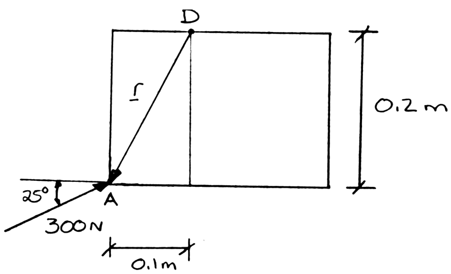
Note that the component of **A** in the *xz*-plane is parallel to **B**.

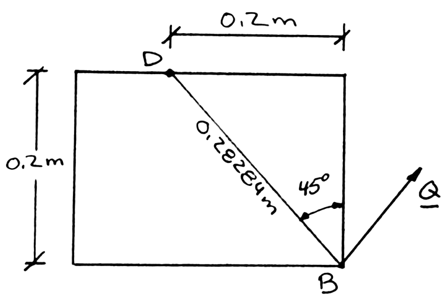


PROBLEM 3.147

A 300-N force is applied at A as shown. Determine (a) the moment of the 300-N force about D, (b) the smallest force applied at B that creates the same moment about D.

SOLUTION





(a)

$$F_x = (300\text{ N})\cos 25° = 271.89\text{ N}$$

$$F_y = (300\text{ N})\sin 25° = 126.785\text{ N}$$

$$\mathbf{F} = (271.89\text{ N})\mathbf{i} + (126.785\text{ N})\mathbf{j}$$

$$\mathbf{r} = \overrightarrow{DA} = -(0.1\text{ m})\mathbf{i} - (0.2\text{ m})\mathbf{j}$$

$$\mathbf{M}_D = \mathbf{r} \times \mathbf{F}$$

$$\mathbf{M}_D = [-(0.1\text{ m})\mathbf{i} - (0.2\text{ m})\mathbf{j}] \times [(271.89\text{ N})\mathbf{i} + (126.785\text{ N})\mathbf{j}]$$

$$= -(12.6785\text{ N}\cdot\text{m})\mathbf{k} + (54.378\text{ N}\cdot\text{m})\mathbf{k}$$

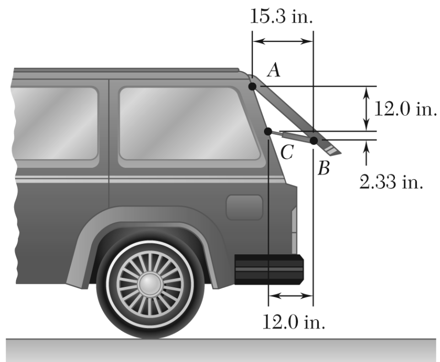
$$= (41.700\text{ N}\cdot\text{m})\mathbf{k}$$

$$\mathbf{M}_D = 41.7\text{ N}\cdot\text{m} \circlearrowleft \blacktriangleleft$$

(b) The smallest force Q at B must be perpendicular to $\overrightarrow{DB}$ at 45° ⦣

$$\mathbf{M}_D = Q(\overrightarrow{DB})$$

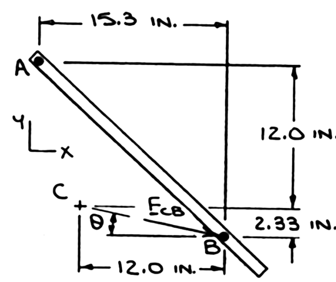
$$41.700\text{ N}\cdot\text{m} = Q(0.28284\text{ m}) \qquad Q = 147.4\text{ N} \measuredangle 45.0° \blacktriangleleft$$



PROBLEM 3.148

The tailgate of a car is supported by the hydraulic lift BC. If the lift exerts a 125-lb force directed along its centerline on the ball and socket at B, determine the moment of the force about A.

SOLUTION



First note

$$d_{CB} = \sqrt{(12.0\text{ in.})^2 + (2.33\text{ in.})^2}$$
$$= 12.2241\text{ in.}$$

Then

$$\cos\theta = \frac{12.0\text{ in.}}{12.2241\text{ in.}}$$

$$\sin\theta = \frac{2.33\text{ in.}}{12.2241\text{ in.}}$$

and

$$\mathbf{F}_{CB} = F_{CB}\cos\theta\,\mathbf{i} - F_{CB}\sin\theta\,\mathbf{j}$$
$$= \frac{125\text{ lb}}{12.2241\text{ in.}}[(12.0\text{ in.})\,\mathbf{i} - (2.33\text{ in.})\,\mathbf{j}]$$

