

# ***300 Solved Problems***

## ***Soil / Rock Mechanics and Foundations Engineering***

**These notes are provided to you by Professor Prieto-Portar, and in exchange, he will be grateful for your comments on improvements.**

**All problems are graded according to difficulty as follows:**

- \* Easy; defines general principles; typical of the PE examination;**
- \*\* Slightly more difficult; typical of Master's level problems;**
- \*\*\* Professional level ("real-life") problems.**

**by**

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## Conversion of Units

<i>Multiplication Factor</i>	<i>Prefix</i>	<i>SI Symbol</i>
1 000 000 000	<i>giga</i>	<i>G</i>
1 000 000	<i>mega</i>	<i>M</i>
1 000	<i>kilo</i>	<i>k</i>
0.001	<i>milli</i>	<i>m</i>
0.000 001	<i>micro</i>	$\mu$
0.000 000 001	<i>Nano</i>	<i>n</i>

## Base SI Units

<i>Quantity</i>	<i>Unit</i>	<i>Symbol</i>
<i>length</i>	<i>meter</i>	<i>m</i>
<i>mass</i>	<i>kilograms (mass)</i>	<i>kg<sub>m</sub></i>
<i>force</i>	<i>Newton</i>	<i>N</i>
<i>time</i>	<i>second</i>	<i>s</i>

## Derived SI Units

<i>Quantity</i>	<i>Derived SI Unit</i>	<i>Name</i>	<i>Symbol</i>
<i>area</i>	<i>square meter</i>		<i>m<sup>2</sup></i>
<i>volume</i>	<i>cubic meter</i>		<i>m<sup>3</sup></i>
<i>density</i>	<i>kilogram per cubic meter</i>		<i>kg<sub>m</sub>/m<sup>3</sup></i>
<i>force</i>	<i>kilogram-meter per square second</i>	<i>Newtons</i>	<i>N</i>
<i>moment of force</i>	<i>Newton-meter</i>		<i>N-m</i>
<i>pressure</i>	<i>Newton per square meter</i>	<i>Pascal</i>	<i>Pa</i>
<i>stress</i>	<i>Newton per square meter</i>	<i>Pascal</i>	<i>Pa or N/m<sup>2</sup></i>
<i>work, energy</i>	<i>Newton-meter</i>	<i>joule</i>	<i>J</i>
<i>power</i>	<i>joule per second</i>	<i>watt</i>	<i>W</i>

## Conversion of SI Units to English Units

Lengths

Multiply by

To

→

↑

From

	inches	feet	yards	miles
mm	3.94 x 10 <sup>-2</sup>	3.28 x 10 <sup>-3</sup>	1.09 x 10 <sup>-3</sup>	6.22 x 10 <sup>-7</sup>
cm	3.94 x 10 <sup>-1</sup>	3.28 x 10 <sup>-2</sup>	1.09 x 10 <sup>-2</sup>	6.22 x 10 <sup>-6</sup>
m	3.94 x 10 <sup>1</sup>	3.28	1.09	6.22 x 10 <sup>-4</sup>
km	3.94 x 10 <sup>4</sup>	3.28 x 10 <sup>3</sup>	1.09 x 10 <sup>3</sup>	6.22 x 10 <sup>-1</sup>

$$1 \mu\text{m} = 1 \times 10^{-6} \text{ m}$$




$$1 \text{ \AA} = 1 \times 10^{-10} \text{ m} = 3.28 \times 10^{-10} \text{ feet}$$

**Area**

	Multiply by	To				
From			square inches	square feet	square yards	square miles
<i>mm<sup>2</sup></i>			$1.55 \times 10^{-3}$	$1.08 \times 10^{-5}$	$1.20 \times 10^{-6}$	$3.86 \times 10^{-13}$
<i>cm<sup>2</sup></i>			$1.55 \times 10^{-1}$	$1.08 \times 10^{-3}$	$1.20 \times 10^{-4}$	$3.86 \times 10^{-11}$
<i>m<sup>2</sup></i>			$1.55 \times 10^3$	$1.08 \times 10^1$	1.20	$3.86 \times 10^{-7}$
<i>km<sup>2</sup></i>			$1.55 \times 10^9$	$1.08 \times 10^7$	$1.20 \times 10^6$	$3.86 \times 10^{-1}$

$$1 \text{ acre} = 43,450 \text{ ft}^2 = 4,047 \text{ m}^2 = 0.4047 \text{ hectares}$$

*Volume*

	<i>Multiply by</i>	<i>To</i>						
<i>From</i>				<i>cubic inches</i>	<i>cubic feet</i>	<i>cubic yards</i>	<i>quarts</i>	<i>gallons</i>
<i>cm<sup>3</sup></i>				$6.10 \times 10^{-2}$	$3.53 \times 10^{-5}$	$1.31 \times 10^{-6}$	$1.06 \times 10^{-3}$	$2.64 \times 10^{-4}$
<i>liter</i>				$6.10 \times 10^1$	$3.53 \times 10^{-2}$	$1.31 \times 10^{-3}$	1.06	$2.64 \times 10^{-1}$
<i>m<sup>3</sup></i>				$6.10 \times 10^4$	$3.53 \times 10^1$	1.31	$1.06 \times 10^3$	$2.64 \times 10^2$

## Conversion of SI Units to English Units

<b>Force</b>					
↑ Multiply by From	To →				
	ounces	pounds	kips	tons (short)	
<i>dynes</i>	$1.405 \times 10^{-7}$	$2.248 \times 10^{-6}$	$2.248 \times 10^{-9}$	$1.124 \times 10^{-9}$	
<i>grams</i>	$3.527 \times 10^{-2}$	$2.205 \times 10^{-3}$	$2.205 \times 10^{-6}$	$1.102 \times 10^{-6}$	
<i>kilograms</i>	$3.527 \times 10^1$	2.205	$2.205 \times 10^{-3}$	$1.102 \times 10^{-3}$	
<i>Newtons</i>	3.597	$2.248 \times 10^{-1}$	$2.248 \times 10^{-4}$	$1.124 \times 10^{-4}$	
<i>kilo-Newtons</i>	$3.597 \times 10^3$	$2.248 \times 10^2$	$2.248 \times 10^{-1}$	$1.124 \times 10^{-1}$	
<i>tons (metric)</i>	$3.527 \times 10^4$	$2.205 \times 10^3$	2.205	1.102	

<b>Pressure (or stress) <math>\sigma</math></b>							
↑ Multiply by From	To →						
	lb/in <sup>2</sup>	lb/ft <sup>2</sup>	kips/ft <sup>2</sup>	tons (short)/ft <sup>2</sup>	feet of water	atmosphere	
<i>gm/cm<sup>2</sup></i>	$1.422 \times 10^{-2}$	2.048	$2.048 \times 10^{-3}$	$1.024 \times 10^{-3}$	$3.281 \times 10^{-2}$	$9.678 \times 10^{-4}$	
<i>kg/cm<sup>2</sup></i>	$1.422 \times 10^1$	$2.048 \times 10^3$	2.048	1.024	$3.281 \times 10^1$	$9.678 \times 10^{-1}$	
<i>kN/m<sup>2</sup></i>	$1.450 \times 10^{-1}$	$2.090 \times 10^1$	$2.088 \times 10^{-2}$	$1.044 \times 10^{-2}$	$3.346 \times 10^{-1}$	$9.869 \times 10^{-3}$	
<i>ton (metric)/m<sup>2</sup></i>	1.422	$2.048 \times 10^2$	$2.048 \times 10^{-1}$	$1.024 \times 10^{-1}$	3.281	$9.678 \times 10^{-2}$	

<b>Torque (or moment) <math>T</math> or <math>M</math></b>				
↑ Multiply by From	To →			
	lb-in	lb-ft	kips-ft	
<i>gm-cm</i>	$8.677 \times 10^{-4}$	$7.233 \times 10^{-5}$	$7.233 \times 10^{-8}$	
<i>kg-m</i>	8.677	7.233	$7.233 \times 10^{-3}$	
<i>kN-m</i>	$9.195 \times 10^3$	$7.663 \times 10^2$	$7.663 \times 10^{-1}$	

## Conversion of SI Units to English Units

<b>Velocity <math>v</math></b>			
↑	Multiply by	To →	
	From	$ft/s$	$ft/min$ $mi/h$
	$cm/s$	$3.281 \times 10^{-2}$	$1.9685$ $2.236 \times 10^{-2}$
	$km/min$	$5.467 \times 10^1$	$3.281 \times 10^3$ $3.728 \times 10^1$
	$km/h$	$9.116 \times 10^{-1}$	$5.467 \times 10^1$ $6.214 \times 10^{-1}$

1 mile = 1,610 meters = 5,282.152 feet

<b>Unit weight <math>\gamma</math></b>		
↑	Multiply by	To →
	From	$lb/in^3$ $lb/ft^3$
	$gm/cm^3$	$3.613 \times 10^{-2}$ $6.248 \times 10^1$
	$kg/m^3$	$3.613 \times 10^{-5}$ $6.248 \times 10^{-2}$
	$kN/m^3$	$3.685 \times 10^{-3}$ $6.368 \times 10^1$
	$tons (metric) / m^3$	$3.613 \times 10^{-2}$ $6.428 \times 10^1$

## **Power $P$**

1 W = 1 J/sec = 1.1622 cal/hr = 3.41 Btu/hr = 0.0013 hp

1 hp = 745.7 W = 0.7457 kW

1 kW = 1.34 hp



# Chapter 1

## Soil Exploration

### **Symbols for Soil Exploration**

$C_B \rightarrow$  *STP* correction factor for the boreholes diameter.

$C_R \rightarrow$  *STP* correction factor for the rod length.

$C_S \rightarrow$  *STP* correction factor for the sampler type used.

$c_u \rightarrow$  Soil's un-drained cohesion.

$D_f \rightarrow$  Depth of the foundation's invert.

$E_m \rightarrow$  The efficiency of the *STP* hammer.

$N \rightarrow$  The "raw" value of the *STP* (as obtained in the field).

$p_o \rightarrow$  The original vertical stress at a point of interest in the soil mass.

$S \rightarrow$  The number of stories of a building.

*SPT*  $\rightarrow$  Stands for "Standard Penetration Test".

$N_{60} \rightarrow$  Corrected *STP* assuming 60% efficiency in the field.

$N_{70} \rightarrow$  Corrected *STP* assuming 70% efficiency in the field.

$\mu \rightarrow$  Correction factor for the shear vane test using the clay's Plasticity Index *PI*.

**\*Exploration–01. Find the required number of borings and their depth.**

(Revised: Sept. 08)

A four story reinforced concrete frame office building will be built on a site where the soils are expected to be of average quality and uniformity. The building will have a 30 m x 40 m footprint and is expected to be supported on spread footing foundations located about 1 m below the ground surface. The site appears to be in its natural condition, with no evidence of previous grading. Bedrock is 30-m below the ground surface. Determine the required number and depth of the borings.

**Solution:**

A reinforced concrete building is heavier than a steel framed building of the same size. Hence, the design engineer will want soil conditions that are at least average or better. From Table-1 below, one boring will be needed for every 300 m<sup>2</sup> of footprint area. Since the total footprint area is 30 m x 40 m = 1,200 m<sup>2</sup>, use **four borings**.

Table-2 provides the minimum depth required for the borings,  $5 S^{0.7} + D = 5(4)^{0.7} + 1 = \mathbf{14\ m}$ . Most design engineers want one boring to go to a slightly greater depth to check the next lower stratum's strength.

In summary, the exploration plan will be 4 borings to a depth of 14 m.

**Table-1 - Spacing of the exploratory borings for buildings on shallow foundations.**

<b><u>Subsurface Conditions</u></b>	<b><u>Structural footprint Area for Each Boring</u></b>	
	(m <sup>2</sup> )	(ft <sup>2</sup> )
Poor quality and / or erratic	200	2,000
Average	300	3,000
High quality and uniform	600	6,000

**Table-2 - Depths of exploratory borings for buildings on shallow foundations.**

<b><u>Subsurface Conditions</u></b>	<b><u>Minimum Depth of Borings</u></b>	
	(S = number of stories and D = the anticipated depth of the foundation)	
	(m)	(ft)
Poor and / or erratic	$6S^{0.7} + D$	$20S^{0.7} + D$
Average	$5S^{0.7} + D$	$15S^{0.7} + D$
High quality and uniform	$3S^{0.7} + D$	$10S^{0.7} + D$

**\*Exploration–02. The sample’s disturbance due to the boring diameter.**

(Revised: Sept. 08)

The most common soil and soft rock sampling tool in the US is the Standard Split Spoon.

Split spoon tubes split longitudinally into halves and permit taking a soil or soft rock sample. The tube size is designated as an NX. The NX outside diameter is  $D_o = 50.8$  mm (2 inches) and its inside diameter is  $D_i = 34.9$  mm (1-3/8 inches). This small size has the advantage of cheapness, because it is relatively easy to drive into the ground. However, it has the disadvantage of disturbing the natural texture of the soil. In soft rocks, such as young limestone, it will destroy the rock to such a degree that it may be classified as a “sand”.

A better sampler is the Shelby (or thin-tube sampler). It has the same outside diameter of 2 inches (although the trend is to use 3 inches).

Compare the degree of sample disturbance of a US standard split-spoon sampler, versus the two Shelby thin-tube samplers (2” and 3” outside diameters) via their area ratio  $A_r$  (a measure of sample disturbance).

**Solution:**

The area ratio for a 2"-standard split-spoon sampler is,

$$A_r(\%) = \frac{D_o^2 - D_i^2}{D_i^2} (100) = \frac{(2.0)^2 - (1.38)^2}{(1.38)^2} (100) = 110\%$$

The area ratio for a 2"-Shelby-tube sampler is,

$$A_r(\%) = \frac{D_o^2 - D_i^2}{D_i^2} (100) = \frac{(2.0)^2 - (1.875)^2}{(1.875)^2} (100) = 13.8\%$$

The area ratio for a 3"-Shelby-tube sampler is,

$$A_r(\%) = \frac{D_o^2 - D_i^2}{D_i^2} (100) = \frac{(3.0)^2 - (2.875)^2}{(2.875)^2} (100) = 8.9\%$$

Clearly, the 3” O-D Shelby-tube sampler is the best tool to use.

### **\*Exploration–03. Correcting the SPT for depth and sampling method.**

(Revision Sept-08)

A standard penetration test (SPT) has been conducted in a loose coarse sand stratum to a depth of 16 ft below the ground surface. The blow counts obtained in the field were as follows: 0 – 6 in = 4 blows; 6 -12 in = 6 blows; 12 -18 in = 8 blows. The tests were conducted using a US-style donut hammer in a 6 inch diameter boring with a standard sampler and liner. The effective unit weight of the loose sand stratum is about 93.8 pcf.

Determine the corrected SPT if the testing procedure is assumed to only be 60% efficient.

#### **Solution:**

The raw SPT value is  $N = 6 + 8 = 14$  (that is, only the last two sets of 6" penetrations).

The US-style donut hammer efficiency is  $E_m = 0.45$ , and the other parameters are obtained from the Tables provided on the next page:  $C_B = 1.05$ ,  $C_S = 1.00$ ,  $C_R = 0.85$ .

With these values, the SPT corrected to 60% efficiency can use Skempton's relation,

$$N_{60} = \frac{E_m C_B C_S C_R N}{0.60} = \frac{(0.45)(1.05)(1.00)(0.85)(14)}{0.60} = 9$$

Notice that the SPT value is always given as a whole number.

That corrected SPT  $N_{60}$  is then corrected for depth. For example, using the Liao and Whitman method (1986),

$$(N)_{60} = N_{60} \sqrt{\frac{2,000 \text{ lb / ft}^2}{(\text{depth})(\text{effective unit weight})}} = (9) \sqrt{\frac{2,000 \text{ lb / ft}^2}{(16 \text{ ft})(93.8 \text{ pcf})}} = 10$$

Other methods for corrections are discussed in Exploration-04.

<b><i>SPT Hammer Efficiencies (adapted from Clayton, 1990).</i></b>			
<i>Country</i>	<i>Hammer Type</i>	<i>Release Mechanism</i>	<i>Hammer Efficiency</i>
Argentina	donut	cathead	0.45
Brazil	pin weight	hand dropped	0.72
China	automatic	trip	0.60
	donut	hand dropped	0.55
	donut	cat-head	0.50
Colombia	donut	cat-head	0.50
Japan	donut	Tombi trigger	0.78 - 0.85
	donut	cat-head + sp. release	0.65 - 0.67
UK	automatic	trip	0.73
US	safety	2-turns on cat-head	0.55 - 0.60
	donut	2-turns on cat-head	0.45
Venezuela	donut	cat-head	0.43

<b><i>Correction Factors for the Boring Diameter, Sampling Method and Boring Rod Length (adapted from Skempton, 1986).</i></b>		
<i>Correction Factor</i>	<i>Equipment Variables</i>	<i>Value</i>
Borehole diameter factor $C_B$	65 – 115 mm (2.5 – 4.5 in)	1.00
	150 mm (6 in)	1.05
	200 mm (8 in)	1.15
Sampling method factor $C_S$	Standard sampler	1.00
	Sampler without liner (not recommended)	1.20
Rod length factor, $C_R$	3 – 4 m (10 – 13 ft)	0.75
	4 – 6 m (13 – 20 ft)	0.85
	6 – 10 (20 – 30 ft)	0.95
	>10 m (>30 ft)	1.00

### \*Exploration–04. Three methods used for SPT depth corrections.

(Revision Sept.-08)

A raw value of  $N = 40$  was obtained from an *SPT* at a depth of 20 feet in a sand stratum that has a unit weight of  $135 \text{ lb/ft}^3$ . Correct it only for depth.

#### Solution:

Any of these three methods will provide acceptable answers. Notice how similar their results are from each other:

#### 1. Using the Bazaraa Method (1967):

$$N_{corrected} = \frac{4N'}{1 + 2p_o} \quad \text{if } p_o \leq 1.5 \text{ kips / ft}^2 \quad \text{and}$$

$$N_{corrected} = \frac{4N'}{3.25 + 0.5p_o} \quad \text{if } p_o \geq 1.5 \text{ kips / ft}^2$$

$$\text{but } p_o = \frac{(20 \text{ ft})(135 \text{ lb / ft}^3)}{1000 \text{ lb / kip}} = 2.70 \text{ kips / ft}^2 > 1.5 \text{ kips / ft}^2$$

$$\text{therefore } N_{corrected} = \frac{4N'}{3.25 + 0.5p_o} = \frac{4(40)}{3.25 + 0.5(2.70 \text{ kips / ft}^2)} = 35$$

#### 2. Using the Peck Method (1974):

$$N_{corrected} = N' C_N \quad \text{where } C_N = 0.77 \log_{10} \frac{20}{p_o} \quad \text{if } p_o \text{ is in tons / ft}^2$$

$$\text{or } C_N = 0.77 \log_{10} \frac{1915}{p_o} \quad \text{if } p_o \text{ is in kN / m}^2$$

$$\text{but } p_o = \frac{(20 \text{ ft})(135 \text{ lb / ft}^3)}{2000 \text{ lb / ton}} = 1.35 \text{ tons / ft}^2 = 2.70 \text{ kips / ft}^2$$

$$\therefore C_N = 0.77 \log_{10} \frac{20}{1.35 \text{ tons / ft}^2} = 0.90 \quad \therefore N_{corrected} = (40)(0.90) = 36$$

#### 3. The Liao-Whitman Method (1986), as used in Exploration-03,

$$N_{corrected} = N' \sqrt{\frac{100}{p_o}} \quad \text{with } p_o \text{ in kN / m}^2 \quad \text{or} \quad = N' \sqrt{\frac{2,000 \text{ psf}}{p_o}} \quad \text{with } p_o \text{ in psf}$$

$$\text{but } p_o = (1.35 \text{ tons / ft}^2) \left( \frac{96.1 \text{ kN / m}^2}{1 \text{ ton / ft}^2} \right) = 129.7 \text{ kN / m}^2$$

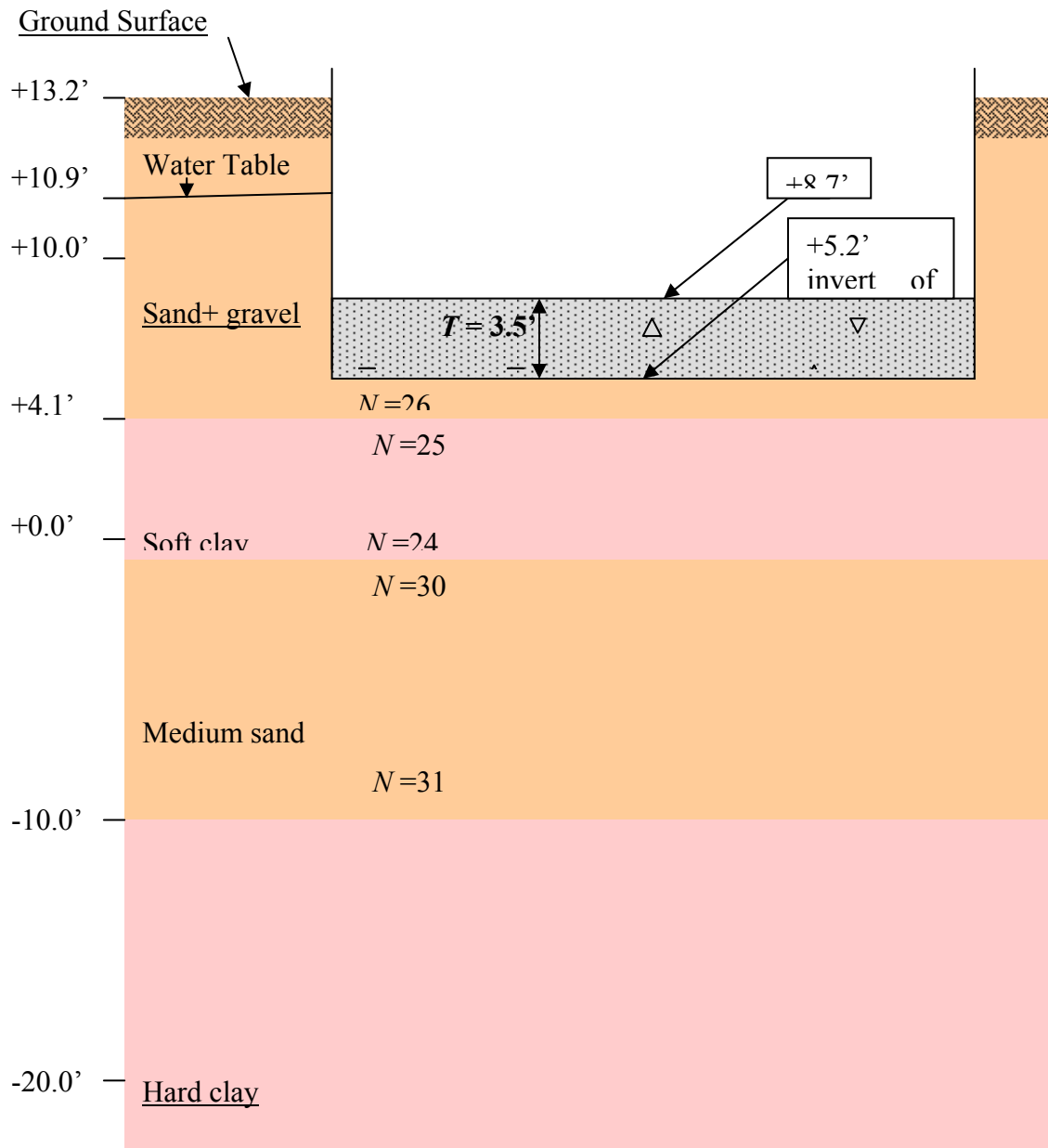
$$\therefore N_{corrected} = 40 \sqrt{\frac{100 \text{ kN / m}^2}{129.7 \text{ kN / m}^2}} = 35$$

**\*Exploration-05. SPT corrections under a mat foundation.**

(Revision Sept.-08)

Correct the *SPT* values shown below for an energy ratio of 60% using a high-efficient US-type donut hammer in a 2"-diameter boring. The invert (bottom) of the mat foundation is at elevation +5.2 feet.

+20'



**Solution:**

Skempton proposed in 1986 the following correction for the sampling methods to the raw *SPT* value,

assuming that only 60% of the energy of the hammer drives the sampler, 
$$N_{60} = \frac{E_m C_B C_S C_R N}{0.60}$$

where:  $N_{60}$  = SPT  $N$ -value corrected for field procedures assuming 60% efficiency

$E_m = 0.60$  → efficiency for a high-efficiency US-style safety hammer

$C_B = 1.00$  → borehole diameter correction

$C_S = 1.00$  → sampler correction, = 0.75 (10'-13')

$C_R = 0.85$  (13'-20') → rod length correction, = 0.95 (20'-30'), = 1.0 (>30')

$N$  = ***SPT***-value recorded in the field by the driller (known as the “raw” ***SPT***).

The depth correction is,

$$(N_1)_{60} = N_{60} \sqrt{\frac{2,000 \text{ lb / ft}^2}{(\text{depth})(\text{effective unit weight})}}$$

At depth of +5.2 feet:

$$N_{60} = \frac{(0.60)(1)(1)(0.75)(26)}{0.60} = 20 \text{ and } (N)_{60} = (20) \sqrt{\frac{2,000 \text{ lb / ft}^2}{(8 \text{ ft})(127 - 62.4 \text{ pcf})}} = 39$$

$$\text{At } +4.1' \quad N_{60} = \frac{(0.60)(1)(1)(0.75)(25)}{0.60} = 19 \text{ and } (N)_{60} = (19) \sqrt{\frac{2,000 \text{ lb / ft}^2}{(9 \text{ ft})(127 - 62.4 \text{ pcf})}} = 35$$

$$\text{At } +2.0' \quad N_{60} = \frac{(0.60)(1)(1)(0.75)(24)}{0.60} = 18 \text{ and } (N)_{60} = (18) \sqrt{\frac{2,000 \text{ lb / ft}^2}{(11 \text{ ft})(125 - 62.4 \text{ pcf})}} = 31$$

$$\text{At } -1.0' \quad N_{60} = \frac{(0.60)(1)(1)(0.85)(30)}{0.60} = 26 \text{ and } (N)_{60} = (26) \sqrt{\frac{2,000 \text{ lb / ft}^2}{(14 \text{ ft})(126 - 62.4 \text{ pcf})}} = 39$$

$$\text{At } -5.0' \quad N_{60} = \frac{(0.60)(1)(1)(0.85)(31)}{0.60} = 26 \text{ and } (N)_{60} = (26) \sqrt{\frac{2,000 \text{ lb / ft}^2}{(18 \text{ ft})(126 - 62.4 \text{ pcf})}} = 34$$

$$\text{At } -10' \quad N_{60} = \frac{(0.60)(1)(1)(0.95)(30)}{0.60} = 29 \text{ and } (N)_{60} = (29) \sqrt{\frac{2,000 \text{ lb / ft}^2}{(23 \text{ ft})(126 - 62.4 \text{ pcf})}} = 34$$

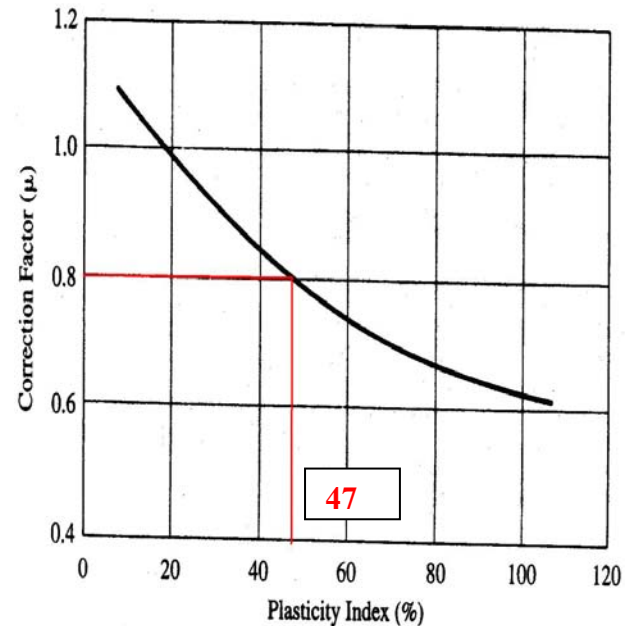
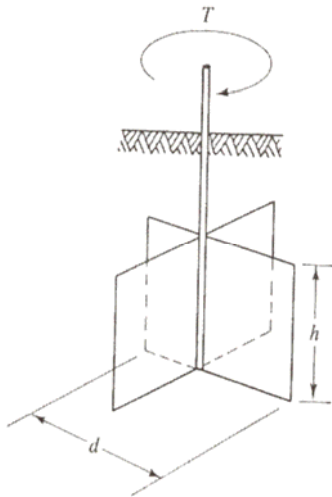
$$\text{At } -21' \quad N_{60} = \frac{(0.60)(1)(1)(1)(43)}{0.60} = 43 \text{ and } (N)_{60} = (43) \sqrt{\frac{2,000 \text{ lb / ft}^2}{(33 \text{ ft})(130 - 62.4 \text{ pcf})}} = 41$$

Notice that the depth correction does not affect the deeper layers.

**\*Exploration–06. The Shear Vane Test determines the in-situ cohesion.**

(Revision Sept.-08)

A shear vane tester is used to determine an approximate value of the shear strength of clay. The tester has a blade diameter  $d = 3.625$  inches and a blade height  $h = 7.25$  inches. In a field test, the vane required a torque of  $17.0 \text{ ft}\cdot\text{lb}$  to shear the clay sample, which has a plasticity index of 47% ( $PI = LL - PL$ ). Determine the un-drained cohesion  $c_u$  corrected for its plasticity.



$$c_u = \frac{T}{\pi \left[ (d^2 h / 2) + (d^3 / 6) \right]} = \frac{17.0 \text{ ft}\cdot\text{lb}}{\pi \left[ \frac{(0.3021 \text{ ft})^2 (0.6042 \text{ ft})}{2} + \frac{(0.3021 \text{ ft})^3}{6} \right]} = 168 \text{ psf}$$

The plasticity index helps correct the raw shear vane test value (Bjerrum, 1974) through the graph shown above. For a plasticity index of 47% read a correction factor  $\mu = 0.80$ . Therefore,

$$c_{u\text{-corrected}} = \mu c_u = (0.80)(168 \text{ psf}) = 134 \text{ psf}$$

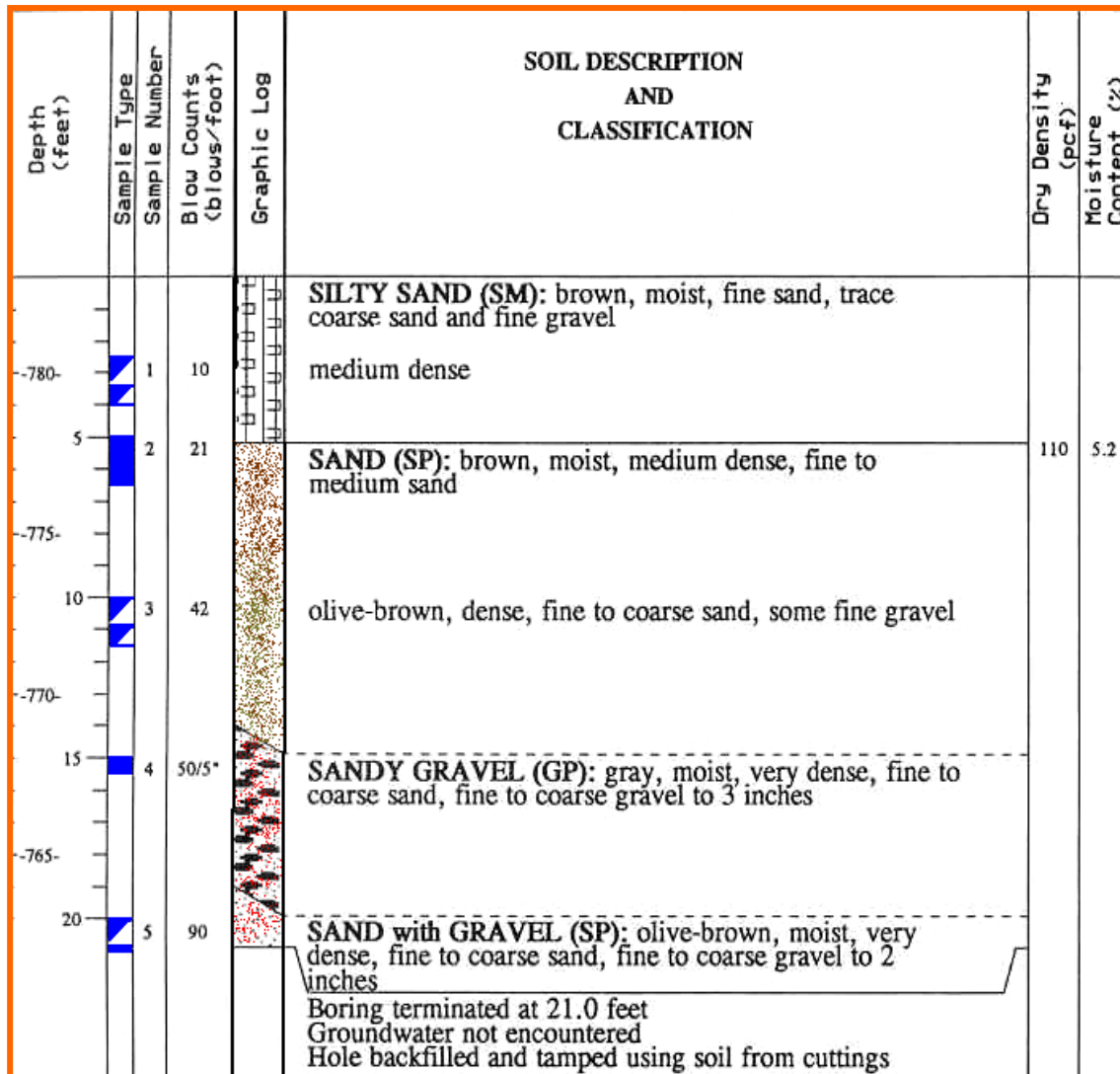
## \*Exploration-07. Reading a soil boring log.

(Revision Sept.-08)

Read the boring log shown below and determine, (1) the location of the phreatic surface, (2) the depth of the boring and (3) the number of samples taken.

### Solution:

- (1) The phreatic surface (the water table) was *not* encountered in this boring and is noted at the bottom of the report;
- (2) The boring was terminated at 21 feet in depth; and
- (3) Five samples were taken. Only one sample (#2) was used for laboratory tests (dry density and moisture content). Samples #1 and #3 were complete split-spoon samples. Samples #4 and #5 were incomplete split-spoon samples.



### **\*Exploration–08: Using a boring log to predict soil engineering parameters.**

(Revision Sept.-08)

Using the boring log and the *SPT versus Soil Engineering Parameters Table* shown on the next two pages, answer these four questions:

- (1) Correct the values of the *SPT* of Sample S-4 to a 70% sampling efficiency with a standard sampling method and a US-donut hammer at elevation – 17 feet;
- (2) Correct the same sample S-4 for depth assuming the unit weight is  $\gamma = 126$  pcf;
- (3) What are your estimates for the angle of internal friction and unit weight  $\gamma$ ?
- (4) What is the elevation (above sea level) of the groundwater and the elevation of the bottom of the boring?

#### **Solution:**

(1) The log shows a value of  $N = 15$  (Sample S-4) at elevation -16.5'; at elevation -17' it has dropped a small amount to  $N = 14$ . Notice that the "Legend" portion denotes that the sampler was a 2" O.D. split spoon. Therefore, the sampling correction is,

$$N_{70} = \frac{EC_B C_S C_R N}{0.70} = \frac{(0.45)(1.0)(1.0)(0.85)(14)}{0.70} \approx 8$$

(2) Correct the same sample S-4 for depth.

$$(N_{70}) = N_{70} \sqrt{\frac{2000 \text{ psf}}{(\gamma h)}} = (8) \sqrt{\frac{2000 \text{ psf}}{(126)(17) \text{ psf}}} \approx 8$$

(3) What are your estimates for the angle of internal friction and unit weight  $\gamma$ ?

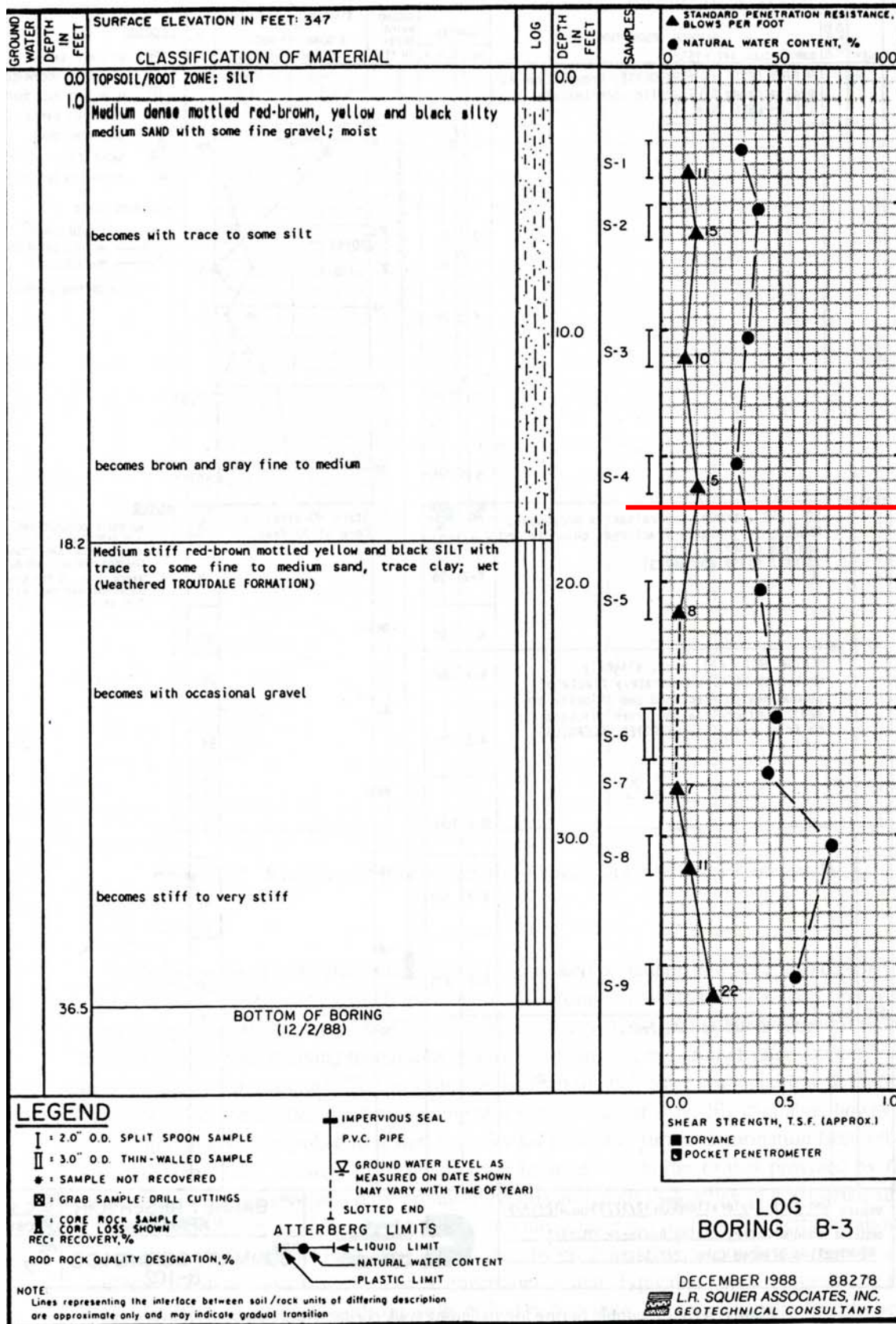
The log identifies this level at -17' as a "brown and grey fine to medium SAND". Use the Table provided on page 23 to obtain an estimate of some of the engineering parameters for granular soils. Read the *SPT* for medium sands; then go to the Medium column and read the value of " $N = 8$ " to obtain the values:

$$\phi = 32^\circ \text{ and } \gamma_{wet} = 17 \text{ kN/m}^2.$$

(4) What are the elevations (above sea level) of the groundwater and of the bottom of the boring?

- The boring did not report finding a ground water table.

- The bottom of the boring was at -36.5' from the surface, or  $347.0' - 36.5' = \underline{+310.5'}$ .



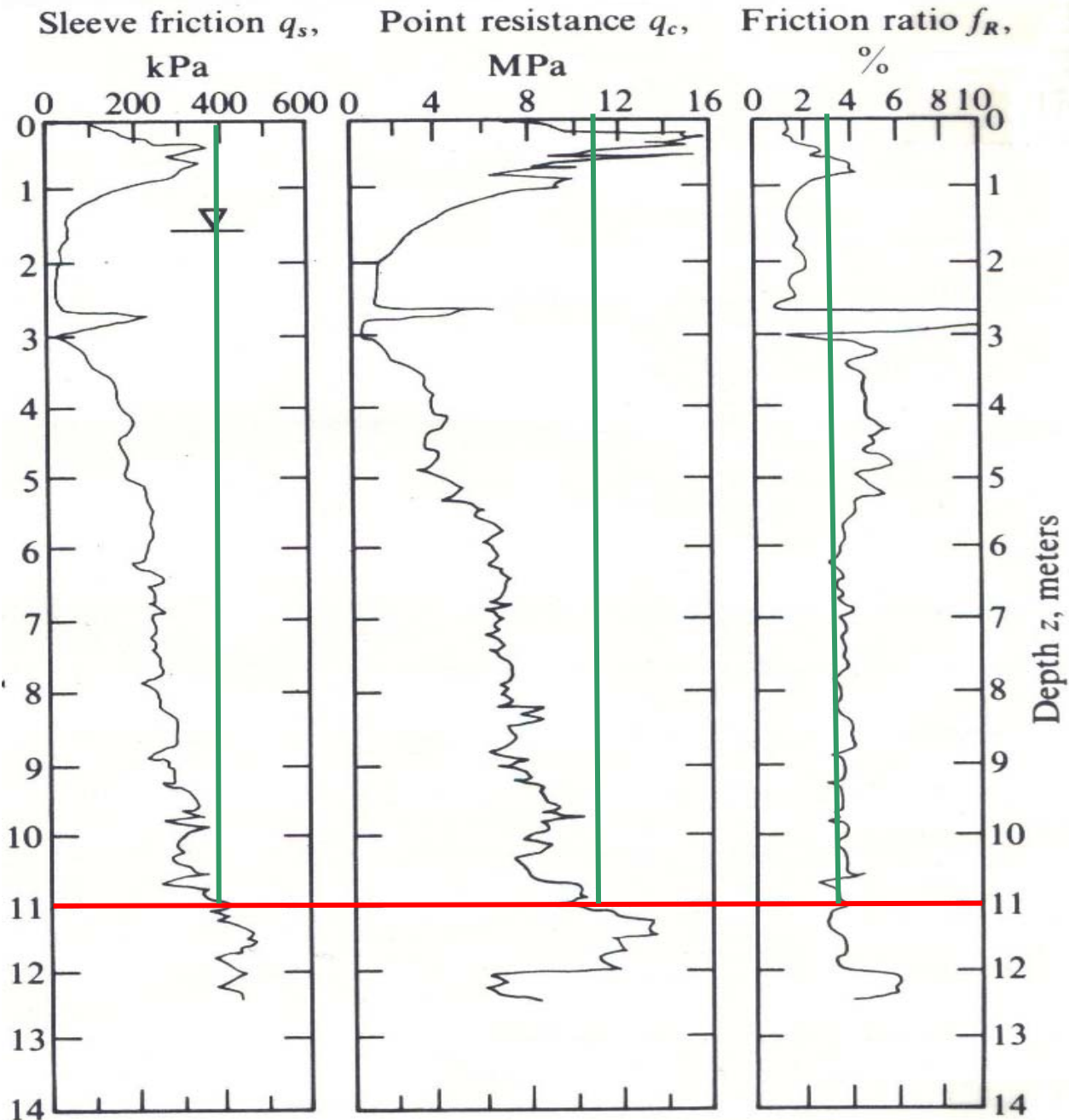
<b>Correlation between SPT values and some Engineering Parameters of Granular Soils</b>						
<b>Description</b>		<b>Very loose</b>	<b>Loose</b>	<b>Medium</b>	<b>Dense</b>	<b>Very dense</b>
<b><math>D_r</math></b>	<i>Relative density</i>	<b>0</b>	<b>0.15</b>	<b>0.35</b>	<b>0.65</b>	<b>0.85</b>
<b>SPT</b>	<i>fine</i>	<b>1 - 2</b>	<b>3 - 6</b>	<b>7 - 15</b>	<b>16 - 30</b>	<b>&gt; 40</b>
<b>(<math>N'_{70}</math>)</b>	<i>medium</i>	<b>2 - 3</b>	<b>4 - 7</b>	<b>8 - 20</b>	<b>21 - 40</b>	
	<i>coarse</i>	<b>3 - 6</b>	<b>5 - 9</b>	<b>10 - 25</b>	<b>26 - 45</b>	
<b><math>\phi</math></b>	<i>fine</i>	<b>26 - 28</b>	<b>28 - 30</b>	<b>30 - 34</b>	<b>33 - 38</b>	<b>&lt; 50</b>
	<i>medium</i>	<b>27 - 28</b>	<b>30 - 32</b>	<b>32 - 36</b>	<b>36 - 42</b>	
	<i>coarse</i>	<b>28 - 30</b>	<b>30 - 34</b>	<b>33 - 40</b>	<b>40 - 50</b>	
<b><math>\gamma_{wet}</math></b>	<i>pcf</i>	<b>70 - 102</b>	<b>89 - 115</b>	<b>108 - 128</b>	<b>108 - 140</b>	<b>128 - 147</b>
	<i>kN/m<sup>3</sup></i>	<b>11 - 16</b>	<b>14 - 18</b>	<b>17 - 20</b>	<b>17 - 22</b>	<b>20 - 23</b>
Note #1: These values are based on tests conducted at depths of about 6 m; Note #2: Typical values of relative densities are about 0.3 to 0.7; values of 0 or 1.0 do not exist in nature; Note #3: The value of the angle of internal friction is based on $\Phi = 28^\circ + 15^\circ D_r$ ; Note #4: The typical value of an excavated soil ranges from 11 to 14 kN/m <sup>3</sup> ;						

<b>Correlation between SPT values and some Engineering Parameters of Cohesive Soils</b>		
<b>SPT - <math>N_{70}</math></b>	<b>Compressive Strength <math>q_u</math></b>	<b>Description</b>
<b>0 - 2</b>	<b>&lt; 25 kPa</b>	<i>Very soft – squeezes between fingers Very young NC clay</i>
<b>3 - 5</b>	<b>25 - 50 kPa</b>	<i>Soft – easily deformed by fingers Young NC clay</i>
<b>6 - 9</b>	<b>50 - 100 kPa</b>	<i>Medium</i>
<b>10 - 16</b>	<b>100 - 200 kPa</b>	<i>Stiff – Hard to deform w/fingers Small OCR – aged clay</i>
<b>17 - 30</b>	<b>200 - 400 kPa</b>	<i>Very Stiff – Very hard w/fingers Increasing OCR – older clays</i>
<b>&gt; 30</b>	<b>&gt; 400 kPa</b>	<i>Hard – Does not deform w/fingers Higher OCR – cemented clays</i>

**\*\*Exploration–09. Find the shear strength of a soil from the CPT Report.**

(Revision: Sept.-08)

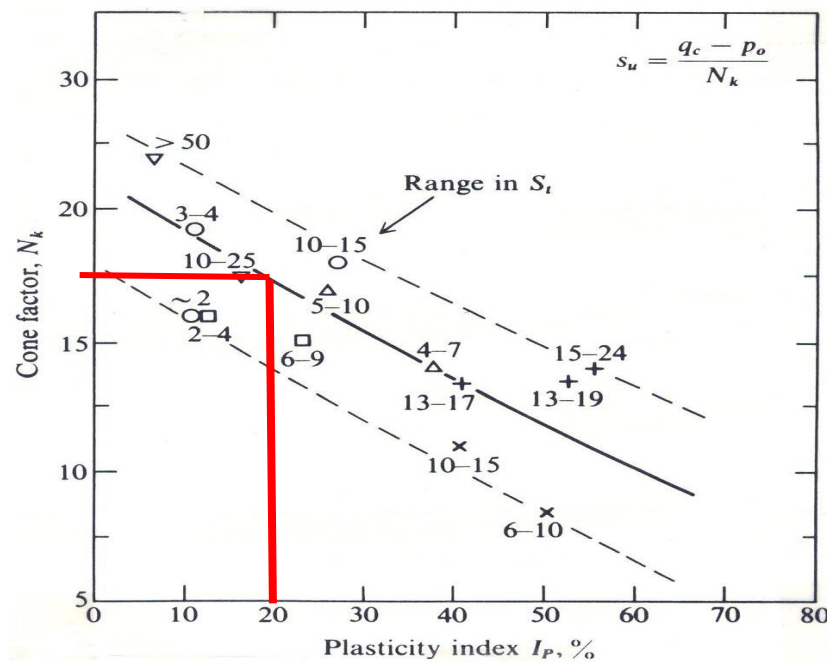
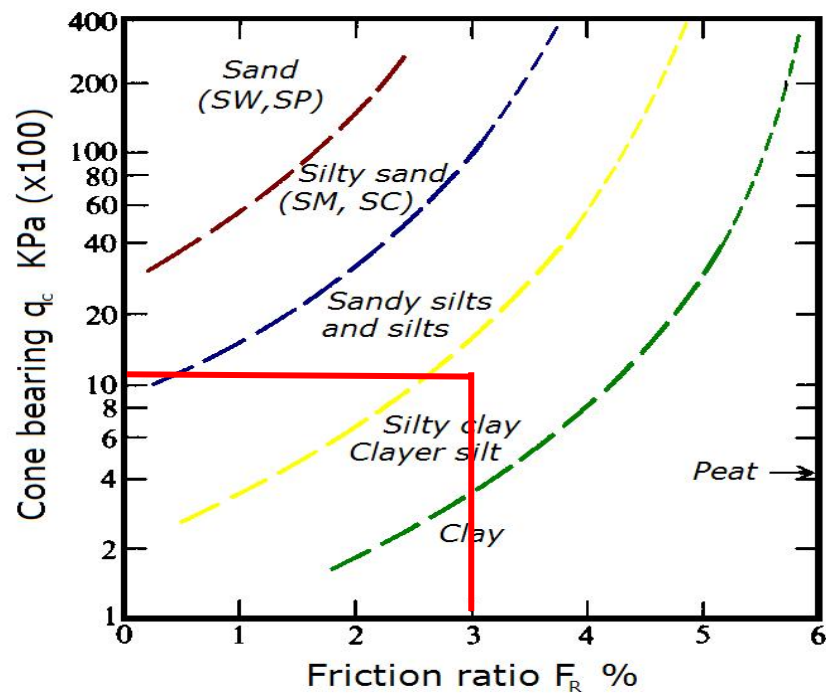
Classify a soil from the data provided by the Cone Penetration Test (CPT) shown below at a depth of 11 m. The clay samples recovered from that depth had  $\gamma = 20 \text{ kN/m}^3$  and  $PI = I_p = 20$ . Compare your estimate of the shear strength versus the lab test value of  $550 \text{ kPa}$ .



**Solution.**

Reading the data,  $q_s \sim 400 \text{ kPa}$  and  $q_c \sim 11 \text{ MPa}$  which results in a  $f_R \sim 3\%$ .

From the next chart, the soil appears to be a silty clay.



At a depth of 11 m, the in-situ pressure  $p_o$  for a NC clay is,

$$p_o = \gamma z = (20 \text{ kN/m}^3)(11 \text{ m}) = 220 \text{ kPa}$$

From the  $N_k$  versus  $I_p$  graph, for  $I_p = 20$  yields an  $N_k \sim 17.5$ .

The un-drained shear strength  $s_u$  is,

$$s_u = \frac{q_c - p_o}{N_k} = \frac{11,000 \text{ kPa} - 220 \text{ kPa}}{17.5} = 616 \text{ kPa versus lab} = 550 \text{ kPa (a 12\% error)}.$$

# Chapter 2

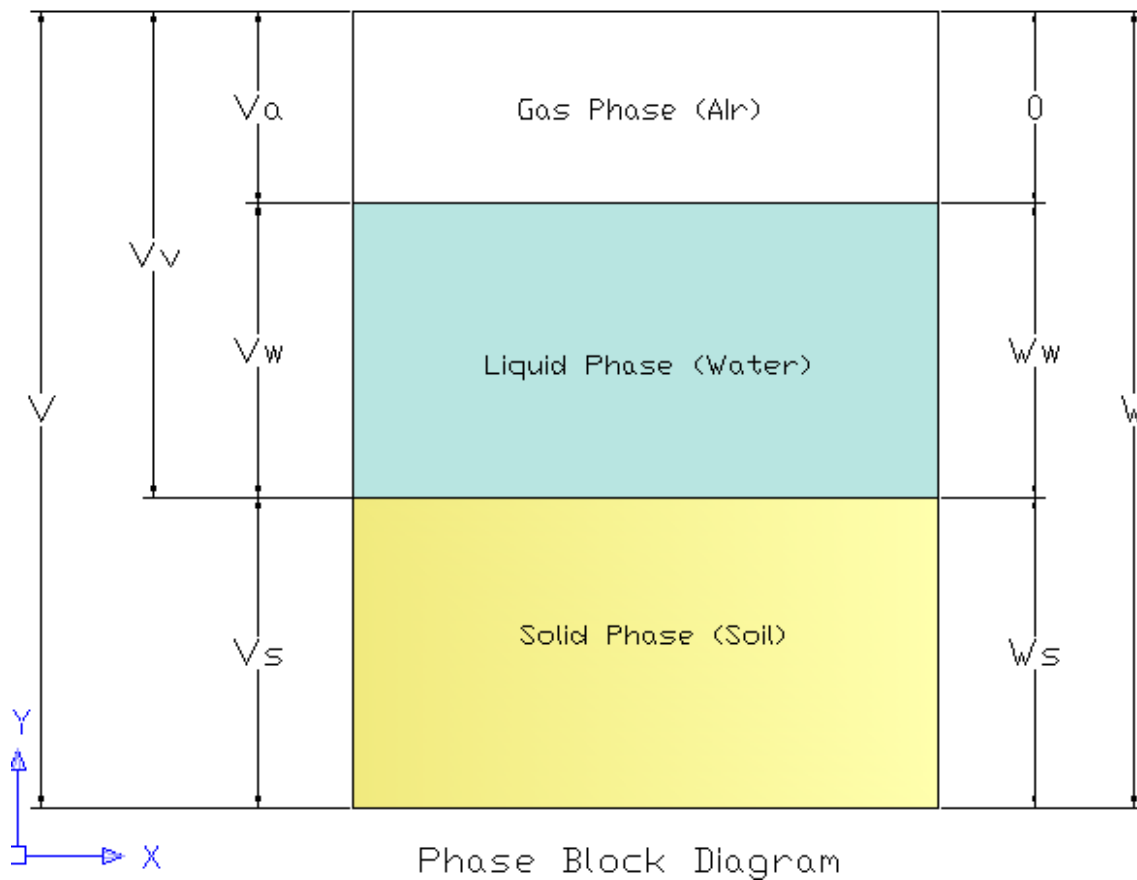
## Phase Relations of Soil

### Symbols for Phase Relations of soils

- $e \rightarrow$  *Voids ratio.*
- $G_s \rightarrow$  *Specific gravity of the solids of a soil.*
- $n \rightarrow$  *Porosity.*
- $S \rightarrow$  *Degree of saturation.*
- $V \rightarrow$  *Total volume (solids + water + air).*
- $V_a \rightarrow$  *Volume of air.*
- $V_v \rightarrow$  *Volume of voids (water + air).*
- $V_s \rightarrow$  *Volume of solids.*
- $V_w \rightarrow$  *Volume of water.*
- $w \rightarrow$  *Water content (also known as the moisture content).*
- $W_s \rightarrow$  *Weight of solids.*
- $W_w \rightarrow$  *Weight of water.*
- $g \rightarrow$  *Unit weight of the soil.*
- $g_d \rightarrow$  *Dry unit weight of the soil.*
- $g_b \rightarrow$  *Buoyant unit weight of the soil (same as  $g'$ ).*
- $g_{SAT} \rightarrow$  *Unit weight of a saturated soil.*
- $g_w \rightarrow$  *Unit weight of water*

## Basic Concepts and Formulas for the Phases of Soils.

### (A) Volumetric Relationships:



#### 1. - Voids ratio $e$

$$e = \frac{V_v}{V_s} \quad 2-1$$

ranges from 0 to infinity.

Typical values of sands are: very dense 0.4 to very loose 1.0

Typical values for clays are: firm 0.3 to very soft 1.5.

#### 2. - Porosity $n$

$$n = \frac{V_v}{V} (100\%) \quad 2.2$$

ranges from 0% to 100%.

The porosity provides a measure of the permeability of a soil.

The interrelationship of the voids ratio and porosity are given by,

$$e = \frac{n}{1-n} \quad \text{and} \quad n = \frac{e}{1+e} \quad 2-3$$

### 3. - Saturation $S$

$$S = \frac{V_w}{V_v} \times 100\% \quad 2-4$$

ranges from 0% to 100%.

## (B) Weight Relationships:

### 4. - Water content $w$

$$w = \frac{W_w}{W_s} \times 100\% \quad 2-5$$

Values range from 0% to over 500%; also known as moisture content.

### 5. - Unit weight of a soil $\gamma$

$$\gamma = \frac{W}{V} = \frac{W_s + W_w}{V_s + V_w + V_a} \quad 2-6$$

The unit weight may range from being dry to being saturated.

Some engineers use “**bulk density  $\rho$** ” to refer to the ratio of mass of the solids and water contained in a unit volume (in  $Mg/m^3$ ). Note that,

$$\gamma = \frac{W}{V} = \rho g = \frac{m}{V} g \quad \text{which is the equivalent of } F = ma. \quad 2-6$$

### 6. - Dry unit weight $\gamma_d$

$$\gamma_d = \frac{W_s}{V} = \frac{\gamma}{1+w} \quad 2-7$$

The soil is perfectly dry (its moisture is zero).

### 7. - The unit weight of water $\gamma_w$

$$\gamma_w = \frac{W_w}{V_w} \quad \text{where } \gamma = \rho g \text{ (} F = ma \text{)}$$

$$\gamma_w = 62.4 \text{ pcf} = 1 \text{ g / ml} = 1 \text{ kg / liter} = 9.81 \text{ kN / m}^3$$

Note that the above is for fresh water. Salt water is 64 pcf, etc.

**8. - Saturated unit weight of a soil  $\gamma_{sat}$**

$$\gamma_{SAT} = \frac{W_s + W_w}{V_s + V_w + 0} \quad 2-8$$

**9. - Buoyant unit weight of a soil  $\gamma_b$**

$$\gamma_b = \gamma' = \gamma_{SAT} - \gamma_w \quad 2-9$$

**10. - Specific gravity of the solids of a soil  $G$**

$$G_s = \frac{\gamma_s}{\gamma_w} \quad 2-10$$

<b><u>Typical Values for the Specific Gravity of Minerals in Soils and Rocks</u></b>		
<b><u>Mineral</u></b>	<b><u>Composition</u></b>	<b><u>Absolute specific gravity <math>G_s</math></u></b>
Anhydrite	CaSO <sub>4</sub>	2.90
Barites	BaSO <sub>4</sub>	4.50
Calcite, chalk	CaCO <sub>3</sub>	2.71
Feldspar	KAlSi <sub>3</sub> O <sub>8</sub>	2.60 to 2.70
Gypsum	CaSO <sub>4</sub> 2H <sub>2</sub> O	2.30
Hematite	Fe <sub>2</sub> O <sub>3</sub>	5.20
Kaolinite	Al <sub>2</sub> Si <sub>2</sub> O <sub>5</sub> (OH) <sub>4</sub>	2.60
Magnetite	Fe <sub>3</sub> O <sub>4</sub>	5.20
Lead	Pb	11.34
Quartz (silica)	SiO <sub>2</sub>	2.65
Peat	Organic	1.0 or less
Diatomaceous earth	Skeletons of plants	2.00

**Other useful formulas dealing with phase relationships:**

$$Se = wG_s$$

$$e = \frac{\gamma_s}{\gamma_{dry}} - 1$$

*Unit weight relationships :*

$$\gamma = \frac{(1+w)G_s\gamma_w}{1+e} = \frac{(G_s + Se)\gamma_w}{1+e} = \frac{(1+w)G_s\gamma_w}{1 + \frac{wG_s}{S}} = G_s\gamma_w(1-n)(1+w)$$

*Saturated unit weights :*

$$\gamma_{SAT} = \frac{(G_s + e)\gamma_w}{1+e} = \left(\frac{e}{w}\right)\left(\frac{1+w}{1+e}\right)\gamma_w$$

$$\gamma_{SAT} = \gamma_d + n\gamma_w = \left[(1-n)G_s + n\right]\gamma_w = \left(\frac{1+w}{1+wG_s}\right)G_s\gamma_w$$

$$\gamma_{SAT} = \gamma' + \gamma_w$$

*Dry unit weights :*

$$\gamma_d = \frac{\gamma}{1+w} = G_s\gamma_w(1-n) = \frac{G_s\gamma_w}{1+e} = \frac{eS\gamma_w}{(1+e)w} = \frac{eG_s\gamma_w}{(S+wG_s)}$$

$$\gamma_d = \gamma_{SAT} - n\gamma_w = \gamma_{SAT} - \left(\frac{e}{1+e}\right)\gamma_w$$

.

**\*Phases of soils-01: Convert from metric units to SI and US units.**

(Revision: Oct.-08)

A cohesive soil sample was taken from an *SPT* and returned to the laboratory in a glass jar. It was found to weigh 140.5 grams. The sample was then placed in a container of  $V = 500 \text{ cm}^3$  and 423  $\text{cm}^3$  of water were added to fill the container. From these data, what was the unit weight of the soil in  $\text{kN/m}^3$  and  $\text{pcf}$ ?

**Solution.**

Notice that the 140.5 grams is a mass. Therefore, the ratio of mass to volume is a density  $\rho$ ,

$$\rho = \frac{m}{V} = \frac{140.5 \text{ g}_f}{(500 - 423) \text{ cm}^3} = 1.82 \frac{\text{g}_f}{\text{cm}^3}$$

$$\gamma = \rho g = \left( 1.82 \frac{\text{g}_f}{\text{cm}^3} \right) \left( \frac{1 \text{ kg}_f}{10^3 \text{ g}_f} \right) \left( 9.806 \frac{\text{m}}{\text{sec}^2} \right) \left( \frac{1 \text{ kN}}{10^3 \text{ N}} \right) \left( \frac{10^2 \text{ cm}}{1 \text{ m}} \right)^3 = 17.9 \frac{\text{kN}}{\text{m}^3} \text{ (SI units)}$$

$$\gamma = \left( 17.9 \frac{\text{kN}}{\text{m}^3} \right) \left( \frac{1000 \text{ N}}{1 \text{ kN}} \right) \left( \frac{0.2248 \text{ lbs}_f}{1 \text{ N}} \right) \left( \frac{1 \text{ m}^3}{35.3 \text{ ft}^3} \right) = 114 \text{ pcf (US units)}$$

### **\*Phases of soils–02: Compaction checked via the voids ratio.**

(Revision: Sept.- 08)

A contractor has compacted the base course for a new road and found that the mean value of the test samples shows  $w = 14.6\%$ ,  $G_s = 2.81$ , and  $\gamma = 18.2 \text{ kN/m}^3$ . The specifications require that  $e \leq 0.80$ . Has the contractor complied with the specifications?

#### **Solution:**

$$\gamma = \frac{G_s \gamma_w (1 + w)}{1 + e} \quad \therefore \quad 1 + e = \frac{G_s \gamma_w (1 + w)}{\gamma}$$

$$1 + e = \frac{2.81 \left( 9.81 \frac{\text{kN}}{\text{m}^3} \right) (1 + 0.146)}{18.2 \frac{\text{kN}}{\text{m}^3}} = 1.74$$

$$e = 1.74 - 1 = 0.74$$

$\therefore e = 0.74 < 0.8$  Yes, the contractor has complied.

### **\*Phases of soils–03: Value of the moisture when fully saturated.**

(Revision: Oct.-08)

(1) Show that at saturation the moisture (water) content is  $w_{sat} = \frac{(n\gamma_w)}{(\gamma_{sat} - n\gamma_w)}$  .

(2) Show that at saturation the moisture (water) content is  $w_{sat} = \gamma_w \left( \frac{1}{\gamma_d} - \frac{1}{\gamma_s} \right)$

#### **Solution:**

(1) In a fully saturated soil the relation,  $Se = wG_s$  becomes simply  $e = wG_s$

$$\text{because } S = 1 \quad \text{or} \quad G_s = \frac{e}{w_{sat}} = \frac{n}{w_{sat}(1-n)}$$

$$\text{but } \gamma_{sat} = \gamma_w [(1-n)G_s + n]$$

$$\text{rearranging } \frac{\gamma_{sat}}{\gamma_w} = [(1-n)G_s + n] = \left[ (1-n) \frac{n}{w_{sat}(1-n)} + n \right] = \frac{n}{w_{sat}} + n$$

$$\text{or } \frac{\gamma_{sat}}{\gamma_w} - n = \frac{n}{w_{sat}} \quad \text{therefore } w_{sat} = \frac{n\gamma_w}{\gamma_{sat} - n\gamma_w}$$

$$(2) \text{ Again, in a fully saturated soil, } w_{sat} = \frac{e}{G_s} = \frac{V_v}{V_s} \frac{\gamma_w}{\gamma_s} = \frac{V_v}{V_s} \frac{\gamma_w}{1} \frac{V_s}{W_s} = \frac{\gamma_w V_v}{W_s}$$

$$\therefore w_{sat} = \frac{\gamma_w V_v}{W_s} = \gamma_w \left( \frac{V_v}{W_s} \right) = \gamma_w \left( \frac{V_v + V_s - V_s}{W_s} \right) = \gamma_w \left( \frac{V_v + V_s}{W_s} - \frac{V_s}{W_s} \right)$$

$$\text{or } w_{sat} = \gamma_w \left( \frac{1}{\gamma_d} - \frac{1}{\gamma_s} \right)$$

### \*Phases of soils–04: Finding the wrong data.

(Revision: Oct.-08)

A geotechnical laboratory reported these results of five samples taken from a single boring. Determine which are not correctly reported, if any.

Sample #1:  $w = 30\%$ ,  $\gamma_d = 14.9 \text{ kN/m}^3$ ,  $\gamma_s = 27 \text{ kN/m}^3$ ; clay.

Sample #2:  $w = 20\%$ ,  $\gamma_d = 18 \text{ kN/m}^3$ ,  $\gamma_s = 27 \text{ kN/m}^3$ ; silt.

Sample #3:  $w = 10\%$ ,  $\gamma_d = 16 \text{ kN/m}^3$ ,  $\gamma_s = 26 \text{ kN/m}^3$ ; sand.

Sample #4:  $w = 22\%$ ,  $\gamma_d = 17.3 \text{ kN/m}^3$ ,  $\gamma_s = 28 \text{ kN/m}^3$ ; silt.

Sample #5:  $w = 22\%$ ,  $\gamma_d = 18 \text{ kN/m}^3$ ,  $\gamma_s = 27 \text{ kN/m}^3$ ; silt.

#### Solution:

$$w_{sat} = \frac{e}{G_s} = \frac{V_v \gamma_w}{V_s \gamma_s} = \frac{V_v \gamma_w}{V_s} \frac{V_s}{W_s} = \frac{\gamma_w V_v}{W_s} = \gamma_w \left( \frac{V_v}{W_s} \right) = \gamma_w \left( \frac{V_v + V_s - V_s}{W_s} \right)$$

$$w_{sat} = \gamma_w \left( \frac{V_v + V_s}{W_s} - \frac{V_s}{W_s} \right) = \gamma_w \left( \frac{1}{\gamma_d} - \frac{1}{\gamma_s} \right)$$

The water content is in error if it is greater than the saturated moisture, that is,

$$\therefore w \leq w_{SAT} = \gamma_w \left( \frac{1}{\gamma_d} - \frac{1}{\gamma_s} \right)$$

$$1) \quad w_{SAT} = (9.81 \text{ kN/m}^3) \left( \frac{1}{14.9} - \frac{1}{27} \right) = 30\% = w = 30\% \text{ GOOD}$$

$$2) \quad w_{SAT} = (9.81 \text{ kN/m}^3) \left( \frac{1}{18} - \frac{1}{27} \right) = 18.5\% < w = 20\% \text{ WRONG}$$

$$3) \quad w_{SAT} = (9.81 \text{ kN/m}^3) \left( \frac{1}{16} - \frac{1}{26} \right) = 24\% > w = 10\% \text{ GOOD}$$

$$4) \quad w_{SAT} = (9.81 \text{ kN/m}^3) \left( \frac{1}{17.3} - \frac{1}{28} \right) = 22.1\% > w = 22\% \text{ GOOD}$$

$$5) \quad w_{SAT} = (9.81 \text{ kN/m}^3) \left( \frac{1}{18} - \frac{1}{27} \right) = 18.5\% < w = 22\% \text{ WRONG}$$

**\*Phases of soils–05: Increasing the saturation of a soil.**

(Revision: Sept.-08)

A soil sample has a unit weight of 105.7 pcf and a saturation of 50%. When its saturation is increased to 75%, its unit weight raises to 112.7 pcf.

Determine the voids ratio  $e$  and the specific gravity  $G_s$  of this soil.

**Solution:**

$$\gamma = \frac{\gamma_w (G_s + Se)}{1 + e}$$

$$\therefore 105.7 \text{ pcf} = \frac{62.4(G_s + 0.50e)}{1 + e} \quad (1)$$

$$\text{and } 112.7 \text{ pcf} = \frac{62.4(G_s + 0.75e)}{1 + e} \quad (2)$$

Solving explicitly for  $G_s$  in equation (1),

$$G_s = \frac{(105.7)(1 + e)}{62.4} - 0.50e$$

Replace  $G_s$  in equation (2) with the above relation from (1),

$$\therefore (112.7)(1 + e) = (105.7)(1 + e) + (62.4)(0.25e)$$

$$\therefore \underline{e = 0.814 \text{ and } G_s = 2.67}$$

**\*Phases of soils–06: Find  $\gamma_d$ ,  $n$ ,  $S$  and  $W_w$ .**

(Revision: Sept.-08)

The moist unit weight of a soil is  $16.5 \text{ kN/m}^3$ . Given that the  $w = 15\%$  and  $G_s = 2.70$ , find:

- (1) Dry unit weight  $\gamma_d$ ,
- (2) The porosity  $n$ ,
- (3) The degree of saturation  $S$ , and
- (4) The mass of water in  $\text{kg}_m/\text{m}^3$  that must be added to reach full saturation.

**Solution:**

$$a) \quad \gamma_d = \frac{\gamma}{(1 + w)} = \frac{16.5}{(1 + 0.15)} = 14.3 \frac{\text{kN}}{\text{m}^3}$$

b) From the table of useful relationships,

$$\gamma_d = \frac{G_s \gamma_w}{1 + e} \quad \therefore 1 + e = \frac{G_s \gamma_w}{\gamma_d} = \frac{(2.70)(9.81)}{(14.3)} = 1.85 \quad \therefore e = 0.85$$

$$n = \frac{e}{1 + e} = \frac{0.85}{1 + 0.85} (100\%) = 46\%$$

$$c) \quad \text{Since } Se = wG_s \quad \therefore S = \frac{wG_s}{e} = \frac{(0.15)(2.70)}{(0.85)} (100) = 48\%$$

$$d) \quad \gamma_{\text{sat}} = \frac{(G_s + e) \gamma_w}{1 + e} = \frac{(2.70 + 0.85)(9.81)}{1 + 0.85} = 18.8 \frac{\text{kN}}{\text{m}^3}$$

The water to be added can be found from the relation  $\gamma = \rho g$

$$\therefore \rho(\text{mass of water}) = \frac{\gamma}{g} = \left[ \frac{(18.8 - 16.5) \text{ kN} / \text{m}^3}{9.81 \text{ kg} \cdot \text{m} / \text{s}^2} \right] \left( \frac{1,000 \text{ N}}{1 \text{ kN}} \right) \left( \frac{9.81 \text{ kg} \cdot \text{m} / \text{s}^2}{\text{N}} \right) = 2,340 \frac{\text{kg}_m}{\text{m}^3}$$

**\*Phases of soils–07: Use the block diagram to find the degree of saturation.**

(Revision: Sept.-08)

A soil has an “in-situ” (in-place) voids ratio  $e_o = 1.87$ ,  $w_N = 60\%$ , and  $G_s = 2.75$ . What are the  $\gamma_{\text{moist}}$  and  $S$ ? (Note: All soils are really “moist” except when dry, that is when  $w = 0\%$ ).

---

**Solution:** Set  $V_s = 1 \text{ m}^3$  (Note: this problem could also be solved by setting  $V = 1.0 \text{ m}^3$ ).

$$\therefore e_o = \frac{V_v}{V_s} = \frac{1.87}{1} = 1.87 \quad \therefore V = V_s + V_v = 1 + 1.87 = 2.87 \text{ m}^3$$

The "natural" water content is  $w_N = \frac{W_w}{W_s} = 0.60 \quad \therefore W_w = 0.60 W_s$

$$G_s = \frac{\gamma_s}{\gamma_w} = \frac{V_s}{V_w} \quad \therefore W_s = V_s (G_s \gamma_w) = (1 \text{ m}^3)(2.75)(9.81 \text{ kN/m}^3) = 26.98 \text{ kN}$$

$$W_w = 0.60(W_s) = (0.60)(26.98) = 16.19 \text{ kN}$$

$$W = W_s + W_w = 26.98 + 16.19 = 43.17 \text{ kN}$$

$$\therefore \gamma_{\text{moist}} = \frac{W}{V} = \frac{43.17 \text{ kN}}{2.87 \text{ m}^3} = 15.0 \frac{\text{kN}}{\text{m}^3}$$

$$\therefore S = \frac{V_w}{V_v} = \frac{\gamma_w}{\gamma_v} = \frac{\left(\frac{16.19}{9.81}\right)}{1.87} = 88.2\%$$

**\*Phases of soils–08: Same as Prob-07 but setting the total volume  $V=1 \text{ m}^3$ .**

(Revision: Oct.-08)

A soil has an “in-situ” (in-place) voids ratio  $e_o = 1.87$ ,  $w_N = 60\%$ , and  $G_s = 2.75$ . What are the  $\gamma_{moist}$  and  $S$ ? (Note: All soils are really “moist” except when dry, that is when  $w = 0\%$ ).

---

**Solution:** Set  $V = 1 \text{ m}^3$  (instead of  $V_s = 1 \text{ m}^3$  used in Phases-07).

$$\text{but } e_o = \frac{V_v}{V_s} = 1.87 \therefore \text{but } V = 1 \text{ m}^3 = V_s + V_v = V_s + 1.87V_s = 2.87V_s \therefore V_s = 0.348 \text{ and } V_v = 0.652$$

$$\text{The "natural" water content is } w_N = \frac{W_w}{W_s} = 0.60 \therefore W_w = 0.60W_s$$

$$G_s = \frac{\gamma_s}{\gamma_w} = \frac{V_s}{V_w} \therefore W_s = V_s (G_s \gamma_w) = (0.348 \text{ m}^3)(2.75)(9.81 \text{ kN} / \text{m}^3) = 9.39 \text{ kN}$$

$$W_w = 0.60(W_s) = (0.60)(9.39) = 5.63 \text{ kN}$$

$$W = W_s + W_w = 9.39 + 5.63 = 15.02 \text{ kN}$$

$$\therefore \gamma_{moist} = \frac{W}{V} = \frac{15.0 \text{ kN}}{1 \text{ m}^3} = 15.0 \frac{\text{kN}}{\text{m}^3}$$

$$\therefore S = \frac{V_w}{V_v} = \frac{\gamma_w}{V_v} = \frac{\left(\frac{5.63}{9.81}\right)}{0.652} = 88.0\%$$

**\*Phases of soils–09: Same as Problem #5 with a block diagram.**

(Revision: Sept.-08)

A soil sample has a unit weight of 105.7 pcf and a water content of 50%. When its saturation is increased to 75 %, its unit weight raises to 112.7 pcf. Determine the voids ratio  $e$  and the specific gravity  $G_s$  of the soil. (NB: This is the same problem as Phase–06, but solved with a block diagram).

**Solution:**

$$\text{Set } V = 1 \text{ ft}^3$$

$$\gamma_2 - \gamma_1 = 112.7 - 105.7 = 7.0 \text{ lbs are 25\% of water}$$

$$\therefore 21.0 \text{ lbs are 75\% of water}$$

$$\therefore W_s = 112.7 - 20.8 = 91.9 \text{ lb}$$

$$V_w = \frac{W_w}{\gamma_w} = \frac{20.8 \text{ lb}}{62.4 \text{ pcf}} = 0.333 \text{ ft}^3$$

$$V_a = \frac{1}{3} V_w = 0.111 \text{ ft}^3 \quad \therefore V_v = V_a + V_w = 0.111 + 0.333 = 0.444$$

$$\therefore V_s = 1 - V_v = 1 - 0.444 = 0.556$$

$$e = \frac{V_v}{V_s} = \frac{0.444}{0.556} = 0.80$$

$$\text{and } G_s = \frac{\gamma_s}{\gamma_w} = \frac{W_s}{V_s \gamma_w} = \frac{91.9 \text{ lb}}{(0.556)(62.4)} = 2.65$$

**\*Phases of soils–10: Block diagram for a saturated soil.**

(Revision: Sept.-08)

A saturated soil sample has a unit weight of 122.5 pcf and  $G_s = 2.70$ . Find  $\gamma_{dry}$ ,  $e$ ,  $n$ , and  $w$ .

**Solution:**

$$V = V_s + V_w = \frac{1}{\gamma_w} \left( \frac{W_s}{G_s} + W_w \right) \quad (1)$$

$$W = W_s + W_w = 122.5 \text{ lb} \quad (2)$$

Combining equations (1) and (2) yields  $1 = \frac{1}{(62.4 \text{ pcf})} \left( \frac{122.5 - W_w}{2.70} + W_w \right)$

$$\therefore W_w = 27.0 \text{ lb} \quad \therefore V_w = \frac{W_w}{\gamma_w} = \frac{27.0 \text{ lb}}{62.4 \text{ pcf}} = 0.433 \text{ ft}^3$$

$$\therefore W_s = 95.5 \text{ lb} \quad \therefore V_s = \frac{W_s}{G_s \gamma_w} = \frac{95.5 \text{ lb}}{(2.70)(62.4 \text{ pcf})} = 0.567 \text{ ft}^3$$

$$\therefore \gamma_{dry} = \frac{W_s}{V} = \frac{95.5 \text{ lb}}{1 \text{ ft}^3} = 95.5 \text{ pcf}$$

$$\therefore e = \frac{V_v}{V_s} = \frac{0.433}{0.567} = 0.764$$

$$\therefore n = \frac{V_v}{V} = \frac{0.433}{1} = 0.433 \quad n = 43.3\%$$

$$\therefore w = \frac{W_w}{W_s} = \frac{27}{95.5} = 0.283 \quad w = 28.3\%$$

**\*Phases of soils–11: Find the weight of water needed for saturation.**

(Revision: Sept.-08)

Determine the weight of water (in  $kN$ ) that must be added to a cubic meter of soil to attain a 95 % degree of saturation, if the dry unit weight is  $17.5 \text{ kN/m}^3$ , its moisture is 4%, the specific gravity of solids is 2.65 and the soil is entirely made up of a clean quartz sand.

**Solution:**

$$\gamma_d = 17.5 \frac{kN}{m^3} = \frac{\gamma}{1+w} = \frac{\gamma}{1+0.04} \therefore \gamma = 18.2 \frac{kN}{m^3}$$

$$W = 18.2 = W_s + W_w = W_s + w W_s = (1.04) W_s$$

$$\therefore W_s = 17.5 \text{ kN}, \quad \text{and} \quad W_w = 0.70 \text{ kN}$$

$$V_s = \frac{W_s}{\gamma_s} = \frac{17.5 \text{ kN}}{G_s \gamma_w} = \frac{17.5 \text{ kN}}{(2.65)(9.81 \text{ kN/m}^3)} = 0.673 \text{ m}^3$$

$$V_w = \frac{W_w}{\gamma_w} = \frac{0.70 \text{ kN}}{(9.81 \text{ kN/m}^3)} = 0.07 \text{ m}^3 \quad \therefore V_a = V - V_s - V_w = 0.257 \text{ m}^3$$

$$e = \frac{V_v}{V_s} = \frac{0.07 + 0.257}{0.673} = 0.49$$

$$\text{The existing } S = \frac{w G_s}{e} = \frac{(0.04)(2.65)}{0.49}(100) = 21.6\%$$

We require a  $S = 95\%$ , therefore,

$$w = \frac{Se}{G_s} = \frac{(0.95)(0.49)}{2.65} = 0.17$$

$$W_w = w W_s = (0.17)(17.5 \text{ kN}) = 2.98 \text{ kN}$$

$$\text{already have } W_w = 0.70 \text{ kN}$$

$$\therefore \text{ must add water} = 2.28 \text{ kN}$$

**Answer:** Add 2.28 kN of water per  $m^3$ .

### **\*Phases of soils–12: Identify the wrong piece of data.**

(Revision: Sept.-08)

A project engineer receives a laboratory report with tests performed on marine marl calcareous silt). The engineer suspects that one of the measurements is in error. Are the engineer's suspicions correct? If so, which one of these values is wrong, and what should be its correct value?

$$\text{Given } \gamma = \text{unit weight of sample} = 18.4 \frac{kN}{m^3}$$

$$\gamma_s = \text{unit weight of solids} = 26.1 \frac{kN}{m^3}$$

$$w = \text{water content} = 40\%$$

$$e = \text{voids ratio} = 1.12$$

$$S = \text{degree of saturation} = 95\%$$

#### **Solution:**

Check the accuracy of 4 out of 5 of the variables using,

$$Se = wG_s \therefore Se = (0.95)(1.12) = 1.06$$

$$wG_s = (w) \frac{\gamma_s}{\gamma_w} = (0.4) \frac{26.1}{9.81} = 1.06 \therefore \text{Therefore, these four are correct.}$$

The only possibly incorrect value is  $\gamma$ . Assume that  $V = 1 \text{ m}^3$ .

$$V = 1 \text{ m}^3 = V_a + V_w + V_s \quad (1)$$

$$\text{but } e = \frac{V_v}{V_s} = 1.12 \therefore 0 = -V_a - V_w + 1.12V_s \quad (2)$$

$$\therefore V_s = 0.472 \text{ m}^3, V_v = 0.528 \text{ m}^3 \text{ but } V_w = 0.95V_v = 0.502 \text{ m}^3$$

$$\therefore V_a = 0.026 \text{ m}^3$$

$$\therefore W_s = \gamma_s V_s = \left( 26.1 \frac{kN}{m^3} \right) (0.472 \text{ m}^3) = 12.3 \text{ kN}$$

$$W_w = wW_s = (0.40) \left( 12.3 \frac{kN}{m^3} \right) = 4.9 \text{ kN}$$

$$W = 12.3 \text{ kN} + 4.9 \text{ kN} = 17.2 \text{ kN}$$

Therefore, the actual unit weight of the soil is,

$$\therefore \gamma = \frac{W}{V} = \frac{17.2 \text{ kN}}{1 \text{ m}^3} = 17.2 \frac{kN}{m^3} \neq 18.4 \frac{kN}{m^3}$$

### ***\*Phases of soils–13: The apparent cheapest soil is not!***

(Revision: Sept.-08)

You are a Project Engineer on a large earth dam project that has a volume of  $5 \times 10^6 \text{ yd}^3$  of select fill, compacted such that the final voids ratio in the dam is 0.80. Your boss, the Project Manager delegates to you the important decision of buying the earth fill from one of three suppliers. Which one of the three suppliers is the most economical, and how much will you save?

Supplier A Sells fill at \$ 5.28/  $\text{yd}^3$  with  $e = 0.90$

Supplier B Sells fill at \$ 3.91/  $\text{yd}^3$  with  $e = 2.00$

Supplier C Sells fill at \$ 5.19/  $\text{yd}^3$  with  $e = 1.60$

#### **Solution:**

Without considering the voids ratio, it would appear that Supplier B is cheaper than Supplier A by \$1.37 per  $\text{yd}^3$ .

Therefore: To put  $1 \text{ yd}^3$  of solids in the dam you would need  $1.8 \text{ yd}^3$  of soil.

For  $1 \text{ yd}^3$  of solids from A you would need  $1.9 \text{ yd}^3$  of fill.

For  $1 \text{ yd}^3$  of solids from B you would need  $3.0 \text{ yd}^3$  of fill.

For  $1 \text{ yd}^3$  of solids from C you would need  $2.6 \text{ yd}^3$  of fill.

The cost of the select fill from each supplier is (rounding off the numbers):

$$A = \frac{1.9}{1.8} (5) (10^6 \text{ yd}^3) \left( \frac{5.28\$}{\text{yd}^3} \right) \approx \$ 27,900,000$$

$$B = \frac{3.0}{1.8} (5) (10^6 \text{ yd}^3) \left( \frac{3.91\$}{\text{yd}^3} \right) \approx \$ 32,600,000$$

$$C = \frac{2.6}{1.8} (5) (10^6 \text{ yd}^3) \left( \frac{5.19\$}{\text{yd}^3} \right) \approx \$ 37,500,000$$

**Therefore Supplier A is the cheapest by about \$ 4.7 Million compared to Supplier B.**

### **\*Phases of soils–14: Number of truck loads.**

(Revision: Sept.-08)

Based on the previous problem (Phases–13), if the fill dumped into the truck has an  $e = 1.2$ , how many truck loads will you need to fill the dam? Assume each truck carries  $10 \text{ yd}^3$  of soil.

#### **Solution:**

Set  $V_s = 1$   $e = \frac{V_v}{V_s} = \frac{V_v}{1} = V_v = 1.2$  which means that there is  $1 \text{ yd}^3$  of solids per  $1.2 \text{ yd}^3$  of voids.

$\therefore 2.2 \text{ yd}^3$  of soil for each  $1 \text{ yd}^3$  of solids.

$10 \text{ yd}^3$  of soil for each  $x \text{ yd}^3$  in a truck load

$\therefore x = 4.54 \text{ yd}^3$  of solids per truck trip.

The required volume of solids in the dam is,

$$V_{\text{solids}} = \frac{(5 \times 10^6 \text{ yd}^3 \text{ of soil})(1 \text{ yd}^3 \text{ of solids})}{1.8 \text{ yd}^3 \text{ of soil}} = 2.8 \times 10^6 \text{ yd}^3 \text{ of solids}$$

Therefore, (rounding off)

$$\text{Number of Truck-trips} = \frac{(2.8 \times 10^6 \text{ yd}^3 \text{ of solids})}{(4.54 \text{ yd}^3 \text{ of solids / truck-trip})} = 616,800$$

### **\*Phases of soils–15: How many truck loads are needed for a project?**

(Revision: Sept.-08)

**You have been hired as the Project Engineer for a development company in South Florida to build 610 housing units surrounding four lakes. Since the original ground is low, you will use the limestone excavated from the lake to fill the land in order to build roads and housing pads. Your estimated fill requirements are 700,000 m<sup>3</sup>, with a dry density equivalent to a voids ratio  $e = 0.46$ . The “in-situ” limestone extracted from the lakes has an  $e = 0.39$ , whereas the limestone dumped into the trucks has an  $e = 0.71$ . How many truckloads will you need, if each truck carries 10 m<sup>3</sup>?**

#### **Solution:**

Assume:  $V_s = 1 \text{ m}^3 \therefore e = \frac{V_v}{V_s} = \frac{V_v}{1} = V_v = 0.46 \text{ m}^3$  in the compacted fill

The required 700,000 m<sup>3</sup> of fill have 1.46 m<sup>3</sup> of voids per each 1 m<sup>3</sup> of solids

Therefore, the 700,000 m<sup>3</sup> of fill have 479,400 m<sup>3</sup> of solids

Each truck carries 1.71 m<sup>3</sup> of fill per 1 m<sup>3</sup> solids

In order for the trucks to carry 479,000 m<sup>3</sup> of solids they must carry 820,000 m<sup>3</sup> of fill

Since each truck carries 10 m<sup>3</sup> of fill,

$$\therefore \text{The number of truck-loads} = \frac{820,000 \text{ m}^3}{10 \text{ m}^3} = 82,000 \text{ truck-loads.}$$

### **\*Phases of soils–16: Choose the cheapest fill supplier.**

(Revised: Sept.-08)

A large housing development requires the purchase and placement of the fill estimated to be 200,000 cubic yards of lime-rock compacted at 95% Standard Proctor with an *OMC* of 10%. Two lime-rock suppliers offer to fill your order: Company A has a borrow material with an in-situ  $\gamma = 115$  pcf,  $w = 25\%$ ,  $G_s = 2.70$ ; Standard Proctor yields a maximum  $\gamma_d = 112$  pcf; at a cost of \$0.20/yd<sup>3</sup> to excavate, and \$ 0.30/yd<sup>3</sup> to haul. Company B has a borrow material with an in-situ  $\gamma = 120$  pcf,  $w = 20\%$ ,  $G_s = 2.70$ ; Standard Proctor yields a maximum  $\gamma_d = 115$  pcf; a cost of \$0.22/yd<sup>3</sup> to excavate, and \$ 0.38/yd<sup>3</sup> to haul.

- (1) What volume would you need from company A?
- (2) What volume would you need from company B?
- (3) Which would be the cheaper supplier?

#### **Solution:**

(1) The key idea: 1 yd<sup>3</sup> of solids from the borrow pit supplies 1 yd<sup>3</sup> of solids in the fill.

(2) Pit A:  $W_s = 92$  lb,  $W_w = 23$  lb  $\longrightarrow V_w = 0.369$  ft<sup>3</sup>,  $V_s = 0.546$  ft<sup>3</sup>,  $V_a = 0.085$  ft<sup>3</sup>

$$e = \frac{V_v}{V_s} = \frac{0.454}{0.546} = 0.83 \quad \therefore 1.83 \text{ yd}^3 \text{ of soil contains } 1.0 \text{ yd}^3 \text{ of solids.}$$

Pit B:  $W_s = 100$  lb,  $W_w = 20$  lb,  $V_w = 0.321$  ft<sup>3</sup>,  $V_s = 0.594$  ft<sup>3</sup>,  $V_a = 0.08$  ft<sup>3</sup>

$$e = \frac{V_v}{V_s} = \frac{0.401}{0.594} = 0.68 \quad \therefore 1.68 \text{ yd}^3 \text{ of soil contains } 1.0 \text{ yd}^3 \text{ of solids.}$$

(3) Material needed for fill from company A:

$$\gamma = 0.95\gamma_d(1+w) = 0.95(112)(1+0.10) = 117 \text{ pcf} \quad \therefore W_s = 106.4 \text{ lb}, \quad W_w = 10.6 \text{ lb}$$

$$e = \frac{V_v}{V_s} = \frac{0.37}{0.63} = 0.59 \quad \therefore 1.59 \text{ yd}^3 \text{ of soil contains } 1.0 \text{ yd}^3 \text{ of solids}$$

$$\therefore \text{Site A requires } \frac{200,000 \text{ yd}^3 \text{ of fill}}{1.59} = 125,800 \text{ yd}^3 \text{ of solids}$$

Material needed for fill from company B:

$$\gamma = 0.95\gamma_d(1+w) = 0.95(115)(1+0.10) = 120 \text{ pcf} \quad \therefore W_s = 109.1 \text{ lb}, \quad W_w = 10.9 \text{ lb}$$

$$e = \frac{V_v}{V_s} = \frac{0.35}{0.65} = 0.54 \quad \therefore 1.54 \text{ yd}^3 \text{ of soil contains } 1.0 \text{ yd}^3 \text{ of solids}$$

$$\therefore \text{Site B requires } \frac{200,000 \text{ yd}^3 \text{ of fill}}{1.54} = 130,000 \text{ yd}^3 \text{ of solids}$$

(4) a) Cost of using Company A:

$$Cost_A = (125,800 \text{ yd}^3)(1.83) \left( \frac{\$0.50}{\text{yd}^3} \right) = \$115,100$$

Cost of using Company B:

$$Cost_B = (130,000 \text{ yd}^3)(1.68) \left( \frac{\$0.60}{\text{yd}^3} \right) = \$131,100$$

*Using Company A will save about \$ 16,000.*

**\*Phases of soils–17: Use a matrix to find the missing data.**

(Revision: Sept.-08)

A contractor obtains prices for 34,000  $yd^3$  of compacted “borrow” material from three pits: Pit #3 is \$11,000 cheaper than Pit #2 and \$39,000 cheaper than Pit #1. The fill must be compacted down to a voids ratio of 0.7. Pit #1 costs \$ 6.00/ $yd^3$  and Pit #3 costs \$ 5.50/ $yd^3$ . Pits #2 and #3 reported their voids ratios as 0.88 and 0.95 respectively. Use a matrix to find,

- The missing unit cost  $C_2$  for Pit #2;
- The missing voids ratio  $e$  for Pit #1;
- The missing volume of fill  $V$  required from each pit; and
- The amount paid by the contractor for each pit.

**Solution:**

A summary of the data provided is herein shown in matrix form,

<u>Item</u>	<u>Borrow Sites</u>			<u>Project</u>
	<u>Pit-1</u>	<u>Pit-2</u>	<u>Pit-3</u>	<u>Site</u>
<i>Unit Cost (\$/<math>yd^3</math>)</i>	6.00	$C_2$	5.50	
<i>Voids ratio <math>e</math></i>	$e_1$	0.88	0.95	0.70
<i>Volume (<math>yd^3</math>)</i>	$V_1$	$V_2$	$V_3$	34,000
<i>Total Cost (\$)</i>	$TC_1 = TC_2 + 28,000$	$TC_2$	$TC_3 = TC_2 - \$11,000$	

The volume of solids  $V_s$  contained in the total volume of fill  $V = 34,000 \text{ } yd^3$  can be found from,

$$V = V_V + V_S = 0.7V_S + V_S = V_S(0.7 + 1) = 34,000 \text{ yd}^3 \therefore V_S = \frac{34,000}{1.7} = 20,000 \text{ yd}^3 \text{ of solids}$$

$$\text{At Pit \#3, } \frac{V_3}{V_S} = 1 + e_3 \therefore V_3 = V_S(1 + e_3) = (20,000 \text{ yd}^3)(1 + 0.95) = 39,000 \text{ yd}^3 \text{ of soil}$$

$$\text{The total cost of Pit \#3 is } TC_3 = (39,000 \text{ yd}^3)(\$ 5.50 / \text{yd}^3) = \$ 214,500$$

$$\text{At Pit \#2: } \frac{V_2}{V_S} = 1 + e_2 \therefore V_2 = V_S(1 + e_2) = (20,000 \text{ yd}^3)(1 + 0.88) = 37,600 \text{ yd}^3 \text{ of soil}$$

$$\text{But, the total cost of Pit \#2 is } TC_2 - \$ 11,000 = TC_3 = \$ 214,500 \therefore TC_2 = \$ 225,500$$

$$\text{The unit cost of Pit \#2 } C_2 = \frac{TC_2}{V_2} = \frac{\$ 225,500}{37,600 \text{ yd}^3} = \$ 6.00 / \text{yd}^3$$

$$\text{At Pit \#1: } V_1 = \frac{TC_1}{\$ 6.00 / \text{yd}^3} = \frac{TC_2 + 28,000}{\$ 6.00 / \text{yd}^3} = \frac{225,500 + 28,000}{\$ 6.00 / \text{yd}^3} = 42,250 \text{ yd}^3 \text{ of soil}$$

$$\text{But, } V_1 = V_S(1 + e_1) = (20,000 \text{ yd}^3)(1 + e_1) = 42,250 \text{ yd}^3 \therefore e_1 = 1.11$$

**\*\*Phases of soils–18: Find the voids ratio of “muck” (a highly organic soil).**

(Revision: Sept.-08)

You have been retained by a local municipality to prepare a study of their “muck” soils. Assume that you know the dry unit weight of the material (solids)  $\gamma_{sm}$  and the dry unit weight of the organic solids  $\gamma_{so}$ . What is the unit weight  $\gamma_s$  of the combined dry organic mineral soil whose organic content is  $M_o$ ? (The organic content is the percentage by weight of the dry organic constituent of the total dry weight of the sample for a given volume.) What is the voids ratio  $e$  of this soil if it is known that its water content is  $w$  and its degree of saturation is  $S$ ?

**Solution:**

$$\text{Set } W_s = 1 \text{ unit and } \gamma_s = \frac{W_s}{V_s} = \frac{1}{(V_{so} + V_{sm})}$$

(a) Assume  $M_o = W_o$  for a unit weight of the *dry soil*

Therefore  $1 - M_o = W_m$

$$\frac{M_o}{\gamma_{so}} = \text{volume of organic } V_{so} \text{ solids}$$

$$\frac{(1 - M_o)}{\gamma_{sm}} = \text{volume of mineral } V_{sm} \text{ solids}$$

The total unit weight is the weight of a unit volume.

$$\text{Therefore } \gamma_s = \frac{1}{\left( \frac{M_o}{\gamma_{so}} + \frac{(1 - M_o)}{\gamma_{sm}} \right)} = \gamma_{so} \left[ \frac{\gamma_{sm}}{M_o (\gamma_{sm} - \gamma_{so}) + \gamma_{so}} \right]$$

$$(b) \quad e = \frac{V_v}{V_s} = \frac{\left( \frac{\text{volume of water}}{S} \right)}{V_s} = \frac{\left( \frac{\text{weight of water}}{\gamma_w S} \right)}{V_s} = \frac{\left( \frac{w (\text{weight of solids})}{\gamma_w S} \right)}{V_s}$$

$$\text{Therefore } e = \frac{\left( \frac{w}{\gamma_w S (1)} \right)}{\left( \frac{M_o}{\gamma_{so}} + \frac{(1 - M_o)}{\gamma_{sm}} \right)} = \frac{w \gamma_{sm} \gamma_{so}}{\gamma_w S [M_o (\gamma_{sm} - \gamma_{so}) + \gamma_{so}]}$$

# Chapter 3

## Classification of Soils and Rocks

### **Symbols for Classification of soils**

$C_c \rightarrow$  Coefficient of gradation (also coefficient of curvature).

$C_u \rightarrow$  Coefficient of uniformity.

$RC \rightarrow$  Relative compaction.

$D_x \rightarrow$  Diameter of the grains (at % finer by weight).

$D_r \rightarrow$  Relative density of a granular soil.

$e \rightarrow$  Voids ratio.

$e_{min} \rightarrow$  Minimum voids ratio.

$e_{max} \rightarrow$  Maximum voids ratio.

$I_p \rightarrow$  Index of plasticity (also referred to as  $PI$ ).

$K \rightarrow$  Constant of the yield value.

$LL \rightarrow$  Liquid limit.

$PL \rightarrow$  Plastic limit.

$SL \rightarrow$  Shrinkage limit.

$V \rightarrow$  Volume of the soil sample.

$W \rightarrow$  Weight of the soil sample.

$\gamma_{d(min)} \rightarrow$  Dry unit weight in loosest condition (voids ratio  $e_{max}$ ).

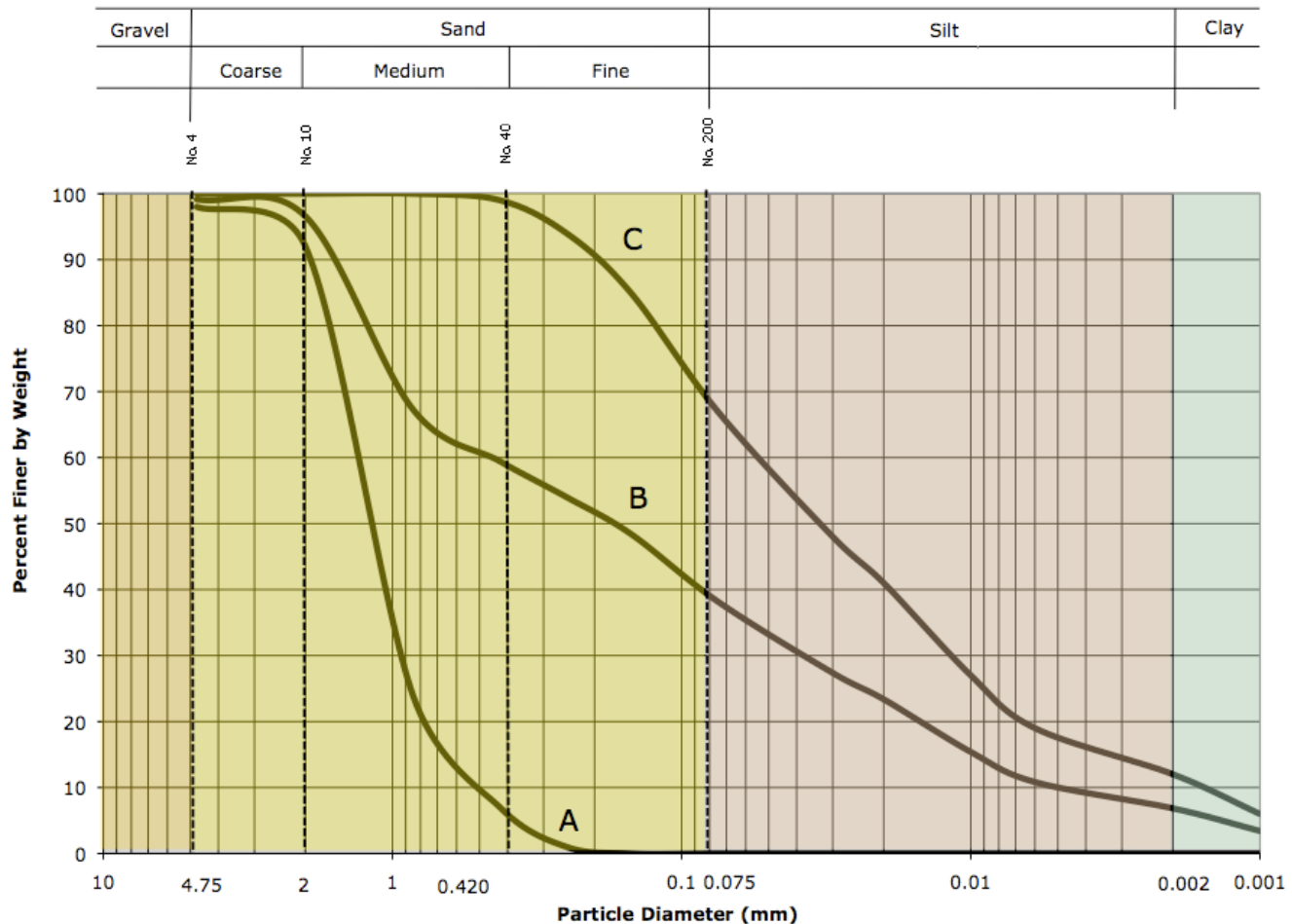
$\gamma_d \rightarrow$  In-situ dry unit weight (voids ratio  $e$ ).

$\gamma_{d(max)} \rightarrow$  Dry unit in densest condition (voids ratio  $e_{min}$ )

### \*Classify–01: Percentage of each of the four grain sizes (G, S, M & C).

(Revision: Sept.-08)

Determine the percentage of gravels (G), sands (S), silts (M) and clays (C) of soils A, B and C



shown below.

#### Solution:

Notice that the separation between gravels (G) and sands (S) is the #4 sieve which corresponds to a particle size of 4.75 mm. The separation between sands (S) and silts (M) is the #200 sieve which corresponds to a particle size of 0.075 mm. Finally, the separation between silts (M) and clays (C) is the 0.002 mm (or 2 micro-meters = 2  $\mu$ m). These divisions are shown above through color differentiation. Each soil A, B and C is now separated into the percentage of each:

**Soil A:** 2% G; 98% S; 0% M; 0%C. This soil is a uniform or poorly-graded sand (SP).

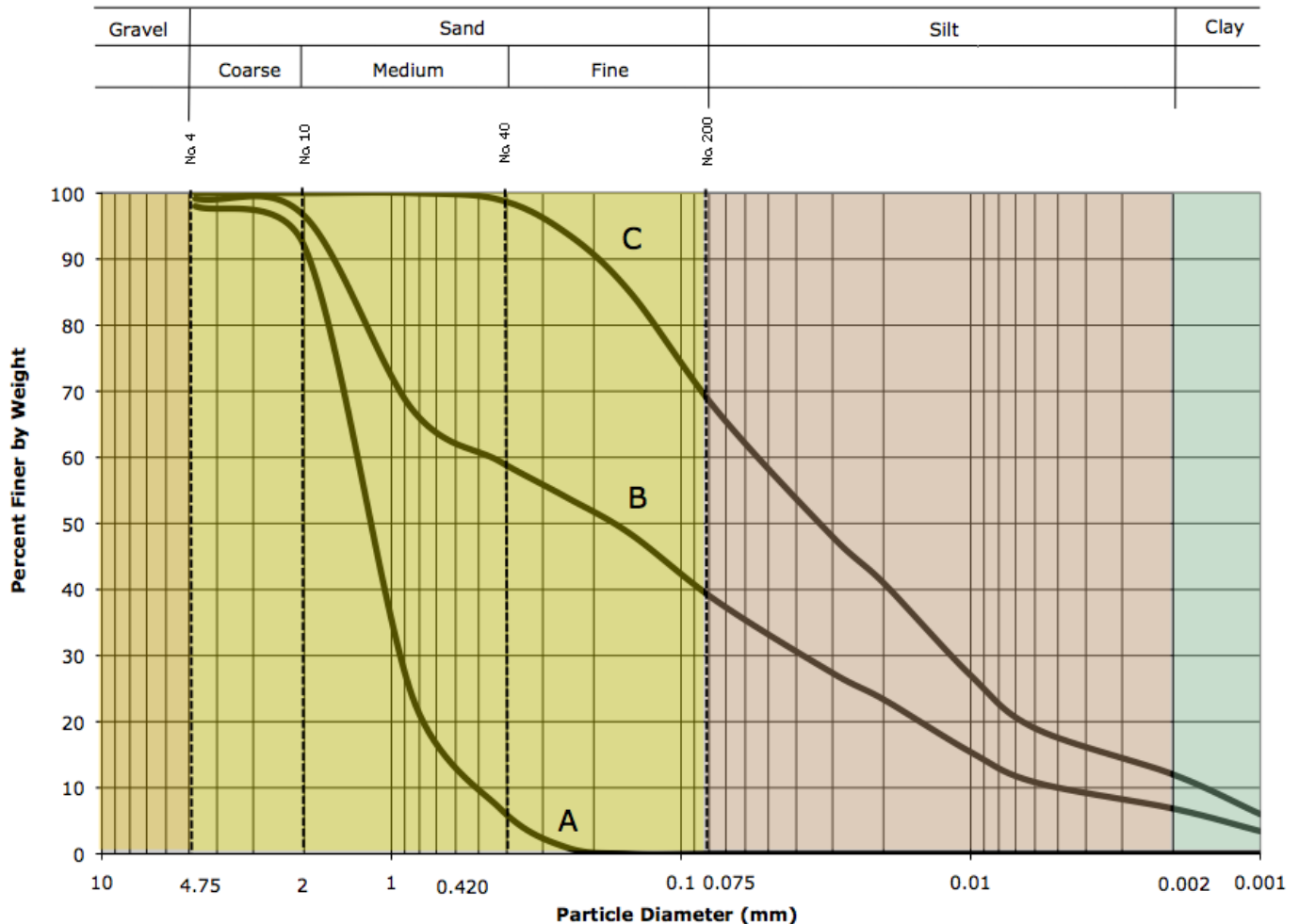
**Soil B:** 1% G; 61% S; 31% M; 7%C. This soil is a well-graded silty sand (SM).

**Soil C:** 0% G; 31% S; 57% M; 12%C. This soil is a well-graded sandy silt (M).

## \*Classify-02: Coefficients of uniformity and curvature of granular soils.

(Revision: Sept.-08)

Determine the uniformity coefficient  $C_u$  and the coefficient of gradation  $C_c$  for soil A.



### Solution:

From the grain distribution curve,  $D_{60} = 1.4$  mm,  $D_{30} = 0.95$  mm and  $D_{10} = 0.50$  mm, therefore the coefficients are,

$$C_u = \frac{D_{60}}{D_{10}} = \frac{1.40 \text{ mm}}{0.50 \text{ mm}} = 2.8 \quad \text{and} \quad C_c = \frac{D_{30}^2}{D_{60}D_{10}} = \frac{(0.95)^2}{(1.40)(0.50)} = 1.29$$

A **uniform** soil has a coefficient of uniformity  $C_u$  less than 4, whereas a **well-graded** soil has a uniformity coefficient greater than 4 for gravels and greater than 6 for sands. Since soil A has a low value of 2.8, and it is sand, this corresponds to a **poorly-graded sand (SP)**. Steep curves are uniform soils (low  $C_u$ ) whereas diagonal curves are well-graded soils (high  $C_u$ ).