Now

$$\mathbf{M}_A = \mathbf{r}_{B/A} \times \mathbf{F}_{CB}$$

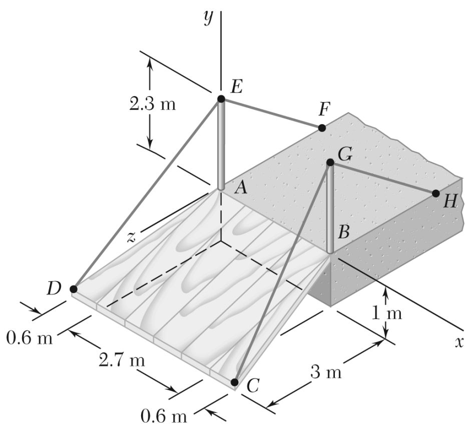
where

$$\mathbf{r}_{B/A} = (15.3\text{ in.})\,\mathbf{i} - (12.0\text{ in.} + 2.33\text{ in.})\,\mathbf{j}$$
$$= (15.3\text{ in.})\,\mathbf{i} - (14.33\text{ in.})\,\mathbf{j}$$

Then

$$\mathbf{M}_A = [(15.3\text{ in.})\mathbf{i} - (14.33\text{ in.})\mathbf{j}] \times \frac{125\text{ lb}}{12.2241\text{ in.}}(12.0\mathbf{i} - 2.33\mathbf{j})$$
$$= (1393.87\text{ lb}\cdot\text{in.})\mathbf{k}$$
$$= (116.156\text{ lb}\cdot\text{ft})\mathbf{k}$$

or $\mathbf{M}_A = 116.2\text{ lb}\cdot\text{ft}$ ↺ ◀



PROBLEM 3.149

The ramp $ABCD$ is supported by cables at corners C and D. The tension in each of the cables is 810 N. Determine the moment about A of the force exerted by (*a*) the cable at D, (*b*) the cable at C.

SOLUTION

(*a*) We have $\quad \mathbf{M}_A = \mathbf{r}_{E/A} \times \mathbf{T}_{DE}$

where

$$\mathbf{r}_{E/A} = (2.3 \text{ m})\mathbf{j}$$

$$\mathbf{T}_{DE} = \boldsymbol{\lambda}_{DE} T_{DE}$$

$$= \frac{(0.6 \text{ m})\mathbf{i} + (3.3 \text{ m})\mathbf{j} - (3 \text{ m})\mathbf{k}}{\sqrt{(0.6)^2 + (3.3)^2 + (3)^2}\text{ m}}(810 \text{ N})$$

$$= (108 \text{ N})\mathbf{i} + (594 \text{ N})\mathbf{j} - (540 \text{ N})\mathbf{k}$$

$$\mathbf{M}_A = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 2.3 & 0 \\ 108 & 594 & -540 \end{vmatrix} \text{N} \cdot \text{m}$$

$$= -(1242 \text{ N} \cdot \text{m})\mathbf{i} - (248.4 \text{ N} \cdot \text{m})\mathbf{k}$$

or $\mathbf{M}_A = -(1242 \text{ N} \cdot \text{m})\mathbf{i} - (248 \text{ N} \cdot \text{m})\mathbf{k}$ ◀

(*b*) We have $\quad \mathbf{M}_A = \mathbf{r}_{G/A} \times \mathbf{T}_{CG}$

where

$$\mathbf{r}_{G/A} = (2.7 \text{ m})\mathbf{i} + (2.3 \text{ m})\mathbf{j}$$

$$\mathbf{T}_{CG} = \boldsymbol{\lambda}_{CG} T_{CG}$$

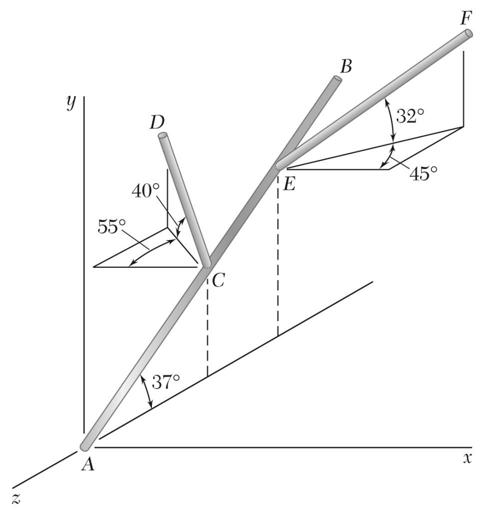
$$= \frac{-(.6 \text{ m})\mathbf{i} + (3.3 \text{ m})\mathbf{j} - (3 \text{ m})\mathbf{k}}{\sqrt{(.6)^2 + (3.3)^2 + (3)^2}\text{ m}}(810 \text{ N})$$

$$= -(108 \text{ N})\mathbf{i} + (594 \text{ N})\mathbf{j} - (540 \text{ N})\mathbf{k}$$

$$\mathbf{M}_A = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2.7 & 2.3 & 0 \\ -108 & 594 & -540 \end{vmatrix} \text{N} \cdot \text{m}$$

$$= -(1242 \text{ N} \cdot \text{m})\mathbf{i} + (1458 \text{ N} \cdot \text{m})\mathbf{j} + (1852 \text{ N} \cdot \text{m})\mathbf{k}$$

or $\mathbf{M}_A = -(1242 \text{ N} \cdot \text{m})\mathbf{i} + (1458 \text{ N} \cdot \text{m})\mathbf{j} + (1852 \text{ N} \cdot \text{m})\mathbf{k}$ ◀



PROBLEM 3.150

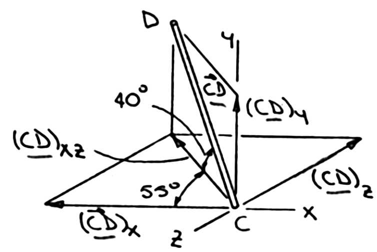
Section *AB* of a pipeline lies in the *yz*-plane and forms an angle of 37° with the *z*-axis. Branch lines *CD* and *EF* join *AB* as shown. Determine the angle formed by pipes *AB* and *CD*.

SOLUTION

First note

$$\overrightarrow{AB} = AB(\sin 37°\mathbf{j} - \cos 37°\mathbf{k})$$

$$\mathbf{CD} = CD(-\cos 40°\cos 55°\mathbf{j} + \sin 40°\mathbf{j} - \cos 40°\sin 55°\mathbf{k})$$



Now

$$\overrightarrow{AB} \cdot \overrightarrow{CD} = (AB)(CD)\cos\theta$$

or

$$AB(\sin 37°\mathbf{j} - \cos 37°\mathbf{k}) \cdot CD(-\cos 40°\cos 55°\mathbf{i} + \sin 40°\mathbf{j} - \cos 40°\sin 55°\mathbf{k})$$

$$= (AB)(CD)\cos\theta$$

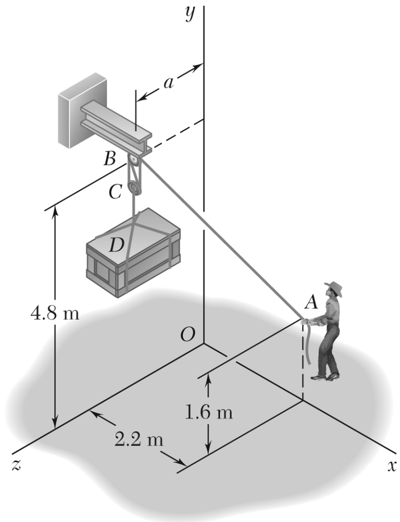
or

$$\cos\theta = (\sin 37°)(\sin 40°) + (-\cos 37°)(-\cos 40°\sin 55°)$$

$$= 0.88799$$

or $\theta = 27.4°$ ◀

PROBLEM 3.151



To lift a heavy crate, a man uses a block and tackle attached to the bottom of an I-beam at hook B. Knowing that the moments about the y and the z axes of the force exerted at B by portion AB of the rope are, respectively, 120 N · m and –460 N · m, determine the distance a.

SOLUTION

First note $\overrightarrow{BA} = (2.2\text{ m})\mathbf{i} - (3.2\text{ m})\mathbf{j} - (a\text{ m})\mathbf{k}$

Now $\mathbf{M}_D = \mathbf{r}_{A/D} \times \mathbf{T}_{BA}$

where $\mathbf{r}_{A/D} = (2.2\text{ m})\mathbf{i} + (1.6\text{ m})\mathbf{j}$

$$\mathbf{T}_{BA} = \frac{T_{BA}}{d_{BA}}(2.2\mathbf{i} - 3.2\mathbf{j} - a\mathbf{k})\ (\text{N})$$

Then
$$\mathbf{M}_D = \frac{T_{BA}}{d_{BA}}\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2.2 & 1.6 & 0 \\ 2.2 & -3.2 & -a \end{vmatrix}$$

$$= \frac{T_{BA}}{d_{BA}}\{-1.6a\,\mathbf{i} + 2.2a\,\mathbf{j} + [(2.2)(-3.2) - (1.6)(2.2)]\mathbf{k}\}$$