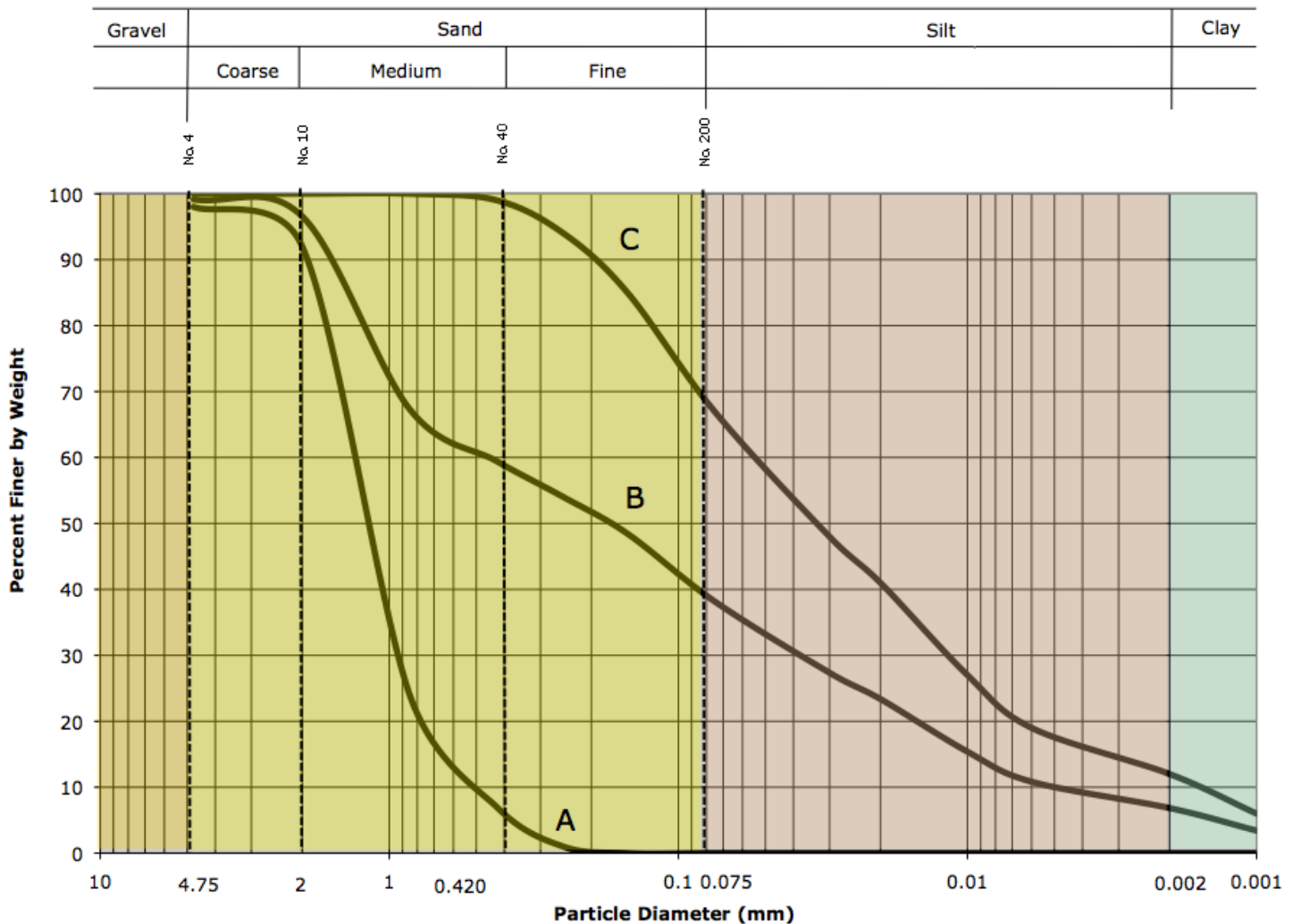
Smooth curved soils have coefficients of curvature  $C_c$  between 1 and 3, whereas irregular curves have higher or lower values. Soils that are missing a type of soil (a gap) are called **gap-graded** ( $C_c$  will be less than 1 or greater than 3 for gap-graded soils).

Therefore, this soil is classified as **poorly-graded sand (or SP)**.

### \*Classify-03: Classify two soils using the USCS.

(Revision: Sept.-08)

Use the grain-size distribution curve shown below to classify soils *A* and *B* using the USCS. Soil *B*'s Atterberg limits are  $LL = 49\%$  and  $PL = 45\%$ ?



#### Solution:

##### Classify Soil A:

For soil A, the distribution is  $G = 2\%$ ,  $S = 98\%$ ,  $M = 0\%$  and  $C = 0\%$ .

$$C_U = \frac{D_{60}}{D_{10}} = \frac{1.40 \text{ mm}}{0.50 \text{ mm}} = 2.8, \text{ therefore, soil A is a poorly graded sand (SP).}$$

##### Classify Soil B:

For soil B, the distribution is  $G = 0\%$ ,  $S = 61\%$ ,  $M = 35\%$  and  $C = 4\%$ .

$$C_U = \frac{D_{60}}{D_{10}} = \frac{0.45 \text{ mm}}{0.005 \text{ mm}} = 90, \text{ therefore, soil A is very well graded silty sand (SM).}$$

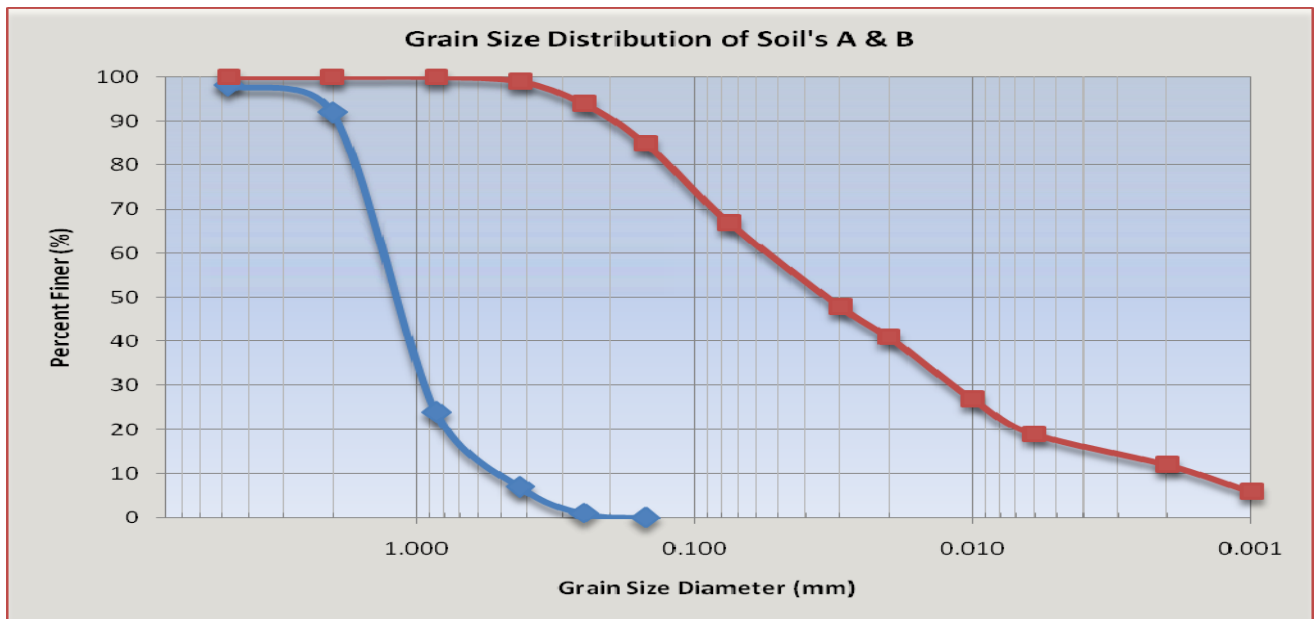
### \*Classify-04: Manufacturing a “new” soil.

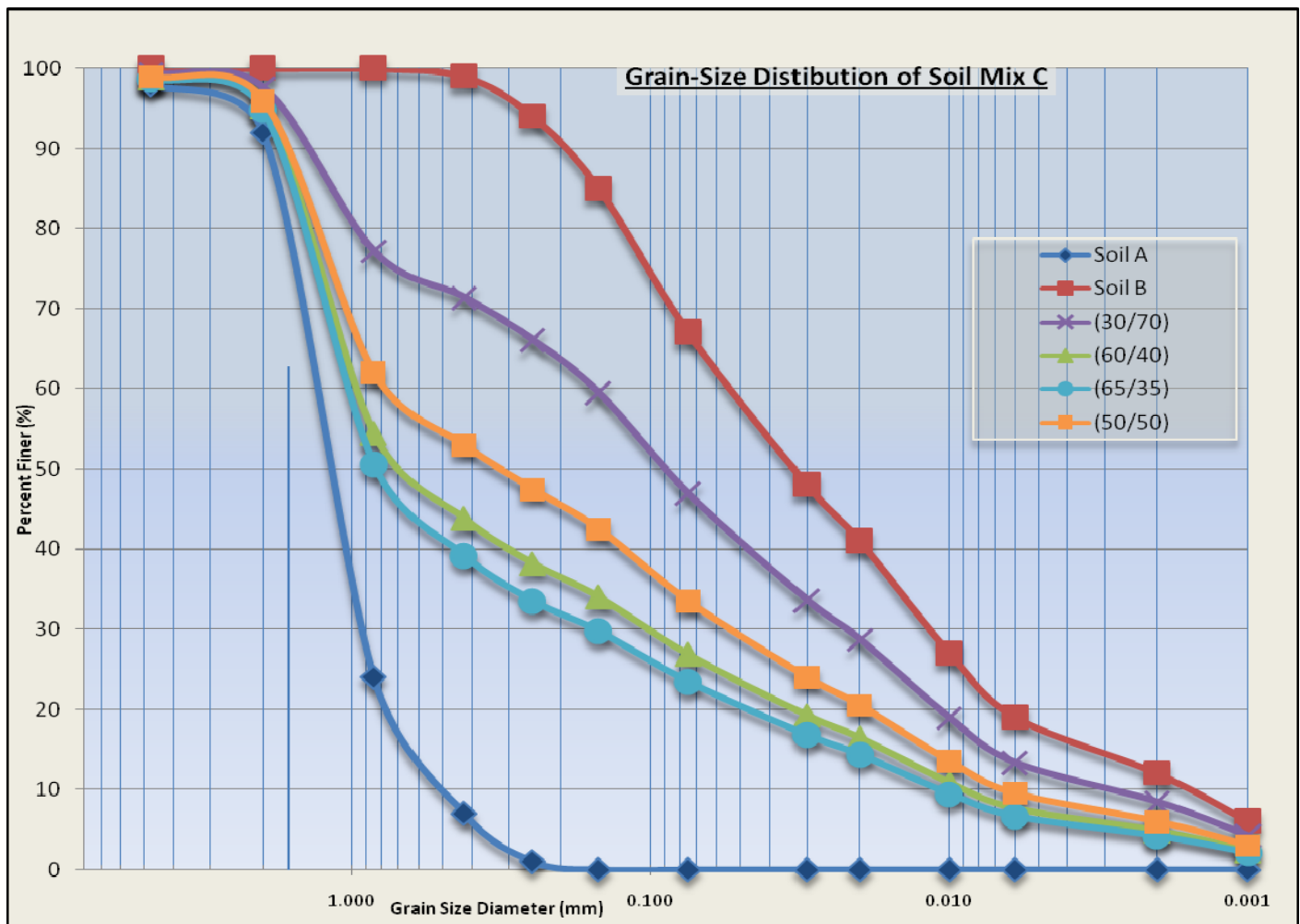
(Revision: Sept.-08)

A site has an unsuitable in-situ soil A that does not compact properly. In lieu of removing that soil A, you have decided to improve it by mixing it with a borrow pit soil B to produce an improved new soil C that will compact better.

You desire a coefficient of uniformity  $C_u$  of about 100 for the new soil C. Determine the relative percentages of these two uniform soils A and B so that they will result in better graded soil C. Plot your results.

The plots of soils A and B are as shown below,





## Classify – 05

(Revision: Sept.-09)

**A sample of soil weighs 1.5 N. Its clay fraction weighs 0.34 N. If its liquid limit is 60% and its plastic limit is 26%, classify the clay.**

### Solution:

$$W = 1.5 \text{ N}$$

$$W_{\text{clay}} = 0.34 \text{ N (or 23\% of W)}$$

$$I_p = PI = LL - PL = 60\% - 26\% = 34 \%$$

$$A = \frac{I_p}{\% \text{ of clay fraction}} = \frac{34\%}{23\%} \approx 1.5$$

The activity number 1.5 falls above the U-line in Skempton's diagram (see Classify-03). Therefore, this is a CH clay, and is probably a member of the Montmorillonite family.

## Classify – 06

(Revision: Sept.-09)

During a hydrometer analysis a soil with a  $G_s = 2.60$  is immersed in a water suspension with a temperature of  $24^\circ\text{C}$ . An  $R = 43$  cm is obtained after 60 minutes of sedimentation. What is the diameter  $D$  of the smallest-size particles that have settled during that time?

### Solution:

Using the table below, for  $G_s = 2.60$  and  $T = 24^\circ\text{C}$ ,  $K = 0.01321$ .

$$\therefore L = 16.29 - (0.164R) = 16.29 - [0.164(43)] = 9.2 \text{ cm}$$

$$D = K \sqrt{\frac{L}{t}} = 0.01321 \sqrt{\frac{9.2 \text{ cm}}{60 \text{ min}}} = 0.00517 \text{ mm} = 5.2 \times 10^{-3} \text{ mm (a silt)}$$

**Table of constant K versus Temperature T ( $^\circ\text{C}$ )**

Temperature ( $^\circ\text{C}$ )	$G_s$							
	2.45	2.50	2.55	2.60	2.65	2.70	2.75	2.80
16	0.01510	0.01505	0.01481	0.01457	0.01435	0.01414	0.01394	0.01374
17	0.01511	0.01486	0.01462	0.01439	0.01417	0.01396	0.01376	0.01356
18	0.01492	0.01467	0.01443	0.01421	0.01399	0.01378	0.01359	0.01339
19	0.01474	0.01449	0.01425	0.01403	0.01382	0.01361	0.01342	0.01323
20	0.01456	0.01431	0.01408	0.01386	0.01365	0.01344	0.01325	0.01307
21	0.01438	0.01414	0.01391	0.01369	0.01348	0.01328	0.01309	0.01291
22	0.01421	0.01397	0.01374	0.01353	0.01332	0.01312	0.01294	0.01276
23	0.01404	0.01391	0.01358	0.01337	0.01317	0.01297	0.01279	0.01261
24	0.01388	0.01365	0.01342	0.01321	0.01301	0.01282	0.01264	0.01246
25	0.01372	0.01349	0.01327	0.01306	0.01286	0.01267	0.01249	0.01232
26	0.01357	0.01334	0.01312	0.01291	0.01272	0.01253	0.01235	0.01218
27	0.01342	0.01319	0.01297	0.01277	0.01258	0.01239	0.01221	0.01204
28	0.01327	0.01304	0.01283	0.01264	0.01244	0.01225	0.01208	0.01191
29	0.01312	0.01290	0.01269	0.01249	0.01230	0.01212	0.01195	0.01178
30	0.01298	0.01276	0.01256	0.01236	0.01217	0.01199	0.01182	0.01169

## Classify – 07

(Revision: Sept.-09)

The fines fraction of a soil to be used for a highway fill was subjected to a hydrometer analysis by placing 20 grams of dry fines in a 1 liter solution of water (dynamic viscosity 0.01 Poise at 20 degrees centigrade). The specific gravity of the solids was 2.65.

- a) Estimate the maximum diameter  $D$  of the particles found at a depth of 5 cm after a sedimentation time of 4 hours has elapsed, if the solution's concentration has reduced to 2 grams/ liter at the level.

At that moment,

- b) What percentage of the sample would have a diameter smaller than  $D$ ?

- c) What type of soil is this?

**Solution:**

a) Using Stoke's relation:  $v = \frac{\gamma_s - \gamma_w}{18\eta} d^2$  or  $d \text{ (mm)} = \sqrt{\frac{18\eta}{\gamma_s - \gamma_w} \left( \frac{L(\text{cm})}{t(\text{min})} \right)}$

where  $t = 4 \text{ hours} = 14,400 \text{ sec}$ ,  $L = 5 \text{ cm}$ ,  $\eta = 10^{-2} \text{ Poise} \left( \text{i.e., } \frac{\text{dyne} \cdot \text{s}}{\text{cm}^2} \right)$

$$\text{but } G_s = \frac{\gamma_s}{\gamma_w} \therefore \gamma_s = G_s \gamma_w = (2.65)(9.81 \text{ dynes/cm}^3) = 26 \text{ dynes/cm}^3$$

$$d = \sqrt{\frac{18 \times 10^{-2} \left( \frac{\text{dynes} \cdot \text{sec}}{\text{cm}^2} \right) (5 \text{ cm})}{9.81 \left( \frac{\text{dynes}}{\text{cm}^3} \right) (2.65 - 1.00)(14,400 \text{ sec})}} = \underline{0.020 \text{ mm}}$$

- b) The unit weight  $\gamma$  of the solution after 4 hours is,

$$\gamma = \frac{\text{weight of soil in solution}}{\text{volume of solution}} = \frac{2\text{g} + [1000\text{cm}^3 - 2\text{g}/2.65] \times 1 \text{ g/cm}^3}{1000 \text{ cm}^3} = 1.001 \text{ g/cm}^3$$

The portion of soil having a diameter smaller than  $D$  is,

$$\text{Portion smaller} = \frac{V}{W} \bullet \frac{\gamma_s \gamma_w}{\gamma_s - \gamma_w} (\gamma - 1) = \frac{1000 \text{ cm}^3}{20 \text{ g}} \left( \frac{2.65 \times 1}{2.65 - 1} \right) (1.001 - 1) = 0.08$$

$\therefore$  The remaining soil is only 8% of the original sample.

- c) The diameter  $d = 0.020 \text{ mm}$  corresponds to a silt.

## Classify – 08

(Revision: Sept.-09)

The formula for the relative compaction  $D_r$  is, 
$$D_r = \frac{e_{\max} - e}{e_{\max} - e_{\min}}$$

Derive an equivalent equation as a function of dry unit weights, such that

$$D_r = \frac{(\gamma_{d(\text{field})} - \gamma_{d \min}) (\gamma_{d \max})}{(\gamma_{d \max} - \gamma_{d \min}) (\gamma_{d(\text{field})})}$$

### Solution:

$\gamma_{d(\min)}$  = dry unit weight in loosest condition (voids ratio  $e_{\max}$ )

$\gamma_d$  = in-situ dry unit weight (voids ratio  $e$ )

$\gamma_{d(\max)}$  = dry unit in densest condition (voids ratio  $e_{\min}$ )

where

$$\gamma_d = \frac{W_s}{V} = \frac{G_s \gamma_w}{1 + e} \quad \text{and} \quad D_r = 0 = \text{loose, to } 1 = \text{very dense}$$

$$D_r = \frac{\frac{1}{\gamma_{d \min}} - \frac{1}{\gamma_d}}{\frac{1}{\gamma_{d \min}} - \frac{1}{\gamma_{d \max}}} = \left[ \frac{\gamma_d - \gamma_{d \min}}{\gamma_{d \max} - \gamma_{d \min}} \right] \left[ \frac{\gamma_{d \max}}{\gamma_d} \right] = \left[ \frac{\gamma_{d(\text{field})} - \gamma_{d \min}}{\gamma_{d \max} - \gamma_{d \min}} \right] \left[ \frac{\gamma_{d \max}}{\gamma_{d(\text{field})}} \right]$$

For example, what is the RC (relative density) of a sand in the field if it was tested to be at 98% Standard Proctor, its maximum unit weight was  $18.8 \text{ kN/m}^3$  and its minimum unit weight was  $14.0 \text{ kN/m}^3$ ?

$$RC = 98\% = \frac{\gamma_{d(\text{field})}}{\gamma_{d(\text{Std. Proct.})}} = \frac{\gamma_{d(\text{field})}}{18.8} \therefore \gamma_{d(\text{field})} = 18.4 \text{ kN/m}^3$$

$$D_r = \left[ \frac{\gamma_d - \gamma_{d \min}}{\gamma_{d \max} - \gamma_{d \min}} \right] \left[ \frac{\gamma_{d \max}}{\gamma_d} \right] = \frac{(18.4 - 14.0)(18.8)}{(18.8 - 14.0)(18.4)} = 94\%$$

## Classify – 09

(Revision: Sept.-09)

The data obtained from relative density tests is shown below. Calculate the range of relative densities.

	<u>Limiting <math>\gamma</math></u>	<u>Average <math>\gamma</math> in kN/m<sup>3</sup></u>
$\gamma_{\max}$	18.07	17.52
$\gamma_{\min}$	14.77	15.56
$\gamma_{\text{field}}$		16.97

**Solution:**

$$D_r = \left( \frac{\gamma_n - \gamma_{\min}}{\gamma_{\max} - \gamma_{\min}} \right) \left( \frac{\gamma_{\max}}{\gamma_n} \right)$$

$$\text{range 1 (low } \gamma_{\min} \text{)(high } \gamma_{\max} \text{)} \quad D_r = \left( \frac{16.97 - 14.77}{18.07 - 14.77} \right) \left( \frac{18.07}{16.97} \right) = 0.71$$

$$\text{range 2 (avg } \gamma_{\min} \text{)(high } \gamma_{\max} \text{)} \quad D_r = \left( \frac{16.97 - 15.56}{18.07 - 15.56} \right) \left( \frac{18.07}{16.97} \right) = 0.60$$

$$\text{range 3 (low } \gamma_{\min} \text{)(avg } \gamma_{\max} \text{)} \quad D_r = \left( \frac{16.97 - 14.77}{17.52 - 14.77} \right) \left( \frac{17.52}{16.97} \right) = 0.83$$

$$\text{range 4 (avg } \gamma_{\min} \text{)(avg } \gamma_{\max} \text{)} \quad D_r = \left( \frac{16.97 - 15.56}{17.52 - 15.56} \right) \left( \frac{17.52}{16.97} \right) = 0.74$$

$$\therefore \quad \underline{60\% \leq D_r \leq 83\%}$$

## Classify – 10

(Revision: Sept.-09)

South Florida has two types of sand, a quartzitic sand ( $\gamma_{s2} = 165.5$  pcf) and calcareous sand ( $\gamma_{s1} = 146.3$  pcf). At a particular site, their voids ratios were found to be:

for the quartzitic sand,  $e_{\max} = 0.98$  and  $e_{\min} = 0.53$

for the calcareous sand,  $e_{\max} = 0.89$  and  $e_{\min} = 0.62$

These voids ratios were measured by using a mold with a diameter of 4 inches and a height of 4.59 inches. The dry quartzitic sand weight was 3.254 lbs, and the dry calcareous sand was 2.868 lbs. Find their relative densities and dry unit weights. Comment on these results.

### Solution:

By definition  $D_r = \frac{e_{\max} - e}{e_{\max} - e_{\min}}$

For the calcareous sand,  $e_1 = \frac{\frac{\pi d^2 h}{4} \frac{P_1}{\gamma_{s1}} - \frac{P_1}{\gamma_{s1}}}{\frac{P_1}{\gamma_{s1}}} = \frac{\frac{\pi(4)^2(4.59)}{4} - \left(\frac{2.868}{146.3}\right) \times 12^3}{\frac{2.868 \times 12^3}{146.3}} = 0.70$

For the quartzitic sand,  $e_2 = \frac{\frac{\pi d^2 h}{4} \frac{P_2}{\gamma_{s2}} - \frac{P_2}{\gamma_{s2}}}{\frac{P_2}{\gamma_{s2}}} = \frac{\frac{\pi(4)^2(4.59)}{4} - \left(\frac{3.254}{165.6}\right) \times 12^3}{\frac{3.254 \times 12^3}{165.6}} = 0.70$

Notice that  $e_1 = e_2$

For the calcareous sand,  $D_{r_1} = \frac{0.89 - 0.70}{0.89 - 0.62} = 0.70$  and

For the quartzitic sand,  $D_{r_2} = \frac{0.98 - 0.70}{0.98 - 0.53} = 0.62$

The two types of sand have different relative densities because the calcareous sand grains are more tightly packed than the quartzitic sand grains.

For the calcareous sand,  $\gamma_{d_1} = \frac{\gamma_{s1}}{1 + e_1} = \frac{146.3}{1 + 0.7} = 86.1$  pcf (but  $D_{r_1} = 0.70$ )

For the quartzitic sand,  $\gamma_{d_2} = \frac{\gamma_{s2}}{1 + e_2} = \frac{165.6}{1 + 0.7} = 97.4$  pcf (but  $D_{r_2} = 0.62$ )

As a result, the dry unit weight is greater for the soil with the lower relative density.

## Classify – 11

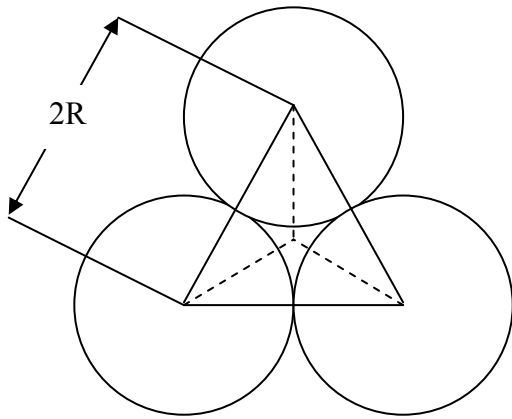
(Revision: Sept.-09)

Prove that  $e_{\min} = 0.35$ .

$$V_{\text{tetr}} = 0.1179a^3 = 0.1179(2R)^3 = 0.943R^3$$

$$V_{\text{sphere}} = \frac{4}{3}\pi R^3$$

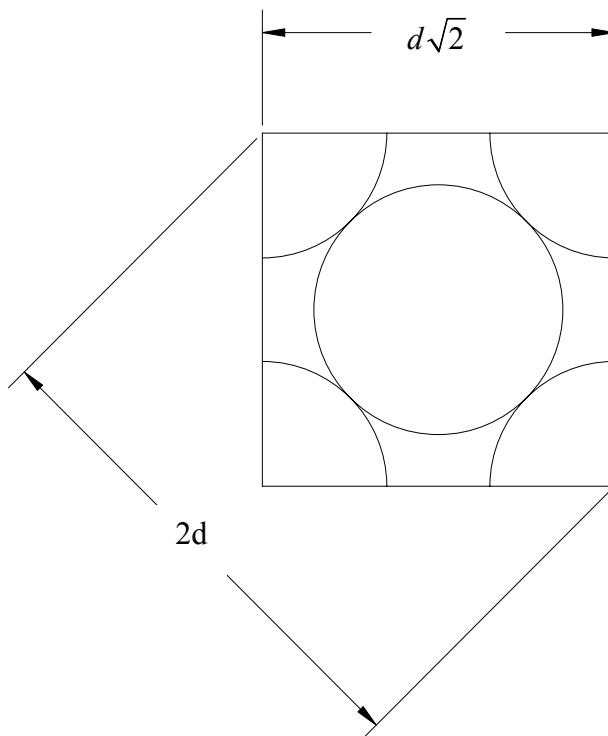
The volume of the sphere occupied by the tetrahedron is:



$$\therefore V_{\text{sphere}(\text{tetr})} \equiv \frac{60}{360} = 0.167 = 16.7\%$$

$$\therefore e = \frac{V_V}{V_S} = \frac{V - V_{\text{sphere}}}{V_{\text{sphere}}} = \frac{0.943R^3 - 0.167(4/3\pi R^3)}{0.167(4/3\pi R^3)} = \underline{0.35}$$

ALTERNATE METHOD:



$$\text{Volume of cube} = (d\sqrt{2})^3 = d^3 2\sqrt{2}$$

$$\text{Volume of sphere} = 4 \left( \frac{\pi d^3}{6} \right) = \frac{2}{3} \pi d^3$$

$$e_{\min} = \frac{V_{\text{cube}} - V_{\text{sphere}}}{V_{\text{sphere}}} = \frac{2d\sqrt{2} - \left( \frac{2}{3} \pi d^3 \right)}{\frac{2}{3} \pi d^3} = 0.35$$

# Chapter 4

## Compaction and Soil Improvement

### **Symbols for Compaction**

$e \rightarrow$  Voids ratio.

$G_s \rightarrow$  Specific gravity of the solids of a soil.

$n \rightarrow$  Porosity of the soil.

$OMC \rightarrow$  Optimum moisture content.

$S \rightarrow$  Degree of saturation.

$V \rightarrow$  ( $\frac{1}{30} ft^3 \equiv 9.44 * 10^{-4} m^3$  Standard Proctor mold, ASTM D-698).

$V_a \rightarrow$  Volume of air.

$V_s \rightarrow$  Volume of solids.

$V_v \rightarrow$  Volume of voids (water + air).

$V_w \rightarrow$  Volume of water.

$w \rightarrow$  Water content.

$V_s \rightarrow$  Volume of soil sample.

$\gamma \rightarrow$  Unit weight of the soil.

$\gamma_d \rightarrow$  Dry unit weight.

$\gamma_b \rightarrow$  Buoyant unit weight of the soil.

$\gamma_{SAT} \rightarrow$  Saturated unit weight of the soil.

$\gamma_s \rightarrow$  Unit weight of the solid.

$\gamma_w \rightarrow$  Unit weight of water.

$\gamma_{d,field} \rightarrow$  Dry unit weight in the field.

**\*Compaction–01: Find the optimum moisture content (OMC).**

(Revision: Aug-08)

A Standard Proctor test has yielded the values shown below. Determine:

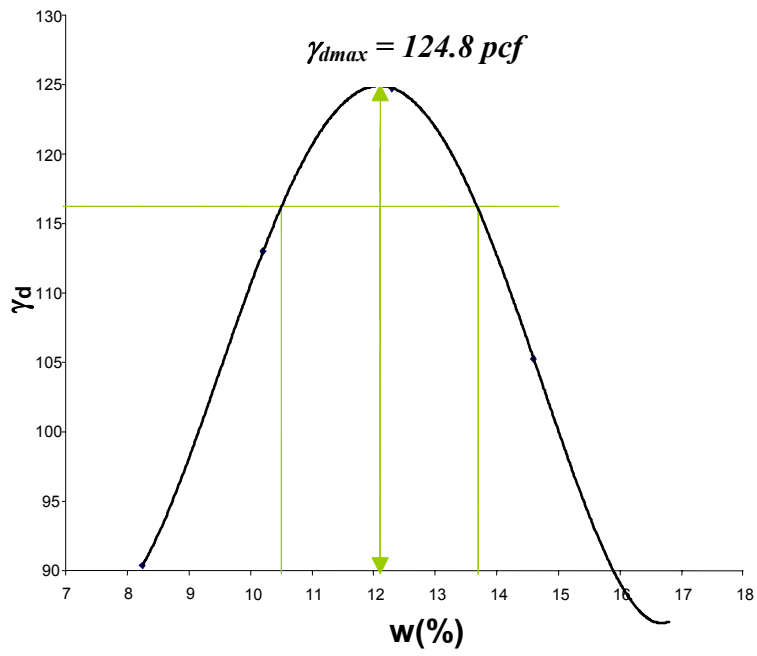
- (5) The maximum dry unit weight and its OMC; remember  $V = 1/30 \text{ ft}^3$ .  
(6) The moisture range for 93% of maximum dry unit weight.

No	Weight of wet soil (lb)	Moisture %
1	3.26	8.24
2	4.15	10.20
3	4.67	12.30
4	4.02	14.60
5	3.36	16.80

**Solution:**

Formulas used for the calculations:  $\gamma = \frac{W}{V}$  and  $\gamma_d = \frac{\gamma}{1 + w}$

$W \text{ (lb)}$	$w(\%)$	$\gamma \text{ (lb/ft}^3\text{)}$	$\gamma_d \text{ (lb/ft}^3\text{)}$
3.26	8.24	97.8	90.35
4.15	10.20	124.5	113.0
4.67	12.30	140.1	124.8
4.02	14.60	120.6	105.2
3.36	16.80	100.8	86.30



**Maximum dry unit weight = 124.8 pcf**

**OMC = 12.3 %**

$$\gamma_{field} = (0.93)(124.8) = \underline{116.1 \text{ pcf}}$$

**\*Compaction–02: Find maximum dry unit weight in SI units.**

(Revision: Aug-08)

Using the table shown below:

(7) Estimate the maximum dry weight of a sample of road base material, tested under Standard Proctor ASTM D-698 (all weights shown are in Newton).

(8) Note that the volume  $V = \frac{1}{30} \text{ ft}^3 \left( \frac{1 \text{ m}^3}{35.32 \text{ ft}^3} \right) = 9.44 \times 10^{-4} \text{ m}^3$

(9) Find the OMC.

(10) What is the appropriate moisture range when attaining 95% of Standard Proctor?

<i>Trial No.</i>	1	2	3	4	5
<i>W(Newton)</i>	14.5	15.6	16.3	16.4	16.1
<i>ω (%)</i>	20	24	28	33	37

**Solution:**

<i>Trial No.</i>	1	2	3	4	5
$\gamma = \frac{W}{V} \left( \frac{\text{kN}}{\text{m}^3} \right)$	15.4	16.5	17.3	17.4	17.1
$\gamma_d = \frac{\gamma}{1 + \omega} \left( \frac{\text{kN}}{\text{m}^3} \right)$	12.8	13.3	13.5	13.1	12.5

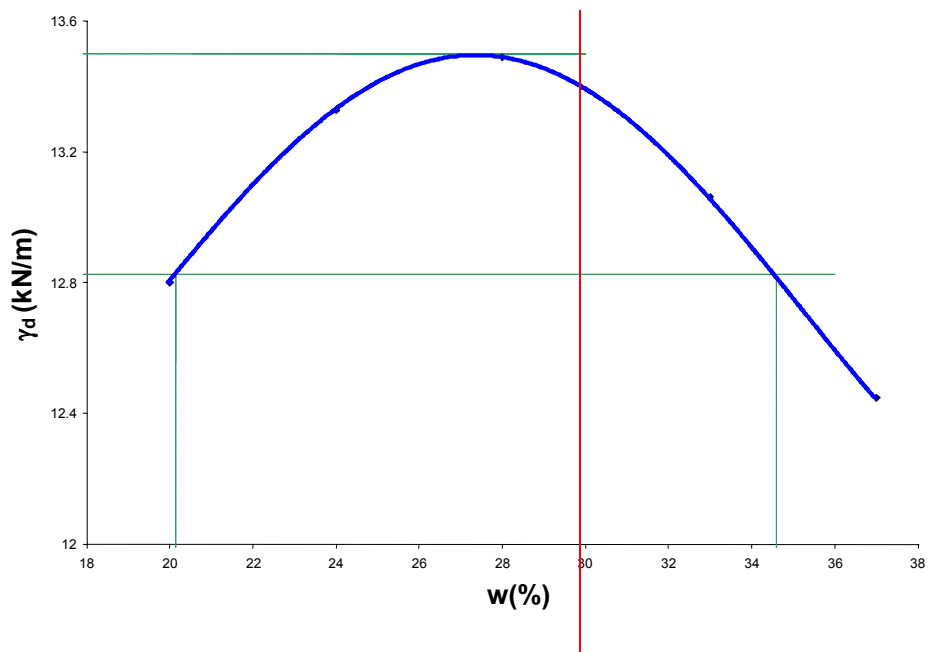
$$\gamma_{d \max} = 13.5 \text{ kN/m}^3$$

$$OMC = 28 \%$$

$$\gamma_{d \max} = 13.5 \text{ kN/m}^3$$

$$OMC = 28\%$$

$$\gamma_{d\text{-field}} = 0.95(13.5) = 12.8 \text{ kN/m}^3$$



### \*Compaction-03: What is the saturation S at the OMC?

(Revision: Sept.-08)

The results of a Standard Compaction test are shown in the table below:

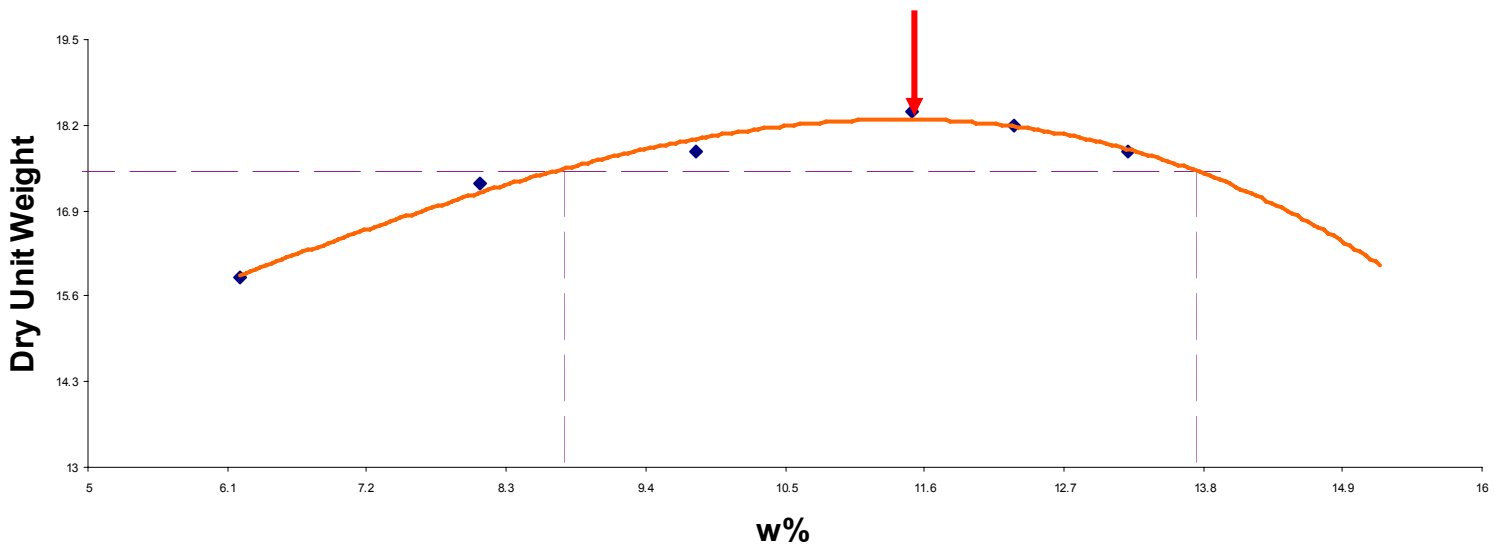
$\omega$ (%)	6.2	8.1	9.8	11.5	12.3	13.2
$\gamma$ ( $kN/m^3$ )	16.9	18.7	19.5	20.5	20.4	20.1
$\gamma_d = \frac{\gamma}{1 + \omega}$	15.9	17.3	17.8	18.4	18.2	17.8

- (11) a) Determine the maximum dry unit weight and the OMC.  
(12) b) What is the dry unit weight and moisture range at 95% RC (Relative Compaction)?  
(13) c) Determine the degree of saturation at the maximum dry density if  $G_s = 2.70$ .

#### Solution:

- a)  $\gamma_{d \max} = 18.4 \text{ kN/m}^3$ ,  $OMC = 11.5\%$   
b)  $\gamma_{d \text{ at } 95\%} = (0.95)(18.4) = 17.5 \text{ kN/m}^3$

The moisture range  $w$  for 95% RC is from 8.75% to 13.75%.



$$\gamma_{d \max} = 13.5 \text{ kN/m}^3 \quad OMC = 11.5 \%$$

c)

$$S = \frac{\frac{wG_s\gamma_{d\max}}{\gamma_w}}{G_s - \frac{\gamma_{d\max}}{\gamma_w}} = \frac{\frac{(0.115)(2.70)(18.4)}{(9.8)}}{2.7 - \left(\frac{18.4}{9.8}\right)} = 0.71 \quad \text{\textit{\textbf{Saturation } S = 71\%}}}$$

### **\*Compaction-04: Number of truck loads required.**

(Revision: Sept.-08)

The in-situ moisture content of a soil is 18% and its moist unit weight is 105 pcf. The specific gravity of the soil solids is 2.75. This soil is to be excavated and transported to a construction site, and then compacted to a minimum dry weight of 103.5 pcf at a moisture content of 20%.

- a) How many cubic yards of excavated soil are needed to produce 10,000 yd<sup>3</sup> of compacted fill?  
b) How many truckloads are needed to transport the excavated soil, if each truck can carry 20 tons?

#### **Solution:**

$$W_{\text{borrow site}} = W_{\text{construction site}}$$

$$V_{\text{borrow site}} \gamma_{\text{borrow site}} = V_{\text{construction site}} \gamma_{\text{construction site}}$$

$$\gamma_{d(\text{borrow site})} = \frac{\gamma}{1 + w} = \frac{105 \text{ pcf}}{1 + 0.18} = 89 \text{ pcf} \text{ versus } \gamma_{d(\text{construction site})} = 103.5 \text{ pcf}$$

$$a) \text{ Volume to be excavated} = 10,000 \text{ yd}^3 \left( \frac{103.5 \text{ pcf}}{89 \text{ pcf}} \right) = 11,630 \text{ yd}^3$$

$$b) \text{ Number of truck loads} = \frac{(11,630 \text{ yd}^3) \left( \frac{27 \text{ feet}^3}{\text{yd}^3} \right) \left( \frac{105 \text{ lb}}{\text{feet}^3} \right)}{\left( \frac{20 \text{ ton}}{\text{truck}} \right) \left( \frac{2,000 \text{ lb}}{\text{ton}} \right)} = 824 \text{ truck} - \text{loads}$$

### \*Compaction-05: What is the saturation S at the OMC?

(Revision: Sept.-08)

A Standard Proctor test was performed on a clayey gravel soil; the test results are shown below. Find the degree of saturation at the optimum condition; assume that  $G_s = 2.60$ .

Test	1	2	3	4	5	6	7
w%	3.00	4.45	5.85	6.95	8.05	9.46	9.90
$\gamma_d / \gamma_w$	1.94	2.01	2.06	2.09	2.08	2.06	2.05
$\gamma_d \text{ kN/m}^3$	19.4	20.1	20.6	20.9	20.8	20.6	20.5

Use  $\gamma_w = 10 \text{ kN/m}^3$  for simplicity.

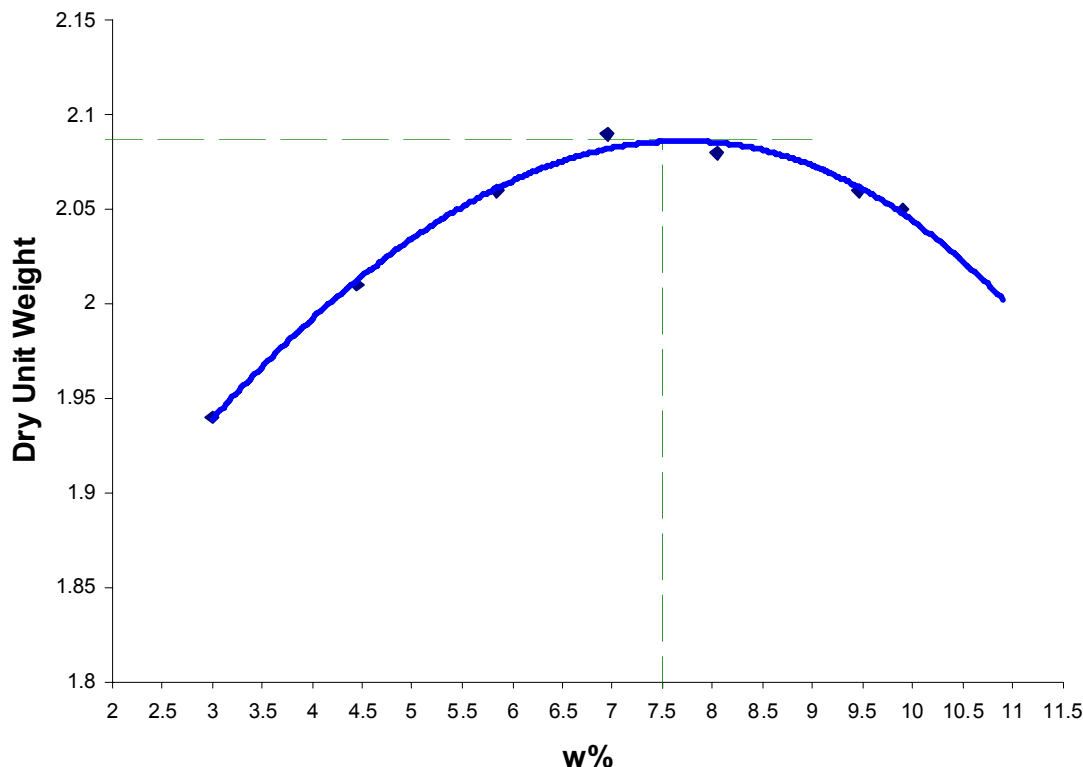
#### Solution:

It is known that  $Se = wG_s \therefore S = \frac{wG_s}{e}$  but  $e = \left( \frac{\gamma_s}{\gamma_d} - 1 \right) = \frac{\gamma_s - \gamma_d}{\gamma_d}$

therefore, at the OMC the saturation is,

$$S_{OMC} = w_{OMC} G_s \left( \frac{\gamma_{d-OMC}}{\gamma_s - \gamma_d} \right) = (0.0695)(2.60) \left( \frac{20.9}{(2.60)(10) - 20.9} \right) = 0.74$$

The degree of saturation at the OMC is 74%.



**\*Compaction-06: Definition of the relative compaction (RC).**

(Revision: Sept.-08)

The relative compaction (RC) of a sandy road base in the field is 90%. The maximum and minimum dry unit weights of the sand are  $\gamma_{d(max)} = 20.4 \text{ kN/m}^3$  and  $\gamma_{d(min)} = 13.9 \text{ kN/m}^3$ . Determine the field values of:

- a) The dry unit weight in the field;
- b) Relative density (of compaction)  $D_r$ ;
- c) The moist unit weight  $\gamma$  when its moisture content is 15%.

**Solution:**

The relative compaction RC is the dry unit weight obtained in the field, as compared to the Standard Proctor obtained in the laboratory.

- a) The relative compaction RC is,

$$RC = 0.90 = \frac{\gamma_{d(field)}}{\gamma_{d(max)}} = \frac{\gamma_{d(field)}}{20.4} \quad \therefore \gamma_{d(field)} = (0.90)(20.4) = 18.4 \text{ kN/m}^3$$

- b) The relative density  $D_r$  is,

$$D_r = \frac{\gamma_{d(field)} - \gamma_{d(min)}}{\gamma_{d(max)} - \gamma_{d(min)}} \times \frac{\gamma_{d(max)}}{\gamma_{d(field)}} = \frac{(18.4 - 13.9)}{(20.4 - 13.9)} \left( \frac{20.4}{18.4} \right) = 0.768 = 76.8\%$$

- c) The moist unit weight  $\gamma$  is,

$$\gamma = \gamma_d (1 + w) = 18.4(1 + 0.15) = 21.2 \text{ kN/m}^3$$

**\*Compaction-07: The relative compaction (RC) of a soil.**

(Revision: Aug-08)

A Standard Proctor compaction test performed on a sample of crushed limestone ( $G_s = 2.70$ ) obtained a maximum dry unit weight of 90 pcf at OMC. A field compacted sample showed a moisture of 28% and a unit weight of 103.7 pcf.

Find the relative compaction (RC).

Find the degree of saturation  $S$  of the field soil sample.

**Solution:**

$$\gamma_{d(field)} = \frac{\gamma_{moist}}{1 + w} = \frac{103.7}{1 + 0.28} = 81.0 \text{ pcf}$$

$$a) \quad RC = \frac{\gamma_{d(field)}}{\gamma_{d \max}} = \left( \frac{81.0 \text{ pcf}}{90.0 \text{ pcf}} \right) = 0.90$$

**The Relative Compaction = 90%**

$$b) \quad \gamma_d = \frac{G_s \gamma_w}{1 + e} \quad \therefore \quad 1 + e = \frac{G_s \gamma_w}{\gamma_d}$$

$$\therefore \quad e = \left( \frac{G_s \gamma_w}{\gamma_d} - 1 \right) = \frac{(2.70)(62.4)}{(81.0)} - 1 = 1.08$$

$$Se = wG_s$$

$$\therefore \quad S = \frac{wG_s}{e} = \frac{(0.28)(2.70)}{(1.08)} = 0.70 \quad \text{Saturation } S = 70\%$$

### **\*Compaction-08: Converting volumes from borrow pits and truck loads.**

(Revision: Oct.-08)

An embankment for a highway 30 m wide and 1.5 m thick is to be constructed from a sandy soil, trucked in from a borrow pit. The water content of the sandy soil in the borrow pit is 15% and its voids ratio is 0.69. Specifications require the embankment to be compacted to a dry unit weight of 18 kN/m<sup>3</sup>. Determine, for 1 km length of embankment, the following:

- The dry unit weight of sandy soil from the borrow pit required to construct the embankment, assuming that  $G_s = 2.70$ ;
- The number of 10 m<sup>3</sup> truckloads of sandy soil required to construct the embankment;
- The weight of water per truck load of sandy soil; and
- The degree of saturation of the in-situ sandy soil.

#### **Solution:**

a) The borrow pit's dry unit weight  $\gamma_d = \frac{G_s \gamma_w}{1 + e} = \frac{2.7(9.8)}{1 + 0.69} = 15.7 \frac{kN}{m^3}$

b) The volume of the finished embankment  $V = 30m(1.5m)(1km \text{ long}) = 45 \times 10^3 \text{ m}^3$

$$\text{Volume of borrow pit soil required} = \frac{\gamma_{d(reqd)}}{\gamma_{d(borrow \text{ pit})}}(V) = \left( \frac{18}{15.7} \right) (45 \times 10^3) \text{ m}^3$$

$$\text{Number of truck trips} = \left( \frac{18}{15.7} \right) \left( \frac{45 \times 10^3 \text{ m}^3}{10 \text{ m}^3} \right) = 5,160 \text{ truck-loads}$$

c) Weight of dry soil in 1 truck-load  $W_d = 10 \text{ m}^3 \left( 15.7 \frac{kN}{m^3} \right) = 157 \text{ kN}$

$$\text{Weight of water} = w W_d = (0.15)(157 \text{ kN}) = 23.6 \text{ kN per truck load}$$

d) Degree of saturation  $S = \frac{w G_s}{e} = \frac{(0.15)(2.70)}{0.69} = 0.59 = 59\%$

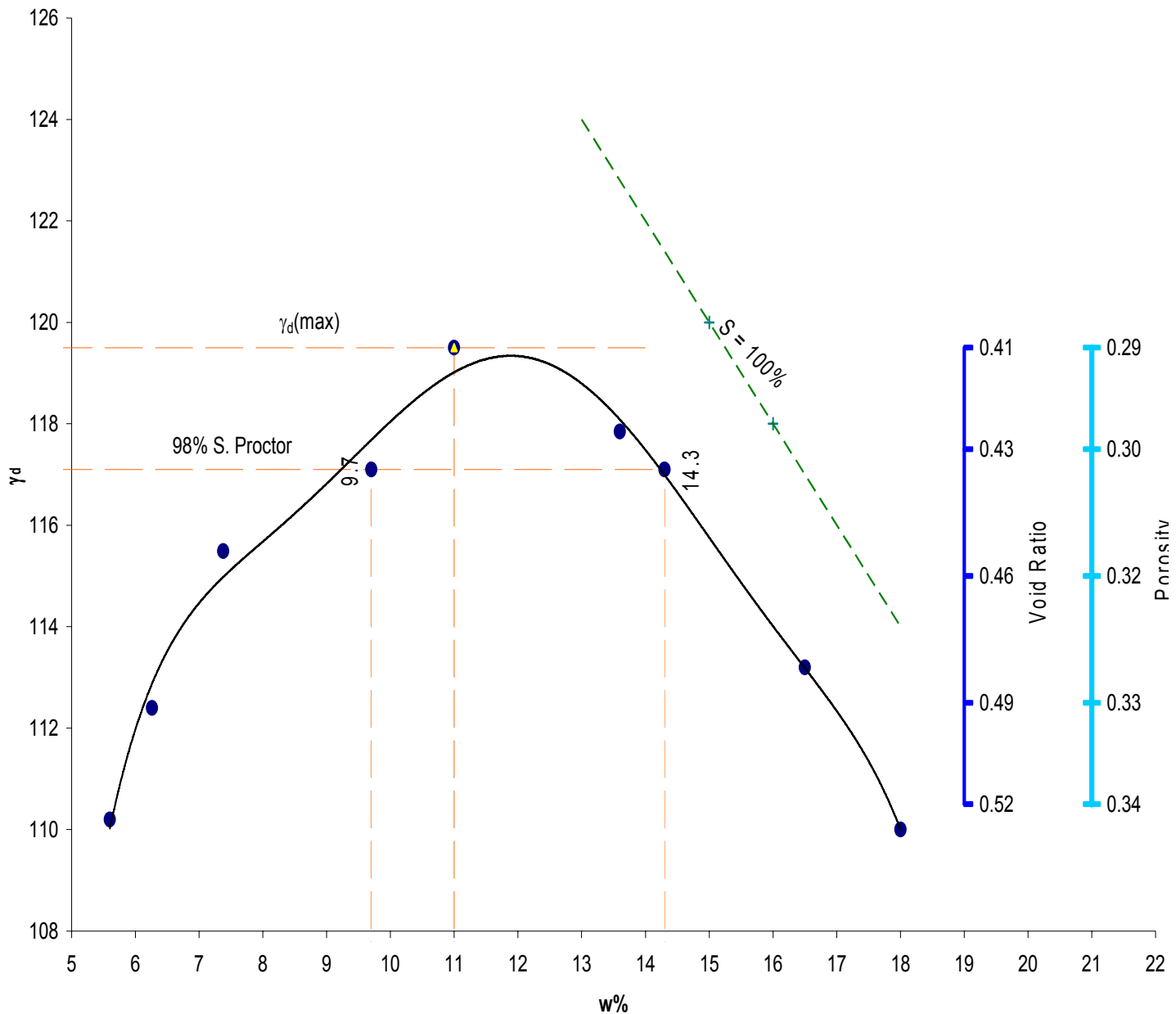
## **\*\*Compaction-09: Ranges of water and fill required for a road.**

(Revision: Octt.-08)

From the Standard Proctor compaction curve shown below:

Give two possible reasons that may cause a Proctor test to cross the ZAV curve?

What is the water content range (in gallons) needed to build a street 1,000 feet long of compacted 16" base at 98% Standard Proctor for two lanes, each 12 ft wide?



Solution:

$$a) \text{ Since } \gamma_{zav} = \frac{G_s \gamma_w}{1 + \left( \frac{w G_s}{S} \right)} \text{ and } \gamma_{zav} = \frac{\gamma_{wet}}{1 + w}$$

The crossing of the ZAV (zero air voids) curve is due to an incorrect assumption of  $G_s$ , and/or a miscalculation of the water content  $w$  and  $\gamma_{wet}$ .

b) The soil volume  $V$  required for the road is,

$$V = (16 \text{ in thick}) \left( \frac{1 \text{ ft}}{12 \text{ in}} \right) (24 \text{ ft wide}) (1,000 \text{ ft long}) = 32,000 \text{ ft}^3$$

The peak dry density  $\gamma_{dmax} = 119.5 \text{ pcf}$  at an OMC=12%, and the range of the water content is from 9.7% to 14.3% for 98% of Standard Proctor or  $(98\%) \gamma_{dmax} = (0.98)(119.5) = 117.1 \text{ pcf}$

The total weight of soil in the pavement is  $W = V\gamma = (32,000 \text{ ft}^3)(117.1 \text{ pcf}) = 3.75 \times 10^6 \text{ lbs}$

The weight of water  $W_w$  at the low end (9.7%) is,

$$V_w = \frac{W_w}{\gamma_w} = \frac{(0.097) \left( \frac{3.75 \times 10^6 \text{ lbs}}{1.097} \right)}{62.4 \text{ pcf}} \left( \frac{7.45 \text{ gal}}{\text{ft}^3} \right) \approx 40,000 \text{ gallons}$$

The weight of water  $W_w$  at the high end (14.3%) is,

$$V_w = \frac{W_w}{\gamma_w} = \frac{(0.143) \left( \frac{3.75 \times 10^6 \text{ lbs}}{1.143} \right)}{62.4 \text{ pcf}} \left( \frac{7.45 \text{ gal}}{\text{ft}^3} \right) \approx 56,000 \text{ gallons}$$

Therefore, the average volume of water is  $(1/2)(56,000 + 40,000) = 48,000 \text{ gallons}$

Therefore, the volume of water required = **48,000 ± 8,000 gallons.**

**\*\*Compaction-10: Find the family of saturation curves for compaction.**

(Revision: Oct.-08)

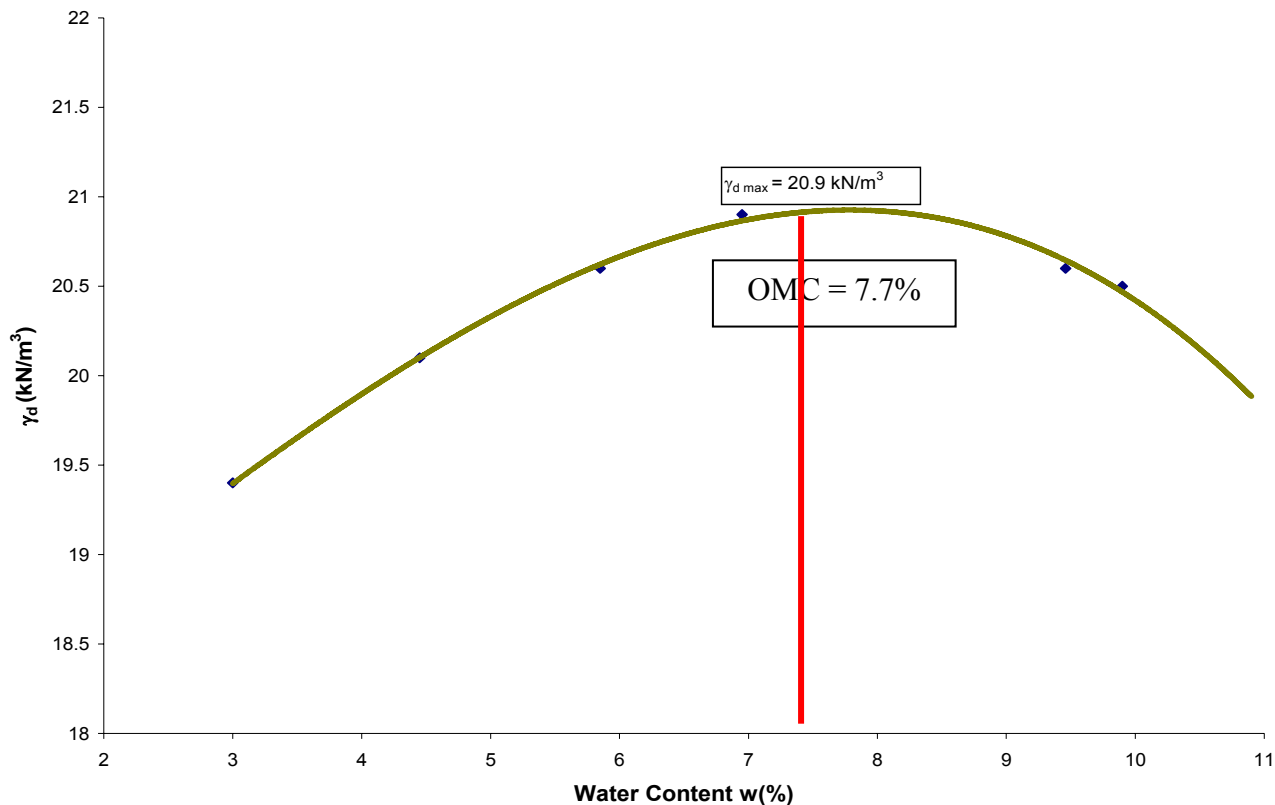
This problem expands Compaction-05: A Modified Proctor compaction test is performed on a clayey gravel road base. The solids have a specific gravity of 2.65. The compaction data yielded the following binomial values for  $\gamma_d/\gamma_w$  versus  $w\%$ :

$w(\%)$	3.00	4.45	5.85	6.95	8.05	9.46	9.90
$\gamma_d / \gamma_w$	1.94	2.01	2.06	2.09	2.08	2.06	2.05

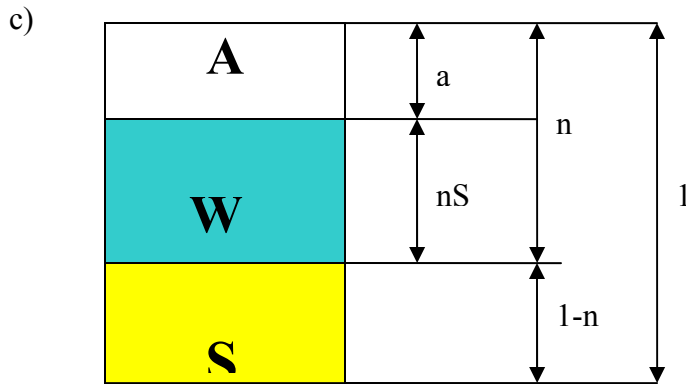
- (14) Find the  $\gamma_{d \max}$  and the OMC from the compaction curve;
- (15) Find the degree of saturation at the above conditions.
- (16) Calculate the percentage of air for a given porosity  $n$  and the saturation  $S$ .
- (17) Find the equation that describes the points of equal saturation.
- (18) Determine the equation for  $S = 100\%$
- (19) Discuss the characteristics of this last curve and equation.

**Solution:**

a)



$$b) \quad Se = wG_s \quad \therefore \quad S = \frac{wG_s}{e} \left( \frac{\gamma_{d\max}}{\gamma_w - \gamma_{d\max}} \right) = (7.7\%)(2.65) \left( \frac{2.09}{2.65 - 2.09} \right) = 76\%$$



$$nS = \frac{V_V}{V} \frac{V_W}{V_V} = \frac{V_W}{V} = \frac{V_W}{1} = V_W$$

$$a = 1 - nS - (1 - n) = 1 - nS - 1 + n$$

$$a = n(1 - S) = V_a$$

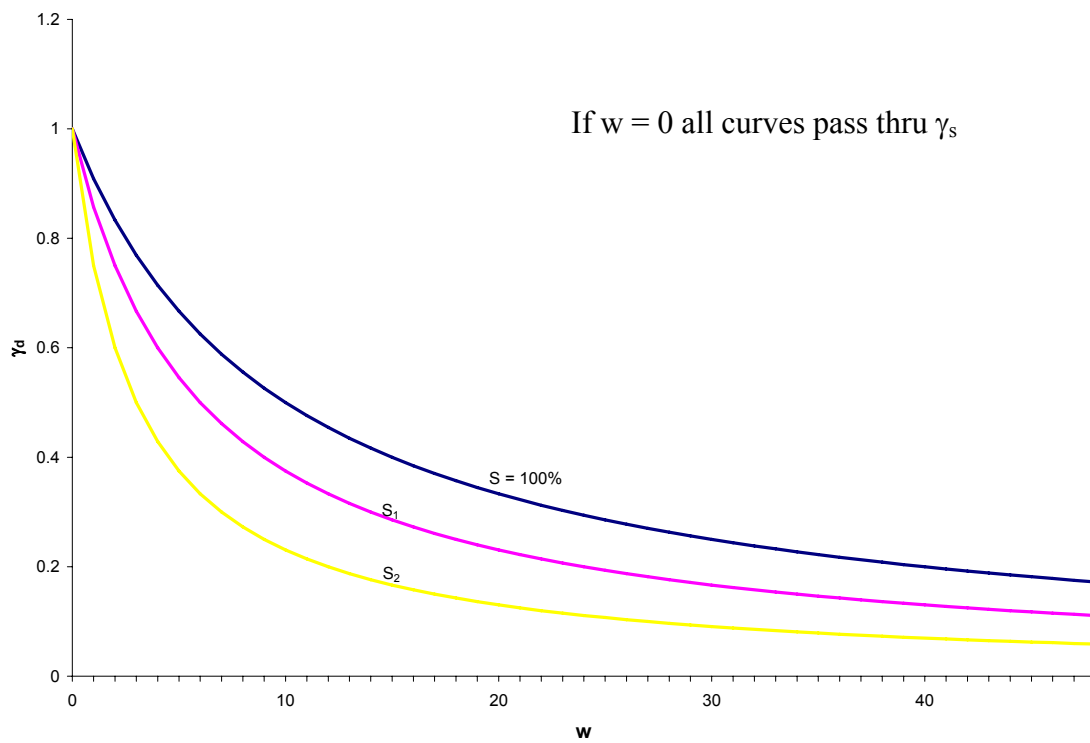
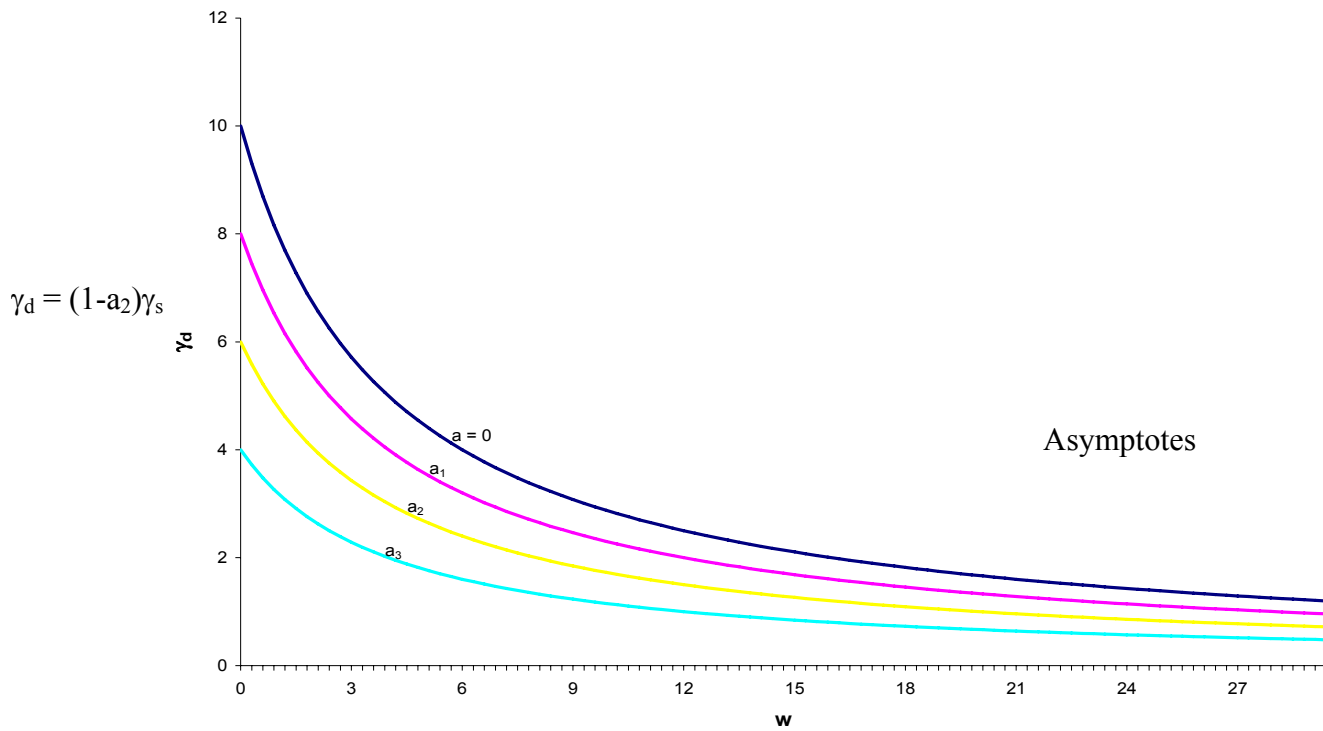
That is, the percentage of air  $a$  is equal to the porosity  $n$  times the factor  $(1 - S)$ , where  $S$  is the degree of saturation.

d) Consider a family of curves of equal saturation,

$$S = \frac{\text{volume of water}}{\text{volume of air} + \text{volume of water}} = \frac{\text{volume of water}}{1 - \text{volume of soil grains}} =$$

$$S = \frac{w \left( \frac{\gamma_d}{\gamma_w} \right)}{1 - \left( \frac{\gamma_d}{\gamma_s} \right)} \quad \therefore \gamma_d \left( \frac{w}{\gamma_w} + \frac{S}{\gamma_s} \right) - S \quad \therefore \frac{\gamma_d}{\gamma_w} = \frac{S\gamma_s}{w\gamma_s + S\gamma_w}$$

Curves can be plotted for varying values of  $a$  and saturation  $S$ .



These are hyperbolas with the  $w$ -axis as an asymptote.

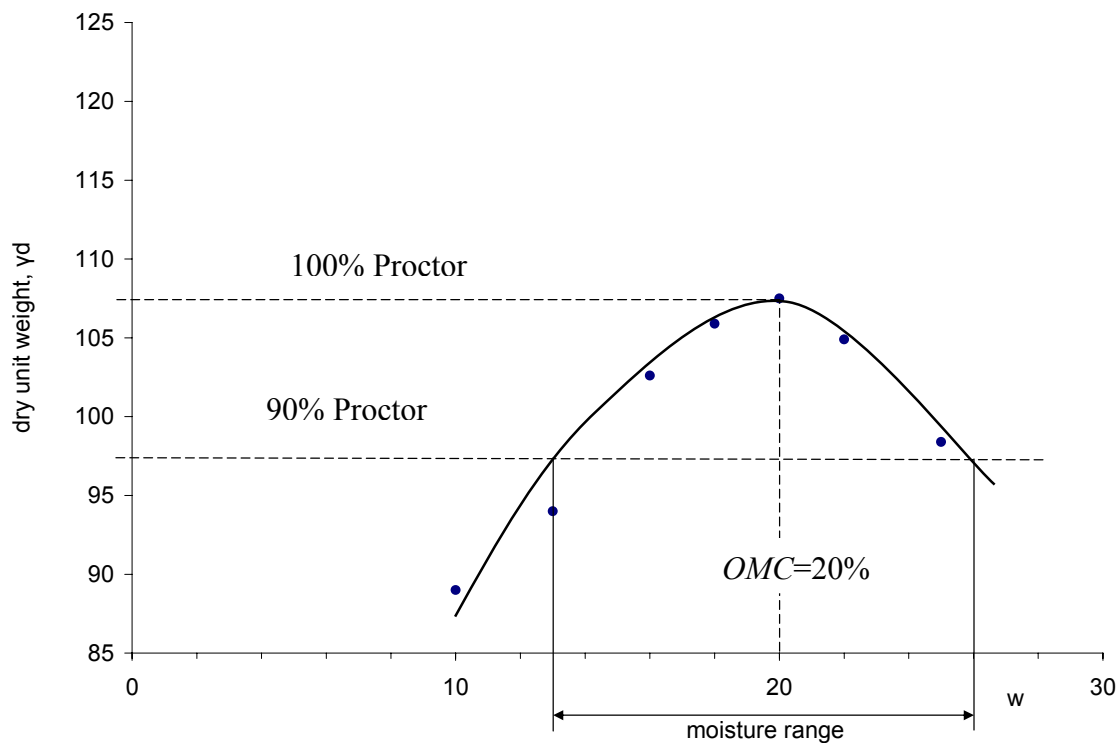
**\*\*Compaction-11: Water needed to reach maximum density in the field.**

(Revision: Aug-08)

A Standard Proctor test yields the values listed below for a soil with  $G_s = 2.71$ . Find:

- (20) The plot of the dry unit weight versus the water content;
- (21) The maximum dry unit weight;
- (22) The optimum moisture content;
- (23) The dry unit weight at 90% of Standard Proctor;
- (24) The moisture range for the 90% value;
- (25) The volume of water (in gallons) that must be added to obtain 1 cubic yard of soil at the maximum density, if the soil was originally at 10% water content.

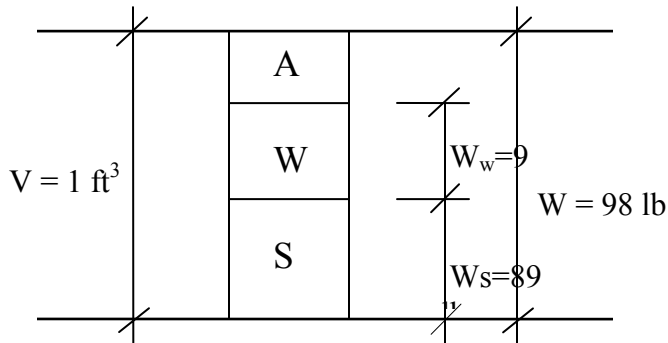
$w$ (%)	10	13	16	18	20	22	25
$\gamma$ (pcf)	98	106	119	125	129	128	123
$\gamma_d$ (pcf)	89	94	102.6	105.9	107.5	104.9	98.4

**Solution:**a) The plot of  $\gamma_d$  (dry unit weight) versus  $w$  (water content):

- (b) From the plot, the maximum dry unit weight is  $\gamma_{dmax} = \underline{107.5 \text{ pcf}}$ .
- (c) From the plot, the optimum moisture content is  $\underline{OMC = 20\%}$ .
- (d) The 90% value of the maximum dry unit weight  $\gamma_{dmax} = (0.9)(107.5) = \underline{96.8 \text{ pcf}}$

(e) The moisture range for the 90% value is approximately from **13% to 26%**.

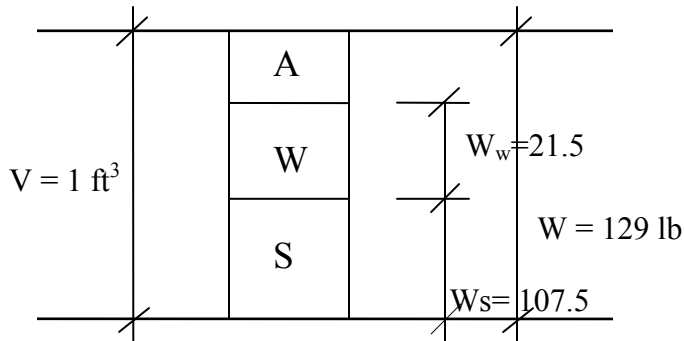
(f) Soil at 10% moisture,



$$w = 0.10 = \frac{W_w}{W_s} \quad \therefore \quad W_w = 0.10W_s$$

$$\text{but } W_s = 89 \text{ lb and } W_w = 0.10W_s = (0.1)(89) = 9 \text{ lb} \therefore W = W_s + W_w = 89 + 9 = 98 \text{ lb}$$

Soil at 20% moisture,



$$w = 0.20 = \frac{W_w}{W_s} \quad \therefore \quad W_w = 0.20W_s$$

$$\text{but } W_s = 107.5 \text{ lb and } W_w = 0.20W_s = (0.2)(107.5) = 21.5 \text{ lb} \therefore W = W_s + W_w = 129 \text{ lb}$$

Therefore, need to add the following water:  $(21.5 \text{ lb}) - (9 \text{ lb}) = 12.5 \text{ lb/ft}^3$

$$\therefore \text{ Added water} = \frac{12.5 \text{ lb}}{\text{ft}^3} \cdot \frac{\text{ft}^3}{62.4 \text{ lb}} \cdot \frac{7.48 \text{ gallons}}{\text{ft}^3} \cdot \frac{27 \text{ ft}^3}{1 \text{ yd}^3} = 40 \text{ gallons / yd}^3$$

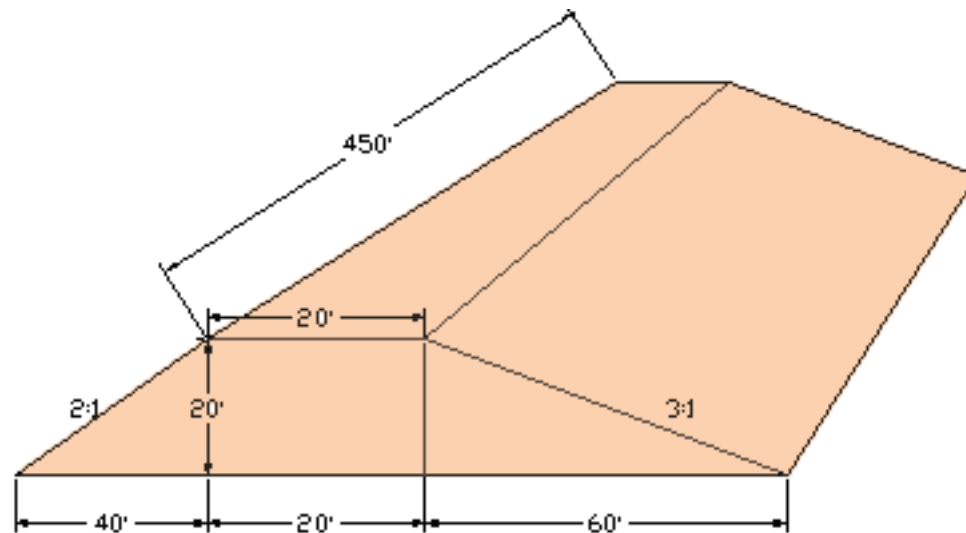
**Answer: Add 40 gallons of water per cubic yard of compacted soil.**

## **\*\*Compaction-12: Fill volumes and truck load requirements for a levee.**

(Revision: Aug-08)

Your company has won a contract to provide and compact the fill material for an earth levee, with the dimensions shown below. The levee fill is a silty clay soil to be compacted to at least 95% of maximum Standard Proctor of  $\gamma_d = 106$  pcf at an *OMC* of 18%. Your borrow pit has a silty clay with an in-situ moist density of 112.1 pcf at 18%, and a  $G_s = 2.68$ . When the soil is excavated and loaded on to your trucks, the voids ratio of the material is  $e = 1.47$ . Your trucks can haul 15 cubic yards of material per trip.

- (26) Determine the volume of fill required for the levee;
- (27) Determine the volume required from the borrow pit;
- (28) Determine the required number of truckloads.



### **Solution:**

$$\text{a) Volume of levee} = \left[ \left( \frac{1}{2} \right) (20') (40') + (20') (20') + \left( \frac{1}{2} \right) (20') (60') \right] (ft^2) \left( \frac{450 \text{ ft long}}{27 \text{ ft}^3 / \text{cy}} \right) = 23,300 \text{ cy}$$

b) To find the volume required from the borrow pit consider that the weight of solids is the same in both,  $W_{s(\text{borrow})} = W_{s(\text{levee})}$

$$\text{But } \gamma_d = \frac{W_s}{V} \text{ or } W_s = \gamma_d V \text{ or } \gamma_{d(\text{borrow})} V_{\text{borrow}} = \gamma_{d(\text{levee})} V_{\text{levee}}$$

$$\text{where } \gamma_{d(\text{borrow})} = \frac{\gamma}{1+w} = \frac{112.1}{1+0.18} = 95 \text{ pcf} \text{ and } \gamma_{d(\text{levee})} = (0.95)(106) = 100.7 \text{ pcf}$$

$$\therefore V_{\text{borrow}} = V_{\text{levee}} \frac{\gamma_{d(\text{levee})}}{\gamma_{d(\text{borrow})}} = (23,300 \text{ cy}) \left[ \frac{100.7 \text{ pcf}}{95 \text{ pcf}} \right] = 24,700 \text{ cy}$$

Number of truck-loads required is based on,

$$W_{s(hauled)} = W_{s(levee)}$$

$$\gamma_{d(hauled)} V_{hauled} = \gamma_{d(levee)} V_{levee}$$

$$\therefore V_{hauled} = V_{levee} \left( \frac{\gamma_{d(levee)}}{\gamma_{d(hauled)}} \right)$$

$$\text{but } \gamma_{d(levee)} = 100.7 \text{ pcf} \quad \text{and} \quad \gamma_{d(hauled)} = \frac{G_s \gamma_w}{1 + e} = \frac{(2.68)(62.4)}{1 + 1.47} = 67.7 \text{ pcf}$$

$$V_{hauled} = V_{levee} \left( \frac{\gamma_{d(levee)}}{\gamma_{d(hauled)}} \right) = (23,300 \text{ yd}^3) \left( \frac{100.7 \text{ pcf}}{67.7 \text{ pcf}} \right) = 34,700 \text{ yd}^3$$

$$\text{Number of truck-loads} = \left( \frac{V_{hauled}}{\text{truck capacity}} \right) = \left( \frac{34,700 \text{ yd}^3}{15 \text{ yd}^3 / \text{truck-load}} \right) = 2,314$$

**\*\*Compaction-13: Multiple choice compaction problem of a levee.**

(Revision: Aug-08)

A town's new reservoir will impound its fresh water with a small earth dam, rectangularly shaped in plan. The perimeter of the dam will be 2,200 ft long by 1,750 ft wide, and its cross-section is shown below in figure B. The dam is to be built with a silty clay soil, with a specific gravity of 2.70, available from three different local sources. Specifications call for a compacted soil at the dam with a dry unit weight of 97 pcf at an OMC of 31 percent. Assume all voids are totally devoid of any gas (or air). The borrow suppliers quoted the following:

Pit	Price (\$/yd <sup>3</sup> )	G <sub>s</sub>	S (%)	w (%)
A	1.05	2.69	65	22
B	0.91	2.71	49	22
C	0.78	2.66	41	18

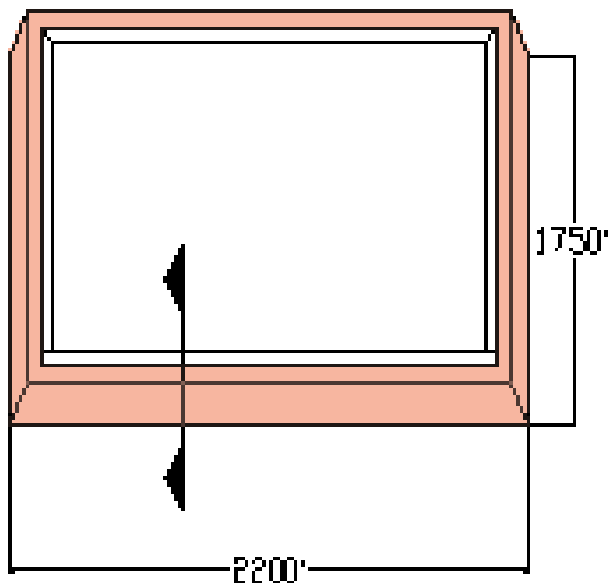


FIGURE A  
WATER SUPPLY RESERVOIR

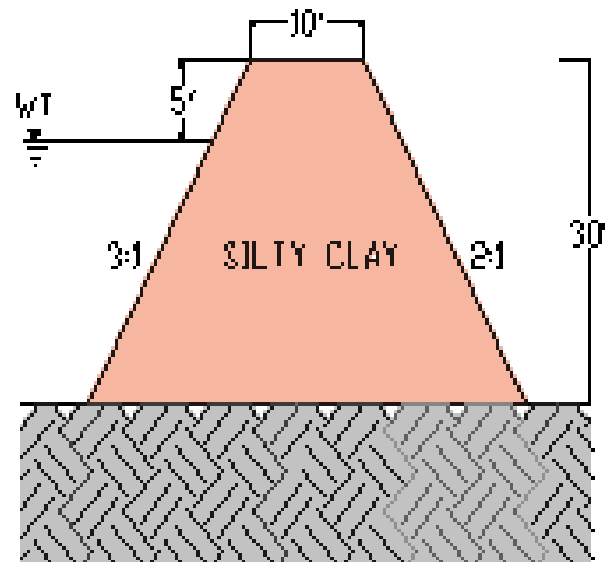


FIGURE B  
DAM CROSS-SECTION

**Questions:**

1. What is the dam's cross-sectional area?
  - a) 675 ft<sup>2</sup>
  - b) 2,100 ft<sup>2</sup>
  - c) 2,350 ft<sup>2</sup>
  - d) 2,700 ft<sup>2</sup>
  - e) 2,550 ft<sup>2</sup>

2. What is the approximate total volume  $V$  of soil required for the dam?
- a)  $2,550 \text{ ft}^3$
  - b)  $300,000 \text{ yd}^3$
  - c)  $900,000 \text{ yd}^3$
  - d)  $1.22 \text{ Myd}^3$
  - e)  $0.75 \text{ Myd}^3$
3. What is the approximate volume of water impounded (stored) in the dam?
- a)  $6,300,000 \text{ ft}^3$
  - b)  $134 \text{ acre-ft}$
  - c)  $7 \text{ Mft}^3$
  - d)  $154 \text{ acre-ft}$
  - e)  $45 \cdot 10^6 \text{ gallons}$
4. What is the unit weight  $\gamma$  of soil of the compacted earth dam?
- a)  $127 \text{ pcf}$
  - b)  $97 \text{ pcf}$
  - c)  $86 \text{ pcf}$
  - d)  $98 \text{ pcf}$
  - e)  $128 \text{ pcf}$
5. What is the designed voids ratio  $e$  of the compacted soil in the dam?
- a)  $1.1$
  - b)  $0.92$
  - c)  $0.84$
  - d)  $0.79$
  - e)  $1.2$
6. What is the voids ratio  $e$  of the material in pit A?
- a)  $0.98$
  - b)  $0.91$
  - c)  $1.02$
  - d)  $1.1$
  - e)  $0.72$
7. What is the voids ratio  $e$  of the material in pit B?
- a)  $0.93$
  - b)  $1.22$
  - c)  $0.81$
  - d)  $1.01$
  - e)  $1.00$

8. What is the voids ratio  $e$  of the material in pit C?
- a) 1.10
  - b) 1.12
  - c) 1.08
  - d) 1.05
  - e) 1.17
9. Assume that the voids ratio  $e$  of pit A is 0.98. What is the equivalent volume required of pit A to place 1.2 million cubic yards of compacted soil in the dam?
- a) 1.29 Myd<sup>3</sup>
  - b) 1.20 Myd<sup>3</sup>
  - c) 0.95 Myd<sup>3</sup>
  - d) 0.97 Myd<sup>3</sup>
  - e) 0.96 Myd<sup>3</sup>
10. Assume that the voids ratio  $e$  of pit B is 0.81, what is the equivalent volume required of pit B to place 1.2 million cubic yards of compacted soil in the dam?
- a) 1.00 Myd<sup>3</sup>
  - b) 1.02 Myd<sup>3</sup>
  - c) 1.18 Myd<sup>3</sup>
  - d) 1.05 Myd<sup>3</sup>
  - e) 1.07 Myd<sup>3</sup>
11. Assume that the voids ratio  $e$  of pit C is 1.10, what is the equivalent volume required of pit C to place 1.2 million cubic yards of compacted soil in the dam?
- a) 1.34 Myd<sup>3</sup>
  - b) 1.37 Myd<sup>3</sup>
  - c) 1.25 Myd<sup>3</sup>
  - d) 1.23 Myd<sup>3</sup>
  - e) 1.21 Myd<sup>3</sup>
12. Which pit offers the cheapest fill?
- a) Pit A
  - b) Pit B
  - c) Pit C
  - d) Both A and C
  - e) Both B and C

# Chapter 5

## Permeability of Soils

### Symbols for Permeability

- $A$  → area of a seepage surface.
- $C$  → Hazen's coefficient.
- $d$  → Diameter of a capillary tube
- $D_x$  → Diameter of a soil % finer (represents % finer by weight)
- $e$  → The voids ratio.
- $G_s$  → Specific gravity of the solids of a soil.
- $h$  → Thickness of the aquifer.
- $H$  → Thickness of the soil layer.
- $h_c$  → Height of the rising capillary
- $i$  → Hydraulic gradient.
- $k$  → coefficient of permeability in D'Arcy's equation .
- $k_H$  → Coefficient of horizontal permeability..
- $k_V$  → Coefficient of vertical permeability
- $L$  → Distance of the hydraulic head loss.
- $L_{\text{dugon}}$  → Standard unit for permeability ( $10^{-4} \text{ mm/sec}$ ).
- $N_f$  → The number of flow channels in Forheimer's equation.
- $N_{eq}$  → The number of equipotentials drops in Forheimer's equation.
- $p_o$  → In-situ vertical pressure at any depth .
- $q$  → Flow rate ( $\text{ft}^3$  per second per foot of width).
- $Q$  → Total seepage (total flow).
- $S_c$  → Seepage capacity.
- $T_s$  → Surface tension (typically given as 0.073 N per meter).
- $u$  → pore water pressure.
- $u_{\text{max}}$  → Maximum pore water pressure.
- $\gamma_s$  → Unit weight of solids.
- $\gamma_w$  → Unit weight of water .
- $\sigma_v$  → Vertical effective stress.

### **\*Permeability–01: Types of permeability tests and common units.**

(Revision: Aug-08)

(a) When is it appropriate to use a constant-head permeability test versus a falling-head permeability test?

(b) What are the “standard” units of permeability versus the “common” unit?

#### **Solution:**

(a) The constant head test is performed for granular soils (gravels **G**, and sands **S**), whereas the falling head test is used for fine-grained (cohesive) soils (silts **M**, and clays **C**).

(b) In Europe, the “standard” unit is the **Ludgeon** ( $10^{-4}$  mm/sec), and in the USA the standard unit is the **Meinzer**, which is the rate of flow in gallons per day through an area of 1 square foot under a hydraulic gradient of unity (1 foot/foot).

The “common” unit of permeability is ***cm / sec***.

**\*Permeability-02: Use of Hazen's formula to estimate the  $k$  of an aquifer.**

(Revision: Aug-08)

A test boring was performed at an elevation 955 feet MSL, and it found the phreatic surface (water table) 5 feet below the ground surface. An aquifer stratum was identified, and a sample of its soil showed the grain size distribution below. Estimate the permeability using Hazen's formula with the coefficient  $C = 12$ . A piezometer (measures the location of the WT) was installed 2500 feet downstream from the boring, and showed its phreatic surface at elevation 942 feet MSL. If the thickness of the aquifer was a uniform 12 feet between both points, estimate the quantity of flow per foot of width in gallons/hour ( $1 \text{ ft}^3 \approx 7.45 \text{ gallons}$ ).

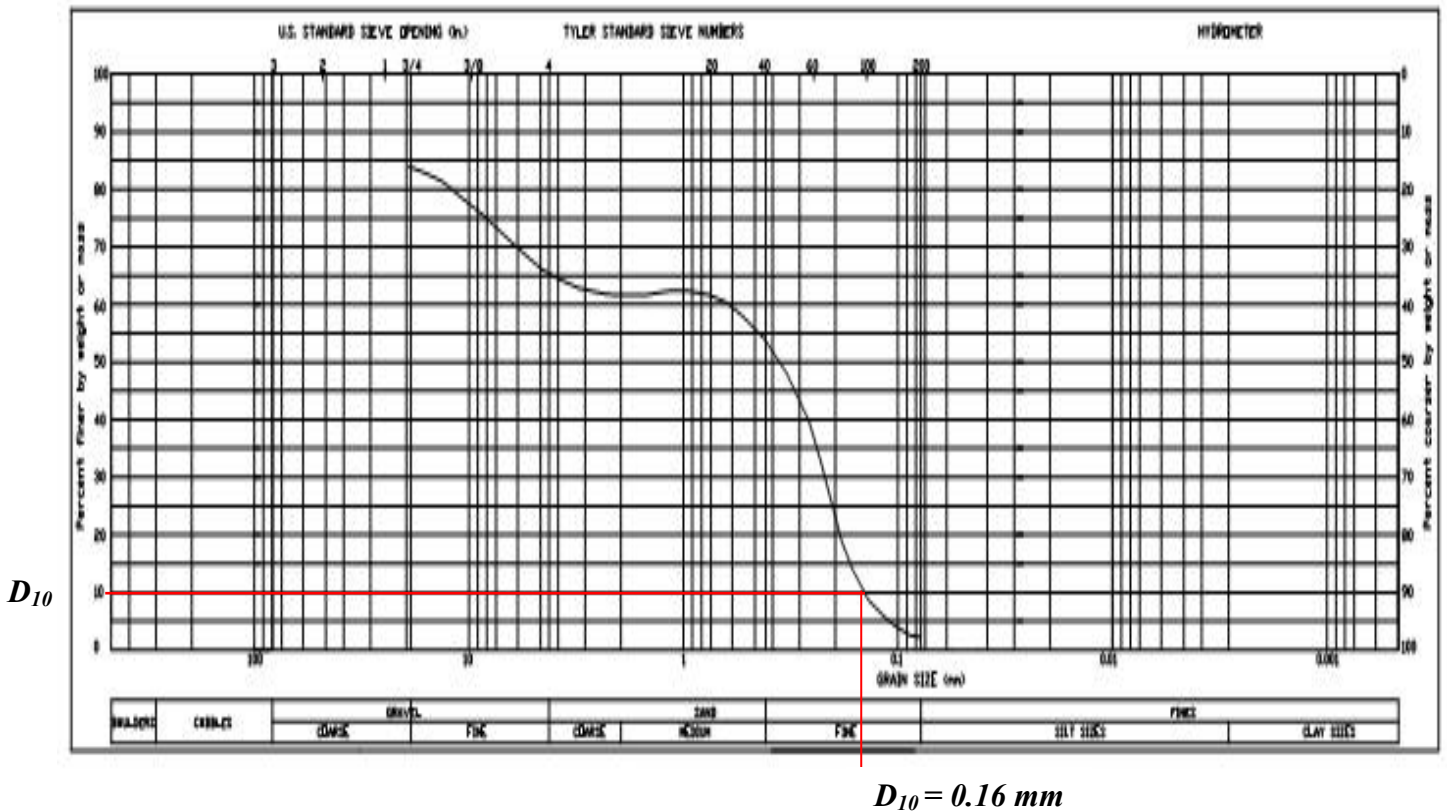
**Solution:**

Allen Hazen's (1893) formula for the permeability  $k = C(D_{10})^2 = (12)(0.16 \text{ mm})^2 = 0.31 \frac{\text{mm}}{\text{s}}$

The hydraulic head drop is  $(955 \text{ ft} - 5 \text{ ft}) - (942 \text{ ft}) = 8 \text{ feet}$

Applying D'Arcy formula,

$$q = k \frac{\Delta h}{L} A = \left(0.31 \frac{\text{mm}}{\text{sec}}\right) \left(\frac{3600 \text{ s}}{\text{hour}}\right) \left(\frac{1 \text{ in}}{25.4 \text{ mm}}\right) \left(\frac{1 \text{ ft}}{12 \text{ in}}\right) \left(\frac{7.45 \text{ gal}}{1 \text{ ft}^3}\right) \left(\frac{8 \text{ ft}}{2,500 \text{ ft}}\right) (12 \text{ ft})(1 \text{ ft}) = 1.05 \frac{\text{gallons}}{\text{hour}} \text{ per ft width}$$

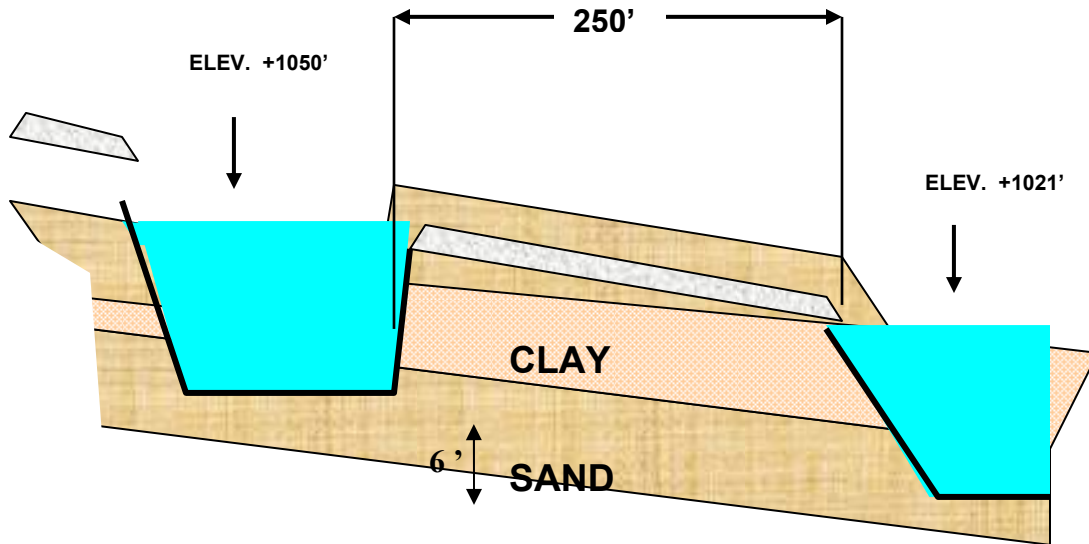


GRAIN SIZE DISTRIBUTION DIAGRAM

**\*Permeability-03: Flow in a sand layer from a canal to a river.**

(Revision: Aug-08)

A canal and a river run parallel an average of 250 feet apart. The elevation of the water surface in the canal is at +1050 feet and in the river at +1021 feet. A stratum of sand intersects both the river and the canal below their water levels. The sand is 6 feet thick, and is sandwiched between strata of impervious clay. Compute the seepage loss  $q$  from the canal in  $\frac{ft^3}{day - mile}$  if the permeability of the sand is  $2 \times 10^{-3} \frac{ft}{sec}$ .



**Solution:**

D'Arcy's formula for  $q$  yields,

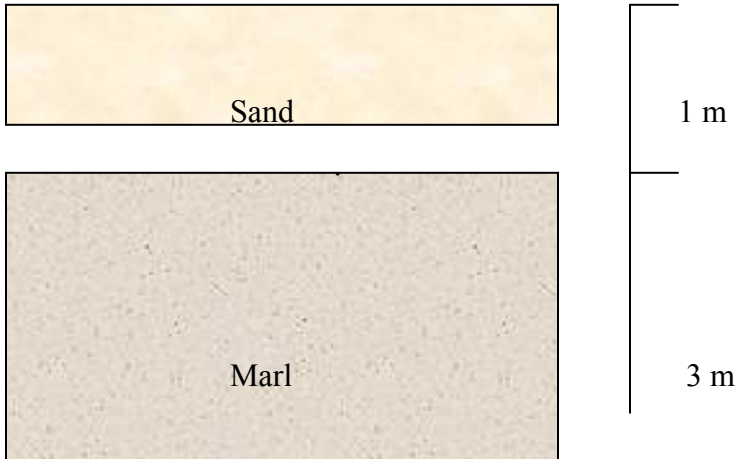
$$q = k \frac{\Delta h}{L} A = (2)(10)^{-3} \frac{ft}{s} \left( \frac{86,400s}{day} \right) \left[ \frac{(1050 - 1021)ft}{250ft} \right] (6.0ft) \left( \frac{5,280ft}{mile} \right)$$

$$q = 635,000 \frac{ft^3}{day - mile}$$

**\*Permeability-04: Find the equivalent horizontal permeability of two layers.**

(Revision: Aug-08)

The topmost layer is loose, clean sand, 1 meter thick. Its vertical permeability  $k_V$  can be estimated using Hazen's formula with  $C = 1.5$  (to over-estimate) and the sieve analysis shown here. Its  $k_H$  is known to be approximately 500% of the  $k_V$ . Below the sand stratum is a marine marl, 3 meters thick, with a  $k_V = k_H = 10^{-6}$  m/s. What is the combined  $k_{Hcomb}$  for the upper 4 m in cm/sec?



**Solution:**

Use Hazen's formula to find the permeability, where  $k$  is in cm/s if  $C$  ranges from 0.8-1.5.  $D_{10}$  is in mm. Use the grain-size distribution curve of Permeability-02.

$$k_{H_1} = 5k_{V_1} = 5(CD_{10}^2) = (5)(1.5)(0.16 \text{ mm})^2 = 1.92 \text{ cm/s} = 1,920(10^{-6}) \text{ m/s}$$

The formula for combining several horizontal layers is,

$$k_{Hcomb} = \frac{k_1 H_1 + k_2 H_2}{H_1 + H_2} = \frac{(1,920 \times 10^{-6} \text{ m/s})(1 \text{ m}) + (10^{-6} \text{ m/s})(3 \text{ m})}{1 \text{ m} + 3 \text{ m}} = 4.8 \times 10^{-4} \text{ cm/s}$$

**\*Permeability-05: Equivalent vertical and horizontal permeabilities.**

(Revision: Aug-08)

The soil profile shown below is typical of Miami-Dade County. Estimate the equivalent permeabilities  $k_{V(eq)}$  and  $k_{H(eq)}$  in cm/sec, and the ratio of  $k_{H(eq)} / k_{V(eq)}$ .

1 m	Fine Sand	$k = 1 \times 10^{-1}$ cm/sec	Pamlico Formation
5 m	Porous Limestone	$k = 2 \times 10^{-3}$ cm/sec	Miami Formation
3 m	Fine Sand	$k = 1 \times 10^{-3}$ cm/sec	Fort Thompson Formation
12 m	Upper sandy Limestone	$k = 2 \times 10^{-4}$ cm/sec	Fort Thompson Formation

**Solution:**

$$k_{V(EQ)} = \frac{H}{\frac{H_1}{k_1} + \frac{H_2}{k_2} + \frac{H_3}{k_3} + \frac{H_4}{k_4}} = \frac{(1+5+3+12) m}{\frac{1 m}{10^{-1} cm/s} + \frac{5 m}{2 \times 10^{-3} cm/s} + \frac{3 m}{10^{-3} cm/s} + \frac{12 m}{2 \times 10^{-4} cm/s}}$$

$$= \frac{(2,100 cm)}{(6,551 \times 10^3) s} = 0.32 \times 10^{-3} cm/s$$

$$k_{H(EQ)} = \frac{1}{H} [H_1 k_1 + H_2 k_2 + H_3 k_3 + H_4 k_4] = \frac{1}{(21 m)} [(1)(10^{-1}) + (5)(2 \times 10^{-3}) + (3)(10^{-3}) + (12)(2 \times 10^{-4})]$$

$$k_{H(EQ)} = 4.8 \times 10^{-3} cm/s$$

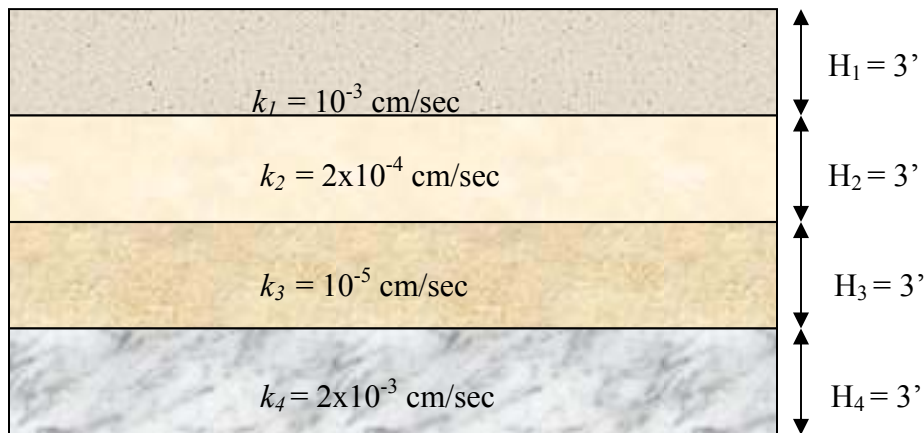
Therefore  $\frac{k_H}{k_V} = \frac{4.8 \times 10^{-3} cm/s}{0.32 \times 10^{-3} cm/s} = 15$

The horizontal permeability is 15 times larger than the vertical permeability.

**\*Permeability-06: Ratio of horizontal to vertical permeabilities.**

(Revision: Aug-08)

Estimate the ratio of the horizontal to the vertical permeability of these four strata.



**Solution:**

The equivalent horizontal permeability of all four layers is:

$$k_{H(eq)} = \frac{1}{H} (k_1 H_1 + k_2 H_2 + k_3 H_3 + k_4 H_4) = \frac{(ft)(cm/s)}{(12ft)} [(3)(10^{-3}) + (3)(2)(10^{-4}) + (3)(10^{-5}) + (3)(2)(10^{-3})]$$

$$k_{H(eq)} = 8 \times 10^{-4} \text{ cm/sec}$$

The equivalent vertical permeability of all four layers is:

$$k_{Veq} = \frac{H}{\frac{H_1}{k_{V_1}} + \frac{H_2}{k_{V_2}} + \frac{H_3}{k_{V_3}} + \frac{H_4}{k_{V_4}}} = \frac{12 \text{ ft}}{\frac{3'}{10^{-3}} + \frac{3'}{2 \times 10^{-4}} + \frac{3'}{10^{-5}} + \frac{3'}{2 \times 10^{-3}}} = 0.37 \times 10^{-4} \text{ cm/sec}$$

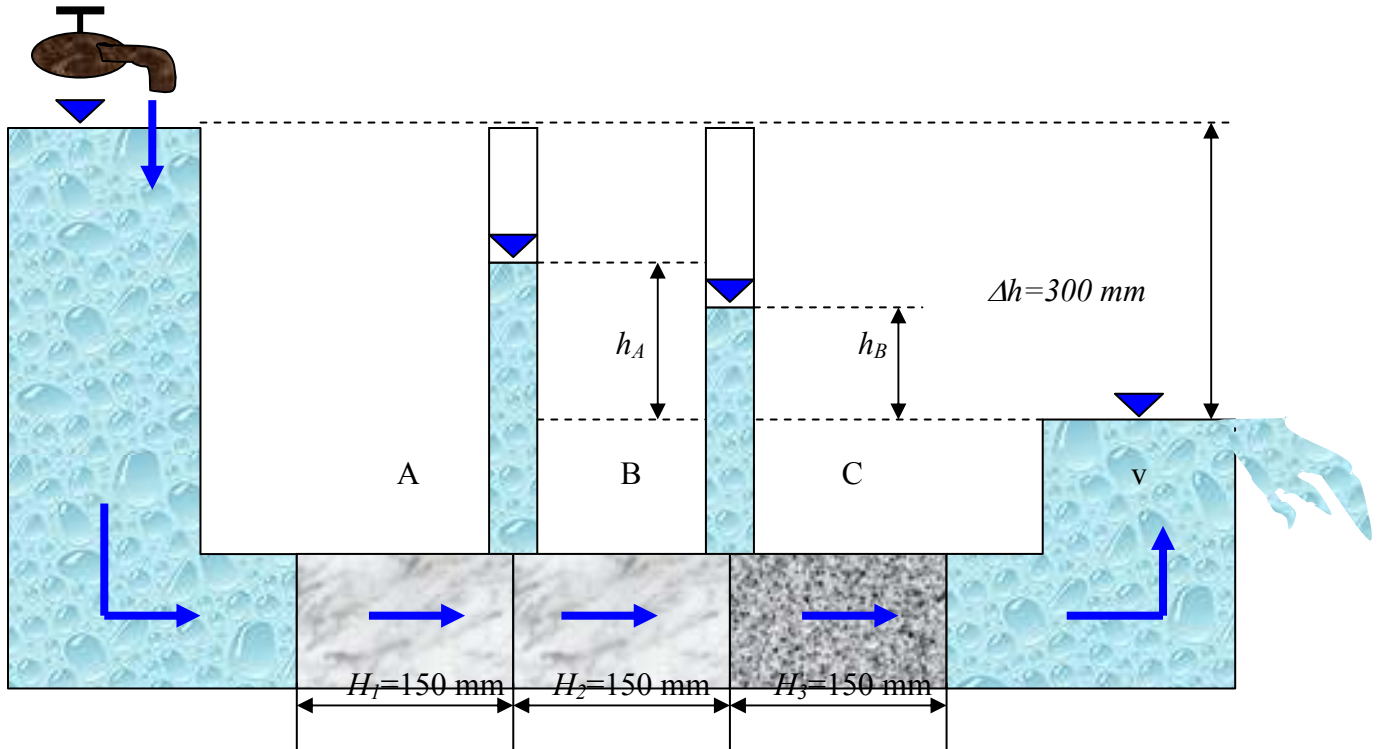
Therefore the ratio of the horizontal to the vertical permeability is:

$$\frac{k_{Heq}}{k_{Veq}} = \frac{8 \times 10^{-4} \text{ cm/s}}{0.37 \times 10^{-4} \text{ cm/s}} = 22$$

**\*Permeability–07: Do not confuse a horizontal with a vertical permeability.**

(Revision: Aug-08)

The soil layers below have a cross section of 100 mm x 100 mm each. The permeability of each soil is:  $k_A = 10^{-2}$  cm/sec.;  $k_B = 3 \times 10^{-3}$  cm/sec;  $k_C = 4.9 \times 10^{-4}$  cm/sec. Find the rate of water supply in  $\text{cm}^3/\text{hr}$ .



**Solution:**

This is a trick drawing: it “looks” like a horizontal flow, but in reality it is a vertical flow because the flow has to cross through every layer; it can not “bypass” any layer. Therefore, every soil layer has the same flow  $v = v_1 = v_2 = v_3$  and the total head  $\Delta h = \Delta h_1 + \Delta h_2 + \Delta h_3$ .

$$k_{V(eq)} = \frac{H}{\frac{H_1}{k_1} + \frac{H_2}{k_2} + \frac{H_3}{k_3}} = \frac{450\text{mm}}{\frac{150\text{mm}}{1 \times 10^{-2} \text{ cm/s}} + \frac{150\text{mm}}{3 \times 10^{-3} \text{ cm/s}} + \frac{150\text{mm}}{4.9 \times 10^{-4} \text{ cm/s}}} = 1.2(10^{-3}) \text{ cm/s}$$

$$\therefore q = k_{eq} i A = k_{eq} \frac{\Delta h}{H} A = (0.0012 \text{ cm/s}) \left( \frac{300 \text{ mm}}{450 \text{ mm}} \right) (10 \text{ cm})(10 \text{ cm}) \left( \frac{3,600 \text{ sec}}{1 \text{ hr}} \right) = 291 \text{ cm}^3 / \text{hour}$$

### **\*Permeability-08: Permeability as a function of the voids ratio $e$ .**

(Revision: Aug-08)

The coefficient of permeability of fine sand is 0.012 cm/sec at a voids ratio of 0.57. Estimate the increased permeability using the Kozeny-Carman formula of this same sand when its voids ratio has increased to 0.72.

#### **Solution:**

Using the Kozeny-Carman formula,

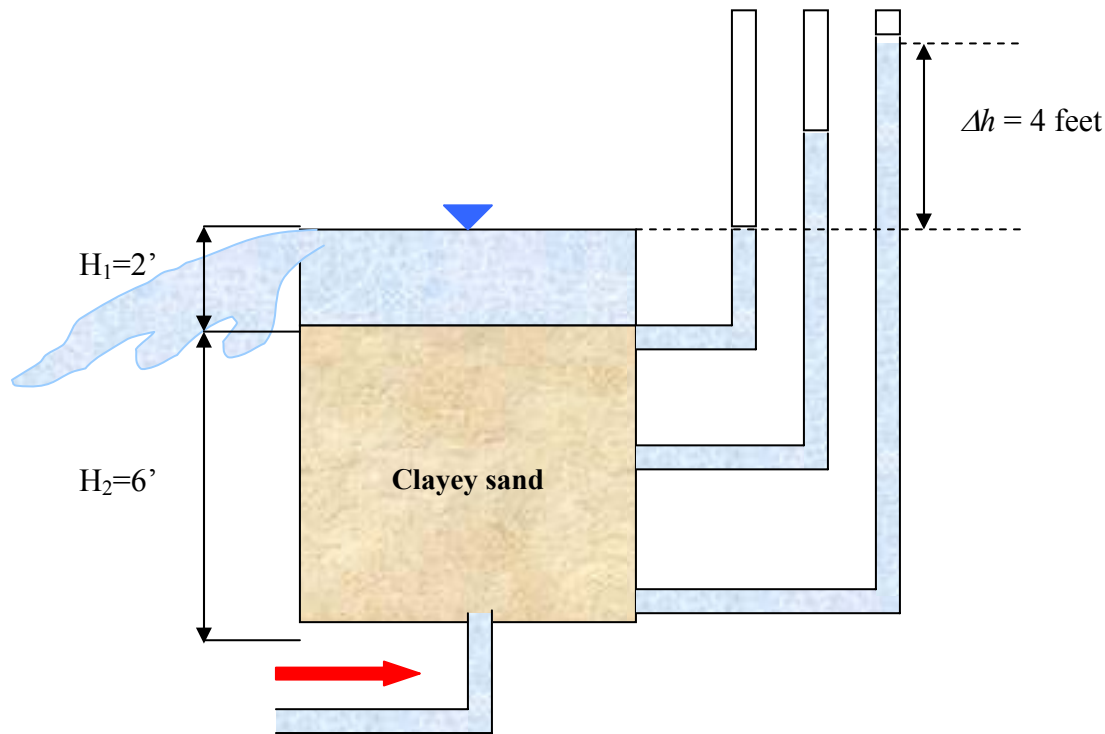
$$k = \frac{e^3}{(1+e)}$$
$$\therefore \frac{k_1}{k_2} = \frac{0.012}{k_2} = \frac{\frac{e_1^3}{1+e_1}}{\frac{e_2^3}{1+e_2}} = \frac{\frac{(0.57)^3}{1+0.57}}{\frac{(0.72)^3}{1+0.72}} = 0.544$$
$$\therefore k_2 = \frac{0.012}{0.544} = 0.022 \text{ cm / s}$$

Notice that since  $k_1 = 0.012$  cm/s the permeability has almost doubled. A 26% increase of the voids ratio has effected a doubling the permeability.

### \*Permeability–09: Uplift pressures from vertical flows.

(Revision: Aug-08)

The soil below is a dense well-graded clayey sand with  $\gamma_d = 112$  pcf and  $G_s = 2.63$ , a permeability  $k = 240$  mm/min at a voids ratio of  $e = 0.85$ ; the cross-sectional area of the tank is  $36 \text{ ft}^2$ . Find (a) the seepage rate  $q$  in  $\text{ft}^3/\text{min}$ ., and (b) the direction of the flow.



#### Solution:

a) The hydraulic gradient  $i$  and the voids ratio  $e$  are:

$$i = \frac{\Delta h}{H_2} = \frac{4 \text{ ft}}{6 \text{ ft}} = 0.667 \quad \text{and} \quad e = \frac{G_s \gamma_w}{\gamma_d} - 1 = \frac{(2.63)(62.4 \text{ pcf})}{112 \text{ pcf}} - 1 = 0.465$$

The Casagrande formula relates the known permeability  $k_{0.85}$  at  $e = 0.85$  to an unknown permeability  $k$  at any voids ratio  $e$ ,

$$k = 1.4e^2 k_{0.85} = 1.4(0.465)^2 \left( 240 \frac{\text{mm}}{\text{min}} \right) \left( \frac{1 \text{ in}}{25.4 \text{ mm}} \right) \left( \frac{1 \text{ ft}}{12 \text{ in}} \right) = 0.239 \text{ ft} / \text{min}$$

Therefore, the seepage rate  $q$  is:

$$q = kiA = \left( 0.239 \frac{\text{ft}}{\text{min}} \right) (-0.667)(36 \text{ ft}^2) = -5.74 \frac{\text{ft}^3}{\text{min}}$$

b) The flow direction is UP.  $\uparrow$

**\*Permeability-10: Capillary rise in tubes of differing diameters.**

(Revision: Sept.-2008)

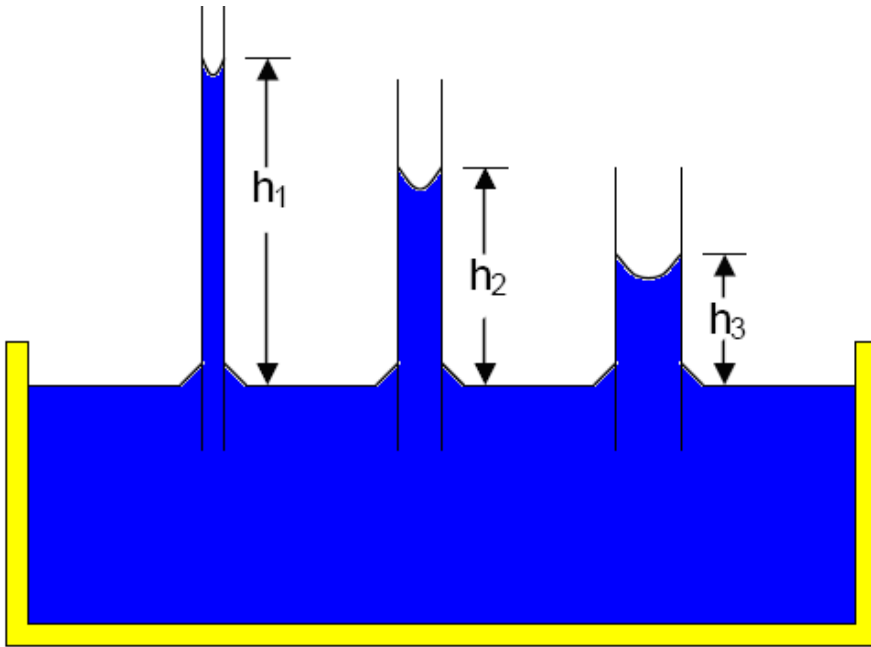
Determine the different heights  $h_c$  that water will raise in three different capillary tubes, with diameters:

$d_1 = 0.00075$  mm (corresponding to a fine clay sized particle),

$d_2 = 0.075$  mm (corresponding to the smallest sand sized particle) and

$d_3 = 0.75$  mm (corresponding to a medium sand sized particle).

Assume that the surface tension is  $0.075$  N/m with an angle  $\alpha = 3^\circ$ .



**Solution:**

The surface tension of water  $T_s$  ranges from about  $0.064$  to  $0.075$  N/m ( $0.0044$  to  $0.0051$  lb/ft). In this problem we have chosen the largest value. Notice that the negative sign indicates that the water has risen due to the capillary tension.

$$h_c = \frac{-4 (T_s)(\cos \alpha)}{d \gamma_w} = \frac{-4 (0.075 N / m)(\cos 3^\circ)}{d (9.81 kN / m^3)}$$

$$h_1 = \frac{-4 (7.5 \times 10^{-2} N / m)(\cos 3^\circ)}{(7.5 \times 10^{-7} m)(9.81 kN / m^3)} = -41 \text{ m}$$

$$h_2 = \frac{-4 (7.5 \times 10^{-2} N / m)(\cos 3^\circ)}{(7.5 \times 10^{-5} m)(9.81 kN / m^3)} = -0.41 \text{ m} = 410 \text{ mm}$$

$$h_3 = \frac{-4 (7.5 \times 10^{-2} N / m)(\cos 3^\circ)}{(7.5 \times 10^{-4} m)(9.81 kN / m^3)} = -0.041 \text{ m} = 41 \text{ mm}$$

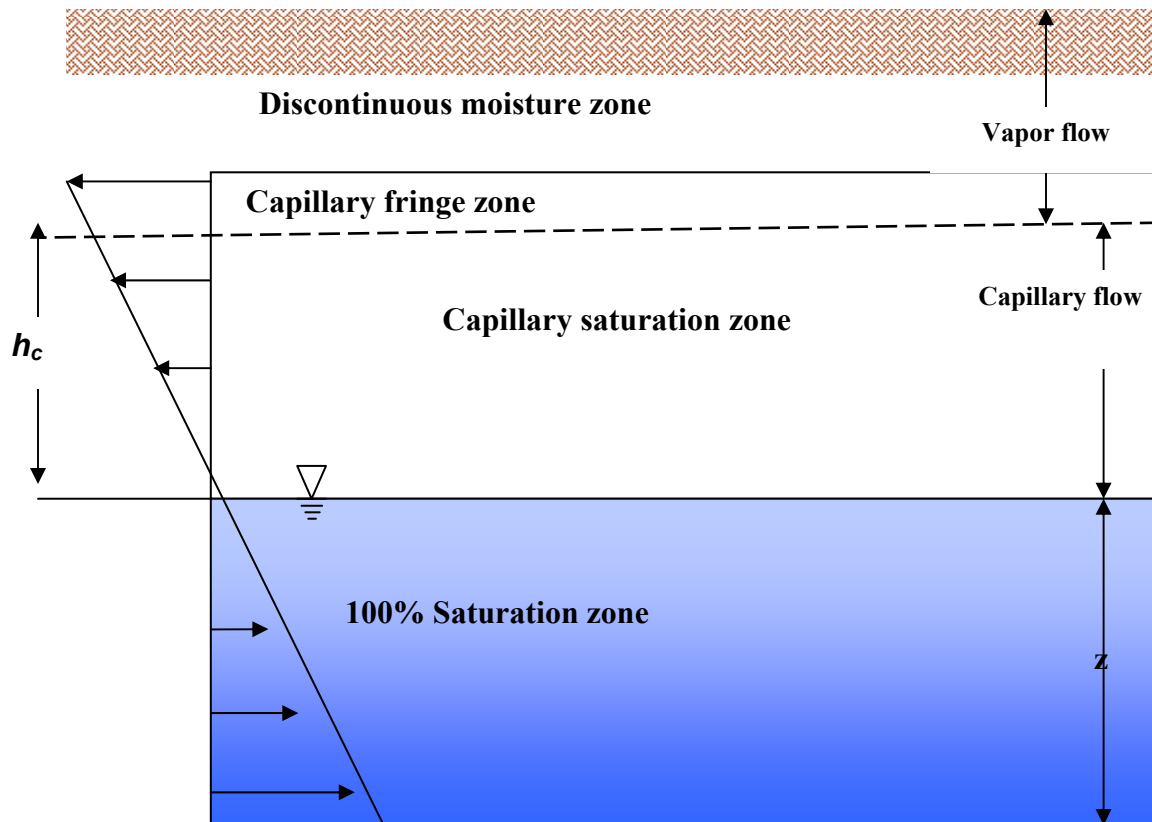
### \*Permeability-11: Rise of the water table due to capillarity saturation.

(Revision: Sept.-08)

How much does the capillary water rise above the water table in a very fine sand ( $d = 0.1 \text{ mm}$ ) if the surface tension force is  $T_o = 0.064 \text{ N/m}$  with an  $\alpha = 3^\circ$ ?

**Solution:**

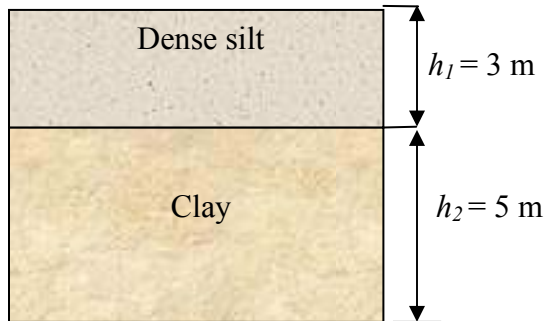
$$h_c = \frac{-4T_o \cos \alpha}{d\gamma_w} = \frac{-4(0.064 \text{ N/m}) \cos 3^\circ}{(10^{-4} \text{ m})(9.81 \text{ kN/m}^3)} = -0.26 \text{ m}$$



**\*Permeability-12: Find the capillary rise  $h_c$  in a silt stratum using Hazen.**

(Revision: Aug-08)

Another method of determining the capillary rise in a soil is to use Hazen's capillary formula. The 3 m thick dense silt layer shown below is the top stratum of a construction site, has an effective diameter of 0.01 mm. What is the approximate height of the capillary rise in that silt stratum? What are the vertical effective stresses at depths of 3 m and 8 m below the surface? The "free ground water" level is 8 meters below the ground surface, the  $\gamma_s = 26.5 \text{ kN/m}^3$ , and the soil between the ground surface and the capillary level is partially saturated to 50%.



**Solution:**

1. The Hazen empirical formula for capillary rise is  $h_c = \frac{0.0306}{0.2D_{10}} = \frac{0.0306}{(0.2)(0.01 \text{ mm})} = 15.3 \text{ m}$

In essence, the entire silt stratum is saturated through capillarity.

2. For full saturation,  $S = 100\%$ ,

$$Se = wG_s \therefore w = \frac{Se}{G_s} = \frac{Se\gamma_w}{\gamma_s} = \frac{(1)(0.40)(9.81)}{26.5} = 0.148$$

$$\therefore \gamma_{SAT} = \gamma_s \left( \frac{1+w}{1+e} \right) = \frac{(26.5)(1.148)}{1.40} = 21.8 \frac{\text{kN}}{\text{m}^3}$$

For 50% saturation,

$$w = \frac{Se\gamma_w}{\gamma_s} = \frac{(0.5)(0.40)(9.81)}{26.5} = 0.074$$

$$\therefore \gamma = \gamma_s \left( \frac{1+w}{1+e} \right) = \frac{(26.5)(1.074)}{1.40} = 20.3 \frac{\text{kN}}{\text{m}^3}$$

Therefore,

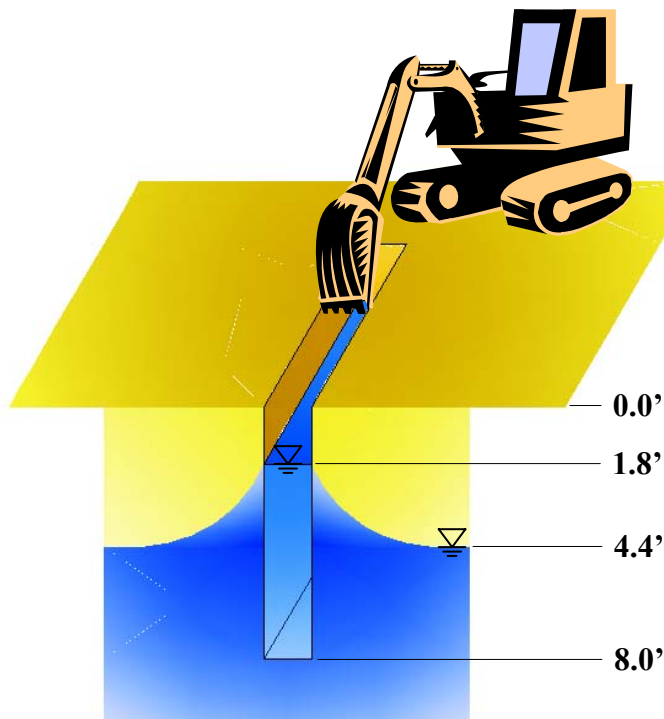
$$\therefore (\sigma_v')_3 = h_1\gamma = (3)(20.3) = \underline{\underline{61 \text{ kPa}}}$$

$$\therefore (\sigma_v')_8 = h_1\gamma + (h_2)\gamma_{SAT} = (3)(20.3) + (5)(21.8) = \underline{\underline{170 \text{ kPa}}}$$

**\*Permeability-13: Back-hoe trench test to estimate the field permeability.**

(Revision: Sept.-08)

A common method of determining a site's drainage capabilities is the constant-head trench percolation test shown below. The trench is dug by a backhoe to roughly the dimensions shown. The testing crew uses a water truck to fill the trench with water above the WT, and then they attempt to maintain the head constant for about 10 minutes. The amount of water that has flowed out during the test is  $Q$  (in gallons/minute). The seepage capacity  $S_c = Q/CL'H$ , where  $C$  is a units conversion factor,  $L'$  is the trench semi-perimeter (length plus width, in ft) and  $H$  is the head. The units of the seepage capacity are commonly given in  $cfs/ft/ft$ . Based on the reported geometric conditions shown below, and that the crew used 1,540 gallons during a 10 minutes test, what is the surface seepage capacity of that site?



**Solution:**

$$Q = \frac{1,540 \text{ gal}}{10 \text{ min}} = 154 \text{ gallons / minute}$$

The perimeter of the trench is  $L' = 2(6 \text{ ft} + 15 \text{ ft}) = 42 \text{ ft}$  and  $H = (4.4 \text{ ft} - 1.8 \text{ ft}) = 2.6 \text{ ft}$

The seepage capacity of the surface stratum is  $S_c$ ,

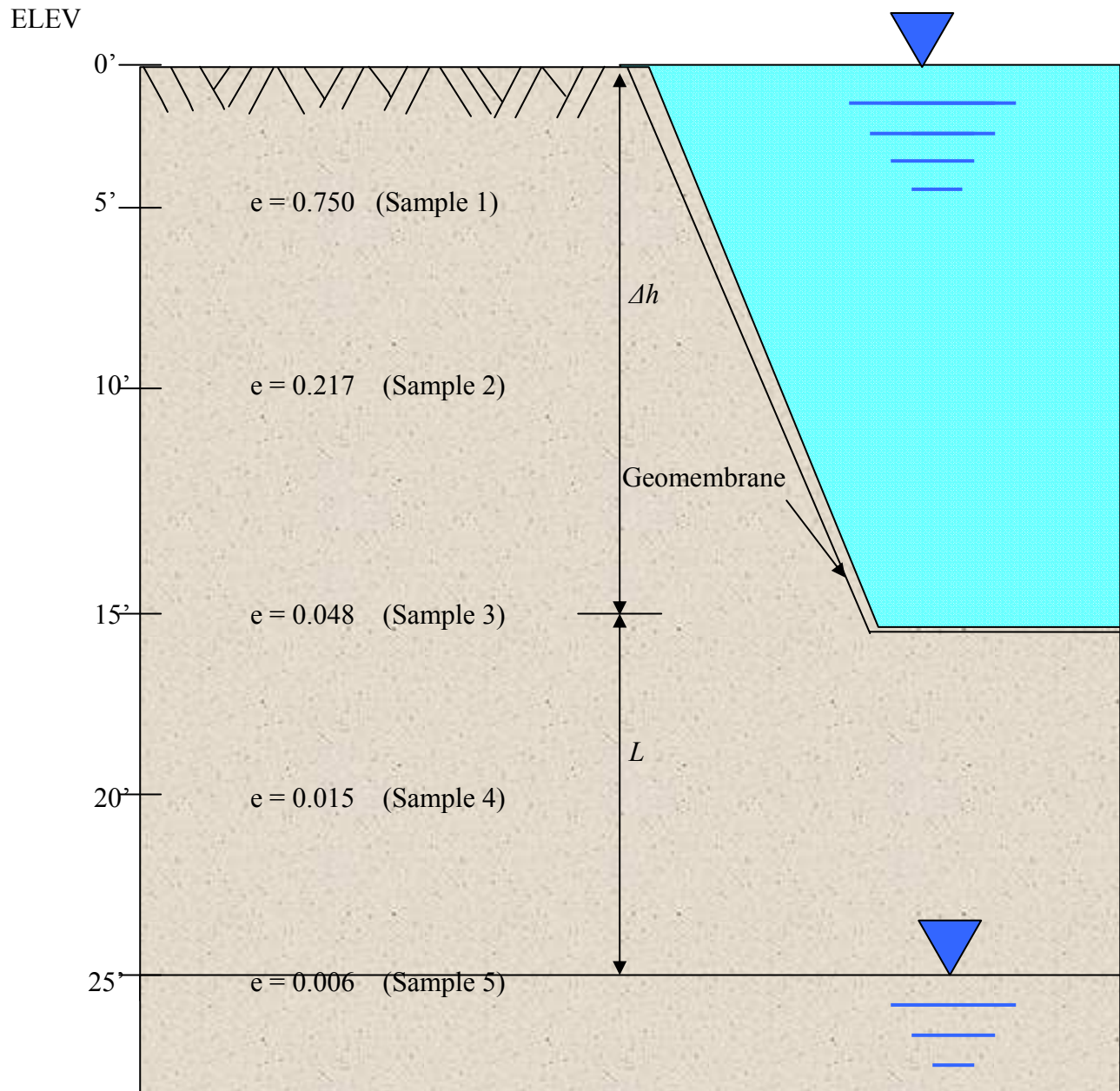
$$S_c = \frac{Q}{C \cdot L' \cdot H} = \frac{154 \text{ gallons / min} \left( \frac{ft^3}{7.45 \text{ gallons}} \right) (min)}{(42 \text{ ft})(2.6 \text{ ft})(7.45 \text{ gallons})(60 \text{ sec})} = 8.8 \times 10^{-3} \text{ cfs / ft / ft}$$

### **\*\*Permeability-14: Seepage loss from an impounding pond.**

(Revision: Aug-08)

Borings were taken at the site of an intended impoundment pond and the in-situ voids ratios at various depths are shown in the figure below. A constant-head permeability test was performed on sample #1 (which was 6" high and 2" in diameter) subjected to a pressure head of 27": after 5 seconds, 50 grams of water were collected through the sample.

Impoundment pond area = 4.4 miles<sup>2</sup>



(29) Determine the permeability of sample #1;

A trial test of a vibroflotation (densification) probe was taken to a depth of 5 feet and showed a densified voids ratio  $e = 0.55$ . If this densification ratio is attained to the full depth of 25 feet, at

what depth (to the nearest 5 feet) would you place the bottom of the pond in order to keep the total seepage  $Q$  below 50,000 gal/min?

Assume seepage only through the bottom of the pond, and that the pond is kept filled.

**Solution:**

**Step 1:** Determine the permeability  $k$  of sample # 1.

From Darcy;  $Q = k i A$ , therefore  $k = \frac{q}{iA} = \frac{4q}{i\pi d^2}$

Since 50 grams of water is equivalent to 50 cm<sup>3</sup> (for  $\gamma_w = 1 \text{ g/cm}^3$ )

but,  $q = \frac{50 \text{ cm}^3}{5 \text{ sec}} = 10 \frac{\text{cm}^3}{\text{sec}}$ , and  $i = \frac{\Delta h}{L} = \frac{27 \text{ in}}{6 \text{ in}} = 4.5$

$$\therefore k_a = \frac{4q}{i\pi d^2} = \frac{(4) \left( 10 \frac{\text{cm}}{\text{in}} \right) (10 \text{ mm/cm})}{4.5\pi (6 \text{ in})^2 \left( 2.54 \frac{\text{cm}}{\text{in}} \right)^2} = 0.122 \frac{\text{mm}}{\text{sec}}$$

**Step 2:** Determine the new permeability of the sand due to vibroflotation densification.

Using Casagrande's relation  $k = 1.4k_{0.85}e^2$

Therefore  $k_b = k_a * \frac{e_a^2}{e_b^2} = 0.122 \frac{(0.55)^2}{(0.75)^2} = 0.066 \text{ mm/sec}$

**Step 3:** The ratio of densified permeability to in-situ permeability

$$\frac{k_b}{k_a} = \frac{0.066}{0.122} = 0.54 \quad \therefore \text{Vibroflotation has reduced the permeability by half.}$$

**Step 4:** Find the densified permeability at each sample depth,

$$k = 1.4 k_{0.85} e^2 \quad \text{therefore} \quad k_{0.85} = \frac{k}{1.4 e^2} = \frac{(0.122 \text{ mm/sec})}{1.4(0.75^2)} = 0.155 \text{ mm/sec}$$

The corresponding densified permeability

$$k = 1.4 k_{0.85} e^2 \quad (\text{x ratio})$$

Depth (feet)	Original $\theta_o$	Original $k$ (mm/sec)	Densified $k$ (mm/sec)
0.	0.	0.	0.
-5	0.750	0.122	0.066
-10	0.217	0.010	0.005
-15	0.048	0.005	$2.76 \times 10^{-4}$
-20	0.015	$5.08 \times 10^{-5}$	$2.74 \times 10^{-5}$
-25	0.006	$1.03 \times 10^{-5}$	$0.56 \times 10^{-5}$

**Step 5:** Estimate the required depth  $h$  of the pond.

$$Q = 50,000 \text{ gallons/min} (1 \text{ ft}^3 / 7.48 \text{ gal}) = 6,680 \text{ ft}^3/\text{min}.$$

Consider this rate to be constant.

$$\text{From Darcy's equation} \quad Q = kiA \quad \text{or} \quad ki = \frac{k\Delta h}{L} = \frac{Q}{A}$$

$$\frac{Q}{A} = \frac{\left(6,680 \text{ ft}^3/\text{min}\right) \left(\frac{1 \text{ mm}}{3.281 \times 10^{-3} \text{ ft}}\right)}{(4.4 \text{ mile}^2) \left(5280 \text{ ft}/\text{mile}\right)^2 (1 \text{ min}) (60 \text{ sec}/\text{min})} = 2.79 \times 10^{-4} \text{ mm/sec}$$

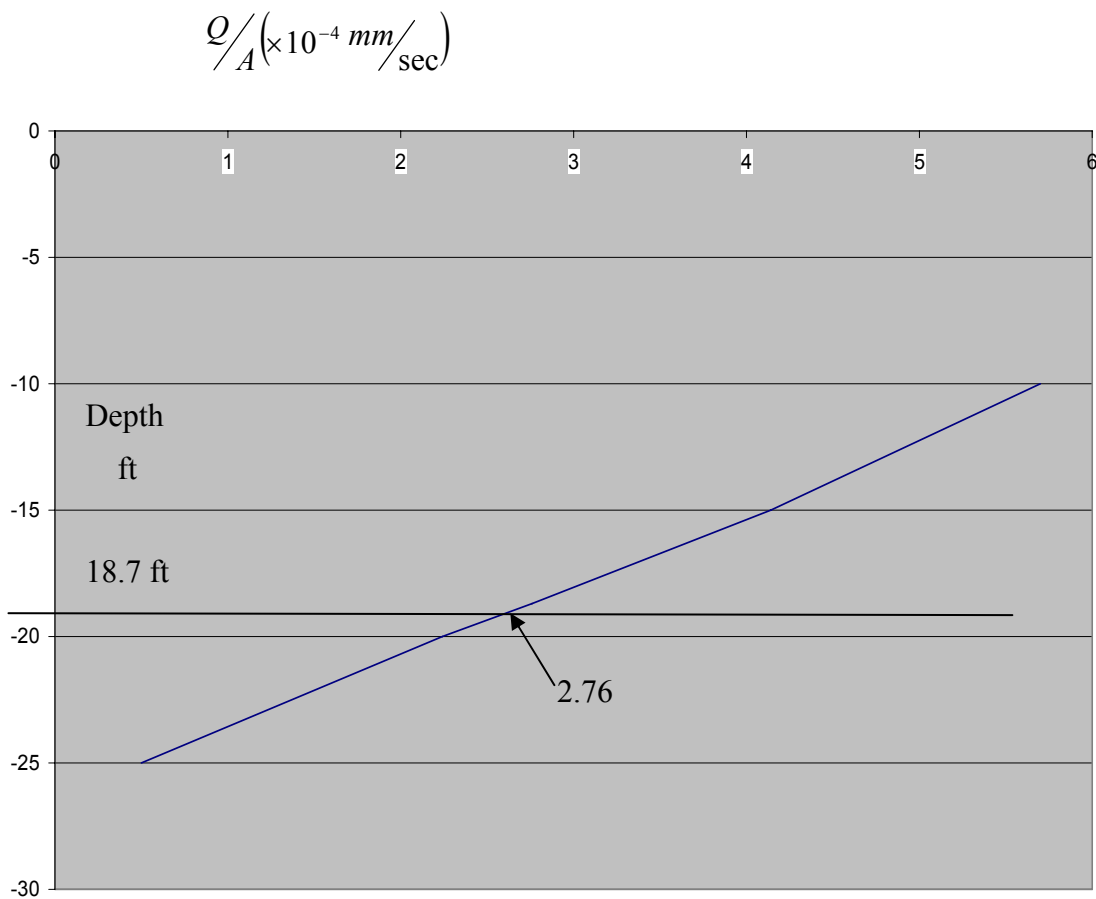
Consider the depth -15 ft.

$$k \frac{\Delta h}{L} = \frac{15}{10} \left( 2.76 \times 10^{-4} \frac{mm}{sec} \right) = 4.14 \times 10^{-4} mm/sec$$

or at a depth of: -20 ft.

$$k \frac{\Delta h}{L} = \frac{20}{5} \left( 0.56 \times 10^{-5} \frac{mm}{sec} \right) = 0.224 \times 10^{-4} \frac{mm}{sec}$$

Therefore, place the bottom of pond at -19 feet.



# Chapter 6

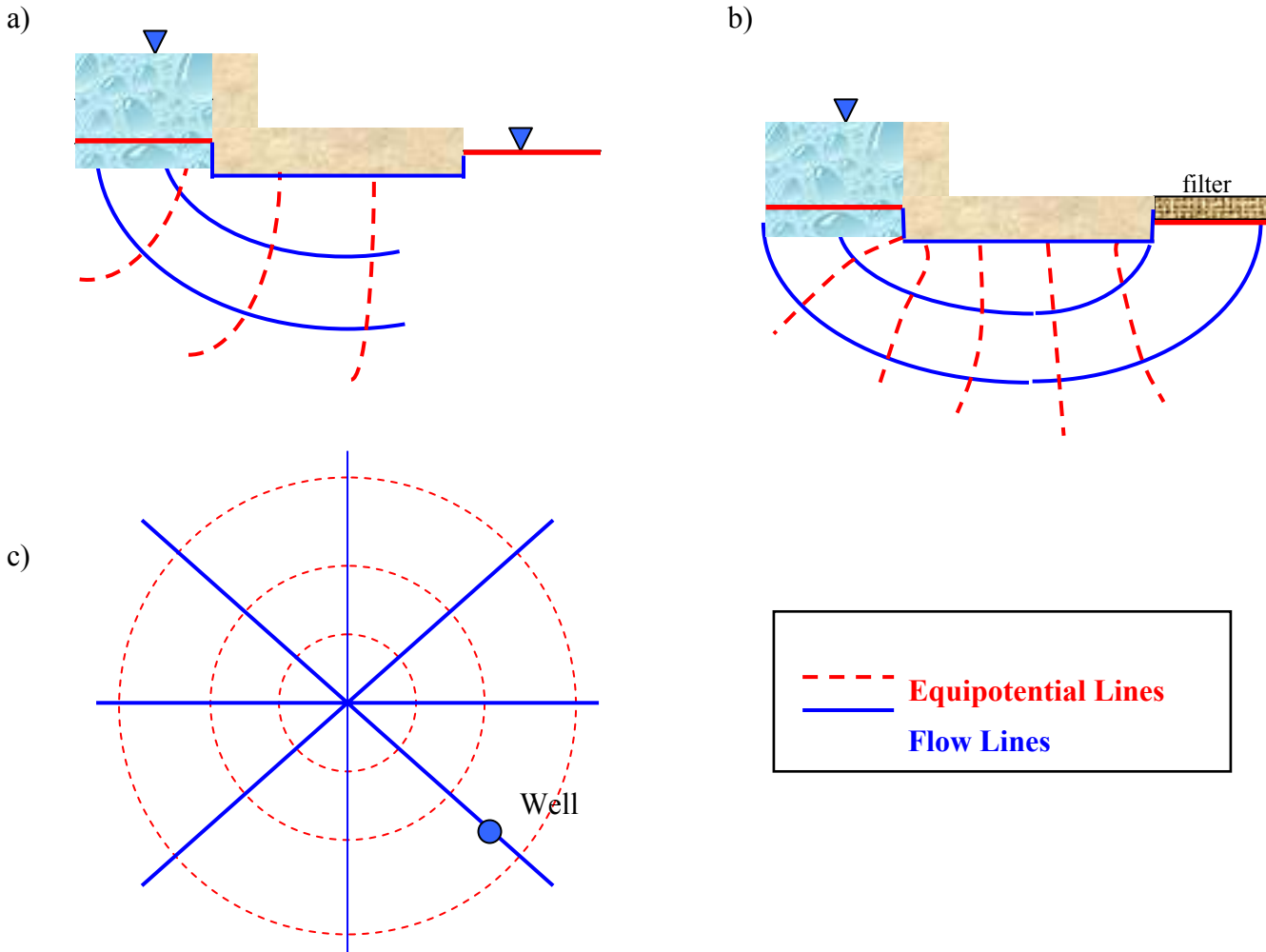
## Seepage and Flow-nets

***Symbols for Seepage and Flow-nets***

## \*Flownets-01: Correcting flawed flow-nets.

(Revision: Aug-08)

Do you recognize something wrong with each of the following flow-nets?



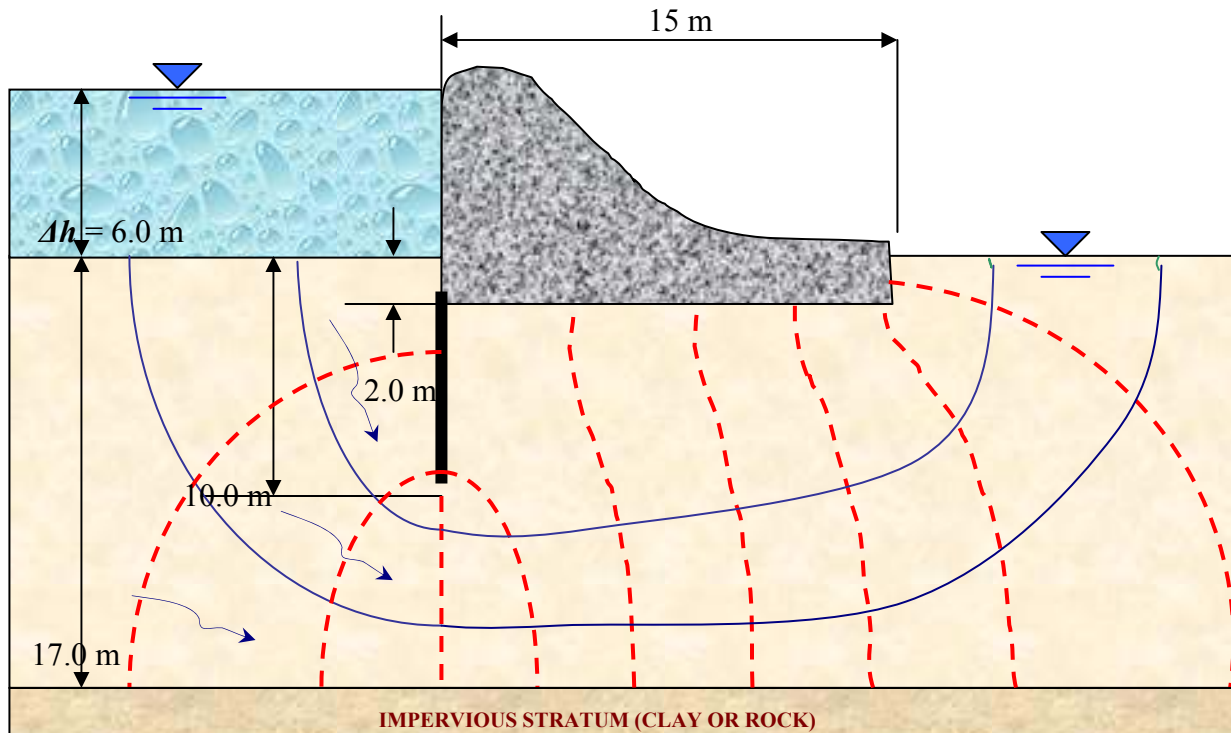
### Solution:

- Incorrectly drawn mesh, because two equipotential lines intersect each other (equipotential lines and flowlines must intersect orthogonally to each other).
- Incorrectly drawn mesh, because two flow-lines intersect each other (same as above).
- The well should be at the center of the net (a sink or a source point).

**\*Flow-nets-02: A flow-net beneath a dam with a partial cutoff wall.**

(Revision: Aug-08)

The completed flow net for the dam shown below includes a steel sheet-pile cutoff wall located at the head-water side of the dam in order to reduce the seepage loss. The dam is half a kilometer in width (shore to shore) and the permeability of the silty sand stratum is  $3.5 \times 10^{-4}$  cm/s. Find, (a) the total seepage loss under the dam in liters per year, and (b) would the dam be more stable if the cutoff wall was placed under its tail-water side?



**Solution:**

(a) Notice that  $\Delta h = 6.0$  m, the number of flow channels  $N_f = 3$  and the equipotentials  $N_{eq} = 10$ .

Using **Forcheimer's** equation,

$$q = k \Delta h \frac{N_f}{N_{eq}} = \left( 3.5 \times 10^{-4} \frac{\text{cm}}{\text{sec}} \right) \left( \frac{\text{m}}{100 \text{ cm}} \right) (6.0 \text{ m}) \left( \frac{3}{10} \right) = 6.3 \times 10^{-6} \text{ m}^2 / \text{sec} / \text{per m of dam width}$$

Since the dam is 500 meters wide (shore-to-shore) the total flow  $Q$  under the dam is,

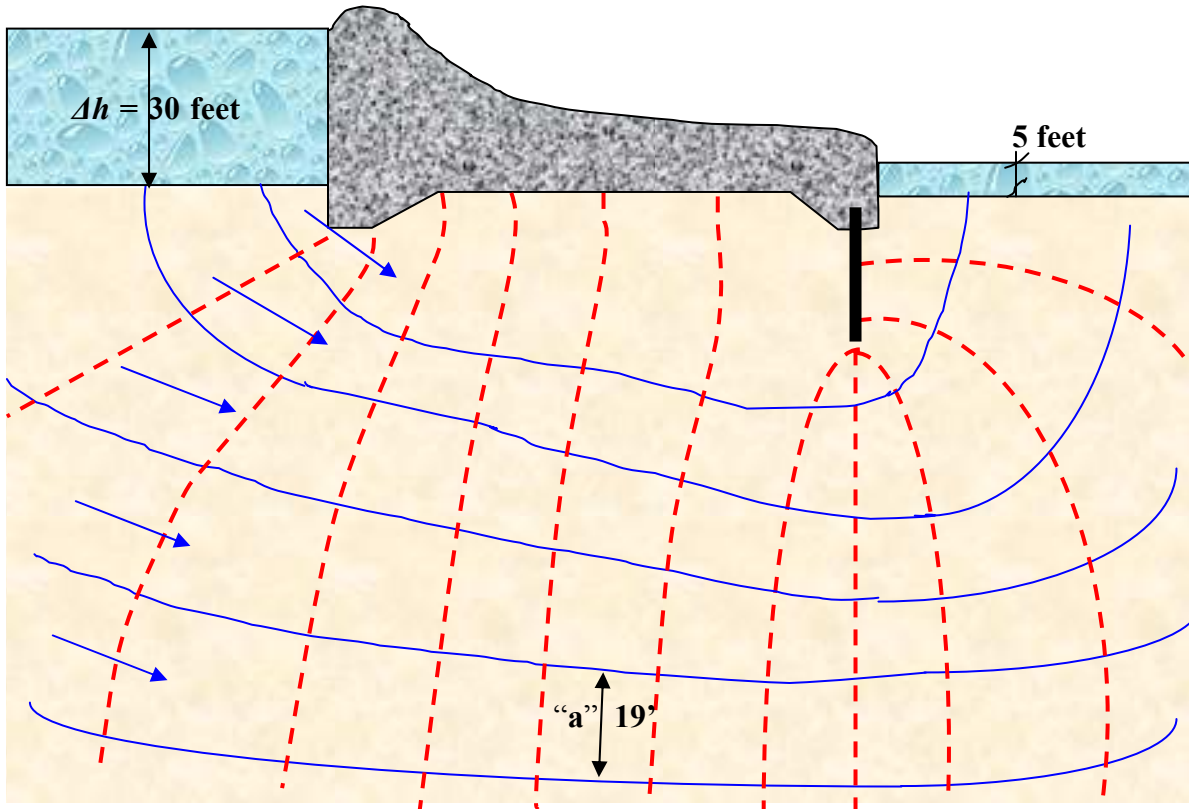
$$Q = Lq = 500 \text{ m} \left[ 6.3 \times 10^{-6} \text{ m}^3 / \text{sec} \right] \left( \frac{10^3 \text{ liters}}{1 \text{ m}^3} \right) \left( 31.5 \times 10^6 \frac{\text{sec}}{\text{year}} \right) = 100 \frac{\text{million liters}}{\text{year}}$$

b) No. Placing the cutoff wall at the toe would allow higher uplift hydrostatic pressures to develop beneath the dam, thereby decreasing the dam's stability against sliding toward the right (down-stream).

### \*Flow-nets-03: The velocity of the flow at any point under a dam.

(Revision: 12 Oct.-08)

Using the flow net shown below, (1) determine the seepage underneath the 1,000 foot wide concrete dam, and (2) the velocity at point “a” in feet/hour, where the height of the net’s square is 19 feet. The soil has a  $G_S = 2.67$ ,  $D_{10} = 0.01$  mm. Overestimate the flow by using Hazen’s coefficient  $C = 15$  to determine the permeability  $k$ .



#### Solution:

Find the permeability  $k$  using Hazen's formula:

$$k = C(D_{10})^2 = 15(10^{-2} \text{ mm})^2 = 0.0015 \frac{\text{mm}}{\text{sec}}$$

Using Forheimer's equation with flow lines  $N_f = 5$  and equipotentials  $N_{eq} = 12$ ,

$$q = k \Delta h \frac{N_f}{N_{eq}} = \left( 0.0015 \frac{\text{mm}}{\text{sec}} \right) \left( \frac{1 \text{ in}}{25.4 \text{ mm}} \right) \left[ \frac{1 \text{ ft}}{12 \text{ in}} \right] \left( \frac{3,600 \text{ sec}}{1 \text{ hour}} \right) (30' - 5') \left( \frac{5}{12} \right) = 0.185 \frac{\text{ft}^3}{\text{hr} - \text{ft of dam}}$$

$$\therefore Q = Lq = (1,000 \text{ ft}) \left( 0.185 \frac{\text{ft}^3}{\text{hr}} \right) = 185 \frac{\text{ft}^3}{\text{hr}}$$

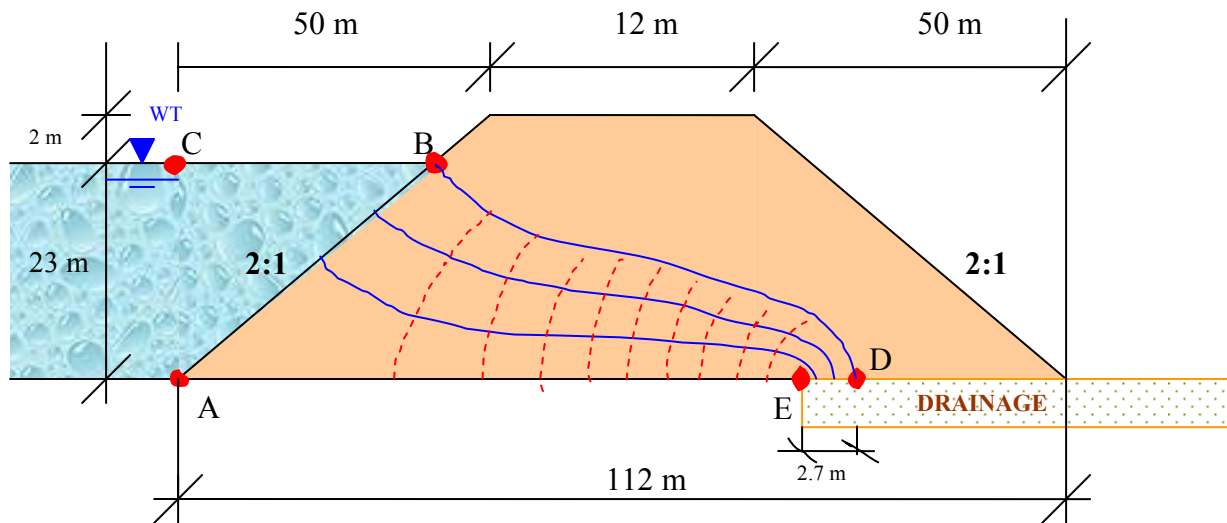
The velocity at "a" has a flow  $q$  in only that channel, or  $q/5$ ,

$$v = \frac{q}{A} = \left( \frac{\frac{0.185 \text{ ft}^3}{5 \text{ hr}}}{(19 \text{ ft high})(1 \text{ ft wide})} \right) \approx 0.002 \frac{\text{ft}}{\text{hr}}$$

### **\*Flow-nets-04: Flow through an earth levee.**

(Revision: Aug-08)

In western Miami-Dade County, the Everglades are contained with levees. Levee #111 runs North-South about 2 kilometers west of Krome Avenue and its cross section is shown below. Laboratory tests indicate that the permeability of the 80-year old levee is 0.30 m/day. What is the volume of water lost through the levee along each kilometer in m<sup>3</sup>/day?



**Cross-section of levee looking north.**

**Solution:**

Using Forheimer's equation,

$$Q = Lq = L \left( k \Delta h \frac{N_f}{N_{eq}} \right) = (1,000 \text{ m}) \left( 0.3 \frac{\text{m}}{\text{day}} \right) (23 \text{ m}) \left( \frac{3}{10} \right) = 2,070 \frac{\text{m}^3}{\text{day}}$$

### \*Flow-nets-05: Finding the total, static and dynamic heads in a dam.

(Revision: Aug-08)

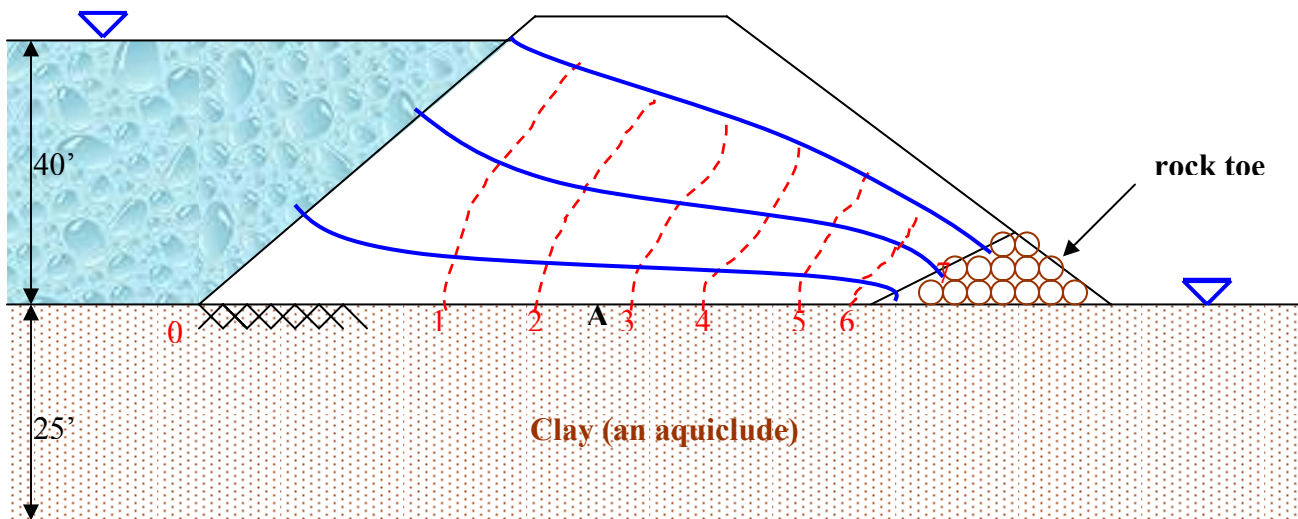
Find the seepage through the earth dam shown below in gallons/day if the sieve analysis shows the  $D_{10}$  to be 0.17 mm, and the dam is 1,200 feet wide. What is the pressure head at the top of the aquiclude and at mid-dam (point A)?

Number of flow channels  $N_f = 3$

Number of equipotential drops  $N_{eq} = 7$

Using Hazen's formula  $k = CD_{10}^2 = 15D_{10}^2 = 15(0.17\text{ mm})^2 = 0.43 \frac{\text{mm}}{\text{sec}}$

Note:  $8 \leq C \leq 15$  for  $D_{10}$  mm; to overestimate flows, use  $C=15$ .



#### Solution:

$$Q = Lq = L \left( k \Delta h \frac{N_f}{N_{eq}} \right) = (1,200 \text{ ft}) \left( 0.43 \frac{\text{mm}}{\text{sec}} \right) \left( \frac{1 \text{ inch}}{25.4 \text{ mm}} \right) \left( \frac{1 \text{ ft}}{12 \text{ inches}} \right) \left( \frac{7.5 \text{ gallons}}{1 \text{ ft}^3} \right) (40 \text{ ft}) \left( \frac{3}{7} \right) \left( \frac{86,400 \text{ sec}}{1 \text{ day}} \right)$$

$$Q = 18.8 \times 10^6 \frac{\text{gallons}}{\text{day}}$$

At point "A" the **dynamic pressure head** is  $\left( \frac{4.4}{7} \right) (40 \text{ ft}) = 25.1 \text{ feet}$

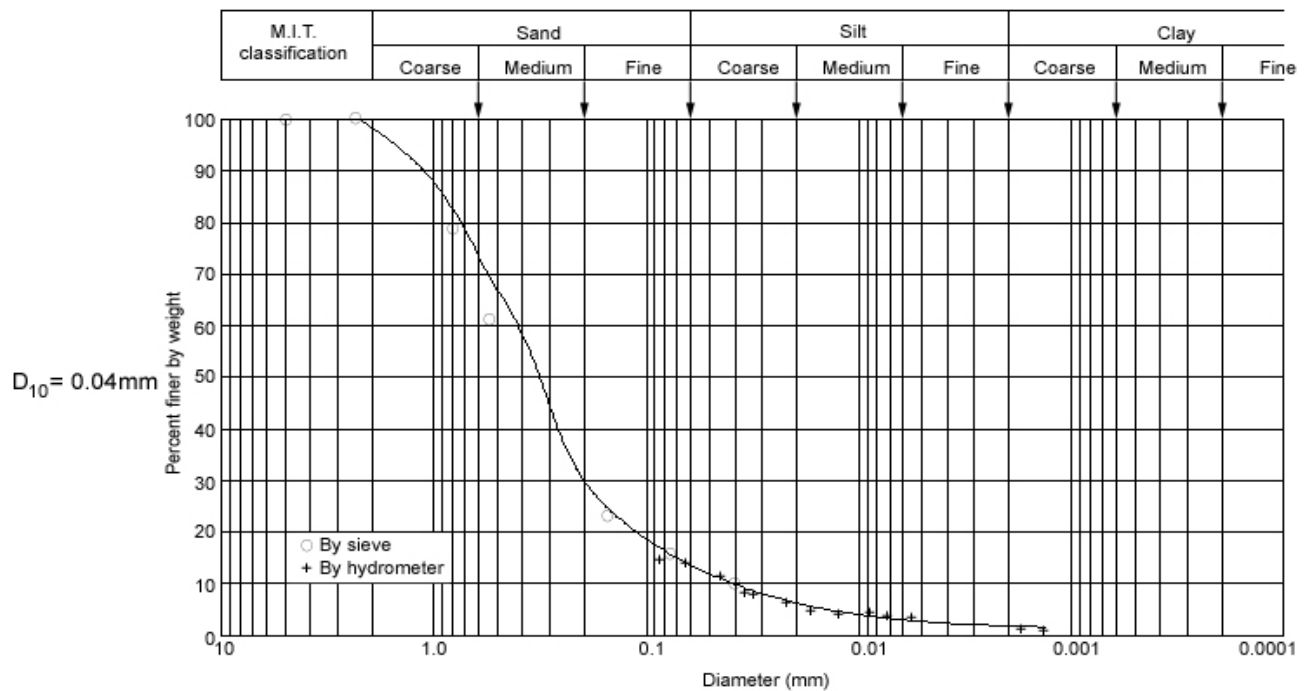
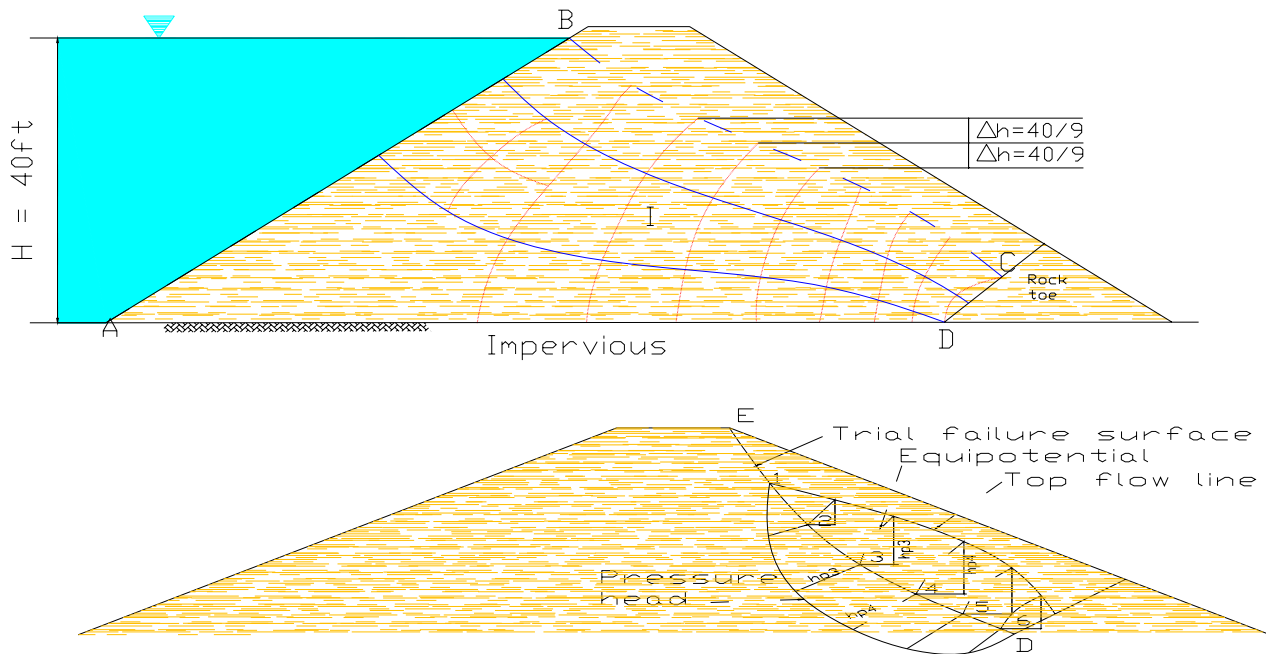
The **static head** at "A" is approximately  $\frac{2}{3} (40 \text{ ft}) = 26.7 \text{ feet}$

Therefore, **the total head** = static + dynamic =  $25.1 \text{ ft} + 26.7 \text{ ft} = 51.8 \text{ feet}$

## \*\*Flow nets-06: Hydraulic gradient profile within an earth levee.

(Revision: Aug-08)

The cross-section of an earth dam 5,000 feet wide is shown below. Determine (a) the seepage flow through the dam, in  $ft^3 / \text{minute}$ , (b) the hydraulic gradient in square I, and (c) the pore pressures along a trial failure surface along the line  $ED$ .



**Solution:**

(a) From graph  $D_{10} = 0.04 \text{ mm}$ . Using Hazen's relation, with  $C = 15$  to overestimate the permeability of the dam,

$$k = C (D_{10})^2 = (15)(0.04 \text{ mm})^2 = 0.024 \text{ mm / sec}$$

$$Q = Lq = L \left( k \Delta h \frac{N_f}{N_{eq}} \right) = (5,000 \text{ ft}) \left( 0.024 \frac{\text{mm}}{\text{sec}} \right) \left( \frac{1 \text{ inch}}{25.4 \text{ mm}} \right) \left( \frac{1 \text{ ft}}{12 \text{ inches}} \right) \left( \frac{86,400 \text{ sec}}{\text{day}} \right) (40 \text{ ft}) \left( \frac{3}{9} \right)$$

$$Q = 454,000 \frac{\text{ft}^3}{\text{day}}$$

(b) The gradient in square  $I$  is,

$$i_I = \frac{\Delta h}{l_I} = \frac{40/9}{11.2} = 0.40$$

(c) The pore pressures along  $ED$  are approximately,

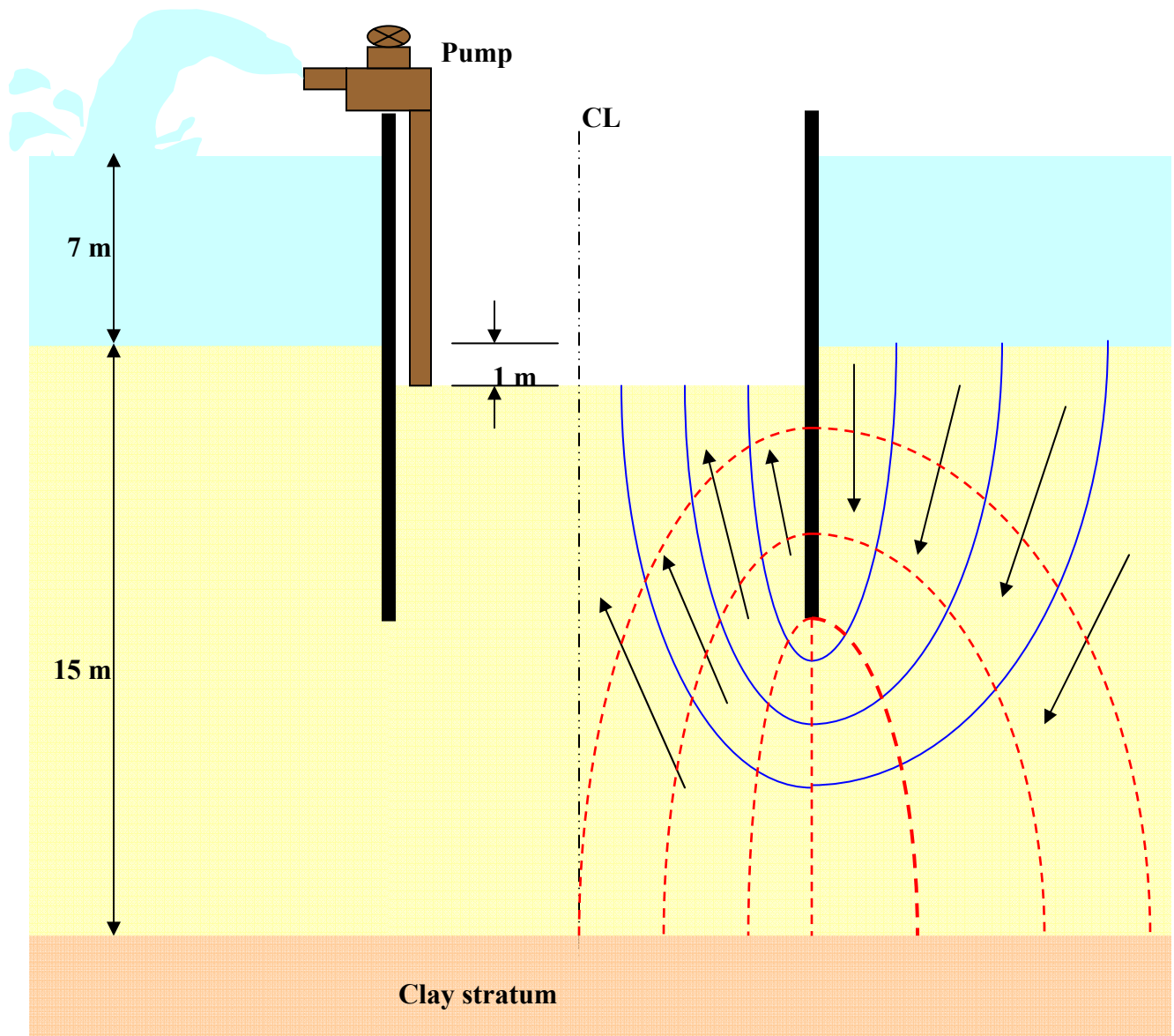
at	pore pressure		
E	0		0
1	$u = \left[ 40' - 2.5 \left( \frac{40}{9} \right) \right] 62.4 =$	1803 psf =	1.80 ksf
2	$u = \left[ 40' - 3 \left( \frac{40}{9} \right) \right] 62.4 =$	1664 psf =	1.66 ksf
3	$u = \left[ 40' - 4 \left( \frac{40}{9} \right) \right] 62.4 =$	1387 psf =	1.39 ksf
4	$u = \left[ 40' - 5 \left( \frac{40}{9} \right) \right] 62.4 =$	1109 psf =	1.11 ksf
5	$u = \left[ 40' - 6 \left( \frac{40}{9} \right) \right] 62.4 =$	832 psf =	0.83 ksf
6	$u = \left[ 40' - 7 \left( \frac{40}{9} \right) \right] 62.4 =$	555 psf =	0.55 ksf
D	$u = \left[ 40' - 8 \left( \frac{40}{9} \right) \right] 62.4 =$	277 psf =	0.28 ksf

### **\*\*Flow-net-07: Flow into a cofferdam and pump size.**

(Revision: Aug-08)

A cofferdam is to be built in the middle of a bay to place the foundations of a tall television tower. A plan area of the cofferdam is 30 m long by 10 m wide. A sample taken from the bay bottom was subjected to a hydrometer analysis: 20 grams of bay bottom dry fines were mixed with 1 liter of water. The specific gravity of the solids was found to be 2.65. The dynamic viscosity of water is  $10^{-2}$  Poise (dynes-sec/cm<sup>2</sup>) at 20°C. After 1 hour of precipitation, the hydrometer dropped 16 cm. The soil is uniform in size, with 80 % passing the # 200 sieve.

- (30) What type of soil was the sample?
- (31) Will a large 3 m<sup>3</sup> per minute pump be adequate to maintain a 1 m draw down below the bay bottom? Use  $FS > 2$ .





**Solution:**

a) Use Stoke's formula to find diameter of the bay bottom particles:

$$D_{10} = d = \sqrt{\frac{18\eta}{\gamma_s - \gamma_w}} \sqrt{\frac{L}{t}} = \left( \sqrt{\frac{18\eta}{\gamma_w (G_s - 1)t}} \right) \left( \sqrt{\frac{18 \times 10^{-2} \text{ dynes-sec} \times \text{cm}^3 \times 16 \text{cm}}{\text{cm}^2 (2.65 - 1)(9.81 \text{ dynes} \times 3600 \text{sec})}} \right)$$
$$d = 0.070 \text{ mm}$$

**Therefore the soil is silt (0.075 mm to 0.002 mm).**

b) Hazen's formula permits us to estimate the permeability  $k$  of the soil:

$$k = CD_{10}^2 \frac{\text{mm}}{\text{sec}} = 15(0.07 \text{ mm})^2 = 0.074 \frac{\text{mm}}{\text{sec}}$$

Use Forheimer's formula to estimate the total flow  $Q$  into the cofferdam:

$$q = k \Delta h \frac{N_f}{N_{eq}} = \left( 7.4 \times 10^{-2} \frac{\text{mm}}{\text{sec}} \right) (8 \text{ m}) \left( \frac{4}{8} \right) \left( \frac{\text{m}}{10^3 \text{ mm}} \right) = 29.6 \times 10^{-5} \frac{\text{m}^3}{\text{m-s}}$$

$$Q = (\text{perimeter})(q) = 80 \text{ m} \left( 29.6 \times 10^{-5} \frac{\text{m}^3}{\text{m-s}} \cdot \frac{60 \text{ s}}{\text{min}} \right) = 1.42 \frac{\text{m}^3}{\text{min}}$$

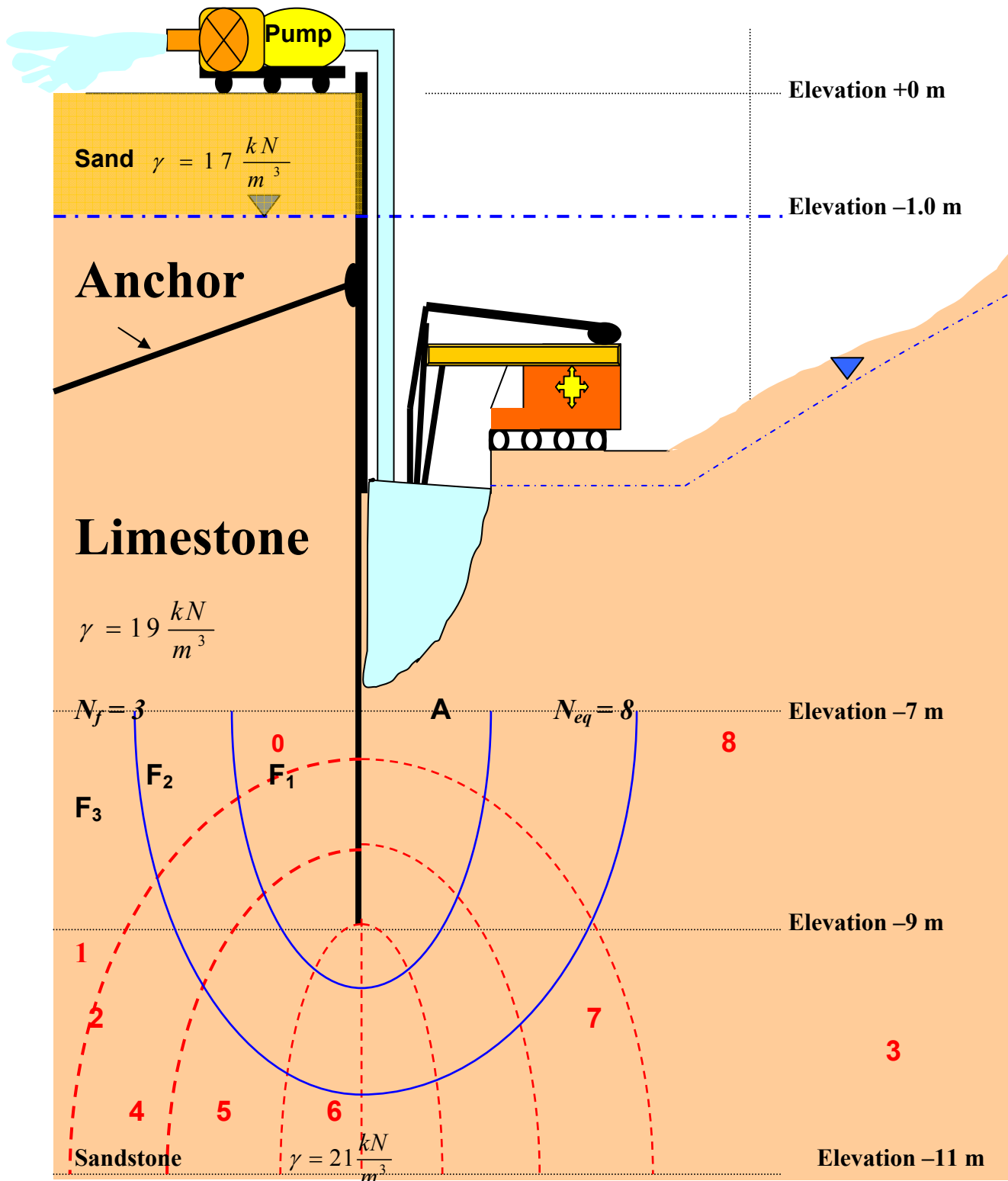
$$\text{Pump FS} = \frac{3 \text{ m}^3 / \text{min}}{1.42 \text{ m}^3 / \text{min}} = 2.11 > 2 \quad \text{OKAY Pump is adequate!}$$

## \*Flow-nets-08: Drainage of deep excavations for buildings.

(Revision: Aug-08)

A new office building will require a two-level underground parking garage. The plan size of the site is 100 x 80 meters. Some of the soil properties are shown below.

- At what depth of the excavation will the limestone (shear strength =  $0.1 \text{ MN/m}^2$ ) have a punching shear failure? Suggest using a 1m x 1m plug as a model.
- What size pump do you need ( $\text{m}^3/\text{minute}$ ) with a factor of safety of 3?



**Solution:**

a) The uplift force at point A is found by,

$$\sum F_y = 0 \quad F_{uplift} - F_{shear\ resistance} = 0$$

$$F_{uplift} = uA = (\Delta h \gamma_w) A = (6\text{ m}) \left( 9.81 \frac{kN}{m^3} \right) (1\text{ m}^2) = 59\text{ kN}$$

$$F_{shear} = \tau A = 0.1 \frac{MN}{m^2} (4x) m^2 = 400x \frac{kN}{m}$$

$$\therefore x = \frac{59}{400} = 0.147\text{ m with } FS = 7 \quad \therefore x = 1\text{ m}$$

This corresponds to elevation - 6 m.

b) Determine the flow quantity with Forheimer's formula,

$$Q = qL = Lk \left( \frac{a}{b} \right) \left( \frac{N_f}{N_q} \right) \Delta h$$

$$\text{where } k = CD_{10}^2 = (12.5)(0.095\text{ mm})^2 = 0.113 \frac{m\text{ m}}{\text{sec}}$$

$$\Delta h = 5\text{ m}$$

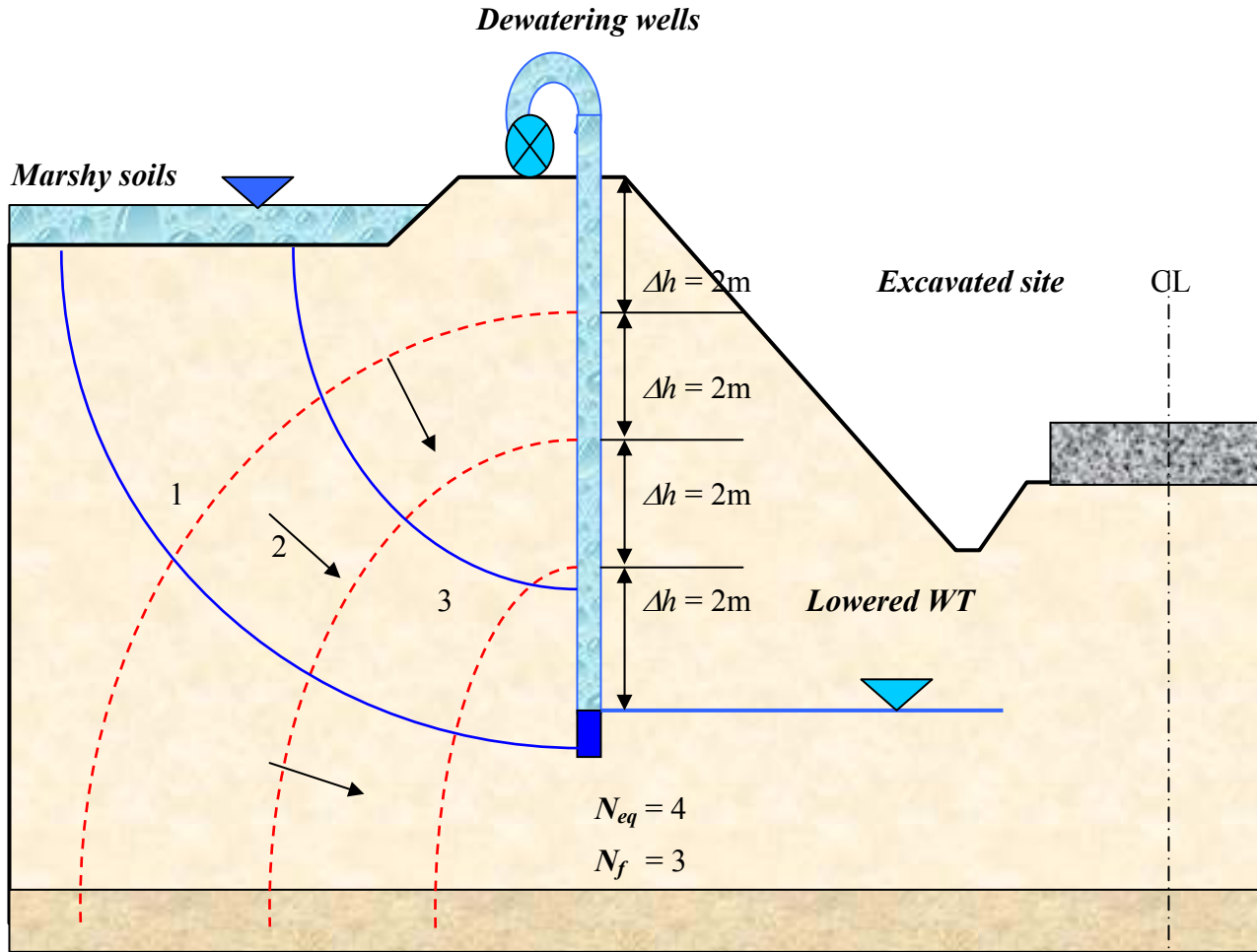
$$\therefore Q = (360\text{ m}) \left( 0.113 \frac{m\text{ m}}{\text{sec}} \right) \left( \frac{3}{8} \right) (5\text{ m}) \left( \frac{60}{m} \right) \left( 10^{-3} \frac{m}{m\text{ m}} \right) = 4.6\text{ m}^3 / \text{min} \approx 5\text{ m}^3 / \text{min}$$

*For a FS = 3 use a 15  $\frac{m^3}{\text{min}}$  pump.*

## \*Flow-nets-09: Dewatering a construction site.

(Revision: Aug-08)

The figure below shows a dewatering plan to build the foundations of an office building below the water table and without sheet-piling. The plan area of the excavation is 400 m long by 100 m wide. The soil has a  $D_{10}$  of 0.02 mm. What size pump do you need (gpm) with a *Factor of Safety* = 2?



### **Solution:**

Notice that  $\Delta h = 8\text{ m}$ ,  $N_f = 3$ ,  $N_{eq} = 4$  and  $L = \text{perimeter} = 1,000\text{ m}$ .

The permeability  $k = C (D_{10})^2 = (15)(0.02\text{ mm})^2 = 0.006\text{ mm/s}$

$$Q = L q = L k \Delta h \left( \frac{N_f}{N_{eq}} \right) = (200 + 800)\text{ m} \left( 0.006 \frac{\text{mm}}{\text{s}} \right) (8\text{ m}) \left( \frac{3}{4} \right) \left( \frac{\text{m}}{10^3 \text{ mm}} \right) \left( \frac{60\text{ s}}{1\text{ min}} \right) = 2.16 \frac{\text{m}^3}{\text{min}}$$

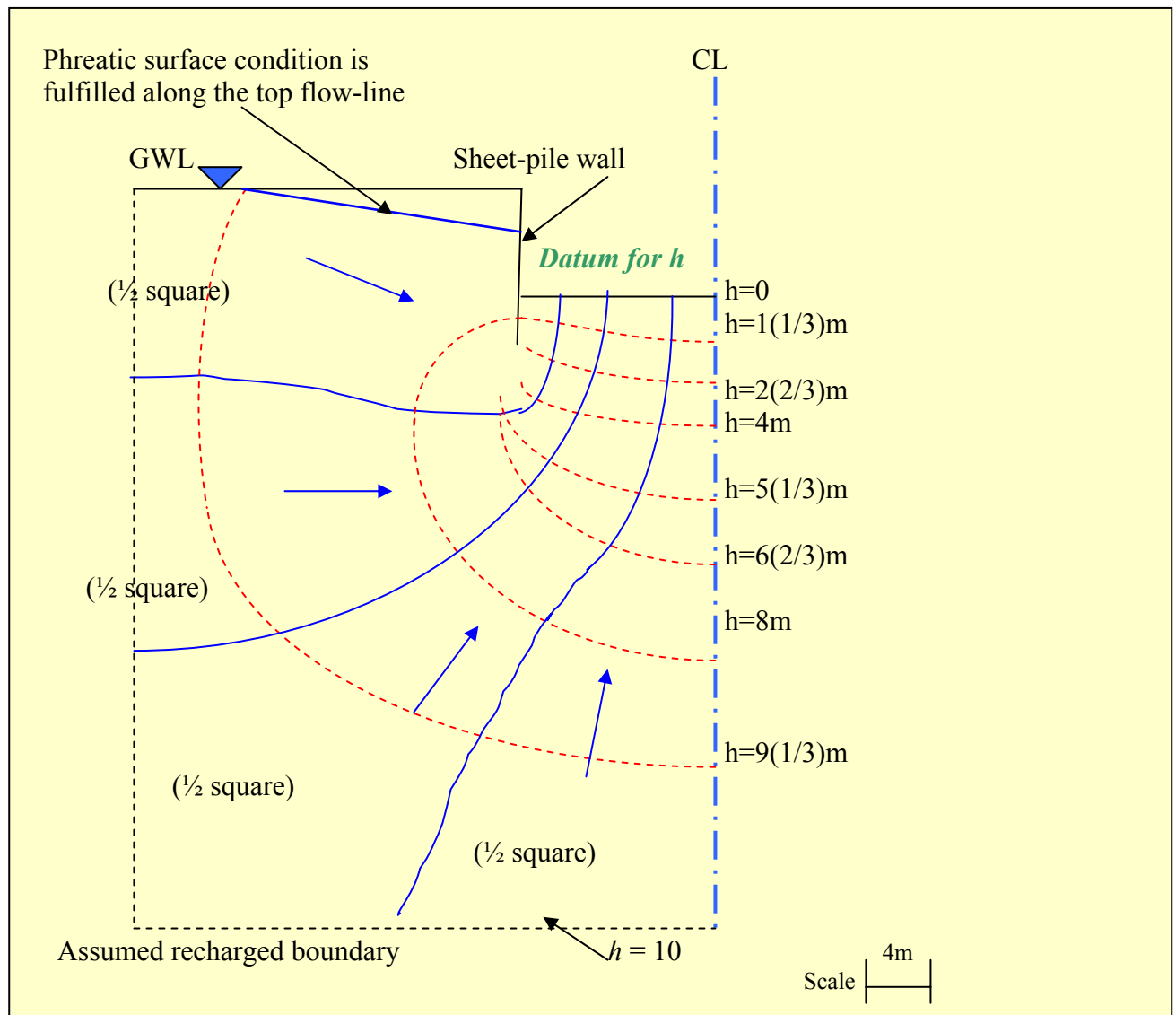
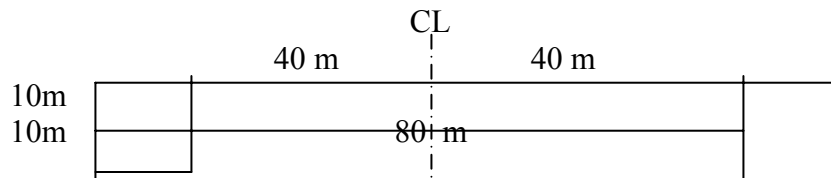
$$Q = \left( 2.16 \frac{\text{m}^3}{\text{min}} \right) \left( \frac{\text{ft}}{0.30\text{ m}} \right)^3 \left( \frac{7.45\text{ gal}}{\text{ft}^3} \right) = 600 \frac{\text{gallons}}{\text{min}}$$

**Therefore, for a factor of safety of 2 use at least a 1,200 gallons per minute pump or two 600 gallons per minute pumps.**

## \*Flow-net-10: Dewatering in layered strata.

(Revision: Aug-08)

The figure below shows the profile of a square excavation (in plan view) in a layered soil, where the vertical permeability is  $5 \times 10^{-5}$  m/s and the horizontal permeability is roughly ten times higher than the vertical. Estimate the dewatering capacity requirements, in  $\text{m}^3/\text{hour}$ , to prevent the excavation from flooding. The value of  $\Delta h$  is to scale, but you may use 10 m.



**Solution:**

$$\Delta h = 10\text{ m}, N_f = 4, N_{eq} = 8$$

$$k = \sqrt{k_x k_y} = \sqrt{(5 \times 10^{-4} \text{ m/s})(5 \times 10^{-5} \text{ m/s})} = 1.6 (10^{-4} \text{ m/s})$$

The perimeter of the cofferdam  $p = (4)(80\text{ m}) = 320\text{ m}$

$$Q = qp = \left[ k \Delta h \left( \frac{N_f}{N_{eq}} \right) \right] p = \left[ (1.6) \left( 10^{-4} \frac{\text{m}}{\text{s}} \right) (10\text{ m}) \left( \frac{4}{8} \right) \right] (320\text{ m})$$

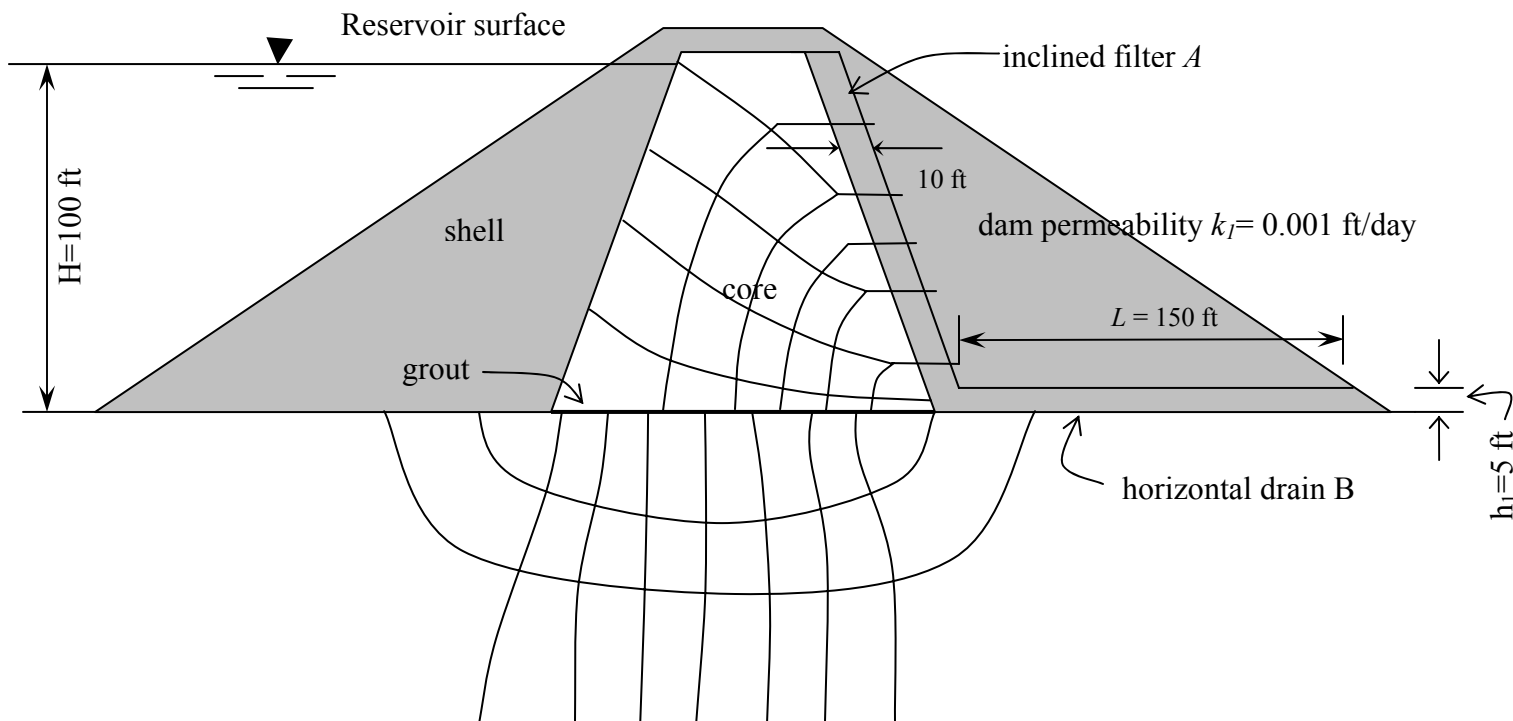
$$Q = 0.253 \left( \frac{\text{m}^3}{\text{s}} \right) \left( \frac{3,600\text{ s}}{\text{hr}} \right) = 911 \frac{\text{m}^3}{\text{hr}}$$

## **\*\*Flownets-11: Flow through the clay core of an earth dam.**

(Revision: Aug-08)

An earth dam on a pervious but strong earth foundation has the cross-section shown in the figure below. The core of the dam is sealed from the jointed rock foundation with a thin layer of grout.

- (32) State the function and properties of the core shell and drains;
- (33) What is the function of the grout between the core and foundation? Under what conditions is it most important?
- (34) Calculate the seepage quantity per foot of length of the dam through the dam, through the foundation, and the total seepage quantity.
- (35) What grading requirements should be specified for the inclined filter A?
- (36) What minimum permeability  $k$  is required in the horizontal drain B to prevent saturation from rising into the random fill zone? Give the results of  $k$  in ft/day.



Note: The grain size of core = 100% passes 1", 15% size = 1/8", and 85% size = 0.001 in.

Foundation layer permeability  $k_2 = 0.1$  ft/day

### **Solution:**

- (a) The **core** is used to retain water within the dam, that is, to resist seepage. The material should be relatively impermeable (clay) and should not shrink or swell excessively.

The **shell** provides the structural strength to support and protect the core. The material must be more permeable than the core material, strong and durable.

The drains are provided to reduce the pore water pressures in the foundation and in the embankment to increase stability. The drains also remove seepage water to reduce soil erosion. The drain material must be permeable enough to permit drainage with a low head loss and yet fine enough to keep the adjacent soil in place.

- (b) The primary function of the grout between the core and foundation is to form an impervious layer which prevents seepage along the contact surface. This becomes most important when the ratio  $k_2/k_1$  becomes large.
- (c) Calculate the seepage  $Q$  by using the flow net shown in the figure.

1) Through the dam:  $Q = k_1(\Delta h/L)b$  in  $\text{ft}^3/\text{day}/\text{ft}$  where  $b$  is the normal distance between streamlines. The flow net divides the core into 4 zones (#1 at the bottom, #4 at the top).

$$Q = k_1 \sum_{j=1}^4 i_j b_j = k_1 \sum_{j=1}^4 (\Delta h)_j (\underline{b})_j$$

In zone #1, the flow net is nearly rectangular, so  $(\underline{b})_1 = 2$ ; for zone # 4,  $(\underline{b})_j = 1$

$\ell$

$\ell$

The average head loss  $h_1$  across the core in each zone

$(\underline{p}) + Z_u = \text{constant} = 100'$  on the upstream face

$\gamma$

On down stream face of core  $(\underline{p}) = 0$  is assumed in the drain,

$\gamma$

so that  $Z_L + h_L = \text{constant} = 100'$  on the downstream face

Using an average  $Z_L$  for each zone by scaling,

$$\text{Zone\#1 } Z_L = 2' \quad h_L = 98' \quad \frac{\Delta h_L}{6} = \frac{98}{6}$$

$$\text{Zone\#2 } Z_L = 10' \quad h_L = 90' \quad \frac{\Delta h_L}{5.5} = \frac{90}{5.5}$$

$$\text{Zone\#3 } Z_L = 25' \quad h_L = 75' \quad \frac{\Delta h_L}{4.5} = \frac{75}{4.5}$$

$$\text{Zone\#4 } Z_L = 55' \quad h_L = 45' \quad \frac{\Delta h_L}{3} = \frac{45}{3}$$

4

$$Q_1 = k_1 \sum_{j=1}^4 \Delta h_j (b/L)_j = 0.001 \left[ \frac{98'}{6}(2) + \frac{90'}{5.5} + \frac{75'}{4.5} + \frac{45'}{3} \right] = \underline{\underline{0.081 \text{ ft}^3/\text{day}/\text{ft of dam}}}$$

2) Through the foundation  $Q_2 = \underline{N}_F kh$

$$N_d$$

where  $N_F$  = number of flow paths,  $N_d$  = number of equipotential drops and  $h$  = total head dissipated.

$$Q_2 = \frac{3}{8} (0.1)(100) = \underline{\underline{3.75 \text{ ft}^3/\text{day}/\text{ft of dam}}}$$

3) The total seepage  $Q$  is therefore,

$$Q = Q_1 + Q_2 = 0.081 + 3.75 = \underline{\underline{3.83 \text{ ft}^3/\text{day}/\text{ft of dam}}}$$

(d) The grading requirements for the inclined filter A,

(1) Free drainage, require  $D_{15}(\text{filter}) \geq 4 D_{15}(\text{soil})$

$$\begin{aligned} D_{15}(\text{filter}) &\geq 4 (0.001 \text{ in}) \\ &\geq 0.004 \text{ in} \end{aligned}$$

(2) To prevent erosion of the core material requires  $D_{15}(\text{filter}) \geq 4 D_{85}(\text{soil})$

$$\begin{aligned} &\geq 4 (0.001 \text{ in}) \\ &\geq 0.004 \text{ in} \end{aligned}$$

So 85% of the filter material must be coarser than 0.01" to 0.2". The filter grain size grading curve should be parallel to or flatter than the core material grading curve. See the graph on the next page for one possible grading curve, which gives

$$\begin{aligned} 100\% \text{ passes} &= 10 \text{ inch} \\ 15\% \text{ passes} &= 2 \text{ inch} \\ 85\% \text{ passes} &= 0.01 \text{ inch} \end{aligned}$$

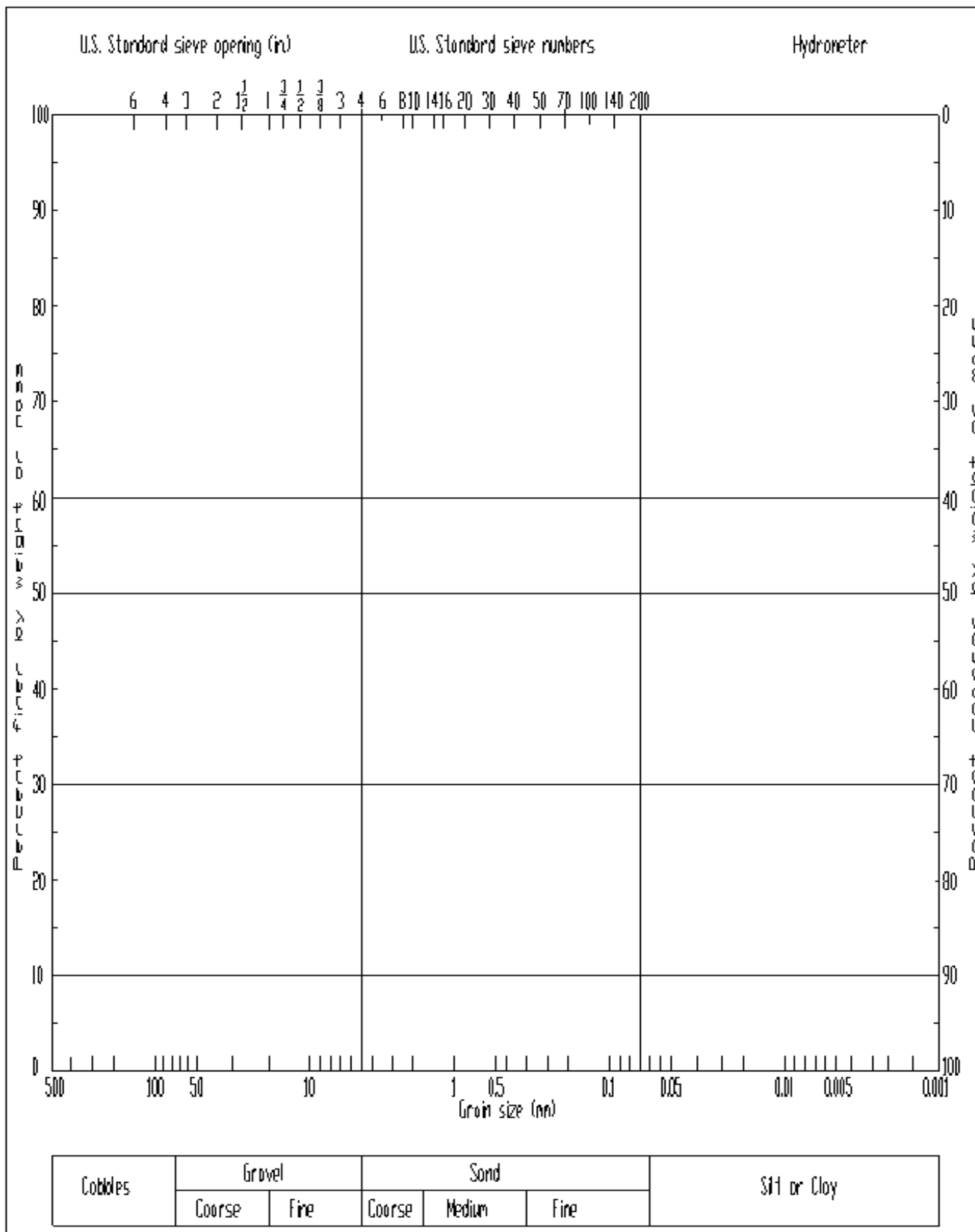
(e) The drain B must carry the total seepage flow  $Q = 3.83 \text{ ft}^3/\text{day}/\text{ft of dam}$  calculated above. The Dupuit formula for two-dimensional flow on a horizontal impervious boundary is

$$Q = \frac{k}{2L} (h_1^2 - h_2^2)$$

where  $Q = 3.83 \text{ ft}^3/\text{day}/\text{ft}$ ,  $L = 150 \text{ ft}$ ,  $h_1 = 5 \text{ ft}$  and  $h_2 < 5 \text{ ft}$ .

At what value of  $h_2$  will it minimize  $k$ ? Clearly it is when  $h_2 = 0$ , although this does seem unrealistic since we are saying that the flow at the lower end of the drain has zero depth. Nevertheless, it gives us a minimum value, which is:

$$k_{\min} = \frac{2LQ}{h_1^2} = \frac{2(150)(3.83)}{5^2} = \underline{\underline{46 \text{ ft/day}}}$$



# Chapter 7

## Effective Stresses and Pore Water Pressure

*Symbols for Effective Stresses and Pore Water Pressure*

**\*Effective Stress–01: The concept of buoyancy.**

(Revision: Aug-08)

**What force is required to hold an empty box that has a volume of 1 cubic foot, just below the water surface?**

**Solution:**

**The volume of the displaced water is  $1 \text{ ft}^3$ .**

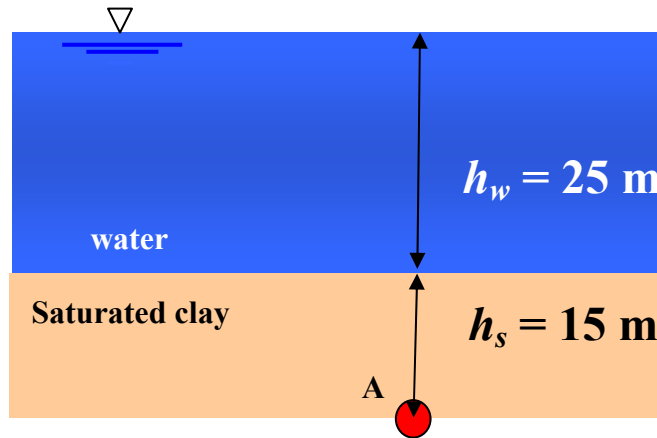
**Therefore, the force is the weight of  $1 \text{ ft}^3$  of water =  $62.4 \text{ lbs} / \text{ft}^3$ .**

**What is the force required to hold the same box 10 feet below the surface?**

### \*Effective Stress–02: The concept of effective stress.

(Revision: Aug-08)

A sample was obtained from point A in the submerged clay layer shown below. It was determined that it had a  $w = 54\%$ , and a  $G_s = 2.78$ . What is the effective vertical stress at A?



#### Solution:

The effective stress  $\sigma'$  at the point A consists solely of the depth of the soil (not of the water) multiplied by the soil buoyant unit weight.

$$\sigma' = \gamma' h_{soil} \quad \text{where} \quad \gamma' = \gamma_b = \gamma_{SAT} - \gamma_w$$

In order to find  $\gamma'$  there are a number of derivations, such as this one,

$$\gamma' = \left[ \frac{(G_s + e) \gamma_w}{1 + e} \right] - \gamma_w \quad \text{where the voids ratio } e \text{ can be found through } Se = wG_s$$

and noticing that  $S = 1$  because the soil is 100% saturated,  $e = wG_s = (0.54)(2.78)$

$$\sigma' = \gamma' h_{soil} = \left\{ \left[ \frac{(G_s + e) \gamma_w}{1 + e} \right] - \gamma_w \right\} h_{soil} = \left[ \frac{[2.78 + (0.54)(2.78)](9.81)}{1 + (0.54)(2.78)} - 9.81 \right] (15 \text{ m})$$

$$\sigma' = 105 \text{ kPa}$$

### **\*Effective Stress–03: The concept of effective stress with multiple strata.**

(Revision: Aug-08)

The City of Houston, Texas has been experiencing a rapid lowering of its phreatic surface (draw-downs) during the past 49 years due to large volumes of water pumped out of the ground by industrial users.

- a) What was the effective vertical stress at a depth of 15 m in 1960?
- b) What is the effective stress at the same depth in 2009?
- c) What happens to the ground surface as a result of the draw-downs?

#### **Solution:**

a)

$$\sigma'_v = [\gamma h + \gamma' h']_{SAND} + [\gamma' h']_{SILT} + [\gamma' h']_{CLAY} \quad \text{where} \quad \gamma' = \gamma_{SAT} - \gamma_w$$
$$\sigma'_v = [(20.4)(3) + (18.8 - 9.81)(3)] + [(14.9 - 9.81)(6)] + [(12.6 - 9.81)(3)]$$
$$\sigma'_v = 128 \text{ kPa}$$

b)  $\sigma'_v = [(20.4)(6) + (16.5)(6)] + [(12.6 - 9.81)(3)] = 230 \text{ kPa}$

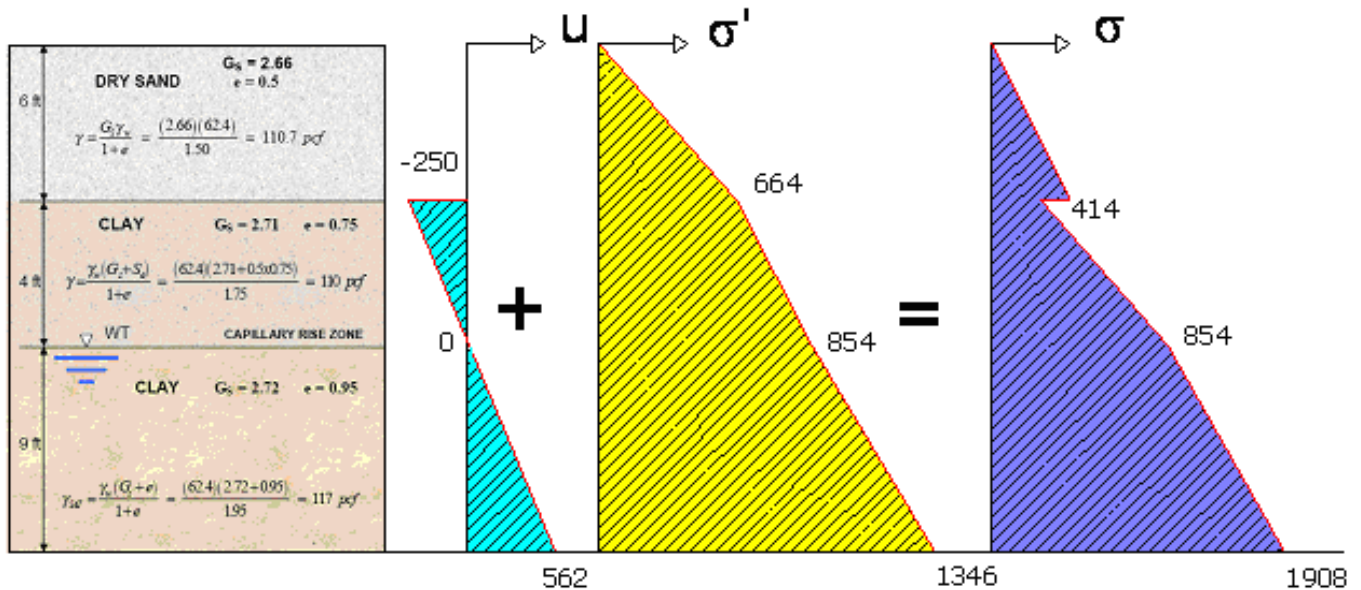
This is an 80% increase in stress due solely to a dropping water table.

c) The ground surface has also been lowered, due to the decreasing thickness of the sand and the silt strata due to their loss of the volume previously occupied by the water.

## Effective Stress-03B

Revision

In the soil profile shown below, show a plot of the pore water pressure and the effective stress along the right margin of the figure, with numerical values at each interface. Pay heed to the capillarity in the upper clay. Assume  $S = 50\%$ .



**Solution:**

$$\Delta h = (62.4) \times (4) = -250$$

$$\Delta h = (62.4) \times (6) = 562$$

$$\Delta h = (110.7) \times (6) = 664$$

$$\Delta h = \{[(110-62.4) \times (4)] + 664\} = 854$$

$$\Delta h = \{[(117-62.4) \times (9)] + 854\} = 854$$

$$\Delta h = 664 - 250 = 414$$

$$\Delta h = 0 + 854 = 854$$

$$\Delta h = 562 + 1346 = 1908$$

Depth (ft)	$u + \sigma'$		$\sigma$
0	0	0	0
6	-250	664	414
10	0	854	854
19	562	1346	1908

# Chapter 8

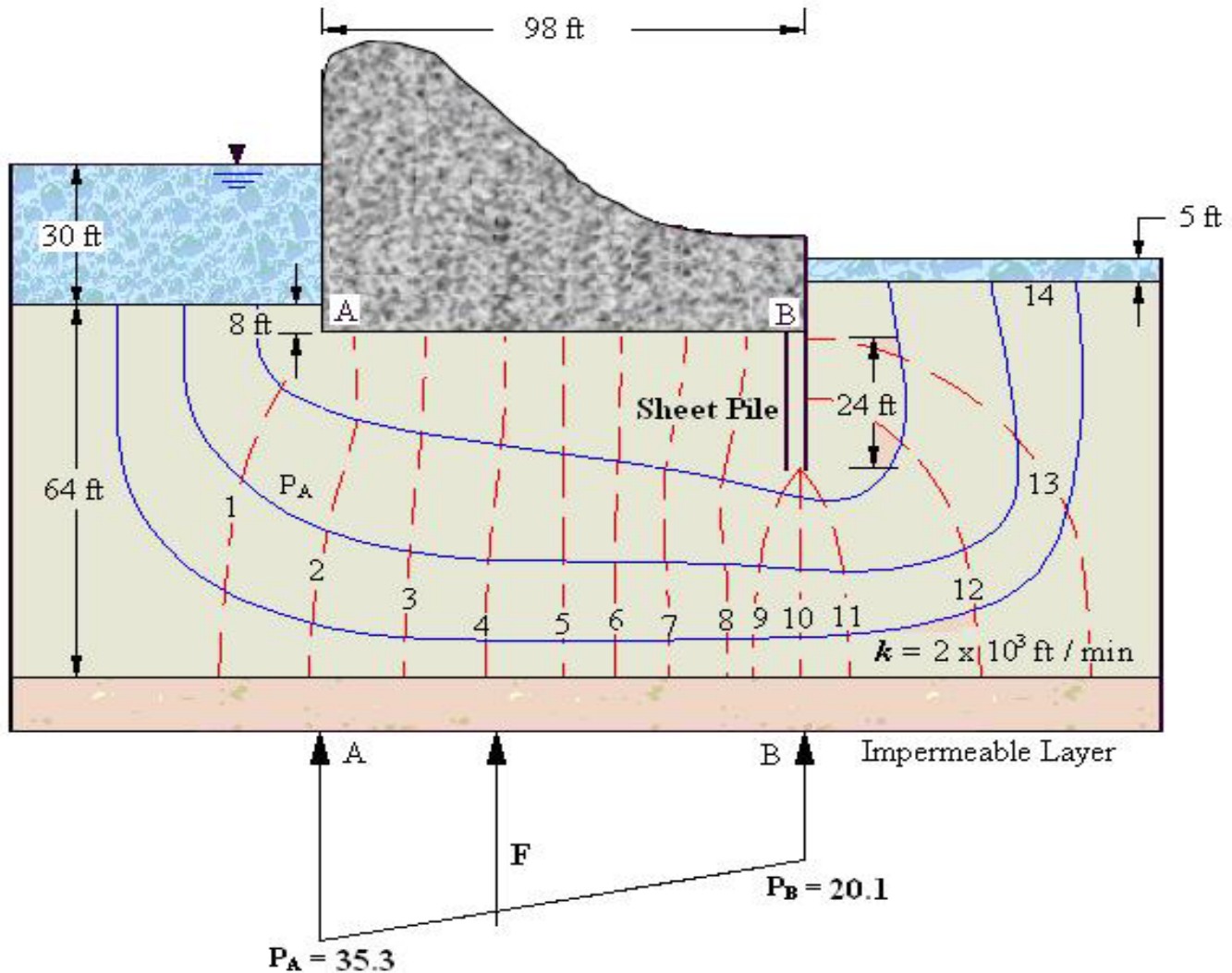
## Dams and Levees

*Symbols for Dams and Levees*

**\*Dams-01: Find the uplift pressure under a small concrete levee.**

(Revision: Sept.-08)

Calculate the uplift force at the base of the weir, per foot of width. Points A and B are at the corners of the concrete levee.



**Solution:**

The dynamic head drop per equipotential is,

$$\Delta(\Delta h) = \frac{H_A - H_B}{N_{eq}} = \frac{30 \text{ ft} - 5 \text{ ft}}{14 \text{ drops}} = 1.8 \text{ ft / drop}$$

$$\text{Pressure head at A} = 30 \text{ ft} + 8 \text{ ft} - \Delta h (1.5) = 35.3 \text{ ft}$$

$$\text{Pressure head at B} = 30 \text{ ft} + 8 \text{ ft} - \Delta h (10) = 20.1 \text{ ft}$$

The uplifting force  $F$  is,

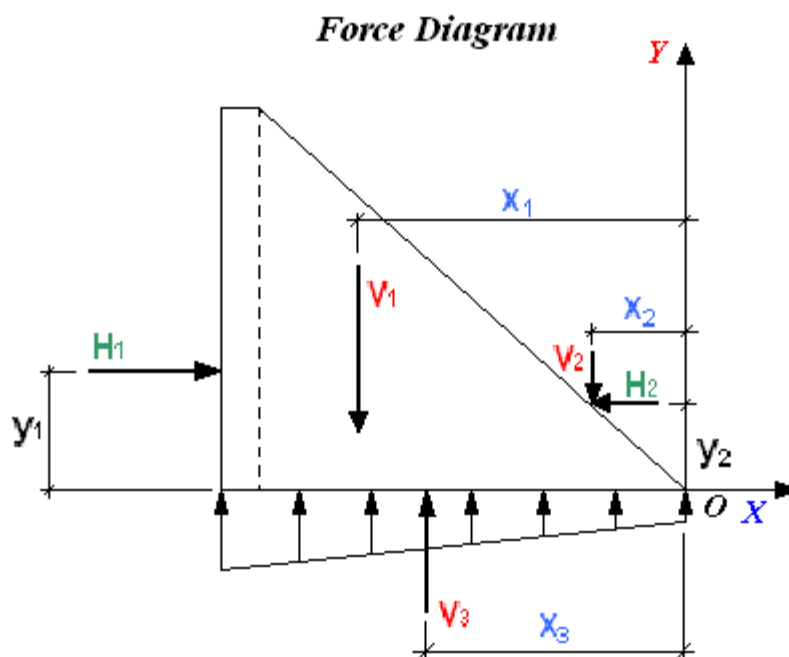
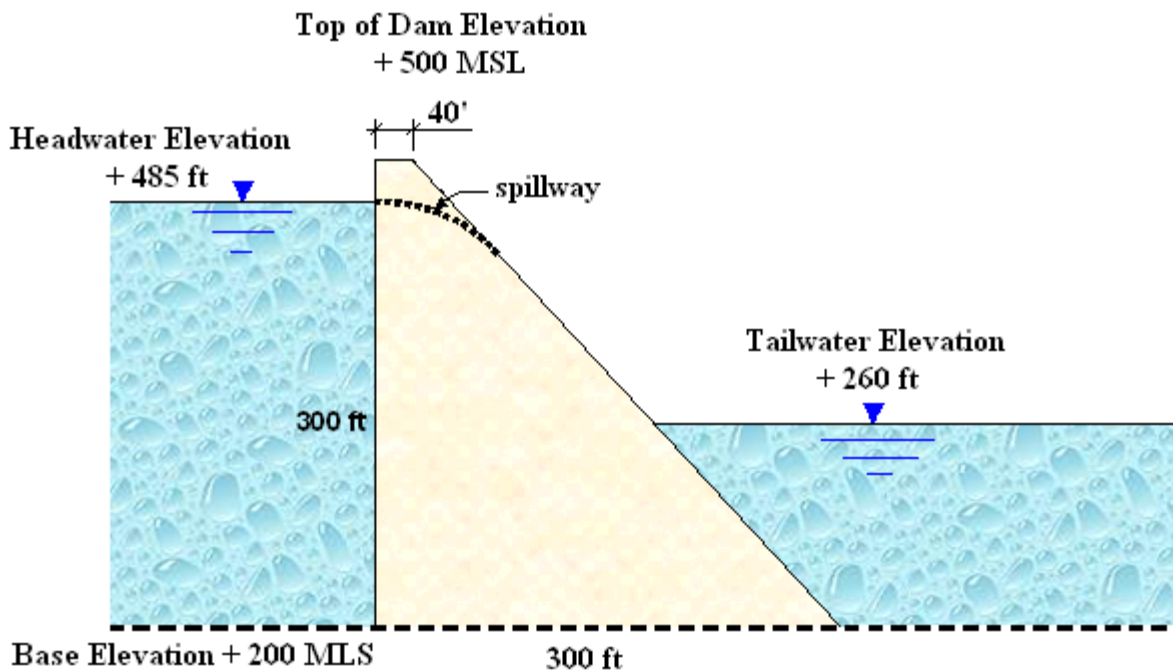
$$F = L \left( \frac{P_A + P_B}{2} \gamma_w \right) = 98 \left( \frac{35.3 + 20.1}{2} \right) (62.4) \frac{\text{lb}}{\text{ft}^3} = 169,000 \frac{\text{lb}}{\text{ft}} = 169 \frac{\text{kip}}{\text{ft}}$$

## \*Dams-02: Determine the uplift forces acting upon a concrete dam.

(Revision: Aug-08)

The uplift (hydrostatic) force under the concrete gravity dam shown below varies as a straight line from 67% of the headwater pressure at the heel, to 100% of the tail-water at the toe. Assume  $\gamma_{\text{Concrete}} = 145 \text{ pcf}$

- Determine the Factor of Safety against overturning; and
- Determine the FS against sliding, if the sand that underlay the dam has  $\phi = 37^\circ$ .