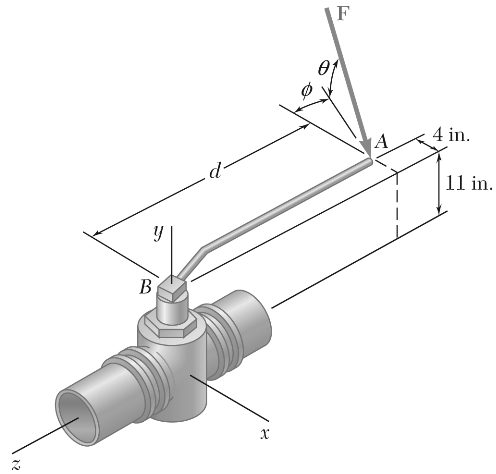
Thus
$$M_y = 2.2\frac{T_{BA}}{d_{BA}}a \qquad (\text{N}\cdot\text{m})$$

$$M_z = -10.56\frac{T_{BA}}{d_{BA}} \qquad (\text{N}\cdot\text{m})$$

Then forming the ratio $\dfrac{M_y}{M_z}$

$$\frac{120\text{ N}\cdot\text{m}}{-460\text{ N}\cdot\text{m}} = \frac{2.2\frac{T_{BA}}{d_{BA}}\,(\text{N}\cdot\text{m})}{-10.56\frac{T_{BA}}{d_{BA}}\,(\text{N}\cdot\text{m})}$$

or $a = 1.252\text{ m}$ ◀



PROBLEM 3.152

To loosen a frozen valve, a force **F** of magnitude 70 lb is applied to the handle of the valve. Knowing that $\theta = 25°$, $M_x = -61\ \text{lb}\cdot\text{ft}$, and $M_z = -43\ \text{lb}\cdot\text{ft}$, determine ϕ and d.

SOLUTION

We have $\Sigma \mathbf{M}_O: \quad \mathbf{r}_{A/O} \times \mathbf{F} = \mathbf{M}_O$

where $\mathbf{r}_{A/O} = -(4\text{ in.})\mathbf{i} + (11\text{ in.})\mathbf{j} - (d)\mathbf{k}$

$$\mathbf{F} = F(\cos\theta\cos\phi\mathbf{i} - \sin\theta\mathbf{j} + \cos\theta\sin\phi\mathbf{k})$$

For $F = 70\text{ lb}, \quad \theta = 25°$

$$\mathbf{F} = (70\text{ lb})[(0.90631\cos\phi)\mathbf{i} - 0.42262\mathbf{j} + (0.90631\sin\phi)\mathbf{k}]$$

$$\mathbf{M}_O = (70\text{ lb})\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -4 & 11 & -d \\ -0.90631\cos\phi & -0.42262 & 0.90631\sin\phi \end{vmatrix}\text{ in.}$$

$$= (70\text{ lb})[(9.9694\sin\phi - 0.42262d)\mathbf{i} + (-0.90631d\cos\phi + 3.6252\sin\phi)\mathbf{j} + (1.69048 - 9.9694\cos\phi)\mathbf{k}]\text{ in.}$$

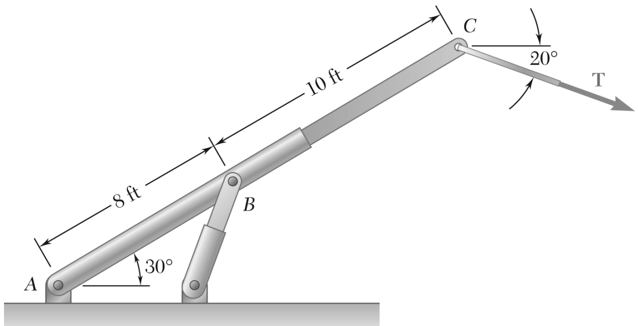
and $M_x = (70\text{ lb})(9.9694\sin\phi - 0.42262d)\text{ in.} = -(61\text{ lb}\cdot\text{ft})(12\text{ in./ft})$ (1)

$M_y = (70\text{ lb})(-0.90631d\cos\phi + 3.6252\sin\phi)\text{ in.}$ (2)

$M_z = (70\text{ lb})(1.69048 - 9.9694\cos\phi)\text{ in.} = -43\text{ lb}\cdot\text{ft}(12\text{ in./ft})$ (3)

From Equation (3): $\phi = \cos^{-1}\left(\dfrac{634.33}{697.86}\right) = 24.636°$ or $\phi = 24.6°$ ◀

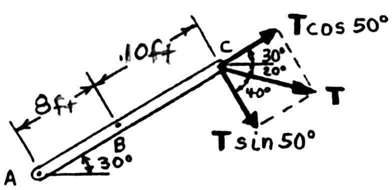
From Equation (1): $d = \left(\dfrac{1022.90}{29.583}\right) = 34.577\text{ in.}$ or $d = 34.6\text{ in.}$ ◀



PROBLEM 3.153

The tension in the cable attached to the end C of an adjustable boom ABC is 560 lb. Replace the force exerted by the cable at C with an equivalent force-couple system (*a*) at A, (*b*) at B.