**Solution:**

**Step 1: Determine all forces on the dam**

$$\Sigma F_v = 0$$

$$V_1 = \text{weight of dam} = \left( 0.145 \frac{k}{ft^3} \right) \left( (300 ft)(40 ft) + \left( \frac{1}{2} \right) (260 ft)(300 ft) \right) = 7,395 \text{ kips / ft}$$

$$\Sigma M_o = 0$$

$$x_1, \text{ from toe } O = \frac{(1740 k)(280 ft) + (5655 k) \left( \frac{2}{3} (260 ft) \right)}{(1740 k + 5655 k)} = 198 \text{ ft left of toe}$$

$$V_2 : \text{vertical weight of water upon toe section} = 0.0624 \frac{k}{ft^3} \left( \frac{1}{2} (60 ft)(52 ft) \right) = 97 \text{ kips / ft}$$

$$x_2, \text{ from toe } O = \frac{1}{3} (52 ft) = 17.3 \text{ ft left of toe}$$

$$H_1 : \text{lateral force from headwater} = \left( \frac{1}{2} 0.0624 \frac{k}{ft^3} \right) (285 ft)^2 = 2,534 \text{ kips / ft}$$

$$y_1, \text{ from toe } O = \frac{1}{3} (285 ft) = 95 \text{ ft above the toe.}$$

$$H_2 : \text{lateral force from tailwater} = \left( \frac{1}{2} 0.0624 \frac{k}{ft^3} \right) (60 ft)^2 = 112 \text{ kips / ft}$$

$$y_2, \text{ from toe } O = \frac{1}{3} (60 ft) = 20 \text{ ft above the toe.}$$

**Step 2: The hydrostatic uplift at the base of the dam.**

$$V_3 : \text{uplift force} = \frac{1}{2} (p_{LEFT} + p_{RIGHT}) (300 ft) = 2,340 \text{ kips / ft}$$

$$\text{where the pressure } p_{LEFT} = (0.67) \gamma_w h = 0.67 (0.0624) (285 ft) = 11.9 \text{ ksf}$$

$$\text{and the pressure } p_{RIGHT} = (1.00) \gamma_w h = 1.0 (0.0624) (60 ft) = 3.7 \text{ ksf}$$

$$\therefore V_3 = \frac{1}{2} (11.9 + 3.7) \frac{k}{ft^2} (300 ft) = 2,340 \text{ kips / ft}$$

$$x_3, \text{ from toe } O = \frac{\left( 3.7 \frac{k}{ft^2} \right) (300 ft) \left( \frac{300 ft}{2} \right) + \frac{1}{2} \left( 11.9 \frac{k}{ft^2} - 3.7 \frac{k}{ft^2} \right) (300 ft) \left( \frac{2}{3} (300 ft) \right)}{\left( 3.7 \frac{k}{ft^2} \right) (300 ft) + \frac{1}{2} \left( 11.9 \frac{k}{ft^2} - 3.7 \frac{k}{ft^2} \right) (300 ft)} = 176.3 \text{ ft left of toe}$$

**Step 3: The factor of safety (FS) against overturning** (taken about the toe),

$$FS = \frac{\text{resisting moments}}{\text{overturning moments}} = \frac{V_1x_1 + V_2x_2 + H_2y_2}{V_3x_3 + H_1y_1}$$

$$FS_{\text{overturning}} = \frac{(7395)(198) + (97)(17.3) + (112)(20)}{(2534)(95) + (2340)(176.3)} = 2.2 > 2 \text{ GOOD}$$

**Step 4: The F.S. against sliding,**

$$FS_{\text{sliding}} = \frac{\text{resisting forces}}{\text{driving forces}} = \frac{H_2 + (V_1 + V_2 - V_3) \tan \phi}{H_1}$$

$$FS_{\text{sliding}} = \frac{(112) + (7395 + 97 - 2340) \tan 37^\circ}{2534} = 1.58 < 2 \text{ NOT GOOD ENOUGH}$$

# Chapter 9

## Stresses in Soil Masses

### ***Symbols for Stresses in Soil Masses***

$\theta \rightarrow$  The angle of the plane of interest angle with respect to the major principal stress ( $\sigma_1$ ).

$\sigma_{max} \rightarrow$  Maximum normal axial stress.

$\sigma_{min} \rightarrow$  Minimum normal axial stress.

$\sigma_\theta \rightarrow$  The normal stress on a plane with an angle  $\theta$  with respect to the major principal stress plane ( $\sigma_1$ ).

$\tau_{max} \rightarrow$  maximum shear stress.

$\tau_{min} \rightarrow$  Minimum shear stress.

$\tau_\theta \rightarrow$  The shear stress on a plane with an angle  $\theta$  respect to the major principal stress plane ( $\sigma_1$ ).

$\phi \rightarrow$  The angle of internal friction of the soil.

$\gamma \rightarrow$  Unit weight of the soil.

$\tau_n \rightarrow$  Shear stress.

$\sigma_n \rightarrow$  Normal stress.

$q_u \rightarrow$  Ultimate shear strength of a soil.

### **Symbols for Boussinesq Stresses**

$B \rightarrow$  Width of the loaded selected region.

$G_s \rightarrow$  Specific gravity of the solids of a soil.

$L \rightarrow$  Length of the loaded selected region.

$m \rightarrow$  The ratio ( $B/Z$ ).

$n \rightarrow$  The ratio ( $L/Z$ ).

$N \rightarrow$  Normal load carried by a foundation.

$Dp \rightarrow$  Increased stress on the soil from a surface loaded area.

$p \rightarrow$  Stress of the loaded area.

$z \rightarrow$  Depth of the soil at the point of interest.

$\gamma \rightarrow$  Unit weight of the soil.

### **Symbols for Newmark**

$IV \rightarrow$  The influence value in the Newmark' chart (for example, a chart divided into 100 areas, each is  $IV=0.01$ ).

$AB \rightarrow$  Scale to the depth of interest to determine the size of the surface structure graph the Newmark's graph.

$C \rightarrow$  consolidation.

$M \rightarrow$  Number of squares (enclosed in the Newmark' chart).

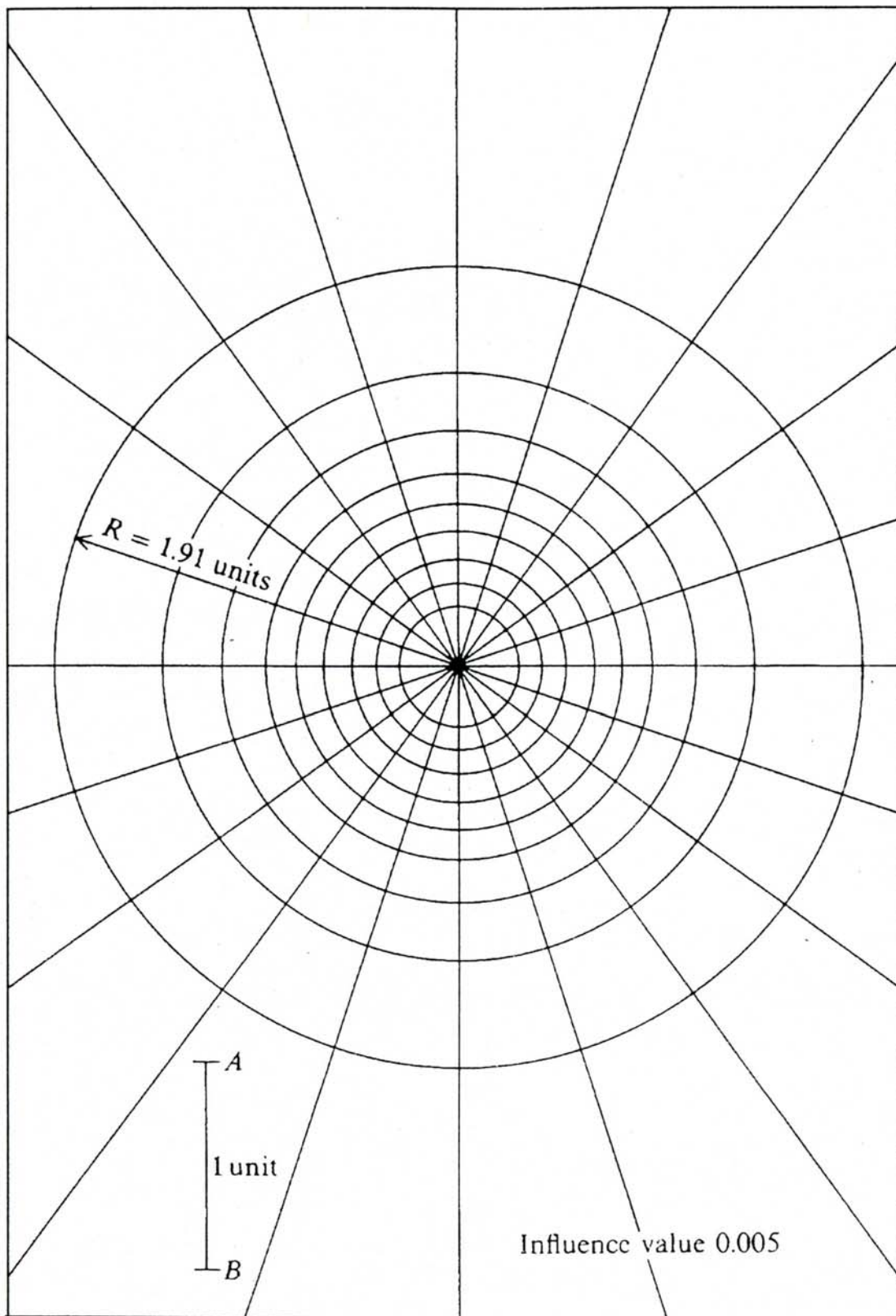
$p_o \rightarrow$  The effective stress at the point of interest.

$q \rightarrow$  The load of the footing.

$q_u \rightarrow$  Ultimate shear strength of a soil.

$\sigma_x \rightarrow$  stress at an specific point (x).

$z \rightarrow$  Depth of the stratum stat of the soil.



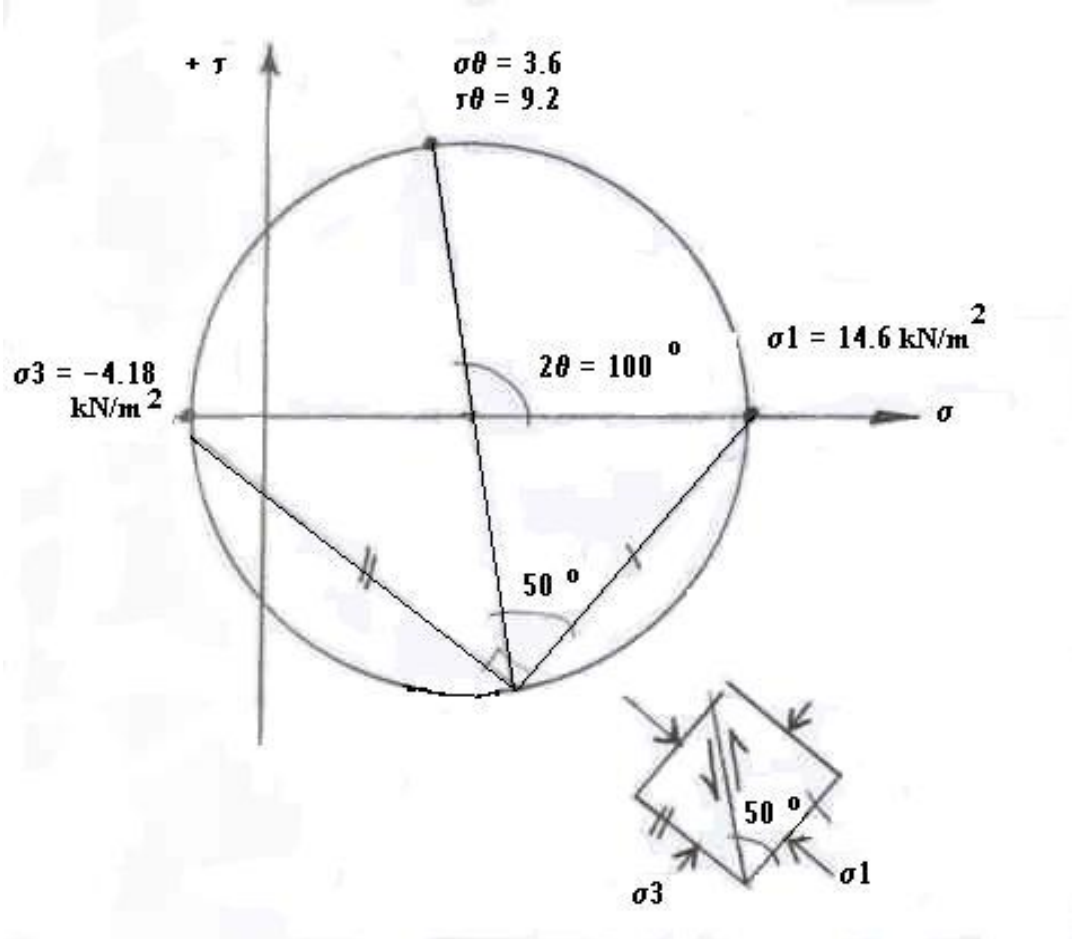
Influence chart for vertical pressure. [After Newmark (1942).]

**\*Mohr-01: Simple transformation from principal to general stress state.**

(Revision: Jan-09)

A soil particle is found to be subjected to a maximum stress of  $14.6 \text{ kN/m}^2$ , and a minimum stress of  $-4.18 \text{ kN/m}^2$ . Find the  $\sigma$  and  $\tau$  on the plane of  $\theta = 50^\circ$  with respect to the major principal stresses, and also find  $\tau_{\max}$ .

(a) The graphical solution,



(b) The calculated solution,

$$\sigma_\theta = \left( \frac{\sigma_1 + \sigma_3}{2} \right) + \left( \frac{\sigma_1 - \sigma_3}{2} \right) \cos 2\theta = \left( \frac{14.6 - 4.18}{2} \right) + \left( \frac{14.6 + 4.18}{2} \right) \cos 2(50^\circ) = 3.6 \frac{\text{kN}}{\text{m}^2}$$

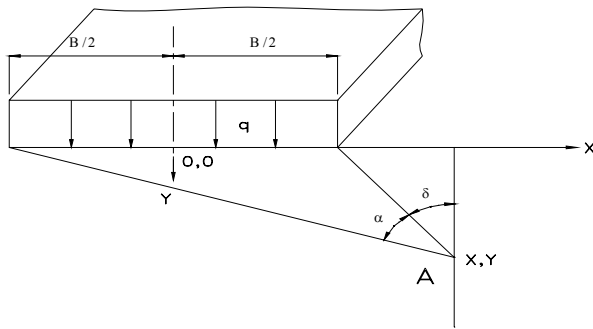
$$\tau_\theta = \left( \frac{\sigma_1 - \sigma_3}{2} \right) \sin 2\theta = \left( \frac{14.6 + 4.18}{2} \right) \sin 2(50^\circ) = 9.2 \frac{\text{kN}}{\text{m}^2}$$

$$\tau_{\text{maximum}} = \left( \frac{\sigma_1 - \sigma_3}{2} \right) = 9.4 \frac{\text{kN}}{\text{m}^2}$$

## \*Mohr – 02: Find the principal stresses and their orientation.

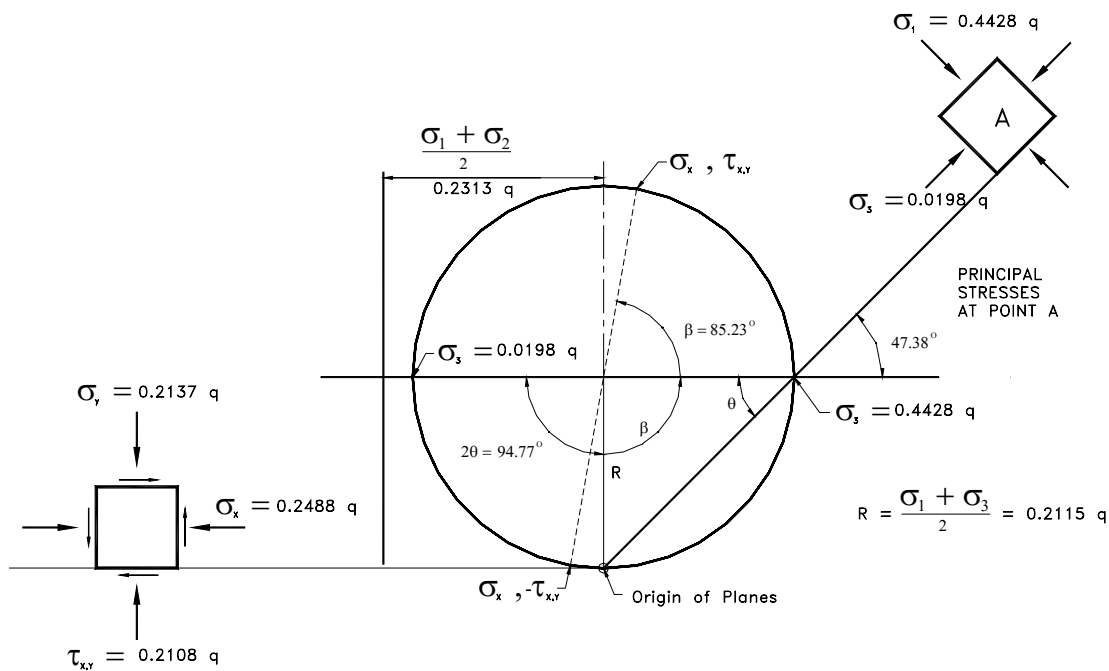
(Revision: Sept.-08)

Equations for the principal stresses in the elastic half-space shown below for a uniformly loaded strip footing are as follows:  $\sigma_1 = q/\pi(\alpha + \sin \alpha)$  and  $\sigma_3 = q/\pi(\alpha - \sin \alpha)$ .



The direction of the major principal stress bisects the angle  $\alpha$ . Calculate the vertical stress  $\sigma_y$ , the horizontal stress  $\sigma_x$ , and  $\tau_{xy}$  at point A if  $x = 0.75B$  and  $y = 0.5B$  using Mohr's diagram.

### Solution:



$$\alpha + \delta = \arctan 2.5 = 68.20^\circ$$

$$2\theta + \beta = 180^\circ$$

$$\delta = \arctan 0.5 = 26.57^\circ$$

$$\beta = 85^\circ$$

$$68.20^\circ - 26.57^\circ = 41.63^\circ$$

if  $q = 432.5 \text{ kPa}$  then

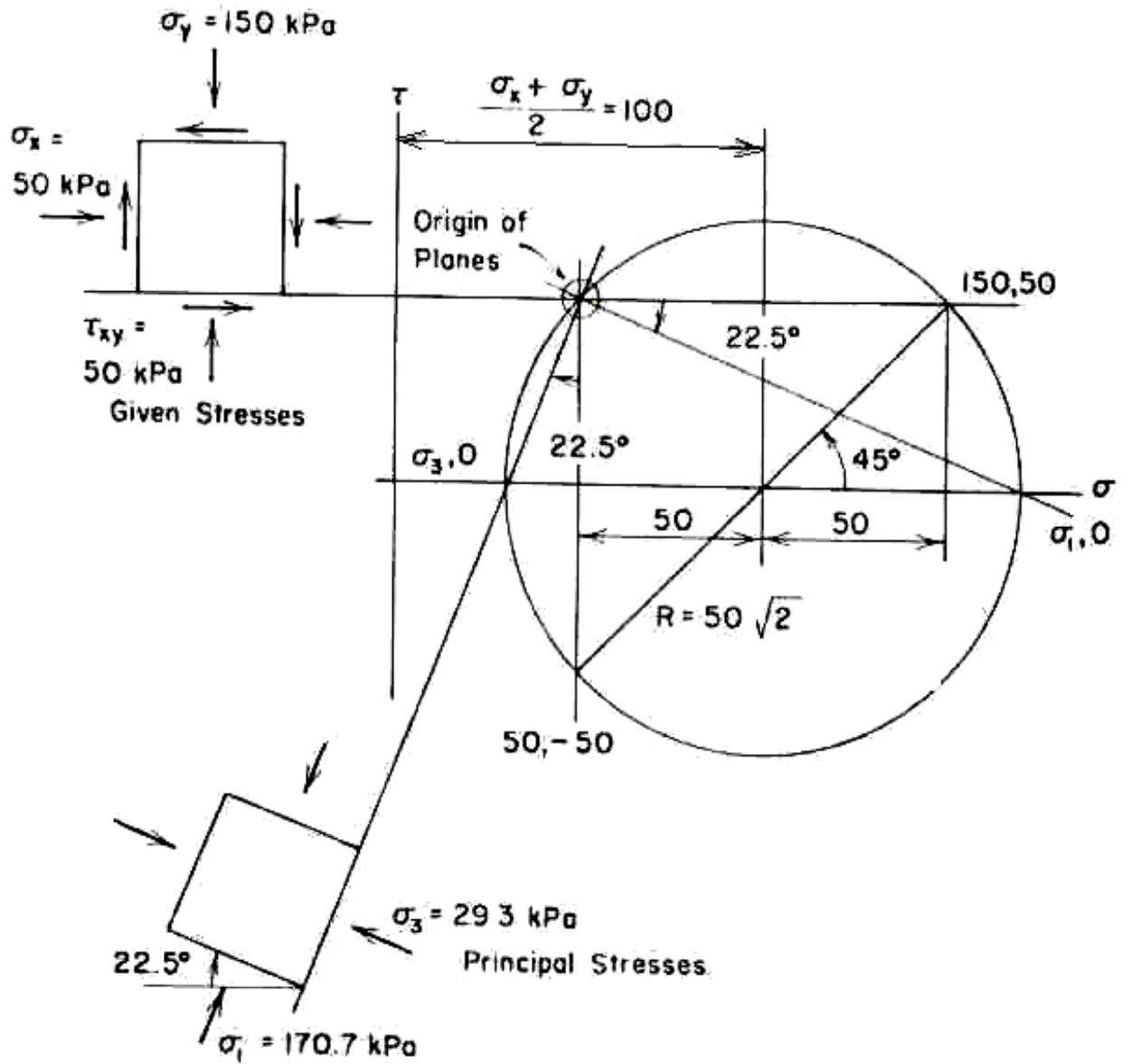
$$\sigma_1 = (\sigma_x + \sigma_y)/2 + R = 100 + 70.7 = 170.7 \text{ kPa}$$

$$\sigma_3 = (\sigma_x + \sigma_y)/2 - R = 100 - 70.7 = 29.3 \text{ kPa}$$

**\*Mohr – 03: Find the principal stresses and their orientation.**

(Revision: Sept-09)

Given the general stresses at a point in a soil, determine the principal stresses and show them on a properly oriented element.

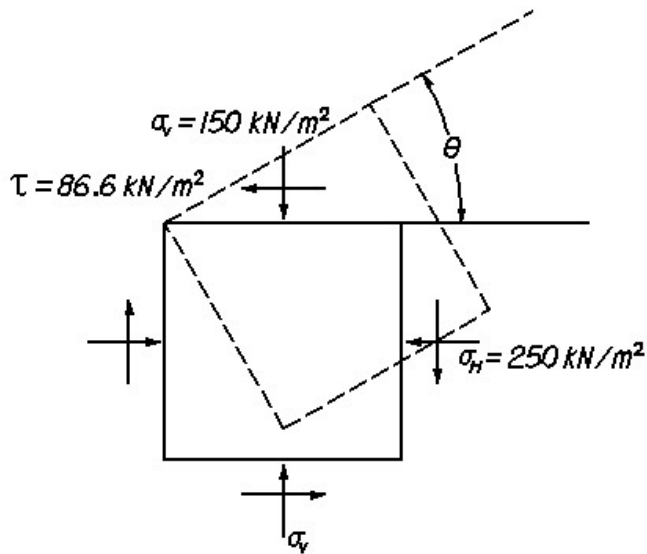


### \*Mohr – 04:

(Revision: Sept-09)

A sample of clean sand was retrieved from 7 m below the surface. The sample had been under a vertical load of  $150 \text{ kN/m}^2$ , a horizontal load of  $250 \text{ kN/m}^2$ , and a shear stress of  $86.6 \text{ kN/m}^2$ . If the angle  $\theta$  between the vertical stress and the principal stress is  $60^\circ$ , what is the angle of internal friction  $\phi$  of this sample?

Solution:

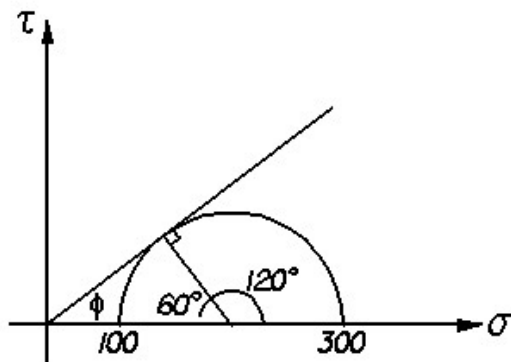


$$\sigma_1 = \frac{\sigma_v + \sigma_h}{2} \pm \sqrt{\left(\frac{\sigma_v - \sigma_h}{2}\right)^2 + (\tau)^2}$$

$$\sigma_1 = \frac{150 + 250}{2} \pm \sqrt{\left(\frac{150 - 250}{2}\right)^2 + (86.6)^2}$$

$$\sigma_1 = 200 + 100 = \boxed{300 \text{ kN/m}^2}$$

$$\therefore \sigma_3 = 200 - 100 = \boxed{100 \text{ kN/m}^2}$$



$$\sin \phi = \frac{1/2(\sigma_1 - \sigma_3)}{1/2(\sigma_1 + \sigma_3)} = \frac{300 - 100}{300 + 100}$$

$$\sin \phi = 0.5$$

$$\phi = \sin^{-1}(0.5)$$

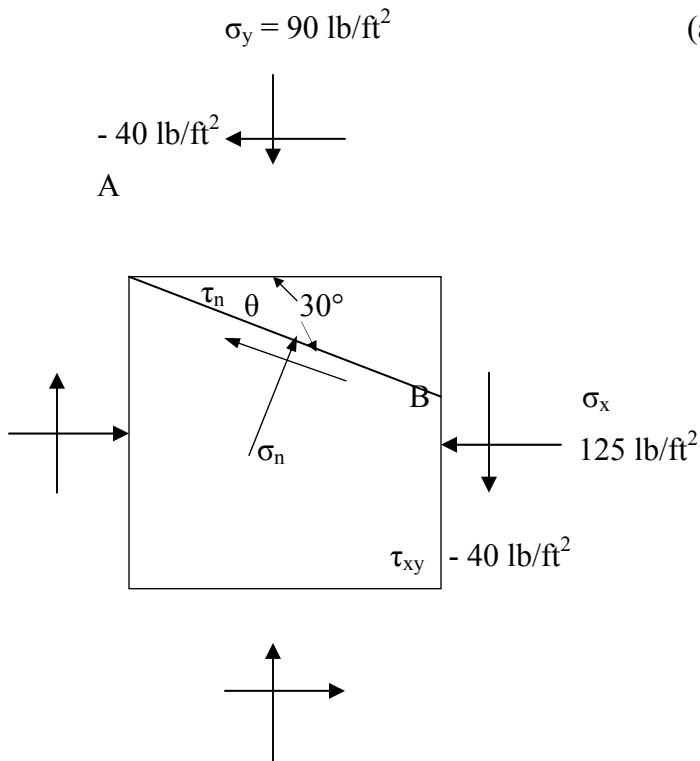
$$\boxed{\phi = 30^\circ}$$

**\*Mohr – 05: Normal and shear stress at a chosen plane.**

(Revision: Sept-09)

Using a Mohr circle, determine the normal and shear stresses on the plane AB.

**Solution:**



$$(a) \quad \sigma_1 = \frac{\sigma_y + \sigma_x}{2} + \sqrt{\left(\frac{\sigma_y - \sigma_x}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_3 = \frac{90 + 125}{2} - \sqrt{\left(\frac{90 - 125}{2}\right)^2 + (-40)^2}$$

$$\boxed{\begin{array}{l} \sigma_1 = 151 \text{ psf} \\ \sigma_3 = 64 \text{ psf} \end{array}}$$

$$(b) \quad \sigma_n = \frac{\sigma_1 + \sigma_3}{2} + \frac{\sigma_1 - \sigma_3}{2} \cos 2\theta$$

$$= \frac{151 + 64}{2} + \frac{151 - 64}{2} \cos 60^\circ$$

$$\boxed{\sigma_n = 129 \text{ psf}}$$

$$\tau_n = \frac{\sigma_1 - \sigma_3}{2} \sin 2\theta$$

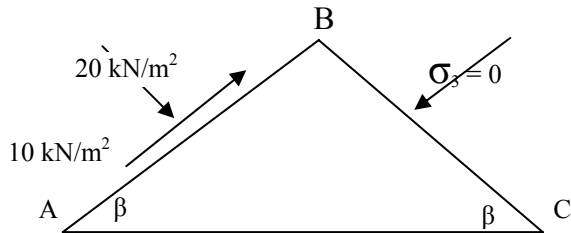
$$= \frac{151 - 64}{2} \sin 60^\circ$$

$$\boxed{\tau_n = 38 \text{ psf}}$$

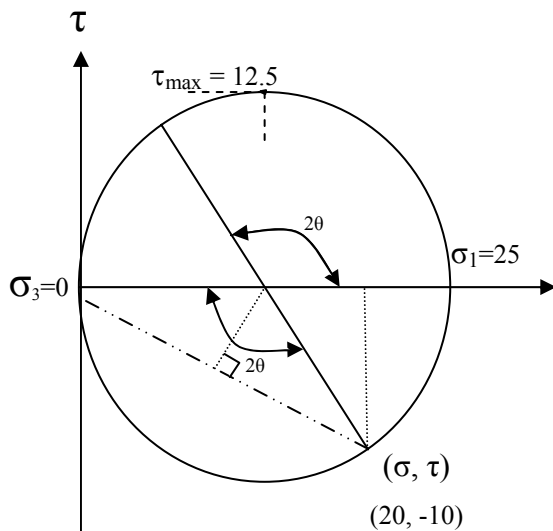
## \*\*Mohr – 07: Back figure the failure angle

(Revised: Sept-09)

From the stress triangle shown below, find (a) the maximum and minimum principle stresses, (b) the angle alpha, as shown, (c) the angle theta, and (d) the value for the maximum shear stress.



(A) Graphical Solution:



(a)  $\sigma_1 = 25 \text{ kN/m}^2$  and  $\sigma_3 = 0$

(b)  $\alpha$  in the figure is in the angle  $2\theta$  between  $(\sigma, \tau)$  and  $\sigma_3$ .

.....  $\alpha = 2\theta = 126.9^\circ$

(c) .....  $\theta = 63.4^\circ$

and  $\tau_{\max} = 12.5 \text{ kN/m}^2$

OR (B) Analytical Solution:

$$\begin{aligned} \textcircled{1} \quad \sigma_1 &= \frac{\sigma_1 + \sigma_1}{2} + \frac{\sigma_1 - \sigma_1}{2} \cos 2\theta \\ 20 &= \frac{\sigma_1}{2} + \frac{\sigma_1}{2} \cos 2\theta \quad \therefore \sigma_1 = \frac{40}{1 + \cos 2\theta} \\ \textcircled{2} \quad \tau_n &= \frac{\sigma_1 - \sigma_3}{2} \sin 2\theta \\ 10 &= \frac{\sigma_1}{2} \sin 2\theta \quad \therefore \theta = \frac{20}{\sin 2\theta} \end{aligned}$$

$$\therefore 1 - 2 \sin 2\theta + \cos 2\theta = 0$$

.....  $\theta = 63.4^\circ$

.....  $\sigma_1 = \frac{20}{\sin 126.9^\circ} = 25 \text{ kN/m}^2, \quad \sigma_3 = 0$

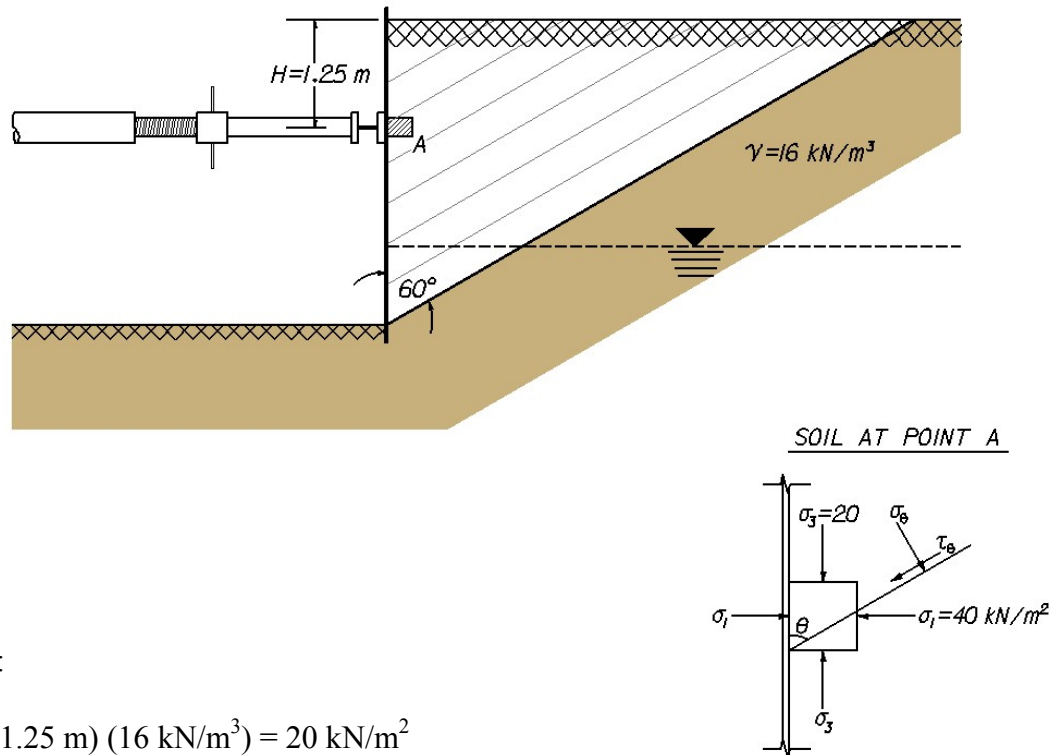
.....  $\tau_{\max} = 12.5 \text{ kN/m}^2$

### \*Mohr – 08: find the Principle pressure using Mohr

(Revised Sept-09)

The temporary excavation shown below is braced with a steel tube strut. Every morning, a misguided foreman tightens the screw mechanism on the strut “just to be safe”. The stress on a soil particle at point A, just behind the wall, has been measured with a pressure sensor installed by the Engineer. It now measures  $40 \text{ kN/m}^2$ . If the potential failure planes in the soil behind the wall sustain  $60^\circ$  angles with respect to the vertical wall, estimate the normal and shear stresses at that point A along a potential failure plane.

#### Solution:



At point A:

$$\sigma_v = h\gamma = (1.25 \text{ m}) (16 \text{ kN/m}^3) = 20 \text{ kN/m}^2$$

$\therefore \sigma_v$  is the minor principal stress at A,

Since  $\theta = 60^\circ$  is with respect to the major principal stress ( $\sigma_1$ ) plane, then  $\sigma_v = \sigma_3$

$$\therefore \sigma_\theta = (\sigma_1 + \sigma_3)/2 + (\sigma_1 - \sigma_3)/2 \cos 2\theta = (40 + 20)/2 + (40 - 20)/2 \cos 120^\circ$$

$$\therefore \sigma_\theta = \boxed{25 \text{ kN/m}^2}$$

$$\text{and } \tau_\theta = (\sigma_1 - \sigma_3)/2 \sin 2\theta = (40 - 20)/2 \sin 120^\circ$$

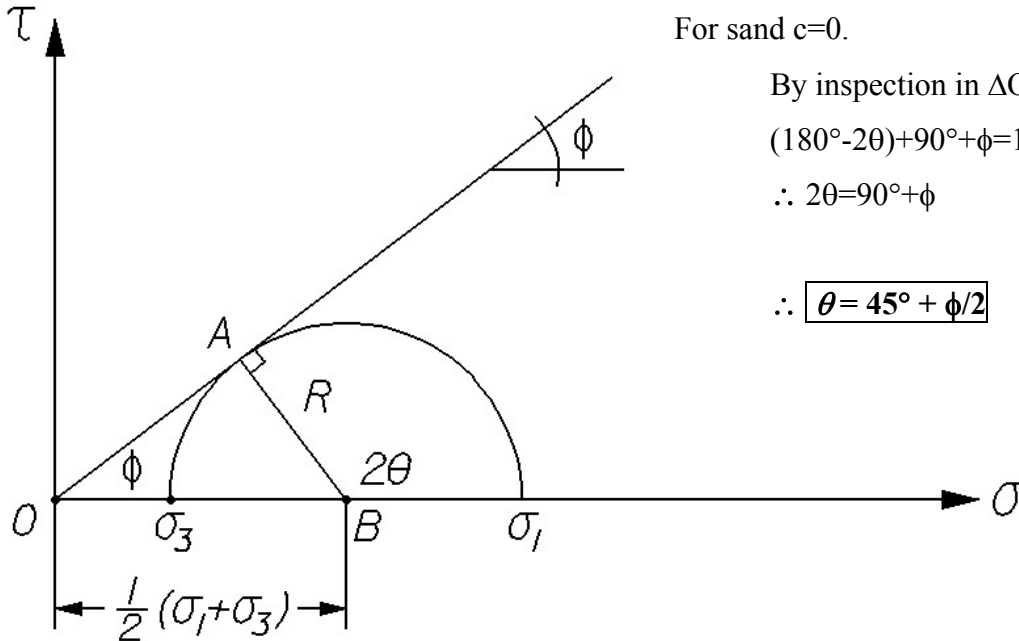
$$\therefore \tau_\theta = \boxed{8.7 \text{ kN/m}^2}$$

### \*Mohr – 09: Relation between $\theta$ and $\phi$ .

(Revised Sept-09)

For a clean sand, prove that  $\theta = 45^\circ + \phi/2$  using Mohr's circle.

Solution:



A failure test on a clean sand (i.e.  $c=0$ ) shows that  $\sigma_1=11.5$  ksf and  $\sigma_3=3.2$  ksf at failure. Find the angle  $\phi$  for this sand.

In the  $\triangle OAB$  above,  $R = \frac{1}{2}(\sigma_1 - \sigma_3)$

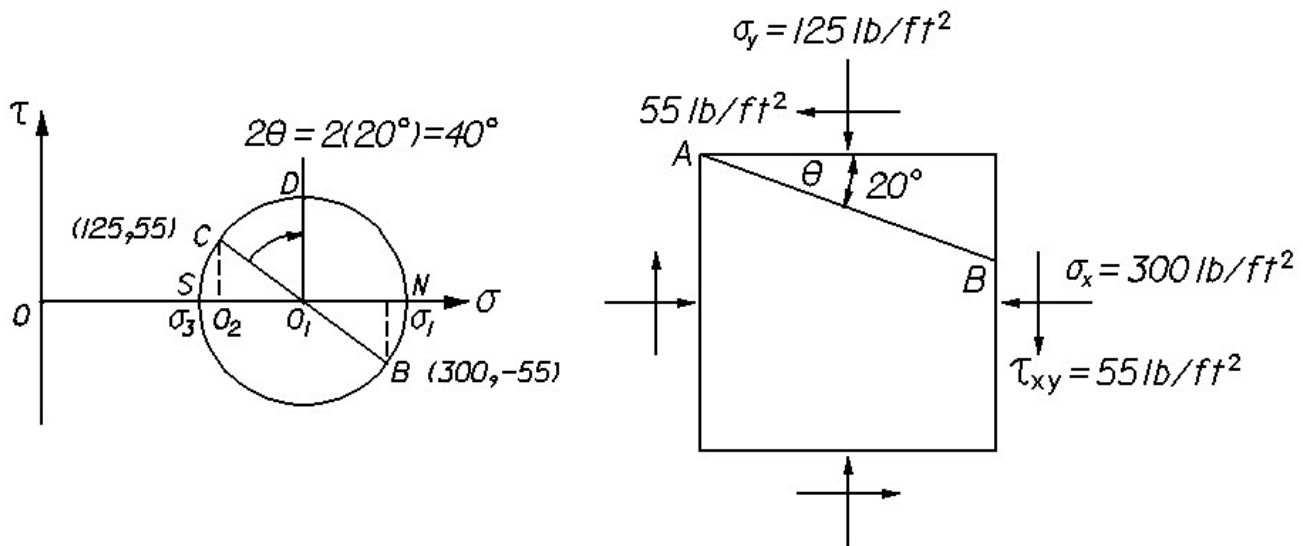
$$\therefore \sin \phi = \frac{\left[ \frac{1}{2}(\sigma_1 - \sigma_3) \right]}{\left[ \frac{1}{2}(\sigma_1 + \sigma_3) \right]} = \frac{(11.5 - 3.2)}{(11.5 + 3.2)} = 0.565$$

$$\therefore \boxed{\phi = 34.5^\circ}$$

### \*Mohr – 10:

(Revised Sept-09)

Determine the normal and shear stresses on the plane AB.



$$\sigma_{O_1} = (300 + 125)/2 = 212.5 \text{ psf}$$

$$\sigma_{O_1 O_2} = (300 - 125)/2 = 87.5 \text{ psf}$$

$$O_1 B = \sqrt{87.5^2 + 55^2} = 103 \text{ psf}$$

$$\sigma_1 = 212.5 + 103 = \boxed{315.5 \text{ psf}}$$

$$\sigma_3 = 212.5 - 103 = \boxed{109.5 \text{ psf}}$$

$$\angle C O_1 O_2 = \tan^{-1} (55/87.5) = 32^\circ$$

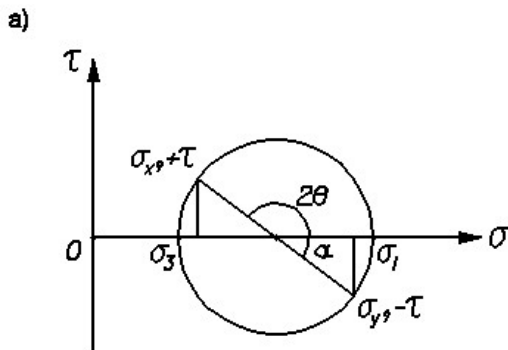
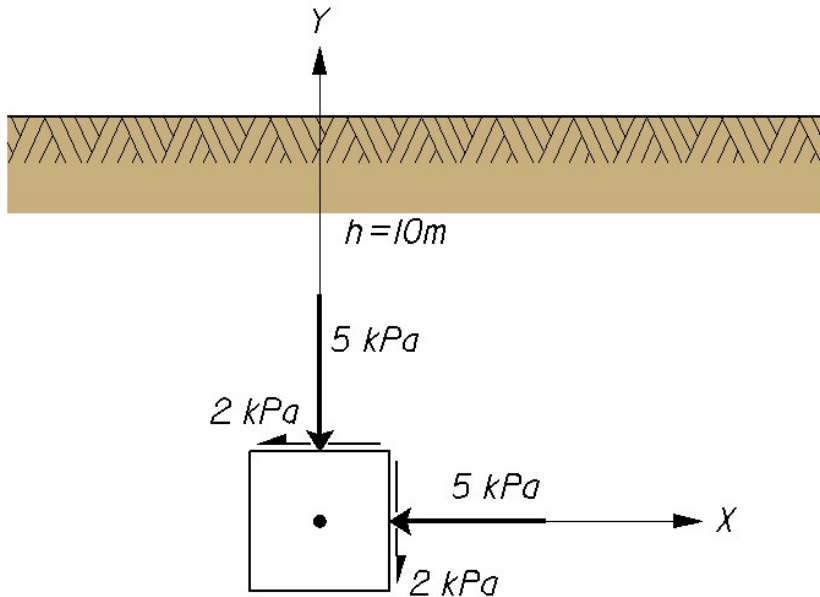
$$\sigma_n = 212.5 - 103 \cos (32 + 40) = \boxed{181 \text{ psf}}$$

$$\tau_n = 103 \sin (32 + 40) = \boxed{98 \text{ psf}}$$

### \*Mohr-11:

(Revised Sept-09)

- (37) Derive the equation that transforms a general state of stress to the principal state of stress.  
(Hint: Use Mohr's circle for a graphical solution).
- (38) Determine the value of the major principal stress.
- (39) Determine the angle  $\theta$  between the major principal stress and the state of stress shown in the figure above.



By inspection,

$$\sigma_1 = \sigma_x + \frac{\sigma_y - \sigma_x}{2} + \sqrt{\left(\frac{\sigma_y - \sigma_x}{2}\right)^2 + \tau^2}$$

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2}$$

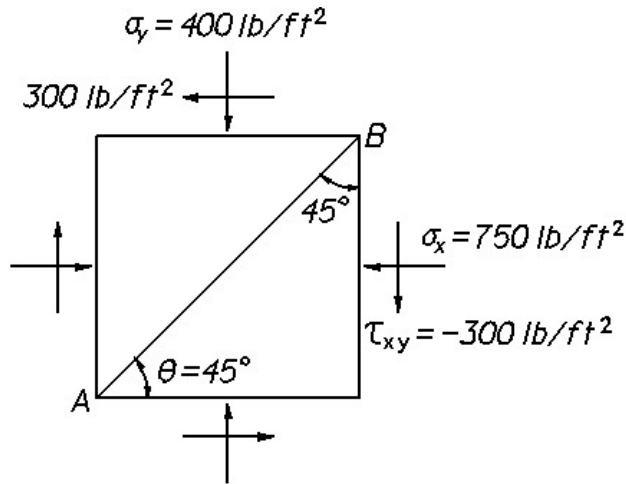
b)  $\sigma_1 = \frac{5+5}{2} + \sqrt{0+2^2} = \boxed{7 \text{ kPa}}$

c)  $\sin \alpha = \frac{\tau}{\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2}} = \frac{2}{\sqrt{0+2^2}} = 1 \quad \therefore \alpha = 90^\circ$   
 but  $\alpha + 2\theta = 180^\circ$   
 $\therefore 2\theta = 180^\circ$   
 $\boxed{\theta = 45^\circ}$

**\*Mohr – 12:**

(Revised Sept-09)

Determine the maximum and minimum principal stresses, and the normal and shear stresses on plane AB.



Maximum and minimum principal stresses:

$$\begin{aligned}\sigma_1 &= \frac{\sigma_y + \sigma_x}{2} \pm \sqrt{\left(\frac{\sigma_y - \sigma_x}{2}\right)^2 + (\tau_{xy})^2} \\ \sigma_3 &= \frac{400 + 750}{2} \pm \sqrt{\left(\frac{400 - 750}{2}\right)^2 + (-300)^2} \\ \therefore \quad &\boxed{\begin{aligned}\sigma_1 &= 922 \text{ psf} \\ \sigma_2 &= 228 \text{ psf}\end{aligned}}$$

Normal and shear stresses:

$$\begin{aligned}\sigma_n &= \frac{\sigma_1 + \sigma_3}{2} + \frac{\sigma_1 - \sigma_3}{2} \cos 2\theta + \tau_{xy} \sin 2\theta \\ &= \frac{400 + 750}{2} + \frac{400 - 750}{2} \cos 90^\circ + (-300) \sin 90^\circ\end{aligned}$$

$$\boxed{\sigma_n = 200 \text{ psf}}$$

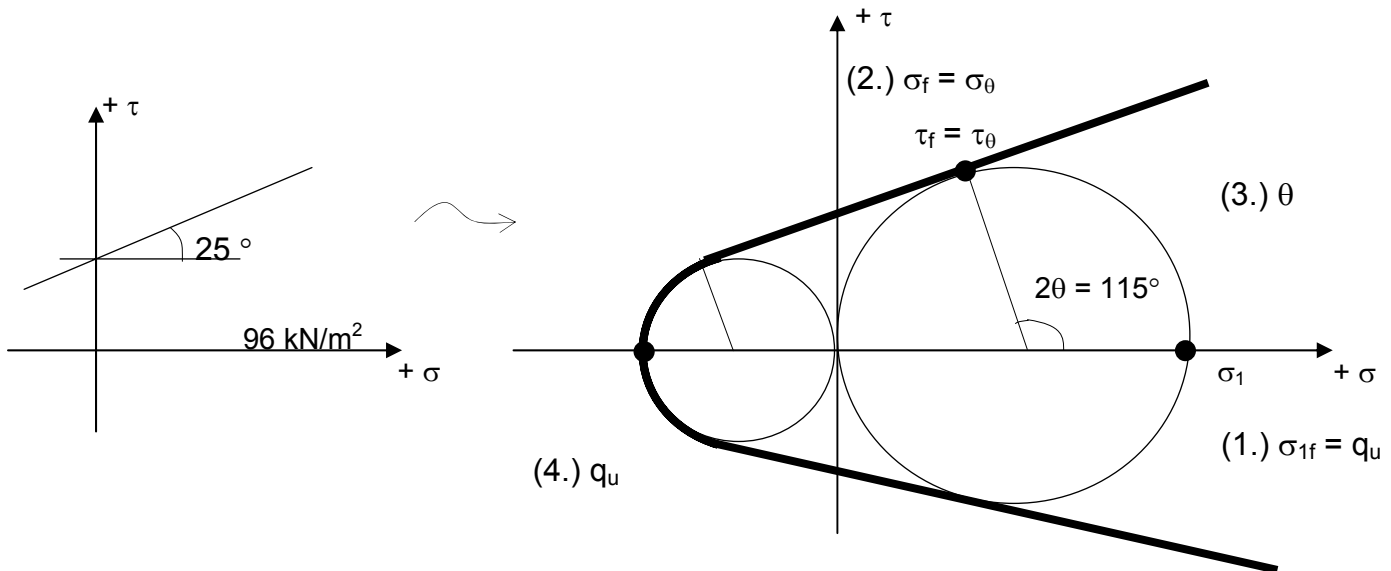
$$\begin{aligned}\tau_n &= \frac{\sigma_1 - \sigma_3}{2} \sin 2\theta - \tau_{xy} \cos 2\theta \\ &= \frac{400 - 750}{2} \sin 90^\circ - (-300) \cos 90^\circ\end{aligned}$$

$$\boxed{\tau_n = -175 \text{ psf}}$$

**\*Mohr – 13: Data from Mohr-Coulomb failure envelope.**

(Revised Sept-09)

A soil sample has been tested and when plotted developed the Mohr-Coulomb envelope of failure shown below. Find (1) axial stresses at failure, (2) the normal and shear stresses on the failure plane, (3) the angle of failure with respect to the principal axis, and (4) the soil tensile strength.



**Solution:**

$$\sigma_{1f} = q_u = 300 \text{ Pa}$$

$$\sigma_f = \sigma_\theta = 86 \text{ Pa}$$

$$\tau_f = \tau_\theta = 136 \text{ Pa}$$

$$\theta = 115^\circ / 2 = 57.5^\circ$$

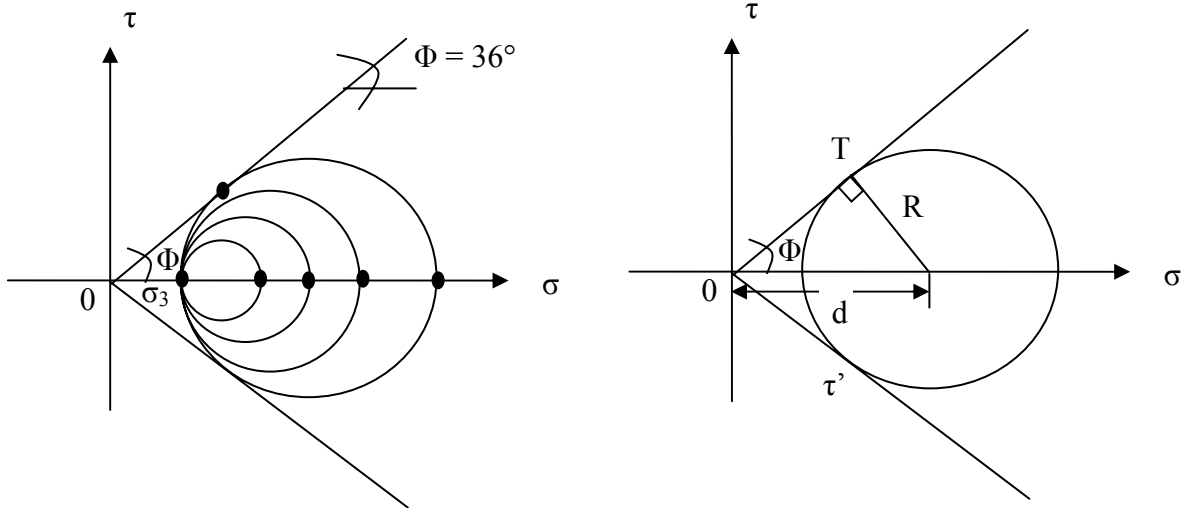
$$q_u = -123 \text{ Pa}$$

## **\*\*Mohr – 14:**

(Revised Sept-09)

A dry sample of sand was tested in a triaxial test. The angle of internal friction was found to be  $36^\circ$ . If the minor principal stress was 300 kPa, at what value of maximum principal stress will the sample fail? The same test was then performed on a clay sample that had the same  $\phi$ , and cohesion of 12 kPa. What was the new maximum principal stress?

### **Solution:**



- a) Failure will occur when the Mohr circle becomes tangent to the failure envelopes.

$$\frac{R}{d} = \sin \phi$$

$$\sigma_3 = d - R = \frac{R}{\sin \phi} - R = R \frac{(1 - \sin \phi)}{\sin \phi}$$

$$\sigma_1 = d + R = \frac{R}{\sin \phi} + R = R \frac{(1 + \sin \phi)}{\sin \phi}$$

$$\therefore \frac{\sigma_1}{\sigma_3} = \frac{(1 + \sin \phi)}{(1 - \sin \phi)} = \tan^2 \left( 45^\circ + \frac{\phi}{2} \right)$$

$$\therefore \sigma_1 = (300 \text{ kPa}) \tan^2 \left( 45^\circ + \frac{36^\circ}{2} \right) = 1160 \text{ kPa}$$

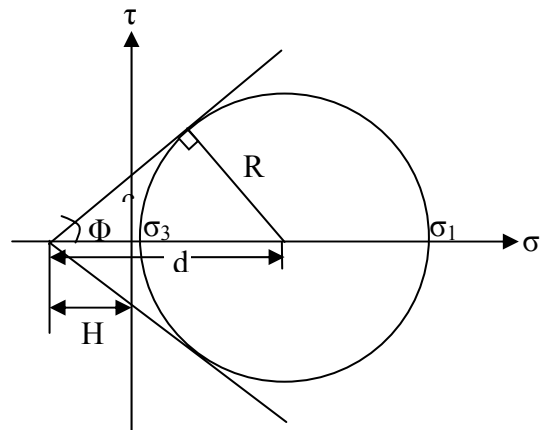
- b)

$$H = c(\cot \phi)$$

$$\sigma_1 + H = (\sigma_3 + H) \tan^2 \left( 45^\circ + \frac{\phi}{2} \right)$$

$$H = (12 \text{ kPa})(\cot 36^\circ) \approx 17 \text{ kPa}$$

$$\sigma_1 = (300 + 17) \tan^2 \left( 45^\circ + \frac{36^\circ}{2} \right) - 17 = 1,200 \text{ kPa}$$

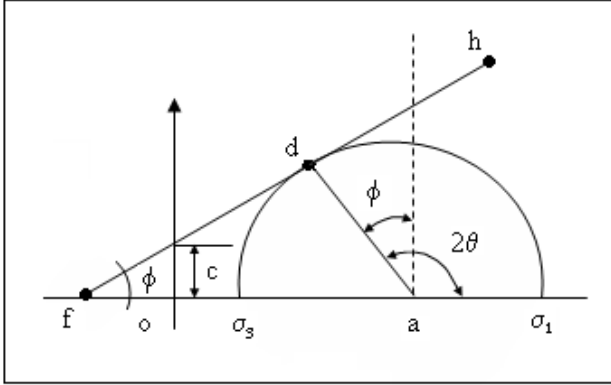


**\*Mohr – 15: Derive the general formula for horizontal stress.**

(Revised Sept-09)

Derive the general formula of the horizontal stress as a function of the vertical stress, cohesion and the angle of internal friction.

**Solution:**



$$\theta = 45^\circ + \frac{\phi}{2} \therefore 2\theta = 90^\circ + \phi$$

$$\overline{ad} = \frac{\sigma_1 - \sigma_3}{2}$$

$$\sin \phi \approx \frac{\overline{ad}}{\overline{af}} \quad (1)$$

$$\overline{af} = \overline{of} + \overline{af} = (c) \cot \theta + \frac{\sigma_1 + \sigma_3}{2} \quad (2)$$

$$\tan \theta = \frac{c}{of} \therefore \frac{of}{c} = \cot \theta \therefore \overline{of} = (c) \cot \theta \quad (3)$$

According to the Mohr's circle properties:

$$\overline{oa} = \sigma_3 + \frac{\sigma_1 - \sigma_3}{2} \therefore \overline{oa} = \frac{\sigma_1 - \sigma_3}{2} \therefore \overline{af} = (c) \cot \phi + \frac{\sigma_1 - \sigma_3}{2} \quad (4) \text{From (1) \& (4):}$$

$$\sin \phi = \frac{\frac{\sigma_1 - \sigma_3}{2}}{(c) \cot \phi + \frac{\sigma_1 + \sigma_3}{2}} \therefore \sin \phi \left( (c) \cot \phi + \frac{\sigma_1 + \sigma_3}{2} \right) = \frac{\sigma_1 - \sigma_3}{2}$$

$$(c) \sin \phi \cot \phi + \frac{\sigma_1 + \sigma_3}{2} \sin \phi = \frac{\sigma_1 - \sigma_3}{2} \therefore (c) \sin \phi \cot \phi = \frac{\sigma_1 - \sigma_3}{2} - \frac{\sigma_1 + \sigma_3}{2} \sin \phi$$

$$\therefore 2 ((c) \sin \phi \cot \phi) = \sigma_1 - (\sin \phi (\sigma_1)) - (\sigma_3 + \sin \phi (\sigma_3))$$

$$\therefore 2 \left( (c) \sin \phi \frac{\cot \phi}{\sin \phi} \right) = \sigma_1 (1 - \sin \phi) - \sigma_3 + (1 + \sin \phi) \sigma_3 \therefore 2 \frac{(c) \cos \phi}{1 - \sin \phi} + \sigma_3 \frac{1 + \sin \phi}{1 - \sin \phi} = \sigma_1$$

Since  $\frac{\cos \phi}{1 - \sin \phi} \approx \tan \left( 45^\circ + \phi \right)$  and  $\frac{1 + \sin \phi}{1 - \sin \phi} \approx \tan^2 \left( 45^\circ + \frac{\phi}{2} \right)$

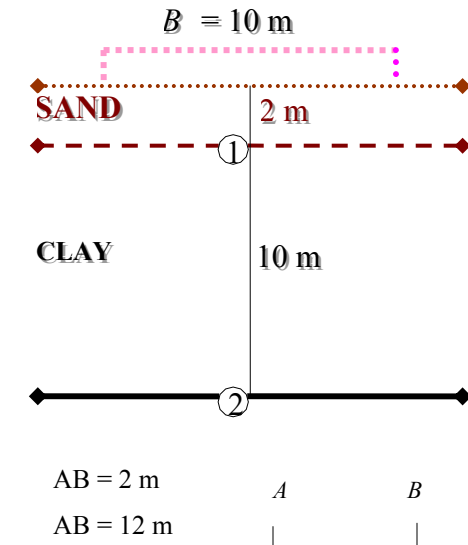
$$\sigma_1 = \sigma_3 \tan^2 \left( 45^\circ + \frac{\phi}{2} \right) + 2c \tan \left( 45^\circ + \frac{\phi}{2} \right)$$

**\*Newmark-01: Stress beneath a tank at different depths.**

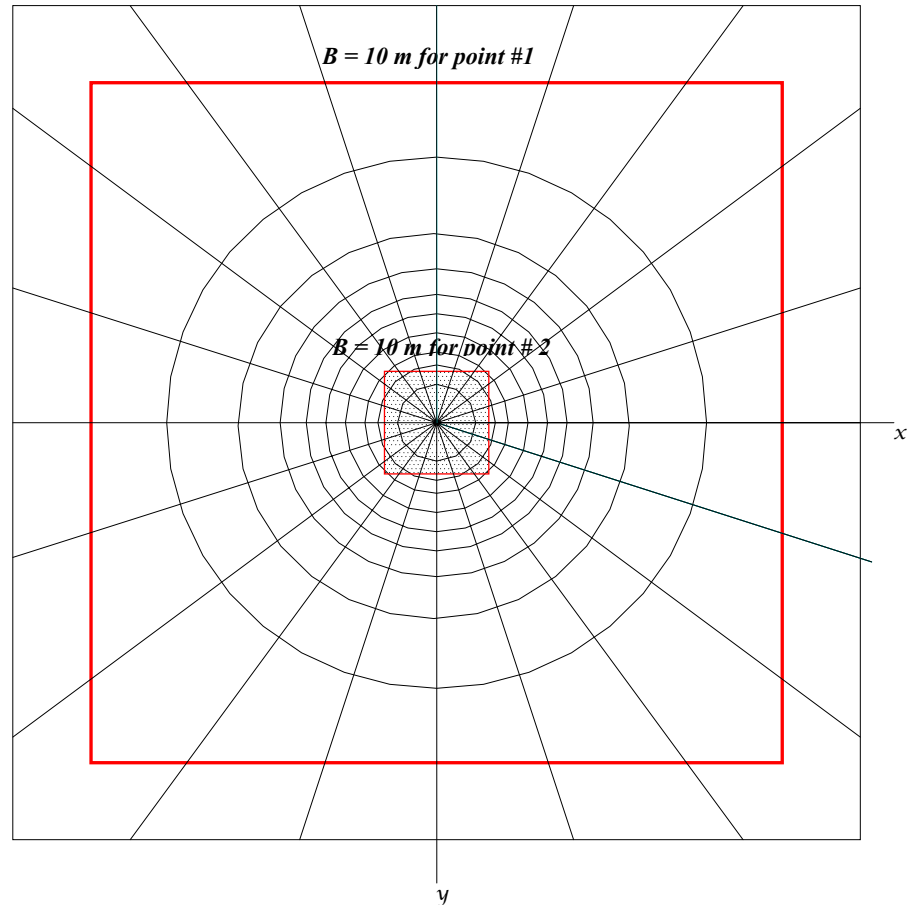
(Revision: Sept.-08)

A construction site has a surface layer of Aeolic sand 2 m thick, underlain by a 10 m thick clay stratum. The project involves placing a wastewater treatment tank, 10 m x 10 m in plan, with a contact pressure  $p_o = 400 \text{ kN/m}^2$ . Find the stress down the centerline of the tank at the top and the bottom of the clay stratum using Newmark's influence chart shown below.

**Solution:**



Influence Value (IV) =  $1/200 = 0.005$



The increase in the vertical stress is found from  $\Delta p = p_o M (IV)$  where  $M$  is the number of “squares” enclosed in the Newmark chart and  $(IV)$  is the influence value.

$$\text{For } \Delta p_1, AB = 2 \text{ m} \rightarrow \Delta p_1 = p_o M (IV) = (400 \text{ kN/m}^2) (190) (0.005) \rightarrow \underline{\Delta p_1 = 380 \text{ kN/m}^2}$$

$$\text{For } \Delta p_2, AB = 12 \text{ m} \rightarrow \Delta p_2 = p_o M (IV) = (400 \text{ kN/m}^2) (42) (0.005) \rightarrow \underline{\Delta p_2 = 84 \text{ kN/m}^2} \text{ (which represents a 78\% reduction in the vertical stress).}$$

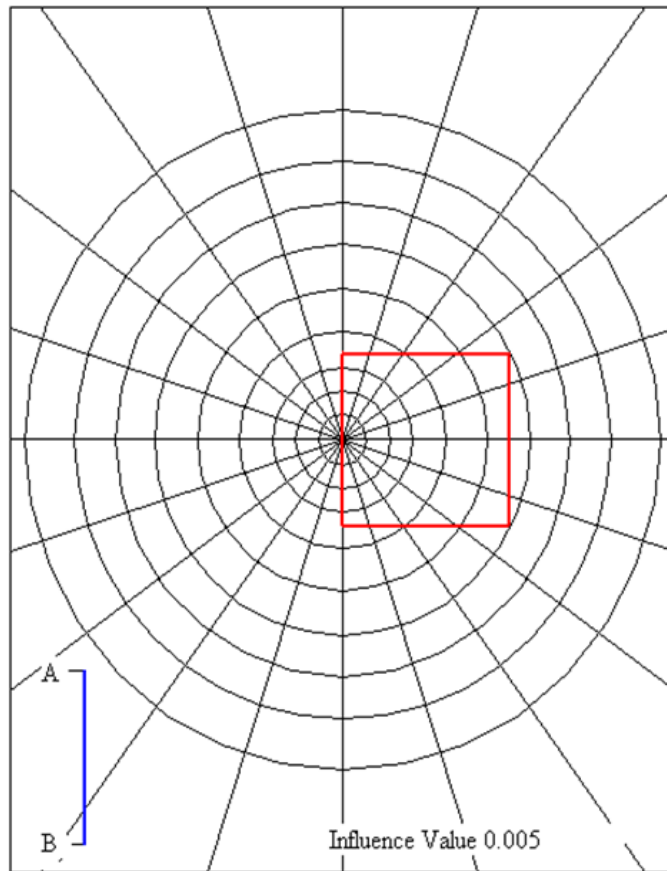
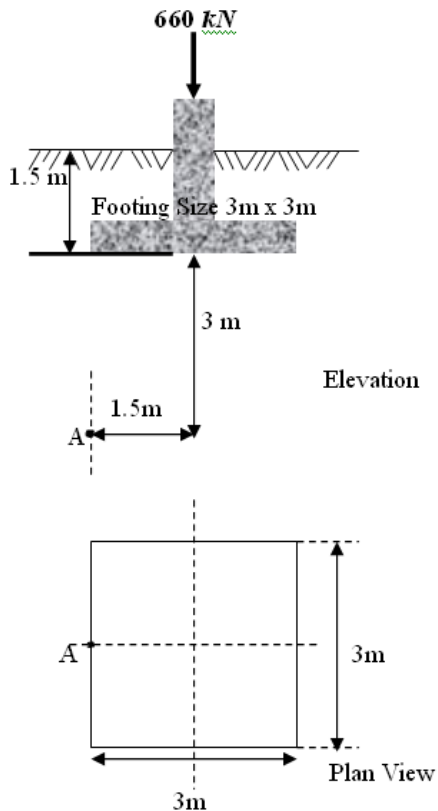
**\*Newmark-02: The stress below the center of the edge of a footing.**

(Revision: Aug-08)

Find the stress at the point A shown below, at a depth of 3 m below the edge of the footing. The plan of the square footing has been plotted on top of the Newmark graph to a scale of  $AB = 3\text{ m}$  and placed in such a way that point A falls directly over the center of the chart.

**Solution:**

The distance  $AB$  is now  $= 3\text{ m}$



Influence Chart for Vertical Pressure [After Newmark (1942)]

The number of elements inside the outline of the plan is about  $M = 45$ . Hence,

$$\Delta p = q M (IV) = \left[ \frac{660\text{ kN}}{(3\text{ m})(3\text{ m})} \right] (45)(0.005) = 16.5\text{ kN / m}^2$$

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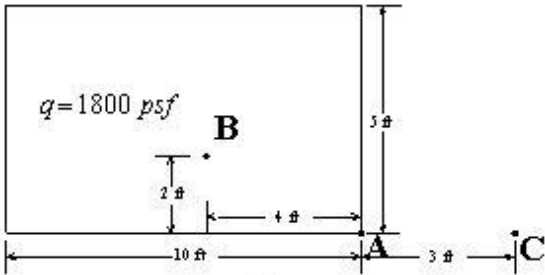
**\*Newmark-03: Stress at a point distant from the loaded footing.**

(Revision: Aug-08)

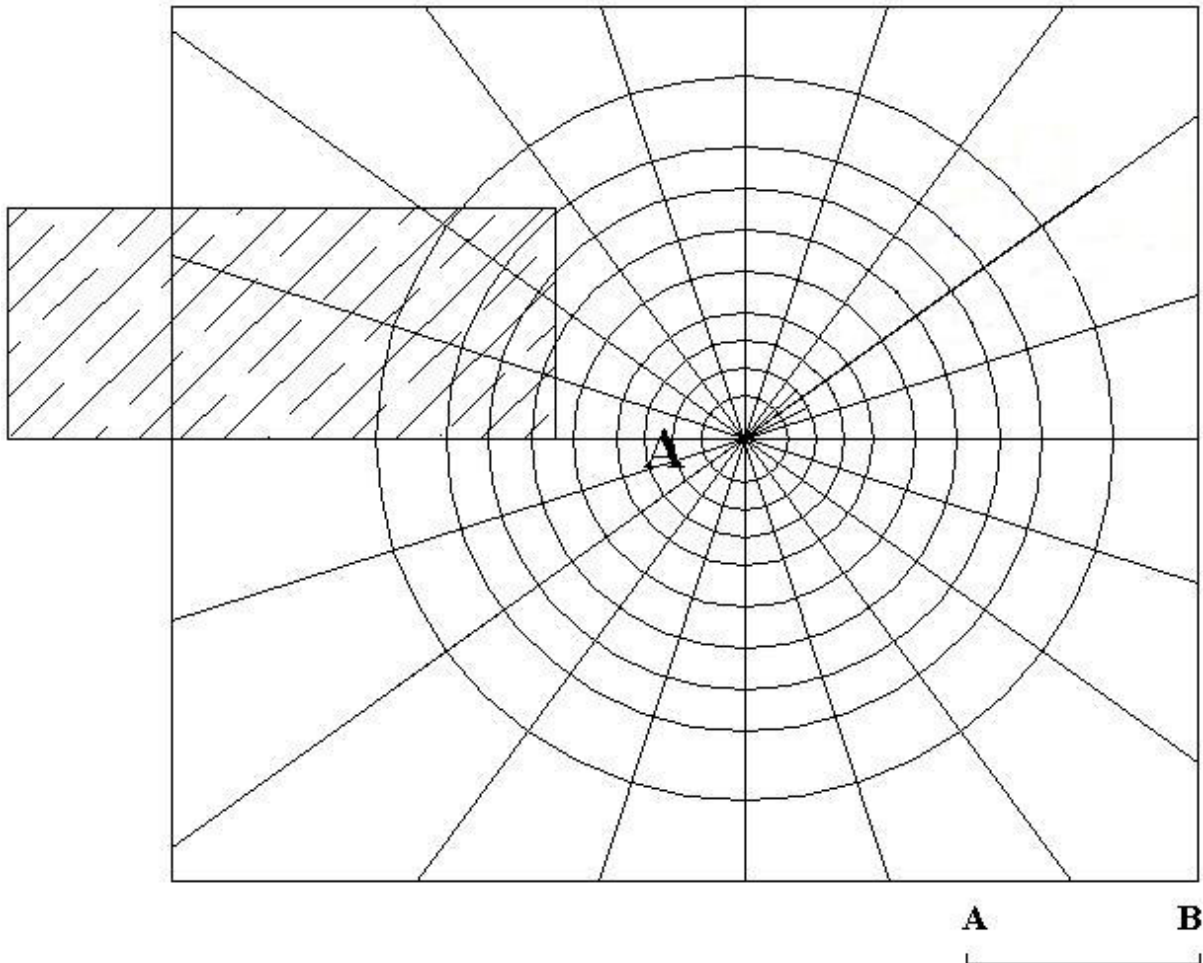
The footing shown below has a load  $q = 1.8 \text{ ksf}$ . Find the stress at a depth of 5 feet below the footing invert, at the point C.

**Influence Value (IV) = 0.005**

**Depth Point = Z**



**Solution:**



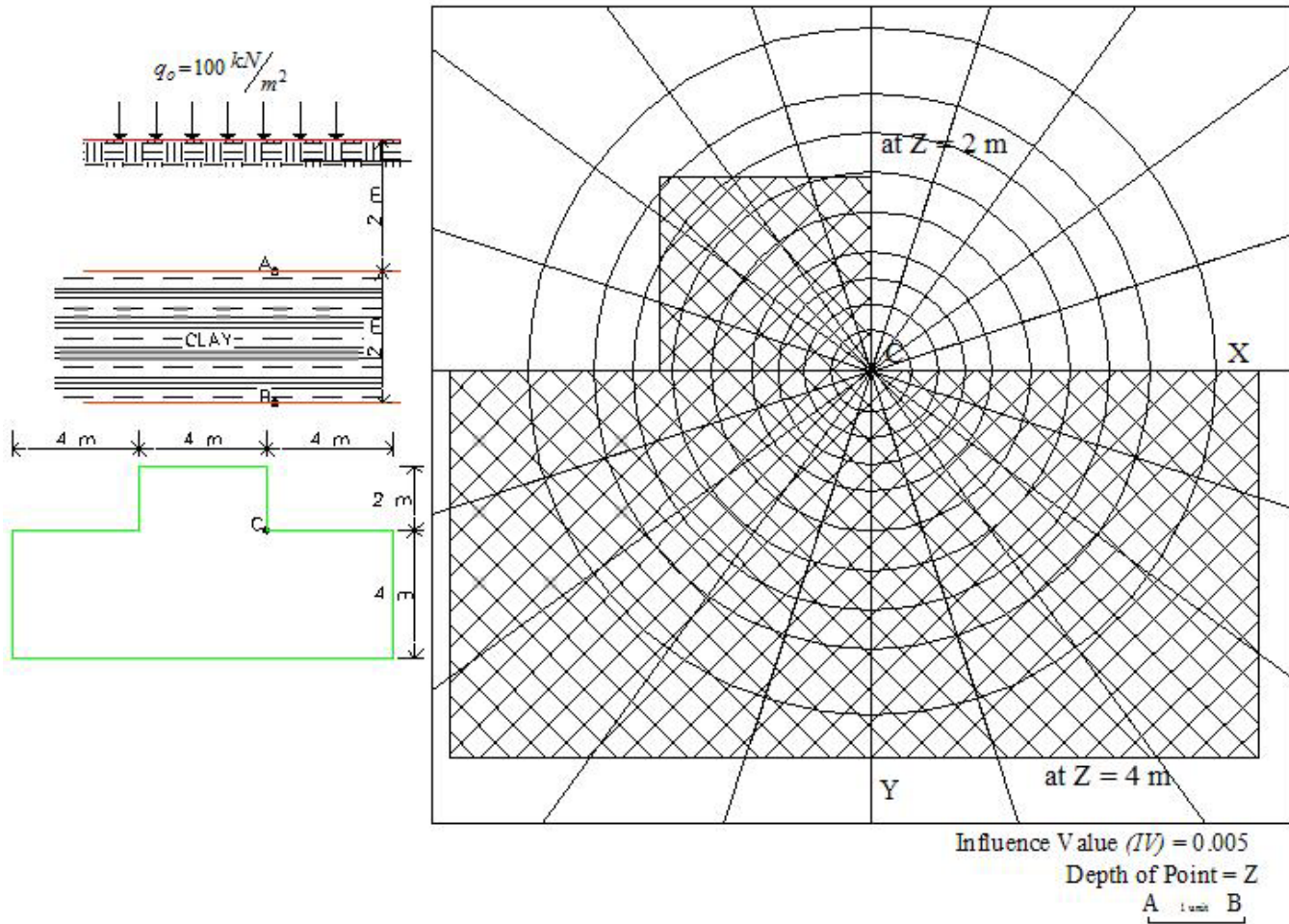
**Set  $AB = 5$  and draw the footing to that scale. The number of affected areas  $M=8$ , therefore**

$$\Delta p = q M (IV) = (1,800 \text{ psf})(8)(0.005) = \underline{72 \text{ psf}}$$

**\*Newmark-04: Stresses coming from complex shaped foundations.**

(Revision: Aug-08)

A small but heavy utility building will be placed over a 2 m thick sand stratum. Below the sand is a clay stratum 2 m thick. Find the stress at points A and B in the clay stratum directly below point C at the surface.



**Solution:**

Point A (top of clay stratum)  $q_A = q M(IV) = (100)(136)(0.005) = 68 \frac{\text{kN}}{\text{m}^2}$

Point B (bottom of clay)  $q_B = q M(IV) = (100)(100)(0.005) = 50 \frac{\text{kN}}{\text{m}^2}$

$$q_{\text{average}} = \frac{q_A + q_B}{2} = \frac{(70 + 50)}{2} = 59 \frac{\text{kN}}{\text{m}^2}$$

**\*Newmark-05: Stress beneath a circular oil tank.**

(Revision: Aug-08)

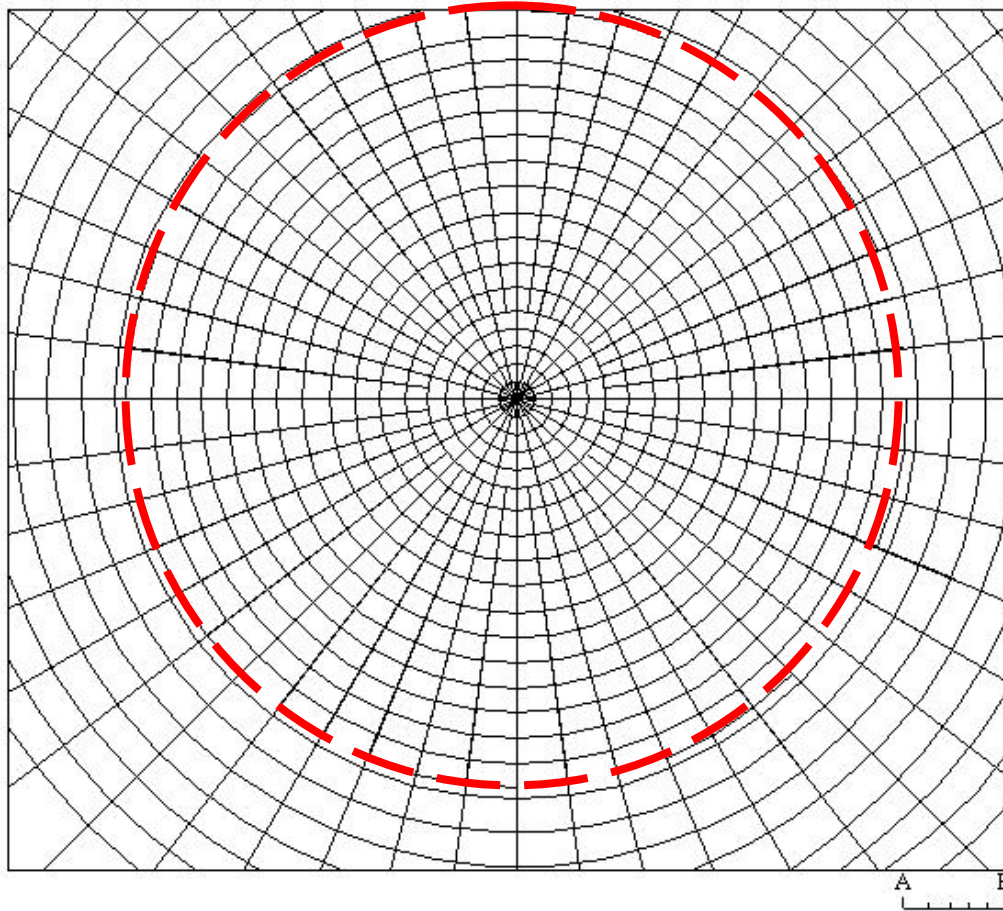
A circular oil storage tank will be built at the shore of Tampa Bay. It will be 20 m in diameter, and 15 m high. The tank sits upon a 2 m thick sand deposit that rests upon a clay stratum 16 m thick. The water table is at practically at the surface. Find the stress increase from a fully loaded tank, at mid-clay stratum, (a) directly under the center of the tank, and (b) at its outer edge, using the Newmark influence chart shown below.

Set AB 10 m      Influence Value ( $IV$ ) = 0.001

**Solution:**

The contact stress is  $q_o = (0.95)(9.81 \text{ kN/m}^3)(15\text{m}) = 140 \text{ kN / m}^2$

At mid-clay depth along the centerline of the tank (depth = 10 m)  $\therefore OQ = 10 \text{ m}$



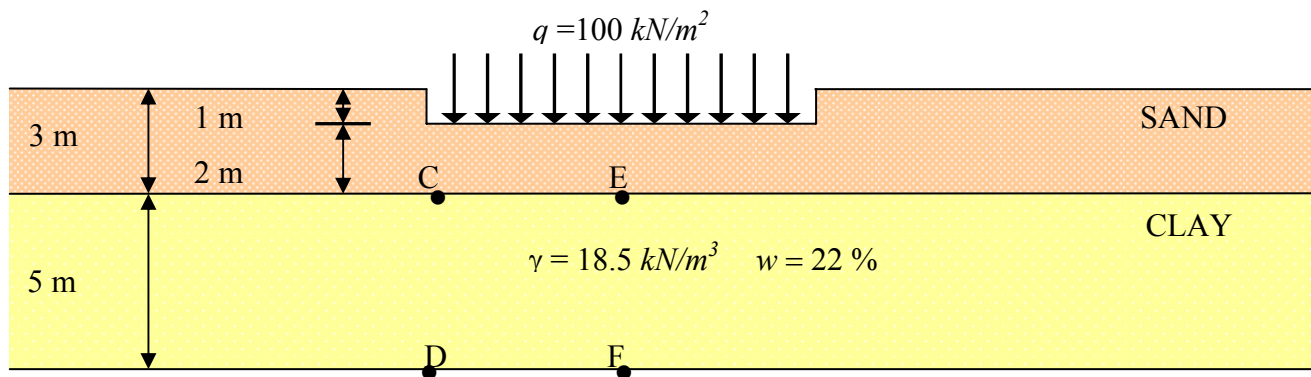
$$\sigma'_v = (q_o - \gamma_w h) M(IV) = [140 - (9.81)(10)](680)(0.001) = 28.5 \frac{\text{kN}}{\text{m}^2}$$

**\*\*Newmark-06: Use Newmark with a settlement problem.**

(Revision: Aug-08)

A small but heavily loaded utility building has dimensions of 20 m x 20 m. It applies a uniform load on its mat foundation of  $100 \text{ kN/m}^2$ . Its mat foundation sits 1 m below the surface. The soil profile consists of 3 m of a dry sand, with  $\gamma = 17.5 \text{ kN/m}^3$  under laid by a 5 m thick clay layer with a  $\gamma = 18.5 \text{ kN/m}^3$ , a moisture content of 22%,  $C_c = 0.30$  and a  $G_s = 2.70$ . The clay stratum is under laid by another sand stratum, and the phreatic surface coincides with the top of the clay stratum.

- (40) Using the Newmark method, what are the new stresses at the top and bottom of the clay stratum due to the building's loading?
- (41) What is the expected differential settlement between the building's center and one of its corners, in mm?
- (42) If a laboratory sample 4" thick of the field clay attained 50% consolidation in 5 hours, what time will the clay layer in the field attain 60% consolidation?

**Solution:**

(1) Set  $AB = 2 \text{ m}$  and observe that the building's foot-print covers the entire graph.

$$\therefore \Delta\sigma_E = 100 \text{ kN/m}^2$$

Set  $AB = 7 \text{ m}$  and the number of units  $M = 175$

$$\therefore \Delta\sigma_F = qM(IV) = (100)(175)(0.005) = 87.5 \text{ kN/m}^2$$

The stress at point C has dropped to 50% of the stress at E,

$$\therefore \Delta\sigma_C = 50 \text{ kN/m}^2 \quad \text{and} \quad \therefore \Delta\sigma_D = (0.96)(50) = 48 \text{ kN/m}^2$$

$$\text{Average stress in the clay stratum beneath the center} = \frac{(100 + 87.5)}{2} = 93.8 \frac{\text{kN}}{\text{m}^2}$$

$$\text{Average stress in the clay stratum beneath the corner} = \frac{(50 + 48)}{2} = 49 \frac{\text{kN}}{\text{m}^2}$$

(2) The differential settlement between the center and a corner of the building is,

$$C_c = 0.30 \text{ and } G_s = 2.70$$

$$Se_o = wG_s \quad \therefore e_o = \frac{wG_s}{S} = \frac{(0.22)(2.70)}{1} = 0.59$$

The in-situ stress at mid-clay stratum before the building is built is,

$$p_o = \gamma_{sand} h_{sand} + \gamma_{clay} h_{at \text{ mid-clay}} = (3m)(17.5) + (2.5m)(18.5 - 9.8) = 74.3 \frac{kN}{m^2}$$

Therefore,

$$\text{the settlement at the center } \Delta H = \frac{C_c H}{1+e} \log \left( \frac{p_o + \Delta \sigma}{p_o} \right) = \frac{(0.30)(5m)}{1+(0.59)} \log \left( \frac{(74.3) + (93.8)}{(74.3)} \right) = 0.33m$$

$$\text{the settlement at the corner } \Delta H = \frac{C_c H}{1+e} \log \left( \frac{p_o + \Delta \sigma}{p_o} \right) = \frac{(0.30)(5m)}{1+(0.59)} \log \left( \frac{(74.3) + (49)}{(74.3)} \right) = 0.21m$$

$$\text{the differential settlement is } \Delta(\Delta H) = 0.33m - 0.21m = 0.12m = 120mm$$

(3) The time required to attain 60% consolidation in the field is,

$$c_v = \frac{T_\sigma H_{drained}^2}{t} = \frac{(0.26)(2,500mm)^2}{t_F} = \frac{(0.18)(2 \times 25.4mm)^2}{(5 \text{ hours})}$$

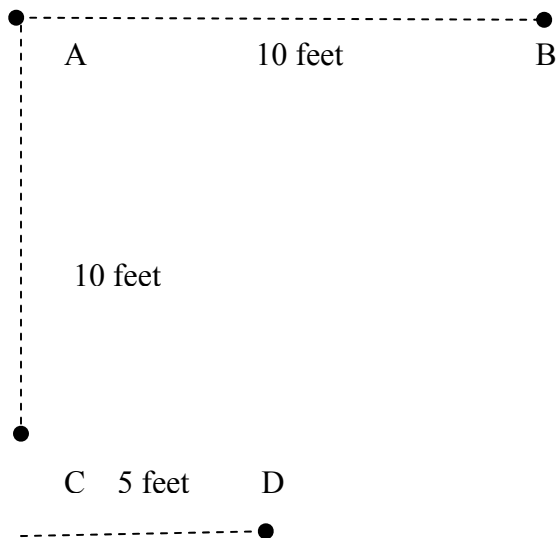
Solving for the time of settlement in the field  $t_f$ ,

$$t_F = \frac{(0.26)(2,500mm)^2}{(0.18)(2 \times 25.4mm)^2} \frac{(5 \text{ hours})}{1} \left( \frac{1 \text{ day}}{24 \text{ hours}} \right) \left( \frac{1 \text{ year}}{365 \text{ days}} \right) \approx 2 \text{ years}$$

**\*Stress-01: Stress increase at a point from several surface point loads.**

(Revision: Aug-08)

Point loads of 2000, 4000, and 6000 lbs act at points A, B and C respectively, as shown below. Determine the increase in vertical stress at a depth of 10 feet below point D.



**Solution.**

Using the Boussinesq (1883) table on page 202 for vertical point loads, the vertical increase in stress contributed by each at a depth  $z = 10$  feet is found by,

$$\Delta p_z = \frac{P}{z^2} \left\{ \frac{3}{2\pi} \frac{1}{\left[ \left( r/z \right)^2 + 1 \right]^{5/2}} \right\} = \frac{P}{z^2} I_1$$

<i>Increase in the load at:</i>	<i>P</i> (lbs)	<i>r</i> (ft)	<i>z</i> (ft)	<i>r/z</i>	<i>I<sub>1</sub></i>	<i>Δp</i> (psf)
<i>Δp</i> from A	2,000	$(10^2 + 5^2)^{1/2} = 11.18$	10	1.12	0.0626	1.25
<i>Δp</i> from B	4,000	$(10^2 + 5^2)^{1/2} = 11.18$	10	1.12	0.0626	2.50
<i>Δp</i> from C	6,000	5	10	0.50	0.2733	16.40
<b>Total = 20.2 psf</b>						

Therefore, the vertical stress increase at D from the three loads A, B and C is 20.2 psf.

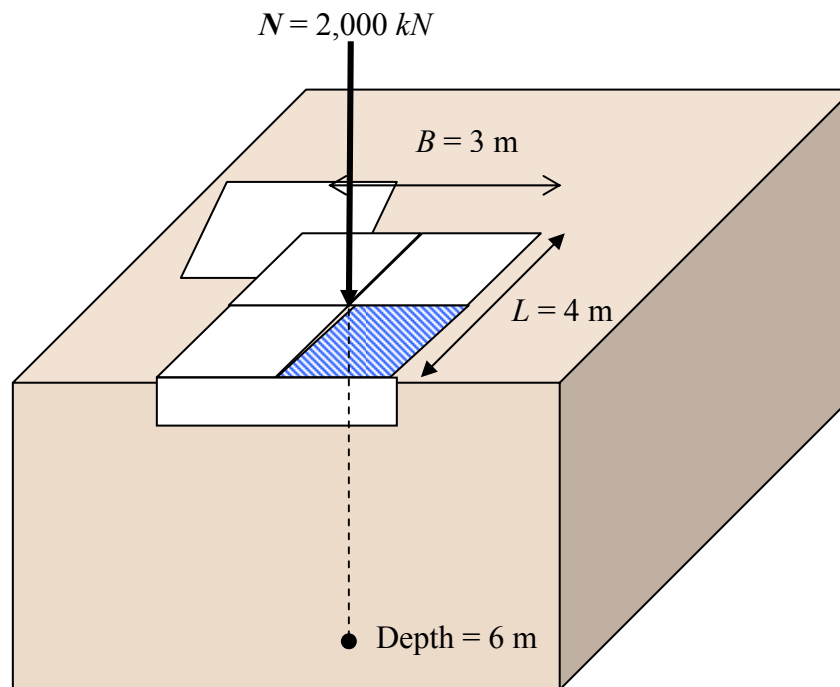
**\*Stress-02: Find the stress under a rectangular footing.**

(Revision: Aug-08)

Determine the vertical stress increase in a point at a depth of 6 m below the center of the invert of a newly built spread footing, 3 m by 4 m in area, placed on the ground surface carrying a columnar axial load of  $N = 2,000 \text{ kN}$ .

**Solution:**

The Boussinesq solution for a rectangular loaded area only admits finding stresses below a corner of the loaded area. Therefore, the footing must be cut so that the load is at a “corner” (shown as the quarter of the area), where the reduced footing dimensions for the shaded area are  $B_1 = 1.5 \text{ m}$  and  $L_1 = 2.0 \text{ m}$ .



$$m = \frac{B_1}{z} = \frac{1.5 \text{ m}}{6.0 \text{ m}} = 0.25 \quad \text{and} \quad n = \frac{L_1}{z} = \frac{2.0 \text{ m}}{6.0 \text{ m}} = 0.33$$

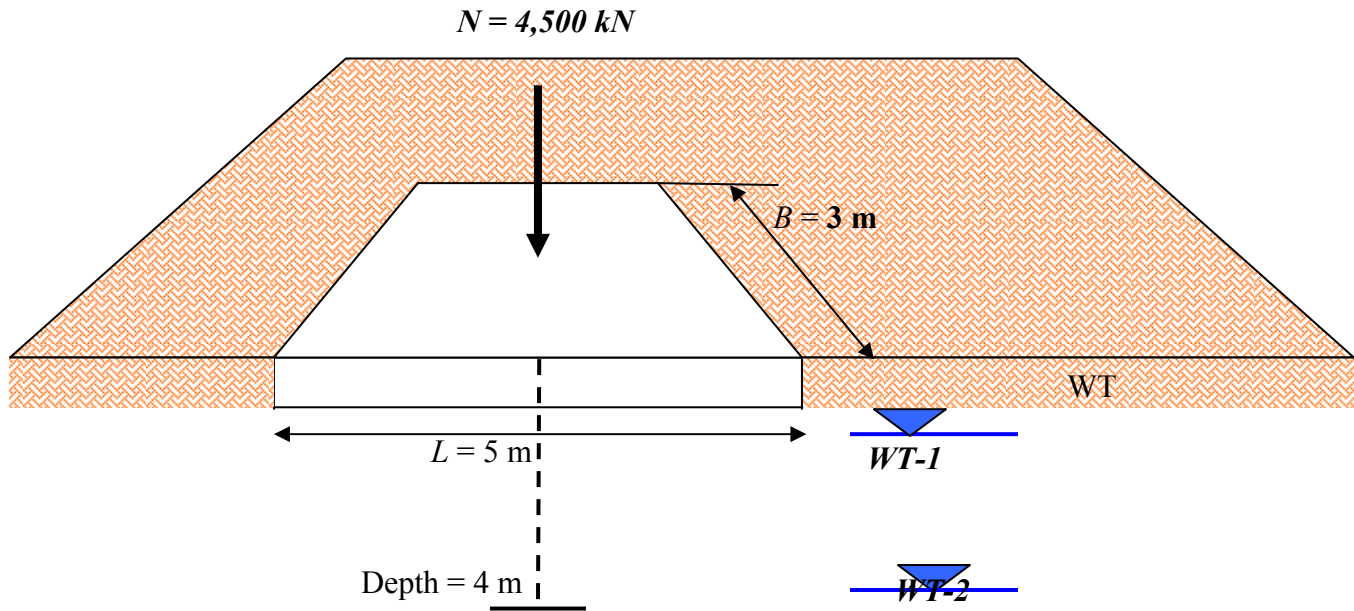
Use the table and extrapolate and find  $I_4 = 0.0345$

$$\Delta q_z = q_o (4 I_4) = \left( \frac{N}{BL} \right) (4 I_4) = \left( \frac{2,000 \text{ kN}}{(3 \text{ m})(4 \text{ m})} \right) (4)(0.0345) = 23 \text{ kN} / \text{m}^2$$

**\*Stress-03: The effect of the WT on the stress below a rectangular footing.**

(Revision: Aug-08)

Find the effective stress increase in the soil at a depth of 4 m below the footing, and then find the increase in the stress due to a drop of the *WT* from originally 1 m below the footing to 5 m below the footing.



**Solution:**

$$q = \frac{N}{A} = \frac{4,500\text{ kN}}{15\text{ m}^2} = 300 \frac{\text{kN}}{\text{m}^2} \quad m = \frac{B}{z} = \frac{1.5\text{ m}}{4\text{ m}} = 0.375 \text{ and } n = \frac{L}{z} = \frac{2.5\text{ m}}{4\text{ m}} = 0.625$$

$$\therefore I_4 = 0.076$$

a) The total stress increase from the footing is,

$$\Delta p_o = q(4I_4) = (300)(4)(0.076) = 91.2\text{ kN/m}^2$$

and the effective stress when the *WT* is 1 m below the footing is,

$$\Delta p_o' = \Delta p_o - u = (91.2) - (3\text{ m})(9.8) = 61.8\text{ kN/m}^2$$

b) When the *WT* drops from -1 m to -5 m below the footing, the effective stress is identical to the total stress. Therefore the effective stress increase is,

$$\Delta p_o' = 91.2\text{ kN/m}^2 \text{ which is a 48\% increase in stress.}$$

### \*Stress-04: Finding the stress outside the footing area.

(Revision: Aug-08)

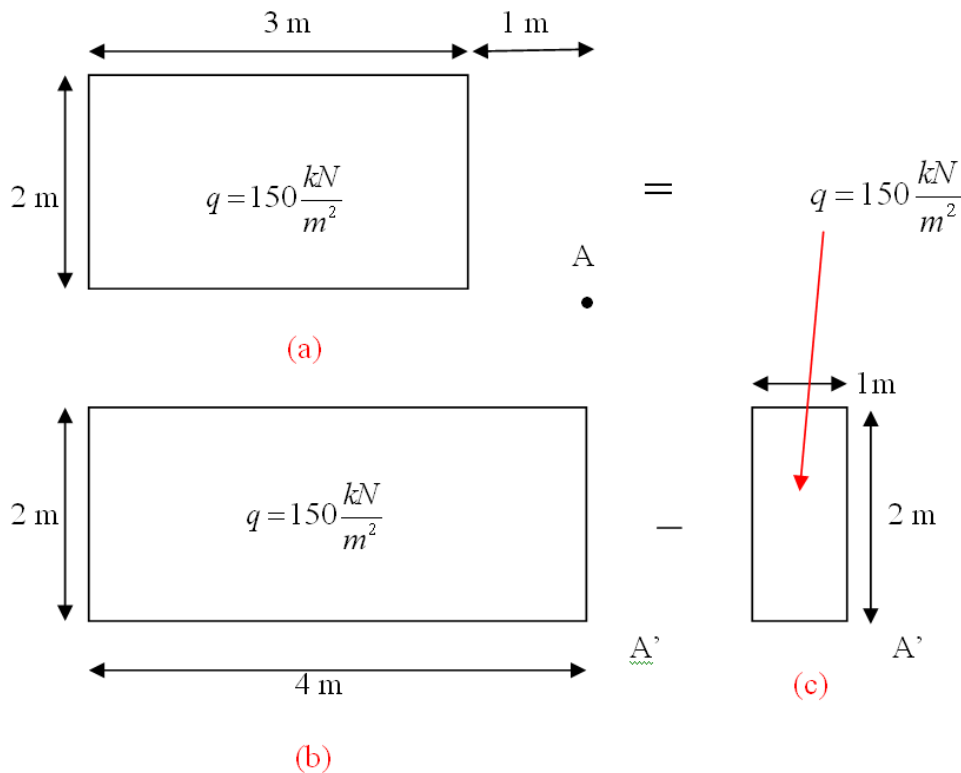
Find the vertical stress increase  $\Delta p$  below the point A at a depth  $z = 4$  m.

#### Solution:

The stress increase,  $\Delta p$ , can be written as :  $\Delta p = \Delta p_1 - \Delta p_2$

where  $\Delta p_1$  = stress increase due to the loaded area shown in (b).

$\Delta p_2$  = stress increase due to the loaded area shown in (c).



For the loaded area shown in (b):

$$m = \frac{B}{Z} = \frac{2}{4} = 0.5 \quad \text{and} \quad n = \frac{L}{Z} = \frac{4}{4} = 1.0$$

$$\Delta p_1 = qI_4 = (150)(0.12) = 18 \text{ kN} / m^2$$

Similarly, for the loaded area shown in (c):

$$m = \frac{B}{Z} = \frac{1}{4} = 0.25 \quad \text{and} \quad n = \frac{L}{Z} = \frac{2}{4} = 0.5$$

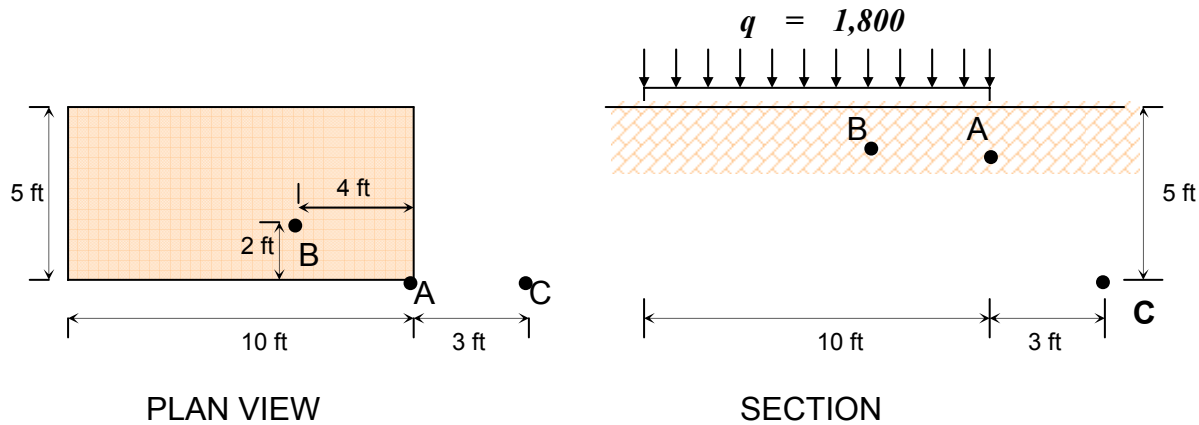
$$\Delta p_2 = (150)(0.048) = 7.2 \text{ kN} / m^2$$

$$\text{Therefore, } \Delta p = \Delta p_1 - \Delta p_2 = 18 - 7.2 = 10.8 \text{ kN} / m^2$$

**\*Stress-05: Stress below a footing at different points.**

(Revision: Sept.-08)

A clay sanitary pipe is located at a point C below the footing shown below. Determine the increase in the vertical stress  $\Delta p$  at the depth of the pipe, which is  $z = 5$  feet below the footing invert, and 3 feet away from its edge. The footing has a uniformly distributed load  $q = 1,800$  psf.



**Solution:**

For the expanded 5' x 13' area,

$$m = \frac{B}{Z} = \frac{5}{5} = 1 \quad \text{and} \quad n = \frac{L}{Z} = \frac{13}{5} = 2.6 \quad \text{therefore, } I_4 = 0.200$$

For virtual 3' x 5' area

$$m = \frac{B}{Z} = \frac{3}{5} = 0.6 \quad \text{and} \quad n = \frac{L}{Z} = \frac{5}{5} = 1 \quad \text{therefore, } I_4 = 0.136$$

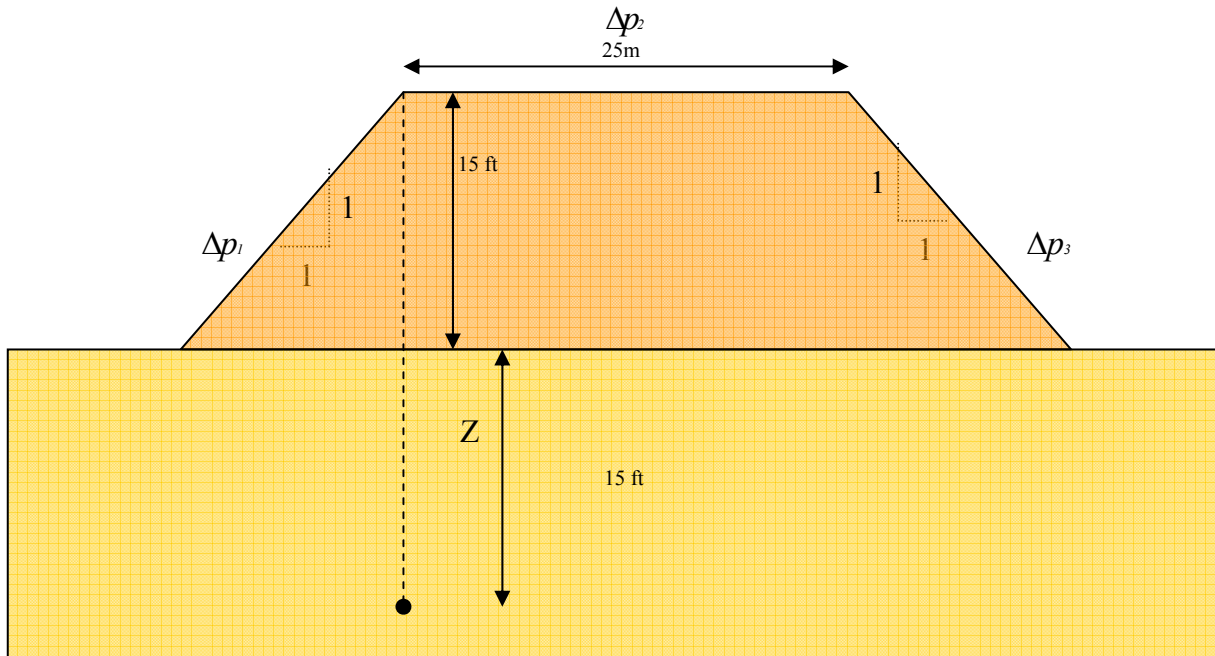
The increase in stress at point C from the footing is therefore,

$$\Delta p = q(I_4 - I'_4) = \left( 1,800 \frac{\text{lb}}{\text{ft}^2} \right) (0.200 - 0.136) = 115 \text{ psf}$$

**\*Stress-06: Stress increase from a surcharge load of limited width.**

(Revision: Aug-08)

Calculate the stress increase at the point A due to the new road embankment.



**Solution:**

The contribution from the central portion of the fill is  $\Delta p_2$ , whereas the contribution from the left and right hand slopes are  $\Delta p_1$  and  $\Delta p_3$  respectively. Using Boussinesq,

$$\Delta p_1 \Rightarrow \frac{2x_1}{B_1} = \frac{2(15')}{15'} = 2 \quad \frac{2z}{B_1} = \frac{2(15')}{15'} = 2 \quad \frac{\Delta p_1}{q} = 0.25$$

$$\therefore \Delta p_1 = (0.25)(15') \left( 120 \frac{lb}{ft^3} \right) = 450 \text{ psf}$$

$$\Delta p_2 \Rightarrow \frac{2x_2}{B_2} = \frac{2(-12.5')}{25'} = -1 \quad \frac{2z}{B_2} = \frac{2(15')}{25'} = 1.2 \quad \frac{\Delta p_2}{q} = 0.47$$

$$\therefore \Delta p_2 = (0.47)(15') \left( 120 \frac{lb}{ft^3} \right) = 846 \text{ psf}$$

$$\Delta p_3 \Rightarrow \frac{2x_3}{B_3} = \frac{2(40')}{15'} = 5.3 \quad \frac{2z}{B_3} = \frac{2(15')}{15'} = 2 \quad \frac{\Delta p_3}{q} = 0.02$$

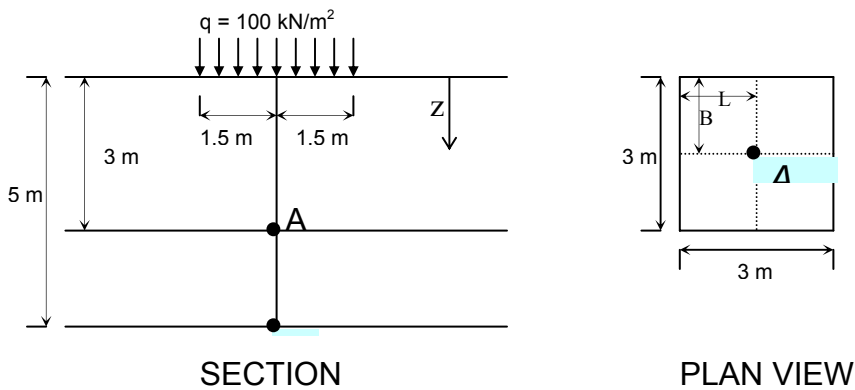
$$\therefore \Delta p_3 = (0.02)(15') \left( 120 \frac{lb}{ft^3} \right) = 36 \text{ psf}$$

$$\Delta p = \Delta p_1 + \Delta p_2 + \Delta p_3 = 450 + 846 + 36 = 1,332 \text{ psf}$$

### \*Stress-07: Finding a stress increase from a surface load of limited width.

(Revision: Aug-08)

Determine the average stress increase below the center of the loaded area, between  $z = 3$  m and  $z = 5$  m.



#### Solution:

The stress increase between the required depths (below the corner of each rectangular area) can be given as:

$$\Delta p_{avg(H_2/H_1)} = q \left[ \frac{(H_2)(I_4(H_2)) - (H_1)(I_4(H_1))}{H_2 - H_1} \right] = 100 \left[ \frac{(5)(I_4(H_2)) - (3)(I_4(H_1))}{5 - 3} \right]$$

$$\text{For } I_4(H_2): \quad m' = B / H_2 = 1.5 / 5 = 0.3$$

$$n' = L / H_2 = 1.5 / 5 = 0.3$$

$$\text{For } m' = n' = 0.3, \quad I_4(H_2) = 0.038$$

$$\text{For } I_4(H_1): \quad m' = B / H_1 = 1.5 / 3 = 0.5$$

$$n' = L / H_1 = 1.5 / 3 = 0.5$$

$$\text{For } m' = n' = 0.5, \quad I_4(H_1) = 0.086$$

Therefore:

$$\Delta p_{av(H_2/H_1)} = 100 \times \frac{(5)(0.038) - (3)(0.086)}{5 - 3} = 3.4 \text{ kN/m}^2$$

The stress increase between  $z = 3$  m and  $z = 5$  m below the center of the load area is equal to:

$$4\Delta p_{avg(H_2/H_1)} = (4)(3.4) = 13.6 \text{ kN/m}^2$$

**\*\*Stress-08: Stress increase as a function of depth.**

(Revision: Aug-08)

The vertical stress  $\sigma_v$  in a soil at any depth below the surface can be estimated as a function of the soil unit weight  $\gamma$  by the equation,

$$\sigma_v = \int_0^Z \gamma(\sigma_v) dz = \int_0^{100} (95 + 0.0007\sigma_v) dz$$

If a particular stratum has a function  $\gamma = 95 + 0.0007 \sigma_v$ , where  $\gamma$  is in pcf and  $\sigma_v$  is in psf, find the vertical stress at a depth of 100 feet below the surface.

**Solution:**

Rearranging, and integrating by parts,

$$\int_0^{\sigma_v} \frac{d\sigma_v}{(95 + 0.0007\sigma_v)} = \int_0^{100} dz$$

$$\frac{1}{0.0007} \ln(95 + 0.0007\sigma_v) \Big|_0^{\sigma_v} = z \Big|_0^{100}$$

$$\sigma_v = 135,800 \left( e^{0.0007z} - 1 \right) \Big|_0^{100} = 9,840 \text{ psf}$$

$$\text{At } Z = 100 \quad \sigma_v = 135,800(1.0725 - 1)$$

$$\therefore \quad \sigma_v = 9,840 \text{ psf}$$

# Chapter 10

## Elastic Settlements

### ***Symbols for Elastic Settlements***

- $N \rightarrow$  Raw value of the STP (obtained in the field).
- $q_o \rightarrow$  Contact pressure.
- $C_1 \rightarrow$  Embedment coefficient.
- $C_2 \rightarrow$  Creep correction factor.
- $E_S \rightarrow$  Soil elastic modulus.
- $E_{eq} \rightarrow$  Equivalent modulus.
- $\Delta(\Delta_o) \rightarrow$  Differential settlement between adjacent foundation.
- $\Delta H_i \rightarrow$  Elastic settlement.
- $I \rightarrow$  Influence factor; essentially equivalent to the strain  $\varepsilon$  in the soil.
- $I_Z \rightarrow$  Simplifying influence factor.
- $\varepsilon \rightarrow$  Strain at mid stratum.
- $\gamma \rightarrow$  Unit weight of the soil.
- $\nu \rightarrow$  Poisson's ratio.
- $M_T \rightarrow$  Transverse moment.

### **\*Elastic Settlement-01: Settlement (rutting) of a truck tire.**

(Revision: Jan-08)

You are required to move a 60-ton truck-mounted crane onto your construction site. The front wheels carry 20% of the load on tires inflated with 80 *psi* air pressure. Calculate the possible rutting depth to your temporary jobsite road built from in-situ compacted medium sand. A surface SPT test shows an  $N = 12$  and the tire's bearing area is roughly square. Use the Schmertmann method to estimate the rutting.

#### **Solution:**

Each front tire has a square bearing area of  $B \times B$  such that:

$$B^2 = \frac{\text{tire's load}}{\text{tire pressure } q_o} = \frac{(0.5)(20\%)(120,000 \text{ lb})}{(80 \text{ lb/in}^2)} = 150 \text{ in}^2 \quad \therefore B = 12.2 \text{ inches}$$

A rough estimate of the soil's elastic modulus is  $E_s = 14N = 14(12) = 168 \text{ ksf}$ .

Since the sand is compacted, it is a dense sand, and the influence factor  $I_z$  is equivalent to the strain  $\varepsilon$ . The strain reaches a maximum value of 0.6. Therefore, the average value of the strain is about 0.3 throughout its depth to  $2B = 2(12.2 \text{ inches}) = 24.4 \text{ inches}$ . Since the crane loads are on the surface and only for a few days, it is permissible to assume that there is no creep and therefore  $C_1 = C_2 = 1$ . Therefore, for the single layer of soil, the rutting is,

$$\Delta = C_1 C_2 q_o \left( \frac{\varepsilon}{E_s} \right) dz = (1)(1) \left( 80 \frac{\text{lb}}{\text{in}^2} \right) \left( \frac{0.30}{168 \text{ k/ft}^2} \right) (24.4 \text{ in}) \times \left( \frac{144 \text{ in}^2}{\text{ft}^2} \right) \left( \frac{k}{1,000 \text{ lb}} \right)$$

$\therefore \Delta \approx 0.5 \text{ inches of rutting.}$

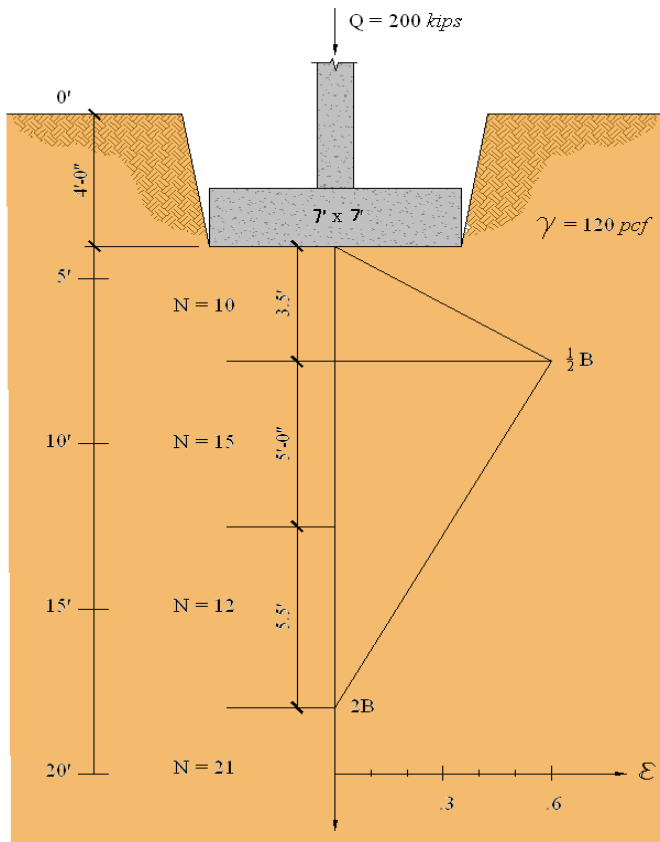
## \*Elastic Settlement-02: Schmertmann method used for granular soils.

(Revision: Aug-08)

Estimate the settlement of a square footing placed on a fine, medium dense sand, embedded 4 ft below the ground surface, for long-term use. Use the Schmertmann method.

Assume  $\frac{E_s}{N} \cong 14$  where  $E_s$  is in ksf; used for fine medium sands.

**Solution:**



Layer	$\Delta z$ (in)	$E_s$ (ksf)	$I_z = \epsilon$	$\frac{I_z \Delta z}{E_s}$
1	42	140	0.30	0.090
2	60	210	0.46	0.130
3	66	168	0.16	0.061
				$\Sigma = 0.281$

The contact pressure on the soil is,  $q_o = \frac{Q}{B^2} - D_f \gamma = \frac{200k}{49ft^2} - (4ft)(0.120 kcf) = 3.60 ksf$

The coefficients for the Schmertmann method are  $C_1$  and  $C_2$  :

$$\text{Depth factor } C_1 = 1.0 - 0.5 \left( \frac{D_f \gamma}{q_o} \right) = 1.0 - 0.5 \left( \frac{0.48}{3.60} \right) = 0.93$$

Creep factor  $C_2 = 1.35$  for a five year period.

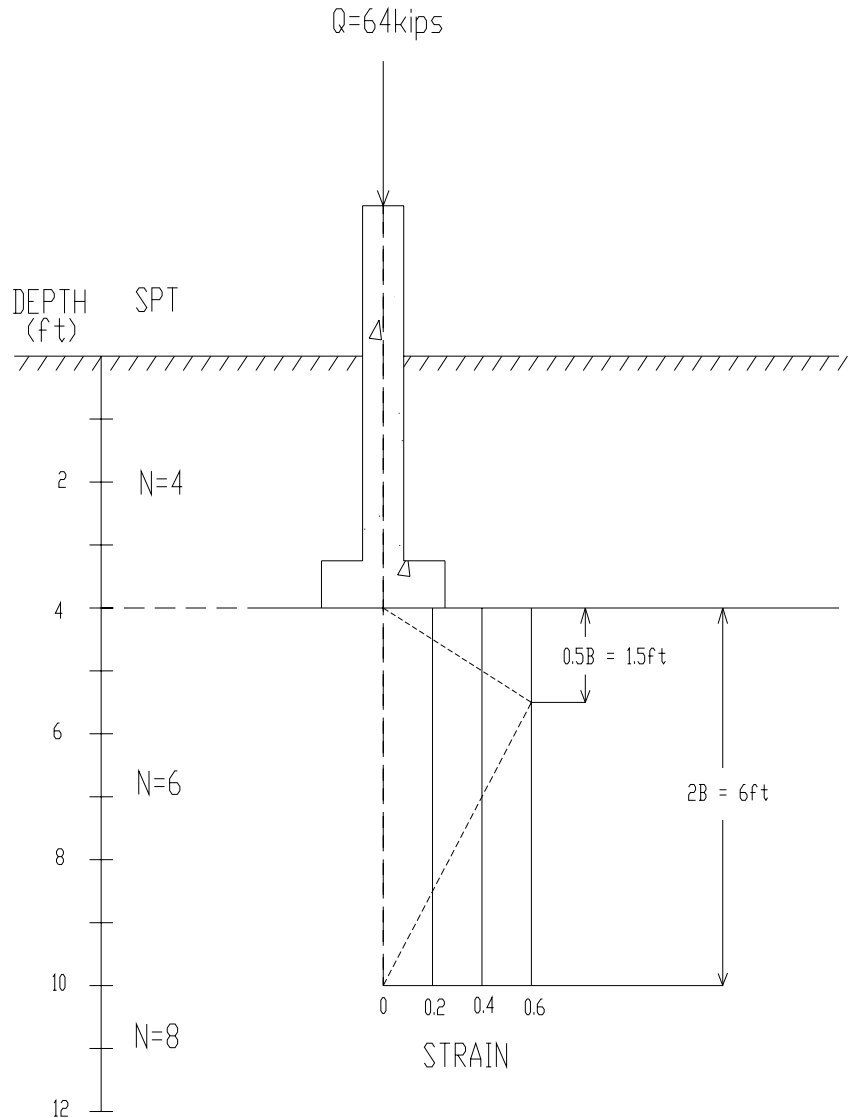
The Schmertmann formula for the elastic settlement  $\Delta$  is,

$$\therefore \Delta = C_1 C_2 q_o \sum_0^{2B} \frac{\epsilon}{E} \Delta z = (0.93)(1.35)(3.60)(0.281) = 1.27 \text{ inches}$$

**\*Elastic Settlement-03: Schmertmann method used for a deeper footings.**

(Revision: Aug-08)

Determine the elastic settlement of a deep spread footing after five years of the 3 ft. x 3 ft. footing when it is placed on a uniform clean sand with  $\gamma = 110$  pcf.



**Solution:**

The contact pressure on the soil is,  $q_0 = \frac{Q}{B^2} = \frac{64 \text{ kips}}{(3 \text{ ft})^2} = 7.11 \text{ ksf}$

The SPT value indicates that the soil is a loose sand. The modulus  $E$  for loose sand can be calculated using the following formula:

$$E_s \approx 10(N + 15) \text{ ksf}$$

The following table summarizes the data and calculations:

Layer number	Layer's thickness $\Delta Z$ (feet)	SPT (average) $N$	Soil's elastic modulus $E$ (ksf)	Average strain at mid stratum $\epsilon$	$\epsilon_z \Delta Z / E$ (ft <sup>3</sup> /kip)
1	1.5	6	210	0.35	0.0025
2	4.5	7	220	0.30	0.0061

$$\Sigma = 0.0086$$

The correction factors are as follows:

$$1. \text{ Depth factor, } C_1 = 1 - \left( \frac{0.5 \gamma D_f}{q_0 - \gamma D_f} \right) = 1 - \left( \frac{(0.5)(0.110)(4)}{7.11 - (0.110 \times 4)} \right) = 0.97$$

Since  $C_1 > 0.5$ , this is **FINE**.

$$2. \text{ Creep factor, } C_2 = 1 + 0.2 \log \left( \frac{t_{yr}}{0.1} \right) = 1 + (0.2) \log \left( \frac{5}{0.1} \right) = 1.34$$

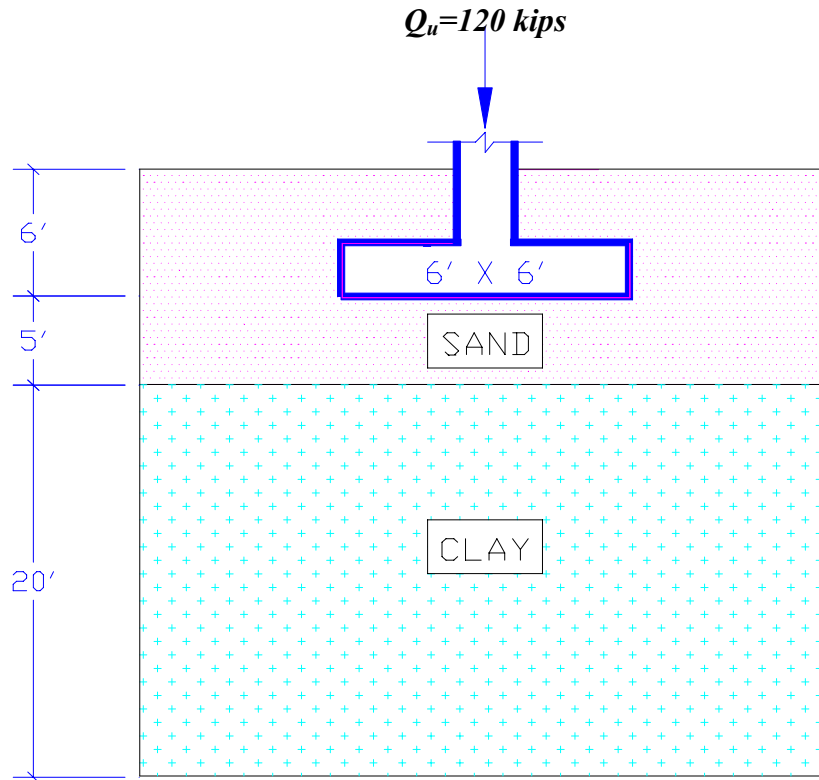
The total elastic settlement is,

$$\Delta = C_1 C_2 (q_o - \gamma D_f) \sum_0^{2B} \frac{\epsilon}{E_s} dz = (0.97)(1.34) [7.11 - (0.110)(4)] (0.0086) = 0.07 \text{ ft} = 0.84 \text{ in}$$

**\*Elastic Settlement-04: The 2:1 method to calculate settlement.**

(Revision: Aug-08)

Use the 2:1 method to find the average stress increase ( $\Delta q$ ) due to the applied load  $Q_u$  in the 5-foot sand stratum directly beneath the footing. If  $E_s = 400$  ksf and  $\nu = 0.3$ , what is the expected immediate settlement  $\Delta H_i$ ?



**Solution:**

The settlement  $\Delta H$  of an elastic media (the 5 foot thick sand stratum in this case) can be found from the theory of elasticity as,

$$\Delta H = q_o B \frac{1-\nu^2}{E_s} I_w$$

For square and flexible footings the influence factor is about  $I_w = 0.95$ . The 2:1 method essentially assumes that the stress reduces vertically by a vertical slope of 2 units vertically to 1 unit horizontally. The stress increase can be found by integrating the above equation,

$$\Delta q = \frac{1}{H} \int_{H_1}^{H_2} \frac{Q_u}{(B+Z)^2} dz$$

where  $H_1 = 0$  feet (the footing's invert) to  $H_2 = 5$  feet (bottom of the sand stratum).

$$\Delta q = \frac{1}{H} \left[ \frac{Q}{B+z} \right]_0^5 = \frac{1}{5ft} \left( \frac{120kip}{6ft} - \frac{120kip}{11ft} \right) = 1.82ksf$$

but

$$\Delta H = q_o B \frac{1-\nu^2}{E_s} I_w = \left( \frac{120 \text{ kip}}{36 \text{ ft}^2} \right) (6ft) \left( \frac{1-0.3^2}{400 \text{ ksf}} \right) (0.95) \left( \frac{12 \text{ in}}{ft} \right) = \mathbf{0.52 \text{ inches}}$$

### **\*Elastic Settlement-05: Differential settlement between two columns.**

(Revision: Aug-08)

The allowable bearing capacity of a 30-ft thick, medium dense sand stratum (with  $\phi = 36^\circ$  and  $\gamma = 112$  pcf) is 3 ksf. Column A has a design load of 430 kips and column B has a design load of 190 kips. Select footing sizes and determine the differential settlement  $\Delta(\Delta H)$  between them. Is this  $\Delta(\Delta H)$  acceptable for columns spaced 30 ft apart?

#### **Solution:**

Footing size for column A:

$$B_A = \sqrt{\frac{Q_A}{q_{all}}} = \sqrt{\frac{430 \text{ kips}}{3 \text{ ksf}}} = 12 \text{ feet}$$

Footing size for column B:

$$B_B = \sqrt{\frac{Q_B}{q_{all}}} = \sqrt{\frac{190 \text{ kips}}{3 \text{ ksf}}} = 8 \text{ feet}$$

A quick estimate of the ratio of settlement to the proportionality is  $\frac{\Delta H_A}{\Delta H_B} = \frac{12 \text{ ft}}{8 \text{ ft}} = 1.5$

Therefore if the settlement at column B is  $\Delta H_B = 1$  in., then the settlement at column A will be  $\Delta H_A = 1.5$  in. Then,

$$\Delta(\Delta H) = 1.5 \text{ in} - 1.0 \text{ in} = 0.5 \text{ inches}$$

and the rotation between the two columns is

$$\theta = \frac{\Delta(\Delta H)}{L} = \frac{(0.5 \text{ in})}{(30 \text{ ft}) \left( \frac{12 \text{ in}}{\text{ft}} \right)} = \underline{\underline{0.0014}}$$

**Both of these values [ $\Delta(\Delta H)$  and  $\theta$ ] are acceptable, since  $\Delta(\Delta H)$  should be  $< 1''$  and  $\theta < 0.0033$ .**

**\*Elastic Settlement-06: Compare the settlements predicted by the Boussinesq, Westergaard, and the 2:1 methods.**

(Revision: Aug-08)

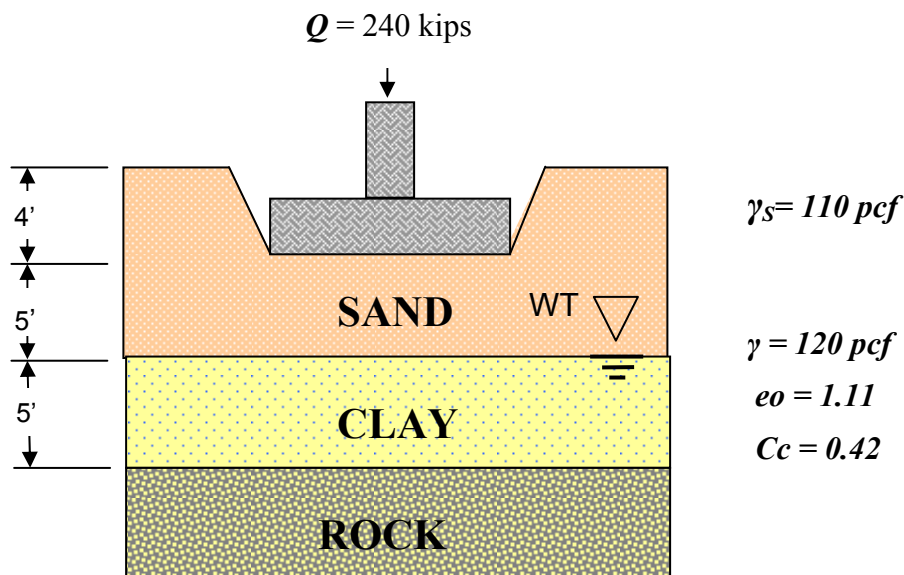
Compute the average stress  $\Delta q$  at mid-clay stratum, for the values shown below, using: (a) Boussinesq's method, (b) Westergaard's method, and (c) the 2:1 method.

Also, determine the size of a square spread footing, in order to limit total settlement  $\Delta H = \Delta H_i + \Delta H_c + \Delta H_s$  to only 1.5 inches.

Estimate the initial settlement  $\Delta H_i = 0.05\sqrt{Z_s}$  where  $Z_s$  is the thickness of the granular stratum beneath the footing in feet, to give  $\Delta H_i$  in inches.

**Solution:**

Assume an initial value of  $B = 10$  feet.



The contact pressure  $q_o$  of the footing is:

$$q_o = \frac{Q}{A_B} = \frac{240 \text{ kips}}{100 \text{ ft}^2} = 2.4 \text{ ksf}$$

(a) Stress at mid-clay stratum using **Boussinesq's method** (use the charts on page 205) for a square footing:

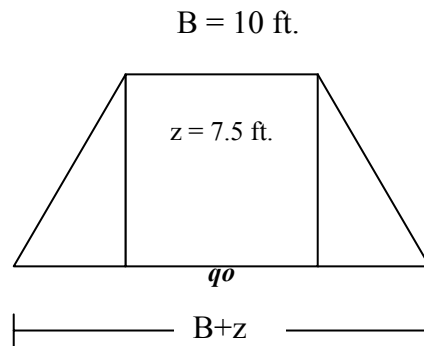
$$\frac{q}{q_o} = 0.52 \quad \therefore \quad q = (0.52)(2.4 \text{ ksf}) = 1.25 \text{ ksf}$$

(b) Stress at mid-clay stratum using Westergaard's method for a square footing:

$$\frac{q}{q_o} = 0.33 \quad \therefore q = (0.33)(2.4 \text{ ksf}) = 0.79 \text{ ksf}$$

(c) Stress at mid-clay stratum using the 2:1 method for a square footing:

The depth (Z) from the footing invert to mid-clay is 7.5 feet:



$$q = \frac{Q}{A_{B+z}} = \frac{240 \text{ kips}}{(10 \text{ ft} + 7.5 \text{ ft})^2} = 0.78 \text{ ksf}$$

Note that the Boussinesq method provides the highest predicted stress. Since this would predict faster consolidation rates, it is the least conservative method. Therefore, for this problem, use the 2:1 method's stress of 0.78 ksf. The instantaneous settlement ( $\Delta H_i$ ):

$$\Delta H_i = 0.05\sqrt{Z_s} = 0.05\sqrt{5 \text{ ft}} = 0.11 \text{ in}$$

The in-situ effective stress  $q_o'$  at mid-clay layer, before placing the footing is:

$$q_o' = \gamma_s h_s + \gamma_c h_c = \left( 0.11 \frac{\text{kip}}{\text{ft}^3} \times 9 \text{ ft} \right) + \left[ \left( 0.120 \frac{\text{kip}}{\text{ft}^3} - 0.0624 \frac{\text{kip}}{\text{ft}^3} \right) \times 2.5 \text{ ft} \right] = 1.13 \text{ ksf}$$

Using  $q = 0.78 \text{ ksf}$  from the 2:1 method, the total settlement  $\Delta H$  is equal to the immediate  $\Delta H_i$ , plus the consolidation  $\Delta H_c$ , and the secondary settlement  $\Delta H_s$ , but limited to no more than 1.50 inches.

$$\Delta H = \Delta H_i + \Delta H_c + \Delta H_s = 1.50 \text{ inches}$$

But  $\Delta H_i = 0.11$  in, and  $\Delta H_s$  is negligible for this problem. Therefore the maximum permissible consolidation settlement  $\Delta H_c$  is limited to:

$$\Delta H_c = 1.50 \text{ in} - \Delta H_i = 1.50 \text{ in} - 0.11 \text{ in} = 1.39 \text{ in}$$

or

$$\Delta H_c = \frac{C_c H}{1 + e_o} \log \frac{q_o' + \Delta q}{q_o'} = 1.39 \text{ in} = \frac{(0.42) \left( 5 \text{ ft} \times \frac{12 \text{ in}}{\text{ft}} \right)}{1 + 1.11} \log \left( \frac{1.13 \text{ ksf} + \Delta q}{1.13 \text{ ksf}} \right) = 1.39 \text{ in}$$

Therefore  $\Delta q = 0.39 \text{ ksf}$

Using the 2:1 method:

$$\Delta q = \frac{Q}{A_{B+Z}} \quad B + Z = \sqrt{\frac{Q}{\Delta q}} = \sqrt{\frac{240 \text{ kips}}{0.34 \text{ ksf}}} = 26.6 \text{ ft}$$

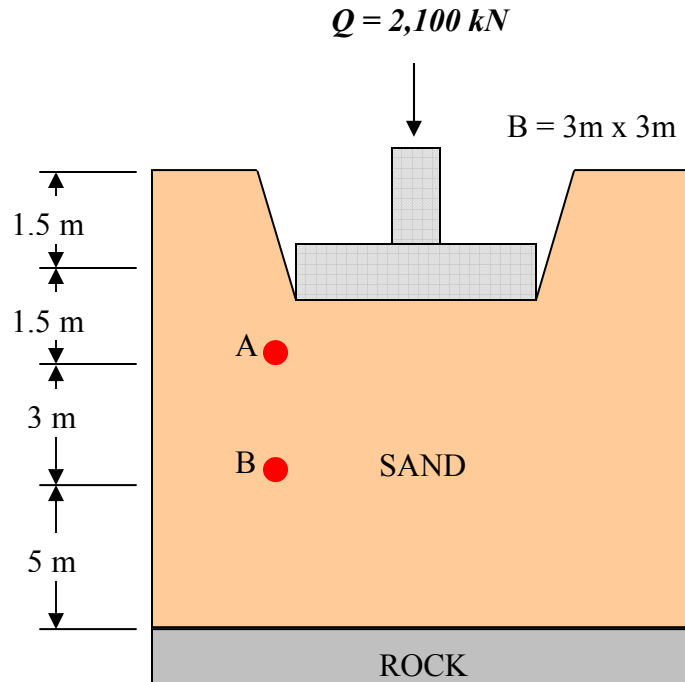
Therefore  $B = 19 \text{ feet}$

Since the initial  $B = 10$  feet, the new value of 19 feet should be used to re-iterate towards a better solution that converges.

## \*Elastic Settlement-07: Schmertmann versus the strain methods.

(Revision: Aug-08)

Compute the immediate settlement  $\Delta H_i$  using the Schmertmann formula using an average  $\Delta q$  value ( $q_{v1} = 233.3 \text{ kPa}$ ,  $q_{v2} = 163.3 \text{ kPa}$ ,  $q_{v3} = 77.0 \text{ kPa}$ ,  $q_{v4} = 44.0 \text{ kPa}$  and  $q_{v5} = 28.0$ ).  $E_s$  at point A is  $20,400 \text{ kPa}$ ,  $\frac{D_f}{B} = 0.5$  and  $C_1 = C_2 = 1$ . Compare the results with an alternate method using  $\Delta H = q_o B \left( \frac{1 - \nu^2}{E_s} \right) I_w$ , where  $\nu = 0.3$  and  $I_w = 0.95$ .



### Solution:

The average stress from multiple layers is solved via this formula,

$$\Delta q_v = \frac{\Delta H}{H} \left[ \left( \frac{q_1 + q_n}{2} \right) + q_2 + q_3 + \dots + q_{n-1} \right] = \frac{1.5m}{6m} \left[ \left( \frac{233.3kPa + 28.0kPa}{2} \right) + 163.3kPa + 77.0kPa + 44.0kPa \right]$$

$$\Delta q_v = 104 \text{ kPa}$$

The elastic settlement via Schmertmann is,

$$\Delta H = C_1 C_2 (0.6B) \frac{\Delta q}{E_s} = (1)(1)(0.6)(3m) \left( \frac{1000 \text{ mm}}{m} \right) \left( \frac{104 \text{ kPa}}{20,400 \text{ kPa}} \right) = 9.2 \text{ mm}$$

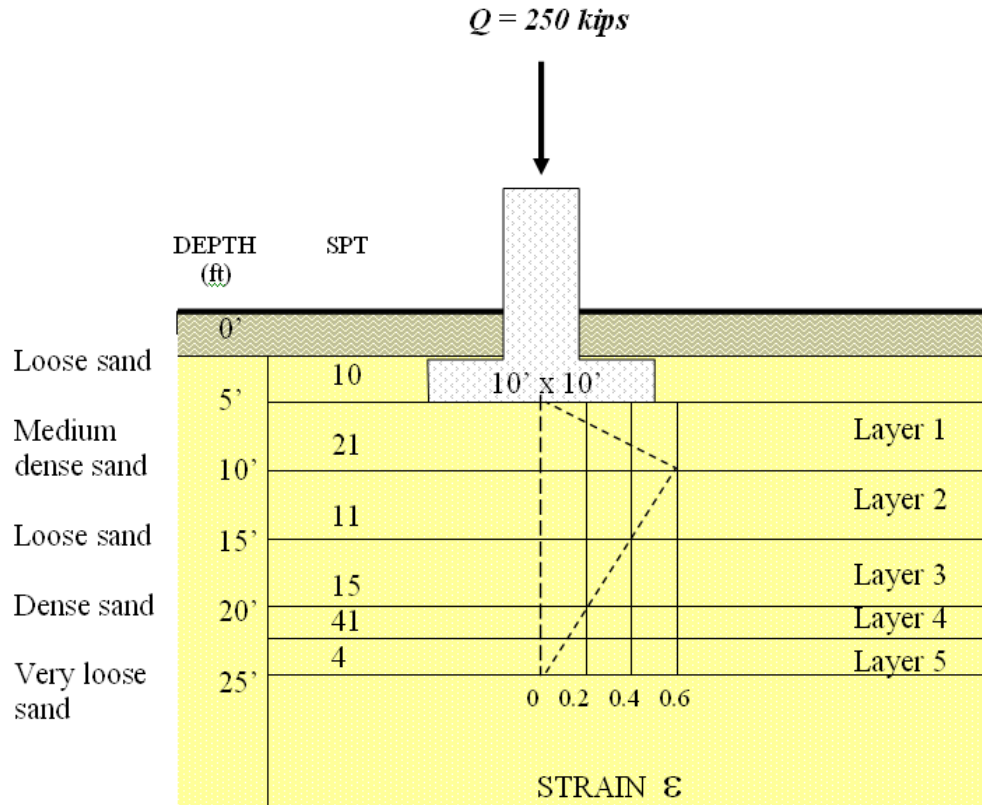
The alternative method from the theory of elasticity would yield,

$$\Delta H = q_{v1} B \left( \frac{1 - \nu^2}{E_s} \right) I_w = (233.3 \text{ kPa})(3m) \left( \frac{1 - 0.3^2}{20,400 \text{ kPa}} \right) (0.95) \left( \frac{1000 \text{ mm}}{m} \right) = 10 \text{ mm}$$

### \*Elastic Settlement-08: The Schmertmann method in multiple strata.

(Revision: Aug-08)

Determine the elastic settlement using the Schmertmann method of the 10'x 10' footing as shown below. Estimate the elastic modulus using  $E_s = 10(N + 15)$ , where  $E_s$  is in ksf and  $N$  is the corrected SPT value.



#### Solution:

The data from these strata are placed into a table below.

Layer No.	Layer Thickness $\Delta Z$ , (feet)	Soil Modulus $E_s$ , (ksf)	$I_z = \epsilon$ (average strain)	$\frac{I_z \Delta Z}{E}$ (ft/kp)
1	5.0	360	0.35	0.00486
2	5.0	260	0.50	0.00962
3	2.5	300	0.35	0.00292
4	2.5	560	0.25	0.00112
5	5.0	190	0.10	0.00263
	$\Sigma = 2B = 20 \text{ ft}$		$\Sigma =$	0.02115

The Schmertmann coefficients are,

$$\text{The depth coefficient } C_1 = 1 - \left[ 0.5 \left( \frac{\gamma D_f}{q - \gamma D_f} \right) \right] = 1 - \left[ \frac{0.5(0.1)(5)}{1.5 - (0.1)(5)} \right] = 0.75$$

$$\text{The time coefficient } C_2 = 1.35$$

$$\Delta H_i = C_1 C_2 (q - \gamma D_f) \sum_0^{2B} \frac{\varepsilon}{E} dz = (0.88)(1.35) [2.5 - (0.1)(5)] (0.02115) = 0.042 \text{ ft} = 0.05 \text{ in}$$

**\*\*Elastic Settlement-09: Settlement of a mat foundation.**

(Revision: Aug-08)

A mat foundation located 8 feet below grade supports a ten story building upon an area of 50 ft by 150 ft, and carries a uniform load of 6 ksf. For the soil profile conditions shown below, determine the total settlement at the center and a corner of the foundation. The structure is of reinforced concrete with column spacing at 25 ft. Is the calculated differential settlement acceptable?

**Solution:**

a) Using the Schmertmann method,

Layer	$\Delta z$ (in)	$E_s$ (ksi)	$I_z = \epsilon$	$(I_z/E_s)\Delta z$ (in <sup>3</sup> /kip)
1	300	3.47	0.35	30.26
2	300	3.47	0.50	43.23
3	300	8.33	0.30	10.80
				$\Sigma = 84.29$

$$\Sigma = 84.29$$

$$\text{The depth factor } C_1 = 1.0 - 0.5 \frac{\gamma D_f}{(q - \gamma D_f)} = 1.0 - 0.5 \left[ \frac{(0.100)(8)}{6 - (0.100)(8)} \right] = 0.92$$

$$\text{The time factor (creep) } C_2 = 1.0$$

$$\Delta = C_1 C_2 (q_o - \gamma D_f) \sum_0^{2B} \frac{\epsilon}{E_s} dz = (0.92)(1.0) [6 - (0.100)(8)] (84.29) = 3.78 \text{ in}$$

b) Consolidation settlement **Method A** (take each layer at a time),Set  $e_{o1} = 1.00$ , and from  $e = 125/\gamma$ ,  $e_{o2} = 0.96$ ,  $e_{o3} = 0.89$ .

$$\text{For clay 1: } q_o = (0.100)(50) + (0.110 - 0.0624)25 + (0.125 - 0.0624)12.5 = 5.95 \text{ ksf}$$

At the mid clay stratum is at 87.5 ft below the surface,

$$(\text{ie. } 87.5/50 = 1.75 \text{ B} = 3.5 \text{ B}/2) \Delta q = 0.12q = 0.12(6) = 0.72 \text{ ksf}$$

$$\Delta_I = \frac{C_c H_I}{1 + e_o} \log_{10} \frac{q_o + \Delta q}{q_o} = \frac{0.20(25 \times 12)}{1 + 1} \log_{10} \frac{5.95 + 0.72}{5.95} = 1.48 \text{ in}$$

$$\text{At corner } \Delta q = 0.09q = 0.09(6) = 0.54$$

$$\Delta_I = \frac{0.20(25 \times 12)}{2} \log_{10} \frac{5.95 + 0.54}{5.95} = 1.13 \text{ in}$$

$$\text{For clay 2: } q_o = 5.95 + (0.125 + 0.0624)12.5 + (0.130 - 0.0624)12.5 = 7.58 \text{ ksf}$$

midlayer at 112 ft = 4.5 B/2,  $\Delta q \cdot 0.07q = 0.42$

$$\Delta_2 = \frac{0.35(25 \times 12)}{1 + 0.96} \log_{10} \frac{7.58 + 0.42}{7.58} = 1.25 \text{ in at the centerline}$$

$\Delta q \cdot 0.05q = 0.30$

$$\Delta_2 = \frac{0.35(25 \times 12)}{1 + 0.96} \log_{10} \frac{7.58 + 0.30}{7.58} = 0.96 \text{ in}$$

For clay 3:  $q_o = 7.58 + (0.130 + 0.0624)12.5 + (0.140 - 0.624)12.5 = \underline{8.65 \text{ ksf}}$

at midlayer (137.5' / 5.5 B/2)  $\Delta q = 0.04q = 0.24 \text{ ksf}$

$$\Delta_3 = \frac{0.15(25 \times 12)}{1 + 0.89} \log_{10} \frac{8.65 + 0.24}{8.65} = 0.28 \text{ in}$$

at corner  $\Delta q = 0.03q = 0.18 \text{ ksf}$

$$\Delta_3 = \frac{0.15(25 \times 12)}{1 + 0.89} \log_{10} \frac{8.65 + 0.18}{8.65} = 0.21 \text{ in}$$

**Method B:** the equivalent layer equation

$$\frac{\Delta}{H} = \frac{\Delta e}{1 + e_o} \quad \text{where} \quad \Delta e = C_c \log \frac{q_o + \Delta q}{q_o}$$

The total settlement on the centerline is,

$$\Delta_6 = 3.78 + 1.48 + 1.25 + 0.28 = 6.78 \text{ inches}$$

and along the foundation edge,

$$\Delta_{\text{edge}} = 3.78 + 1.13 + 0.96 + 0.21 = \underline{6.08 \text{ inches}}$$

The differential settlement is  $\Delta(\Delta) = \underline{0.70 \text{ inches}}$

The allowable for reinforced concrete buildings  $\Delta(\Delta) < 0.003(\text{span}) = 0.003(25 \times 12 \text{ inches})$   
 $= 0.90 \text{ inches}$

**Therefore, the design is acceptable.**

# Chapter 11

## Plastic Settlements

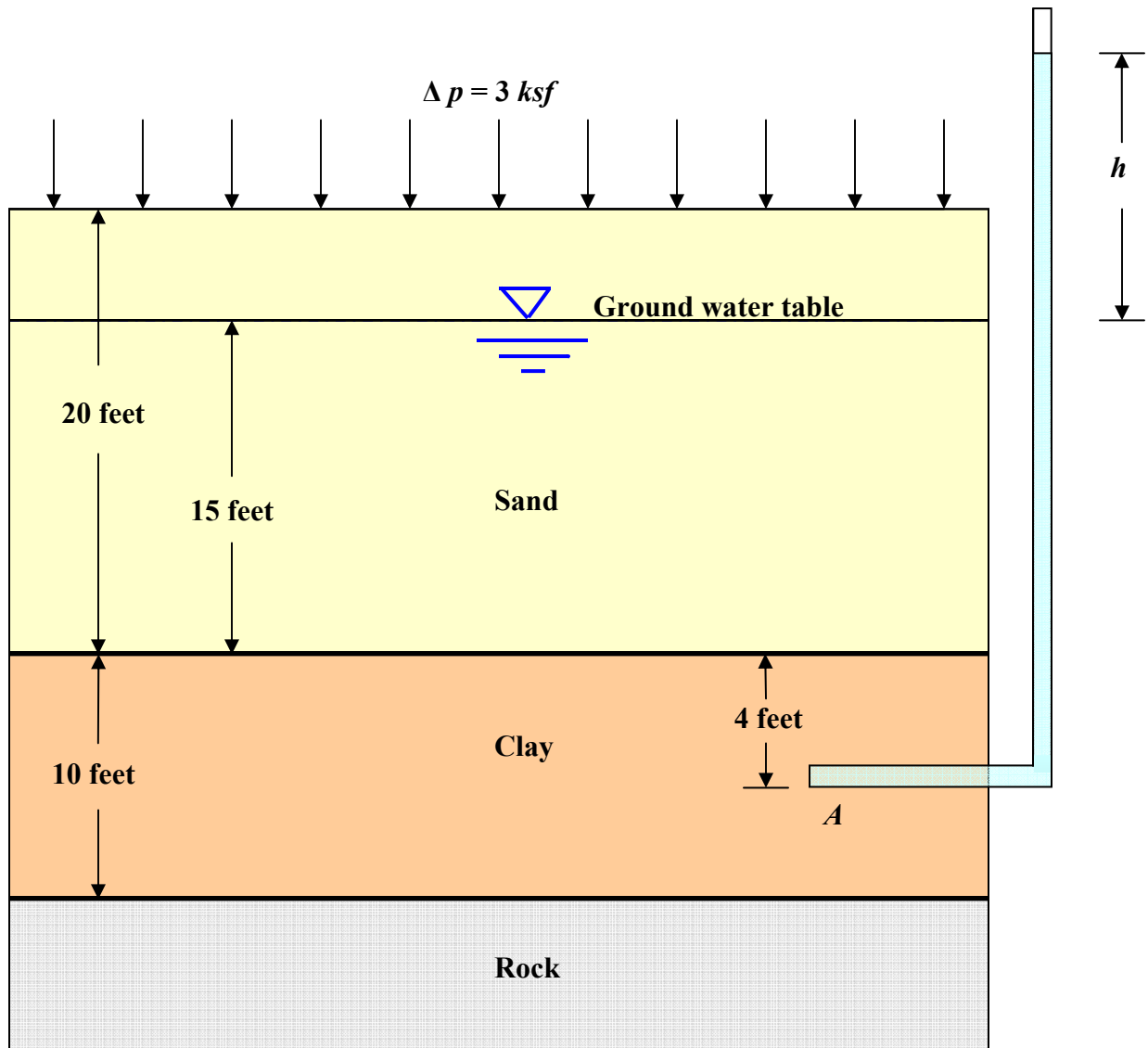
### Symbols for Plastic Settlements

- $e \rightarrow$  Voids ratio.
- $E_s \rightarrow$  Soil elastic modulus.
- $C_c \rightarrow$  Compression index.
- $G_s \rightarrow$  Specific gravity of the solids of a soil.
- $OCR \rightarrow$  Over-consolidation ratio (ratio of in-situ stress divided by the overburden stress).
- $H \rightarrow$  Depth of zone influence.
- $DH_c \rightarrow$  Plastic settlement (also called primary consolidation).
- $DH_{total} \rightarrow$  Total settlement of a structure.
- $DH_s \rightarrow$  Second consolidation settlement.
- $Dp \rightarrow$  Increasing pressure on the surface.
- $u \rightarrow$  Pore water pressure.
- $P_e \rightarrow$  Pre-consolidation pressure of a specimen.
- $\gamma_{SAT} \rightarrow$  Saturated unit weight of the soil.
- $\gamma_w \rightarrow$  Unit weight of water.
- $U \rightarrow$  Degree of consolidation.

### \*Plastic Settlement–01: Porewater pressure in a compressible soil.

(Revision: Oct.-08)

- How high will the water rise in the piezometer immediately after the application of the surface load of 3 ksf?
- What is the degree of consolidation from the 3 ksf at point A, when  $h = 15$  ft.?
- Find  $h$  when the degree of consolidation at A is 60%.



#### Solution:

- Assume a uniform increase of the initial excess pore water pressure throughout the 10-foot thickness of the clay layer:

$$\text{The pore water pressure is } u_0 = \Delta p = \gamma_w h = 3,000 \text{ lb/ft}^2 \quad \therefore h = \frac{\Delta p}{\gamma_w} = \frac{3,000}{62.4} = 48.1 \text{ feet}$$

b) The degree of consolidation at A is  $U_A$  (%) when  $h = 15$  feet:

$$U_A \% = \left(1 - \frac{u_A}{u_0}\right) 100 = \left(1 - \frac{(15 \text{ ft})(62.4 \text{ pcf})}{(48.1 \text{ ft})(62.4 \text{ pcf})}\right) 100 = 69\%$$

c) When  $U_A = 60$  %, what is the value of  $h$ ?

$$U_A = 0.6 = \left(1 - \frac{u_A}{u_0}\right) = \left(1 - \frac{u_A}{3,000 \text{ psf}}\right)$$

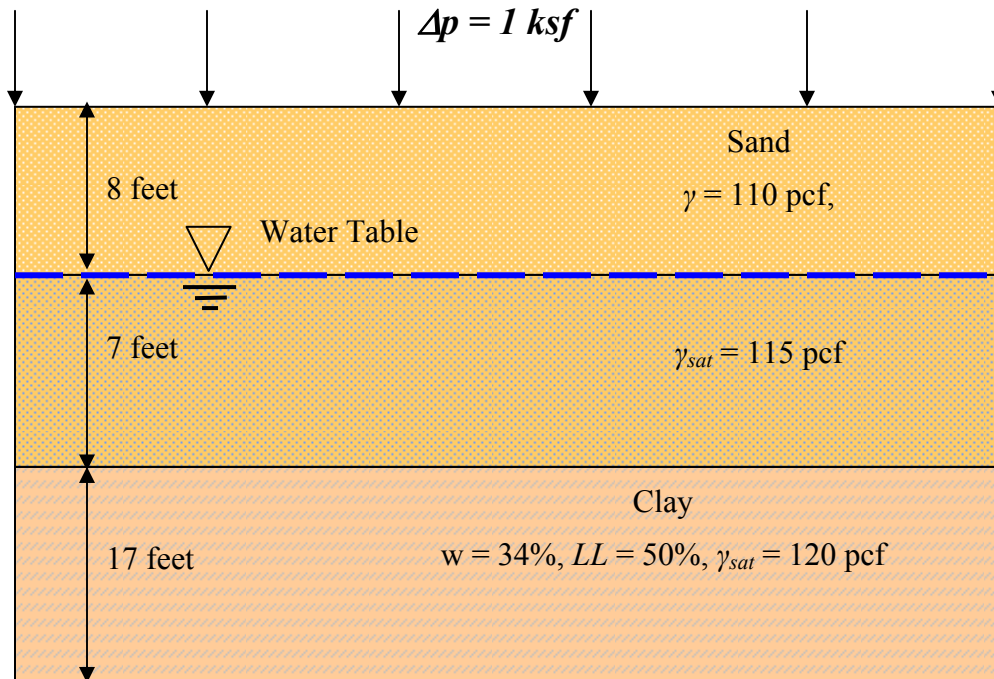
$$\therefore u_A = (1 - 0.6)(3,000 \text{ psf}) = 1,200 \text{ psf}$$

$$h = \frac{u_A}{\gamma_w} = \frac{(1,200 \text{ psf})}{(62.4 \text{ pcf})} = 19.2 \text{ feet}$$

### \*Plastic Settlement-02: Total settlement of a single layer.

(Revision: Aug-08)

A new building is planned upon the site shown below. Assume that the clay solids have a specific gravity of 2.67. Find the primary consolidation settlement if the clay is normally consolidated.



#### Solution:

Skempton formula for the  $C_c = 0.009(LL - 10) = 0.009(50 - 10) = 0.36$

$Se = wG_s$  but  $S = 1 \therefore e = wG_s = (0.34)(2.67) = 0.91$

The stress of the clay at its mid-stratum before the building was built is,

$$\begin{aligned}
 p_o &= \gamma_{d-sand} H_{dry-sand} + (\gamma_{sat} - \gamma_w) H_{sat-sand} + (\gamma_{sat} - \gamma_w) H_{mid-clay} \\
 &= (0.11 kcf)(8 ft) + (0.115 - 0.0624) kcf(7 ft) + (0.120 - 0.0624) kcf(8.5 ft) \\
 p_o &= 1.74 ksf
 \end{aligned}$$

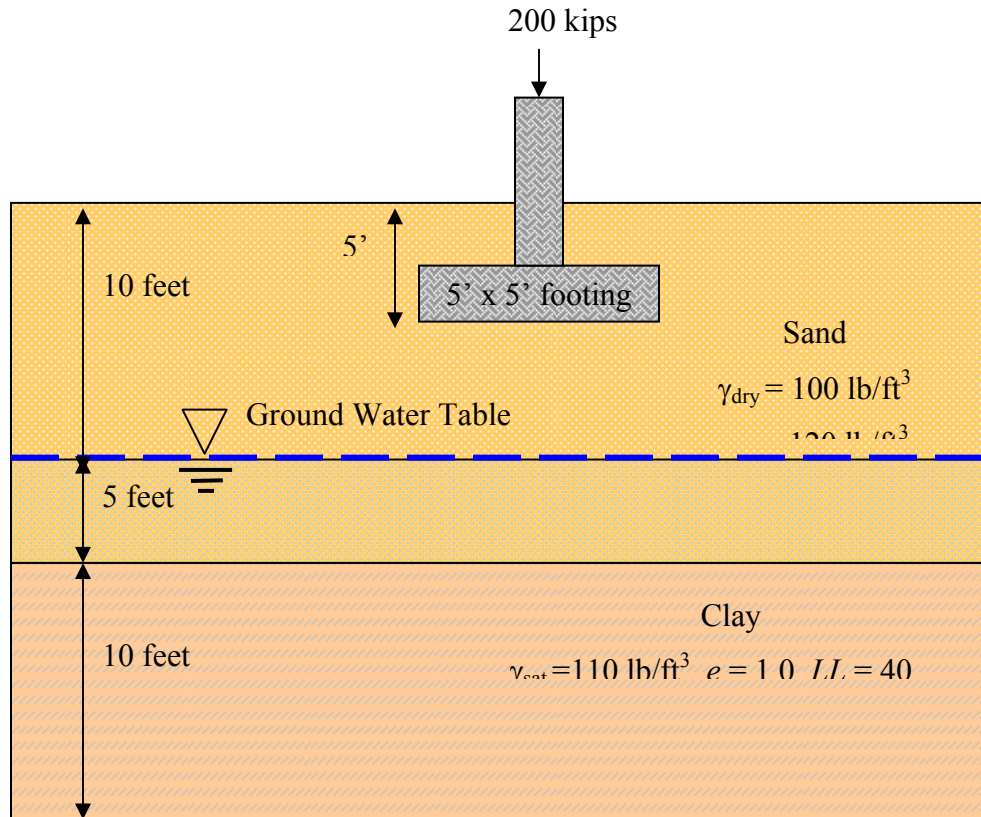
The consolidation settlement is,

$$\Delta H = \frac{HC_c}{1 + e_o} \log_{10} \frac{p_o + \Delta p}{p_o} = \frac{(17)(12)(0.36)}{1 + 0.91} \log_{10} \frac{1.74 + 1}{1.74} = 7.6 \text{ inches}$$

### \*Plastic Settlement-03: Boussinesq to reduce the stress with depth.

(Revision: Aug-08)

Calculate the settlement of the 10-foot thick clay layer shown below that will result from the column's load carried by a 5-foot square footing. The clay is normally consolidated. Apply Boussinesq's formula to find the reduction of the vertical stress with depth.



#### Solution:

$$\text{Skempton formula } C_c = 0.009(LL - 10) = 0.009(40 - 10) = 0.27$$

The stress of the clay at its mid-stratum before the building was built is,

$$p_o = \gamma_{d-sand} H_{dry-sand} + (\gamma_{sat} - \gamma_w) H_{sat-sand} + (\gamma_{sat} - \gamma_w) H_{mid-clay}$$

$$p_o = (0.10 \text{ kcf})(10 \text{ ft}) + (0.120 - 0.0624) \text{ kcf}(5 \text{ ft}) + (0.110 - 0.0624) \text{ kcf}(5 \text{ ft})$$

$$p_o = 1.53 \text{ ksf}$$

The  $\Delta p_{avg}$  below the center of the footing between  $z_1 = 15 \text{ feet}$  to  $z_2 = 25 \text{ feet}$  is given as,

$$\Delta p_{avg} = 4q \left[ \frac{H_2 I_{4(H_2)} - H_1 I_{4(H_1)}}{H_2 - H_1} \right] \quad \text{where} \quad I_{4(H_2)} = f \left[ m' = \frac{B}{H_2}, n' = \frac{L}{H_2} \right]$$

$$B = L = \frac{5}{2} = 2.5 \text{ feet}$$

$$m' = \frac{B}{H_1} = \frac{2.5}{15} = 0.167 \quad \text{and} \quad n' = \frac{L}{H_1} = \frac{2.5}{15} = 0.167$$

From Boussinesq's Table for  $I_4$

$$I_{4(H_2)} = 0.05 \quad \text{and} \quad I_{4(H_1)} = 0.075$$

The average pressure at mid-clay layer is thus given by,

$$\Delta p_{avg} = 4q \left[ \frac{H_2 I_{4(H_2)} - H_1 I_{4(H_1)}}{H_2 - H_1} \right] = 4 \left( \frac{200 \text{ kips}}{(5')(5')} \right) \left[ \frac{(25)(0.05) - (15)(0.075)}{25' - 15'} \right] = 0.4 \text{ ksf} = 400 \text{ psf}$$

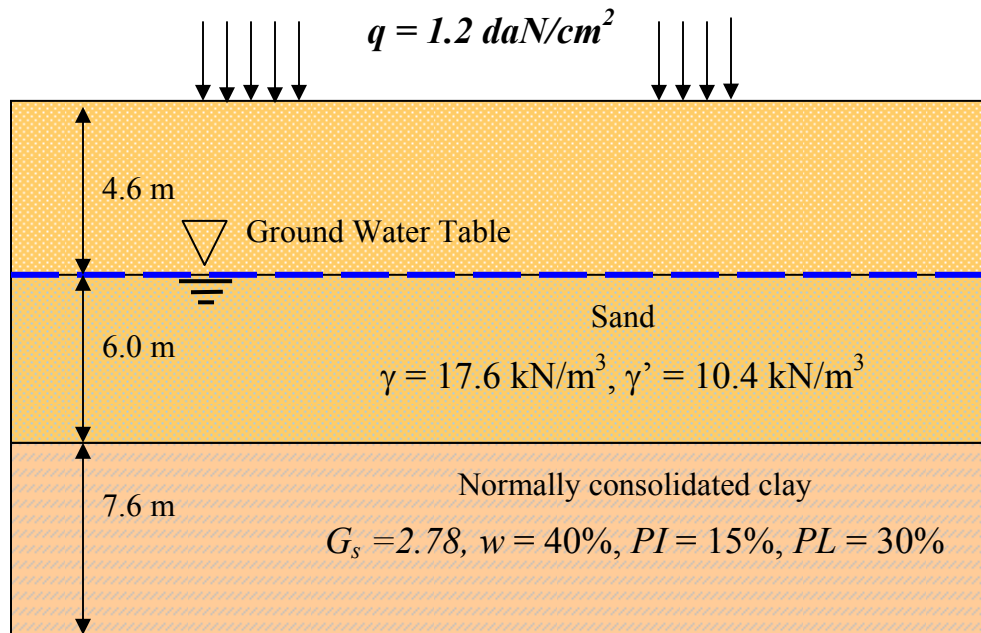
The consolidation settlement is,

$$\Delta H = \frac{HC_c}{1 + e_o} \log_{10} \frac{p_o + \Delta p_{avg}}{p_o} = \frac{(10)(12)(0.27)}{1 + 1} \log_{10} \frac{1.53 + 0.40}{1.53} = 1.6 \text{ inches}$$

### \*Plastic Settlement -04: Surface loads with different units.

(Revision: Aug-08)

Find the total settlement under a building that applies the load shown below.



#### Solution:

Notice that the data provided does not include a unit weight for the clay stratum. Therefore, this value must be determined through the other information provided.

$$\gamma_s = \gamma_w G_s = 27.8 \text{ kN/m}^3 \quad Se = (1)e = wG_s = 0.40(2.78) = 1.11$$

$$\text{Therefore, the clay unit weight is } \gamma_{sat} = \frac{(G_s + e_o)\gamma_w}{1 + e_o} = \frac{[(2.78) + (1.11)](9.81)}{1 + (1.11)} = 18.1 \text{ kN/m}^3$$

The effective unit weight for the clay is,

$$\gamma' = \gamma_{sat} - \gamma_w = 18.1 - 9.81 = 8.3 \text{ kN/m}^3$$

Also, the Skempton relation is,  $C_c = 0.009(w_L - 10) = 0.009(45 - 10) = 0.32$

The stress at the mid-clay stratum,

$$\sigma' = (4.6\text{m})(17.6 \text{ kN/m}^3) + (6.0\text{m})(10.4 \text{ kN/m}^3) + (7.6\text{m}/2)(8.3 \text{ kN/m}^3) = 175 \text{ kN/m}^2$$

The consolidation settlement is,

$$\Delta H = \frac{HC_c}{1 + e_o} \log_{10} \frac{\sigma' + \Delta\sigma}{\sigma'_o} = \frac{(7.6\text{m})(0.32)}{1 + 1.11} \log_{10} \frac{175 + 120}{175} = 0.26 \text{ m} = 260 \text{ mm}$$

### \*Plastic Settlement-05: Pre-consolidation pressure $p_c$ and index $C_c$ .

(Revision: Aug-08)

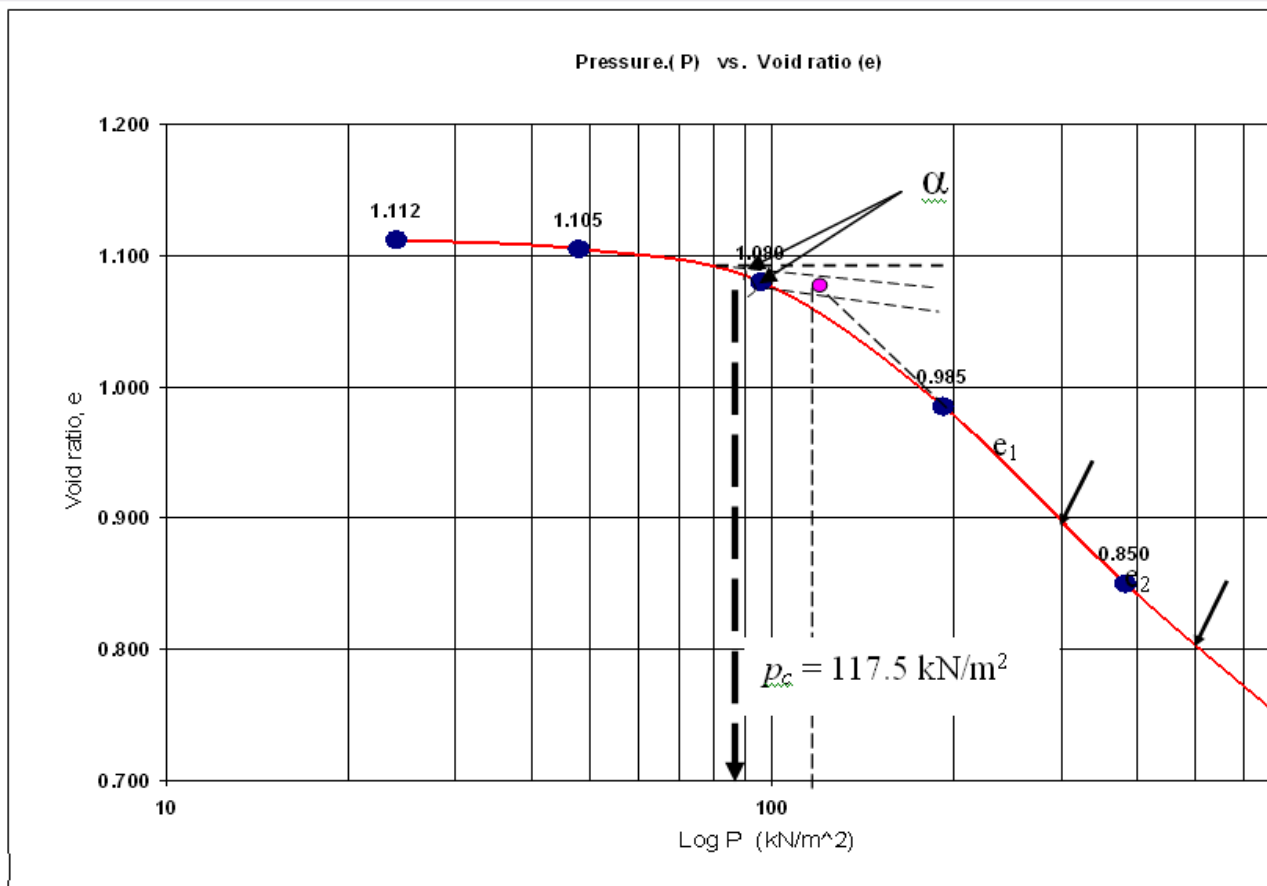
The results of a laboratory consolidation test on a clay sample are given below:

Pressure, $p$ ( $\text{kN/m}^2$ )	Void ratio, $e$
--------------------------------------	-----------------

23.94	1.112
47.88	1.105
95.76	1.080
191.52	0.985
383.04	0.850
766.08	0.731

- (43) a) Draw an  $e$ -log  $p$  plot.  
 (44) b) Determine the pre-consolidation pressure,  $p_c$ .  
 (45) c) Find the compression index,  $C_c$ .

Solution:



b) Determine the pre-consolidation pressure,  $p_c$ . From the e-log p plot,

$$p_2 = 500 \text{ kN/m}^2 \text{ and } e_2 = 0.8 \quad p_1 = 300 \text{ kN/m}^2 \text{ and } e_1 = 0.9 \quad \therefore p_c = 117.5 \text{ kN/m}^2$$

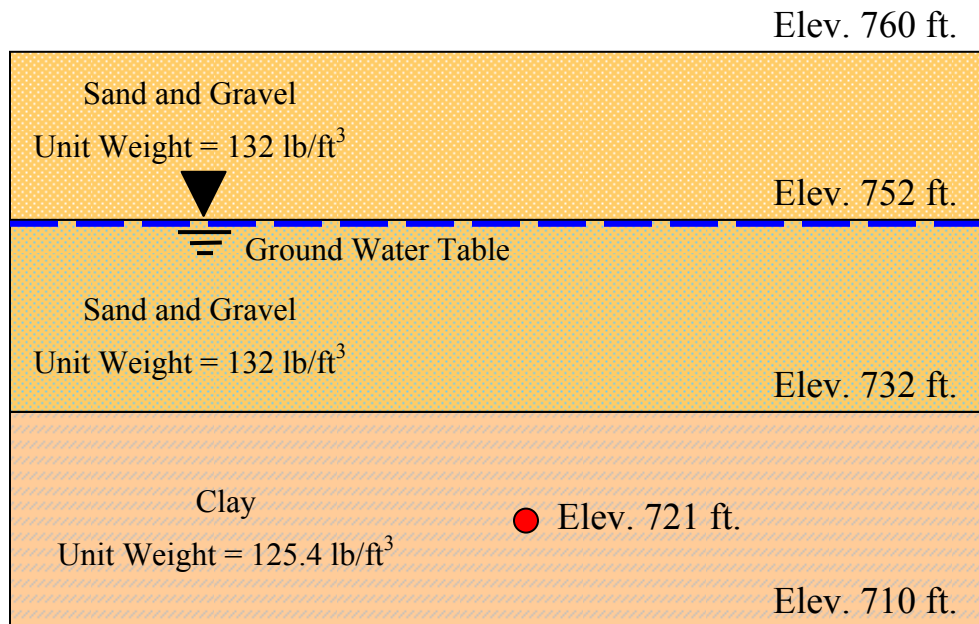
c) Find the compression index,  $C_c$ . From the slope of the graph,

$$C_c = \frac{e_1 - e_2}{\log\left(\frac{p_2}{p_1}\right)} = \frac{0.9 - 0.8}{\log\left(\frac{500}{300}\right)} = 0.451$$

**\*Plastic Settlement-06: Final voids ratio after consolidation.**

(Revision: Aug-08)

The clay stratum shown in the profile below has a total vertical stress of 200 kN/m<sup>2</sup> at its mid-height with a voids ratio of 0.98. When the vertical stress increases to 500 kN/m<sup>2</sup> the voids ratio decreases to 0.81. Find (a) the effective overburden pressure at mid-height of the compressible clay layer, and (b) the voids ratio of the clay if the total pressure at its mid-height is 1000 kN/m<sup>2</sup>.



**Solution:**

$$a) p_o = [\gamma h]_{\text{dry sand-gravel}} + [\gamma' h]_{\text{saturated sand-gravel}} + [\gamma' h]_{\text{mid-clay stratum}}$$

$$p_o = (132 \text{ lb/ft}^3)(8 \text{ ft}) + (132 - 62.4) \text{ lb/ft}^3 (20 \text{ ft}) + (125.4 - 62.4) \text{ lb/ft}^3 (11 \text{ ft})$$

$$p_o = 3.14 \text{ ksf}$$

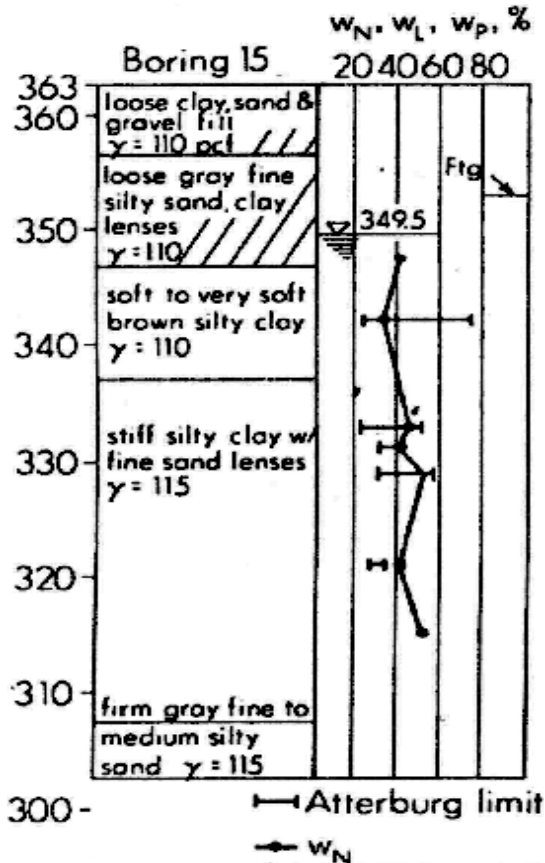
$$b) C_c = \frac{e_1 - e_2}{\log(p_2/p_1)} = \frac{0.98 - 0.81}{\log(500/200)} = 0.427$$

$$0.427 = \frac{0.98 - e_2}{\log(1000/200)} \quad \therefore e_2 = 0.68$$

### \*Plastic Settlement-07: Settlement due to a lowered WT.

(Revision: Aug-08)

Find the settlement due to lowering of the phreatic surface from elevation 349.5' to 344.0' using the boring report shown below.



#### Solution:

$$\bar{p}_o = (0.110 \text{ pcf})(7 \text{ ft}) + (0.110 \text{ pcf})(6 \text{ ft}) + (0.110 - 0.0624 \text{ pcf})(3 \text{ ft}) + (0.110 - 0.0624 \text{ pcf})(3 \text{ ft})$$

$$\bar{p}_o = 1.71 \text{ ksf}$$

$$\Delta p = \frac{\Delta q}{H} + \Delta p \text{ due to the lowering WT}$$

$$\Delta q_2 = \Delta p - \Delta q_1 = 1.853 - 1.16 = 0.693$$

$$\Delta H_1 = \frac{C_c H}{1 + e_0} \log \frac{p_o + \Delta q_1}{p_o}$$

$$\Delta H_1 = \frac{0.030(10)}{1 + 0.96} \log \frac{3 + 1.16}{3} = 0.032 \text{ ft}$$

$$\Delta H_2 = \frac{0.034(10)}{1 + 0.96} \log \frac{3 + 0.693}{3} = 0.157 \text{ ft}$$

versus 0.083 ft the additional settlement.

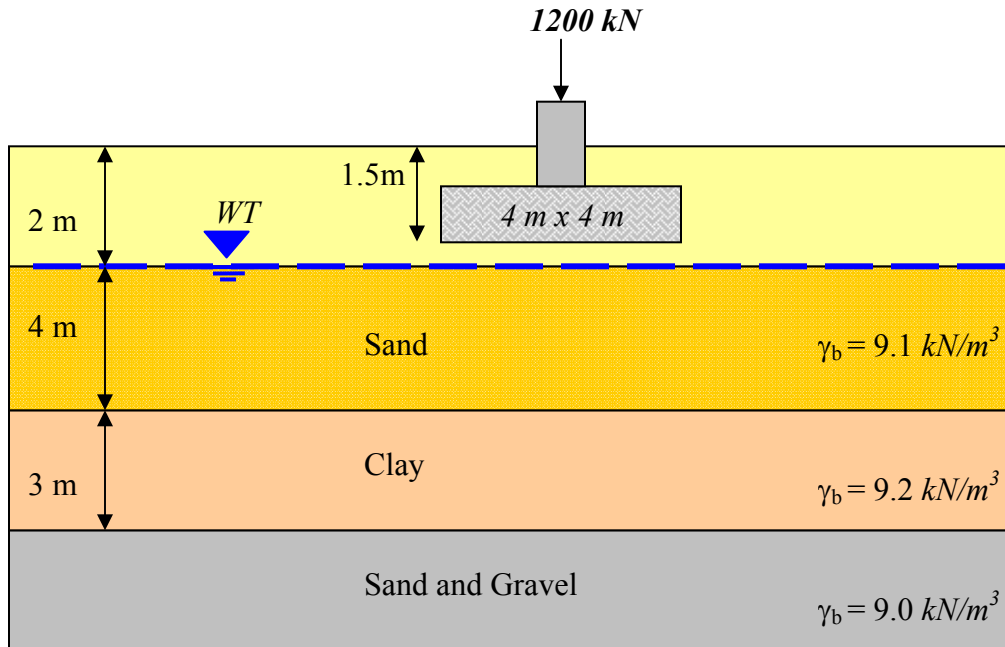
The rising WT may reduce settlement.

**\*Plastic Settlement-08: The over-consolidation ratio (OCR).**

(Revision: Sept-08)

Oedometer (consolidation) tests of several samples from the clay stratum yields the consolidation curve shown below. Given that  $G_s = 2.65$ , find (a) the value of  $p_o$ , (b) The value of  $p_c$  and (c) the OCR of the clay.

**Solution:**



a) In order to find the in-situ stress  $p_o$  before the footing was built, we need to find the unit weight of the sand stratum,

$$\gamma_d = \frac{\gamma_b G_s}{G_s - 1} = \frac{(9.1 \text{ kN/m}^3)(2.65)}{1.65} = 14.6 \text{ kN/m}^3$$

The stress  $p_o$  is found at mid-clay stratum,

$$p_o = [h(\gamma_d)]_{\text{dry(sand)}} + [h(\gamma')]_{\text{sand}} + [h(\gamma')]_{\text{clay}}$$

$$p_o = [(2\text{m})(14.6 \text{ kN/m}^3)] + [(4\text{m})(9.1 \text{ kN/m}^3)] + [(1.5\text{m})(9.2 \text{ kN/m}^3)] = 79.4 \text{ kPa}$$

### **\*\*Plastic Settlement-09: Coefficient of consolidation $C_v$ .**

(Revision: Aug-08)

An odometer test was performed on a peat soil sample from an FDOT project in the Homestead area. The results are shown below. The initial sample thickness is 20 mm, with two-way drainage through porous stones, simulating field conditions. The vertical stress increment is 10 kPa. Estimate of the coefficient of consolidation  $c_v$  as,

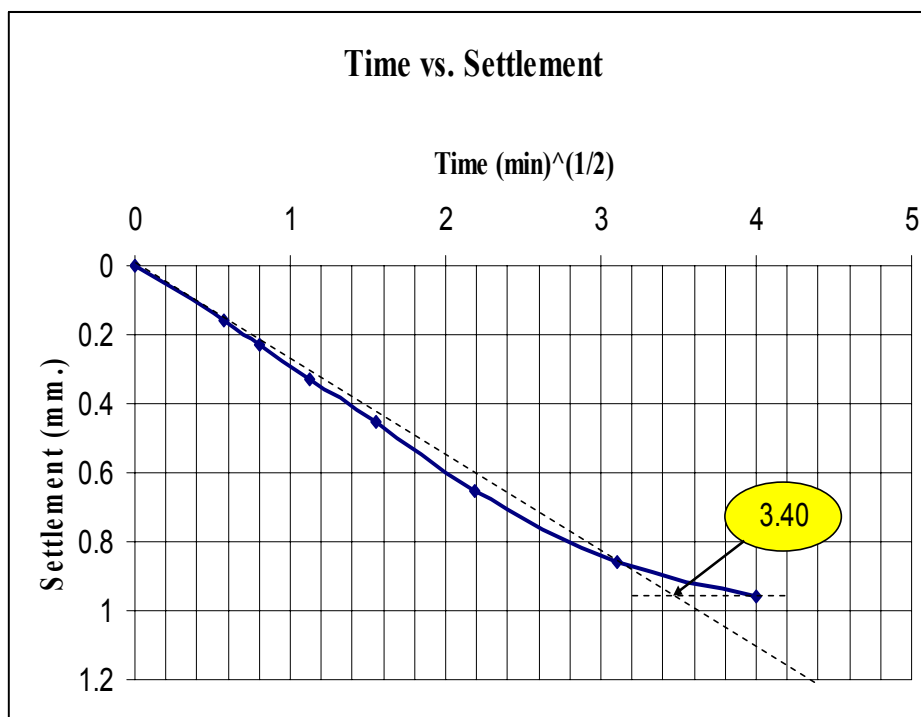
$$\text{Coefficient of consolidation}(C_v) = \frac{3H_{dr}^2}{4t}$$

Time (minutes)	0	0.32	0.64	1.28	2.40	4.80	9.60	16.0
Settlement (mm.)	0	0.16	0.23	0.33	0.45	0.65	0.86	0.96
$\sqrt{\text{time}} = (\text{min})^{(1/2)}$	0	0.57	0.8	1.13	1.55	2.19	3.10	4.0

- (1) Plot a graph of settlement against the square root of time.
- (2) Determine the value of the coefficient of consolidation  $c_v$  (in  $\text{m}^2/\text{s}$ ).
- (3) Estimate what could be a good estimate of the elastic modulus of this soil  $E_o$  (kPa)
- (4) What sort of permeability  $k$  (in  $\text{m/s}$ ) could you estimate for this soil (from  $E_o$ )?

#### **Solution:**

(1)



(2)

$$vt_x = 3.40 \text{ min}^{1/2} \quad \text{and} \quad \text{time}(t_x) = 11.56 \text{ min}$$

$$c_v = \frac{3d^2}{4t_x} = \frac{3(10\text{mm})^2 (1\text{min})(1\text{m})^2}{4(11.56 \text{ min})(60\text{sec})(1000\text{mm})^2} = 1.08 \times 10^{-7} \frac{\text{m}^2}{\text{sec}}$$

(3)

$$\text{Voids - ratio } (e_v) = \frac{1 \text{ mm}}{20 \text{ mm}} = 0.05 \quad \therefore e_v = 5\%$$

$$E_o = \frac{\sigma_v}{e_v} = \frac{10 \times 10^3 \text{ N}}{0.05 \text{ m}^2} = 200 \times 10^3 \text{ Pa} = 200 \text{ kPa}$$

(4)

$$e \approx \frac{C_c}{1+e_o} \quad \therefore c_v = \left[ \frac{2.30\sigma_z' k}{\gamma_w} \right] \left[ \frac{1+e}{C_c} \right] \quad \therefore c_v = \left( \frac{\sigma_z}{e_z} \right) \left( \frac{k}{\gamma_w} \right) = E_o \left( \frac{k}{\gamma_w} \right)$$

$$k = \frac{c_v \gamma_w}{E_o} = \frac{(1.08 \times 10^{-7} \text{ m}^2/\text{s})(9.81 \text{ kN}/\text{m}^3)}{(200 \text{ kN}/\text{m}^2)} = 5.3 \times 10^{-9} \frac{\text{m}}{\text{sec}}$$

### **\*Plastic Settlement -10: Secondary rate of consolidation.**

(Revision: Aug-08)

An oedometer (consolidation) test is performed on a normally consolidated clay stratum that is 8.5 feet thick, and it found that the clay's initial voids ratio was  $e_o = 0.8$  and its primary compression index is  $C_c = 0.28$ . The in-situ stress at mid-clay layer is  $p_o = 2,650$  psf, and the building exerts a pressure through its mat foundation of 970 psf. The secondary compression index  $C_\alpha = 0.02$ .

The time of completion of the primary settlement is approximately 18 months. What is the total consolidation of the 8.5 foot clay stratum 5 years after the primary consolidation?

#### **Solution:**

The primary consolidation  $\Delta H_p$  is,

$$\Delta H_p = \frac{C_c H}{1 + e_o} \log \left( \frac{p'_o + \Delta p'}{p'_o} \right) = \frac{(0.28)(8.5 \text{ ft})(12 \text{ in / ft})}{1 + (0.8)} \log \left( \frac{2,650 + 970}{2,650} \right) = 2.15 \text{ inches}$$

The secondary consolidation  $\Delta H_s$  is,

$$\Delta H_s = \frac{C_\alpha H}{1 + e_p} \log \left( \frac{t_2}{t_1} \right)$$

We must find  $e_p$  first, by finding the change in the voids ratio during primary consolidation,

$$e_p = e_o - \Delta e = e_o - C_c \log \left( \frac{p'_o + \Delta p'}{p'_o} \right) = 0.8 - (0.28) \log \left( \frac{2,650 + 970}{2,650} \right) = 0.76$$

$$\therefore \Delta H_s = \frac{C_\alpha H}{1 + e_p} \log \left( \frac{t_2}{t_1} \right) = \frac{(0.02)(8.5 - 0.18)(12 \text{ in / ft})}{1 + (0.76)} \log \left( \frac{5}{1.5} \right) = 0.59 \text{ inches}$$

The total consolidation settlement is thus  $= \Delta H_p + \Delta H_s = 2.15 + 0.59 = 2.74 \text{ inches}$

### **\*Plastic Settlement-11: Using the Time factor $T_v$ .**

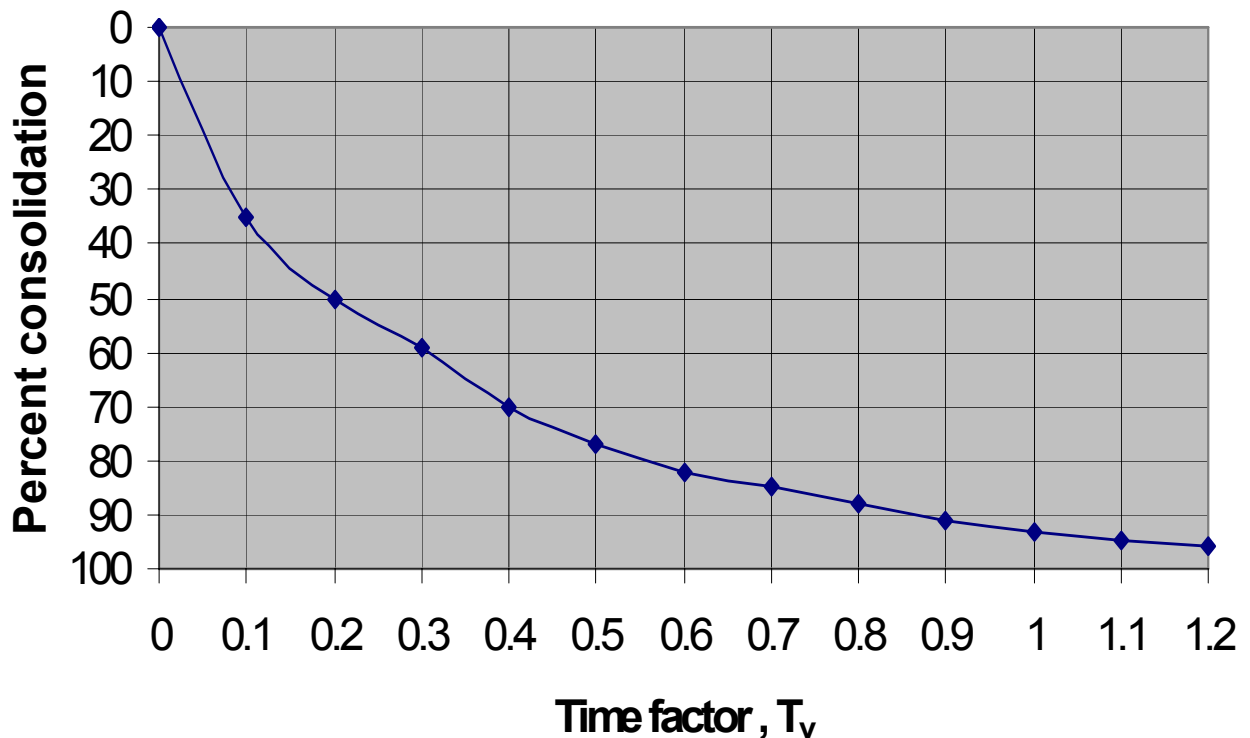
(Revision: Aug-08)

A 3-m thick, doubly-drained saturated stratum of clay is under a surcharge loading that underwent 90% primary consolidation in 75 days. Find the coefficient of consolidation  $c_v$  of this clay in  $\text{cm}^2/\text{sec}$ .

#### **Solution:**

The clay layer has two-way drainage, and  $T_v = 0.848$  for 90% consolidation.

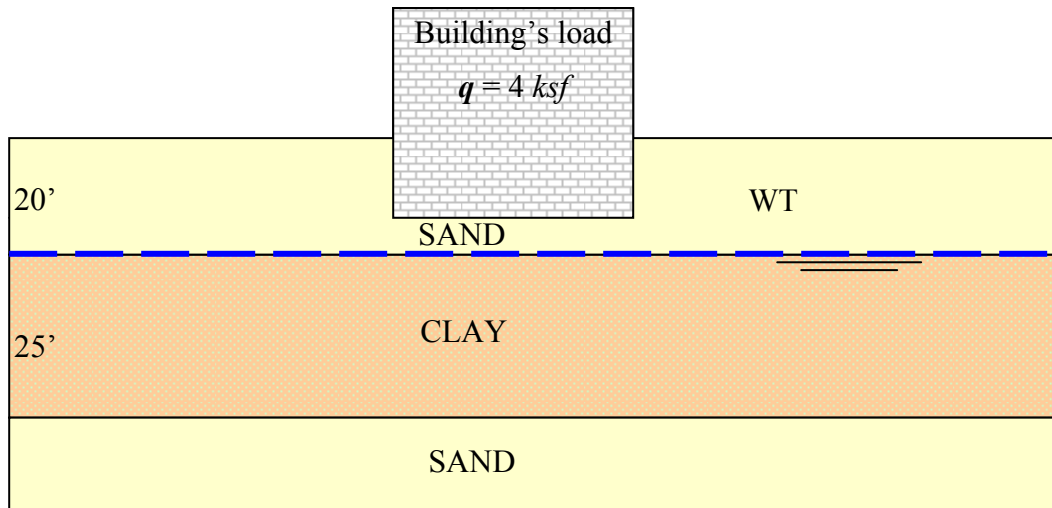
$$c_v = \frac{T_v H_{dr}^2}{t} = \frac{(0.848)(150 \text{ cm})^2}{(75 \text{ days} \times 24 \times 60 \times 60)} = \mathbf{0.00294 \text{ cm}^2/\text{sec}}$$



### \*Plastic Settlement-12: The time rate of consolidation.

(Revision: Aug-08)

An oedometer (consolidation) test is performed on a 4" thick specimen, drained on top and bottom. It was observed that 45 percent consolidation ( $T_v = 0.15$ ) was attained in 78 hours. Determine the time required to attain 70 percent consolidation ( $T_v = 0.40$ ) in a job site where the clay stratum is shown below.



#### Solution:

The coefficient of consolidation  $c_v = \frac{T_v H_{dr}^2}{t}$  is the same for the lab and field samples.

$$c_v = \frac{T_v H_{dr}^2}{t} = \left( \frac{T_v H_{dr}^2}{t} \right)_{\text{laboratory}} = \left( \frac{T_v H_{dr}^2}{t} \right)_{\text{field}}$$

$$\therefore \frac{(0.15)(2 \text{ in})^2}{(78 \text{ hours})} = \frac{(0.40)(12.5 \text{ ft} \times 12 \text{ in / ft})^2}{t_{\text{field}}}$$

$$\therefore t_{\text{field}} = (78 \text{ hours}) \left( \frac{1 \text{ day}}{24 \text{ hours}} \right) \left( \frac{1 \text{ year}}{365 \text{ days}} \right) \left( \frac{0.40}{0.15} \right) \left( \frac{(12.5 \text{ ft} \times 12 \text{ in / ft})^2}{(2 \text{ in})^2} \right)$$

$$\therefore t_{\text{field}} = 134 \text{ years to attain 70\% consolidation}$$

### \*Plastic Settlement-13: Time of consolidation t.

(Revision: Oct.-08)

Using the information derived from Problem 11, how long will it take a 30-mm thick undisturbed clay sample obtained from the field to undergo 90% consolidation in the laboratory?

#### Solution:

The Time Factor  $T_v$  is the same 90% in the field as in the laboratory, therefore,

$$T_{90} = \frac{c_v t_{90(\text{field})}}{H_{dr(\text{field})}^2} = \left[ \frac{c_v (75 \text{ days} \times 24 \times 60 \times 60)}{\left(\frac{3,000 \text{ mm}}{2}\right)^2} \right]_{\text{field}} = \left[ \frac{c_v t_{90}}{\left(\frac{30 \text{ mm}}{2}\right)^2} \right]_{\text{laboratory}}$$

$$\therefore t_{90(\text{lab})} = \frac{(75 \text{ days} \times 24 \times 60 \times 60)(15 \text{ mm})^2}{(1,500 \text{ mm})^2} = 648 \text{ seconds} = 10 \text{ minutes}$$

### **\*Plastic Settlement-14: Laboratory versus field time rates of settlement.**

(Revision: Aug-08)

Laboratory tests on a 25mm thick clay specimen drained at both the top and bottom show that 50% consolidation takes place in 8.5 minutes.

- (1) How long will it take for a similar clay layer in the field, 3.2 m thick, but drained at the top only, to undergo 50% consolidation?
- (2) Find the time required for the clay layer in the field as described in part (a) above, to reach a 65% consolidation.

#### **Solution:**

(1)

$$\frac{t_{lab}}{H_{dr(lab)}^2} = \frac{t_{(field)}}{H_{dr(field)}^2}$$
$$\therefore t_{(field)} = \frac{(3.2m)^2 (8.5 \text{ min})}{\left(\frac{25mm}{2 \times 1000mm}\right)^2} = 557,000 \text{ min} = 387 \text{ days}$$

(2)

$$c_v = \frac{T_v H_{dr}^2}{t_{50}} = \frac{(0.197) \left(\frac{25}{2 \times 1000}\right)^2}{(8.5 \text{ min})} = 0.36 \times 10^{-5}$$
$$\therefore t_{(field 65)} = \frac{T_{65} H_{dr}^2}{c_v} = \frac{(0.34)(3.2m)^2}{0.36 \times 10^{-5} m^2 / \text{min}} = 961,400 \text{ min} = 668 \text{ days}$$

### \*Plastic Settlement-15: Different degrees of consolidation.

(Revision: Aug-08)

A clay layer 20 feet thick sitting on top of granite bedrock, experiences a primary consolidation of 8.9 inches. Find:

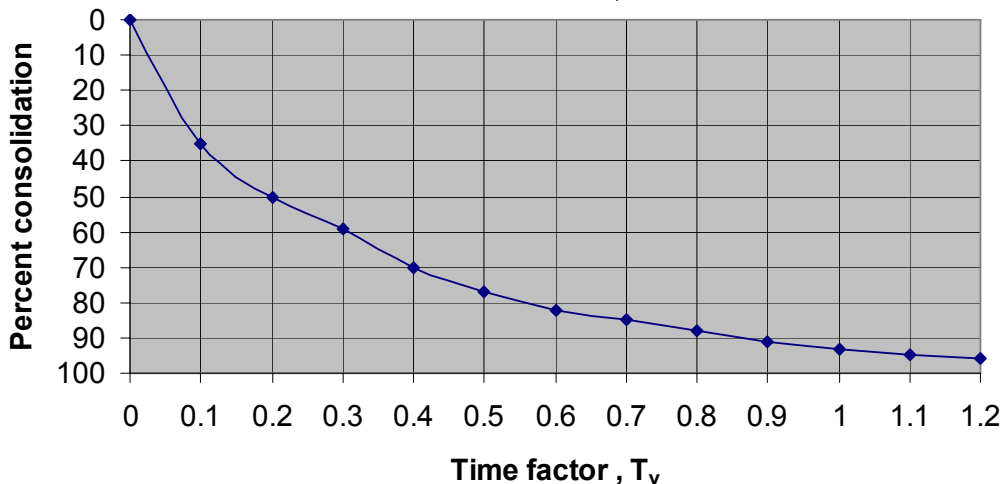
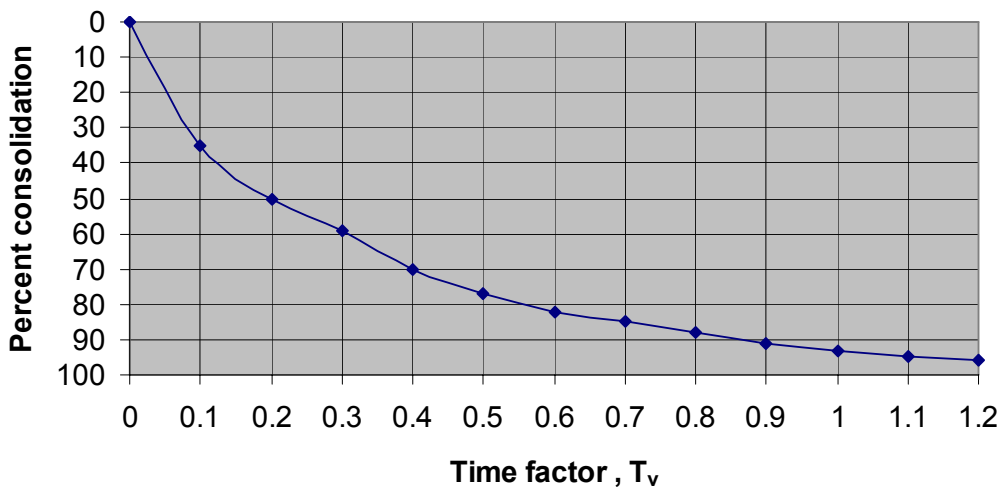
- The degree of consolidation when the settlement reaches 2 inches.
- The time to reach 50% settlement if  $c_v$  is  $0.002 \text{ cm}^2/\text{sec}$ .
- The time for 50% consolidation if the clay stratum is doubly-drained?

**Solution:**

$$(a) \quad U\% = \left( \frac{2}{8.9} \right) 100 = 22.5\%$$

$$(b) \quad T_{50} = 0.197 \quad \text{and} \quad t = \frac{T_v}{c_v} H_{dr}^2 = \frac{(0.197) [(20 \text{ ft})(30.48 \text{ cm / ft})]^2}{(0.002 \text{ cm}^2 / \text{sec}) \left( \frac{1 \text{ hr}}{3600 \text{ sec}} \right) \left( \frac{\text{days}}{24 \text{ h}} \right)} = 424 \text{ days}$$

$$(c) \quad t = \frac{T_v}{c_v} H_{dr}^2 = \frac{(0.197) [(10 \text{ ft})(30.48 \text{ cm / ft})]^2}{(0.002 \text{ cm}^2 / \text{sec}) \left( \frac{1 \text{ hr}}{3600 \text{ sec}} \right) \left( \frac{\text{days}}{24 \text{ h}} \right)} = 106 \text{ days}$$

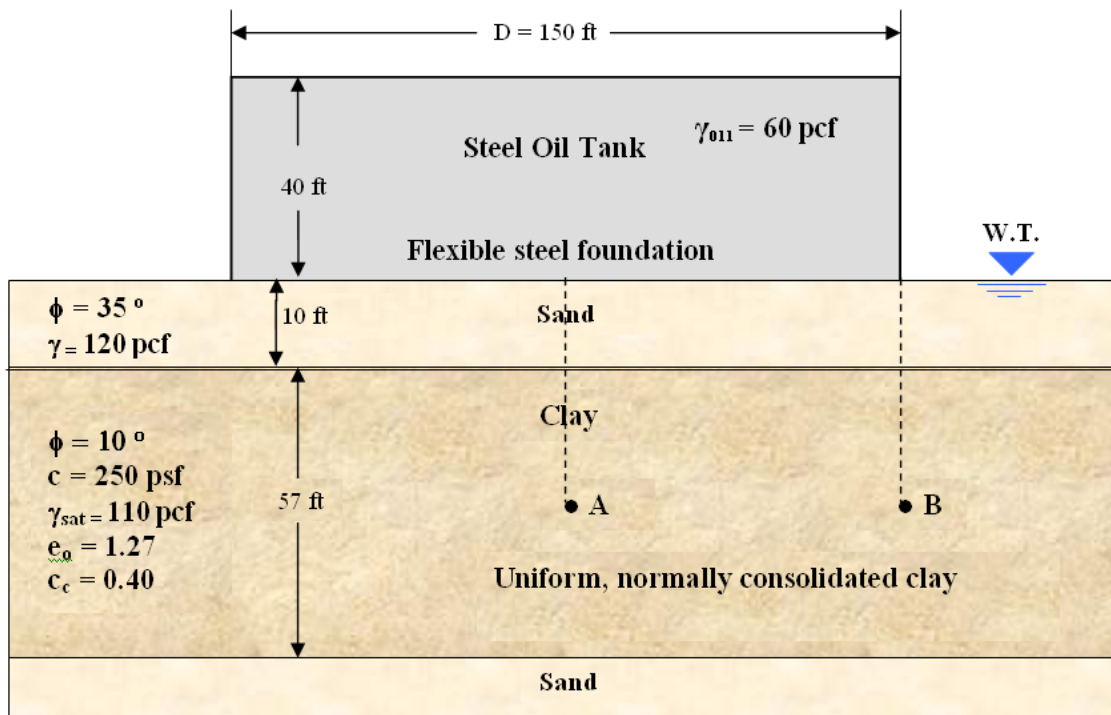


## **\*\*Plastic Settlement-16: Excavate to reduce the settlement.**

(Revision: Oct.-08)

An oedometer test on a 1" thick, doubly-drained sample from the clay stratum (shown below) attained 50% consolidation in 6.5 minutes. Find:

- The total differential settlement of the fully loaded tank.
- The time required for 75% consolidation in the field.
- The depth of excavation for minimal settlement.



### **Solution:**

- The surface load from the oil tank (neglecting the weight of the tank) is  $\Delta p$ :

$$\Delta p = h\gamma_{oil} = (40 \text{ ft})(60 \text{ pcf}) = 2.4 \text{ ksf}$$

Using the Boussinesq pressure diagram (next two pages) for  $B = 75'$ , provides the stress levels at any point in the soil mass, thus,

$$\text{The stress at point A} \Rightarrow 0.91\Delta p = 0.90(2.4) = 2.2 \text{ ksf}$$

$$\text{point B} \Rightarrow 0.43\Delta p = 0.43(2.4) = 1.0 \text{ ksf}$$

The in-situ effective stress at point A was  $p_o$ , before the tank was built:

$$p_o = (10')(0.120 - 0.062) + (28.5')(0.110 - 0.062) = 1.95 \text{ ksf}$$

The settlements at point A (below the center of the tank), and point B (at the edge of the tank) are,

$$\Delta H_A = \frac{C_c H}{1 + e_o} \log \left( \frac{p_o' + \Delta p}{p_o'} \right) = \frac{0.4(57')}{(1 + 1.27)} \log \left( \frac{1.95 + 2.2}{1.95} \right) = 3.34 \text{ ft}$$

$$\Delta H_B = \frac{C_c H}{1 + e_o} \log \left( \frac{p_o' + \Delta p}{p_o'} \right) = \frac{0.4(57')}{(1 + 1.27)} \log \left( \frac{1.95 + 1.0}{1.95} \right) = 1.85 \text{ ft}$$

$\therefore$  The differential settlement between A and B is  $= 3.34 \text{ ft} - 1.85 \text{ ft} = 1.49 \text{ ft} = 18 \text{ inches}$

b) Since the time required for consolidation is,

$T_v$  for 50% = 0.197, and

$T_v$  for 75% = 0.480, using the relationship between field and lab conditions, through the coefficient of consolidation,

$$\frac{T_{v_{field}} H_{field}^2}{t_{75}} = \frac{T_{v_{lab}} H_{lab}^2}{t_{lab}} \quad \therefore \quad c_v = \frac{(0.2) \left[ 0.5 \ln \left( \frac{ft}{12in} \right) \right]^2}{6.5 \text{ min}} = \frac{(0.480)(28.5 \text{ ft})^2}{t_{75}}$$

$\therefore t_{75} = 13.8 \text{ years for 75\% consolidation.}$

c) The settlement can be reduced by excavating weight of soil equal to the weight of the structure (note: total, not effective, since the water is also removed). Assume need to excavate “x” feet from the clay layer.

$$\sigma_{total} = (10')(0.120) + x(0.110) = 2.4 \text{ ksf}$$

Solving for x:  $\therefore x = 10.9 \text{ ft} \cong 11 \text{ feet of clay}$

Add the 11 feet of the clay to the 10 feet of the sand on top of the clay for a

$\therefore$  Total excavation depth  $= 11 \text{ ft} + 10 \text{ ft} = 21 \text{ feet}$

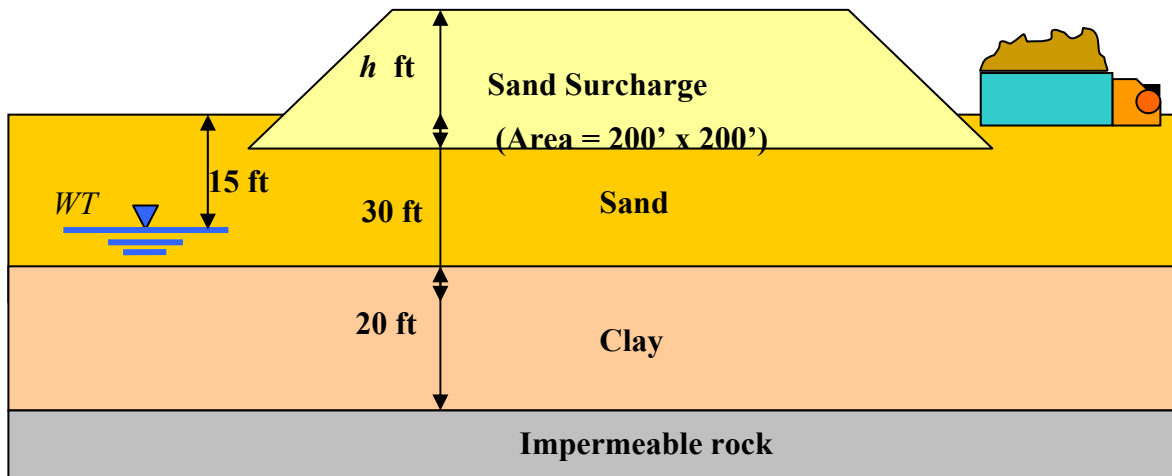
### **\*\*Plastic Settlement-17: Lead time required for consolidation of surcharge.**

(Revision: Aug-08)

A common method used to accelerate the consolidation of a clay stratum is a sand surcharge, as shown below. The surcharge load will force the clay to attain a large part of its settlement before the structure with the same load is built. This method minimizes the settlement of the structure.

An office building is planned to be built on the site shown below. The total weight of the building is 136,000 kips, spread over a square foundation 200 ft by 200 ft. Field tests showed that the clay stratum has a liquid limit of 28 percent, an initial void ratio of 0.95, a  $\gamma = 130$  pcf and a consolidation coefficient of  $10^{-3}$  in<sup>2</sup>/second. The sand stratum has a  $C_c = 0.01$ , a  $\gamma = 125$  pcf and an initial void ratio of 0.70. The sand surcharge has a  $\gamma = 115$  pcf.

- (d) Determine the total settlement at mid-clay under the center of the surcharge.
- (e) The time required to attain 60% consolidation of the clay stratum (i.e.  $T_v = 0.30$ ). This is the lead time required to place the surcharge before construction.
- (f) The SPT in the sand stratum is  $N = 15$ .



#### **Solution:**

- (a) The weight of the new building is estimated at  $Q = 136,000$  kips. The surcharge will have to weigh the same, spread over an area  $= 200' \times 200' = 40,000$  ft<sup>2</sup>, using a fill with a unit weight of  $\gamma = 115$  pcf. The unit pressure of the surcharge  $\gamma_{surcharge}$  is,

$$q_{sur} = \frac{Q}{A} = \frac{136,000 \text{ kips}}{40,000 \text{ ft}^2} = 3.4 \text{ ksf}$$

$$\text{but, } q_{sur} = \gamma_{sur} h = 115 \frac{\text{lb}}{\text{ft}^3} (h) = 3.4 \text{ ksf} \quad \therefore h = 30 \text{ feet high}$$

Both the sand and clay strata contribute to the settlement. However, the settlement from the clay stratum is a consolidation settlement, such that,

Using Skempton's formula  $C_c = 0.009(LL - 10) = 0.009(28 - 10) = 0.162$

Also  $e_o = 0.95$  and  $H_{\text{clay stratum}} = 20 \text{ ft} = 240 \text{ in}$ .

The initial stress at mid-clay stratum is,

$$p_0 = \gamma_s h + \gamma_s^b h = \gamma_c^b h = (0.125 \text{ kcf})(15 \text{ ft}) + (0.125 - 0.0624)(15) + (0.130 - 0.0624)(10) = 3.5 \text{ ksf}$$

The increased load from the surcharge  $\Delta p$  is,

$$\Delta p = \gamma h = (0.115 \text{ kcf})(30 \text{ ft}) = 3.45 \text{ ksf}$$

The settlement created by the surcharge is,

$$\Delta H_c = \frac{HC_c}{(1 + e_o)} \log_{10} \left( \frac{p_0 + \Delta p}{p_0} \right) = \frac{(240 \text{ in})(0.162)}{(1 + 0.95)} \log_{10} \left( \frac{3.5 + 3.45}{3.5} \right) = 5.94 \text{ inches}$$

The settlement from the sand stratum is an elastic settlement,

$$H_{\text{sand}} = 30 \text{ ft} = 360 \text{ in} \quad E_s \approx 14 \text{ N} = (14)(15) = 210 \text{ k / ft}^2; \text{ depth factor } C_1 = 1 \text{ and creep factor } C_2 = 1$$

$$\Delta = C_1 C_2 q_o \left( \frac{\varepsilon}{E} \right) dz = (1)(1)(3.4 \text{ ksf}) \left( \frac{0.30}{210 \text{ ksf}} \right) (360 \text{ in}) = 1.75 \text{ in}$$

The total settlement =  $5.94 \text{ in} + 1.75 \text{ in} = 7.69 \text{ in}$

b) The lead time  $t$  required for the surcharge to accomplish its task is,

$$t = \frac{T_v H_{dr}^2}{c_v} = \frac{(0.30) \left[ (20 \text{ ft})(12 \text{ in} / 1 \text{ ft}) \right]^2 \left( \frac{1 \text{ min}}{60 \text{ sec}} \right) \left( \frac{1 \text{ hr}}{60 \text{ min}} \right) \left( \frac{1 \text{ day}}{24 \text{ hr}} \right)}{(10^{-3} \text{ in}^2 / \text{ sec})} = 200 \text{ days for 60\% consolidation}$$

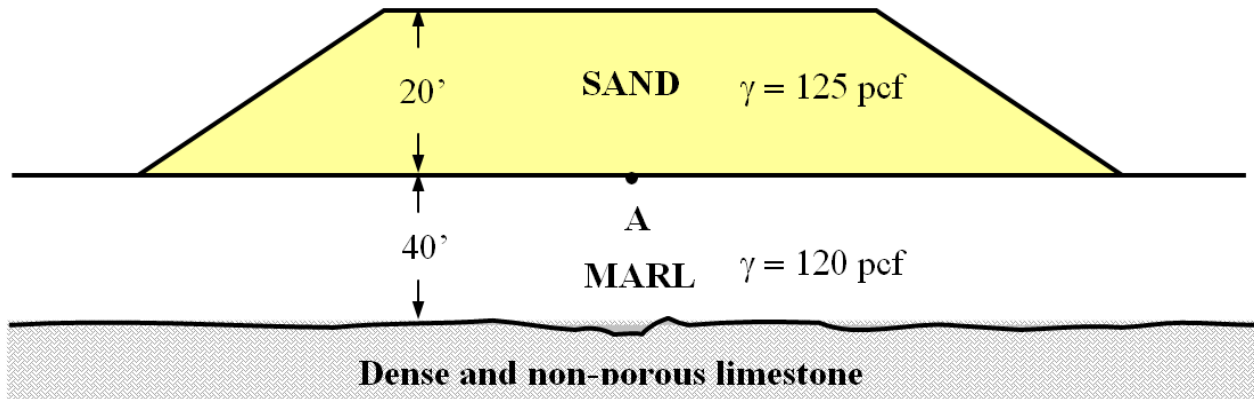
The total settlement that has taken place at 200 days (60% consolidation) is,

$$\text{Total Time} = \Delta H_s + (60\%) \Delta H_c = 1.73 \text{ in} + (0.6)(5.94 \text{ in}) = 5.3 \text{ inches}$$

### **\*\*Plastic Settlement-18: Settlement of a canal levee.**

(Revision: Aug-08)

A uniform surcharge of sand 20 feet in height will be placed over the marl stratum as shown below, in order to preconsolidate that layer for a future building. The in-situ voids ratio of the marl is 0.59, and its index of compression can be found from a relation proposed by Sowers as  $C_c = 0.75 (e_o - 0.30)$ . Find the total settlement of the surcharge at its point A.

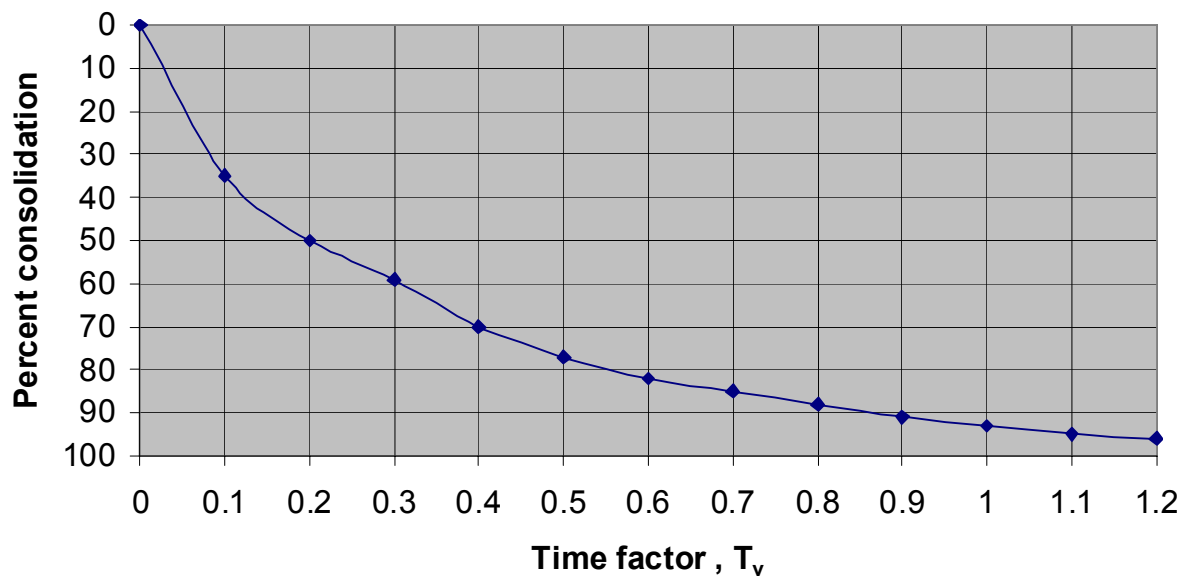


The coefficient of consolidation  $c_v$  for the marl can be found from the relation,

$$c_v = \frac{k(1 + e_o)}{a_v \gamma_w} \text{ ft}^2/\text{day}$$

where the permeability  $k = 1.0 \times 10^{-5} \text{ cms/s}$ , and  $a_v = 2.9 \times 10^{-4} \text{ ft}^2/\text{lb}$ .

Find the time required (in days) for the marl to attain 50% consolidation.



**Solution.**

$$c_c = 0.75(e_o - 0.30) = 0.75(0.59 - 0.30) = 0.218$$

At midpoint of the marl, the in-situ stress  $\sigma_o$  is,

$$\sigma_o = \gamma h = \left(120 \frac{lb}{ft^3}\right)(20 ft) = 2,400 psf$$

The increase in stress due to the surcharge  $\Delta\sigma$  is,

$$\Delta\sigma = \gamma \Delta h = \left(125 \frac{lb}{ft^3}\right)(20 ft) = 2,500 psf$$

Therefore, the consolidation settlement of the marl is,

$$\therefore \Delta = \frac{HC_c}{1 + e_o} \log_{10} \left( \frac{\sigma_o + \Delta\sigma}{\sigma_o} \right) = \frac{(40 ft)(12 in / ft)(0.218)}{(1 + 0.59)} \log_{10} \left( \frac{2.4 + 2.5}{2.4} \right) = 20.4 inches$$

The time required for 50% of consolidation to take place is found through the coefficient of consolidation  $c_v$ ,

$$c_v = k \left( \frac{1 + e_o}{a_v \gamma_w} \right) = \frac{(1.0 \times 10^{-5} cm / sec)(1 + 0.59)}{(2.9 \times 10^{-4} ft^2 / lb)(62.4 lb / ft^3)} \left( \frac{1 in}{2.54 cm} \right) \left( \frac{1 ft}{12 in} \right) \left( \frac{86.4 \times 10^3 sec}{day} \right) = 2.5 ft^2 / day$$

The time factor for 50% consolidation is  $T_v = 0.2$ , therefore the time  $t$  required is,

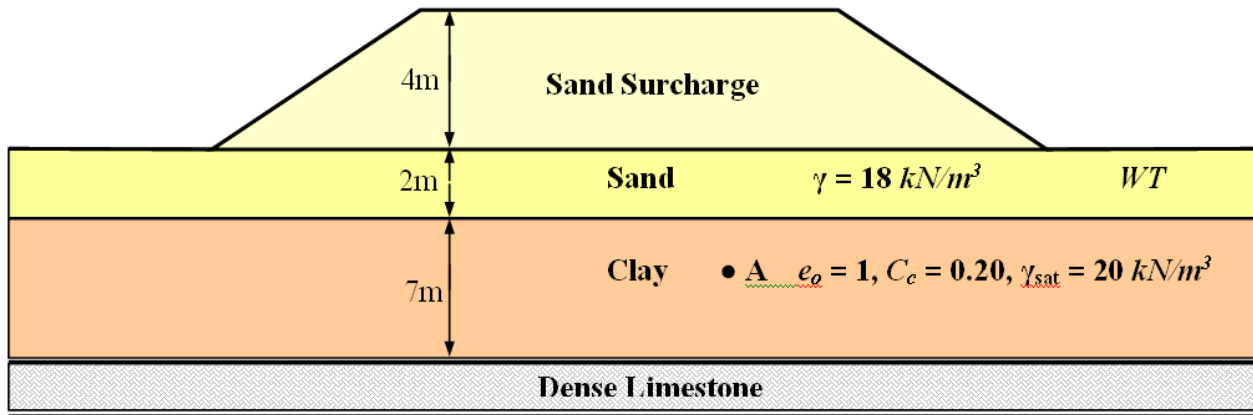
$$t = \frac{T_v H_{dr}^2}{c_v} = \frac{(0.2)(40 ft)^2}{2.5 ft^2 / day} = 128 days$$

## **\*\*Plastic Settlement-19: Differential settlements under a levee.**

(Revision: Aug-08)

An oedometer (consolidation) test was performed on a clay sample 3 cm high, drained on both sides, and taken from mid-stratum shown below. Seventy percent consolidation was attained in 6.67 minutes. Find:

- The time required to attain 70% consolidation of the clay stratum.
- The magnitude of that settlement in that time.