SOLUTION



(*a*) Based on ΣF: $F_A = T = 560$ lb

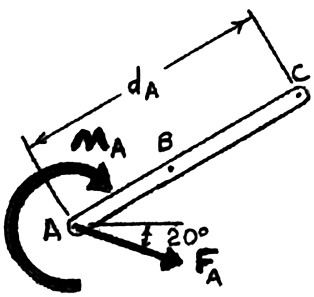
or $\mathbf{F}_A = 560$ lb $\measuredangle$ 20.0° ◀

ΣM_A: $M_A = (T \sin 50°)(d_A)$

$= (560 \text{ lb}) \sin 50° (18 \text{ ft})$

$= 7721.7 \text{ lb} \cdot \text{ft}$

or $\mathbf{M}_A = 7720 \text{ lb} \cdot \text{ft}$ ↻ ◀



(*b*) Based on ΣF: $F_B = T = 560$ lb

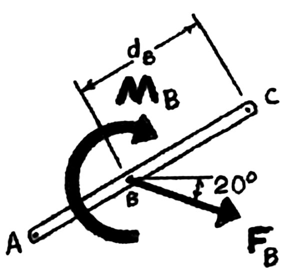
or $\mathbf{F}_B = 560$ lb $\measuredangle$ 20.0° ◀

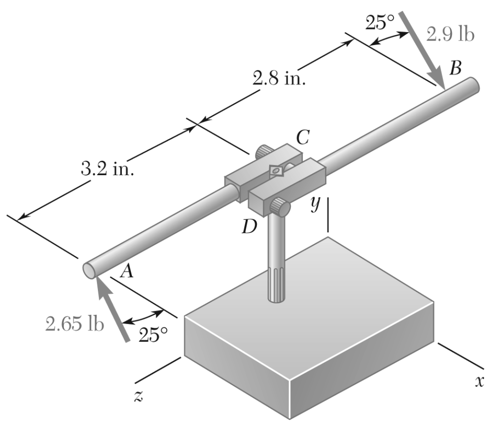
ΣM_B: $M_B = (T \sin 50°)(d_B)$

$= (560 \text{ lb}) \sin 50° (10 \text{ ft})$

$= 4289.8 \text{ lb} \cdot \text{ft}$

or $\mathbf{M}_B = 4290 \text{ lb} \cdot \text{ft}$ ↻ ◀





PROBLEM 3.154

While tapping a hole, a machinist applies the horizontal forces shown to the handle of the tap wrench. Show that these forces are equivalent to a single force, and specify, if possible, the point of application of the single force on the handle.

SOLUTION

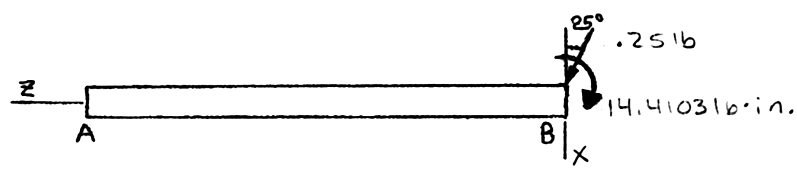
Since the forces at *A* and *B* are parallel, the force at *B* can be replaced with the sum of two forces with one of the forces equal in magnitude to the force at *A* except with an opposite sense, resulting in a force-couple.