### **Solution.**

- (a) Since the soil is the same clay in the laboratory and the field, and both are 70% consolidation,

$$c_v = \frac{T_v H_{dr}^2}{t} = \left[ \frac{T_v H_{dr}^2}{t} \right]_{\text{laboratory}} = \left[ \frac{T_v H_{dr}^2}{t} \right]_{\text{field}} \quad \therefore \frac{t_{\text{field}}}{t_{\text{lab}}} = \frac{H_{\text{field}}^2}{H_{\text{lab}}^2}$$

$$\text{or } t_{\text{field}} = \frac{t_{\text{lab}} H_F^2}{H_L^2} = (6.67 \text{ min}) \left( \frac{700 \text{ cm}}{1.5 \text{ cm}} \right)^2 = 1.45 \times 10^6 \text{ min} = \mathbf{2.76 \text{ years}}$$

- (b) The amount of settlement that takes place at 70% consolidation is,

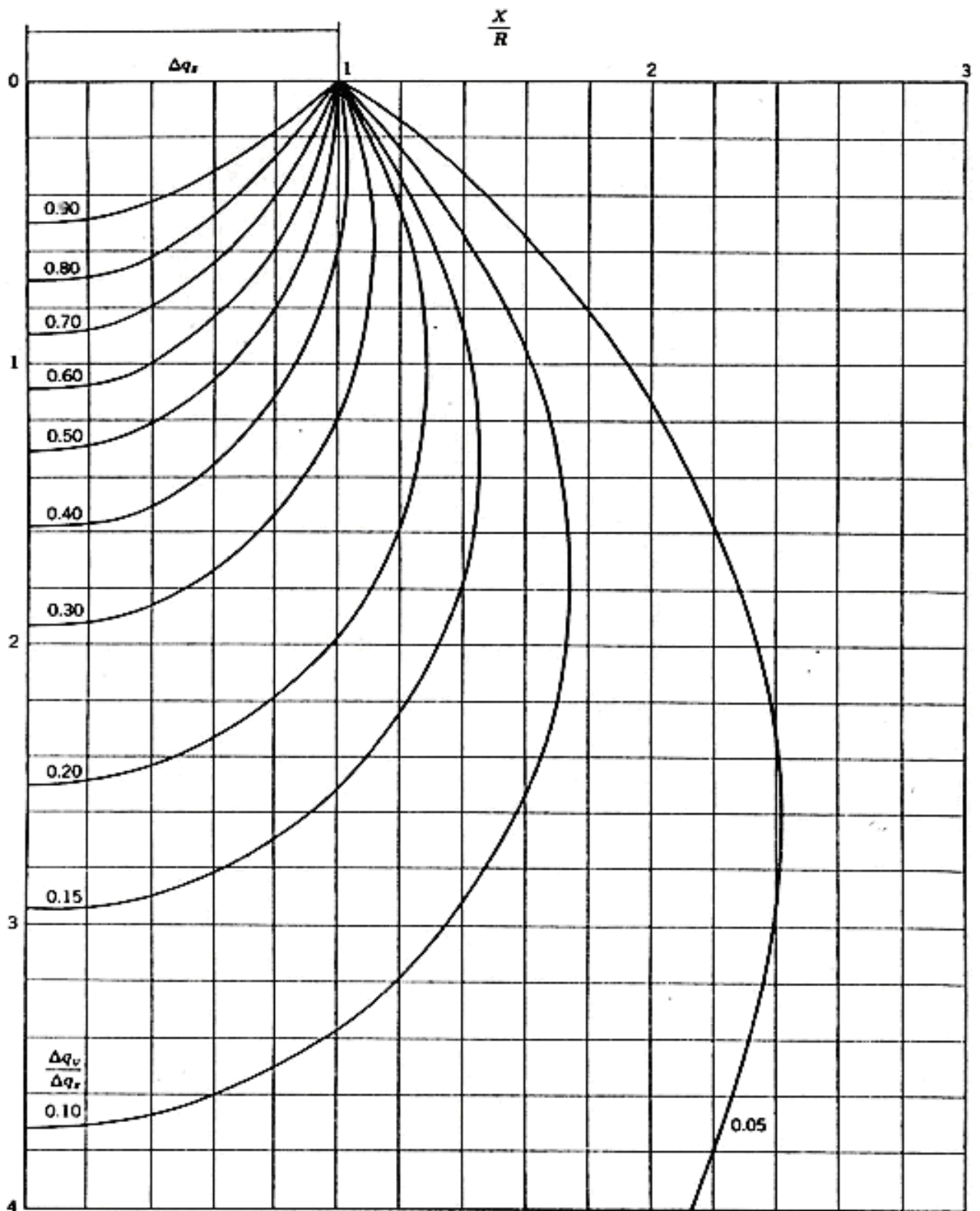
$$\Delta H_{70\%} = (0.70) \left\{ \frac{C_c H}{1 + e_o} \log_{10} \left( \frac{p_o' + \Delta p'}{p_o'} \right) \right\}$$

The in-situ stress at mid-clay stratum before the surcharge was applied was,

$$\therefore p_o' = \sum_i \gamma_i h_i = (18 \text{ kN/m}^3)(2 \text{ m}) + (20 - 9.81) \text{ kN/m}^3 (3.5 \text{ m}) = 71.7 \text{ kN/m}^2$$

$$\text{and } \Delta p' = 72 \text{ kN/m}^2$$

$$\therefore 70\% \Delta H = \frac{0.7 C_c H}{1 + e_o} \log_{10} \left( \frac{p_o' + \Delta p'}{p_o'} \right) = \frac{0.7(0.20)(700 \text{ cm})}{(1+1)} \log_{10} \left( \frac{71.7 + 72}{71.7} \right) = \mathbf{14.8 \text{ cms}}$$



Vertical stresses induced by a uniform load on a circular area

### \*\*\*Plastic Settlement-20: Estimate of the coefficient of consolidation $c_v$ .

(Revised Oct-09)

An oedometer test was performed on a peat soil sample from an FDOT project in the Homestead area. The results are shown below. The initial sample thickness was 20 mm with two way drainage through porous stones, simulating field conditions. The vertical stress increment is 10 kPa. Use an estimate of the coefficient of consolidation  $c_v$  as,

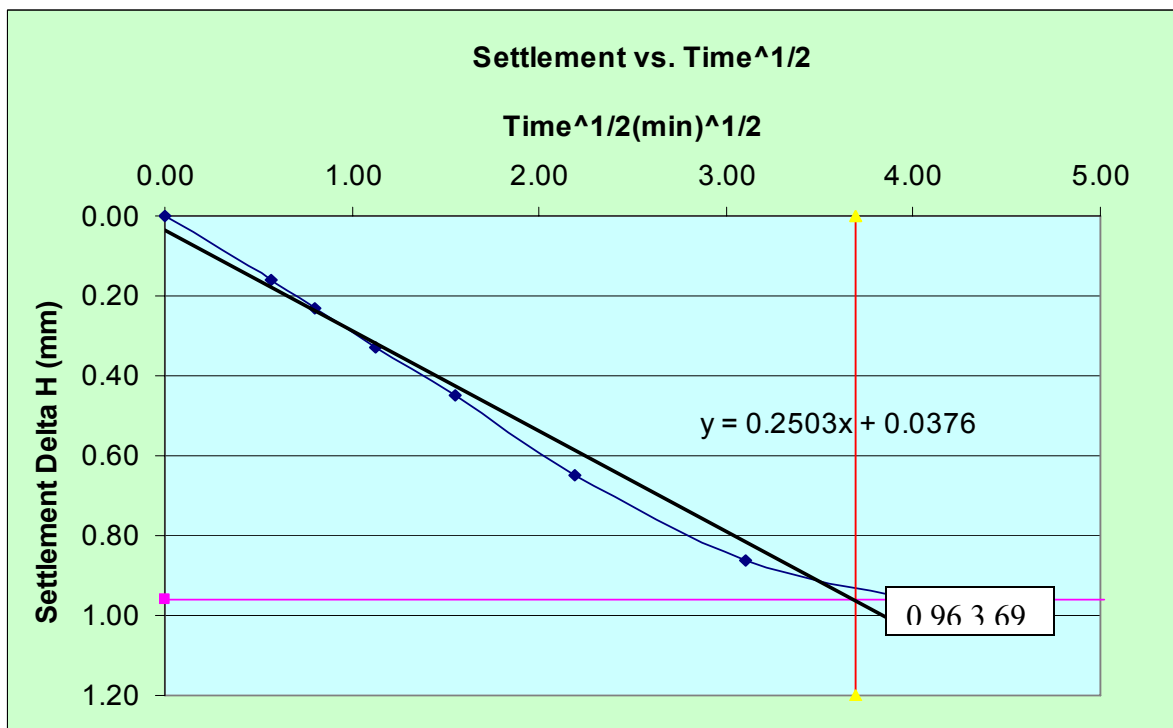
$$c_v = \frac{3H_{dr}^2}{4t}$$

Given Data								
Time(min)	0.00	0.32	0.64	1.28	2.40	4.80	9.60	16.00
Settlement (mm)	0.00	0.16	0.23	0.33	0.45	0.65	0.86	0.96
Time <sup>1/2</sup> (min) <sup>1/2</sup>	0.00	0.57	0.80	1.13	1.55	2.19	3.10	4.00

- (a) Plot a graph of settlement against the square root of time.
- (b) Determine the value of  $c_v$  ( $\frac{m^2}{s}$ ).
- (c) Estimate what could be a good estimate of the elastic modulus of this soil,  $E_o$  (in kPa).
- (d) What sort of permeability  $k$  ( $\frac{m}{s}$ ) could you estimate for this soil (from  $E_o$ )?

#### Solution:

(a)



(b)  $\sqrt{t_x} = 3.69 \text{ min}^{1/2} \rightarrow t_x = 13.62 \text{ min}$

$$c_v = \frac{3d^2}{4t_x} = \frac{3(10 \text{ min})^2 (1 \text{ min})(1 \text{ m})^2}{4(13.62 \text{ min})(60 \text{ sec})(1000 \text{ mm})^2} = 9.178 * 10^{-8} \frac{\text{m}^2}{\text{sec}}$$

(c) The strain at the end of the loading is:

$$\varepsilon_v = \frac{\Delta}{L} = \frac{(1 \text{ mm})}{(20 \text{ mm})} = 0.05 \text{ or } 5\%$$

$$E_o = \frac{\sigma_v}{\varepsilon_v} = \frac{10 * 10^3 \text{ N}}{0.05 \text{ m}^2} = 200 \text{ kPa} \text{ Vertical Modulus}$$

From Coduto (pages 390 and 391)  $\varepsilon_v \approx \frac{C_c}{1 + e_o}$  but Coduto (page 424)

$$c_v = \frac{2.30 \sigma_z k}{\gamma_w} \left( \frac{1 + e_o}{C_c} \right) \text{ or } c_v = \frac{\sigma_z}{\varepsilon_z} \left( \frac{k}{\gamma_w} \right) = E_o \frac{k}{\gamma_w}$$

\*note: (2.30 is for ln; use 1.0 for log)

$$k = \frac{c_v \gamma_w}{E_o} = \frac{\left( \left( 9.178 * 10^{-8} \frac{\text{m}^2}{\text{sec}} \right) \left( 9.81 \frac{\text{kN}}{\text{m}^3} \right) \right)}{\left( 200 \frac{\text{kN}}{\text{m}^2} \right)} = 4.50 * 10^{-9} \frac{\text{m}}{\text{s}}$$

**\*\*Plastic Settlement-21: The apparent optimum moisture content.**

(Revised Oct-09)

Find the  $C_c$  and the “apparent” OCR for a superficial clay stratum that is 4m thick. The water is at the surface. A sample of the clay has a water content of 27.9%, and a specific gravity of 2.65. An oedometer test showed the following results:

Voids Ratio	Vertical Effective Stress
0.736	15
0.733	28
0.73	60
0.675	230
0.638	480
0.6	930

**Solution:**

$$C_c = [C_a - C_b] / [\log(O'_{z_b}) - \log(O'_{z_a})] = [0.675 - 0.600] / [\log(930) - \log(230)] = 0.123$$

$$S_e = wG_s \quad (\text{Saturation means } S = 1)$$

$$e = wG_s = (0.279)(2.65) = 0.73935 = 0.739$$

$$\gamma = [(G_s + e) \gamma_w] / [1 + e] = [(2.65 + 0.739)(9.81)] / [1 + 0.739] = 33.25 / 1.739 = 19.12$$

$$\text{OCR} = (98) / (19.12) = 5.13$$

## ***\*\*Plastic Settlement-22: The differential settlement between two buildings.***

(Revised Oct-09)

Two tall buildings sit next to each other in the downtown area of Boston. They are separated by a narrow 5 foot alley and are both 50 stories high (550 feet tall). They also have similar foundations, which consist of a simple mat foundation. Each mat is a thick reinforced concrete slab, 5 feet thick and 100 feet by 100 feet in plan view. The total load (dead + live + wind) of each building is 150,000 kips.

The mats are sitting upon a thick prepared stratum of carefully improved soil 40 feet thick, that has an allowable bearing capacity of 17 ksf. Below the compacted fill stratum lays a medium to highly plastic clay stratum 38 feet thick. Below the clay stratum is a thick layer of permeable sand. The water table coincides with the interface between the improved soil and the clay. The dry unit weight of improved soil is 110 pcf, whereas the in-situ unit weight of the clay is 121 pcf.

The clay has a specific gravity of 2.68. Also a moisture content of 32%, a PI of 52% and a PL of 12% before building “A” was built in 1975. The clay has a consolidation coefficient of  $10^{-4}$  in<sup>2</sup>/sec. The second building “B” was finished by early 1995. Assume that each building’s mat rotates as a rigid plate.

How much do you predict will building ‘A’ drift towards building “B” by early 1996, in inches?

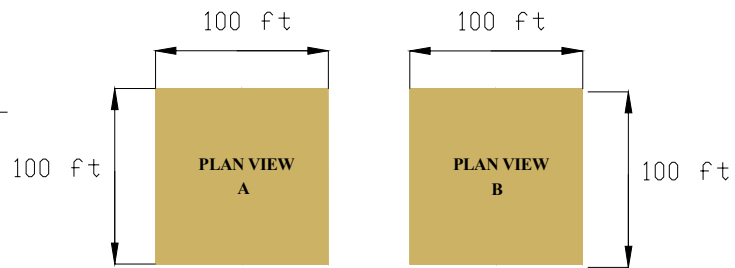
(Note: The drift of a building is its horizontal movement at the edge of the roof level due to lateral loads such as wind or earthquakes, or the differential settlement due to unequal pressures.)

**Given:**      $P = 150,000$  kips  
                $q_a = 17$  ksf;

For clay:  $G_s = 2.68$ ,  
              $w = 32\%$ ,  
              $P.I = 52\%$ ,  
              $P.L = 12\%$ ,  
              $\gamma = 121$  pcf,  
              $c_v = 10^{-4}$  in<sup>2</sup>/sec

For the improved soil:  $\gamma_d = 110$  pcf

# Diagram 1



BUILDING A

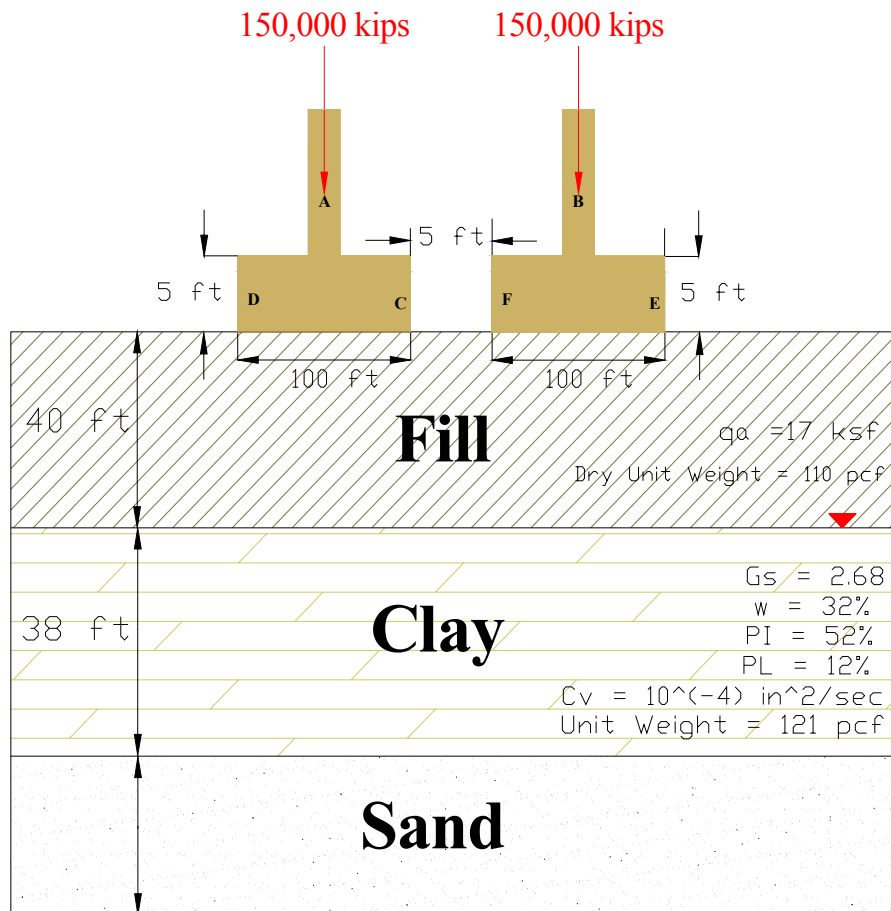
50 Stories = 550 ft

Completed 1975

BUILDING B

50 Stories = 550 ft

Completed 1975



### **Solution:**

Initial Condition: Geostatic Stresses

$$\sigma'_{zo} = \sum H\gamma - u = (40)(121) - (40)(62.4) = 2,344 \text{ lb/ft}^2$$

Induced Stresses:

To solve the induced stresses we will use the Boussinesq's Method:

$$(\sigma_z)_{\text{induced}} = \frac{q \left[ 2BLz_f (B^2 + L^2 + z_f^2)^{1/2} \right]}{4\pi (z_f^2 (B^2 + L^2 + z_f^2) + B^2 L^2)} \times \frac{B^2 + L^2 + 2z_f^2}{B^2 + L^2 + z_f^2} + \Pi - \sin^{-1} \frac{2BLz_f (B^2 + L^2 + 2z_f^2)^{1/2}}{z_f^2 (B^2 + L^2 + z_f^2) + B^2 L^2}$$

$$q = \frac{p}{A} = \frac{150,000 \times 10^3}{(100)(100)} = 15,000 \text{ lbs}$$

$$z_f = 58 \text{ ft (at mid-clay)}$$

Replacing the values of q, B, L and  $z_f$  into the Boussinesq's equation yields:

$$(\sigma_z)_{\text{induced}} = 8008.65 \text{ lb/ft}^2, 1059.79 \text{ lb/ft}^2$$

Exists differential stresses at the corners of the foundation.

Stresses due to fill:

$$(\sigma_z)_{\text{fill}} = H_{\text{fill}} \gamma_{\text{fill}} = (110)(38) = 4,180 \text{ lb/ft}^2$$

- The total final stresses at corner "C"

$$\begin{aligned} (\sigma'_{zf})_C &= \sigma'_{zo} + (\sigma_z)_{\text{induced}} + (\sigma_z)_{\text{fill}} \\ &= 2344 \text{ lb/ft}^2 + 8008.65 \text{ lb/ft}^2 + 4180 \text{ lb/ft}^2 = 14,532.65 \text{ lb/ft}^2 \approx 14,533 \text{ lb/ft}^2 \end{aligned}$$

- The total final stresses at corner "D"

$$\begin{aligned} (\sigma'_{zf})_D &= \sigma'_{zo} + (\sigma_z)_{\text{induced}} + (\sigma_z)_{\text{fill}} \\ &= 2344 \text{ lb/ft}^2 + 1059.79 \text{ lb/ft}^2 + 4180 \text{ lb/ft}^2 = 7,583.79 \text{ lb/ft}^2 \approx 7,584 \text{ lb/ft}^2 \end{aligned}$$

(Note: Both buildings A and B will experience the same final stress due to fill, loadings and geostatic factors).

### **Time Factor**

Determining the time factor for building A:

$$T_v = \frac{4C_v t}{H_{\text{dr}}^2}$$

For 21 years:  $t = 662,256,000 \text{ sec}$

$$T_{v_A} = \frac{4(10^{-4} \text{ in}^2 / \text{sec})(662,256,000 \text{ sec})}{(40)^2} = 165.56$$

### **Degree of Consolidation**

$$U = [1 - 10^{-(0.085 + T_v)/0.933}] \times 100\%$$

The degree of consolidation at  $T_{v_A} = 165.56$  is:

$$U = [1 - 10^{-(0.085 + 166)/0.933}] \times 100\% = 100\%$$

By early 1996, 21 yrs after the load was applied, the soils are completely consolidated.

Therefore,

$$U = \frac{\delta_c}{(\delta_c)_{ult}}$$

$$\delta_c = (\delta_c)_{ult}$$

Assuming normally consolidated soils

$$\delta_c = \sum \frac{C_c}{1 + e_0} H \log \left( \frac{\sigma_{zf}}{\sigma_{z0}} \right) \quad (\text{Eqn 1-1})$$

$$1 + e_0 \quad \sigma'_{z0}$$

$$C_c = 0.009 (LL - 10)$$

$$LL = PL + PL = 52 + 12 = 64 \%$$

$$\text{Therefore, } C_c = 0.009 (64 - 10) = 0.486$$

$$e_0 = \frac{G_s \gamma_w}{\gamma_d} - 1$$

$$\gamma_d = \frac{\gamma}{1 + w} = \frac{121}{1 + 0.32} = 91.67 \quad \therefore e_0 = \frac{(2.68)(62.4)}{91.67} - 1 = 0.82$$

Recall that uniform stress does not exist beneath the corners of the mat foundation. Therefore, we will have differential settlement.

At corner C:

$$H = 40 \text{ ft} \quad (\sigma'_{zf})_C = 14,533 \text{ lb/ft}^2 \quad \sigma'_{z0} = 2,344 \text{ lb/ft}^2$$

Replacing values of H,  $(\sigma'_{zf})_C$ ,  $\sigma'_{z0}$ ,  $C_c$  and  $e_0$  into (Eqn 1-1):

$$\delta_c = \frac{0.486}{1 + 0.82} (40 \text{ ft}) \log \left( \frac{14,533}{2,344} \right) = 8.46 \text{ ft} = 101.55 \text{ in}$$

At corner D:

$$H = 40 \text{ ft} \quad (\sigma'_{zf})_D = 7584 \text{ lb/ft}^2 \quad \sigma'_{z0} = 2,344 \text{ lb/ft}^2$$

$$\delta_c = \frac{0.486}{1+0.82}(40 \text{ ft}) \log\left(\frac{7,584}{2,344}\right) = 5.45 \text{ ft} = 65.35 \text{ in}$$

**Diagram 2**

550 ft

DRIFT = x

?

5.45 ft

100 ft

3.01 ft

8.46 ft

A

D

C

D'

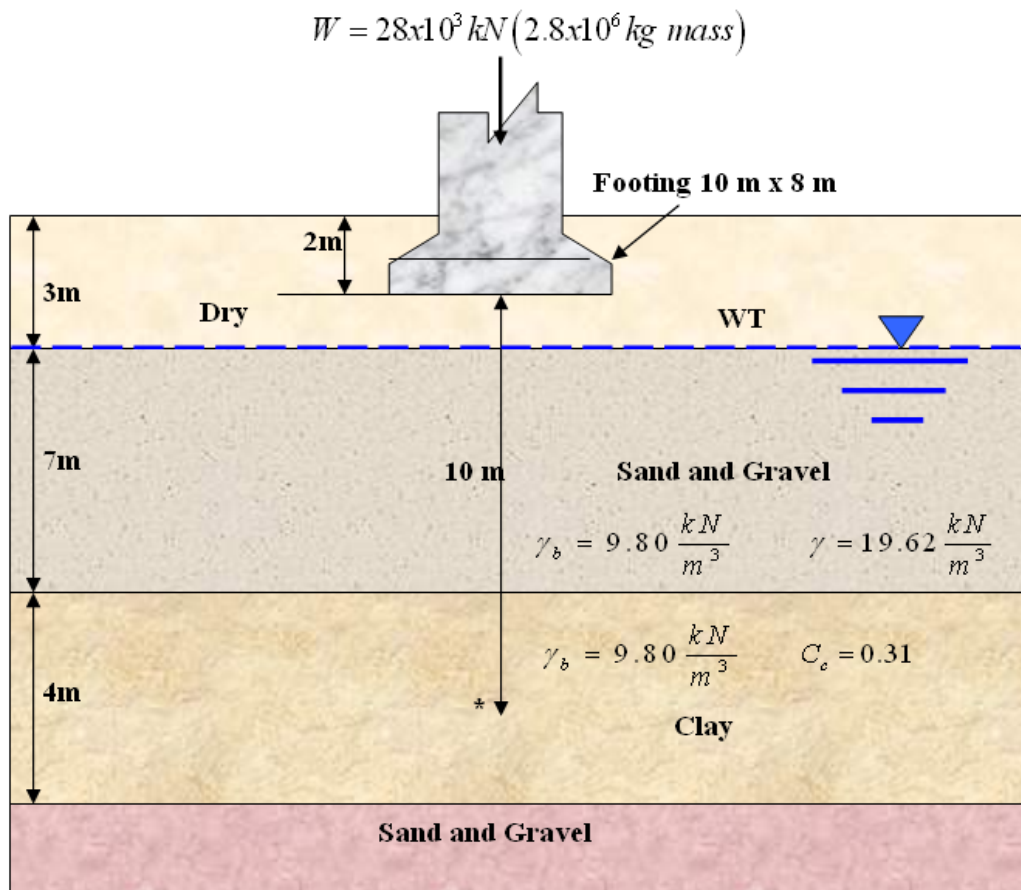
C'

$$\text{Drift} = x = 550 \tan 1.72 = \underline{\underline{16.52 \text{ ft} = 200 \text{ inches.}}}$$

## \*\*Plastic Settlement-23: Settlement of a bridge pier.

(Revision: Aug-08)

Estimate the average settlement from primary consolidation of the clay stratum under the center of the bridge pier.



### Solution:

Stress at mid-clay stratum:

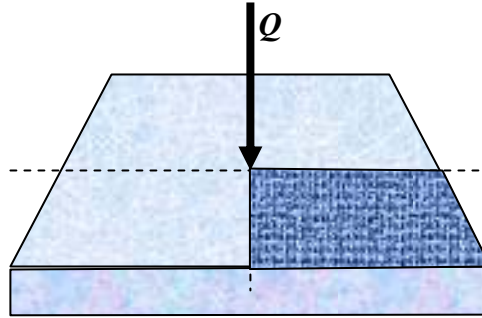
$$\begin{aligned}
 3 \text{ m} \times 19.62 \frac{\text{kN}}{\text{m}^3} &= 58.9 \frac{\text{kN}}{\text{m}^2} \\
 7 \text{ m} \times 9.80 \frac{\text{kN}}{\text{m}^3} &= 68.6 \frac{\text{kN}}{\text{m}^2} \\
 2 \text{ m} \times 9.60 \frac{\text{kN}}{\text{m}^3} &= 19.2 \frac{\text{kN}}{\text{m}^2}
 \end{aligned}
 \quad \text{for a total} \quad \rho_o = 146.7 \frac{\text{kN}}{\text{m}^2}$$

Determine  $e_o$ : From chart  $p_c \sim 150 < p_o$  normally consolidated,  $e_o = 0.81$ .

Determine  $\Delta\sigma$  or  $\Delta p$ :

$$m = \frac{4m}{10m} = 0.286 \quad \text{and} \quad n = \frac{5m}{10m} = 0.500 \quad \therefore I_4 = 0.071$$

$$\Delta \sigma = 4I_4 \frac{Q}{A} = 4(0.071) \left( \frac{28 \times 10^3 \text{ kN}}{8m \times 10m} \right) = 99.5 \frac{\text{kN}}{\text{m}^2}$$



Determine the settlement  $\Delta H$  :

$$\Delta H = H \left( \frac{C_c}{1 + e_o} \right) \log_{10} \left( \frac{p_o + \Delta p}{p_o} \right) = 4m \left( \frac{0.31}{1 + 0.81} \right) \log_{10} \left( \frac{146.7 + 99.5}{146.7} \right) = 0.15 \text{ m}$$

# Chapter 12

## Shear Strength of Soils

### Symbols for Shear Strength of Soils

$c \rightarrow$  The cohesion of a soil particle.

$c_u \rightarrow$

$p \rightarrow$

$p' \rightarrow$

$q \rightarrow$

$q' \rightarrow$

$q_u \rightarrow$  Ultimate shear strength of a soil.

$u \rightarrow$  Pore water pressure.

$u_c \rightarrow$

$u_d \rightarrow$

$\sigma \rightarrow$  The normal axial stress.

$\sigma_1 \rightarrow$  stress

$\sigma_1' \rightarrow$  stress

$\sigma_3' \rightarrow$  stress

$\sigma_3 \rightarrow$  Confining pressure

$\sigma_d \rightarrow$

$\tau \rightarrow$  The shear stress.

$\tau_f \rightarrow$  The normal shear stress at a failure.

$\phi \rightarrow$  The angle of internal friction of the soil.

$\phi \rightarrow$  The angle of inclination of the plane of failure caused by the failure shear stress.

$\sigma' \rightarrow$  Effective stress.

**\*Shear strength–01: Maximum shear on the failure plane.**

(Revision: Oct-08)

A consolidated un-drained triaxial test was performed on a specimen of saturated clay with a chamber pressure  $\sigma_3 = 2.0 \frac{kg}{cm^2}$ . At failure,

$$\sigma_1 - \sigma_3 = 2.8 \frac{kg}{cm^2}, \quad u = 1.8 \frac{kg}{cm^2} \quad \text{and the failure plane angle } \theta = 57^\circ.$$

Calculate (1) the normal stress  $\sigma$  and (2) shear stress  $\tau$  on the failure surface and (3) the maximum shear stress on the specimen.

**Solution:**

$$\sigma_1 = 2.8 + 2.0 = 4.8 \frac{kg}{cm^2}$$

$$\sigma_3 = 2.0 \frac{kg}{cm^2}$$

$$\text{Shear stress} \quad \tau = \frac{\sigma_1 - \sigma_3}{2} \sin 2\theta = \left( \frac{4.8 - 2}{2} \right) \sin 114^\circ = 1.27 \frac{kg}{cm^2}$$

$$\text{Normal stress} \quad \sigma = \left( \frac{\sigma_1 + \sigma_3}{2} \right) + \left( \frac{\sigma_1 - \sigma_3}{2} \right) \cos 2\theta = \left( \frac{4.8 + 2}{2} \right) + \left( \frac{4.8 - 2}{2} \right) \cos 114^\circ = 2.83 \frac{kg}{cm^2}$$

$$\text{Maximum shear} \quad \tau_{MAX} = \frac{\sigma_1 - \sigma_3}{2} \sin 2\theta = \left( \frac{4.8 - 2}{2} \right) = 1.4 \frac{kg}{cm^2} \quad \text{at } \theta = 45^\circ$$

### **\*Shear strength–02: Why is the maximum shear not the failure shear?**

(Revised Oct-09)

Using the results of the previous Problem 01, and  $\phi = 24^\circ$ ,  $c' = 0.80 \frac{kg}{cm^2}$ , show why the sample failed at  $57^\circ$  instead of the plane of maximum shear stress.

#### **Solution:**

On failure plane

$$\sigma' = (\sigma - u) = 2.83 - 1.8 = 1.03 \frac{kg}{cm^2}$$

$$S_{57^\circ} = c' + \sigma' \tan \phi = 0.8 + (1.03 \tan 24^\circ) = 1.27 \frac{kg}{cm^2}$$

Compare that to  $\tau = 1.27 \text{ kg/cm}$  from the previous problem, and note that they are equal, and so for both,  $S_{57^\circ} = \tau_{57^\circ}$  failed. .

Now at the plane of maximum shear stress  $\theta = 45^\circ$

$$\sigma = \left( \frac{4.8 + 2}{2} \right) + \left( \frac{4.8 - 2}{2} \right) \cos 90^\circ = 3.4 \frac{kg}{cm^2}$$

$$\sigma' = 3.4 - 1.8 = 1.6 \frac{kg}{cm^2}$$

$$S_{45^\circ} = c' + \sigma' \tan \phi = 0.8 + (1.60 \tan 24^\circ) = 1.51 \frac{kg}{cm^2}$$

**The shear strength at  $45^\circ$  is much larger than at  $57^\circ$ , therefore failure does not occur.**

**\*Shear strength–03: Find the maximum principal stress  $\sigma_1$ .**

(Revised Oct-09)

Continuing with the data from the two previous problems, the same soil specimen is now loaded slowly to failure in a drained test, that is  $u = 0$ , with  $\sigma_3 = 2.0 \frac{kg}{cm^2}$ . What will be the major principal stress at failure?

**Solution:**

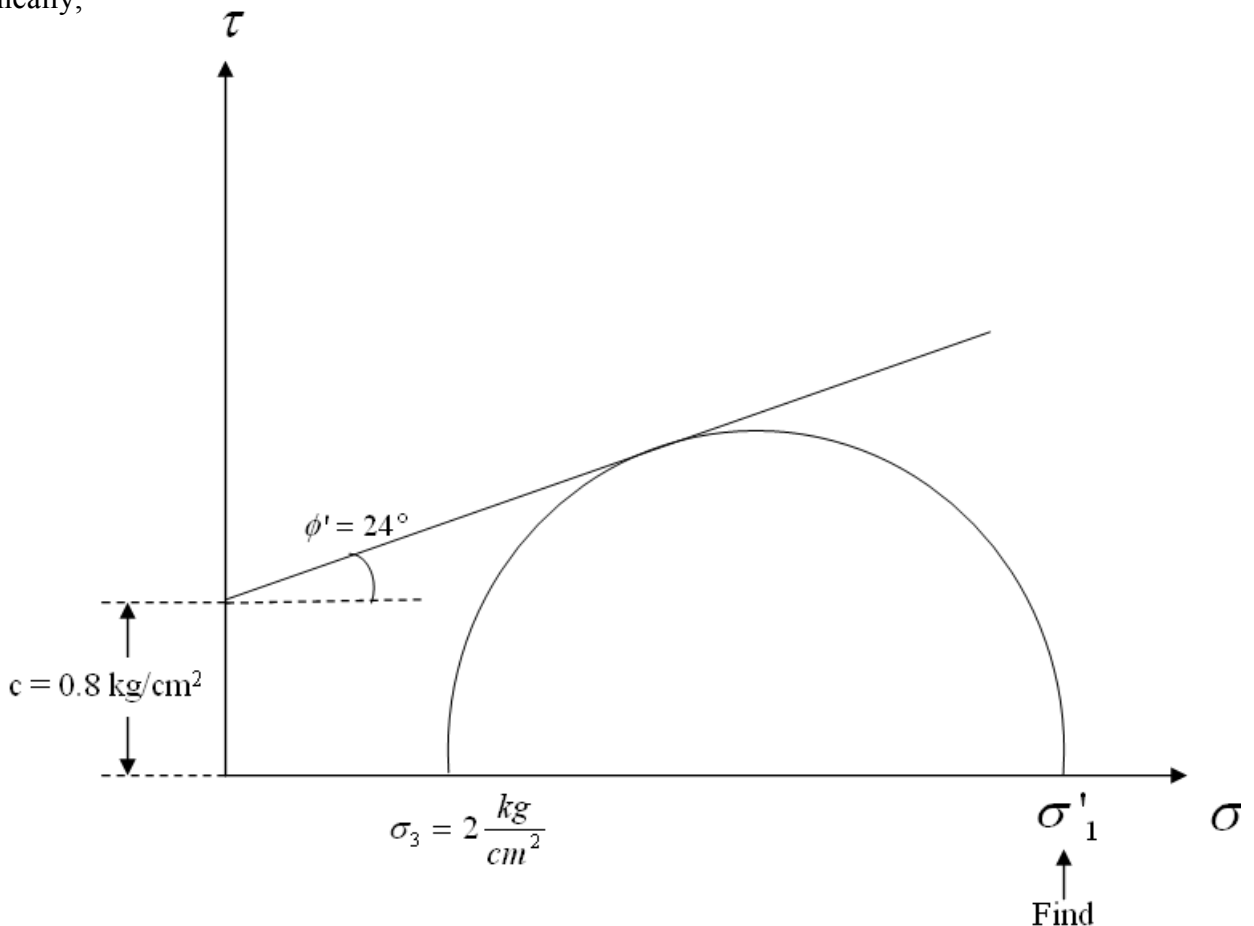
a) Analytically, in a drained test  $u = 0$ ; at failure  $\sigma'_3 = \sigma_3 = 2 \frac{kg}{cm^2}$ , on the failure plane  $\theta = 57^\circ$ .

$$s = c' + \sigma' \tan \phi = 0.80 + \left[ \left( \frac{\sigma'_1 + 2}{2} \right) + \left( \frac{\sigma'_1 - 2}{2} \right) \cos 114^\circ \right] \tan 24^\circ = (1.426) + (0.132) \sigma'_1$$

$$\text{and } \tau = \left( \frac{\sigma'_1 - 2}{2} \right) \sin 114^\circ = (0.457) \sigma'_1 - (0.914)$$

$$\text{At failure } s = \tau \quad \therefore \quad \sigma'_1 = 7.31 \frac{kg}{cm^2}; \quad \sigma_\theta = 3.6 \frac{kg}{cm^2} \text{ and } \tau_\theta = 2.38 \frac{kg}{cm^2}$$

b) Graphically,



**\*Shear strength–04: Find the effective principal stress.**

(Revised Oct-09)

A drained triaxial test on a normally consolidated clay showed that the failure plane makes an angle of  $58^\circ$  with the horizontal. If the sample was tested with a chamber confining pressure of  $103.5 \text{ kN/m}^2$ , what was the major principal stress at failure?

**Solution:**

$$\theta = 45^\circ + \frac{\phi}{2} \quad \therefore \quad 58^\circ = 45^\circ + \frac{\phi}{2} \quad \therefore \quad \phi = 26^\circ$$

Using the equation that relates the major principal stress  $\sigma_1$  to the minor principal stress  $\sigma_3$ , and with  $c = 0$  (the value of cohesion for a normally consolidated clay),

$$\sigma_1' = \sigma_3' \tan^2 \left( 45^\circ + \frac{\phi}{2} \right) = (103.5) \left[ \tan^2 \left( 45^\circ + \frac{26}{2} \right) \right] = 265 \frac{\text{kN}}{\text{m}^2}$$

### \*Shear strength–05: Using the $p$ - $q$ diagram.

(Revised Oct-09)

Triaxial tests performed on samples from our Miami Pamlico formation aeolian sand, showed the peak stresses listed below. Plot these values on a  $p$ - $q$  diagram to find the value of the internal angle of friction.

$$\sigma_1 = 76 \text{ psi} \quad \text{for} \quad \sigma_3 = 15 \text{ psi} \quad p = 45.5, \quad q = 30.5 \text{ psi} \quad (1)$$

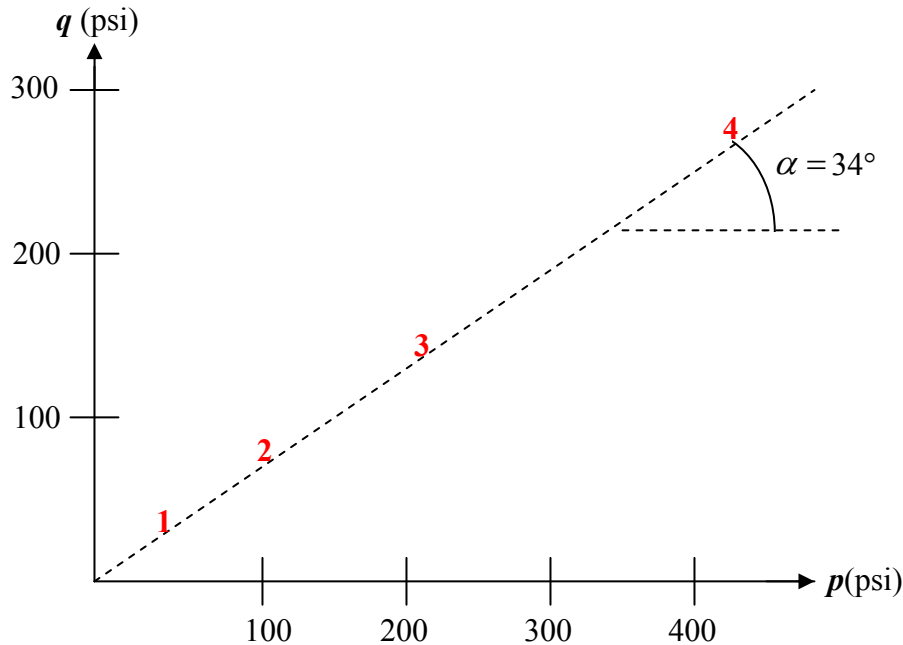
$$\sigma_1 = 148 \text{ psi} \quad \text{for} \quad \sigma_3 = 30 \text{ psi} \quad p = 89.0, \quad q = 59.0 \text{ psi} \quad (2)$$

$$\sigma_1 = 312 \text{ psi} \quad \text{for} \quad \sigma_3 = 60 \text{ psi} \quad p = 186.0, \quad q = 126.0 \text{ psi} \quad (3)$$

$$\sigma_1 = 605 \text{ psi} \quad \text{for} \quad \sigma_3 = 120 \text{ psi} \quad p = 362.5, \quad q = 242.5 \text{ psi} \quad (4)$$

#### Solution:

Remember that  $p = (\sigma_1 + \sigma_3)/2$  and  $q = (\sigma_1 - \sigma_3)/2$



From the  $p$ - $q$  diagram:

$$\tan \alpha \approx \frac{q_4}{p_4} = \frac{242.5}{362.5} = 0.668$$

$$\sin \phi = \tan \alpha = 0.668 \quad \therefore \quad \phi = 42^\circ$$

## **\*\*Shear strength–06: Consolidated-drained triaxial test.**

(Revised Oct-09)

A consolidated-drained triaxial test was conducted on a normally consolidated clay. The results are as follows:

$$\sigma'_3 = 276 \frac{kN}{m^2} \quad (\Delta \sigma_d)_f = 276 \frac{kN}{m^2}$$

Determine:

- (a) The angle of friction  $\phi$ ;
- (b) The angle  $\theta$  that the failure plane makes with the major principal plane, and
- (c) The normal stress  $\sigma'$  and shear stress  $\tau_f$  on the failure plane.

### **Solution:**

For normally consolidated soil the failure envelope equation is:

$$\tau_f = \sigma' \tan \phi \quad \text{because } c = 0$$

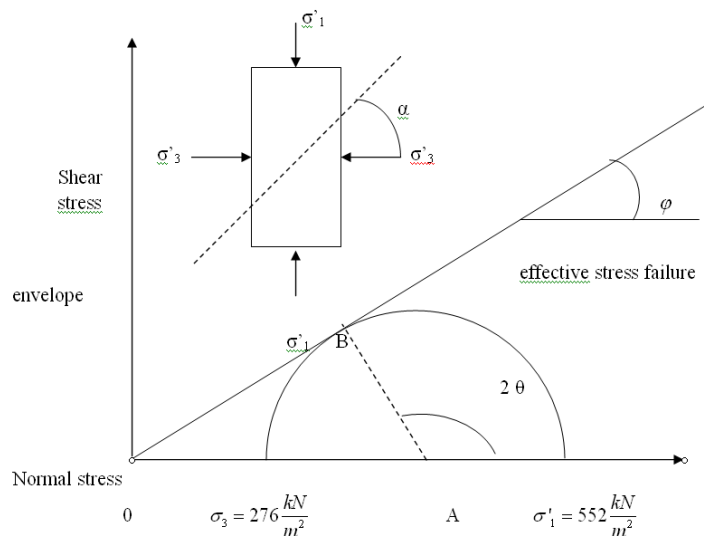
For the triaxial test, the effective major and minor principal stresses at failure are as follows:

$$\sigma'_1 = \sigma_1 = \sigma_3 + (\Delta \sigma_d)_f = 276 + 276 = 552 \frac{kN}{m^2} \quad \text{and} \quad \sigma'_3 = \sigma_3 = 276 \frac{kN}{m^2}$$

### **Part A.**

The Mohr circle and the failure envelope are shown in the figure below, from which:

$$\sin \phi = \frac{AB}{OA} = \frac{\frac{\sigma'_1 - \sigma'_3}{2}}{\frac{\sigma'_1 + \sigma'_3}{2}} = \frac{\sigma'_1 - \sigma'_3}{\sigma'_1 + \sigma'_3} = \frac{552 - 276}{552 + 276} = 0.333 \quad \therefore \phi = 19.45^\circ$$



**Part B.**

$$\theta = 45^\circ + \frac{\phi}{2} = 45^\circ + \frac{19.45^\circ}{2} = 54.7^\circ$$

**Part C.**

$$\sigma'_{(on\ the\ failure\ plane)} = \frac{\sigma'_1 + \sigma'_3}{2} + \frac{\sigma'_1 - \sigma'_3}{2} \cos 2\theta$$

$$\text{and } \tau_f = \frac{\sigma'_1 - \sigma'_3}{2} \sin 2\theta$$

Substituting the values of  $\sigma'_1 = 552 \frac{kN}{m^2}$ ,  $\sigma'_3 = 276 \frac{kN}{m^2}$  and  $\theta = 54.7^\circ$  in the preceding equations,

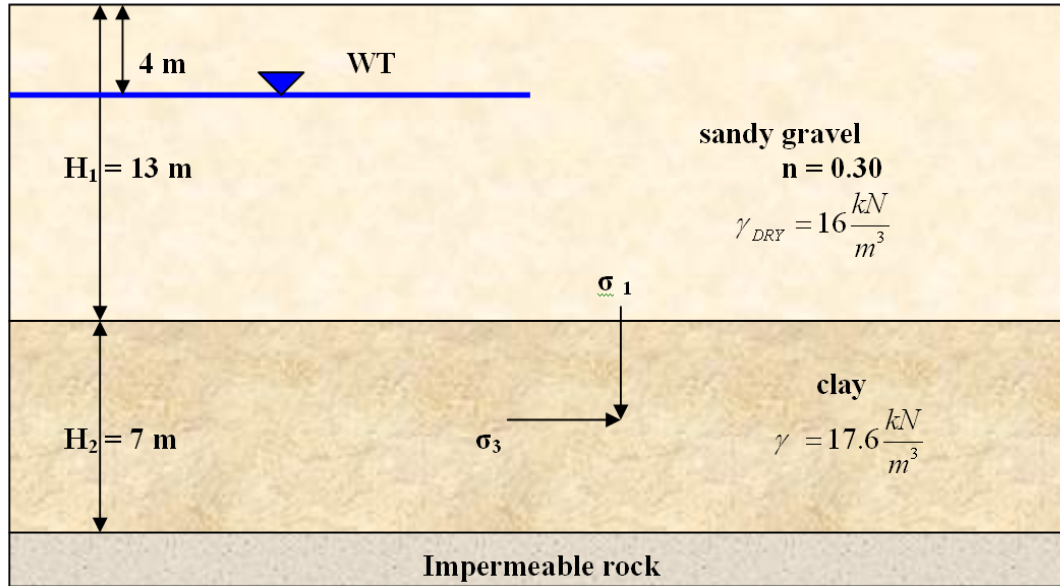
$$\sigma' = \frac{552 + 276}{2} + \frac{552 - 276}{2} \cos 2(54.7) = 368 \frac{kN}{m^2}$$

$$\text{and } \tau_f = \frac{552 - 276}{2} \sin 2(54.7) = 130 \frac{kN}{m^2}$$

## **\*\*Shear strength–07: Triaxial un-drained tests.**

(Revised Oct-09)

Triaxial un-drained tests were performed on clay samples taken from the stratum shown below. The tests were taken with pore water pressure measurements, and  $c' = 20 \frac{kN}{m^2}$ , and  $\phi = 24^\circ$



- Find the clay shear strength at its mid-stratum, and
- Find the effective and total stresses at the same level acting on a vertical face of a soil element.

### **Solution:**

- For the gravel:

$$\gamma_{sat} = \gamma_d + n\gamma_w = (16 + 0.3 \times 10) = 19 \frac{kN}{m^3}$$

$$\therefore \text{For } \sigma_1 = 4m \left( 16 \frac{kN}{m^3} \right) + 9m \left( 19 \frac{kN}{m^3} \right) = 3.5 \left( 17.6 \frac{kN}{m^3} \right) = 297 \frac{kN}{m^2}$$

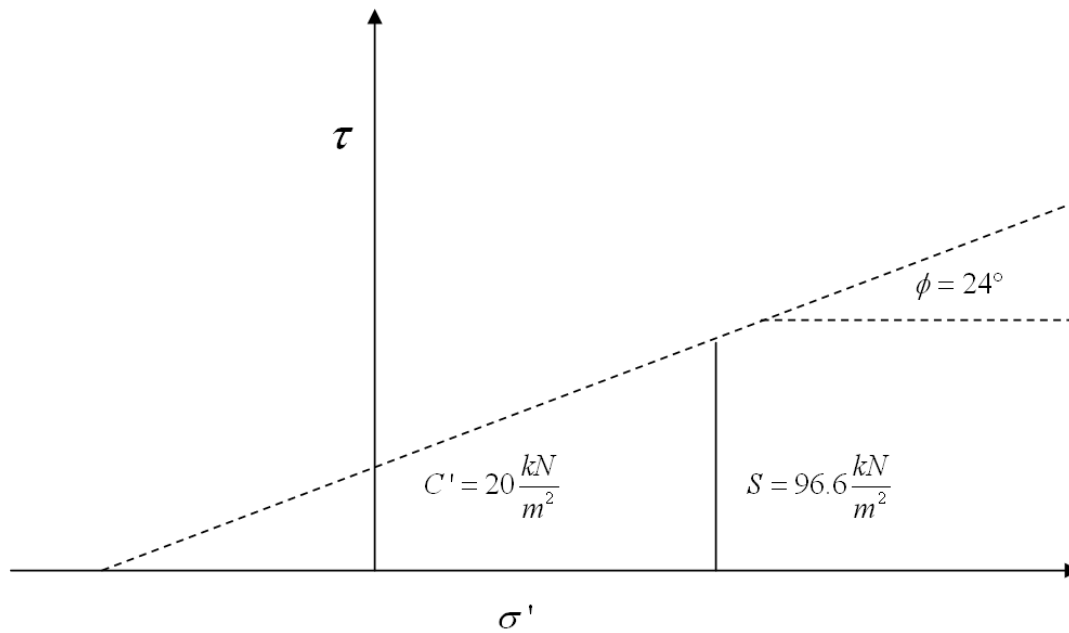
$$\therefore \text{For } \sigma'_1 = \sigma_1 - u = \sigma_1 - \gamma_w(9 + 3.5) = 297 - \left( 10 \frac{kN}{m^3} \right) (12.5m) = 173 \frac{kN}{m^2}$$

$$\therefore S = c' + \sigma'_1 \tan \phi' = 20 \frac{kN}{m^2} + 173 \frac{kN}{m^2} \tan 24^\circ = 96.6 \frac{kN}{m^2}$$

(b) Since the clay is saturated,

$$\sigma'_3 = K_o \sigma'_1 = 0.5 \sigma'_1 = 0.5(173) = 86 \frac{kN}{m^2}$$

$$\text{and } \sigma_3 = \sigma'_3 + u = 86 \frac{kN}{m^2} + (13m + 3.5m) 10 \frac{kN}{m^3} = 251 \frac{kN}{m^2}$$



## **\*\*Shear strength-08: Consolidated and drained triaxial test.**

(Revised Oct-09)

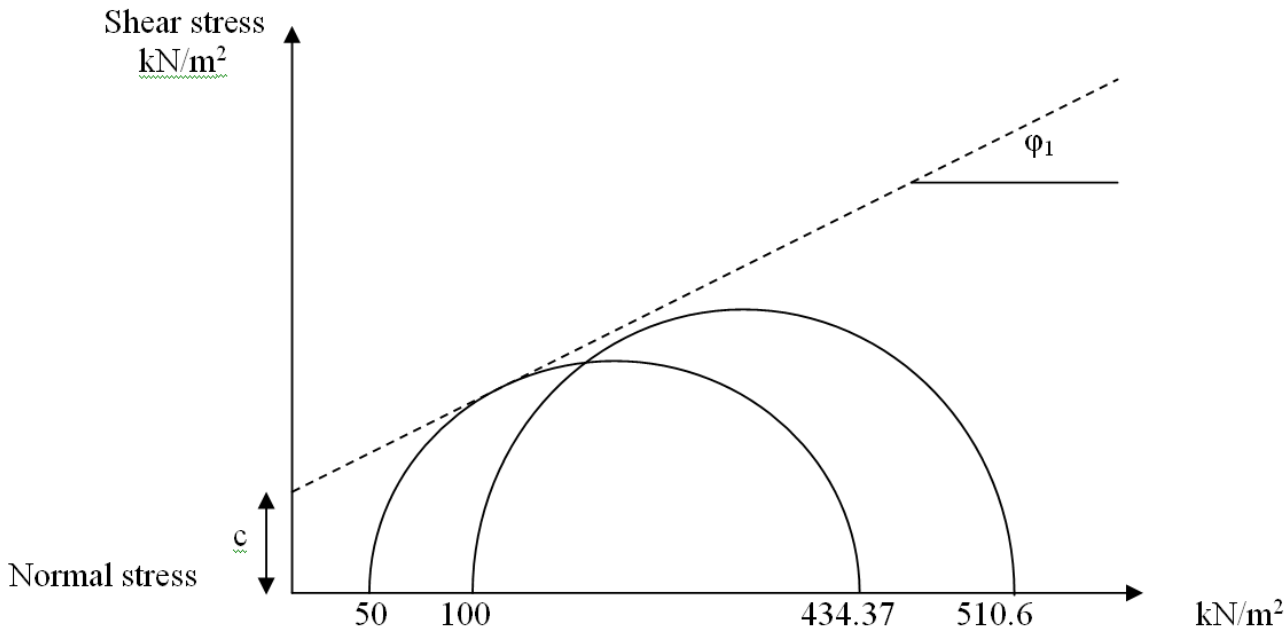
Two similar clay soil samples were pre-consolidated in triaxial equipment with a chamber pressure of  $600 \text{ kN/m}^2$ . Consolidated-drained triaxial tests were conducted on these two specimens. The following are the results of the tests:

**Specimen 1:**

$$\sigma_3 = 100 \frac{\text{kN}}{\text{m}^2}$$
$$(\Delta \sigma_d)_f = 410.6 \frac{\text{kN}}{\text{m}^2}$$

**Specimen 2:**

$$\sigma_3 = 50 \frac{\text{kN}}{\text{m}^2}$$
$$(\Delta \sigma_d)_f = 384.37 \frac{\text{kN}}{\text{m}^2}$$



**Find the values of the cohesion  $c$  and the angle of internal friction  $\phi$ .**

### **Solution:**

For Specimen 1, the principal stresses at failure are,

$$\sigma_3' = \sigma_3 = 100 \frac{\text{kN}}{\text{m}^2} \quad \text{and} \quad \sigma_1' = \sigma_1 = \sigma_3 + (\Delta \sigma_d)_f = 100 + 410.6 = 510.6 \frac{\text{kN}}{\text{m}^2}$$

Similarly, the principal stresses at failure for specimen 2 are

$$\sigma_3' = \sigma_3 = 50 \frac{\text{kN}}{\text{m}^2} \quad \text{and} \quad \sigma_1' = \sigma_1 = \sigma_3 + (\Delta \sigma_d)_f = 50 + 384.4 = 434.4 \frac{\text{kN}}{\text{m}^2}$$

These two samples are over-consolidated. Using the relationship given by equation

$$\sigma_1' = \sigma_3' \tan^2 \left( 45^\circ + \frac{\phi_1}{2} \right) + 2c \tan \left( 45^\circ + \frac{\phi_1}{2} \right)$$

Thus, for specimen 1

$$(510.6) = (100) \tan^2 \left( 45^\circ + \frac{\phi_1}{2} \right) + 2c \tan \left( 45^\circ + \frac{\phi_1}{2} \right)$$

and for specimen 2

$$(434.4) = (50) \tan^2 \left( 45^\circ + \frac{\phi_1}{2} \right) + 2c \tan \left( 45^\circ + \frac{\phi_1}{2} \right)$$

$$\text{Subtracting both equations } (76.2) = (50) \tan^2 \left( 45^\circ + \frac{\phi_1}{2} \right) \therefore \phi_1 = 12^\circ$$

Substituting  $\phi_1$  into the equation,

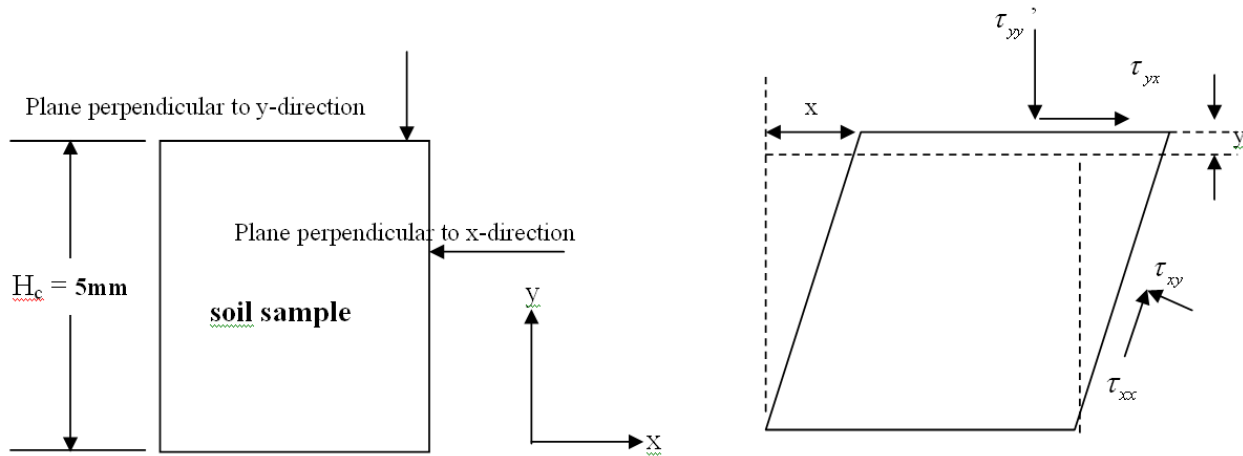
$$(510.6) = (100) \tan^2 \left[ 45^\circ + \left( \frac{12}{2} \right) \right] + 2c \tan \left[ 45^\circ + \left( \frac{12}{2} \right) \right]$$

$$510.6 = 152.5 + 2.47c \therefore c = 145 \frac{kN}{m^2}$$

### \*\*\*Shear strength-09: Plots of the progressive failure in a shear-box.

(Revised Oct-09)

A soil test is performed in the shear-box shown below. The test data lists the stresses and displacements. Assign positive normal stresses to compression and positive shear stresses are counter-clockwise. Plot the Mohr circles of stress for each stage.



Stage	DISPLACEMENTS(mm)		$\sigma'_{xx}$ (kpa)	$\sigma'_{yy}$ (kpa)	$\tau_{xy}$ (kpa)	$\tau_{yx}$ (kpa)
	x	y				
a	0	0	30	70	0	0
b	0.30	-0.50	71	70	31.0	-31.3
c (peak)	2.50	-0.60	145	70	43.3	=49.0
d	3.00	-0.82	-	-	-	-
e	10.00	1.50	90.6	70	24.5	-32.0

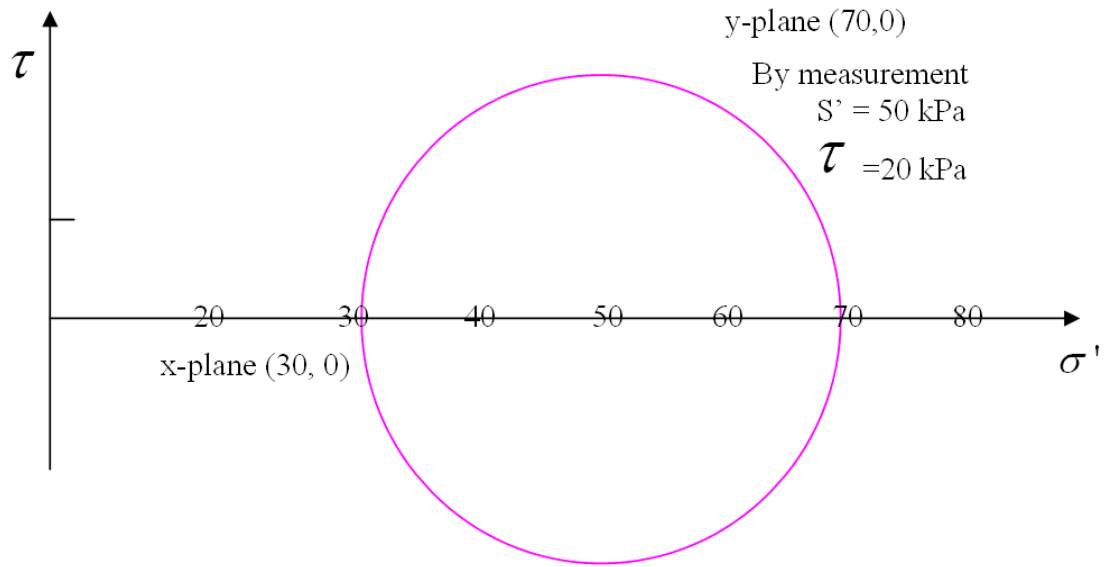
#### Solution:

For small displacements, the x and y planes remain perpendicular. Use a compass to locate by trial and error the center of the Mohr circle. The center of the circle must lie on the  $\sigma'$  axis, and it must be equidistant from the two stress points  $(\sigma'_{xx}, \tau_{xy})$  and  $(\sigma'_{yy}, \tau_{yx})$ .

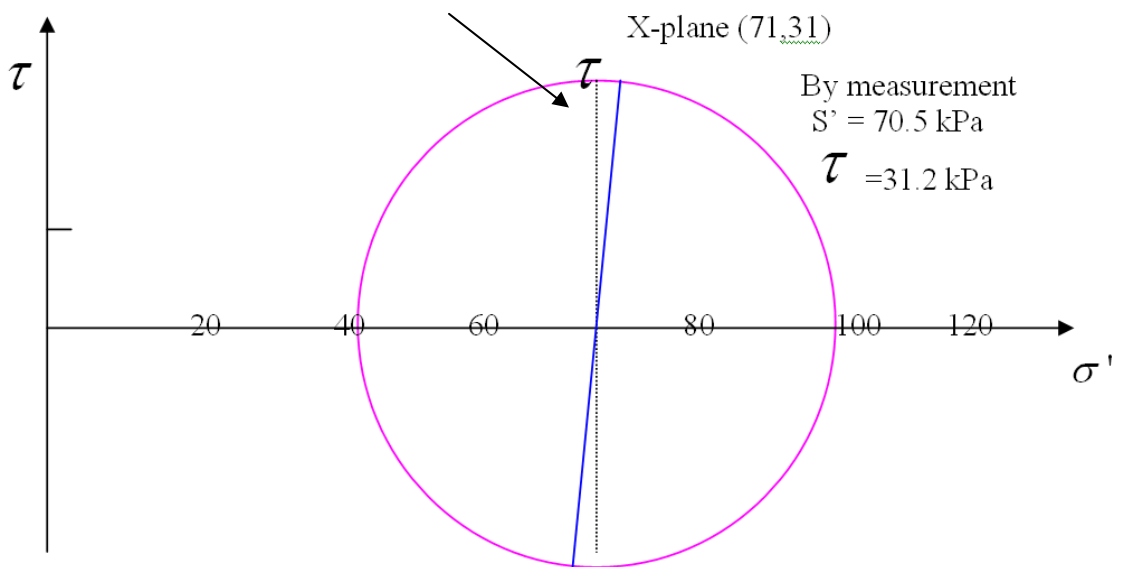
TABLE OF VALUES

STAGE	$S' = \frac{\sigma'_1 + \sigma'_3}{2}$ kPa	$\tau = \frac{\sigma'_1 - \sigma'_3}{2}$ kPa	$\frac{\tau}{S'}$	Change in angle between x + y plan (in degrees)
a	50	20	0.40	0°
b	70.5	31.2	0.44	0°
c (peak)	103	60	0.58	5.25°
e	70	32	0.46	21°

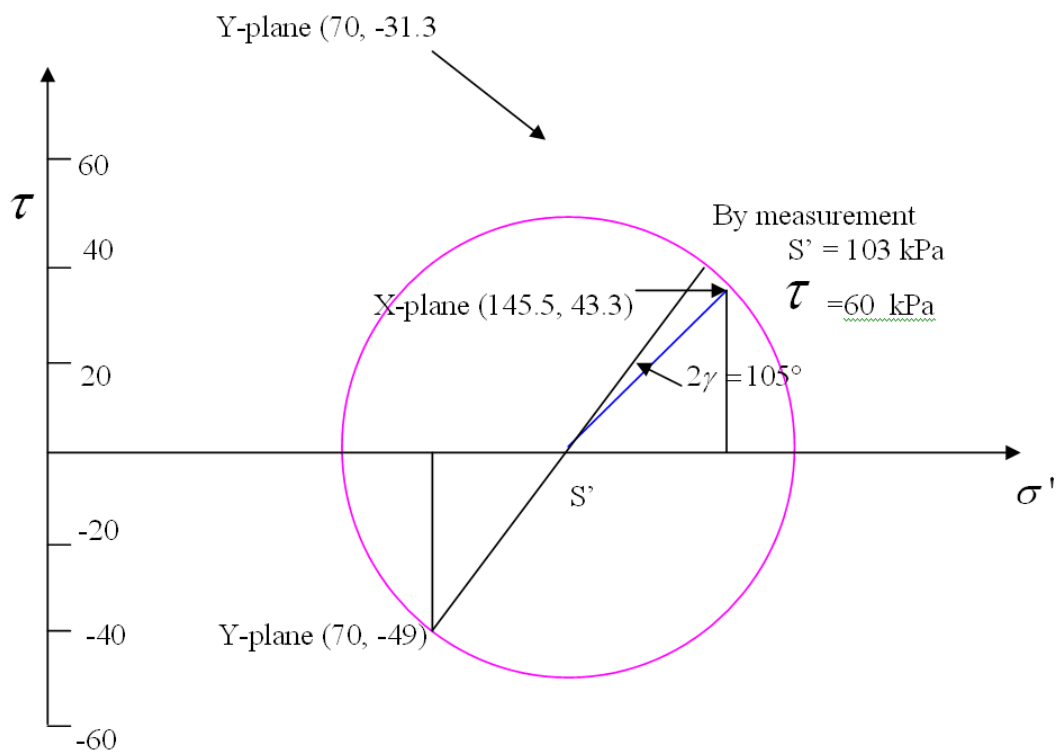
STAGE a



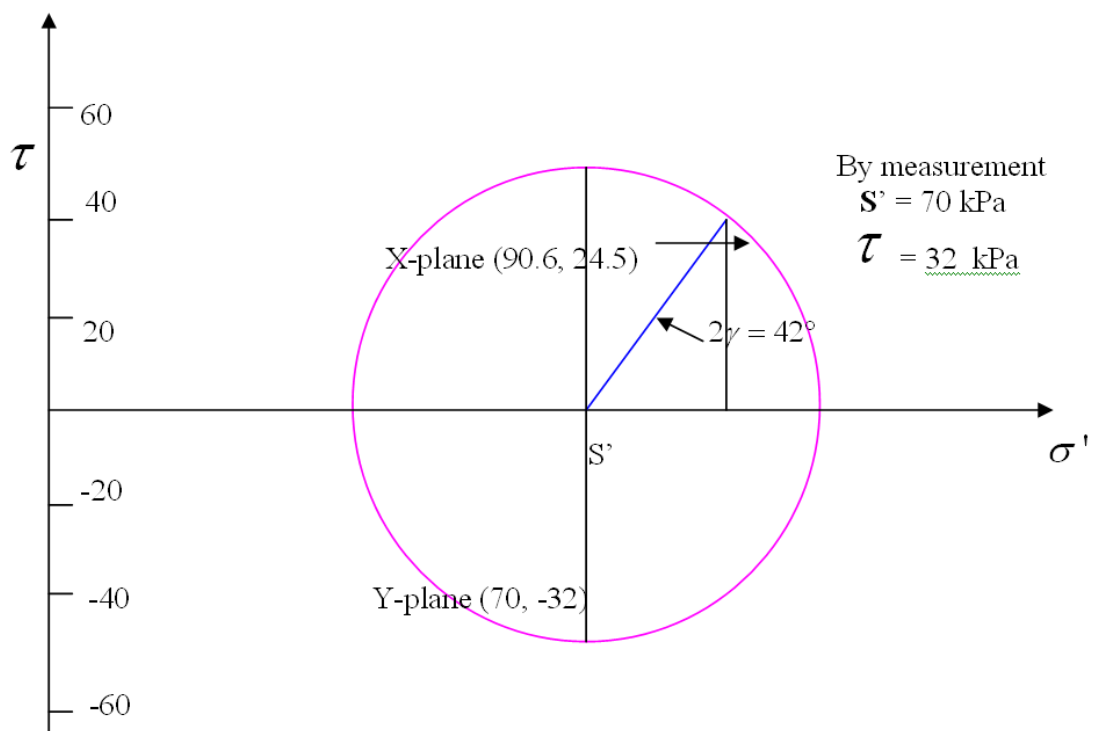
STAGE b.



STAGE c.



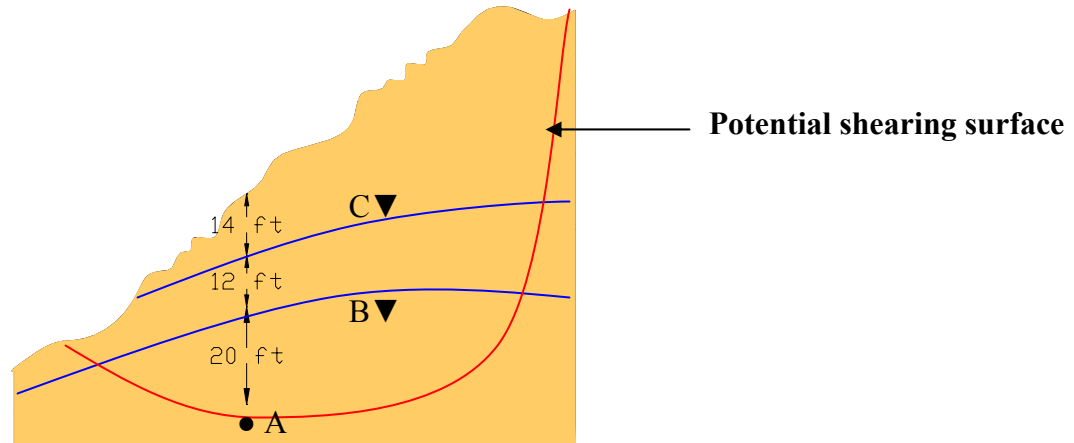
STAGE e.



## **\*\*Shear strength-10: Shear strength along a potential failure plane.**

(Revision Oct-09)

An engineer is evaluating the stability of the slope in the figure below, and considers that the potential for a shear failure occurs along the shear surface shown. The soil has an angle  $\phi' = 30^\circ$  and no cohesive strength. Compute the shear strength at point A along this surface when the groundwater table is at level B, then compute the new shear strength if it rose to level C. The unit weight of the soil is  $120 \text{ lb/ft}^3$  above the WT and  $123 \text{ lb/ft}^3$  below.



### **Solution:**

When the groundwater table is at B:

$$u = \gamma_z z_w = \left( 62.4 \frac{\text{lb}}{\text{ft}^3} \right) (20 \text{ ft}) = 1248 \frac{\text{lb}}{\text{ft}^2}$$

$$\sigma'_z = \sum \gamma H - u = \left( 120 \frac{\text{lb}}{\text{ft}^3} \right) (26 \text{ ft}) + \left( 123 \frac{\text{lb}}{\text{ft}^3} \right) (20 \text{ ft}) - 1248 \frac{\text{lb}}{\text{ft}^2} = 4332 \frac{\text{lb}}{\text{ft}^2}$$

The potential shear surface is horizontal, so  $\sigma' = \sigma'_z$

$$s = \sigma' \tan \phi' = \left( 4332 \frac{\text{lb}}{\text{ft}^2} \right) \tan 30^\circ = 2501 \frac{\text{lb}}{\text{ft}^2}$$

When the groundwater table is at C:

$$u = \gamma_z z_w = \left( 62.4 \frac{\text{lb}}{\text{ft}^3} \right) (32 \text{ ft}) = 1997 \frac{\text{lb}}{\text{ft}^2}$$

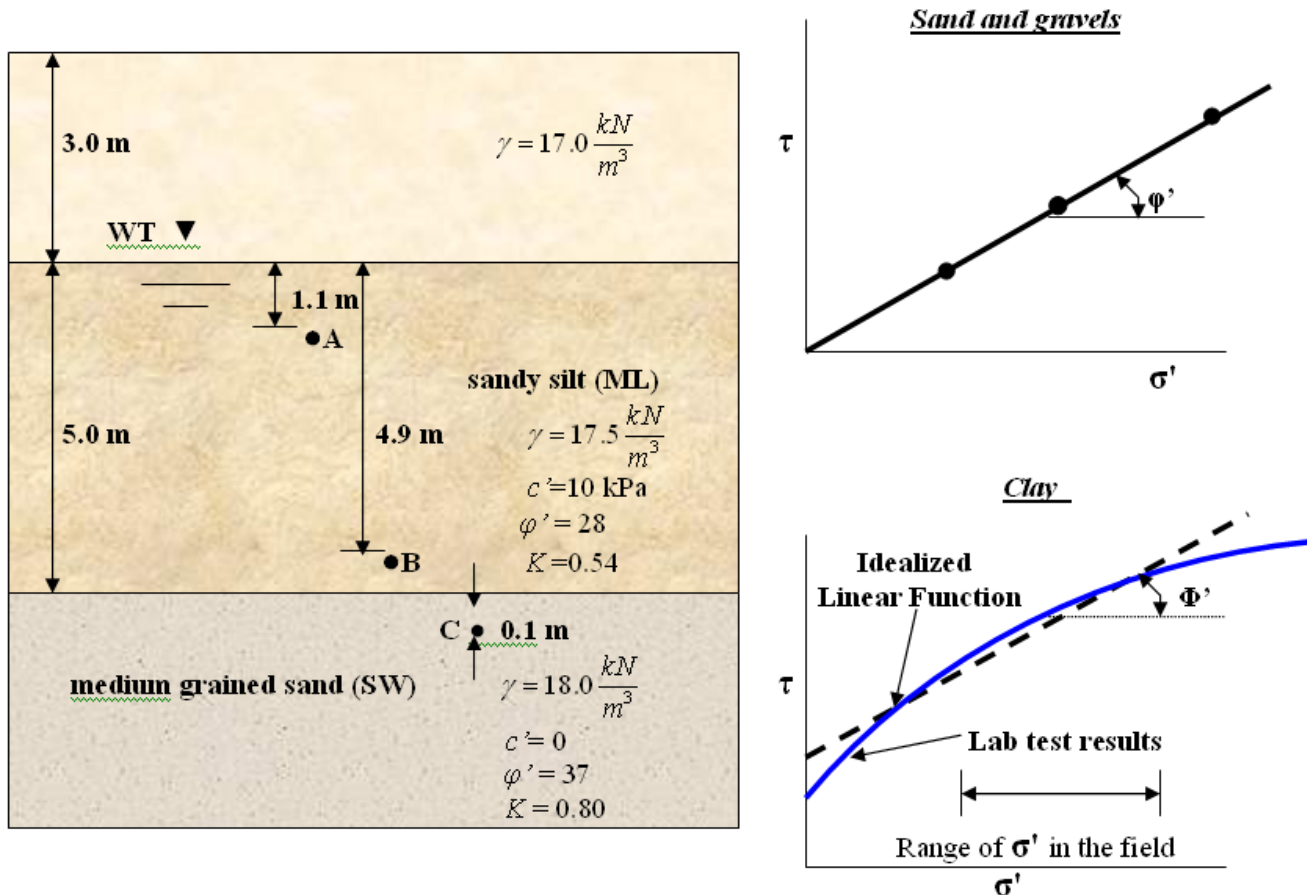
$$\sigma'_z = \sum \gamma H - u = \left( 120 \frac{\text{lb}}{\text{ft}^3} \right) (14 \text{ ft}) + \left( 123 \frac{\text{lb}}{\text{ft}^3} \right) (32 \text{ ft}) - 1997 \frac{\text{lb}}{\text{ft}^2} = 3619 \frac{\text{lb}}{\text{ft}^2}$$

$$s = \sigma' \tan \phi' = \left( 3619 \frac{\text{lb}}{\text{ft}^2} \right) \tan 30^\circ = 2089 \frac{\text{lb}}{\text{ft}^2}$$

### \*\*\*Shear strength-11: Use of the Mohr-Coulomb failure envelope.

(Revised Oct-09)

Samples have been obtained from both soil strata shown in the figure below. A series of shear strength tests were then performed on both samples and plotted in diagrams below. The  $c'$  and  $\phi'$  values obtained from these diagrams are shown in the figure below. Using this data, compute the shear strength on the horizontal and vertical planes at points A, B, and C.



#### Solution:

Point A - Horizontal plane:

$$\sigma'_z = \sum \gamma H - u = (17.0 \frac{kN}{m^3})(3.0m) + (17.5 \frac{kN}{m^3})(1.1m) - (9.8 \frac{kN}{m^3})(1.1m)$$

$$\sigma'_z = 59.8 \text{ kPa}$$

$$s = c' + \sigma'_z \tan \phi' = 10 \text{ kPa} + (59.8 \text{ kPa}) \tan 28^\circ = 41.6 \text{ kPa}$$

Point A - Vertical plane:

$$\sigma'_x = K \sigma'_z = (0.54)(59.8 \text{ kPa}) = 32.1 \text{ kPa}$$

$$s = c' + \sigma'_x \tan \phi' = 10 \text{ kPa} + (32.1 \text{ kPa}) \tan 28^\circ = 27.1 \text{ kPa}$$

Using similar computations:

Point **B** vertical plane  $s = 57.2 \text{ kPa}$

Point **B** horizontal plane  $s = 35.5 \text{ kPa}$

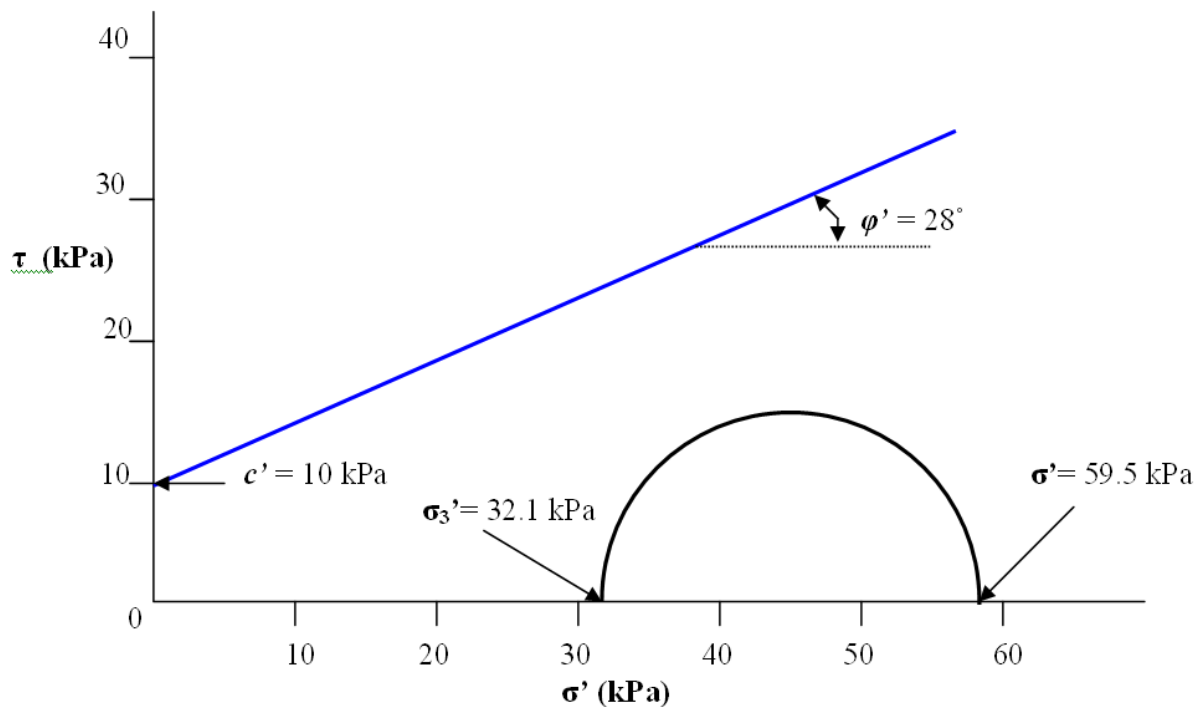
Point **C** vertical plane  $s = 68.1 \text{ kPa}$

Point **C** horizontal plane  $s = 54.4 \text{ kPa}$

### Commentary

At each point the shear strength on a vertical plane is less than that on a horizontal plane because  $K < 1$ . In addition, the shear strength at point B is greater than that at point A, because the effective strength is greater. The strength at point C is even higher than at point B because it is in a new strata with different  $c'$ ,  $\phi'$ , and  $K$  values. Thus, the strength would increase gradually with depth within each stratum, but change suddenly at the boundary between the two strata.

Draw the shear strength envelope for the ML stratum and then plot the upper half of the Mohr circle for point A on this diagram. Assume the principal stresses act vertically and horizontally.



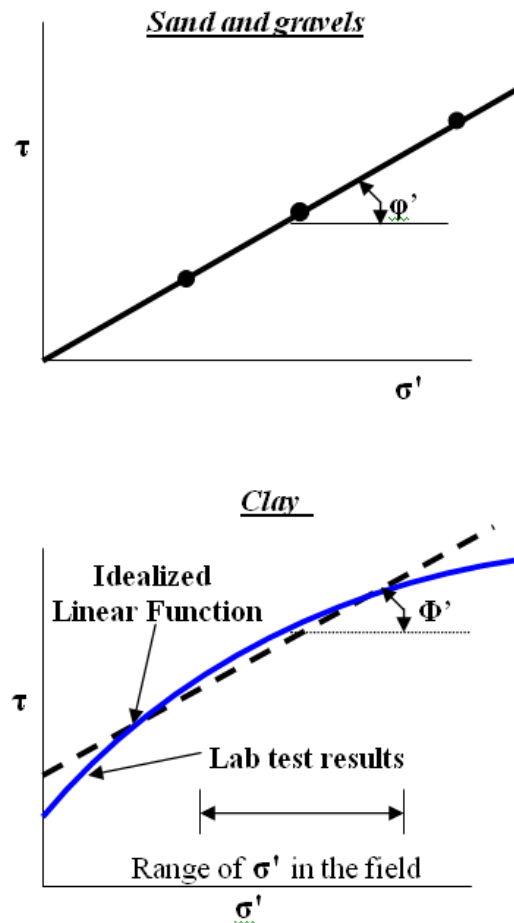
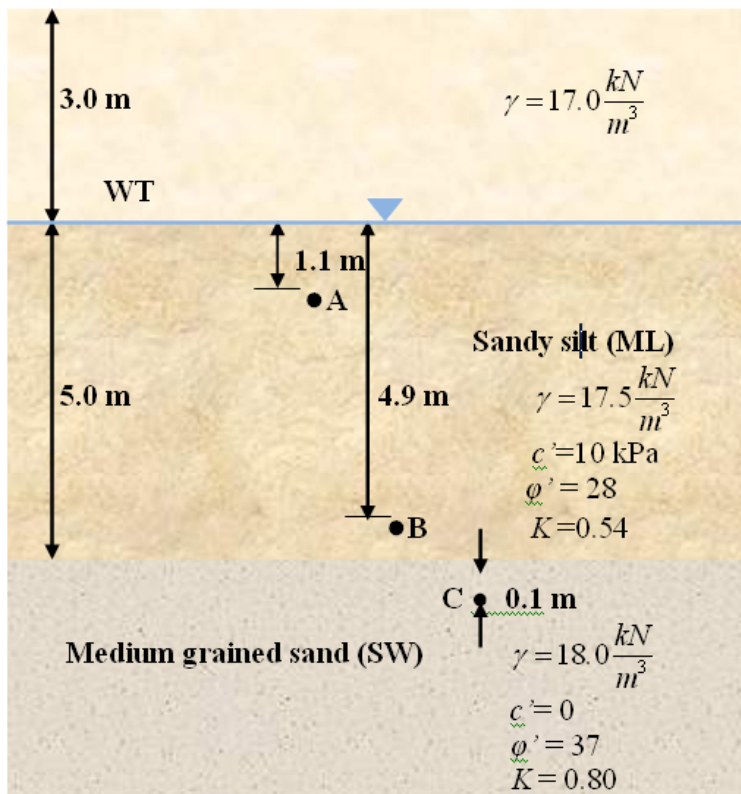
*Failure envelope and Mohr's circle*

### \*\*\*Shear strength-11b: Use of the Mohr-Coulomb failure envelope.

(Revised Oct-09)

Samples have been obtained from both soil strata shown in the figure below. A series of shear strength tests were then performed on both samples and plotted in diagrams below. The  $c'$  and  $\phi'$  values obtained from these diagrams are shown in the figure below. Using this data, compute the shear strength on the horizontal and vertical planes at points A, B, and C.

#### Solution:



Point A - Horizontal plane:

$$\sigma'_z = \sum \gamma H - u = (17.0 \frac{kN}{m^3})(3.0m) + (17.5 \frac{kN}{m^3})(1.1m) - (9.8 \frac{kN}{m^3})(1.1m)$$

$$\sigma'_z = 59.8 \text{ kPa}$$

$$s = c' + \sigma' \tan \phi' = 10 \text{ kPa} + (59.5 \text{ kPa}) \tan 28^\circ = 41.6 \text{ kPa}$$

Point A - Vertical plane:

$$\sigma'_z = K \sigma'_z = (0.54)(59.5 \text{ kPa}) = 32.1 \text{ kPa}$$

$$s = c' + \sigma' \tan \phi' = 10 \text{ kPa} + (32.1 \text{ kPa}) \tan 28^\circ = 27.1 \text{ kPa}$$

Using similar computations:

Point **B** vertical plane  $s = 57.2 \text{ kPa}$

Point **B** horizontal plane  $s = 35.5 \text{ kPa}$

Point **C** vertical plane  $s = 68.1 \text{ kPa}$

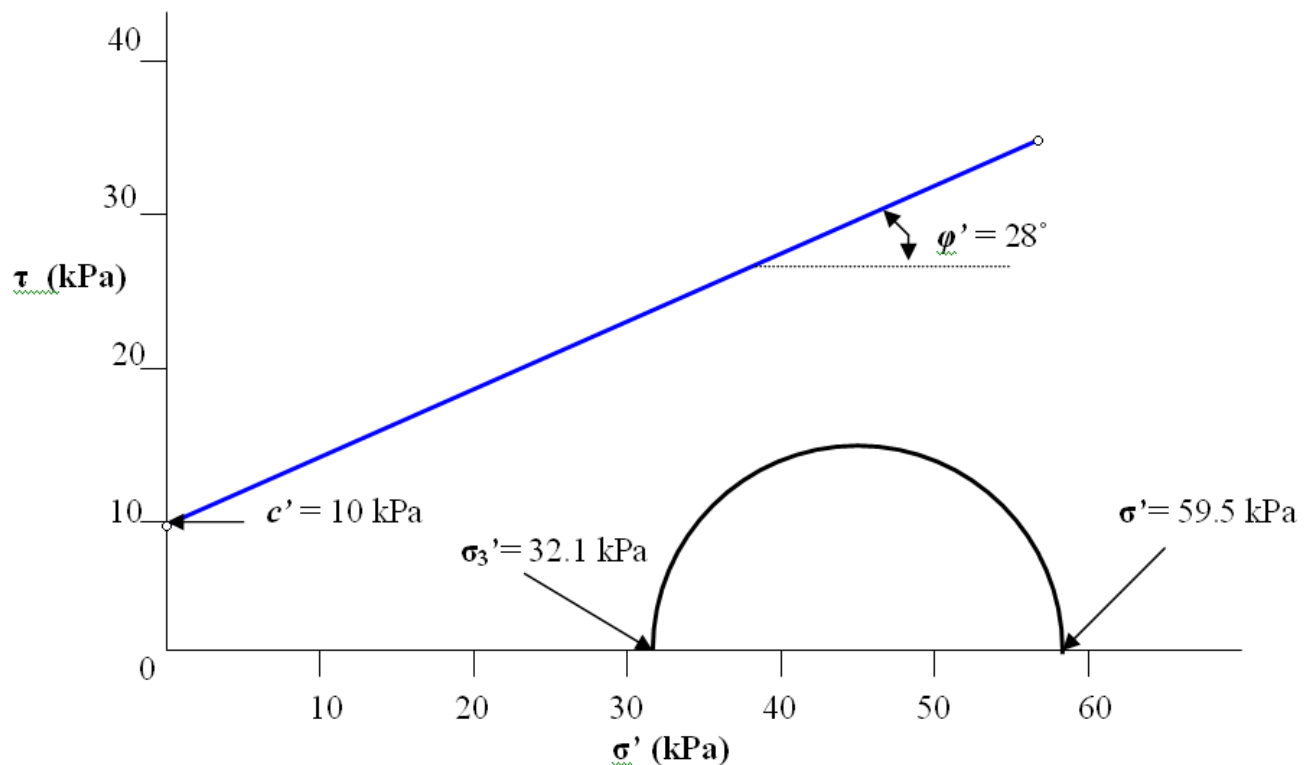
Point **C** horizontal plane  $s = 54.4 \text{ kPa}$

### Commentary

At each point the shear strength on a vertical plane is less than that on a horizontal plane because

$K < 1$ . In addition, the shear strength at point B is greater than that at point A, because the effective strength is greater. The strength at point C is even higher than at point B because it is in a new strata with different  $c'$ ,  $\phi'$ , and  $K$  values. Thus, the strength would increase gradually with depth within each stratum, but change suddenly at the boundary between the two strata.

Draw the shear strength envelope for the ML stratum and then plot the upper half of the Mohr circle for point A on this diagram. Assume the principal stresses act vertically and horizontally.

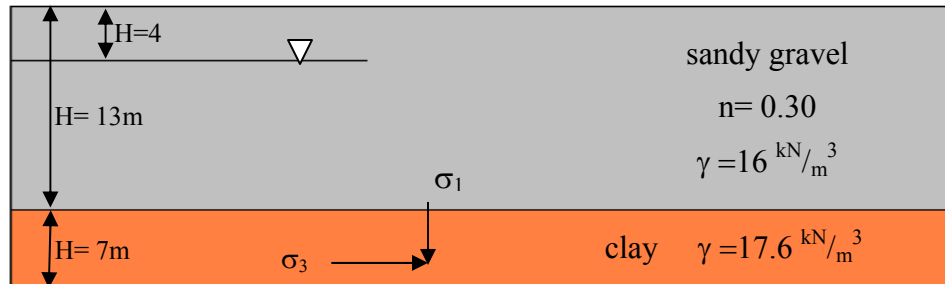


*Failure envelope and Mohr's circle*

## \*\*Shear strength-12: Triaxial un-drained tests.

(Revised Oct-09)

Triaxial un-drained tests were performed on clay samples taken from the stratum shown below. The test were taken with pure water pressure measurements and yield a  $c' = 20 \text{ kN/m}^3$ , and  $\phi = 24^\circ$ . Find (1) the clay shear strength at mid-stratum, and (2) the effective and total stresses at that same level acting on a vertical face of a small element.



Impermeable rock stratum

### Solution:

For the gravel:  $\gamma_{\text{sat}} = \gamma_{\text{sat}} + n\gamma_w = [16 + (0.3)10] = 19 \text{ kN/m}^3$

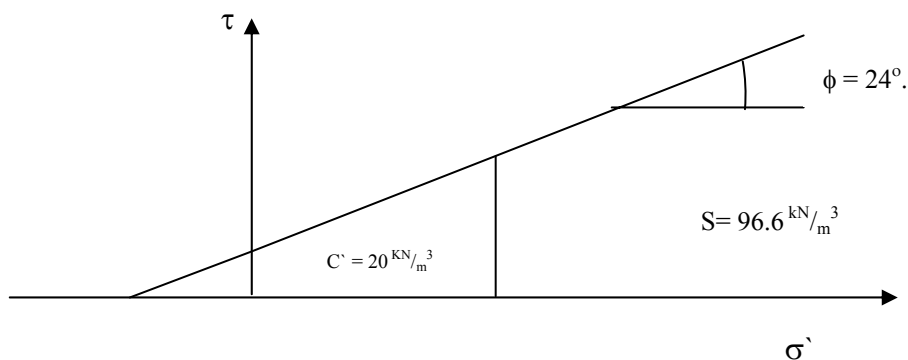
For the clay:  $\sigma_1 = (4\text{m})(16 \text{ kN/m}^3) + (9\text{m})(19 \text{ kN/m}^3) + (3.5\text{m})(17.6 \text{ kN/m}^3) = 297 \text{ kN/m}^3$

$$\sigma_1' = \sigma_1 - u = \sigma_1 - \gamma_w(9 + 3.5) = 297 - 10 \text{ kN/m}^3 [12.5\text{m}] = 172 \text{ kN/m}^3$$

$$S = c' + \sigma_1' \tan \phi = (20 \text{ kN/m}^3 + 172 \text{ kN/m}^3 \tan 24^\circ) = 96.6 \text{ kN/m}^3$$

Since the clay is saturated,  $\sigma_3' = k \sigma_1' = 0.5(172) = 86 \text{ kN/m}^3$

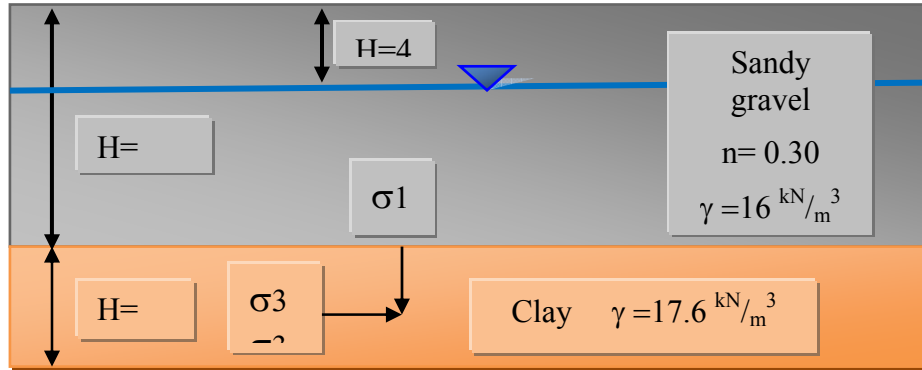
$$\sigma_3 = \sigma_3' + u = 86 \text{ kN/m}^3 + (86\text{m} + 86\text{m}) 10 \text{ kN/m}^3 = 251 \text{ kN/m}^3$$



## **\*\*Shear strength-12b: Triaxial un-drained tests.**

(Revised Oct-09)

Triaxial un-drained tests were performed on clay samples taken from the stratum shown below. The test were taken with pure water pressure measurements and yield a  $c' = 20 \text{ kN/m}^3$ , and  $\phi = 24^\circ$ . Find (1) the clay shear strength at mid-stratum, and (2) the effective and total stresses at that same level acting on a vertical face of a small element.



### **Solution:**

For the gravel:  $\gamma_{\text{sat}} = \gamma_{\text{sat}} + n\gamma_w = [16 + (0.3)10] = 19 \text{ kN/m}^3$

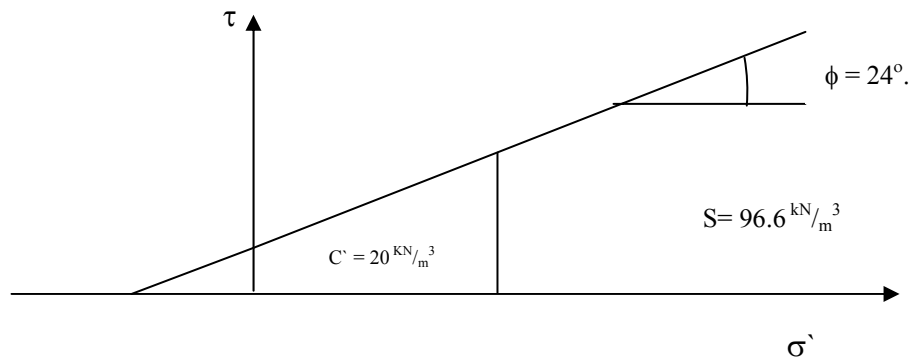
For the clay:  $\sigma_1 = (4\text{m})(16 \text{ kN/m}^3) + (9\text{m})(19 \text{ kN/m}^3) + (3.5\text{m})(17.6 \text{ kN/m}^3) = 297 \text{ kN/m}^3$

$$\sigma_1' = \sigma_1 - u = \sigma_1 - \gamma_w(9 + 3.5) = 297 - 10 \text{ kN/m}^3 [12.5\text{m}] = 172 \text{ kN/m}^3$$

$$S = c' + \sigma_1' \tan \phi = (20 \text{ kN/m}^3 + 172 \text{ kN/m}^3 \tan 24^\circ) = 96.6 \text{ kN/m}^3$$

Since the clay is saturated,  $\sigma_3' = k \sigma_1' = 0.5(172) = 86 \text{ kN/m}^3$

$$\sigma_3 = \sigma_3' + u = 86 \text{ kN/m}^3 + (86\text{m} + 86\text{m}) 10 \text{ kN/m}^3 = 251 \text{ kN/m}^3$$

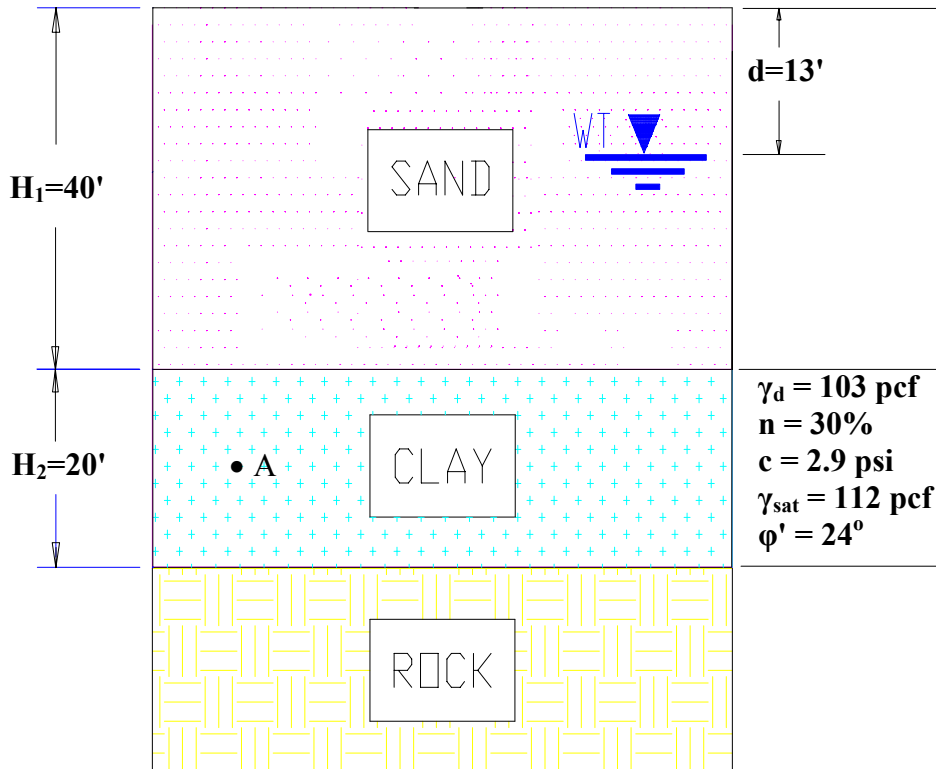


**\*\*Shear strength-13: Determine the principal stresses of a sample.**

(Revised Oct-09)

A clay layer, 20 feet thick is covered by a 40 foot sandy gravel stratum with a porosity of 30%, and a dry unit weight of 103 pcf. Tests on the un-drained samples of the clay gave  $c = 2.9$  psi,  $\gamma_{SAT} = 112$  psf and  $\phi' = 24^\circ$ . Find:

- the soil shear strength  $s = c + \sigma' \tan \phi'$  at the clay's midlevel (point A), and
- the effective and total stress acting on the vertical face of a soil element at the clay midlevel (point A).



**Solution:**

- In order to find  $s$ , it is required to know the  $\gamma_{sat}$  of the sand.

$$e = \frac{n}{1-n} = \frac{0.30}{1-0.30} = \frac{0.30}{0.70} = 0.429 \quad \text{and} \quad G_s = \frac{\gamma_d(1+e)}{\gamma_w} = \frac{103(1+0.429)}{62.4} = 2.36$$

$$\gamma_{sat} = \frac{(G_s + e)\gamma_w}{1+e} = \frac{(2.36 + 0.429)62.4}{1.429} = 122 \text{ pcf}$$

\*Assume that the clay was normally consolidated to find  $\sigma'$  at midlevel in the clay (point A), that is  $c = 0$ .

$$\sigma'_A = \gamma h_S + \gamma' h_S + \gamma' h_C = (0.103)(13) + [(0.122 - 0.0624)(27)] + [(0.122 - 0.0624)(10)] = 3.4 \text{ksf}$$

and

$$s = c + \sigma'_A \tan \phi'$$

$$s = 2.9 \frac{\text{lb}}{\text{in}^2} \times \frac{\text{kip}}{10^3 \text{lb}} \times \frac{144 \text{in}^2}{\text{ft}^2} + 3.44 \frac{\text{k}}{\text{ft}^2} (\tan 24^\circ) = 0.42 \text{ksf} + 1.53 \text{ksfs} = 1.95 \text{ksf}$$

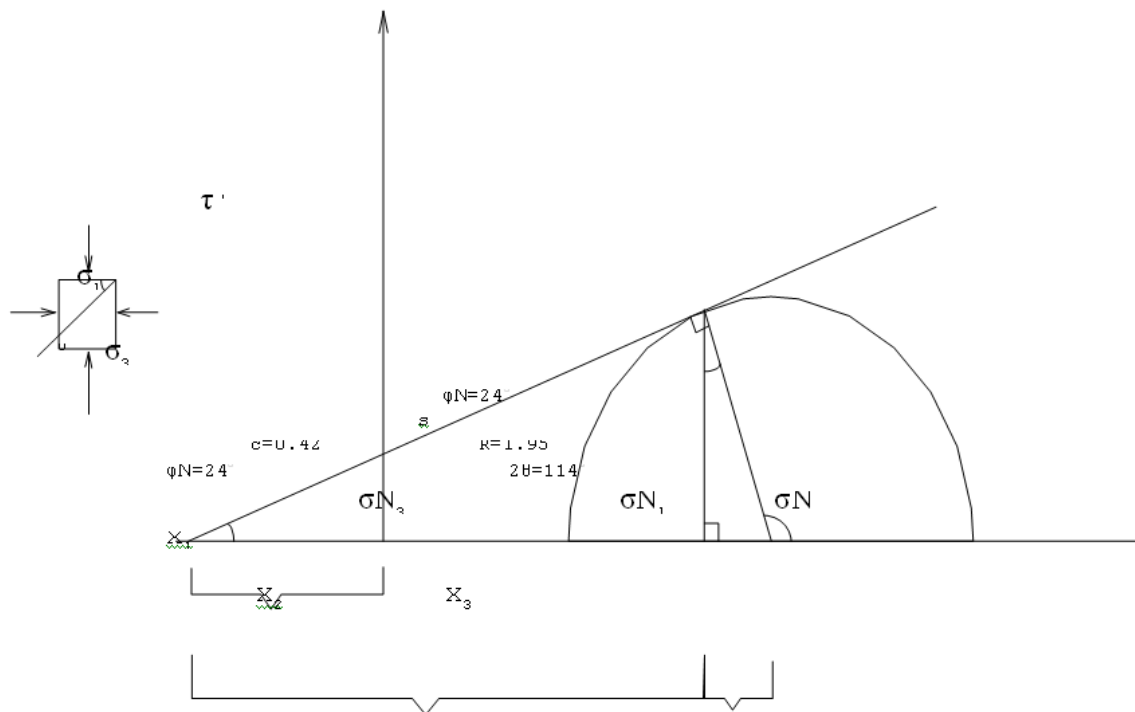
The pore water pressure  $u$  is,

$$u = (10 + 27)(0.0624) = 2.3 \text{ksf}$$

Therefore, the total stress is,

$$\sigma_A = \sigma'_A + u = 3.4 + 2.3 = 5.7 \text{ksf}$$

- (b) To find the stress on the vertical face of the soil element at A, we find  $\theta$  through a graphical solution as follows,



$$\theta = 45^\circ + \frac{\phi}{2} = 45^\circ + \frac{24^\circ}{2} = 57^\circ \quad 2\theta = 114^\circ$$

$$x_1 = \frac{c}{\tan \phi'} = \frac{0.42}{\tan 24^\circ} = 0.939 \text{ksf}$$

$$x_2 = \frac{s}{\tan \phi'} = \frac{1.95}{\tan 24^\circ} = 4.38 \text{ksf}$$

$$x_3 = s(\tan 24^\circ) = 1.95(\tan 24^\circ) = 0.868 \text{ksf}$$

$$R = \frac{s}{\cos \phi'} = \frac{1.95}{\cos 24^\circ} = 2.13 \text{ksf}$$

$$\sigma N_3 = x_2 + x_3 - x_1 - R = 4.38 + 0.868 - 0.939 - 2.13 = \underline{\underline{2.18 \text{ ksf}}}$$

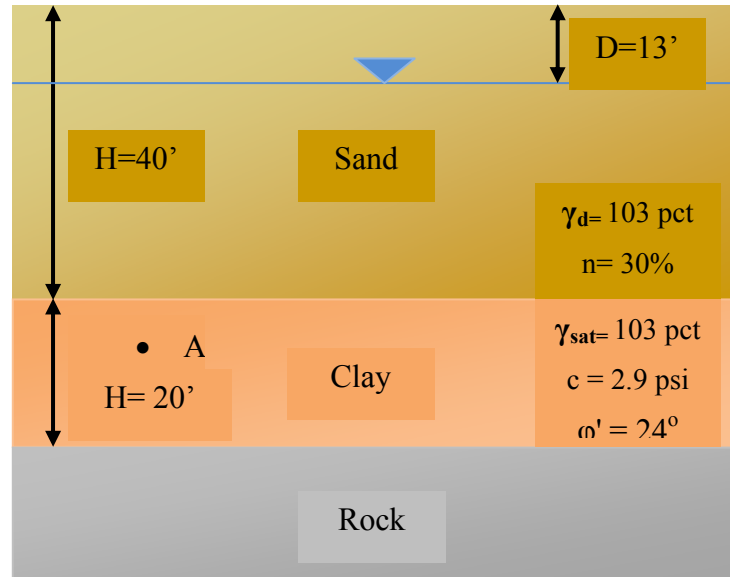
$$\sigma N_1 = 2.18 + 2R = 2.18 + 2(2.13) = \underline{\underline{6.44 \text{ ksf}}}$$

$$\sigma_3 = \sigma N_3 + u = 2.18 + 2.31 = \underline{\underline{4.5 \text{ ksf}}}$$

**\*\*Shear strength-13b: Determine the principal stresses of a sample.**

(Revised Oct-09)

A clay layer, 20 feet thick is covered by a 40 foot sandy gravel stratum with a porosity of 30%, and a dry unit weight of 103 pcf. Tests on the un-drained samples of the clay gave  $c = 2.9$  psi,  $\gamma_{SAT} = 112$  psf and  $\phi' = 24^\circ$ . Find (1) the soil shear strength  $s = c + \sigma' \tan \phi'$  at the clay's midlevel (point A), and (2) the effective and total stress acting on the vertical face of a soil element at the clay midlevel (point A).



**Solution:**

In order to find  $s$ , it is required to know the  $\gamma_{sat}$  of the sand.

$$e = \frac{n}{1-n} = \frac{0.30}{1-0.30} = \frac{0.30}{0.70} = 0.429 \quad \text{and} \quad G_s = \frac{\gamma_d(1+e)}{\gamma_w} = \frac{103(1+0.429)}{62.4} = 2.36$$

$$\gamma_{sat} = \frac{(G_s + e)\gamma_w}{1+e} = \frac{(2.36 + 0.429)62.4}{1.429} = 122 \text{ pcf}$$

\* Assume that the clay was normally consolidated to find  $\sigma'$  at midlevel in the clay (point A), that is  $c = 0$ .

$$\sigma'_A = \gamma h_s + \gamma' h_s + \gamma' h_c = (0.103)(13) + [(0.122 - 0.0624)(27)] + [(0.122 - 0.0624)(10)] = 3.4 \text{ksf}$$

and

$$s = c + \sigma'_A \tan \phi'$$

$$s = 2.9 \frac{\text{lb}}{\text{in}^2} \times \frac{\text{kip}}{10^3 \text{lb}} \times \frac{144 \text{in}^2}{\text{ft}^2} + 3.44 \frac{\text{k}}{\text{ft}^2} (\tan 24^\circ) = 0.42 \text{ksf} + 1.53 \text{ksfs} = 1.95 \text{ksf}$$

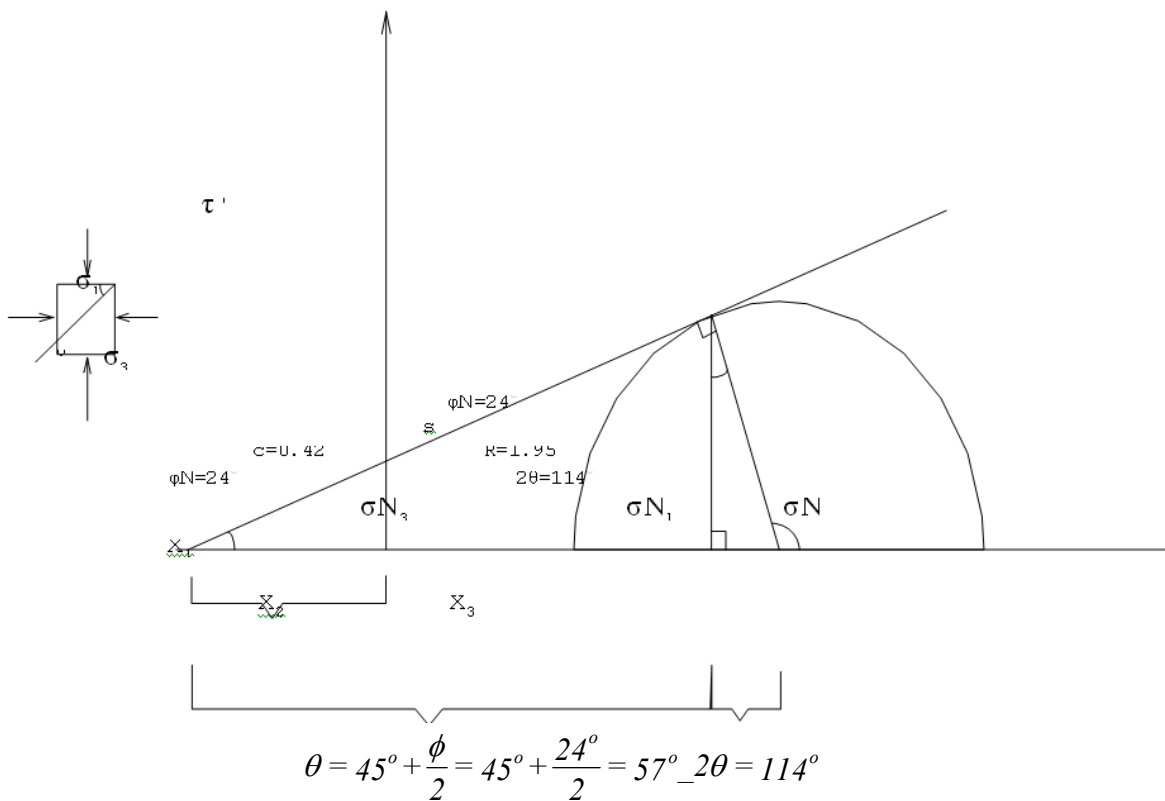
The pore water pressure  $u$  is,

$$u = (10 + 27)(0.0624) = 2.3 \text{ksf}$$

Therefore, the total stress is,

$$\sigma_A = \sigma'_A + u = 3.4 + 2.3 = 5.7 \text{ksf}$$

**To find the stress on the vertical face of the soil element at A, we find  $\theta$  through a graphical solution as follows,**



$$x_1 = \frac{c}{\tan \phi'} = \frac{0.42}{\tan 24^\circ} = 0.939 \text{ksf}$$

$$x_2 = \frac{s}{\tan \phi'} = \frac{1.95}{\tan 24^\circ} = 4.38 \text{ksf}$$

$$x_3 = s(\tan 24^\circ) = 1.95(\tan 24^\circ) = 0.868 \text{ksf}$$

$$R = \frac{s}{\cos \phi'} = \frac{1.95}{\cos 24^\circ} = 2.13 \text{ksf}$$

$$\sigma_{N_3} = x_2 + x_3 - x_1 - R = 4.38 + 0.868 - 0.939 - 2.13 = \underline{\underline{2.18 \text{ ksf}}}$$

$$\sigma_{N_1} = 2.18 + 2R = 2.18 + 2(2.13) = \underline{\underline{6.44 \text{ ksf}}}$$

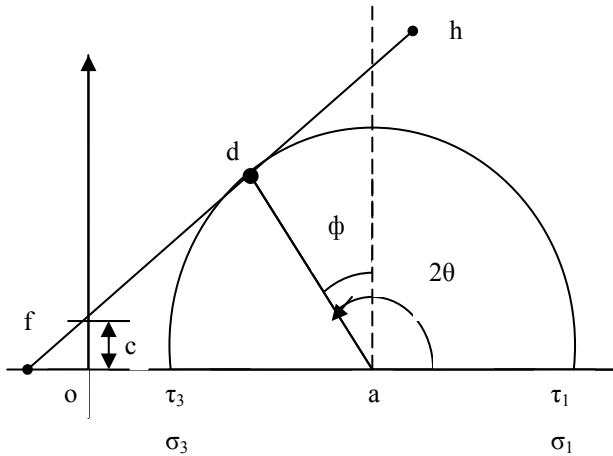
$$\sigma_3 = \sigma_{N_3} + u = 2.18 + 2.31 = \underline{\underline{4.5 \text{ ksf}}}$$

## **\*\*Shear strength-14: Formula to find the maximum principal stress.**

(Revised Oct-09)

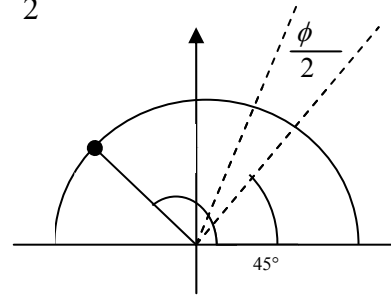
Derive the general formula that gives the value of the major principal stress  $\sigma_1$  as a function of the minor principal stress  $\sigma_3$ , the cohesion  $c$  and the angle of internal friction  $\phi$ .

**Solution:**



$$\theta = 45^\circ + \frac{\phi}{2} \rightarrow 2\theta = 90^\circ + \phi$$

$$\overline{ad} = \frac{\sigma_1 - \sigma_3}{2}$$



From the figure,

$$\sin \phi = \frac{\overline{ad}}{\overline{fa}} \quad (1)$$

$$\overline{fa} = \overline{fo} + \overline{oa} = (c) \cot \phi + \frac{\sigma_1 + \sigma_3}{2}$$

$$\tan \phi = \frac{c}{fo} \rightarrow \frac{fo}{c} = \cot \phi \rightarrow \overline{fo} = (c)(\cot \phi)$$

Using the properties of the Mohr circle,

$$\overline{oa} = \sigma_3 + \frac{(\sigma_1 - \sigma_3)}{2} \rightarrow \overline{oa} = \frac{(\sigma_1 + \sigma_3)}{2} \rightarrow \overline{fa} = (c) \cot \phi + \frac{(\sigma_1 + \sigma_3)}{2} \quad (2)$$

$$\text{Introducing (1) into (2): } \sin \phi = \frac{\frac{(\sigma_1 - \sigma_3)}{2}}{(c) \cot \phi + \frac{(\sigma_1 + \sigma_3)}{2}} \rightarrow (\sin \phi) \left[ (c) \cot \phi + \frac{\sigma_1 + \sigma_3}{2} \right] = \frac{\sigma_1 - \sigma_3}{2}$$

$$\rightarrow [(c) \sin \phi \cot \phi] + \left[ \frac{\sigma_1 + \sigma_3}{2} \sin \phi \right] = \frac{\sigma_1 - \sigma_3}{2} \rightarrow (c) \sin \phi \cot \phi = \left[ \frac{\sigma_1 - \sigma_3}{2} \right] - \left[ \frac{\sigma_1 + \sigma_3}{2} \sin \phi \right]$$

$$\rightarrow (2) [(c) \sin \phi \cot \phi] = [\sigma_1 - (\sin \phi)(\sigma_1)] - [\sigma_3 + (\sin \phi)(\sigma_3)]$$

$$\rightarrow (2) \left[ (c) \sin \phi \frac{\cos \phi}{\sin \phi} \right] = \sigma_1 [1 - (\sin \phi)] - \sigma_3 [1 + (\sin \phi)]$$

$$\frac{2(c) \cos \phi}{(1 - \sin \phi)} + (\sigma_3) \frac{(1 + \sin \phi)}{(1 - \sin \phi)} = \sigma_1$$

$$\text{Since } \frac{\cos \phi}{(1 - \sin \phi)} \approx \tan \left( 45^\circ + \frac{\phi}{2} \right) \quad \text{and} \quad \frac{(1 + \sin \phi)}{(1 - \sin \phi)} \approx \tan^2 \left( 45^\circ + \frac{\phi}{2} \right)$$

$$\sigma_1 = \sigma_3 \tan^2 \left( 45^\circ + \frac{\phi}{2} \right) + 2c \tan \left( 45^\circ + \frac{\phi}{2} \right)$$

# Chapter 13

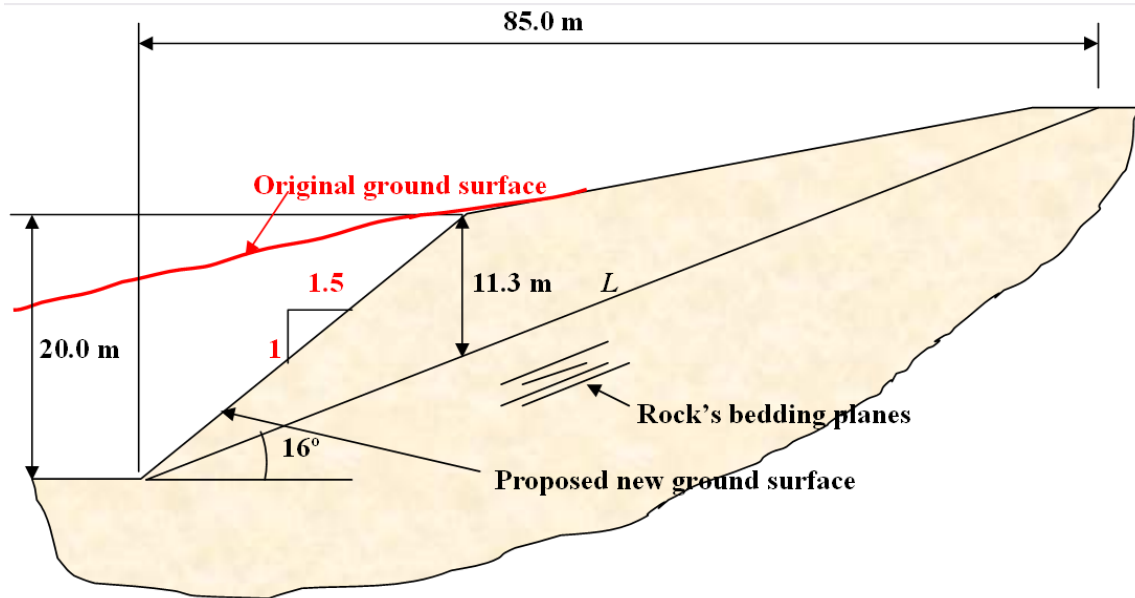
## Slope Stability

### *Symbols for Slope Stability*

### \*Slope-01: Factor of Safety of a straight line slope failure.

(Revision: Oct.-08)

A slope cut to 1.5H:1V will be made in a shale rock stratum that has bedding planes that have an apparent dip of  $16^\circ$  (see the figure below). If the acceptable factor of safety against failure is at least 2 along the lower-most bedding plane, is this slope stable? Use a unit weight of  $20.1 \text{ kN/m}^3$ , and bedding strength parameters of  $c = 22 \text{ kPa}$  and  $\phi = 30^\circ$ .



#### Solution:

The triangle of rock above the potential slip plane has a weight  $W$  per unit width,

$$W = \frac{1}{2}(85.0 \text{ m})(11.3 \text{ m})\left(20.1 \frac{\text{kN}}{\text{m}^3}\right) = 9,650 \frac{\text{kN}}{\text{m}}$$

The length  $L$  of the slip plane is,

$$L = \frac{85.0 \text{ m}}{\cos 16^\circ} = 88.4 \text{ m}$$

Therefore,

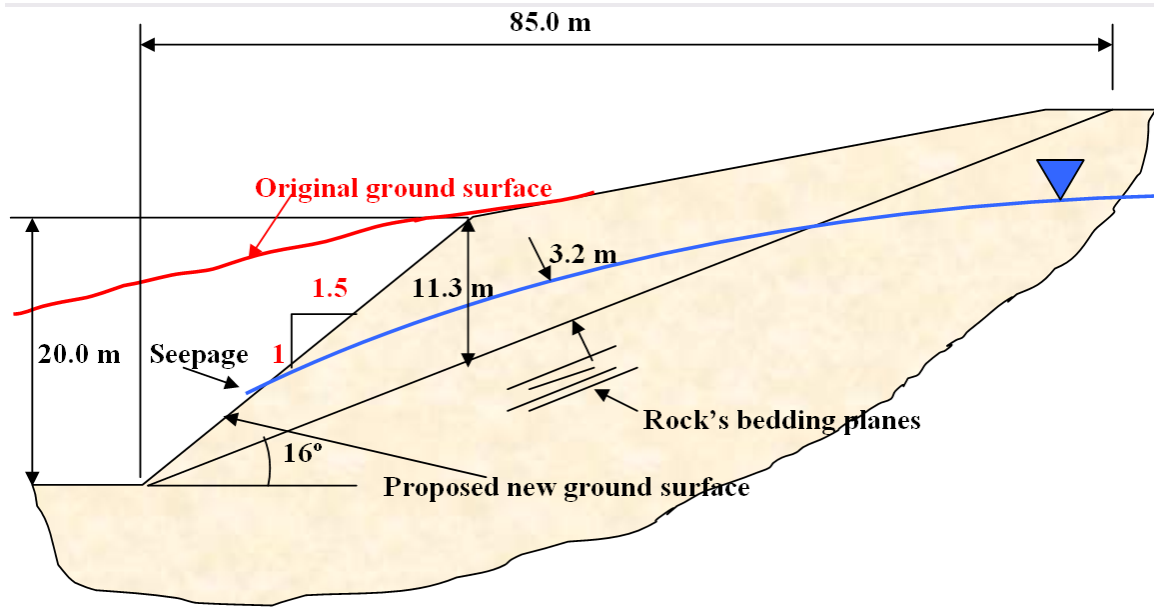
$$FS = \frac{\text{Resisting Forces}}{\text{Driving Forces}} = \frac{cL + [(W) \cos \alpha] \tan \phi}{(W) \sin \alpha} =$$

$$FS = \frac{\left(22 \frac{\text{kN}}{\text{m}^2}\right)(88.4 \text{ m}) + \left[\left(9,650 \frac{\text{kN}}{\text{m}}\right) \cos 16^\circ\right] \tan 30^\circ}{\left(9,650 \frac{\text{kN}}{\text{m}}\right) \sin 16^\circ} = 2.7 > 2 \quad \text{OK}$$

**\*Slope-02: Same as Slope-01 but with a raising WT.**

(Revision: Oct.-08)

In the previous problem the slope appeared to be stable with a factor of safety = 2.7. What happens to that factor of safety if the water table rises to the level shown below? Use a unit weight of  $20.1 \text{ kN/m}^3$ , and bedding strength parameters are reduced by the effective parameters of  $c' = 15 \text{ kPa}$  and  $\phi' = 20^\circ$ .



**Solution:**

The weight  $W$  of the rock triangle per unit width is still  $9,650 \frac{\text{kN}}{\text{m}}$

The length  $L$  of the slip plane is still  $88.4 \text{ m}$ .

The pore water pressure is based on an estimate of its value along the length  $L$ , at water depth  $z_w$  above the plane that range from 0 to  $3.2 \text{ m}$ ; conservatively,

$$u = \gamma_w z_w = \left( 9.81 \frac{\text{kN}}{\text{m}^3} \right) (3.2 \text{ m}) = 31.4 \text{ kPa}$$

$$FS = \frac{\text{Resisting Forces}}{\text{Driving Forces}} = \frac{c' L + [(W) \cos \alpha - u] \tan \phi}{(W) \sin \alpha} =$$

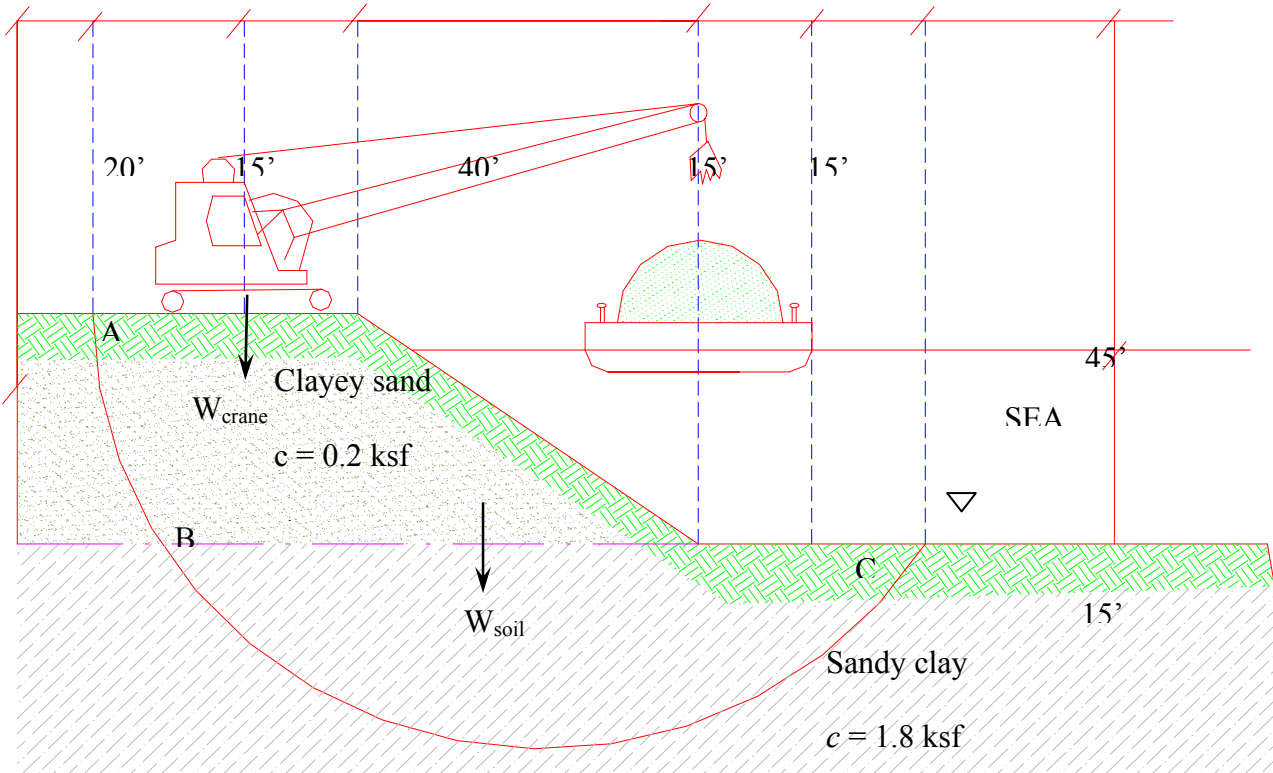
$$FS = \frac{\left( 15 \frac{\text{kN}}{\text{m}^2} \right) (88.4 \text{ m}) + \left[ \left( 9,650 \frac{\text{kN}}{\text{m}} \right) \cos 16^\circ - (31.4 \text{ kPa}) \right] \tan 20^\circ}{\left( 9,650 \frac{\text{kN}}{\text{m}} \right) \sin 16^\circ} = 1.76 < 2 \quad \text{NG}$$

The computed factor of safety of 1.76 is less than the minimum acceptable value of 2, therefore this design is NOT acceptable. *Notice that a rising WT decreases the stability of the slope.*

### \*Slope-03: Is a river embankment safe with a large crane?

(Revision: Oct.-08)

Determine if the work site shown below is safe, provided you consider the minimum acceptable factor of safety for the man-made waterfront slope shown below to be 2. Assume the arc radius is 80 feet; the circular lengths are AB = 22 feet and BC = 102 feet. The total weight of the soil per unit width are  $W_{soil} = 205$  kips and  $W_{crane} = 70$  kips. The site is located in a seismic zone with a seismic coefficient of 0.15.



#### Solution:

$$M_r = R[S_1(AB) + S_2(BC)] = R[(C_1' + \sigma_1' \tan \phi_1)AB + (C_2' + \sigma_2' \tan \phi_2)BC]$$

$$= 80'[(0.2 + 0.125(8')(\tan 40^\circ))22' + (1.8 + (0.130 - 0.064)(21)(\tan 15^\circ))(102)] = 80[23k + 221k] = \underline{19,500 \text{ k-ft}}$$

$$M_o = Wb_1 - W_{wH}(d_1) - W_{wV}(b_2) + Vb_3 + \frac{W_{ae}(d_2)}{g}$$

$$= 205(40) - (1/2)(0.064)(15)^2 - (0.064)[30(15) + (1/2)(40)(15)]15 + 70(55) + 205(0.15)(50)$$

$$M_o = 8,200 - 7.2 - 720 + 3850 + 1540 = \underline{12,900 \text{ k-ft}}$$

$$\text{Therefore: } FS = M_r/M_o = 19,500/12,900 = \underline{1.51}$$

➔ **Not Good!**

$$\text{Removing Crane} \rightarrow M_o = 9,050 \text{ k-ft}$$

$$\text{Therefore: } FS = M_r/M_o = 19,500/9,050 = 2.15$$

➔ **GOOD!**

**\*Slope-04: Simple method of slices to find the FS.**

(Revision: Oct.-08)

The stability of a slope was analyzed by the method of slices. One of the trial curved surfaces through the soil mass yielded the shearing and normal components of each slice as listed below. The curved length of the trial curved surface is 40 feet, the soil parameters are  $c = 225 \text{ lb/ft}^2$  and  $\phi = 15^\circ$ . Determine the factor of safety along this trial surface.

**Solution:**

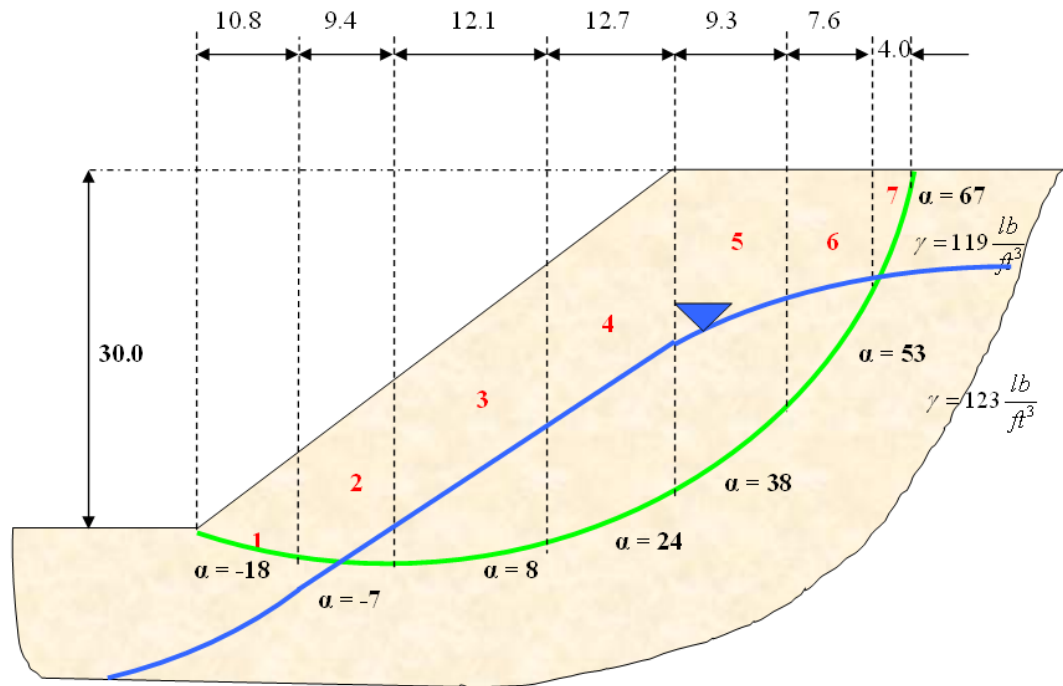
Slice Number	Shearing Component ( $W \sin \alpha$ ) (lb/ft)	Normal Component ( $W \cos \alpha$ ) (lb/ft)
1	-38	306
2	-74	1410
3	124	2380
4	429	3050
5	934	3480
6	1570	3540
7	2000	3210
8	2040	2190
9	766	600
$\Sigma = 7,751 \text{ lb/ft}$		$\Sigma = 20,166 \text{ lb/ft}$

$$FS = \frac{cL + \sum (W \cos \alpha) \tan n\phi}{\sum W \sin \alpha} = \frac{(225 \text{ psf})(40 \text{ ft}) + 20,166 \text{ plf}}{7,751 \text{ plf}} = 1.86 < 2 \quad \text{NG}$$

## **\*\*Slope-05: Method of slices to find the factor of safety of a slope with a WT.**

(Revision: Oct.-08)

A 30 ft tall, 1.5H:1V slope is to be built as shown below. The soil is homogeneous, with  $c' = 400 \frac{lb}{ft^2}$  and  $\phi' = 29^\circ$ . The unit weight is 119 pcf above the groundwater table, and 123 pcf below. Using the ordinary method of slices, compute the factor of safety along the trial circle.



### **Solution:**

Weights:

$$\frac{W_1}{b} = 10.8 \left( \frac{10.3}{2} \right) 119 = 6,620 \frac{lb}{ft}$$

$$\frac{W_2}{b} = 9.4 \left( \frac{10.3 + 12.5}{2} \right) 119 + 9.4 \left( \frac{5.2}{2} \right) 123 = 15,800 \frac{lb}{ft}$$

$$\frac{W_3}{b} = 12.1 \left( \frac{12.5 + 14.6}{2} \right) 119 + 12.1 \left( \frac{5.2 + 10.0}{2} \right) 123 = 30,800 \frac{lb}{ft}$$

$$\frac{W_4}{b} = 2.9 \left( \frac{5.0}{2} \right) 17.0 + 7.1 \left( \frac{12.9 + 8.0}{2} \right) 17.8 = 1,620 \frac{lb}{ft}$$

$$\frac{W_5}{b} = 9.3 \left( \frac{16.8 + 12.8}{2} \right) 119 + 9.3 \left( \frac{10.7 + 7.3}{2} \right) 123 = 39,900 \frac{lb}{ft}$$

$$\frac{W_6}{b} = 7.6 \left( \frac{12.8 + 9.9}{2} \right) 119 + 7.6 \left( \frac{7.3}{2} \right) 123 = 26,700 \frac{lb}{ft}$$

$$\frac{W_7}{b} = 4.0 \left( \frac{9.9}{2} \right) 119 = 2,400 \frac{lb}{ft}$$

Average pore water pressure at base of each slice:

$$u_1 = 0$$

$$u_2 = \left( \frac{5.2}{2} \right) 62.4 = 160 \frac{lb}{ft^2}$$

$$u_3 = \left( \frac{5.2 + 10.0}{2} \right) 62.4 = 470 \frac{lb}{ft^2}$$

$$u_4 = \left( \frac{10.0 + 10.7}{2} \right) 62.4 = 650 \frac{lb}{ft^2}$$

$$u_5 = \left( \frac{10.7 + 7.3}{2} \right) 62.4 = 560 \frac{lb}{ft^2}$$

$$u_6 = \left( \frac{7.3}{2} \right) 62.4 = 230 \frac{lb}{ft^2}$$

$$u_7 = 0$$

Slice	$\frac{W}{b}(lb)$	$\alpha$ (Deg)	$c' \left( \frac{lb}{ft^2} \right)$	$\phi$ (Deg)	$u \left( \frac{lb}{ft^2} \right)$	$l$ (ft)	$c'l + \left( \frac{W}{b} \right) (\cos \alpha - ul) \tan \phi'$	$\frac{W}{b}(lb)$
1	6620	-18	400	29	0	11.4	8,000	-2,000
2	15,800	-7	400	29	160	9.5	11,700	-1,900
3	30,800	8	400	29	470	12.2	18,600	4,300
4	39,900	24	400	29	650	13.9	20,800	16,200
5	26,700	38	400	29	560	11.8	12,700	16,400
6	13,700	53	400	29	230	12.6	8,000	10,900
7	24,00	67	400	29	0	10.2	4,600	2,200
							$\Sigma = 84,400$	$\Sigma = 46,100$

Therefore, the factor of safety is,

$$FS = \frac{c'l + \left( \frac{W}{b} \right) (\cos \alpha - ul) \tan \phi'}{\left( \frac{W}{b} \right)} = \frac{84,400}{46,100} = 1.83 < 2 \quad NG$$

**Note how slices # 1 and 2 have a negative  $\alpha$  because they are inclined backwards.**

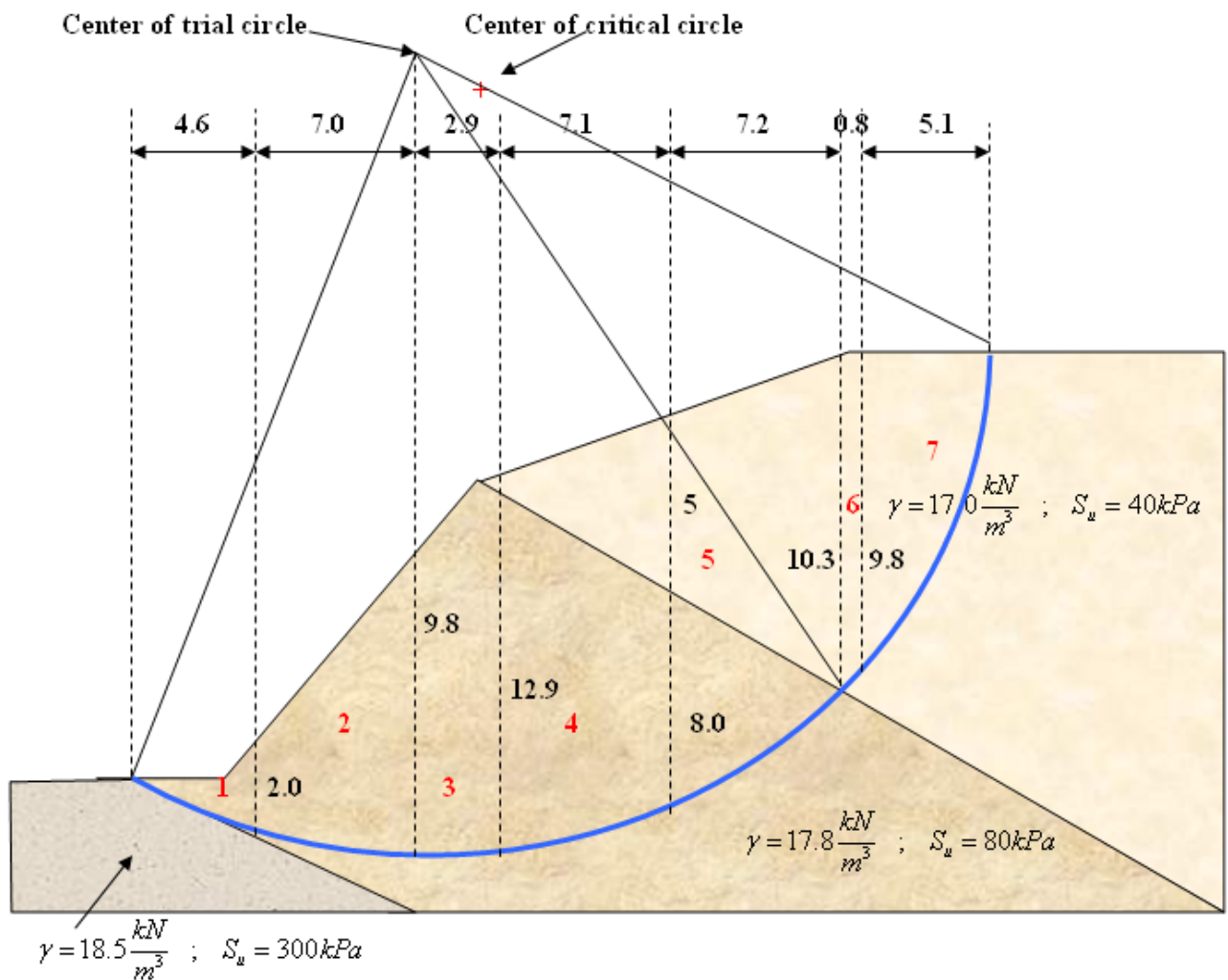
## **\*\*Slope-06: Swedish slip circle solution of a slope stability.**

(Revision: Oct.-08)

Using the Swedish slip circle method, compute the factor of safety along the trial circle shown in the figure below.

### **Solution:**

Divide the slide mass into vertical slices as shown. One of the slice borders should be directly below the center of the circle (in this case, the border between slices 2 and 3). For convenience of computations, also draw a slice border wherever the slip surface intersects a new soil stratum and whenever the ground surface has a break in slope. Then, compute the weight and moment arm for each slice using simplified computations as follows:



**Solution:**

*Weights*

$$\frac{W_1}{b} = 4.6 \left( \frac{2.0}{2} \right) 17.8 = 80 \frac{kN}{m}$$

$$\frac{W_2}{b} = 7.0 \left( \frac{2.0 + 9.8}{2} \right) 17.8 = 130 \frac{kN}{m}$$

$$\frac{W_3}{b} = 2.9 \left( \frac{9.8 + 12.9}{2} \right) 17.8 = 590 \frac{kN}{m}$$

$$\therefore \frac{W_4}{b} = 2.9 \left( \frac{5.0}{2} \right) 17.0 + 7.1 \left( \frac{12.9 + 8.0}{2} \right) 17.8 = 1620 \frac{kN}{m}$$

$$\frac{W_5}{b} = 7.2 \left( \frac{5.0 + 10.3}{2} \right) 17.0 + 7.2 \left( \frac{8.0}{2} \right) 17.8 = 1450 \frac{kN}{m}$$

$$\frac{W_6}{b} = 0.8 \left( \frac{10.3 + 9.8}{2} \right) 17.0 = 140 \frac{kN}{m}$$

$$\frac{W_7}{b} = 5.1 \left( \frac{9.8}{2} \right) 17.0 = 420 \frac{kN}{m}$$

Moment arms:

$$d_1 = -7.0 - \frac{4.6}{3} = -8.5 \text{ m}$$

$$d_2 = \frac{-7.0}{2} = -3.5 \text{ m}$$

$$d_3 = \frac{2.9}{2} = 1.5 \text{ m}$$

$$d_4 = 2.9 + \frac{7.1}{2} = 6.5 \text{ m}$$

$$d_5 = 2.9 + 7.1 + \frac{7.1}{2} = 10.9 \text{ m}$$

$$d_6 = 2.9 + 7.1 + 7.2 + \frac{0.8}{2} = 17.6 \text{ m}$$

$$d_7 = 2.9 + 7.1 + 7.2 + 0.8 + \frac{5.1}{3} = 19.7 \text{ m}$$

Slice	$S_u$ (kPa)	$\theta$ (Deg)	$S_u \theta$	$\frac{W}{b} \left( \frac{kN}{m} \right)$	$d$ (m)	$\left( \frac{W}{b} \right) d$
1	80	76	6080	80	-8.5	-690
2				130	-3.5	-450
3				390	1.5	890
4				1620	6.5	10,530
5				1450	10.9	15,800
6	40	30	1200	140	17.6	2,460
7				420	19.7	8,280
			$\Sigma = 7280$			$\Sigma = 36,830$

$$FS = \frac{\pi R^2}{180} \frac{\sum S_u \theta}{\sum \left( \frac{W}{b} \right) d} = \frac{\pi (23.6)^2}{180} \frac{7,280}{36,830} = 1.92 < 2 \quad \text{Not Good}$$

# Chapter 14

## Statistical Analysis of Soils

*Symbols for the Statistical Analysis of Soils*

# Chapter 15

## Lateral Pressures from Soils

### Symbols for Lateral Pressures from Soils

$D_x \rightarrow$  Diameter of the grains distributed (represent % finer by weight).

$e \rightarrow$  The voids ratio.

$G_s \rightarrow$  Specific gravity of the solids of a soil.

$H \rightarrow$  Maximum depth of excavation or thickness of a soil layer.

$h_{\text{soil}} \rightarrow$  depth of the soil.

$i_{\text{critical}} \rightarrow$  Critical hydraulic gradient.

$k_H \rightarrow$  Horizontal permeability.

$k_V \rightarrow$  Vertical permeability.

$u \rightarrow$  pore water pressure.

$\sigma' \rightarrow$  Effective stress.

$\sigma_V' \rightarrow$  Vertical Effective stress.

$g' \rightarrow$  Bouyant unit weight of a soil.

$\gamma_{\text{SAT}} \rightarrow$  Saturated unit weight of a soil

$\gamma_W \rightarrow$  Unit weight of water.

$V_W \rightarrow$  Volume of water.

$w \rightarrow$  water content.

## Formulas and Figures for Lateral Stresses.

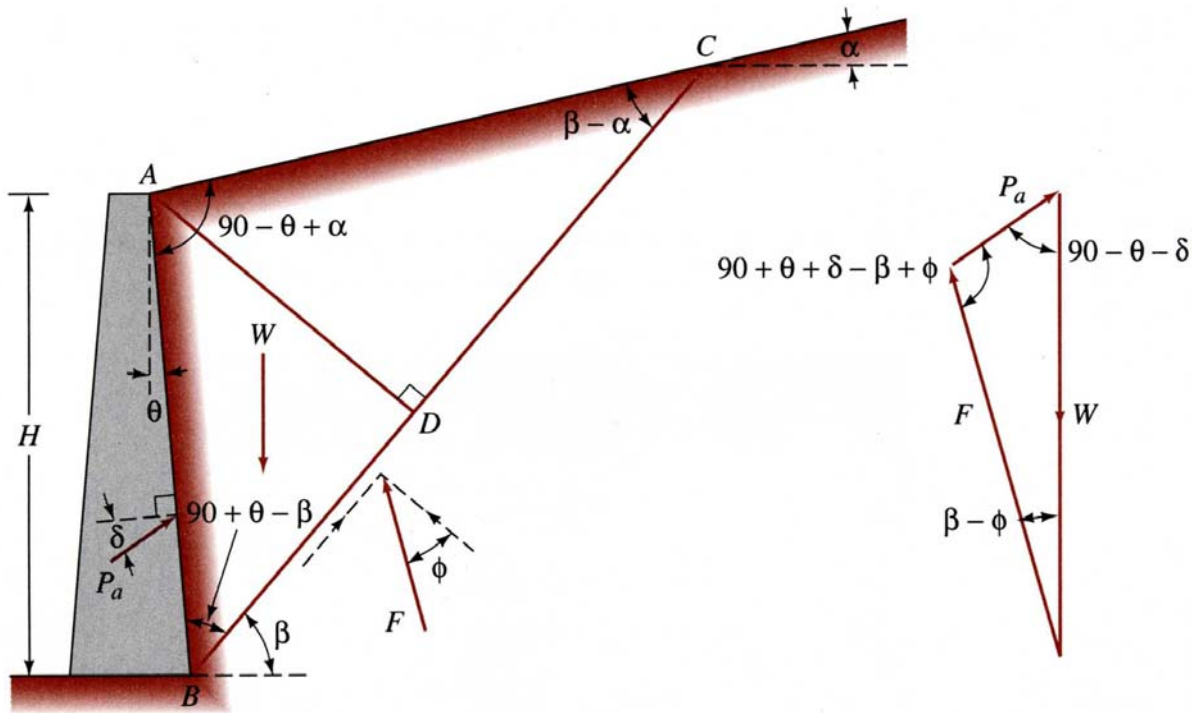


Figure for symbols used in the Coulomb earth pressures.

Coulomb's lateral pressure coefficients  $K_a$  and  $K_p$ .

$$K_a = \frac{\cos^2(\phi - \theta)}{\cos^2\theta \cos(\delta + \theta) \left[ 1 + \sqrt{\frac{\sin(\delta + \phi) \sin(\phi - \alpha)}{\cos(\delta + \theta) \cos(\theta - \alpha)}} \right]^2}$$

$$K_p = \frac{\cos^2(\phi + \theta)}{\cos^2\theta \cos(\delta - \theta) \left[ 1 - \sqrt{\frac{\sin(\phi - \delta) \sin(\phi + \alpha)}{\cos(\delta - \theta) \cos(\alpha - \theta)}} \right]^2}$$

$K_a$  for the case of  $\theta = 0^\circ$  and  $\alpha = 0^\circ$ .

$\downarrow \phi$ (deg)	$\delta$ (deg) $\rightarrow$					
	0	5	10	15	20	25
28	0.3610	0.3448	0.3330	0.3251	0.3203	0.3186
30	0.3333	0.3189	0.3085	0.3014	0.2973	0.2956
32	0.3073	0.2945	0.2853	0.2791	0.2755	0.2745
34	0.2827	0.2714	0.2633	0.2579	0.2549	0.2542
36	0.2596	0.2497	0.2426	0.2379	0.2354	0.2350
38	0.2379	0.2292	0.2230	0.2190	0.2169	0.2167
40	0.2174	0.2089	0.2045	0.2011	0.1994	0.1995
42	0.1982	0.1916	0.1870	0.1841	0.1828	0.1831

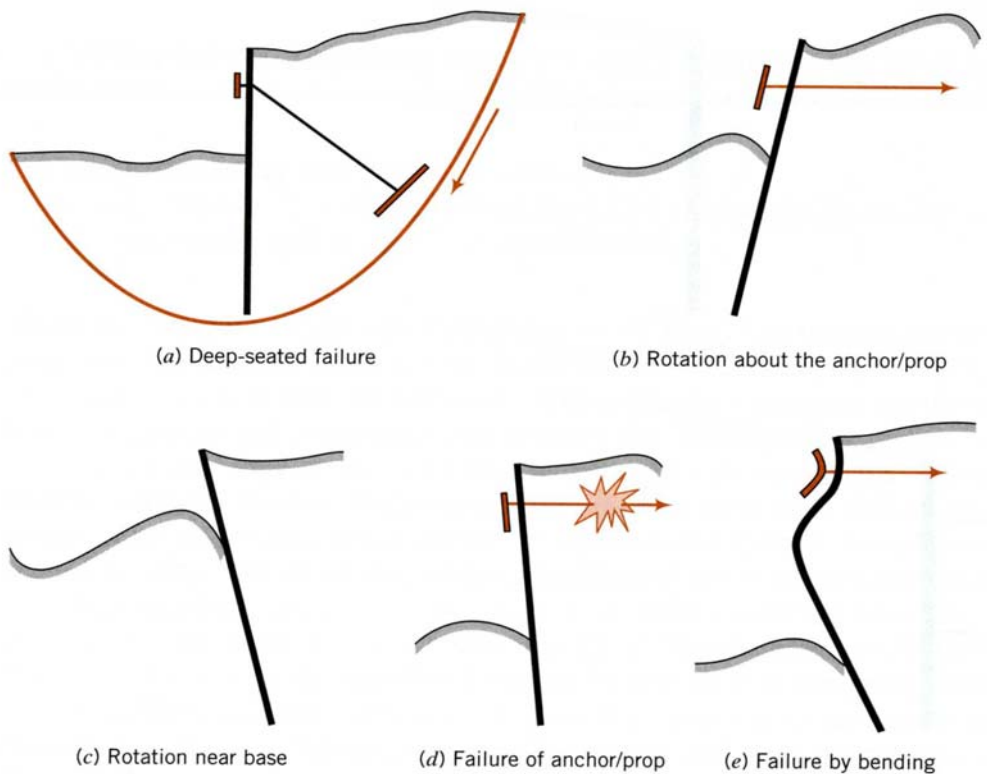
$K_a$  for the case where  $\delta = 2/3 \phi$ .

$\alpha$ (deg)	$\phi$ (deg)	$\theta$ (deg)					
		0	5	10	15	20	25
0	28	0.3213	0.3588	0.4007	0.4481	0.5026	0.5662
	30	0.2973	0.3349	0.3769	0.4245	0.4794	0.5435
	32	0.2750	0.3125	0.3545	0.4023	0.4574	0.5220
	34	0.2543	0.2916	0.3335	0.3813	0.4367	0.5017
	36	0.2349	0.2719	0.3137	0.3615	0.4170	0.4825
	38	0.2168	0.2535	0.2950	0.3428	0.3984	0.4642
	40	0.1999	0.2361	0.2774	0.3250	0.3806	0.4468
5	28	0.1840	0.2197	0.2607	0.3081	0.3638	0.4303
	30	0.3431	0.3845	0.4311	0.4843	0.5461	0.6191
	32	0.3165	0.3578	0.4043	0.4575	0.5194	0.5926
	34	0.2919	0.3329	0.3793	0.4324	0.4943	0.5678
	36	0.2691	0.3097	0.3558	0.4088	0.4707	0.5443
	38	0.2479	0.2881	0.3338	0.3866	0.4484	0.5222
	40	0.2282	0.2679	0.3132	0.3656	0.4273	0.5012
10	28	0.2098	0.2489	0.2937	0.3458	0.4074	0.4814
	30	0.1927	0.2311	0.2753	0.3271	0.3885	0.4626
	32	0.3702	0.4164	0.4686	0.5287	0.5992	0.6834
	34	0.3400	0.3857	0.4376	0.4974	0.5676	0.6516
	36	0.3123	0.3575	0.4089	0.4683	0.5382	0.6220
	38	0.2868	0.3314	0.3822	0.4412	0.5107	0.5942
	40	0.2633	0.3072	0.3574	0.4158	0.4849	0.5682
15	28	0.2415	0.2846	0.3342	0.3921	0.4607	0.5438
	30	0.2214	0.2637	0.3125	0.3697	0.4379	0.5208
	32	0.2027	0.2441	0.2921	0.3487	0.4164	0.4990
	34	0.4065	0.4585	0.5179	0.5869	0.6685	0.7671
	36	0.3707	0.4219	0.4804	0.5484	0.6291	0.7266
	38	0.3384	0.3887	0.4462	0.5134	0.5930	0.6895
	40	0.3091	0.3584	0.4150	0.4811	0.5599	0.6554
20	28	0.2823	0.3306	0.3862	0.4514	0.5295	0.6239
	30	0.2578	0.3050	0.3596	0.4238	0.5006	0.5949
	32	0.2353	0.2813	0.3349	0.3981	0.4740	0.5672
	34	0.2146	0.2595	0.3119	0.3740	0.4491	0.5416
	36	0.4602	0.5205	0.5900	0.6715	0.7690	0.8810
	38	0.4142	0.4728	0.5403	0.6196	0.7144	0.8303
	40	0.3742	0.4311	0.4968	0.5741	0.6667	0.7800
	28	0.3388	0.3941	0.4581	0.5336	0.6241	0.7352
	30	0.3071	0.3609	0.4233	0.4970	0.5857	0.6948
	32	0.2787	0.3308	0.3916	0.4637	0.5587	0.6580
	34	0.2529	0.3035	0.3627	0.4331	0.5185	0.6243
	36	0.2294	0.2784	0.3360	0.4050	0.4889	0.5931
	38						
	40						

$K_p$  for  $\theta = 0^\circ$  and  $\alpha = 0^\circ$ .

$\downarrow \phi$ (deg)	$\delta$ (deg) $\rightarrow$				
	0	5	10	15	20
15	1.698	1.900	2.130	2.405	2.735
20	2.040	2.313	2.636	3.030	3.525
25	2.464	2.830	3.286	3.855	4.597
30	3.000	3.506	4.143	4.977	6.105
35	3.690	4.390	5.310	6.854	8.324
40	4.600	5.590	6.946	8.870	11.772

Failure modes for flexible walls (sheet-piling).

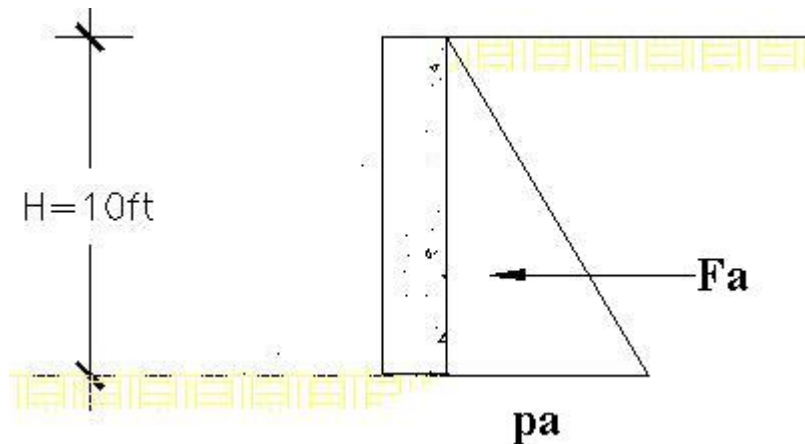


**\*Lateral-01: A simple wall subjected to an active pressure condition.**

(Revision: Sept.-08)

Consider a small 10-foot tall and 3 feet thick concrete retaining wall. The backfill behind the wall will be from local sandy gravel with a dry unit weight of 115 pcf and an angle of internal friction of 30 degrees. The wall will not have to retain water.

Estimate, (a) the lateral force on the wall from the backfill in an active pressure condition, (b) its stability against overturning, and (c) its stability against sliding (use a Factor of Safety  $\geq 2$ ).



**Solution:**

(a) The Rankine active earth pressure coefficient is,  $K_a = \tan^2 \left( 45^\circ - \frac{\phi}{2} \right) = \tan^2 \left( 45^\circ - \frac{30^\circ}{2} \right) = 0.33$

The lateral pressure at the bottom of the wall is  $p_a = \gamma h K_a = (0.115 \text{ kcf})(10 \text{ ft})(0.33) = 0.38 \text{ ksf}$

The force against the wall is  $F_a = \frac{1}{2}(p_a)h = (0.5)(0.38)(10) = 1.9 \text{ kips per foot of wall}$

(b) The stability of the wall against overturning is found by taking moments about the point "O" at the toe of the wall,

$$\text{Factor of Safety (FS)} = \frac{\text{resisting moment}}{\text{overturning moment}} = \frac{(3')(10')(1')(0.150 \text{ kcf})(1.5 \text{ ft})}{(1.9 \text{ kips})(10/3 \text{ ft})} = 1.07 < 2 \quad \text{NG}$$

(c) The stability of the wall against sliding towards the left is found by,

$$\text{Factor of Safety (FS)} = \frac{\text{resisting force}}{\text{driving force}} = \frac{(3')(10')(1')(0.150 \text{ kcf})(\tan 30^\circ)}{(1.9 \text{ kips})} = 1.37 < 2 \quad \text{NG}$$

**\*Lateral–02: Compare the Rankine and Coulomb lateral coefficients.**

(Revision: Sept-2008)

- (a) Compare the Rankine and Coulomb lateral earth pressure coefficients for a wall that retains a granular backfill soil with  $\phi = 35^\circ$ ,  $\delta = 12^\circ$ ,  $\theta = 0^\circ$  and  $\alpha = 20^\circ$ . (Note:  $\delta$  is the angle of friction between the soil and the backside of the wall;  $\alpha$  is the angle of the slope for the backfill behind the wall and  $\theta$  is the back of the wall's angle with respect to the vertical).
- (b) What is the passive earth force on the wall at failure if the wall is 10 m high,  $\gamma = 18.1 \text{ kN/m}^3$  and  $c = 9 \text{ kN/m}^2$ ?

**Solution:**

- (a) Rankine's active and passive earth pressure coefficients,

$$K_a = \tan^2\left(45^\circ - \frac{\phi}{2}\right) = \tan^2\left(45^\circ - \frac{35^\circ}{2}\right) = 0.271$$

$$K_p = \tan^2\left(45^\circ + \frac{\phi}{2}\right) = \tan^2\left(45^\circ + \frac{35^\circ}{2}\right) = 3.690 \quad \text{Note that} \quad K_a = \frac{1}{K_p}$$

Coulomb's active and passive earth pressure coefficients,

$$K_a = \frac{\cos^2(\phi - \theta)}{\cos^2\theta \cos(\delta + \theta) \left[1 + \sqrt{\frac{\sin(\delta + \phi)\sin(\phi - \alpha)}{\cos(\delta + \theta)\cos(\theta - \alpha)}}\right]^2} = \frac{\cos^2(35 - 0)}{\cos^2 0 \cos(12 + 0) \left[1 + \sqrt{\frac{\sin(12 + 35)\sin(35 - 20)}{\cos(12 + 0)\cos(0 - 20)}}\right]^2} = 0.323$$

$$K_p = \frac{\cos^2(\phi + \theta)}{\cos^2\theta \cos(\delta - \theta) \left[1 - \sqrt{\frac{\sin(\phi - \delta)\sin(\phi + \alpha)}{\cos(\delta - \theta)\cos(\alpha - \theta)}}\right]^2} = \frac{\cos^2(35 + 0)}{\cos^2 0 \cos(12 - 0) \left[1 - \sqrt{\frac{\sin(35 - 12)\sin(35 + 20)}{\cos(12 - 0)\cos(20 - 0)}}\right]^2} = 3.517$$

When  $\alpha = 0^\circ$ ,  $\theta = 0^\circ$  and  $\delta = 0^\circ$  the Coulomb formula becomes identical to Rankine's.

- (b) Therefore, the Rankine coefficient is 3.690 versus 3.517 for Coulomb's. Using these values, the total passive force
- $F_p$
- on the wall per unit length is,

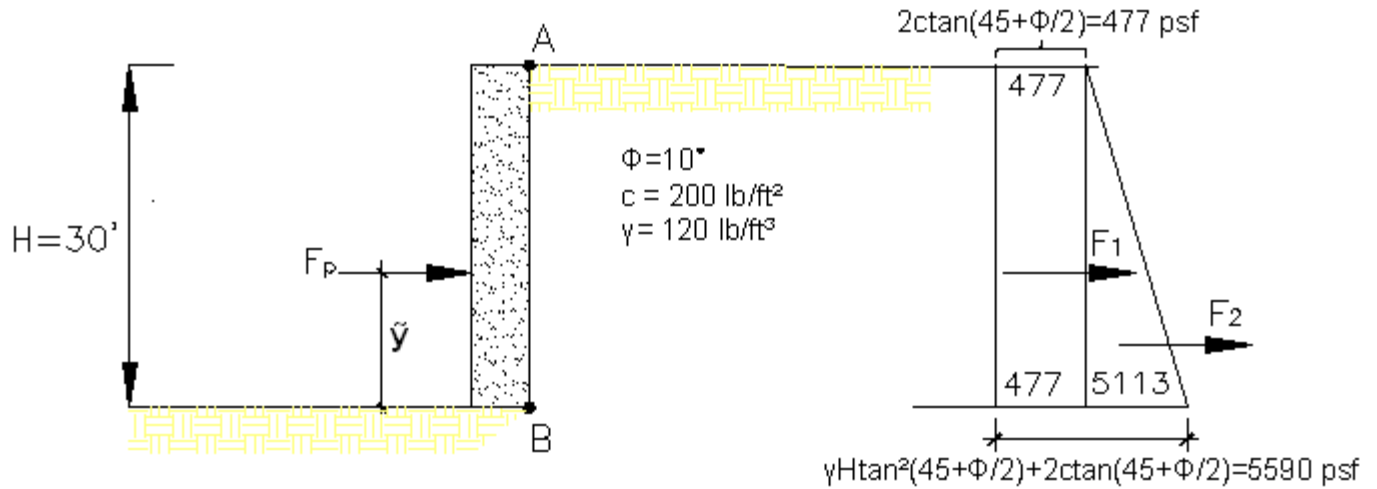
$$\text{Rankine's } F_p = 0.5\gamma h^2 K_p + 2ch\sqrt{K_p} = (0.5)(18.1)(10)^2 (3.690) + 2(9)(10)\sqrt{3.690} = 3,685 \text{ kN/m}^2$$

$$\text{Coulomb's } F_p = 0.5\gamma h^2 K_p + 2ch\sqrt{K_p} = (0.5)(18.1)(10)^2 (3.517) + 2(9)(10)\sqrt{3.517} = 3,520 \text{ kN/m}^2$$

**\*Lateral-03: Passive pressures using the Rankine theory.**

(Revision: Sept-08)

Using the Rankine method, find the magnitude and location of the passive pressure force  $F_p$  with respect to the heel of the wall (point B), exerted upon a temporary retaining wall by a large jacking system (which is not shown in the figure).



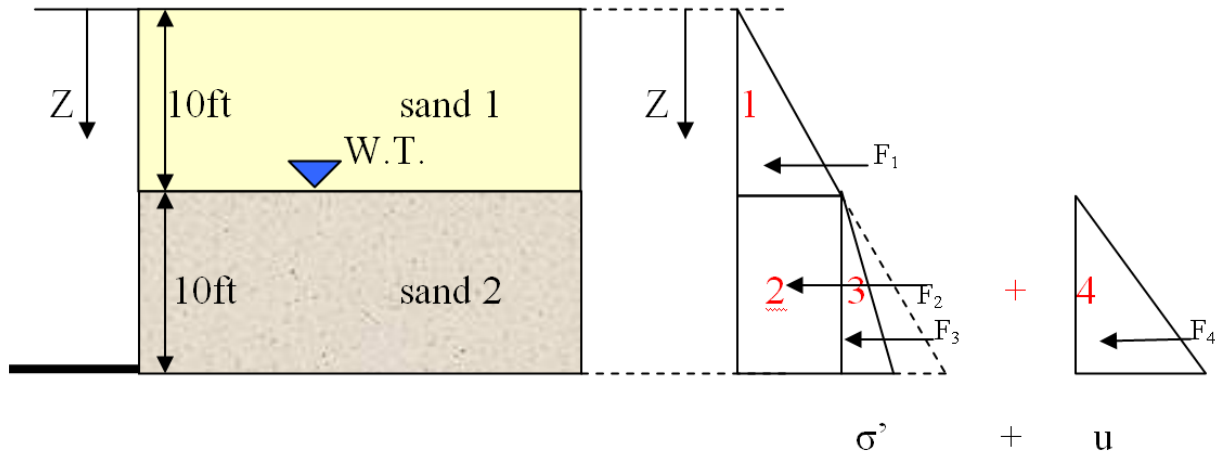
**Solution:**

**\*Lateral-04: The “at-rest” pressure upon an unyielding wall.**

(Revision: Sept-08)

Find the lateral “at-rest” force  $F_o$  on the wall and its location with respect to the top of the wall.

Given: Sand #1 has a unit weight of 105 pcf,  $c = 0$  psf and  $\phi = 30^\circ$ ; Sand #2 has a unit weight of 122 pcf,  $c = 0$  psf and  $\phi = 30^\circ$ .



**Solution:**

From Jaky's empirical relation,  $K_o = 1 - \sin \phi' = 1 - \sin 30^\circ = 0.50$

at  $z = 0$  feet  $\sigma' = 0$  ksf, because there is no surcharge loading upon the surface of Sand #1.

at  $z = 10$  feet  $\sigma'_h = K_o \sigma'_v = (0.5)(0.105 \text{ kcf})(10 \text{ ft}) = 0.525$  ksf

at  $z = 20$  feet  $\sigma'_h = (0.5)[(0.105)(10) + (0.122 - 0.0624)10] = 0.823$  ksf

$$\sigma_w = \gamma_w h = (0.0624 \text{ pcf})(10 \text{ ft}) = 0.624 \text{ ksf}$$

$$F_o = \sum_i f_i = F_1 + F_2 + F_3 + F_4 = \frac{1}{2}(0.525)(10) + (0.525)(10) + \frac{1}{2}(0.302)(10) + \frac{1}{2}(0.624)(10)$$

$$F_o = 2.63 + 5.25 + 1.49 + 3.12 = 12.5 \frac{\text{kip}}{\text{ft}}$$

$$z = \frac{(2.63)(6.67) + (5.25)(15) + (1.49)(16.67) + (3.12)(16.67)}{12.5 \text{ kip}} = \frac{173.1 \text{ kip-ft}}{12.5 \text{ kip}}$$

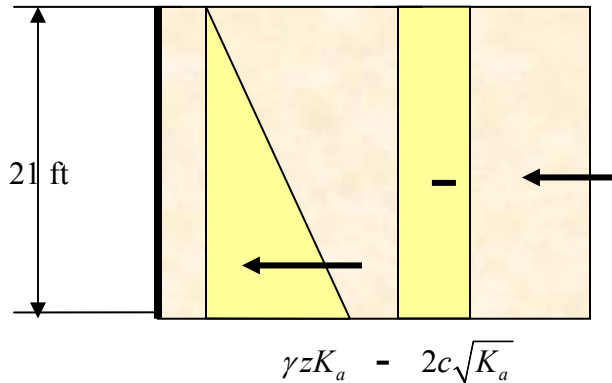
$z = 13.8$  ft from the top of the wall.

**\*Lateral-05: The contribution of cohesion to reduce the force on the wall.**

(Revision: Sept-08)

A 21 foot high retaining wall supports a purely cohesive soil ( $\phi = 0^\circ$ ) with a cohesion of 630 psf and a unit weight of 113 pcf. Find:

- The Rankine active earth pressure on the wall.
- Estimate the depth of separation of the clay from the wall, and (c) find the lateral force upon the wall whilst considering the clay separation.



**Solution:**

- a) The coefficient of active earth pressure is,

$$K_a = \tan^2 \left( 45^\circ - \frac{\phi}{2} \right) = \tan^2 \left( 45^\circ - \frac{0^\circ}{2} \right) = \tan^2 45^\circ = 1$$

The net active earth pressure  $p_a$  on the wall is,

$$p_a = \sigma_3 = \gamma h K_a - 2c\sqrt{K_a}$$

$$= (0.113 \text{ kcf})(21 \text{ ft})(1) - 2(0.630 \text{ ksf})\sqrt{1} = 2.37 - 1.26 = 1.11 \text{ ksf}$$

- b) The crack stops where the pressure is zero,  $p_a = 0$ ,

$$p_a = \gamma h K_a - 2c\sqrt{K_a} \quad \therefore \quad \gamma h K_a = 2c\sqrt{K_a}$$

$$\therefore h_{crack} = \frac{2c\sqrt{K_a}}{\gamma K_a} = \frac{2c}{\gamma\sqrt{K_a}} = \frac{2(0.630 \text{ ksf})}{(0.113 \text{ kcf})(1)} = 11.2 \text{ feet}$$

- c) The total (Rankine) active earth force upn the wall  $F_a$  is,

$$F_a = \frac{1}{2} \gamma H^2 K_a - 2cH\sqrt{K_a}$$

but there is no contact on the wall where the tension crack exists, therefore

$$F_a = \frac{1}{2} (\gamma H K_a - 2c\sqrt{K_a}) \left( H - \frac{2c}{\gamma\sqrt{K_a}} \right) = \frac{1}{2} \gamma H^2 - 2cH\sqrt{K_a} + \frac{2c^2}{\gamma}$$

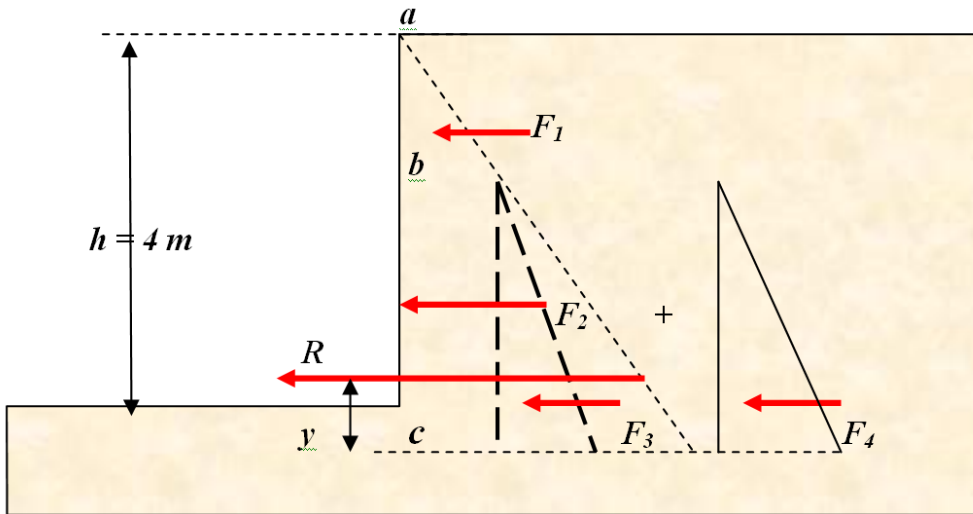
$$F_a = \frac{1}{2} (0.113 \text{ kcf})(21 \text{ ft})^2 (1) - 2(0.63 \text{ ksf})(21 \text{ ft})(1) + \frac{2(0.63 \text{ ksf})^2}{(0.113 \text{ kcf})} = 5.48 \text{ k / ft of wall}$$

**\*\*Lateral-06: The effect of a rising WT upon a wall's stability.**

(Revision: Sept-08)

A 4 m wall retains a dry sand backfill with a unit weight of  $18.3 \text{ kN/m}^3$ , an angle of internal friction of  $36^\circ$  and a porosity of 31%. The backfill is fully drained through weep holes.

- 1) What is the magnitude of the backfill force on a 1 m wide slice of wall if it is not allowed to deflect?
- 2) What is the magnitude of the backfill force on the same 1 m wide slice, if the wall does deflect enough to develop a Rankine active earth pressure condition?
- 3) What is the new force on the wall, and its location from its heel, if the wall's weep holes are clogged and the water table now rises to within 1 m of the ground surface behind the wall?



**Solution:**

- 1) No deflection of the wall means the soil is "at rest" and  $K_o = 1 - \sin \phi = 1 - \sin 36^\circ = 0.41$

$$\text{The force } F_o = \frac{1}{2} \gamma_d h^2 K_o = \frac{1}{2} \left( 18.3 \frac{\text{kN}}{\text{m}^3} \right) (4\text{m})^2 (0.41) = 60 \text{ kN per meter of wall}$$

- 2) When the wall deflects to the left sufficiently to develop an active pressure condition,

$$K_a = \tan^2 \left( 45^\circ - \frac{\phi}{2} \right) = \tan^2 \left( 45^\circ - \frac{36^\circ}{2} \right) = 0.26$$

$$\text{The force } F_a = \frac{1}{2} \gamma_d h^2 K_a = \frac{1}{2} \left( 18.3 \frac{\text{kN}}{\text{m}^3} \right) (4\text{m})^2 (0.26) = 38 \text{ kN per meter of wall}$$

- 3) The buoyant weight  $\gamma'$  of the flooded sand is,

$$\gamma' = \gamma_{sat} - \gamma_w = \gamma_d - n\gamma_w - \gamma_w = 18.3 + (0.31)(9.81) - (9.81) = 11.5 \frac{\text{kN}}{\text{m}^3}$$

The stress at point "a" is  $\sigma_a = 0$ , and at "b" which is 1 meter below the surface,

$$\sigma_b = \gamma_d h K_a = (18.3 \frac{\text{kN}}{\text{m}^3})(1\text{m})(0.26) = 4.8 \text{ kN/m}^2 \quad \therefore F_1 = \frac{1}{2}(4.8 \text{ kN/m}^2)(1\text{m}) = 2.4 \text{ kN/m}$$

$$\sigma_{bc} = \gamma_d h K_a = (18.3 \frac{kN}{m^3})(1m)(0.26) = 4.8 \text{ kN} / m^2 \quad \therefore F_2 = (4.8 \text{ kN} / m^2)(3m) = 14.4 \text{ kN} / m$$

$$\sigma_c = \gamma' h K_a = (11.5 \frac{kN}{m^3})(3m)(0.26) = 9.0 \text{ kN} / m^2 \quad \therefore F_3 = \frac{1}{2}(9.0 \text{ kN} / m^2)(3m) = 13.5 \text{ kN} / m$$

The water pressure and force,

$$\sigma_w = \gamma_w h = (9.81 \frac{kN}{m^3})(3m) = 29.4 \text{ kN} / m^2 \quad \therefore F_4 = \frac{1}{2}(29.4 \text{ kN} / m^2)(3m) = 44.1 \text{ kN} / m$$

Therefore

$$R = \sum_{i=1}^4 F = 74.4 \text{ kN} / m$$

The location of the resultant is  $\bar{y}$ ,

$$\bar{y} = \frac{F_1 d_1 + F_2 d_2 + F_3 d_3 + F_4 d_4}{R} = \frac{(2.4)(3.33m) + (14.4)(1.5m) + (13.5)(1m) + (44.1)(1)}{74.4}$$

$$\bar{y} = 1.17m \text{ from the bottom of the wall.}$$

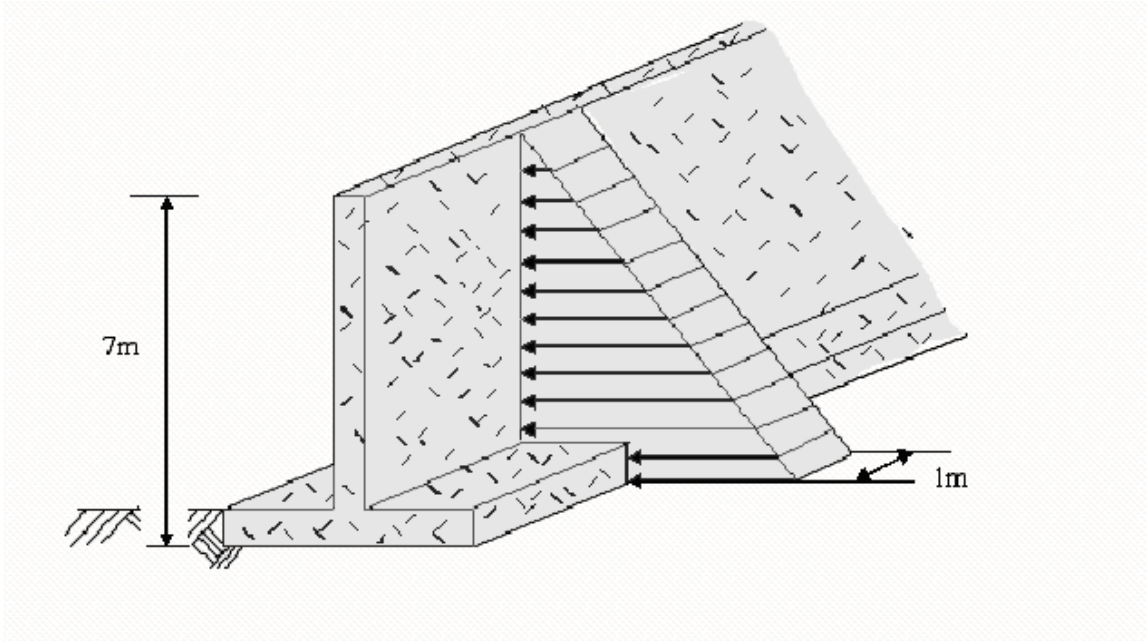
The percent increase in load upon the wall due to flooding is,

$$\Delta F = \frac{(74.4 \text{ kN} - 38 \text{ kN})}{38 \text{ kN}} = 96\% \text{ increase.}$$

**\*Lateral-07: The effects of soil-wall friction upon the lateral pressure.**

(Revision: Sept-08)

A 7.0 m high retaining wall has a horizontal backfill of dry sand with a unit weight of  $17.2 \text{ kN/m}^3$  and an angle of internal friction  $\phi = 32^\circ$ . The wall is cast-in-place concrete, with a friction angle  $\delta = 20^\circ$ . Ignoring the effect of the passive pressure upon the toe of the footing, find the magnitude of the active earth force upon a length of wall equal to 3.5 m assuming Rankine conditions.



**Solution:**

The force applied to the wall first requires the coefficient of active earth pressure,

$$K_a = \tan^2(45^\circ - \frac{\phi}{2}) = \tan^2(45^\circ - \frac{32^\circ}{2}) = 0.307$$

The horizontal force  $F_H$  per unit width of wall is,

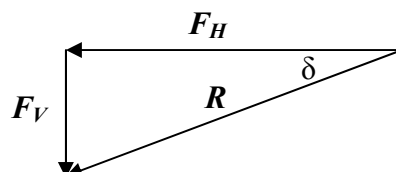
$$F_H = \frac{1}{2} \gamma h^2 K_a = \frac{1}{2} (17.2 \text{ kN/m}^3) (7 \text{ m})^2 (0.307) = 129.5 \text{ kN/m}$$

The  $F_H$  is related to the total force  $R$  on the wall as a function of the angle of wall friction  $\delta$ ,

$$F_H = R \cos \delta \quad \therefore \quad R = \frac{F_H}{\cos \delta} = \frac{129.5 \text{ kN/m}}{\cos 20^\circ} = 138 \text{ kN/m}$$

We are asked what is the total force every 3.5 m,

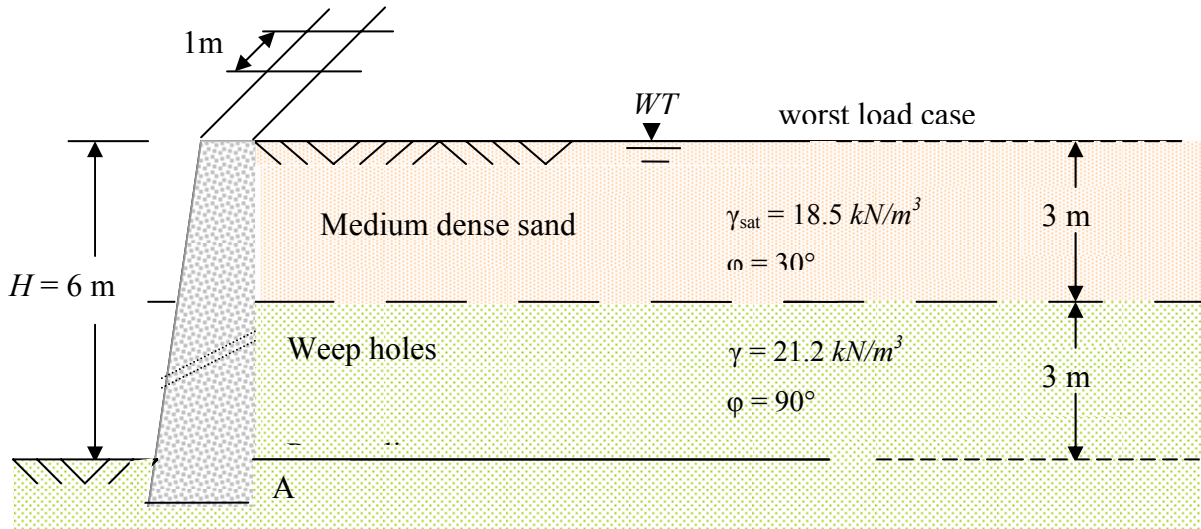
$$\text{Total Active Force every 3.5 m} = (138)(3.5 \text{ m}) = 482 \text{ kN}$$



### \*Lateral-08: What happens when the lower stratum is stronger?

(Revision: Sept-08)

Calculate the active force  $F_a$  and its location  $\bar{y}$  with respect to the heel of the 6 m wall (point A), for the worst case (clogged weep holes).



*Solution.*

The worst active pressure load occurs when the water table raises to the top of the wall.

$$K_{a \text{ sand}} = \tan^2(45^\circ - \frac{\phi}{2}) = \tan^2(45^\circ - \frac{30^\circ}{2}) = 0.333$$

$$K_{a \text{ limestone}} = \tan^2(45^\circ - \frac{\phi}{2}) = \tan^2(45^\circ - \frac{90^\circ}{2}) = 0 \quad \therefore \text{the limestone does not load the wall.}$$

The  $\phi = 90^\circ$  is really a combination of shear and cohesion ("cementation").

$$p_1 = \gamma' h_1 K_a = (\gamma_{SAT} - \gamma_w) h_1 K_a = (18.5 - 9.8)(3)(0.33) = 8.7 \text{ kN/m}^2$$

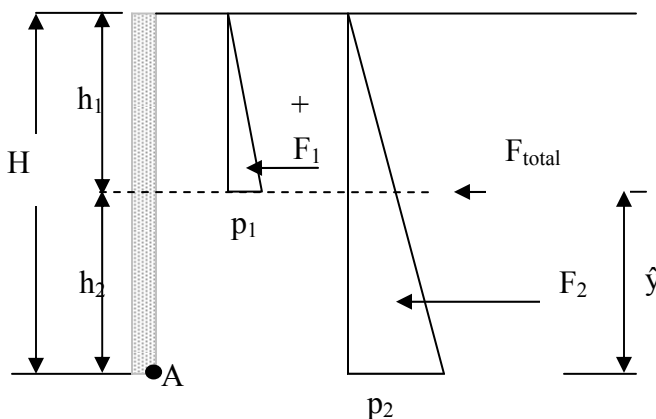
$$p_2 = \gamma_w H = (9.8)(6 \text{ m}) = 58.8 \text{ kN/m}^2$$

$$F_1 = \frac{1}{2} p_1 h_1 = (0.5)(8.7)(3 \text{ m}) = 13.1 \text{ kN/m}$$

$$F_2 = \frac{1}{2} p_2 H = (0.5)(58.8)(6 \text{ m}) = 176.4 \text{ kN/m}$$

$$F_{total} = 189.5 \text{ kN/m}$$

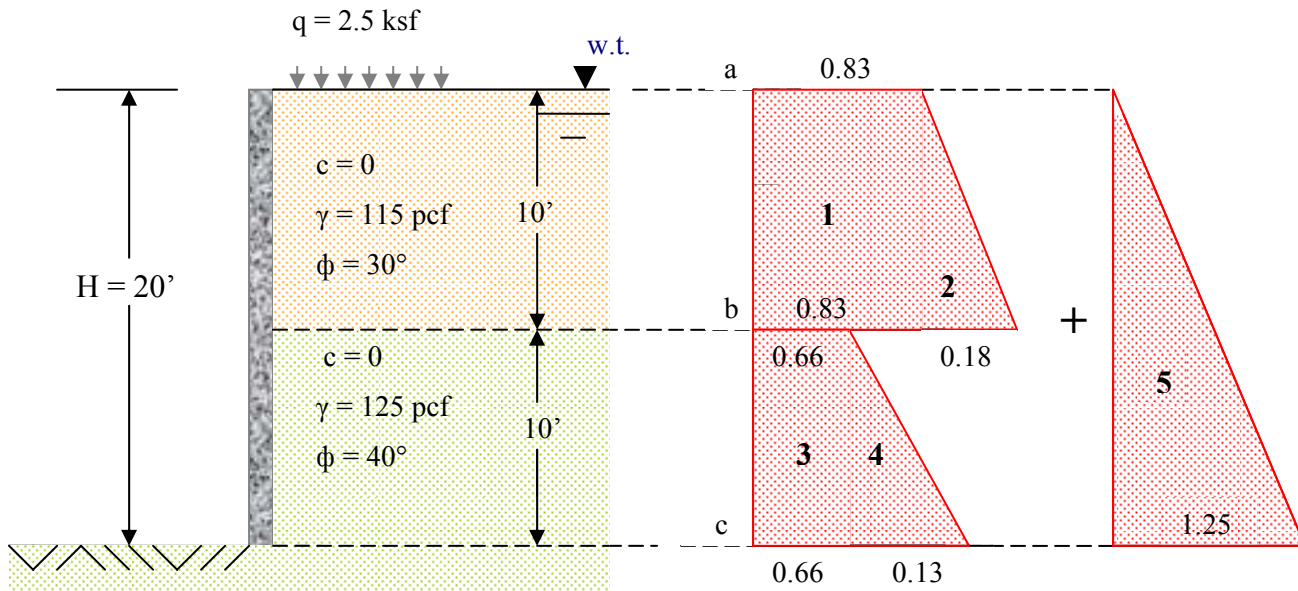
$$\text{The location } \bar{y} = \frac{y_1 F_1 + y_2 F_2}{F_{total}} = \frac{(4 \text{ m})(13.1) + (2 \text{ m})(176.4)}{(189.5)} = 2.1 \text{ m from A.}$$



### \*Lateral-09: Strata with different parameters.

(Revised Oct-09)

Draw the pressure diagram on the wall in an active pressure condition, and find the resultant  $F_{\text{total}}$  on the wall and its location with respect to the top of the wall.



#### Solution:

Step 1

$$K_{a1} = \tan^2 (45^\circ - 30^\circ/2) = 0.333$$

$$K_{a2} = \tan^2 (45^\circ - 40^\circ/2) = 0.217$$

Step 2

The stress on the wall at point a is:  $p_a = q K_{a1} = (2.5) (0.333) = 0.83 \text{ ksf}$

The stress at b (within the top stratum) is:  $p_{b'} = (q + \gamma' h) K_{a1}$

$$= [2.5 + (0.115 - 0.0624) (10')] [0.333] = 1.01 \text{ ksf}$$

The stress at b (within bottom stratum) is:  $p_{b'} = (q + \gamma' h) K_{a2}$

$$= [2.5 + (0.115 - 0.0624) (10')] [0.217] = 0.66 \text{ ksf}$$

The stress at point c is:  $p_c = [q + (\gamma' h)_1 + (\gamma' h)_2] K_{a2}$

$$= [2.5 + (0.115 - 0.0624) (10') + (0.125 - 0.0624)(10')] [0.217] = 0.79 \text{ ksf}$$

The pressure of the water upon the wall is:  $p_w = \gamma_w h = (0.0624) (20') = 1.25 \text{ ksf}$

Step 3

The forces from each area:

$$F_1 = (10') (0.83) = 8.30 \text{ kips/ft}$$

$$F_2 = \frac{1}{2} (10')(0.18) = 0.90 \text{ kips/ft}$$

$$F_3 = (10') (0.66) = 6.60 \text{ kips/ft}$$

$$F_4 = \frac{1}{2} (10')(0.13) = 0.65 \text{ kips/ft}$$

$$F_5 = \frac{1}{2} (1.25) (20') = 12.5 \text{ kips/ft}$$

$$\mathbf{F_{total} = 29.0 \text{ kips/ft}}$$

Step 4

The location of forces  $\hat{y}$  is at:

$$\hat{y} = \frac{5 \cdot 8.3 + \frac{20}{3} \cdot 0.9 + 15 \cdot 6.6 + \frac{50}{3} \cdot 0.65 + \frac{40}{3} \cdot 12.5}{29} = 0.66 \text{ ksf}$$

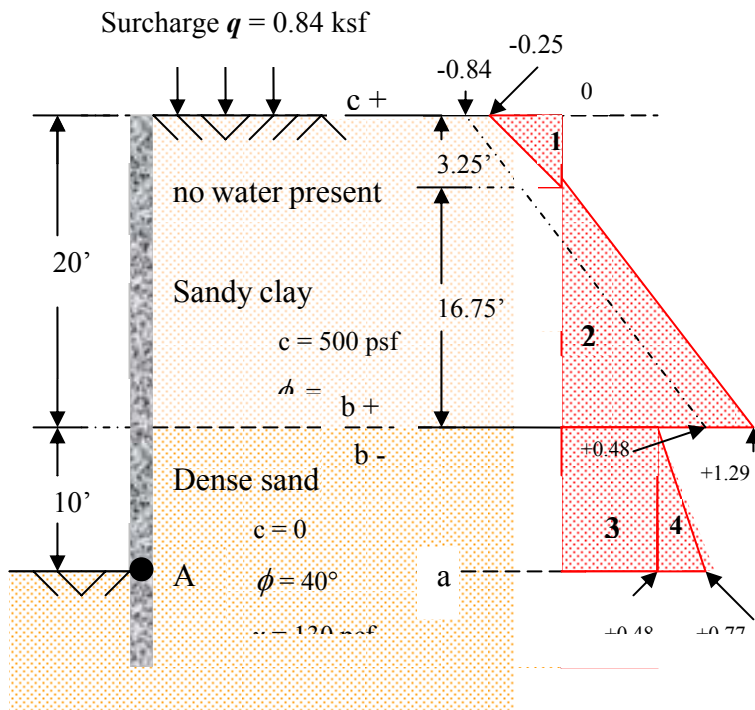
The stress at point  $c$  is:  $\mathbf{\hat{y} = 11.2 \text{ feet from top of wall}}$

**\*Lateral-10: The effects of a clay stratum at the surface.**

The sheet pile wall shown below is flexible enough to permit the retained soil to develop an active earth pressure condition. Calculate the magnitude of the resultant  $F_{total}$  of the active force above the point "A" upon the wall. Assume Rankine conditions.

**Solution:**

Notice that the vertical pressure diagram will always increase in magnitude, but the horizontal pressures are governed by the  $K_a$  coefficient, which may increase or decrease the pressures on the wall.



Lateral load from the surcharge

$$\sigma_{c+} = K_{a1} q = (0.70)(0.84 \text{ ksf}) = 0.59 \text{ ksf}$$

$$\sigma_{c-} = -2c\sqrt{K_{a1}} = -2(0.5)\sqrt{0.70} = -0.84 \text{ ksf}$$

$$\therefore \sum \sigma_c = 0.59 - 0.84 = -0.25 \text{ ksf}$$

$$\begin{aligned} \sigma_{b+} &= K_{a1} \gamma h - 2c\sqrt{K_{a1}} + q K_a \\ &= (0.7)(0.11)(20') - 2(0.50)\sqrt{0.70} \\ &= 1.29 \text{ ksf} \end{aligned}$$

$$\sigma_b = K_{a2} \gamma h - 0 = (0.22) (0.11) (20') = 0.48 \text{ ksf}$$

$$\sigma_a = 0.48 + K_{a2} \gamma h = 0.48 + (0.22)(0.13)(10') = 0.48 + 0.29 = 0.77 \text{ ksf}$$

$$K_{a1} = \tan^2 (45^\circ - \phi/2) = \tan^2 40^\circ = 0.70$$

$$K_{a2} = \tan^2 (45^\circ - 40^\circ / 2) = \tan^2 25^\circ = 0.22$$

$$F_1 = \frac{1}{2} (-0.25)(3.25') = -0.41 \text{ k/ft (tension).}$$

$$F_2 = \frac{1}{2} (1.29)(16.75') = +10.80 \text{ k/ft}$$

$$F_3 = (0.48)(10') = +4.80 \text{ k/ft}$$

$$F_4 = \frac{1}{2} (0.29)(10') = \underline{+1.45 \text{ k/ft}}$$

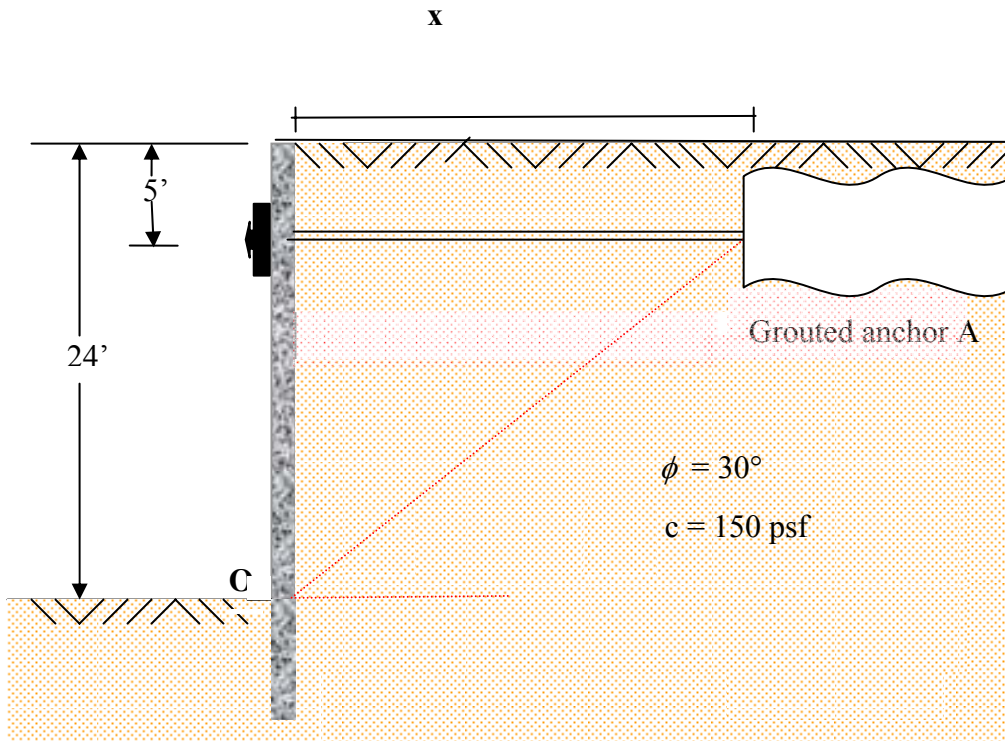
$$F_{total} = \underline{\underline{+16.6 \text{ kip/ft}}}$$

### **\*\*Lateral-11: Anchoring to help support a wall.**

(Revision: Sept.-08)

The wall shown below will be used to retain the sides of an excavation for the foundations of a large building. The engineer has decided to use earth anchors in lieu of braces or rakers to stabilize the wall.

- (1) What is the minimum distance  $x$  from the anchor to behind the wall?
- (2) What is your recommended factor of safety for the anchor? What is an economical load for the anchor?



#### **Solution:**

(1) The anchor must be beyond the passive slip plane, or  $(x) \tan 30^\circ = 19'$  or  $x = 33$  feet.

(2)  $K_a = \tan^2(45^\circ - \phi/2) = 0.33$  and  $K_p = \tan^2(45^\circ + \phi/2) = 3.0$

The active force upon the wall per unit width  $F_a$  is:

$$F_a = \frac{1}{2} \gamma H^2 K_a - 2cH \sqrt{K_a} = \frac{1}{2} (0.105) (24')^2 (0.33) - 2(0.15)(24) \sqrt{0.33} = 5.84 \text{ kip/ft} \quad \text{with the force located at } \hat{y} = \frac{1}{3}(19') = 6.33' \text{ above point O (note that the tensile portion does not load the wall).}$$

The potential passive failure force (from the anchor) on the wall  $F_p$  is:

$$F_p = \frac{1}{2} \gamma H^2 K_p + 2cH \sqrt{K_p} = \frac{1}{2} (0.105) (24')^2 (3) + 2(0.15)(24) \sqrt{3} = 103 \text{ kip/ft}$$

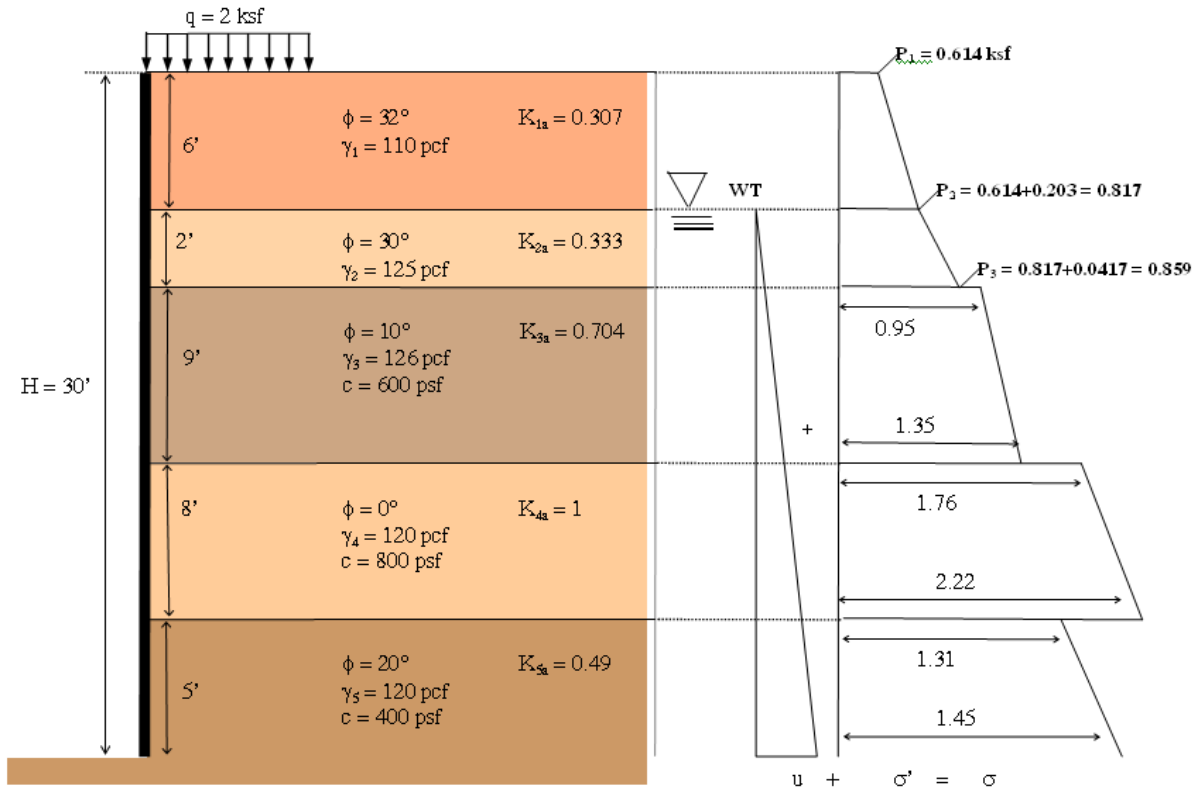
The factor of safety should be the same for an active failure as a passive failure. Therefore, a simple equation could be written as,  $F_a(FS) = \frac{F_p}{(FS)}$  or  $(FS)^2 = \frac{F_p}{F_a} = \frac{103 \text{ kips}}{5.84 \text{ kips}} = 17.6 \therefore FS = 4.2$

Note that this corresponds to a load in the anchor of  $(5.84)(4.2) = \underline{24.5 \text{ kips/ft}}$  (which is the same as using the passive force  $= (103)/(4.2) = 24.5 \text{ kips/ft}$ ). The horizontal spacing of the anchors is not influenced by this analysis, and depends on cost factors. A common spacing would be 10 feet, which means  $A = 245 \text{ kips}$ .

## **\*\*Lateral-12: The effect of five strata have upon a wall.**

(Revision Oct-09)

**Plot the pressure diagram and find the resultant force F and its location under an active pressure condition.**



$$\text{At } h=0' \quad p_1 = q K_{1a} = (2) (0.307) = 0.614 \text{ ksf}$$

$$\text{at } h = -6' \quad \Delta p_2 = \gamma_1 h K_{1a} = (0.110)(6) (0.307) = 0.203 \text{ ksf}$$

$$\text{at } h = -8' \quad \Delta p_3 = (\gamma_2 - \gamma_w) h K_{2a} = (0.125 - 0.0624)(2)(0.333) = 0.417 \text{ ksf}$$

$$\begin{aligned} \text{at } h = -(8+dh)' \quad &= [q + (\gamma_1) 6' + (\gamma_2 - \gamma_w) 2'] K_{3a} - 2c \sqrt{K_{3a}} \quad \text{from } p = \gamma h K_a - 2c\sqrt{K_a} \\ &= [2 + (0.11)6' + (0.125 - 0.0624)2'](0.704) - 2(0.6)(0.84) = 0.95 \text{ ksf} \end{aligned}$$

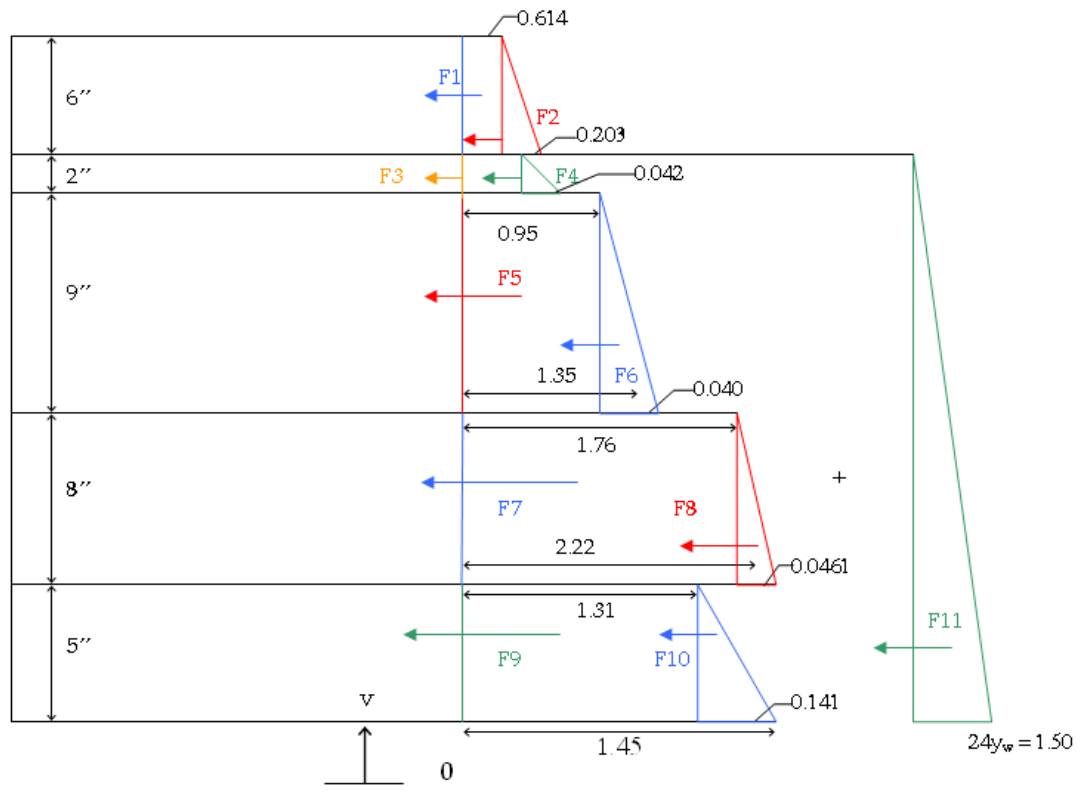
$$\text{at } h = -17' \quad \Delta p_4 = (\gamma_3 - \gamma_w) h K_{3a} = (0.126 - 0.0624)(9)(0.704) = 0.403 \text{ ksf} \quad \therefore 0.95 + 0.403 = 1.35 \text{ ksf}$$

$$\text{at } h = -(17 + dh)' \quad = [2 + 0.66 + 0.125 + (0.0626) (9)](1) - 2(0.8)(1) = 1.76 \text{ ksf}$$

$$\text{at } h = -25' \quad \Delta p_5 = (\gamma_4 - \gamma_w) h K_{4a} = (0.120 - 0.0624)(8)(1) = 0.46 \text{ ksf} \quad \therefore 1.76 + 0.46 = 2.22 \text{ ksf}$$

$$\text{at } h = -(25 + dh)' \quad = [2 + 0.66 + 0.125 + 0.572 + 8(0.120 - 0.0624)](0.49) - 2(0.4)(0.7) = 1.13 \text{ ksf}$$

$$\text{at } h = -30' \quad \Delta p_6 = (\gamma_5 - \gamma_w) h K_{5a} = (0.120 - 0.0624)(5) (0.49) = 0.141 \text{ ksf} \quad \therefore 1.31 + 0.14 = 1.45 \text{ ksf}$$



$$F1 = (0.614)(6) = 3.68 \text{ kips}$$

The resultant R is,  **$R = \sum Fi = 57.1 \text{ kips}$**

$$F2 = 0.5(0.203)(6) = 0.61 \text{ kips}$$

$$F3 = (0.817)(2) = 1.63 \text{ kips}$$

$$F4 = 0.5(0.042)(2) = 0.04 \quad \text{The location of R is } \dots \sum M_0 = 0 \text{ (about 0)}$$

$$F5 = (0.95)(9) = 8.55 \text{ kips} \quad 57.09(y) = (3.68)(27) + (0.61)(26) = (1.63)(23)$$

$$F6 = 0.5(0.40)(9) = 1.80 \text{ kip}$$

$$F7 = (1.758)(8) = 14.1 \text{ kips}$$

$$F8 = 0.5(0.461)(8) = 1.84 \text{ kips}$$

$$F9 = (1.31)(5) = 6.55 \text{ kips}$$

$$F10 = 0.5(0.141)(5) = 0.35 \text{ kips}$$

$$F11 = 0.51(1.50)(24) = 18.0 \text{ kips}$$

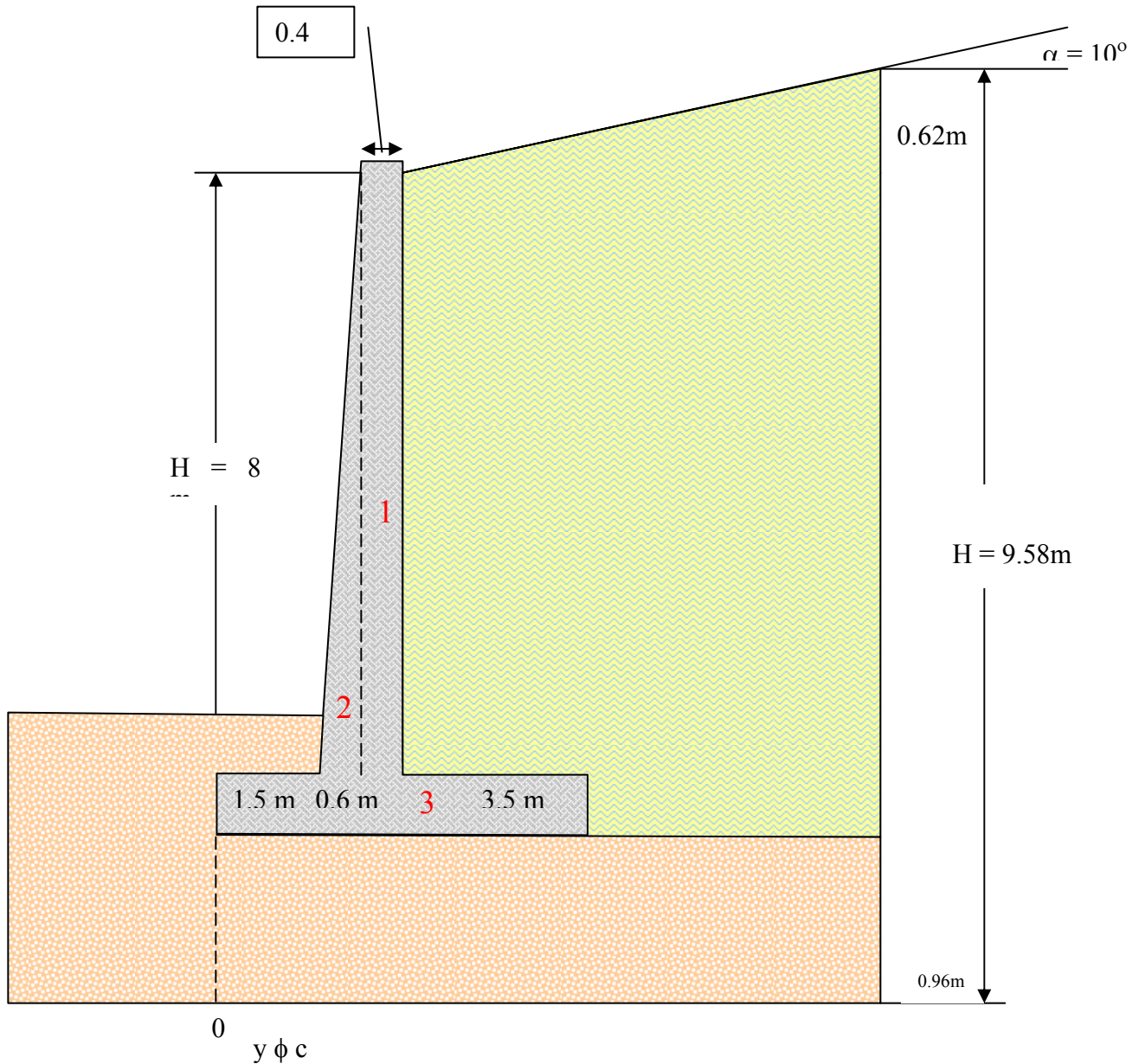
$$57.1 \text{ kips}$$

$$\therefore y = 611 / 57.1 = \underline{\underline{10.7 \text{ feet above "0"}}$$

**\*\*Lateral-13: The stability of a reinforced concrete wall.**

(Revised Oct-09)

Calculate the Factor of Safety against, (a) overturning, (b) sliding, and (c) bearing capacity failures.



$$\gamma_1 = 16.8 \frac{\text{kN}}{\text{m}^3}$$

$$\gamma_{\text{conc}} = 23.6 \frac{\text{kN}}{\text{m}^3}$$

$$\phi_1 = 32^\circ$$

$$c_1 = 0$$

$$\gamma_2 = 17.6 \frac{\text{kN}}{\text{m}^3}$$

$$\phi_2 = 28^\circ$$

$$c_2 = 30 \frac{\text{kN}}{\text{m}^2}$$

$$K_a = \cos \alpha \frac{\cos \beta - \sqrt{\cos^2 \beta - \cos^2 \phi'}}{\cos \beta + \sqrt{\cos^2 \beta - \cos^2 \phi'}} = \cos 10^\circ \frac{\cos 10^\circ - \sqrt{\cos^2 10^\circ - \cos^2 32^\circ}}{\cos 10^\circ + \sqrt{\cos^2 10^\circ - \cos^2 32^\circ}} = 0.322$$

$$F_a = (1/2) H^2 \gamma_1 K_a = (1/2)(9.58 \text{ m})^2 (16.8 \text{ kN/m}^3)(0.322) = 248 \text{ kN/m}$$

$$F_v = F_a \sin 10^\circ = (248 \text{ kN/m})(0.174) = 43.1 \text{ kN/m}$$

$$F_h = F_a \cos 10^\circ = (248 \text{ kN/m})(0.985) = 244 \text{ kN/m}$$

a) The factor of safety against overturning is found by taking moments about point “O”.

The resisting moment against overturning is  $M_R$ ,

$$\begin{aligned} M_R &= 23.6 \text{ kN/m}^3 [(0.4\text{m})(8\text{m})(1.90\text{m}) + (1/2)(0.2\text{m})(8\text{m})(1.63\text{m}) + (0.96\text{m})(5.6\text{m})(2.8\text{m})] (1\text{m}) \\ &\quad + 16.8 \text{ kN/m}^3 [(3.5\text{m})(8\text{m})(3.85\text{m}) + (1/2)(0.617\text{m})(3.5\text{m})(4.43\text{m})] (1\text{m}) \\ &\quad + 43.1 \text{ kN/m} (5.6\text{m})(1\text{m}) = 2661 \text{ kN-m} \end{aligned}$$

and the overturning moment is  $M_O = F_h (1/3) H' = 244 \text{ kN/m} (9.58\text{m})(1/3) = 777 \text{ kN-m}$

$$\boxed{FS_O = M_R / M_O = 3.42}$$

b) The factor of safety ( $FS_S$ ) against sliding failure,

$$K_1 = K_2 = 2/3$$

$$K_p = \tan^2(45^\circ + 28^\circ/2) = 2.77$$

$$F_p = (1/2) \gamma_2 H^2 K_p + 2 c_2 H \sqrt{K_p}$$

$$= (0.5)(2.77)(17.6 \text{ kN/m}^3)(1.75 \text{ m})^2 + (2)(30 \text{ kN/m}^2) \sqrt{2.77} (1.75 \text{ m}) = 249 \text{ kN/m}$$

the driving force  $= F_h = 244 \text{ kN/m}$

the resisting force  $= F_R = \Sigma V \tan(2/3)(28) + (5.6)(2/3)(30) = 355 \text{ kN/m}$

$$\boxed{FS_S = F_h / F_R = 1.46}$$

c) the factor of safety ( $FS_{BC}$ ) against a bearing capacity failure,

$$e = \frac{B}{2} - \frac{\Sigma Mr - \Sigma Mo}{\Sigma V} = e = 2.8 - \frac{2661 - 777}{749} = 0.31 \text{ m}$$

$$q_{\text{toe}} = \frac{\Sigma V}{B} \left[ 1 + \frac{6e}{B} \right] = \frac{749}{5.6} \left[ 1 + \frac{6(0.31)}{5.6} \right] = 178 \text{ kN/m}^2$$

$$B' = B - 2e = 5.6\text{m} - 2(0.31\text{m}) = 4.98 \text{ m}$$

$$q_u = (1/2) \gamma_2 B' N_\gamma F_{\gamma d}^s F_{\gamma i}^d + C_2 N_c F_{cd}^s F_{ci}^d + q N_q F_{qd}^s F_{qi}^d$$

$$q = \gamma_2 D = (17.6)(1.75) = 30.8 \text{ kN/m}^2$$

$$\text{using } \phi_2 = 28^\circ \rightarrow N_c = 25.8$$

$$N_q = 14.7$$

$$N_\gamma = 16.7$$

$$F_{cd} = 1 + 0.4 \frac{Df}{B'} = 1 + 0.4 \frac{1.75}{4.98} = 1.14$$

$$\psi = \tan^{-1} \left( \frac{Ph}{\Sigma V} \right) = \tan^{-1}(244 / 749) = 18.04^\circ$$

$$F_{qi} = F_{ci} = \left( 1 - \frac{\psi}{90} \right)^2 = 0.96$$

$$F_{\gamma i} = \left( 1 - \frac{\psi}{\phi} \right)^2 = \left( 1 - \frac{18.04}{28} \right)^2 = 0.58$$

$$F_{\gamma d} = 1$$

$$F_{qd} = 1 + 2 \tan \phi (1 - \sin \phi)^2 \left( \frac{Df}{B} \right) = 1 + 2 \tan 28^\circ (1 - \sin 28^\circ)^2 (1.75/5.6) =$$

$$= 1.08$$

$$q_u = (1/2)(17.6)(4.98)(16.7)(1)(0.58) + (30)(25.8)(1.14)(0.96) + (30.8)(14.7)(1.08)(0.96) =$$

$$= 1740 \text{ kN/m}^2$$

$$\boxed{FS_{BC} = q_u / q_{\text{toe}} = 1740 / 178.00 = 9.78}$$

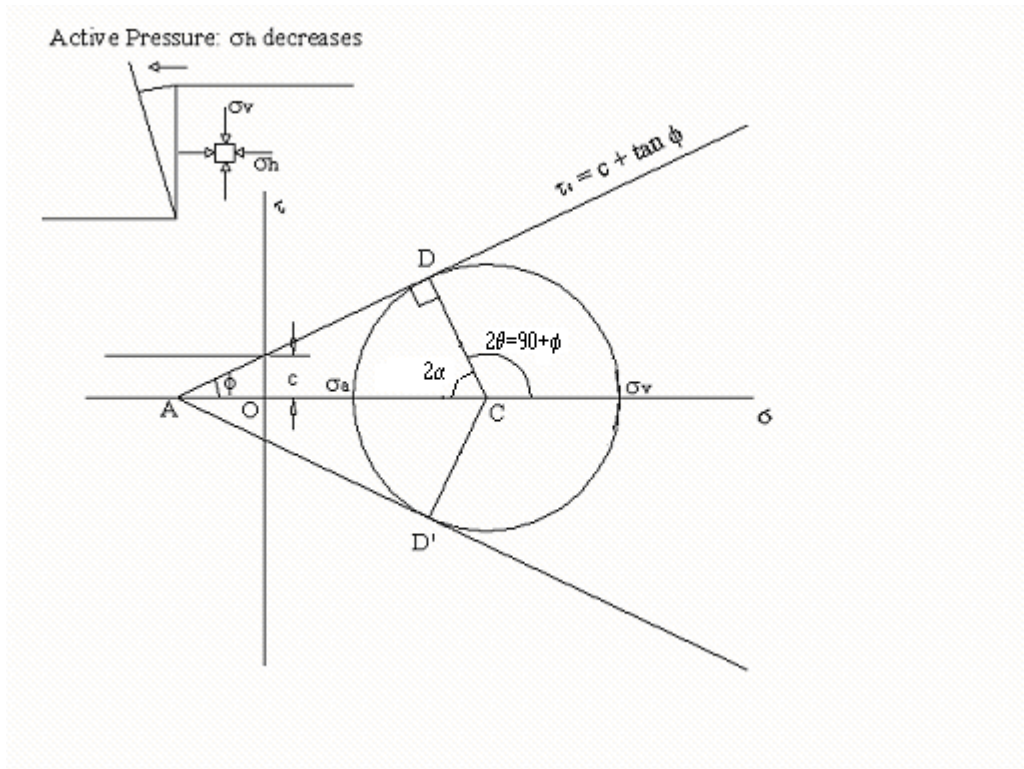
**\*\*\*Lateral-14:** Derive a formula that provides  $K$  and  $\sigma_H$  as a function of  $\sigma_v$ .

(Revised Oct-09)

**Using the Mohr-Coulomb failure criterion combined with Rankine's theory, find the coefficient of active earth pressure  $K_a$  as an investigation of the stress conditions in soil at a state of plastic equilibrium (in other words, when the soil mass is on the verge of failure).**

**Solution:**

The definition of an “active pressure” condition is when  $\sigma_h$  decreases until it touches point D on the Mohr-Coulomb failure envelope.