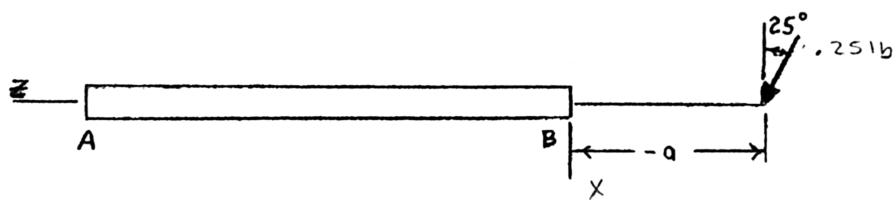
We have $F_B = 2.9 \text{ lb} - 2.65 \text{ lb} = 0.25 \text{ lb}$, where the 2.65-lb force is part of the couple. Combining the two parallel forces,

$$M_{\text{couple}} = (2.65 \text{ lb})[(3.2 \text{ in.} + 2.8 \text{ in.})\cos 25°]$$
$$= 14.4103 \text{ lb} \cdot \text{in.}$$

and

$$\mathbf{M}_{\text{couple}} = 14.4103 \text{ lb} \cdot \text{in.}$$





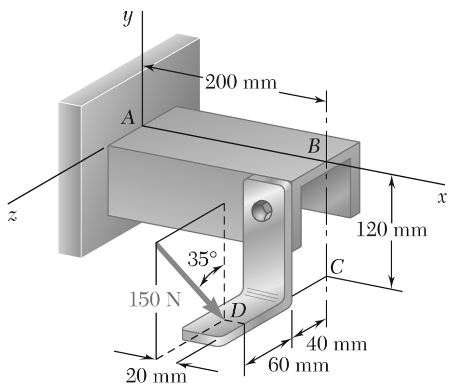
A single equivalent force will be located in the negative *z* direction.

Based on $\quad \Sigma M_B: \quad -14.4103 \text{ lb} \cdot \text{in.} = [(0.25 \text{ lb})\cos 25°](a)$

$$a = 63.600 \text{ in.}$$

$$\mathbf{F}' = (0.25 \text{ lb})(\cos 25°\mathbf{i} + \sin 25°\mathbf{k})$$

$\mathbf{F}' = (0.227 \text{ lb})\mathbf{i} + (0.1057 \text{ lb})\mathbf{k}$ and is applied on an extension of handle *BD* at a distance of 63.6 in. to the right of *B*. ◀



PROBLEM 3.155

Replace the 150-N force with an equivalent force-couple system at A.

SOLUTION

Equivalence requires $\Sigma\mathbf{F}$: $\mathbf{F} = (150\text{ N})(-\cos 35°\mathbf{j} - \sin 35°\mathbf{k})$

$$= -(122.873\text{ N})\mathbf{j} - (86.036\text{ N})\mathbf{k}$$

$$\Sigma\mathbf{M}_A: \quad \mathbf{M} = \mathbf{r}_{D/A} \times \mathbf{F}$$

where $\mathbf{r}_{D/A} = (0.18\text{ m})\mathbf{i} - (0.12\text{ m})\mathbf{j} + (0.1\text{ m})\mathbf{k}$

Then

$$\mathbf{M} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.18 & -0.12 & 0.1 \\ 0 & -122.873 & -86.036 \end{vmatrix} \text{N}\cdot\text{m}$$

$$= [(-0.12)(-86.036) - (0.1)(-122.873)]\mathbf{i}$$
$$+ [-(0.18)(-86.036)]\mathbf{j}$$
$$+ [(0.18)(-122.873)]\mathbf{k}$$
$$= (22.6\text{ N}\cdot\text{m})\mathbf{i} + (15.49\text{ N}\cdot\text{m})\mathbf{j} - (22.1\text{ N}\cdot\text{m})\mathbf{k}$$

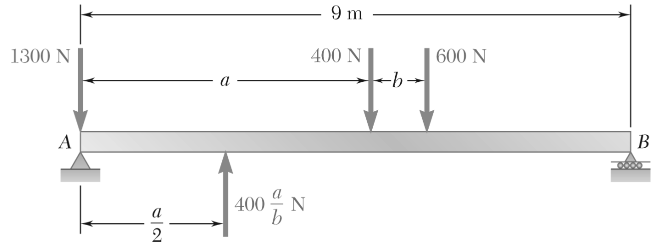
The equivalent force-couple system at A is

$$\mathbf{F} = -(122.9\text{ N})\mathbf{j} - (86.0\text{ N})\mathbf{k} \blacktriangleleft$$

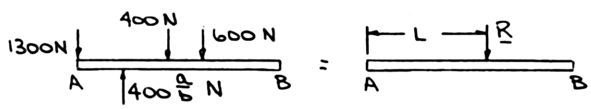
$$\mathbf{M} = (22.6\text{ N}\cdot\text{m})\mathbf{i} + (15.49\text{ N}\cdot\text{m})\mathbf{j} - (22.1\text{ N}\cdot\text{m})\mathbf{k} \blacktriangleleft$$

PROBLEM 3.156

A beam supports three loads of given magnitude and a fourth load whose magnitude is a function of position. If $b = 1.5$ m and the loads are to be replaced with a single equivalent force, determine (a) the value of a so that the distance from support A to the line of action of the equivalent force is maximum, (b) the magnitude of the equivalent force and its point of application on the beam.



SOLUTION



For equivalence,

$$\Sigma F_y: \quad -1300 + 400\frac{a}{b} - 400 - 600 = -R$$

or
$$R = \left(2300 - 400\frac{a}{b}\right) \text{ N} \qquad (1)$$

$$\Sigma M_A: \quad \frac{a}{2}\left(400\frac{a}{b}\right) - a(400) - (a+b)(600) = -LR$$

or
$$L = \frac{1000a + 600b - 200\frac{a^2}{b}}{2300 - 400\frac{a}{b}}$$

Then with
$$b = 1.5 \text{ m} \qquad L = \frac{10a + 9 - \frac{4}{3}a^2}{23 - \frac{8}{3}a} \qquad (2)$$

where a, L are in m.

(a) Find value of a to maximize L.

$$\frac{dL}{da} = \frac{\left(10 - \frac{8}{3}a\right)\left(23 - \frac{8}{3}a\right) - \left(10a + 9 - \frac{4}{3}a^2\right)\left(-\frac{8}{3}\right)}{\left(23 - \frac{8}{3}a\right)^2}$$

PROBLEM 3.156 (Continued)

or $$230-\frac{184}{3}a-\frac{80}{3}a+\frac{64}{9}a^2+\frac{80}{3}a+24-\frac{32}{9}a^2=0$$

or $$16a^2-276a+1143=0$$

Then $$a=\frac{276\pm\sqrt{(-276)^2-4(16)(1143)}}{2(16)}$$

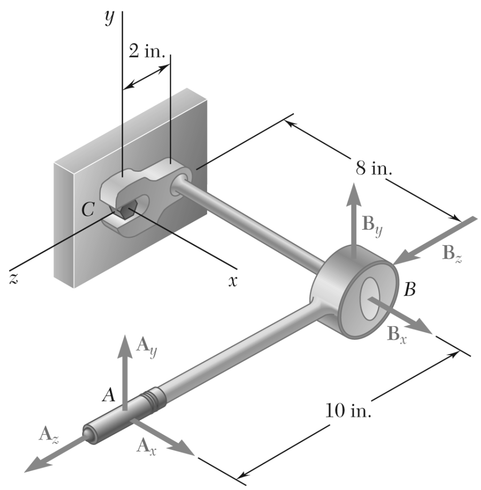
or $a=10.3435$ m and $a=6.9065$ m

Since $AB=9$ m, a must be less than 9 m $a=6.91$ m ◀

(*b*) Using Eq. (1), $$R=2300-400\frac{6.9065}{1.5}$$ or $R=458$ N ◀

and using Eq. (2), $$L=\frac{10(6.9065)+9-\frac{4}{3}(6.9065)^2}{23-\frac{8}{3}(6.9065)}=3.16\text{ m}$$

R is applied 3.16 m to the right of *A*. ◀



PROBLEM 3.157

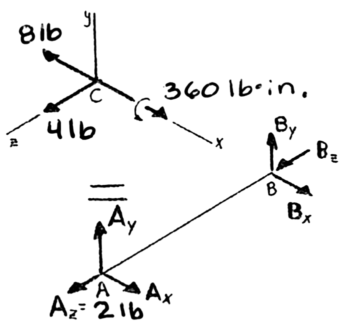
A mechanic uses a crowfoot wrench to loosen a bolt at *C*. The mechanic holds the socket wrench handle at Points *A* and *B* and applies forces at these points. Knowing that these forces are equivalent to a force-couple system at *C* consisting of the force $\mathbf{C} = (8\text{ lb})\mathbf{i} + (4\text{ lb})\mathbf{k}$ and the couple $\mathbf{M}_C = (360\text{ lb}\cdot\text{in.})\mathbf{i}$, determine the forces applied at *A* and at *B* when $A_z = 2$ lb.

SOLUTION

We have $\quad \Sigma\mathbf{F}: \quad \mathbf{A} + \mathbf{B} = \mathbf{C}$

or $\quad F_x: \quad A_x + B_x = 8\text{ lb}$

$$B_x = -(A_x + 8\text{ lb}) \qquad (1)$$



$\Sigma F_y: \quad A_y + B_y = 0$

or
$$A_y = -B_y \qquad (2)$$

$\Sigma F_z: \quad 2\text{ lb} + B_z = 4\text{ lb}$

or
$$B_z = 2\text{ lb} \qquad (3)$$

We have $\quad \Sigma\mathbf{M}_C: \quad \mathbf{r}_{B/C} \times \mathbf{B} + \mathbf{r}_{A/C} \times \mathbf{A} = \mathbf{M}_C$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 8 & 0 & 2 \\ B_x & B_y & 2 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 8 & 0 & 8 \\ A_x & A_y & 2 \end{vmatrix} \text{lb}\cdot\text{in.} = (360\text{ lb}\cdot\text{in.})\mathbf{i}$$

or
$$(2B_y - 8A_y)\mathbf{i} + (2B_x - 16 + 8A_x - 16)\mathbf{j}$$
$$+(8B_y + 8A_y)\mathbf{k} = (360\text{ lb}\cdot\text{in.})\mathbf{i}$$

From $\quad$ **i**-coefficient: $\quad 2B_y - 8A_y = 360\text{ lb}\cdot\text{in.} \qquad (4)$

j-coefficient: $\quad -2B_x + 8A_x = 32\text{ lb}\cdot\text{in.} \qquad (5)$

k-coefficient: $\quad 8B_y + 8A_y = 0 \qquad (6)$

PROBLEM 3.157 (Continued)

From Equations (2) and (4): $2B_y - 8(-B_y) = 360$

$$B_y = 36 \text{ lb} \quad A_y = 36 \text{ lb}$$

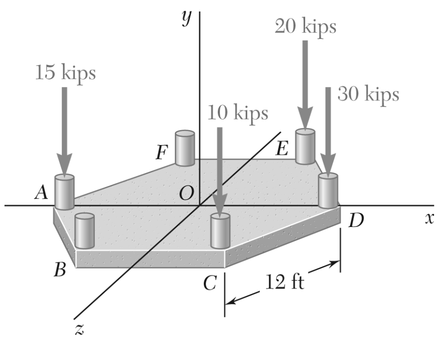
From Equations (1) and (5): $2(-A_x - 8) + 8A_x = 32$

$$A_x = 1.6 \text{ lb}$$

From Equation (1): $B_x = -(1.6 + 8) = -9.6 \text{ lb}$

$$\mathbf{A} = (1.600 \text{ lb})\mathbf{i} - (36.0 \text{ lb})\mathbf{j} + (2.00 \text{ lb})\mathbf{k} \blacktriangleleft$$

$$\mathbf{B} = -(9.60 \text{ lb})\mathbf{i} + (36.0 \text{ lb})\mathbf{j} + (2.00 \text{ lb})\mathbf{k} \blacktriangleleft$$



PROBLEM 3.158

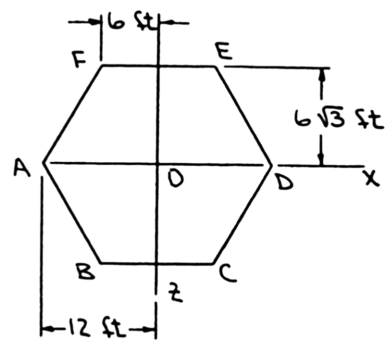
A concrete foundation mat in the shape of a regular hexagon of side 12 ft supports four column loads as shown. Determine the magnitudes of the additional loads that must be applied at B and F if the resultant of all six loads is to pass through the center of the mat.

SOLUTION

From the statement of the problem, it can be concluded that the six applied loads are equivalent to the resultant **R** at O. It then follows that

$$\Sigma \mathbf{M}_O = 0 \quad \text{or} \quad \Sigma M_x = 0 \quad \Sigma M_z = 0$$

For the applied loads:



Then $\Sigma \mathbf{M}_x = 0$: $(6\sqrt{3}\text{ ft})F_B + (6\sqrt{3}\text{ ft})(10\text{ kips}) - (6\sqrt{3}\text{ ft})(20\text{ kips})$

$$-(6\sqrt{3}\text{ ft})F_F = 0$$

or $$F_B - F_F = 10 \qquad (1)$$

$\Sigma \mathbf{M}_z = 0$: $(12\text{ ft})(15\text{ kips}) + (6\text{ ft})F_B - (6\text{ ft})(10\text{ kips})$

$$-(12\text{ ft})(30\text{ kips}) - (6\text{ ft})(20\text{ kips}) + (6\text{ ft})F_F = 0$$

or $$F_B + F_F = 60 \qquad (2)$$

Then Eqs. (1) + (2) $\Rightarrow$ $\mathbf{F}_B = 35.0\text{ kips}\downarrow$ ◀

and $\mathbf{F}_F = 25.0\text{ kips}\downarrow$ ◀