Find  $\sigma_a$ :

From the figure,

$$\sin \phi = \frac{CD}{AC} = \frac{CD}{AO + OC}$$

$$CD = \text{radius of failure circle} = \left( \frac{\sigma_1 - \sigma_3}{2} \right) = \left( \frac{\sigma_v - \sigma_a}{2} \right)$$

$$AO = c \cot \phi$$

$$OC = \left( \frac{\sigma_1 + \sigma_3}{2} \right) = \left( \frac{\sigma_v - \sigma_a}{2} \right)$$

Substituting values into the equation for  $CD$ ,  $AO$  and  $OC$  gives:

$$\sin \phi = \frac{\left( \frac{\sigma_v - \sigma_a}{2} \right)}{\left[ c \cot \phi + \left( \frac{\sigma_v + \sigma_a}{2} \right) \right]}$$

Rearrange the equation to make  $\sigma_a$  the subject:

This gives:

$$2c \cos \phi + \left( \frac{\sigma_v + \sigma_a}{2} \right) \sin \phi = \frac{\sigma_v - \sigma_a}{2}$$

This then gives:

$$\sigma_a (\sin \phi + 1) = \sigma_v (1 - \sin \phi) - 2c \cos \phi$$

$$\therefore \sigma_a = \sigma_v \left( \frac{1 - \sin \phi}{1 + \sin \phi} \right) - 2c \left( \frac{\cos \phi}{1 + \sin \phi} \right)$$

Solution of the trigonometric expressions:

$$2\alpha + (90^\circ + \phi) = 180^\circ$$

$$\alpha = \frac{90^\circ + \phi}{2} = 45^\circ - \frac{\phi}{2}$$

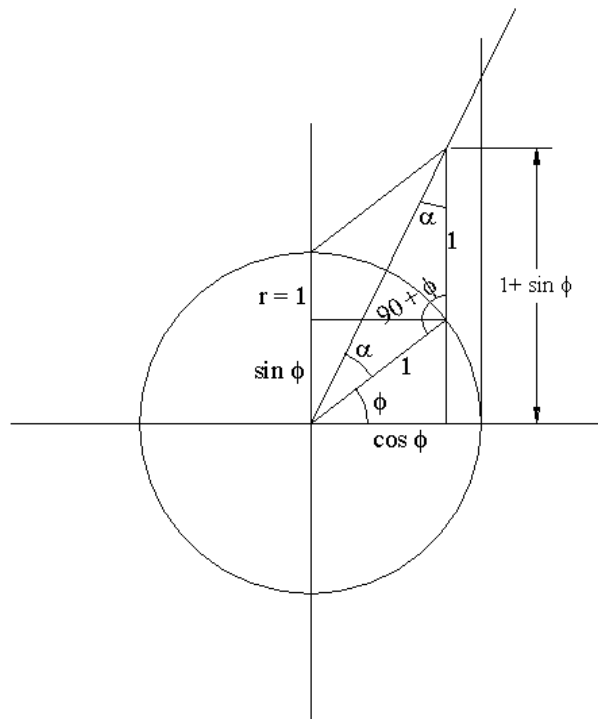
$$\frac{1 + \sin \phi}{\cos \phi} = \tan (\alpha + \phi)$$

$$= \tan \left( 45^\circ - \frac{\phi}{2} + \phi \right)$$

$$= \tan \left( 45^\circ + \frac{\phi}{2} \right)$$

$$= \frac{\sin \left( 45^\circ + \frac{\phi}{2} \right)}{\cos \left( 45^\circ + \frac{\phi}{2} \right)}$$

$$\therefore \frac{\cos \phi}{1 + \sin \phi} = \frac{\cos \left( 45^\circ + \frac{\phi}{2} \right)}{\sin \left( 45^\circ + \frac{\phi}{2} \right)}$$



But for any complementary angles  $\beta$  and  $(90^\circ - \beta)$ ,  $\cos \beta = \sin (90^\circ - \beta)$ .

Thus,  $\cos \left( 45^\circ - \frac{\phi}{2} \right) = \sin \left( 45^\circ + \frac{\phi}{2} \right)$  and  $\sin \left( 45^\circ - \frac{\phi}{2} \right) = \cos \left( 45^\circ + \frac{\phi}{2} \right)$ .

$$\therefore \frac{\cos \phi}{1 + \sin \phi} = \frac{\sin\left(45^\circ - \frac{\phi}{2}\right)}{\cos\left(45^\circ - \frac{\phi}{2}\right)} = \tan\left(45^\circ - \frac{\phi}{2}\right)$$

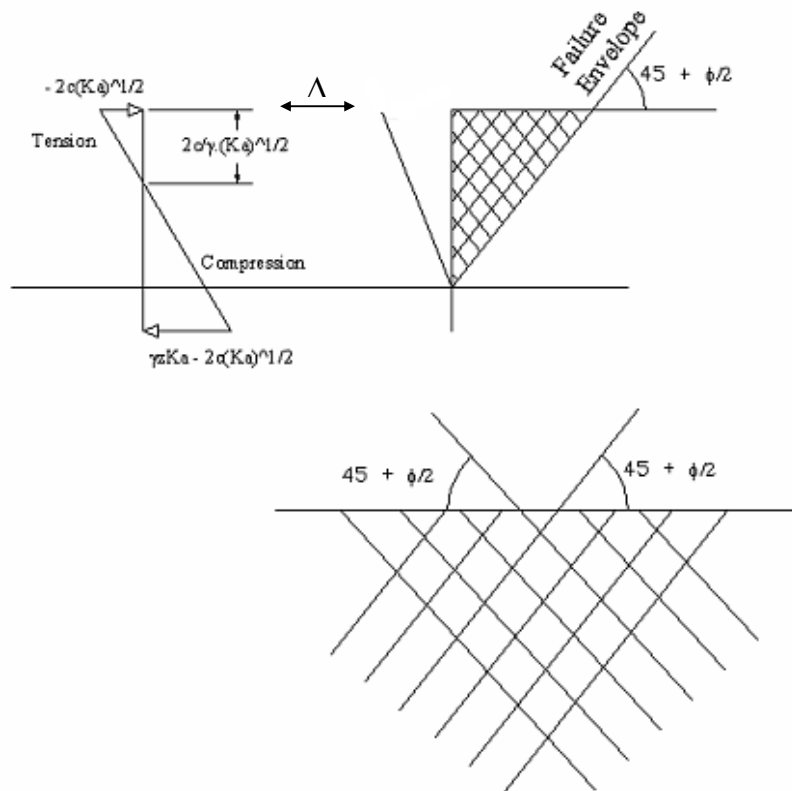
$$\therefore \frac{1 - \sin \phi}{1 + \sin \phi} = \left(\frac{1 - \sin \phi}{1 + \sin \phi}\right) \times \left(\frac{1 + \sin \phi}{1 + \sin \phi}\right) = \frac{1^2 - \sin^2 \phi}{(1 + \sin \phi)^2} = \frac{\cos^2 \phi}{(1 + \sin \phi)^2} = \tan^2\left(45^\circ - \frac{\phi}{2}\right)$$

$$\sigma_a = \sigma_v \tan^2\left(45^\circ - \frac{\phi}{2}\right) - 2c \tan\left(45^\circ - \frac{\phi}{2}\right) \leftarrow$$

Rankine's expression gives the effective

$$\therefore \sigma_a = (\gamma z) K_a - 2c (K_a)^{1/2} ; \text{ where } K_a = \tan^2\left(45^\circ - \frac{\phi}{2}\right)$$

Using this equation, the slip planes can be described by the grid of lines shown below:

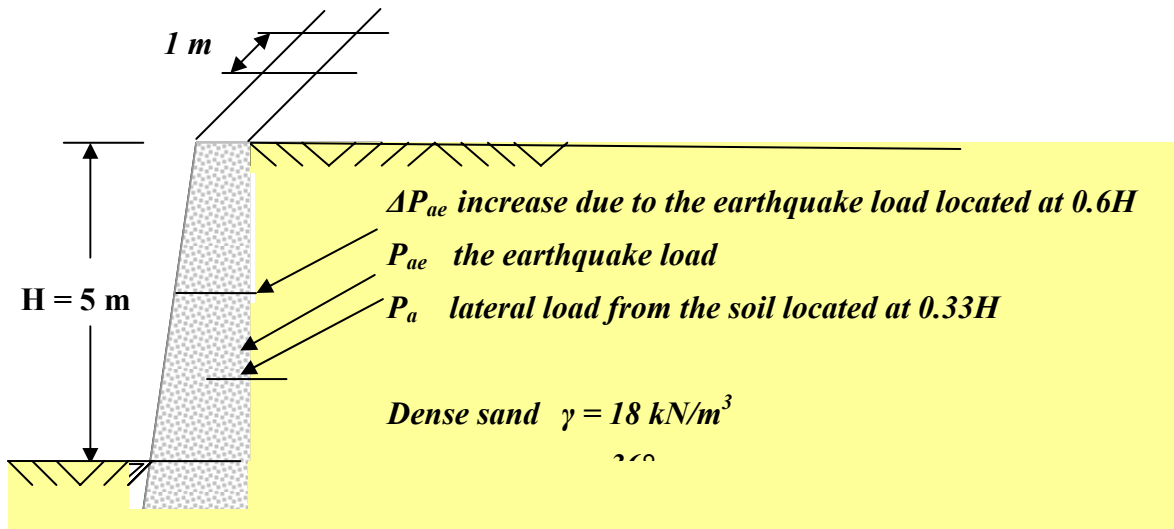


**\*\*Lateral-15: The magnitude and location of a seismic load upon a retaining wall.**

(Revision: Sept-08)

The reinforced concrete retaining wall shown below will be subjected to a horizontal seismic load of 0.2 g without a vertical component. Determine,

- The magnitude of the active earth force  $P_a$  on the wall;
- The magnitude of the earthquake active earth force  $P_{ae}$  on the wall;
- The location of the resultant of both forces.



**Solution.**

Calculate the coefficient of active earth pressure (Coulomb)  $K_a$  using,

$\phi = 36^\circ$ ,  $\alpha = 0^\circ$ ,  $\beta = 90^\circ$  and  $\delta = 2/3\phi = 24^\circ$ ,

$$K_a = \frac{\sin^2(\beta + \phi')}{\sin^2 \beta \sin(\beta - \delta) \left[ 1 + \sqrt{\frac{\sin(\phi' + \delta) \sin(\phi' - \alpha)}{\sin(\beta - \delta) \sin(\alpha + \beta)}} \right]^2}$$

$$K_a = \frac{\sin^2(90^\circ + 36^\circ)}{\sin^2(90^\circ) \sin(90^\circ - 24^\circ) \left[ 1 + \sqrt{\frac{\sin(36^\circ + 24^\circ) \sin(36^\circ - 0^\circ)}{\sin(90^\circ - 24^\circ) \sin(0^\circ + 90^\circ)}} \right]^2}$$

$$K_a = \frac{\sin^2(126^\circ)}{\sin(66^\circ) \left[ 1 + \sqrt{\frac{\sin(60^\circ) \sin(36^\circ)}{\sin(66^\circ)}} \right]^2} = \frac{0.654}{(0.914) \left[ 1 + \sqrt{\frac{(0.866)(0.588)}{(0.914)}} \right]^2} = 0.2346$$

The active earth force  $P_a$  is,

$$P_a = \frac{1}{2} \gamma H^2 K_a = (0.5)(18 \text{ kN/m}^3)(5 \text{ m})^2 (0.2346) = 53 \text{ kN/m}$$

Calculate the earthquake coefficient of active earth pressure (Coulomb)  $K_{ae}$ ,

$$k_h = 0.2, k_v = 0, \delta = 2/3\phi = 2/3(36^\circ) = 24^\circ \text{ and } \theta' = \tan^{-1} \left[ \frac{k_h}{1-k_v} \right] = \tan^{-1}(0.2) = 11.3^\circ$$

$$\therefore K_{ae} = \frac{\sin^2(\phi + \beta - \theta')}{\cos \theta' \sin^2 \beta \sin(\beta - \theta' - \delta) \left[ 1 + \sqrt{\frac{\sin(\phi + \delta) \sin(\phi - \theta' - \alpha)}{\sin(\beta - \delta - \theta') \sin(\alpha + \beta)}} \right]^2}$$

$$K_{ae} = \frac{\sin^2(36^\circ + 90^\circ - 11.3^\circ)}{\sin^2(90^\circ) \sin(90^\circ - 24^\circ) \left[ 1 + \sqrt{\frac{\sin(36^\circ + 24^\circ) \sin(36^\circ - 0^\circ)}{\sin(90^\circ - 24^\circ) \sin(0^\circ + 90^\circ)}} \right]^2} = 0.372$$

The Mononobe-Okabe earthquake active earth force  $P_{ae}$  is,

$$P_{ae} = \frac{1}{2} \gamma H^2 (1 - k_v) K_{ae} = (0.5)(18 \text{ kN/m}^3)(5 \text{ m})^2 (1 - 0)(0.372) = 83.7 \text{ kN/m}$$

The earthquake force is  $\Delta P_{ae} = P_{ae} - P_a = 83.7 - 53 = 30.7 \text{ kN/m}$

The location of the resultant earthquake force  $\bar{z}$  is, found by locating the force  $P_{ae}$  at a height  $0.6H$  above the base of the wall; the active earth force  $P_a$  is obviously  $(0.33)H$  above the base.

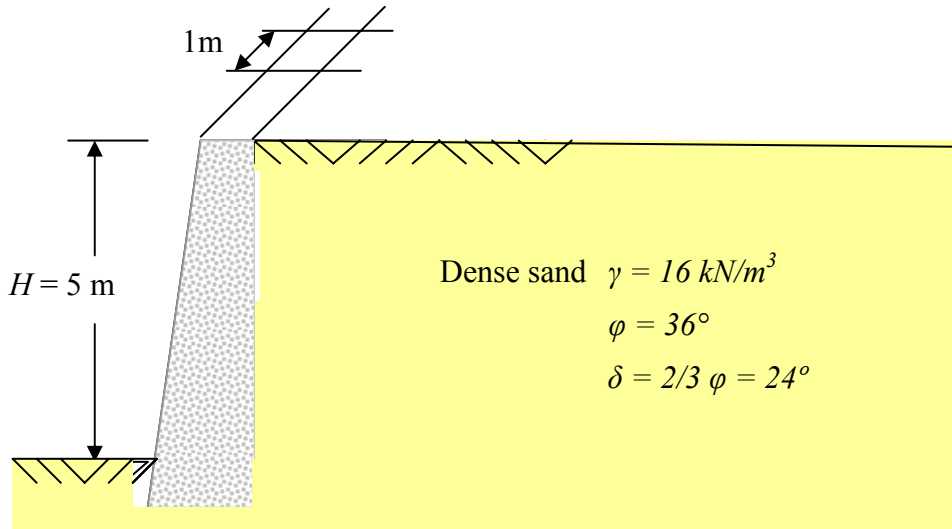
$$\bar{z} = \frac{(0.6H)(\Delta P_{ae}) + (1/3)H(P_a)}{P_{ae}} = \frac{(0.6)(5 \text{ m})(30.7 \text{ kN/m}^2) + (1/3)(5 \text{ m})(53 \text{ kN/m}^2)}{(83.7 \text{ kN/m}^2)} = 2.1 \text{ m}$$

**\*\*Lateral-16: Seismic loading upon a retaining wall.**

(Revision: Aug-08)

The reinforced concrete retaining wall shown below will be designed to a horizontal seismic loading of 0.2 g. Assume no vertical seismic component ( $k_v=0$ ). Determine,

- (a) The weight of the wall  $W_w$  under static conditions;
- (b) The weight of the wall under seismic conditions, for zero lateral displacement;
- (c) The weight of the wall under seismic conditions, for a lateral displacement = 1.5 inches.



**Solution.**

# Chapter 16

## Braced Cuts for Excavations

***Symbols for Braced Cuts for Excavations***

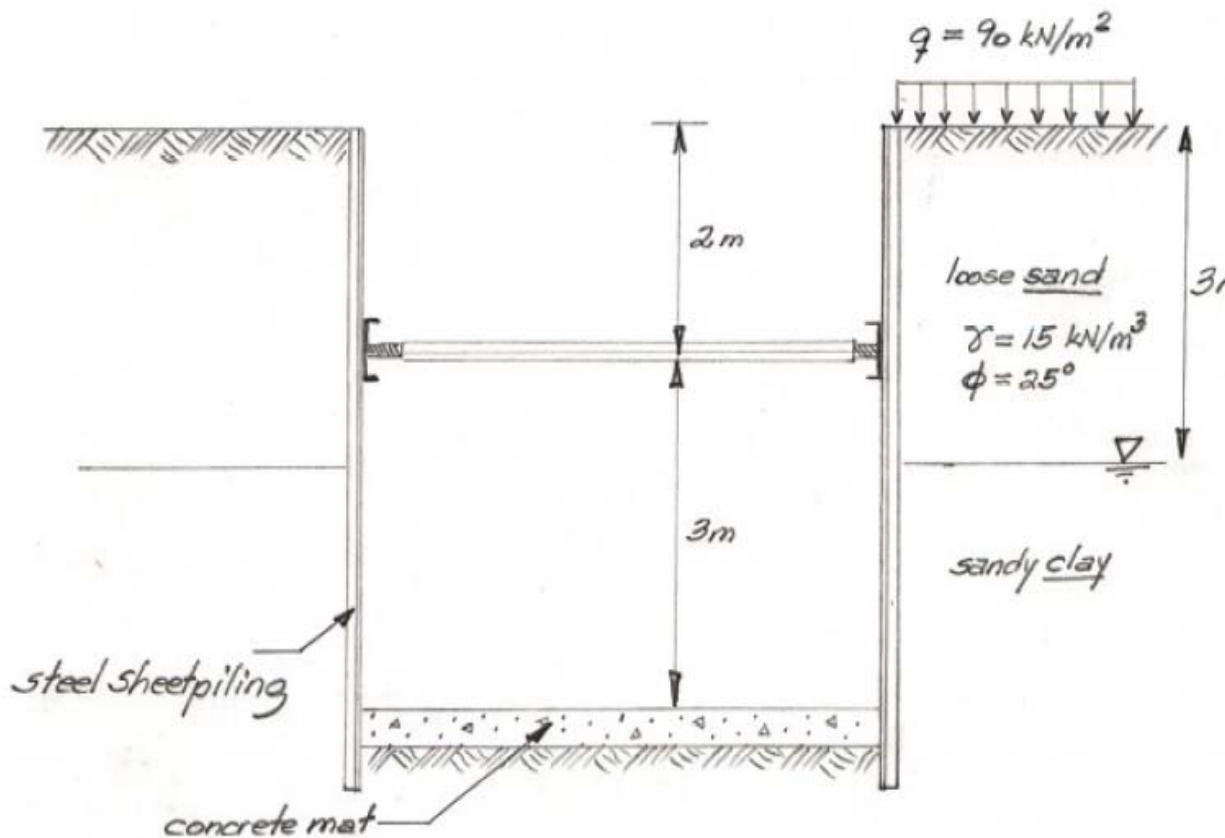
**\*Braced-cuts-01: Forces and moments in the struts of a shored trench.**

(Revision: Sept-08)

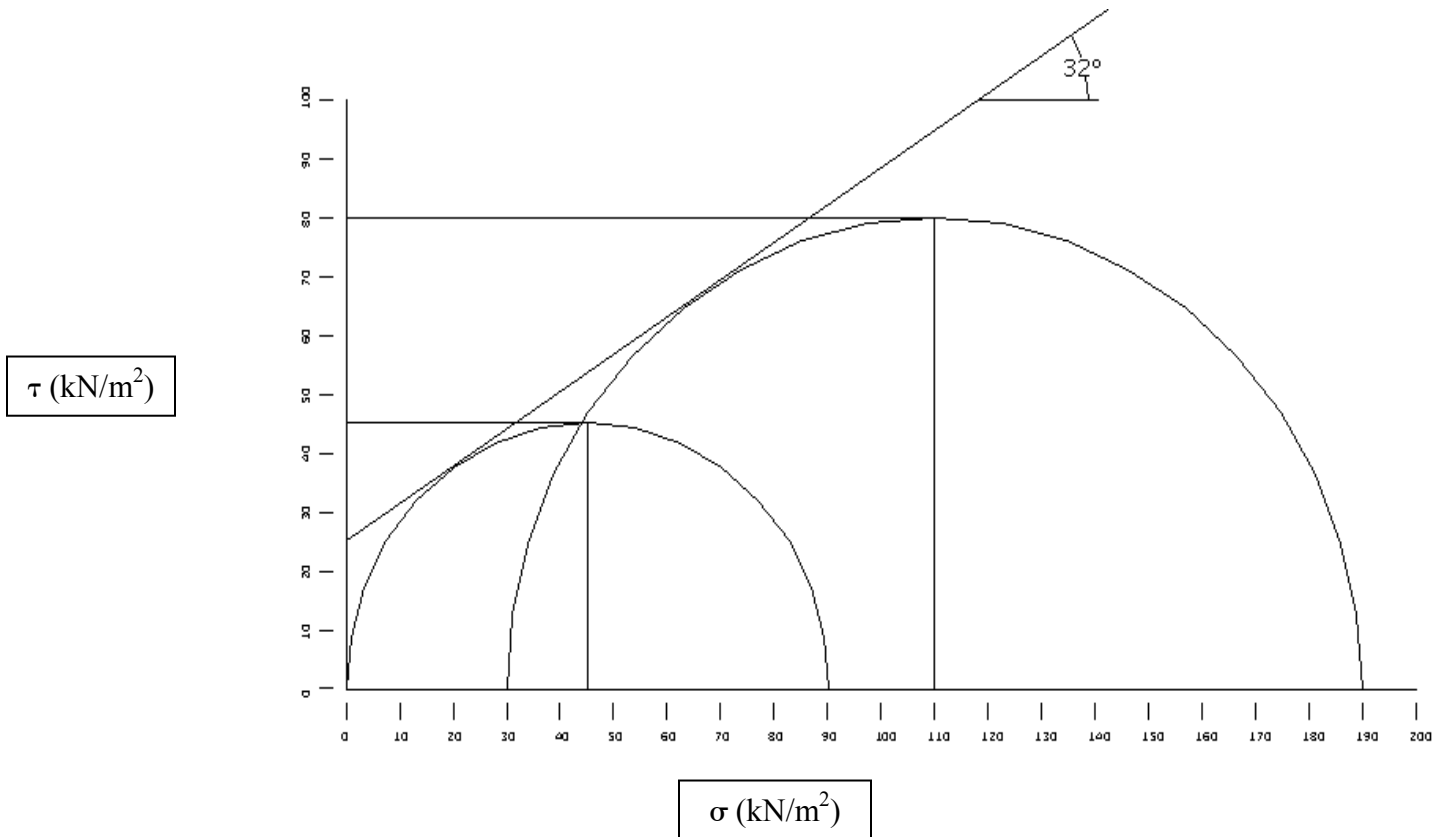
You have been asked by a contractor to design the internal supports (struts) of a temporary utility trench, as shown below. In order to design the steel horizontal strut shown, you must first find the force and moment on one of them, if they are spaced every 4 m horizontally.

Two triaxial laboratory tests were performed on samples of the clayey sand. The first sample had a confining pressure of  $0 \text{ kN/m}^2$ , and the sample reached failure with a deviator stress of  $90 \text{ kN/m}^2$ . (N.B.: the deviator stress is the additional vertical stress required to reach failure, i.e.  $s_1$  to  $s_3$ ). The second sample had its confining stress increased to  $30 \text{ kN/m}^2$ . The deviator stress needed to attain failure was  $160 \text{ kN/m}^2$ .

Further laboratory tests show that this clayey sand had an in-situ voids ratio of  $0.46$  at a moisture of  $34\%$  (assume  $G_s = 2.65$ ). Show all your calculations.



## Effective Stress Mohr's Circle for failure Angle



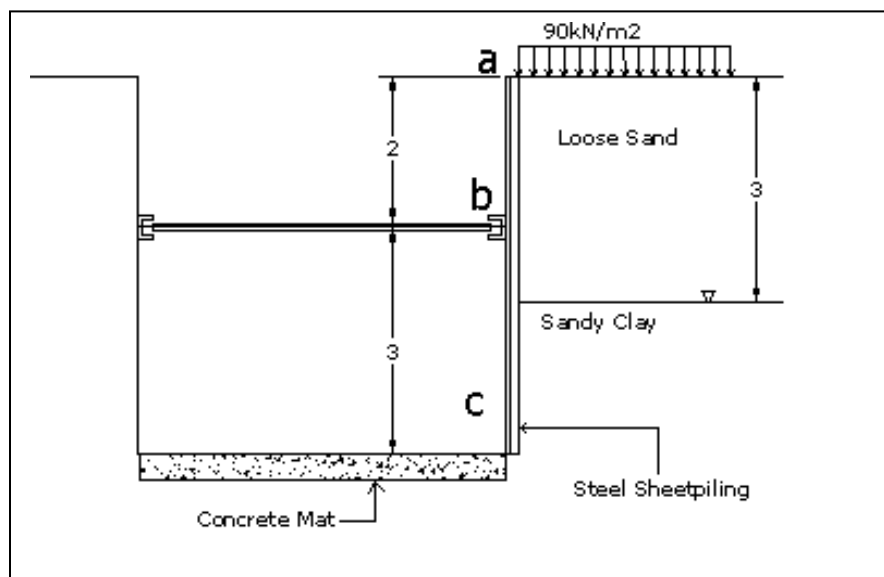
From the Mohr's Circle, we can get that  $\phi_2 = \underline{32^\circ}$

$$G_s = 2.65 ; \gamma_w = 9810 \text{ N/m}^2 \rightarrow \gamma_s = \frac{(G_s)(\gamma_w)}{1+e} = \frac{(2.65)(9810)}{1+0.46} \rightarrow \gamma_s = \gamma_2 = \underline{17.8 \text{ kN/m}^2}$$

$$K_A = \tan^2\left(45^\circ - \frac{\phi}{2}\right)$$

$$K_{A1} = \tan^2\left(45^\circ - \frac{25^\circ}{2}\right) = \underline{0.406}$$

$$K_{A2} = \tan^2\left(45^\circ - \frac{32^\circ}{2}\right) = \underline{0.307}$$



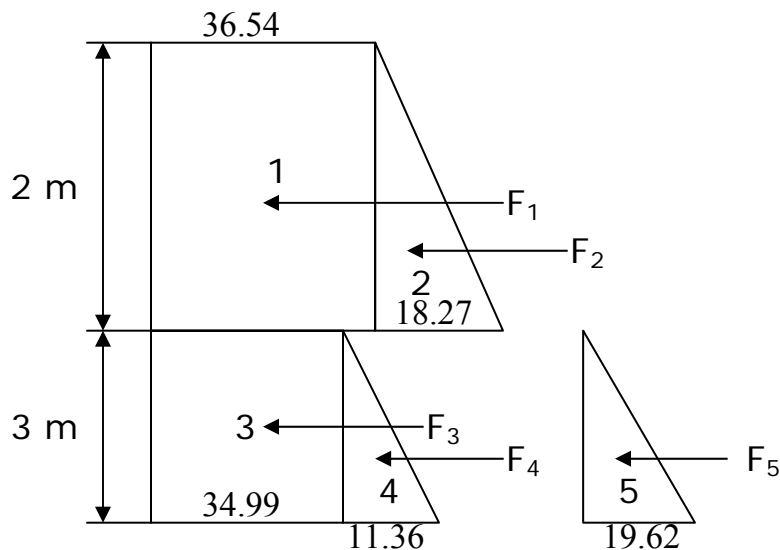
$$P_a = (q) (K_{A1}) = (90 \text{ kN/m}^2) (.406) \rightarrow 36.54 \text{ kN/m}^2$$

$$P_{b,+} = [K_{A1} (q + \gamma_1 h_1)] = [(.406) (15 \text{ kN/m}^2 \times 3 \text{ m})] \rightarrow 54.81 \text{ kN/m}^2$$

$$P_{b,-} = [K_{A2} (q + (\gamma_2 - \gamma_w) h)] = [(.307) (90 + (17.8 - 9.81) (3))] \rightarrow 34.99 \text{ kN/m}^2$$

$$P_c = [(q + \gamma_1 h_1 + (\gamma_2 - \gamma_w) h_2) K_{A2} = .307 [90 + (15)(3) + (17.8 - 9.81)(2)] \rightarrow 46.35 \text{ kN/m}^2$$

$$P_w = \gamma_w h_w = (9.81)(2) \rightarrow 19.62 \text{ kN/m}^2$$



**Location of the Forces (with respect to the top datum):**

$$F_1: 3 \text{ m} (1/2) = 1.5 \text{ m}$$

$$F_2: 3 \text{ m} (2/3) = 2.0 \text{ m}$$

$$F_3: 3 \text{ m} + 2 \text{ m} (1/2) = 4.0 \text{ m}$$

$$F_4: 3 \text{ m} + 2 \text{ m} (2/3) = 4.33 \text{ m}$$

$$F_5: 3 \text{ m} + 2 \text{ m} (2/3) = 4.33 \text{ m}$$

**Magnitude of the Forces:**

$$F_1 = (P_a)(h_1) = (36.54 \text{ kN/m}^2)(3 \text{ m}) = 109.6 \text{ kN/m}$$

$$F_2 = (P_{b,+} - P_a)(h_1/2) = (54.81 - 36.54)(3/2) = 27.4 \text{ kN/m}$$

$$F_3 = (P_b)(h_2) = (34.99 \text{ kN/m}^2)(2\text{m}) = \mathbf{69.98 \text{ kN/m}}$$

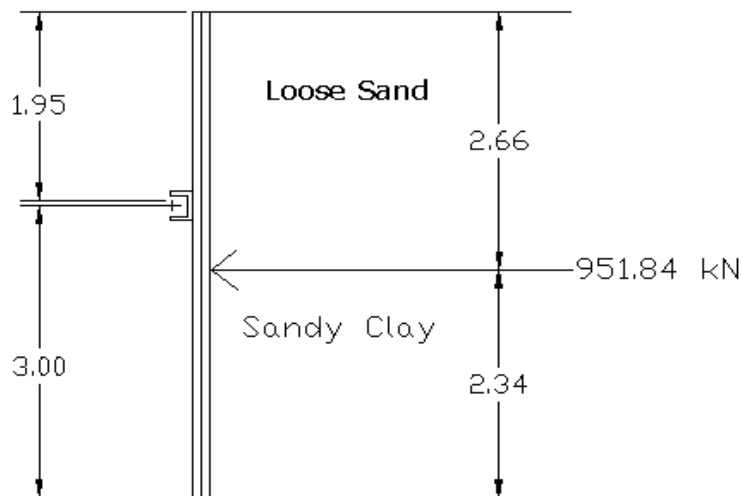
$$F_4 = (P_c - P_b)(h_2/2) = (46.35 - 34.99)(2/2) = \mathbf{11.36 \text{ kN/m}}$$

$$F_5 = (P_w)(h_w/2) = (19.62)(2/2) = \mathbf{19.62 \text{ kN/m}}$$

$$\sum F = F_1 + F_2 + F_3 + F_4 + F_5 = \mathbf{237.96 \text{ kN/m}}$$

$$F_{\text{tot}} = (\sum F)(\text{space b/t struts}) = (237.96 \text{ kN/m})(4\text{m}) \rightarrow \mathbf{951.84 \text{ kN}}$$

$$\text{Located at } y_f = \frac{109.6(1.5) + 27.4(2) + 69.98(4) + 11.36(4.33) + 19.62(4.33)}{237.96} = 2.66\text{m}$$



$$\boxed{\sum M_c = 0} \text{ Where C is located at the bottom of the trench along with } R_A$$

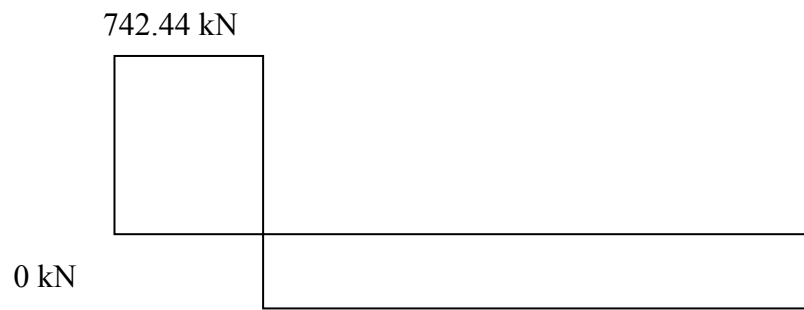
$R_B$  is located at the end of the strut.

$$\rightarrow R_B(3\text{m}) - 951.84 \text{ kN}(2.34\text{m}) = 0$$

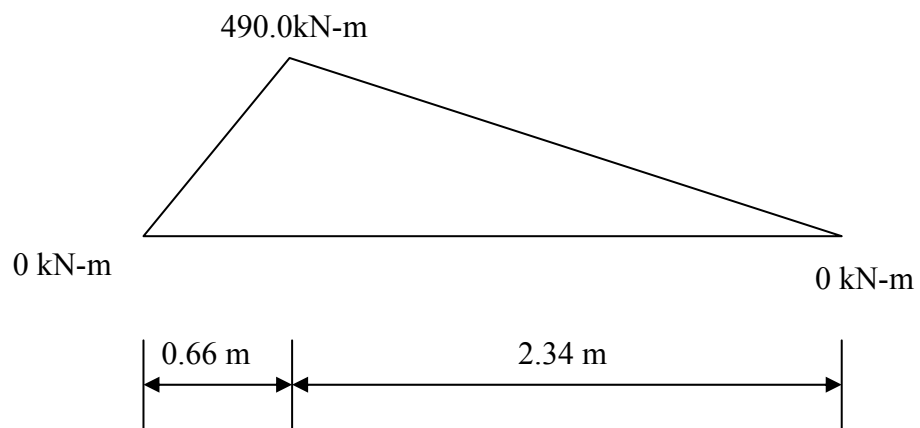
$$\rightarrow R_B = \mathbf{742.44 \text{ kN}}$$

$$\rightarrow R_A = \mathbf{209.40 \text{ kN}}$$

Shear Diagram



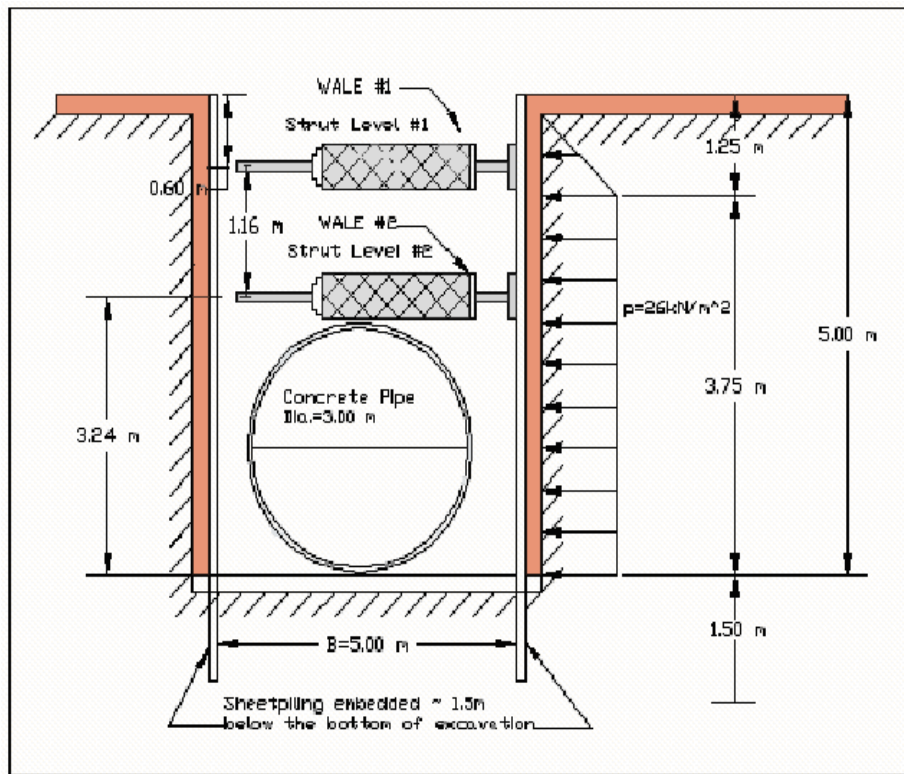
Moment Diagram



**\*\*Braced cuts-02: A 5 m deep excavation with two struts for support.**

(Revision: Sept-08)

Design a braced excavation for a large sanitary sewer force-main, which is a reinforced concrete pipe with a diameter of 3 m. The trench should be 5 m deep and 5 m wide. The phreatic surface is below bottom of excavation. The SPT for the silty clay is  $N_{avg} = 20$ , and  $\gamma = 17 \text{ kN/m}^3$ . Assume  $\phi = 0$ .



**Solution:**

Use Stroud's relation to estimate the un-drained cohesion of the soil (the previous problem provided the shear strength):

$$c_u = KN = (3.5 \text{ kN/m}^2) (20) = 70 \text{ kN/m}^2.$$

Therefore,

$$\text{if } \frac{\gamma H}{c_u} > 4 \quad \text{the clay is soft to medium}$$

$$\text{if } \frac{\gamma H}{c_u} \leq 4 \quad \text{the clay is stiff}$$

$$\text{In this problem, } \frac{\gamma H}{c_u} = \frac{(17)(5)}{(70)} = 1.21 < 4 \therefore \text{this is a } \underline{\text{stiff clay}}$$

Also, since  $\gamma H / c_u < 6$ , the sheet-piling should extend at least 1.5 m below bottom.

**Step 1. Establish the lateral earth pressure distribution.**

Using Peck's (1967) apparent pressure envelope, we must choose the larger of,

$$(1) p_a = \gamma H \left[ 1 - \left( \frac{4c_u}{\gamma H} \right) \right]$$

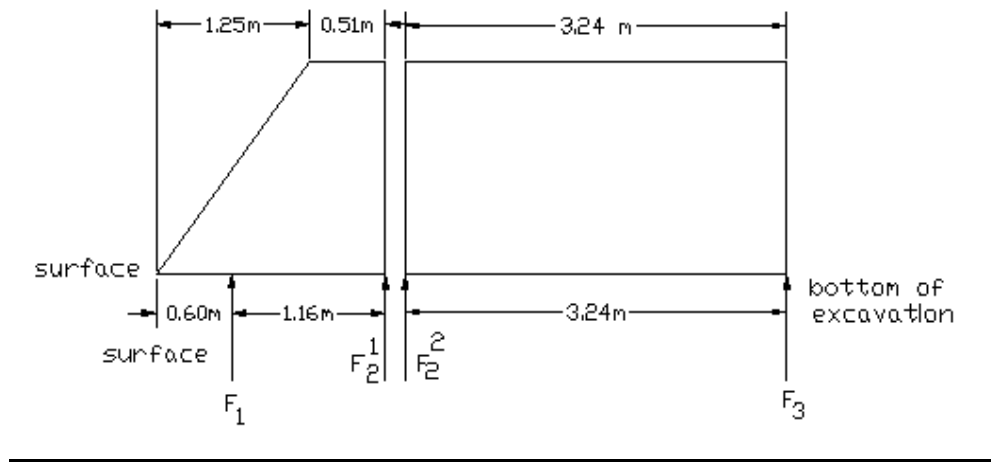
$$(2) p_a = 0.3\gamma H = 0.3(17)(5) = 25.5 \text{ kN/m}^2$$

The location of the top strut should be less than the depth of the tensile crack  $z_c$ . Since  $\phi = 0$ ,

$$K_a \rightarrow \sqrt{K_a} = 1. \text{ therefore } \sigma_3 = \sigma_a = (\gamma)(z_c)K_a - 2c\sqrt{K_a}$$

$$\text{therefore } z_c = 2c/\gamma = 2(70 \text{ kN/m}^2)/17 \text{ kN/m}^3 = 8.2 \text{ m} >> 0.6 \text{ m} \quad \textbf{OK}$$

**Step 2: Determine the lateral loads at strut locations and excavation bottom.**



Isolation the left portion between the surface and strut #2.

$$\sum M_{F_2} = 0 = F_1(1.16\text{m}) - (0.5)(1.25\text{m})(26)[0.51 + 1.25/3] - (0.51)(26)[0.51/2] = 0$$

$$\text{therefore, } F_1 = 15.9 \text{ kN/m}$$

$$\sum F_y = 0 = -15.9 + 1/2 (1.25)(26) + (0.51)(26) - F'_2 = 0 \quad \text{therefore, } F'_2 = 13.6 \text{ kN/m}$$

Isolating the right portion between strut #2 and the trench bottom, by symmetry

$$F_2^2 = F_1^1 = 13.6 \text{ kN/m} \quad \sum F_y = 0 = -F_2^2 + (3.75 - 0.51)(26) - F_3$$

$$\text{therefore, } F_3 = 70.6 \text{ kN/m}$$

### **Step 3: Find the maximum moment $M_{\max}$ in the sheet-piling.**

Finding moments at A, B, & C (that is, the areas under the shear diagram):

$$M_A = \frac{1}{2}(0.60)(12.48)(0.60/3) = 0.75 \text{ kN-m/m}$$

$$M_B = \frac{1}{2}(1.25)(26)(1.25/3) - 15.9(0.65) = 3.56 \text{ kN-m/m}$$

$$M_C = (2.71)(26)(2.71/2) = 96 \text{ kN-m/m}$$

Obviously  $M_{\max} = 96 \text{ kN-m/m}$

### **Step 4: Select the steel-piling .**

Assume  $f_y = 50 \text{ ksi} = 345 \text{ MN/m}^2$ , therefore  $\sigma_{\text{allow}} = 50\%f_y = 172 \text{ MN/m}^2$

The required section modulus S

$$S = M_{\max} / \sigma_{\text{all}} = 96 \text{ kN-m} / 172,000 \text{ kN/m}^2 = 0.00056 \text{ m}^3 = 56 \text{ m}^3 / \text{m} \cdot 10^5$$

Choose a PDA-27 section, which provides  $57.46 \text{ m}^3 / \text{m} \cdot 10^5$ .

### **Step 5: Select the horizontal waler at each strut level.**

At strut level #1 the load  $F_1$  is  $16 \text{ kN/m}$ . Select the horizontal spacings to be  $4 \text{ m}$ . (May use  $3 \text{ m}$  to reduce steel size, but increases the difficulty of placing the concrete pipes).

$$M_{\max} = F_1 s^2 / 8 = (16)(4)^2 / 8 = 32 \text{ kN-m (where } s \text{ is the spacing)}$$

$$\text{therefore, } S_{\text{wale at 1}} = M_{\max} / \sigma_{\text{allow}} = 32 \text{ kN-m} / 172,000 \text{ kN/m}^2 = 18.6 \text{ m}^3 / \text{m} \cdot 10^5$$

At strut level #2 the load is  $27.2 \text{ kN/m}$ ; the spacing  $s$  is  $4 \text{ m}$ .

$$M_{\max} = F_2 s^2 / 8 = (27.2)(4)^2 / 8 = 54.4 \text{ kN-m}$$

$$\text{Therefore, } S_{\text{wale at 2}} = M_{\max} / \sigma_{\text{allow}} = 54.4 \text{ kN-m} / 172,000 \text{ kN/m}^2 = 31.6 \text{ m}^3 / \text{m} \cdot 10^5$$

*Notes: 1. The bottom of the trench has the highest lateral load, with  $70.6 \text{ kN}$  per every meter. Propose to cast a concrete "mud" slab at the bottom of the trench. Design the thickness of the slab (diaphragm).*

*2. Wales are commonly channels or WF beams. Design the steel pipe wales and the struts, calculated in Step 6 below.*

### **Step 6: Select the struts.**

$$\text{Level \# 1 strut} = F_1 s = (16 \text{ kN/m})(4\text{m}) = 64 \text{ kN}$$

$$\text{Level \# 2 strut} = 2 F_2 s = (27.2 \text{ kN/m})(4\text{m}) = 109 \text{ kN (Design the steel for the struts).}$$

### **Step 7: Check for possible heave of the excavation bottom.**

Braced cuts in clay may become unstable if the bottom heaves upward and fails a section of wall.

$$FS_{\text{against heaving}} = [cN_c(0.84 + 0.16 B/L)] / \gamma H = (70)(6.4)(0.84) / (17)(5) = 4.4 > 2 \quad \underline{\text{O.K.}}$$

**Step 8: Expected lateral yielding of the sheet-piling and ground settlement behind the wall.**

Expect  $\delta_h$  from 5 to 10 cms.

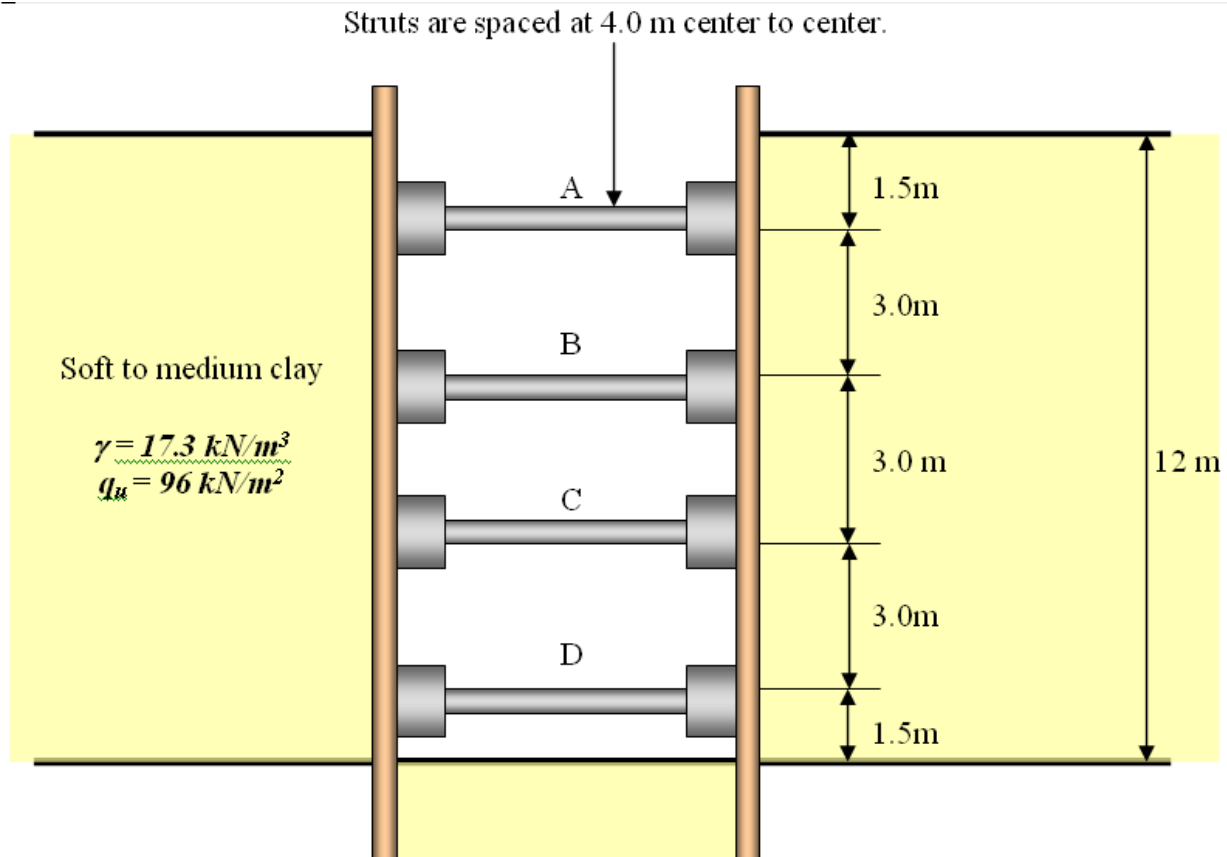
$\delta_\gamma$  from 1 to 5 cms.

**\*Braced cuts-03: Four-struts bracing a 12 m excavation in a soft clay.**

(Revision: Sept-08)

A four-strut braced sheet pile installation is designed for an open cut in a clay stratum, as shown below. The struts are spaced longitudinally (in plan view) at 4.0 m center to center. Assume that the sheet piles are pinned or hinged at strut levels B and C.

- Find: 1. The lateral earth pressure diagram for the braced sheet pile system.  
2. The loads on struts A, B, C, and D.



**Solution:**

From Terzaghi and Peck (1967), a clay is soft, medium or stiff,

$$\text{if } \frac{\gamma H}{c_u} > 4 \text{ the clay is soft to medium then } \sigma_a = \gamma H \left( 1 - \frac{4c_u}{\gamma H} \right)$$

$$\text{if } \frac{\gamma H}{c_u} \leq 4 \text{ the clay is stiff then } \sigma_a = 0.2\gamma H \text{ to } 0.4\gamma H$$

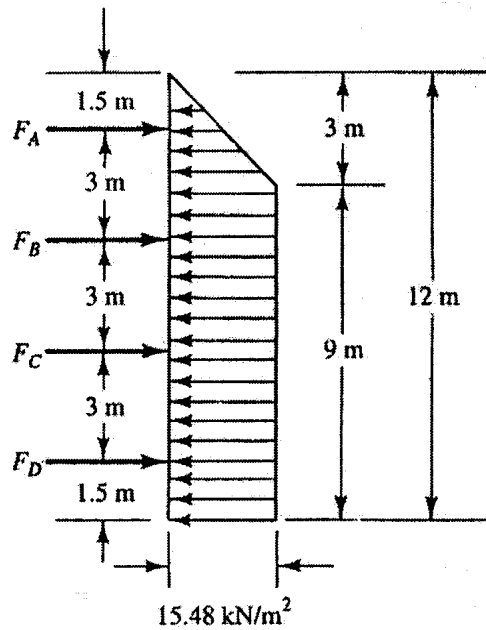
Determine the cohesion from Mohr's circle  $c_u = \frac{q_u}{2} = \frac{96 \text{ kN/m}^2}{2} = 48 \text{ kN/m}^2$

$$\therefore \frac{\gamma H}{c_u} = \frac{(17.3 \text{ kN/m}^3)(12 \text{ m})}{(48 \text{ kN/m}^2)} = 4.33 > 4 \quad \therefore \text{this is a soft to medium clay}$$

Peck (1969) provided a criterion for soft to medium clays,

$$p_a = \gamma H \left( 1 - \frac{4c_u}{\gamma H} \right) = (17.3 \text{ kN/m}^3)(12 \text{ m}) \left[ 1 - \frac{(4)(48 \text{ kN/m}^2)}{(17.3 \text{ kN/m}^3)(12 \text{ m})} \right] = 15.48 \text{ kN/m}^2$$

The lateral earth pressure diagram for the braced sheet pile system in soft clays is,



2. In the free body diagram, part (a),  $\sum M_B = 0$

$$\left( \frac{1}{2} \right) (15.48 \text{ kN/m}^2) (3.0 \text{ m}) (4.0 \text{ m}) \left( 1.5 \text{ m} + \frac{3.0 \text{ m}}{3} \right) + (1.5 \text{ m}) (15.48 \text{ kN/m}^2) (4.0 \text{ m}) \left( \frac{1.5 \text{ m}}{2} \right) - (F_A) (3.0 \text{ m}) = 0$$

$$\therefore F_A = 100.6 \text{ kN}$$

From  $\sum H = 0$

$$F_{B1} = \left( \frac{1}{2} \right) (1.5 \text{ m} + 4.5 \text{ m}) (15.48 \text{ kN/m}^2) (4.0 \text{ m}) - 100.6 \text{ kN} = 85.2 \text{ kN}$$

In the free body diagram, part (b)

$$F_{B2} = F_{C1} = \left( \frac{1}{2} \right) (3.0 \text{ m}) (15.48 \text{ kN/m}^2) (4.0 \text{ m}) = 92 \text{ kN}$$

In the free body diagram, part (c),  $\sum M_C = 0$

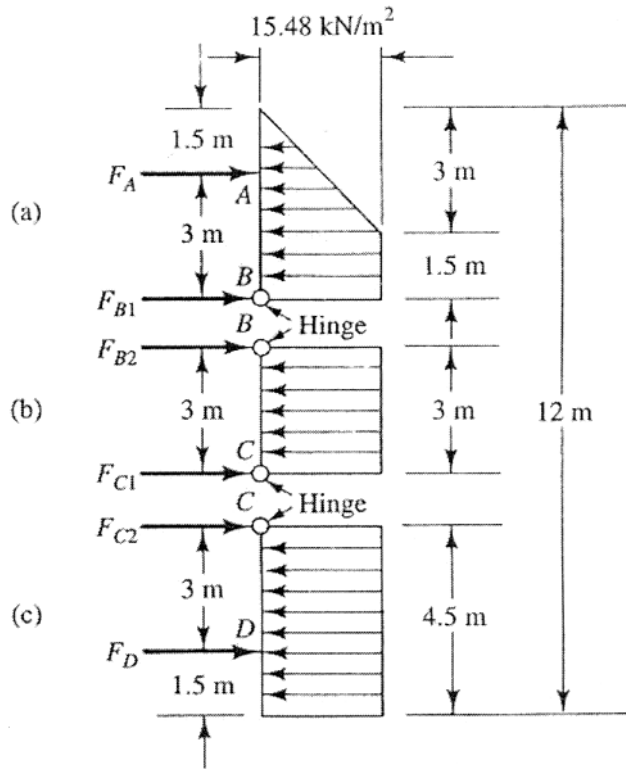
$$(F_D) (3.0 \text{ m}) - (4.5 \text{ m}) (15.48 \text{ kN/m}^2) (4.0 \text{ m}) \left( \frac{4.5 \text{ m}}{2} \right) = 0$$

$$\therefore F_D = 209.0 \text{ kN}$$

From  $\sum H = 0$

$$F_{C2} + F_D - (4.5 \text{ m})(15.48 \text{ kN/m}^2)(4.0 \text{ m}) = 0$$

$$F_{C2} = (4.5 \text{ m})(15.48 \text{ kN/m}^2)(4.0 \text{ m}) - 209.0 \text{ kN} = 69.6 \text{ kN}$$



Therefore,

$$F_A = 100.6 \text{ kN}$$

$$F_B = 85.2 \text{ kN} + 92.9 \text{ kN} = 178.1 \text{ kN}$$

$$F_C = 92.9 \text{ kN} + 69.6 \text{ kN} = 162.5 \text{ kN}$$

$$F_D = 209.0 \text{ kN}$$

# **Chapter 17**

## **Bearing Capacity of Soils**

***Symbols for the Bearing Capacity of Soils***

<b>Bearing Capacity Factors for General Shear</b>											
Angle $\phi$ (Degrees)	Angle $\phi$ (Radians)	Terzaghi				Meyerhof			Hansen		
		$K_{\phi\gamma}$	$N_c$	$N_q$	$N_{\gamma}$	$N_c$	$N_q$	$N_{\gamma}$	$N_c$	$N_q$	$N_{\gamma}$
0	0.0000	10.18	5.70	1.00	0.00	5.10	1.00	0.00	5.10	1.00	0.00
1	0.0175	10.61	6.00	1.10	0.08	5.38	1.09	0.00	5.38	1.09	0.00
2	0.0349	11.07	6.30	1.22	0.18	5.63	1.20	0.01	5.63	1.20	0.01
3	0.0524	11.56	6.62	1.35	0.28	5.90	1.31	0.02	5.90	1.31	0.02
4	0.0698	12.07	6.97	1.49	0.39	6.19	1.43	0.04	6.19	1.43	0.05
5	0.0873	12.61	7.34	1.64	0.51	6.49	1.57	0.07	6.49	1.57	0.07
6	0.1047	13.19	7.73	1.81	0.65	6.81	1.72	0.11	6.81	1.72	0.11
7	0.1222	13.80	8.15	2.00	0.80	7.16	1.88	0.15	7.16	1.88	0.16
8	0.1396	14.44	8.60	2.21	0.96	7.53	2.06	0.21	7.53	2.06	0.22
9	0.1571	15.13	9.09	2.44	1.15	7.92	2.25	0.28	7.92	2.25	0.30
10	0.1745	15.87	9.60	2.69	1.35	8.34	2.47	0.37	8.34	2.47	0.39
11	0.1920	16.65	10.16	2.98	1.58	8.80	2.71	0.47	8.80	2.71	0.50
12	0.2094	17.49	10.76	3.29	1.84	9.28	2.97	0.60	9.28	2.97	0.63
13	0.2269	18.38	11.41	3.63	2.12	9.81	3.26	0.74	9.81	3.26	0.78
14	0.2443	19.33	12.11	4.02	2.44	10.37	3.59	0.92	10.37	3.59	0.97
15	0.2618	20.36	12.86	4.45	2.79	10.98	3.94	1.13	10.98	3.94	1.18
16	0.2793	21.46	13.68	4.92	3.19	11.63	4.34	1.37	11.63	4.34	1.43
17	0.2967	22.65	14.56	5.45	3.63	12.34	4.77	1.66	12.34	4.77	1.73
18	0.3142	23.92	15.52	6.04	4.13	13.10	5.26	2.00	13.10	5.26	2.08
19	0.3316	25.30	16.56	6.70	4.70	13.93	5.80	2.40	13.93	5.80	2.48
20	0.3491	26.80	17.69	7.44	5.34	14.83	6.40	2.87	14.83	6.40	2.95
21	0.3665	28.42	18.92	8.26	6.07	15.81	7.07	3.42	15.81	7.07	3.50
22	0.3840	30.18	20.27	9.19	6.89	16.88	7.82	4.07	16.88	7.82	4.13
23	0.4014	32.10	21.75	10.23	7.83	18.05	8.66	4.82	18.05	8.66	4.88
24	0.4189	34.19	23.36	11.40	8.90	19.32	9.60	5.72	19.32	9.60	5.75
25	0.4363	36.49	25.13	12.72	10.12	20.72	10.66	6.77	20.72	10.66	6.76
26	0.4538	39.01	27.09	14.21	11.53	22.25	11.85	8.00	22.25	11.85	7.94
27	0.4712	41.78	29.24	15.90	13.15	23.94	13.20	9.46	23.94	13.20	9.32
28	0.4887	44.85	31.61	17.81	15.03	25.80	14.72	11.19	25.80	14.72	10.94
29	0.5061	48.26	34.24	19.98	17.21	27.86	16.44	13.24	27.86	16.44	12.84
30	0.5236	52.05	37.16	22.46	19.75	30.14	18.40	15.67	30.14	18.40	15.07
31	0.5411	56.29	40.41	25.28	22.71	32.67	20.63	18.56	32.67	20.63	17.69
32	0.5585	61.04	44.04	28.52	26.20	35.49	23.18	22.02	35.49	23.18	20.79
33	0.5760	66.40	48.09	32.23	30.33	38.64	26.09	26.17	38.64	26.09	24.44
34	0.5934	72.48	52.64	36.50	35.23	42.16	29.44	31.15	42.16	29.44	28.77
35	0.6109	79.40	57.75	41.44	41.08	46.12	33.30	37.15	46.12	33.30	33.92
36	0.6283	87.33	63.53	47.16	48.11	50.59	37.75	44.43	50.59	37.75	40.05
37	0.6458	96.49	70.07	53.80	56.62	55.63	42.92	53.27	55.63	42.92	47.38
38	0.6632	107.13	77.50	61.55	67.00	61.35	48.93	64.07	61.35	48.93	56.17
39	0.6807	119.59	85.97	70.61	79.77	67.87	55.96	77.33	67.87	55.96	66.76
40	0.6981	134.31	95.66	81.27	95.61	75.31	64.20	93.69	75.31	64.20	79.54
41	0.7156	151.89	106.81	93.85	115.47	83.86	73.90	113.99	83.86	73.90	95.05
42	0.7330	173.09	119.67	108.75	140.65	93.71	85.37	139.32	93.71	85.37	113.96
43	0.7505	198.99	134.58	126.50	173.00	105.11	99.01	171.14	105.11	99.01	137.10
44	0.7679	231.10	151.95	147.74	215.16	118.37	115.31	211.41	118.37	115.31	165.58
45	0.7854	271.57	172.29	173.29	271.07	133.87	134.87	262.74	133.87	134.87	200.81
46	0.8029	323.57	196.22	204.19	346.66	152.10	158.50	328.73	152.10	158.50	244.65
47	0.8203	391.94	224.55	241.80	451.28	173.64	187.21	414.33	173.64	187.21	299.52
48	0.8378	484.34	258.29	287.85	600.15	199.26	222.30	526.45	199.26	222.30	368.67
49	0.8552	613.53	298.72	344.64	819.31	229.92	265.50	674.92	229.92	265.50	456.40
50	0.8727	801.95	347.51	415.15	1155.97	266.88	319.06	873.86	266.88	319.06	568.57

The bearing capacity of a soil is its ability to carry loads without failing in shear. There are four major methods to predict failure. The first method was developed by Karl Terzaghi in 1943. Field tests in Canada by Meyerhof (1963) lead to modification factors. Finally, Brinch Hansen in Denmark (1970) and Vesic in the USA modified these factors to a greater refinement.

These bearing capacity factors are based on these three authors:

### **Terzaghi (1943):**

For square footings,  $q_{ult} = 1.3c'N_c + \bar{q}N_q + 0.4\gamma BN_\gamma$

For continuous or wall footings  $q_{ult} = c'N_c + \bar{q}N_q + 0.5\gamma BN_\gamma$

where,  $\bar{q} = \gamma D_f$  and the factors are,

$$N_q = \frac{a^2}{a \cos^2(45^\circ - \phi/2)} \quad \text{where } a = e^{(0.75\pi - \phi/2)\tan\phi}$$

$$N_c = (N_q - 1) \cot \phi$$

$$N_\gamma = \frac{\tan \phi}{2} \left( \frac{K_{p\gamma}}{\cos^2 \phi} - 1 \right)$$

### **Meyerhof (1963):**

For vertical loads,  $q_{ult} = cN_c F_{sc} F_{dc} + \bar{q}N_q F_{sq} F_{dq} + 0.4\gamma BN_\gamma F_{s\gamma} F_{d\gamma}$

and for inclined loads,  $q_{ult} = cN_c F_{ic} F_{dc} + \bar{q}N_q F_{iq} F_{dq} + 0.4\gamma BN_\gamma F_{i\gamma} F_{d\gamma}$

and the factors are,

$$N_q = e^{\pi \tan \phi} \tan^2(45^\circ - \phi/2)$$

$$N_c = (N_q - 1) \cot \phi$$

$$N_\gamma = (N_q - 1) \tan(1.4\phi)$$

### **Brinch Hansen (1970):**

The general equation,  $q_{ult} = cN_c F_{sc} F_{dc} F_{ic} + \bar{q}N_q F_{sq} F_{dq} F_{iq} + 0.4\gamma BN_\gamma F_{s\gamma} F_{d\gamma} F_{i\gamma}$

and the factors are,

$$N_q = e^{\pi \tan \phi} \tan^2(45^\circ - \phi/2)$$

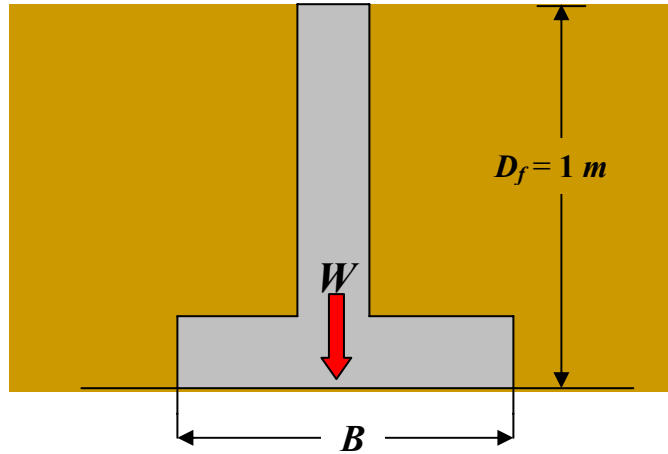
$$N_c = (N_q - 1) \cot \phi$$

$$N_\gamma = 1.5(N_q - 1) \tan \phi$$

**\*Bearing–01: Terzaghi's bearing capacity formula for a square footing.**

(Revision: Sept-08)

The square footing shown below must be designed to carry a 294 kN load. Use *Terzaghi's* bearing capacity formula to determine  $B$  of the square footing with a Factor of Safety =3.



$$W = 294 \text{ kN}$$

$$\gamma = 18.15 \text{ kN/m}^3$$

$$\phi = 35^\circ$$

$$c = 0$$

**Solution:**

Terzaghi's formula for the ultimate bearing capacity  $q_{ult}$  of a square footing is,

$$q_{ult} = 1.3c'N_c + \bar{q}N_q + 0.4\gamma BN_\gamma \quad \text{where} \quad \bar{q} = D_f\gamma$$

The allowable bearing capacity  $q_{all}$  with the factor of safety of 3 is,

$$q_{all} = \frac{q_{ult}}{3} = \frac{1}{3}(1.3c'N_c + \bar{q}N_q + 0.4\gamma BN_\gamma) \quad \text{and} \quad q_{all} = \frac{W}{B^2} = \frac{294 \text{ kN}}{B^2}$$

$$\text{or} \quad \frac{294}{B^2} = \frac{1}{3}(1.3c'N_c + \bar{q}N_q + 0.4\gamma BN_\gamma)$$

For  $\phi=35^\circ$ ,  $N_c=57.8$ ,  $N_q=41.4$ , and  $N_\gamma=41.1$ .

Substituting these values into Terzaghi's equation, we get

$$\frac{294}{B^2} = \frac{1}{3}[(0) + (18.15)(1)(41.4) + (0.4)(18.15)B(41.1)]$$

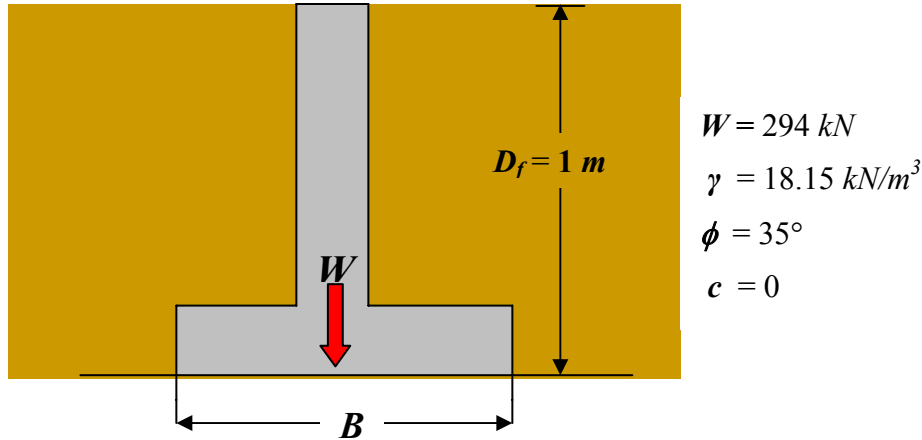
$$\frac{294}{B^2} = 250.5 + 99.5B$$

$$B^3 + 2.52B^2 - 2.96 = 0 \quad \therefore \quad B = 0.90 \text{ m}$$

### \*Bearing–02: Meyerhof's bearing capacity formula for a square footing.

(Revision: Sept-08)

The square footing shown below must be designed to carry a 294 kN load. Use Meyerhof's bearing capacity formula to determine  $B$  with a factor of safety =3.



#### Solution:

Meyerhof's formula for the ultimate bearing capacity  $q_{ult}$  of a square footing is,

$$q_{ult} = c' N_c F_{sc} F_{dc} F_{ic} + \bar{q} N_q F_{sq} F_{dq} F_{iq} + 0.4 \gamma B N_\gamma F_{s\gamma} F_{d\gamma} F_{i\gamma} \quad \text{where} \quad \bar{q} = D_f \gamma$$

Since the load is vertical, all three inclination factors  $F_{ic} = F_{iq} = F_{i\gamma} = 1$ .

$$F_{sq} = 1 + \left( \frac{B}{L} \right) \tan \phi = 1 + \left( \frac{1}{1} \right) \tan 35^\circ = 1.70 \quad \text{and} \quad F_{s\gamma} = 1 - 0.4 \left( \frac{B}{L} \right) = 1 - 0.4(1) = 0.6$$

$$F_{dq} = 1 + 2 \tan \phi (1 - \sin \phi)^2 \left( \frac{D_f}{B} \right) = 1 + 2 (\tan 35^\circ) (1 - \sin 35^\circ)^2 \left( \frac{1}{B} \right) \approx 1.25 \quad \text{and} \quad F_{d\gamma} = 1$$

The allowable bearing capacity  $q_{all}$  with the factor of safety of 3 is,

$$q_{all} = \frac{q_{ult}}{3} = \frac{1}{3} (c' N_c F_{sc} F_{dc} + \bar{q} N_q F_{sq} F_{dq} + 0.4 \gamma B N_\gamma F_{s\gamma} F_{d\gamma}) \quad \text{and} \quad q_{all} = \frac{W}{B^2} = \frac{294 \text{ kN}}{B^2}$$

$$\text{or} \quad \frac{294}{B^2} = \frac{1}{3} (c' N_c F_{sc} F_{dc} + \bar{q} N_q F_{sq} F_{dq} + 0.4 \gamma B N_\gamma F_{s\gamma} F_{d\gamma})$$

For  $\phi = 35^\circ$ ,  $N_c = 46.12$ ,  $N_q = 33.30$ , and  $N_\gamma = 37.15$ .

Substituting these values into Meyerhof's equation, we get

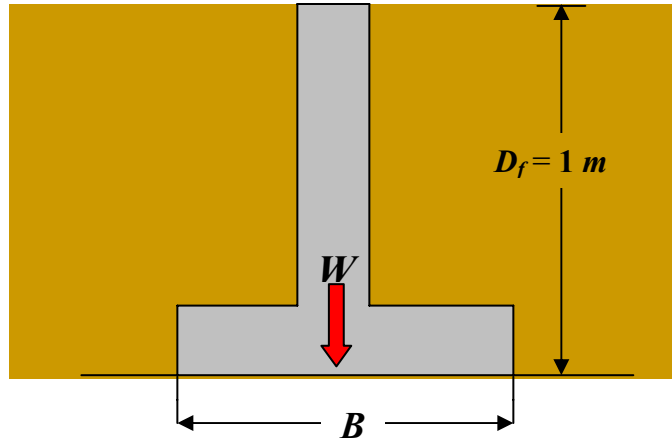
$$\frac{294}{B^2} = \frac{1}{3} [(0) + (18.15)(1)(33.3)(1.7)(1.25) + (0.4)(18.15)B(37.15)(0.6)(1)]$$

$$\frac{294}{B^2} = 428.1 + 53.94B \quad \text{or} \quad B^3 + 7.94B - 5.45 = 0 \quad \therefore B = 0.65 \text{ m}$$

**\*Bearing–03: Hansen's bearing capacity formula for a square footing.**

(Revision: Sept-08)

The square footing shown below must be designed to carry a 294 kN load. Use Brinch Hansen's bearing capacity formula to determine  $B$  with a factor of safety =3.



$$\begin{aligned} W &= 294 \text{ kN} \\ \gamma &= 18.15 \text{ kN/m}^3 \\ \phi &= 35^\circ \\ c &= 0 \end{aligned}$$

**Solution:**

Hansen's formula for the ultimate bearing capacity  $q_{ult}$  of a square footing is,

$$q_{ult} = c' N_c F_{sc} F_{dc} F_{ic} + \bar{q} N_q F_{sq} F_{dq} F_{iq} + 0.4 \gamma B N_\gamma F_{s\gamma} F_{d\gamma} F_{i\gamma} \quad \text{where} \quad \bar{q} = D_f \gamma$$

Since the load is vertical, all three inclination factors  $F_{ic} = F_{iq} = F_{i\gamma} = 1$ .

$$F_{sq} = 1 + \left( \frac{B}{L} \right) \tan \phi = 1 + \left( \frac{1}{1} \right) \tan 35^\circ = 1.7 \quad \text{and} \quad F_{s\gamma} = 1 - 0.4 \left( \frac{B}{L} \right) = 1 - 0.4(1) = 0.6$$

$$F_{dq} = 1 + 2 \tan \phi (1 - \sin \phi)^2 \left( \frac{D_f}{B} \right) = 1 + 2 (\tan 35^\circ) (1 - \sin 35^\circ)^2 \left( \frac{1}{B} \right) \approx 1.255 \quad \text{and} \quad F_{d\gamma} = 1$$

The allowable bearing capacity  $q_{all}$  with the factor of safety of 3 is,

$$q_{all} = \frac{q_{ult}}{3} = \frac{1}{3} \left( c' N_c F_{sc} F_{dc} + \bar{q} N_q F_{sq} F_{dq} + 0.4 \gamma B N_\gamma F_{s\gamma} F_{d\gamma} \right) \quad \text{and} \quad q_{all} = \frac{W}{B^2} = \frac{294 \text{ kN}}{B^2}$$

$$\text{or} \quad \frac{294}{B^2} = \frac{1}{3} \left( c' N_c F_{sc} F_{dc} + \bar{q} N_q F_{sq} F_{dq} + 0.4 \gamma B N_\gamma F_{s\gamma} F_{d\gamma} \right)$$

For  $\phi = 35^\circ$ ,  $N_c = 46.12$ ,  $N_q = 33.30$ , and  $N_\gamma = 33.92$ .

Substituting these values into Hansen's equation, we get

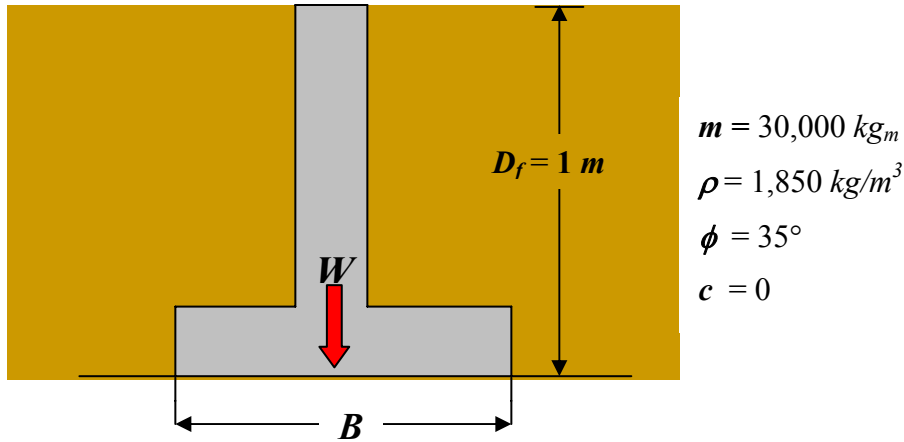
$$\frac{294}{B^2} = \frac{1}{3} \left[ (0) + (18.15)(1)(33.3)(1.7)(1.255) + (0.4)(18.15)B(33.92)(0.6)(1) \right]$$

$$\frac{294}{B^2} = 429.8 + 49.25B \quad \text{or} \quad B^3 + 8.73B - 5.97 = 0 \quad \therefore B = 0.70 \text{ m}$$

**\*Bearing–04: Same as #01 but requiring conversion from metric units.**

(Revision: Sept-08)

The square footing shown below must be designed to a load of 30,000  $kg_m$ . Using a factor of safety of 3 and using *Terzaghi's* method, determine the size  $B$  of the square footing.



**Solution:**

The soil density  $\rho = 1,850 \text{ kg/m}^3$  converts to a unit weight via  $\gamma = \rho g$  (like  $F = ma$ ),

$$\gamma = \rho g = \frac{\left(1,850 \frac{\text{kg}_m}{\text{m}^3}\right) \left(9.81 \frac{\text{m}}{\text{s}^2}\right)}{(1,000 \text{ N/kN})} = 18.15 \text{ kN/m}^3 \text{ and the load to be supported by the footing is,}$$

$$W = ma = \frac{(30,000 \text{ kg}_m) \left(9.81 \frac{\text{m}}{\text{s}^2}\right)}{(1,000 \text{ N/kN})} = 294 \text{ kN}$$

Terzaghi's ultimate bearing capacity of a square footing is given by,

$$q_{ult} = 1.3c'N_c + \bar{q}N_q + 0.4\gamma BN_\gamma$$

$$\therefore q_{all} = \frac{q_{ult}}{3} = \frac{1}{3} (1.3c'N_c + \bar{q}N_q + 0.4\gamma BN_\gamma) \text{ and } q_{all} = \frac{P}{B^2} = \frac{294}{B^2}$$

$$\text{or } \frac{294}{B^2} = \frac{1}{3} (1.3c'N_c + \bar{q}N_q + 0.4\gamma BN_\gamma)$$

For  $\phi = 35^\circ$ ,  $N_c = 57.8$ ,  $N_q = 41.4$ , and  $N_\gamma = 41.1$ ,

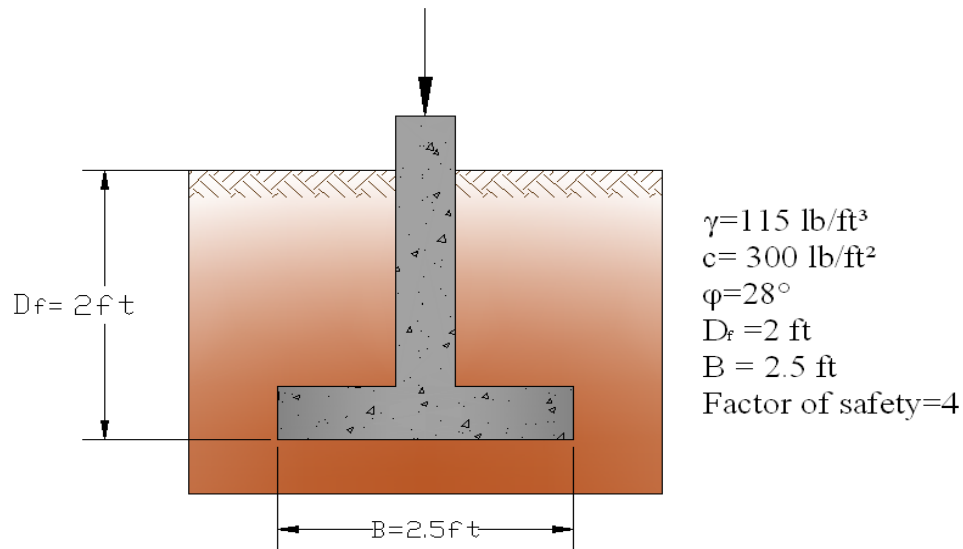
$$\frac{294}{B^2} = \frac{1}{3} [(0) + (18.15)(1)(41.4) + (0.4)(18.15)B(41.1)] \therefore B^3 + 2.52B^2 - 2.96 = 0$$

$$B = 0.90 \text{ m}$$

### \*Bearing–05: General versus local bearing capacity failures.

(Revision: Sept-08)

Using Terzaghi's method, distinguish between the value of the local shear failure versus the general shear failure.



#### Solution:

Terzaghi's general bearing capacity failure of a square footing is,

$$q_{ult} = 1.3c'N_c + \bar{q}N_q + 0.4\gamma BN_\gamma$$

For  $\phi = 28^\circ$   $N_c = 31.6$ ,  $N_q = 17.8$ ,  $N_\gamma = 15.0$  and  $\bar{q} = \gamma D_f = (0.115)(2) = 0.23 \text{ ksf}$

Therefore,

$$q_{ult} = 1.3(0.30)(31.6) + (0.23)(17.8) + 0.4(0.115)(2.5)(15.0) = 18.1 \text{ ksf}$$

To find the value of the bearing capacity of a local shear failure, the cohesion and angle of internal friction are reduced by two-thirds,

$$q'_{ult-local} = 1.3c'N'_c + \bar{q}'N'_q + 0.4\gamma BN'_\gamma \quad \text{where } c' = \frac{2}{3}c = \frac{2}{3}(0.30) = 0.2 \text{ ksf}$$

and  $\phi' = \frac{2}{3}(\phi) = \frac{2}{3}(28^\circ) = 18.7^\circ$  which give  $N'_c = 16.2$ ,  $N'_q = 6.5$  and  $N'_\gamma = 4.52$

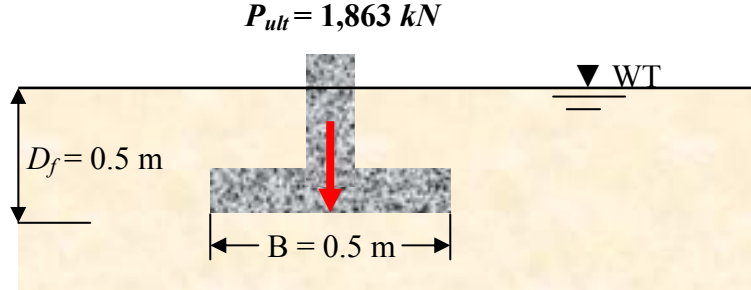
$$\therefore q'_{ult-local} = (1.3)(0.2)(16.2) + (0.23)(6.5) + (0.4)(0.115)(2.5)(4.52) = 6.2 \text{ ksf}$$

$q_{ult-general \text{ failure}} = 18.1 \text{ ksf}$  versus  $q_{ult-local \text{ failure}} = 6.2 \text{ ksf}$  (Almost a three-to-one)

### \*Bearing–06: Comparing the Hansen and Meyerhof bearing capacities.

(Revision: Sept-08)

Compare the results of the *Hansen* and the *Meyerhof* bearing capacity formulas to the results of a field test that took a rectangular footing to failure when the load reached 1,863 kN. Given  $B = 0.5$  m,  $L = 2.0$  m,  $c = 0$ ,  $\phi_{\text{triaxial}} = 42^\circ$  and  $\gamma' = 9.31$  kN/m<sup>3</sup> (the WT is at the surface).



#### Solution:

$$q_{ult} = \frac{P_{ult}}{BL} = \frac{1,863 \text{ kN}}{(0.5 \text{ m})(2.0 \text{ m})} = 1,863 \text{ kPa} \quad \text{was the field measured failure load.}$$

(1) The Hansen formula predicts an ultimate bearing capacity of,

$$q_{ult} = 0 + \bar{q}N_q F_{qs} F_{qd} + 0.5\gamma BN_\gamma F_{\gamma s} F_{\gamma d}$$

$$\text{Lee's adjustment formula is } \phi_{ps} = 1.5\phi_{\text{triaxial}} - 17^\circ = 1.5(42^\circ) - 17^\circ = 46^\circ$$

$$\text{For } \phi = 46^\circ, N_q = 158.5 \text{ and } N_\gamma = 244.65$$

$$F_{qs} = 1 + \left(\frac{B}{L}\right) \tan \phi = 1 + \left(\frac{0.5}{2}\right) \tan 46^\circ = 1.26$$

$$F_{\gamma s} = 1 - 0.4 \left(\frac{B}{L}\right) = 1 - 0.4 \left(\frac{0.5}{2}\right) = 0.9$$

$$F_{qd} = 1 + 2 \tan \phi (1 - \sin \phi)^2 \left(\frac{D_f}{B}\right) = 1 + 2 \tan 46^\circ (1 - \sin 46^\circ)^2 \left(\frac{0.5}{0.5}\right) = 1.16$$

$$F_{\gamma d} = 1.0$$

$$\therefore q_{ult} = 0 + (9.31)(0.5)(159)(1.27)(1.16) + (0.5)(9.31)(0.5)(245)(0.9)(1.0)$$

$$\therefore q_{ult} = 1,485 \text{ kPa versus } 1,863 \text{ kPa measured (Hansen underestimates by 20\%)}$$

(2) The Meyerhof formula with  $\phi = 46^\circ$ ,  $N_q = 158.5$  and  $N_\gamma = 328.73$ ,

$$q_{ult} = 0 + \bar{q}N_q F_{qs} F_{qd} + 0.5\gamma BN_\gamma F_{\gamma s} F_{\gamma d}$$

$$\therefore q_{ult} = 0 + (9.31)(0.5)(158.5)(1.27)(1.16) + (0.5)(9.31)(0.5)(328.73)(0.9)(1.0)$$

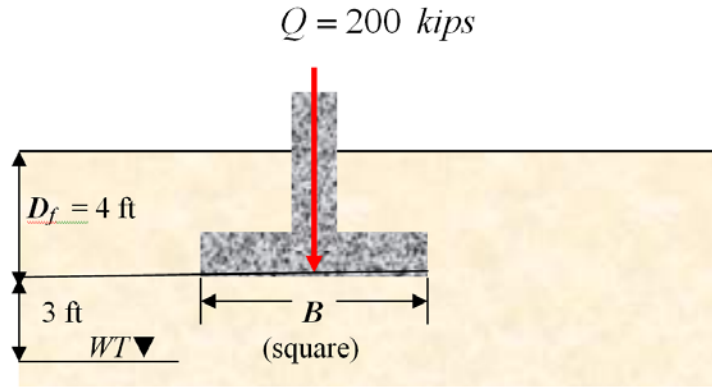
$$\therefore q_{ult} = 1,782 \text{ kPa versus } 1,863 \text{ kPa (Meyerhof underestimates by 4 \% )}.$$

**\*Bearing-07: Increase a footing's width if the WT is expected to rise.**

(Revision: Sept-08)

Use Meyerhof's bearing capacity formula (with a factor of safety = 3) to select a footing's width  $B$  if, (a) the water table is as shown below, and (b) if the water table rises to the ground surface?

The soil has a unit weight of 112 pcf, a moisture of 10%,  $\phi = 25^\circ$ , a cohesion  $c_u = 240$  psf and a specific gravity of solids of  $G_s = 2.68$ .



**Solution:**

(a) Find  $\gamma_{SAT}$  to determine  $\gamma'$ ,

$$\gamma_{dry} = \frac{\gamma}{1 + w_N} = \frac{112}{1.10} = 101.8 \text{ pcf and } V_s = \frac{W_s}{G_s \gamma_w} = \frac{\gamma_{dry}}{G_s \gamma_w}$$

$$\text{set } V = 1 \text{ ft}^3 \therefore V_s = \frac{101.8}{2.68(62.4)} = 0.61 \text{ ft}^3 \therefore V_v = V - V_s = 1 - 0.61 = 0.39 \text{ ft}^3$$

$$\text{but } \gamma_{sat} = \gamma_{dry} + n\gamma_w = \gamma_{dry} + \left(\frac{V_v}{V}\right)\gamma_w \therefore \gamma_{sat} = 101.8 + (0.39)(62.4) = 126.2 \text{ pcf}$$

$$\text{and } \gamma' = \gamma_{sat} - \gamma_w = 126.2 - 62.4 = 63.8 \text{ pcf}$$

Try  $B = 5.7$  feet with Meyerhof's equation,

$$q_{ult} = c' N_c (F_{cs} F_{cd} F_{ci}) + \bar{q} N_q (F_{qs} F_{qd} F_{qi}) + 0.5 \gamma B N_\gamma (F_{\gamma s} F_{\gamma d} F_{\gamma i})$$

where the load inclination factors  $F_{ci}$ ,  $F_{qi}$  and  $F_{\gamma i} = 1$

$$\text{For } \phi \geq 10^\circ \quad K_p = \tan^2 \left( 45^\circ + \frac{\phi}{2} \right) = \tan^2 \left( 45^\circ + \frac{25^\circ}{2} \right) = 2.46, \text{ therefore}$$

$$F_{cs} = 1 + (0.2) \left( \frac{B}{L} \right) K_p = 1 + (0.2) \left( \frac{5.7}{5.7} \right) (2.46) = 1.49$$

$$F_{cd} = 1 + (0.2) \left( \frac{D_f}{B} \right) \sqrt{K_p} = 1 + (0.2) \left( \frac{4}{5.7} \right) \sqrt{2.46} = 1.22$$

$$F_{qd} = F_{\gamma d} = 1 + (0.1) \left( \frac{D_f}{B} \right) \sqrt{K_p} = 1 + (0.1) \left( \frac{4}{5.7} \right) \sqrt{2.46} = 1.11$$

$$F_{qs} = F_{\gamma s} = 1 + (0.1) \left( \frac{B}{L} \right) K_p = 1 + (0.1) \left( \frac{5.7}{5.7} \right) (2.46) = 1.25$$

The Meyerhof bearing capacity factors for  $\phi = 25^\circ$  are

$$N_c = 20.7, N_q = 10.7, \text{ and } N_\gamma = 6.77$$

$$q_{ult} = c' N_c (F_{cs} F_{cd} F_{ci}) + \bar{q} N_q (F_{qs} F_{qd} F_{qi}) + 0.5 \gamma B N_\gamma (F_{\gamma s} F_{\gamma d} F_{\gamma i})$$

$$q_{ult} = (0.24)(20.7)(1.49)(1.22)(1) + (0.112)(4)(10.7)(1.25)(1.11)(1) + (0.5)(0.112)(5.7)(6.67)(1.25)(1.11)(1)$$

$$q_{ult} = 18.6 \text{ ksf}$$

$$q_{all} = \frac{q_{ult}}{FS} = \frac{18.6}{3} = 6.2 \text{ ksf} \text{ therefore } B^2 = \frac{Q}{q_{all}} = \frac{200}{6.2} = 32.25 \text{ ft}^2 \therefore B = 5.7 \text{ ft}$$

Therefore the choice of  $B = 5.7 \text{ ft}$  was a good choice.

(b) When the water table rises to the ground surface, need a larger footing; try  $B = 7.0$  feet.

$$F_{cd} = 1 + 0.2 \left( \frac{B}{L} \right) K_p = 1 + 0.2 \left( \frac{7}{7} \right) (2.46) = 1.49$$

$$F_{cs} = 1.49 \text{ same as above}$$

$$F_{qd} = F_{\gamma d} = 1 + 0.1 \left( \frac{D}{B} \right) \sqrt{K_p} = 1 + 0.1 \left( \frac{4}{7} \right) \sqrt{2.46} = 1.09$$

$$F_{qs} = F_{\gamma s} = 1.25 \text{ same as above}$$

$$q_{ult} = (0.24)(20.7)(1.49)(1.18) + (0.062)(4)(10.7)(1.25)(1.09) + (0.5)(0.062)(7)(6.67)(1.09)(1.25)$$

$$q_{ult} = 16.62 \text{ ksf}$$

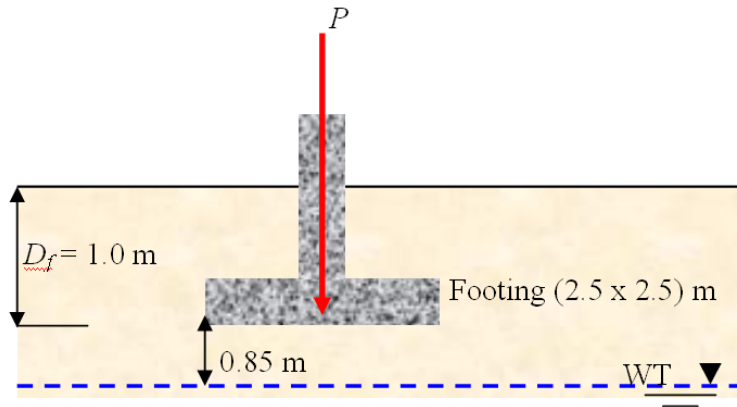
$$q_{all} = \frac{q_{ult}}{FS} = \frac{16.62}{3} = 5.54 \text{ ksf and } B^2 = \frac{Q}{q_{all}} = \frac{200}{5.54} = 36.1 \text{ ft}^2 \therefore B = 6.01 \text{ ft}$$

Iterate once more, and find  $B = 7.5 \text{ feet}$ .

**\*\*Bearing–08: The effect of the WT upon the bearing capacity.**

(Revision: Sept-08)

Using the Hansen method, what are the ultimate and allowable bearing capacities for the footing shown below if you require a factor of safety of at least 2?



$$c = 0$$

$$\phi_t = 35^\circ$$

$$\gamma = 18.10 \frac{kN}{m^3}$$

$$w_N = 10 \%, \text{ and } G_s = 2.68$$

**Solution:**

Always use the effective unit weight of water in the bearing capacity formulas. The average effective weight  $\gamma_e$  of the soil can also be given by the formula:

$$\gamma_e = (2H - d_w) \frac{d_w}{H^2} \gamma_{wet} + \frac{\gamma'}{H^2} (H - d_w)^2$$

$$\text{where } H = (0.5)B \tan \left( 45^\circ + \frac{\phi}{2} \right) = (0.5)(2.5) \tan \left( 45^\circ + \frac{35^\circ}{2} \right) = 2.40 \text{ m}$$

$$\text{and } d_w = \text{depth to the WT below the footing invert} = 0.85 \text{ m}$$

$$\text{Set the total volume } V = 1 \text{ m}^3$$

$$\gamma_{dry} = \frac{\gamma_{wet}}{1 + w} = \frac{18.10}{1 + 0.10} = 16.5 \frac{kN}{m^3} \quad \text{and} \quad V_s = \frac{\gamma_{dry}}{G_s \gamma_{wet}} = \frac{16.5}{(2.68)(9.8)} = 0.63 \text{ m}^3$$

$$V_v = 1.0 - V_s = 1 - 0.63 = 0.37 \text{ m}^3 \quad \text{and} \quad \gamma_{sat} = \gamma_{dry} + n\gamma_{wet} = 16.5 + (0.37)(9.8) = 20.1 \frac{kN}{m^3}$$

$$\therefore \gamma_e = ((2)(2.40 - 0.85)) \left[ \frac{0.85(18.10)}{(2.4)^2} \right] + \left[ \frac{20.1 - 9.8}{(2.4)^2} \right] (2.40 - 0.85)^2 = 12.6 \frac{kN}{m^3}$$

Using Hansen's method with  $\phi = 35^\circ$ , the bearing capacity factors are  $N_q = 33.3$  and  $N_\gamma = 33.92$ .

$$F_{qs} = 1 + \frac{B}{L} \tan \phi = 1 + \left( \frac{2.5}{2.5} \right) \tan 35^\circ = 1.70$$

$$F_{qd} = 1 + 2 \tan \phi (1 - \sin \phi)^2 \left( \frac{D_f}{B} \right) = 1 + 2 \tan 35^\circ (1 - \sin 35^\circ)^2 \left( \frac{1}{2.5} \right) = 1.10$$

$$F_{\gamma s} = 1 - 0.4 \frac{B}{L} = 1 - 0.4 \left( \frac{2.5}{2.5} \right) = 0.6$$

$$F_{\gamma d} = 1.0$$

Therefore, the ultimate and allowable bearing capacities are,

$$q_{ult} = 0 + \bar{q} N_q (F_{qs} F_{qd}) + 0.5 \gamma_e B N_\gamma (F_{\gamma s} F_{\gamma d})$$

$$q_{ult} = (18.1)(1.0)(33)(1.70)(1.10) + (0.5)(12.6)(2.5)(34)(0.6)(1)$$

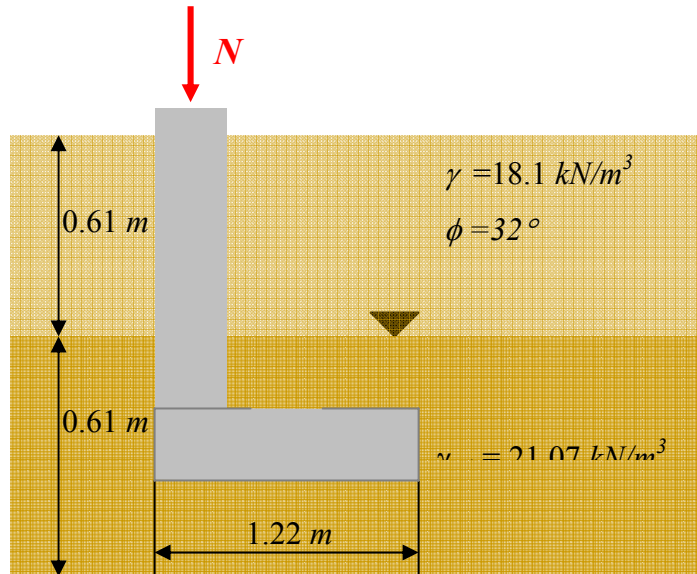
$$q_{ult} = 1,497 \text{ kPa}$$

$$q_{all} = \frac{1,497}{2} = 749 \text{ kPa}$$

**\*Bearing–09: Finding the gross load capacity.**

(Revision: Sept-08)

Use the Hansen formula to determine the gross normal load  $N$  on the column shown below using a factor of safety of 3.



**Solution:**

The Hansen formula for a footing is,

$$q_{ult} = cN_c F_{cs} F_{cd} + qN_q F_{qs} F_{qd} + 0.5\gamma B N_\gamma F_{ys} F_{yd}$$

The inclination factors  $F_{ci}$ ,  $F_{qi}$ , and  $F_{\gamma i}$  are all equal to 1 because the load is vertical.

For  $\phi = 32^\circ$ ,  $N_c = 35.49$ ,  $N_q = 23.18$  and  $N_\gamma = 20.79$  and  $B / L = 1$

$$F_{cs} = 1 + \left( \frac{N_q}{N_c} \right) = 1 + (23.20 / 35.50) = 1.65$$

$$F_{qs} = 1 + \tan \phi = 1 + 0.62 = 1.62$$

$$F_{ys} = 1 - 0.4 \left( \frac{B}{L} \right) = 1 - 0.4 = 0.60$$

$$F_{qd} = 1 + 2 \tan \phi (1 - \sin \phi)^2 \left( \frac{D_f}{B} \right) = 1 + (2)(0.62)(0.22)(1) = 1.273 \text{ for } D_f / B \leq 1$$

$$F_{yd} = 1$$

$$F_{cd} = F_{qd} - \left[ \frac{(1 - F_{qd})}{N_q \tan \phi} \right] = 1.273 - \left[ \left( \frac{1 - 1.273}{23.20 \times 0.62} \right) \right] = 1.292$$

The  $WT$  is located above the footing, therefore,

$$\bar{q} = (0.61m)(18.1 \text{ kN} / m^3) + (0.61m)(21.07 - 9.81) = 17.9 \text{ kN} / m^2$$

$$\therefore q_{ult} = (17.9)(1.62)(1.273)(23.20) + (0.5)(0.6)(21.07 - 9.81)(1.22)(20.8)(1) = 981 \text{ kN} / m^2$$

Therefore,

$$q_{all} = \left( \frac{q_{ult}}{3} \right) = \left( \frac{981 \text{ kN} / m^2}{3} \right) = 327 \text{ kN} / m^2$$

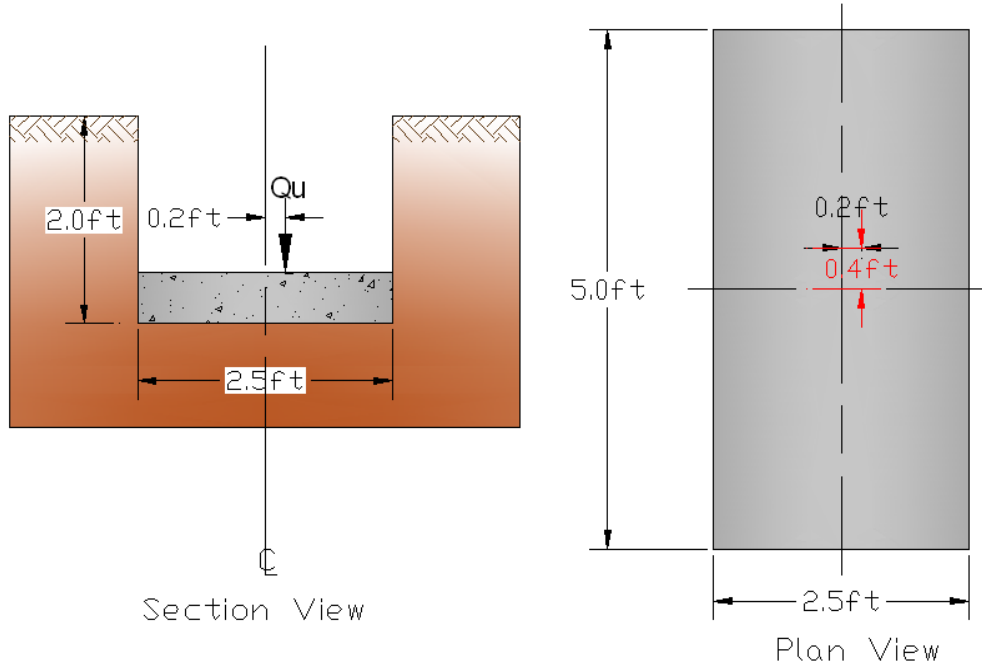
Hence, the total gross load  $N$  is,

$$N = q_{all} B^2 = (327 \text{ kN} / m^2)(1.22m)^2 = 487 \text{ kN}$$

**\*\*Bearing–10: The effect of an eccentric load upon bearing capacity.**

(Revision: Sept-08)

A rectangular footing measures 5 feet by 2.5 feet. Determine the gross ultimate load  $Q_{ult}$  applied eccentrically upon the footing, and the ultimate bearing capacity of the soil  $q_{ult}$ , given that  $\gamma = 115$  pcf,  $c = 0$  and  $\phi = 30^\circ$ .



**Solution:**

The effective width footing width  $B' = B - 2e_x = (2.5) - 2(0.2) = 2.1$  ft

and the effective length  $L' = L - 2e_y = (5) - 2(0.4) = 4.2$  ft.

Meyerhof's ultimate bearing capacity formula with  $c = 0$  is,

$$q_{ult} = 0 + \bar{q}N_q F_{qs} F_{qd} + 0.5\gamma B'N_\gamma F_{ys} F_{yd}$$

For  $\phi = 30^\circ$ ,  $N_q = 18.4$  and  $N_\gamma = 15.67$

$$F_{qs} = 1 + \left( \frac{B'}{L'} \right) \tan \phi = 1 + \left( \frac{2.1}{4.2} \right) (0.58) = 1.29$$

$$F_{qd} = 1 + 2(\tan 30^\circ)(1 - \sin 30^\circ)^2 \left( \frac{2}{2.1} \right) = 1.275$$

$$F_{ys} = 1 - 0.4 \left( \frac{B'}{L'} \right) = 1 - 0.4 \left( \frac{2.1}{4.2} \right) = 0.8$$

$$F_{yd} = 1$$

$$q_{ult} = (2)(0.115)(18.4)(1.29)(1.275) + (0.5)(0.115)(2.1)(15.67)(0.8)(1) = 8.47 \text{ ksf}$$

$$\text{Hence, } Q_{ult} = q_{ult} (B'L') = (8.47)(2.1)(4.2) = 74.73 \text{ kips}$$

**\*\*Bearing–11: The effect of an inclined load upon the bearing capacity.**

(Revision: Sept-08)

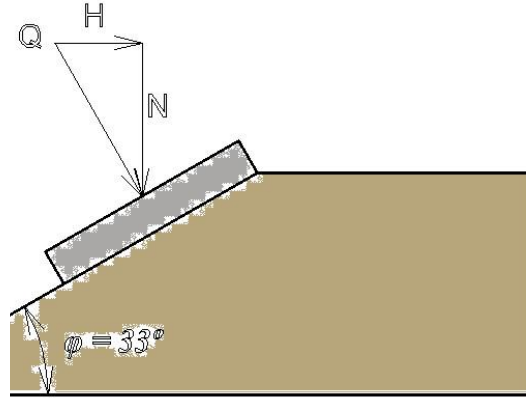
A square 8' x 8' footing is loaded with an axial load of 400 kips and  $M_x = 200$  ft-kips,  $M_y = 120$  ft-kips. Un-drained triaxial tests (the soil is not saturated) gave  $\phi = 33^\circ$  and  $c = 200$  psf. The footing depth  $D_f = 6.0$  feet, the soil unit weight is 115 pcf, and the  $WT$  was not found.

Use the *Hansen* equation with the *Meyerhof* reduction factors and a  $FS = 3$  to find the

Vertical axial load = 400 kips

$M_x = 200$  ft-kips

$M_y = 120$  ft-kips



**Solution:**

$$\text{Eccentricities} \quad e_x = \frac{M_y}{Q} = \frac{120 \text{ ft-k}}{400} = 0.3 \text{ feet} \quad \text{and} \quad e_y = \frac{M_x}{Q} = \frac{200 \text{ ft-k}}{400} = 0.5 \text{ feet}$$

$$\therefore B_r = B - 2e_y = 8' - 1' = 7 \text{ feet} \quad \text{and} \quad L_r = L - 2e_x = 8' - 0.6' = 7.4 \text{ feet} \quad (\text{ie. } L_r > B_r)$$

Adjusting the  $\phi$  from triaxial ( $\phi_{tr}$ ) to a plane-strain value ( $\phi_{ps}$ ) via Lee's formulation,

$$\phi_{ps} \cong 1.1\phi_{tr} = 1.1(32.7^\circ) = 36^\circ$$

$$N_q = e^{\pi \tan 36^\circ} \tan^2 \left( 45^\circ + \frac{36^\circ}{2} \right) = 37.8$$

$$N_c = (N_q - 1) \cot \phi = (36.8) \cot 36^\circ = 50.6$$

$$N_\gamma = (N_q - 1) \tan(1.4\phi) = (36.8) \tan 50.4^\circ = 44.4$$

$$N_\gamma = 1.5(N_q - 1) \tan \phi = 1.5(36.8) \tan 36^\circ = 40.1$$

$$\therefore S_c = 1 + 0.2K_p \left( \frac{B_r}{L_r} \right) = 1 + 0.2(3.85) \frac{7}{7.4} = 1.73$$

$$\text{and} \quad d_c = 1 + 0.2\sqrt{K_p} \left( \frac{D}{B_r} \right) = 1 + 0.2(\sqrt{3.85}) \left( \frac{6}{7} \right) = 1.34$$

$$\text{Since } \phi > 10^\circ, S_q = S_\gamma \cong 1.0 \quad \text{and} \quad d_q = d_\gamma = 1.0.$$

$$\text{Hansen's } q_{ult} = 0.5\gamma B N_\gamma S_\gamma d_\gamma i_\gamma g_\gamma b_\gamma + c N_c S_c d_c i_c g_c b_c + q_q N_q S_q d_q i_q g_q b_q$$

Also  $i = g = b = 1.0$  for this problem, since  $\alpha = 0 = i$  (inclination factor  $f$  / load  $Q$  with  $t$  vertical)  $\eta = 4$   
 $g$  (ground factor with  $t$  inclined ground on side of footing)  
 $b$  (base factor with  $t$  inclined ground under the footing)

$$q_{ult} = 0.5(0.115)(7)(40.1)(1) + (0.200)(50.6)(1.73)(1.34) + (0.115)(6)(37.8)(1) =$$

$$q_{ult} = 16.1 + 23.5 + 26.1 = 65.7 \text{ ksf}$$

$$q_{all} = \frac{q_{ult}}{FS} = \frac{65.7}{3} = 21.9 \text{ ksf}$$

$$R_{ex} = 1 - \left( \frac{e_x}{B} \right)^{\frac{1}{2}} = 1 - \left( \frac{0.3}{8} \right)^{\frac{1}{2}} = 0.81$$

$$R_{ey} = 1 - \left( \frac{e_y}{B} \right)^{\frac{1}{2}} = 1 - \left( \frac{0.5}{8} \right)^{\frac{1}{2}} = 0.75$$

$$Q_{all} = q_{all} (B^2) (R_{ex}) (R_{ey}) = 21.9(8 \times 8)(0.81)(0.75) = 851 \text{ kips}$$

$$q_{all} = \frac{Q_{all}}{B^2} = \left( \frac{851}{64} \right) = 13.3 \text{ ksf}$$

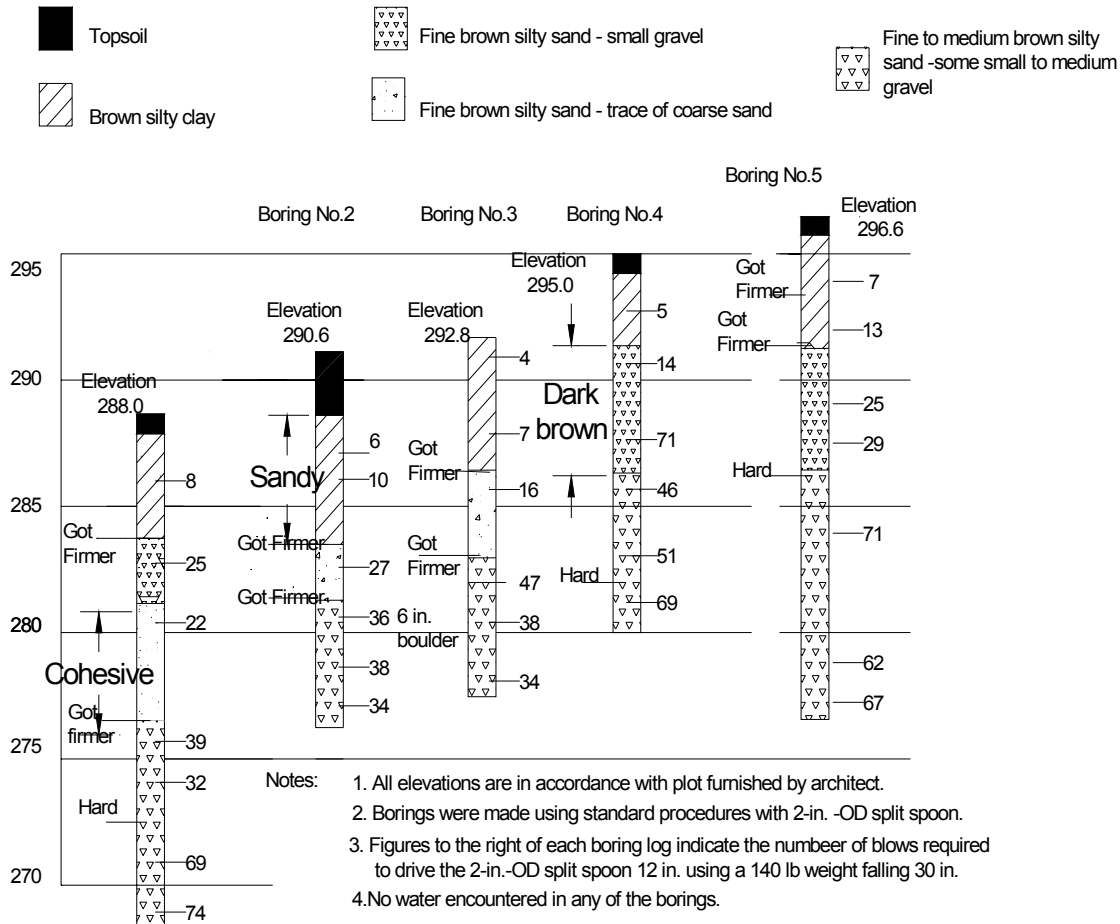
$$(\text{The contact load } q_o = 13 \left( \frac{400}{851} \right) = 6.1 \text{ ksf})$$

## **\*\*Bearing-12: Interpretation of borings to estimate a bearing capacity.**

(Revision: Sept-08)

Use the boring logs show below to recommend an allowable soil pressure  $q_{all}$  for the footings located in the vicinity of elevation 284, boring No. 2?

The building is a four-story (five on the low side) office building with column loads around 160 kips. State your reasons.



### **Solution:**

It is presumed that all the building's footings will be placed at roughly elevation 284 or thereabouts. This is fine for the building area covered by borings # 3, 4 and 5 because they have good SPT values.

Meyerhof has proposed formulas for the allowable bearing capacity adjusted so that the settlement is limited to 1-inch. These formulas are:

$$q_{all} = \frac{N}{4} (K_D) \text{ for } B \leq 4 \text{ ft}$$

$$q_{all} = \frac{N}{6} \left( \frac{B+1}{B} \right)^2 (K_D) \text{ for } B > 4\text{ft}$$

$$\text{where } K_D = 1 + 0.33 \left( \frac{D_f}{B} \right) \leq 1.33$$

$$\text{For the silty sand use } N = \frac{47 + 51 + 71}{3} = 56.33 \approx 56 \text{ (\#3, 4, and 5)}$$

Let's assume  $B = 4.5$  ft and  $D_f = 0$

$$q_{all} = \frac{56}{6} \left( \frac{4.5+1}{4.5} \right)^2 (1) = 13.9 \text{ ksf} \quad \text{This suggests that a } B = 4.5 \text{ feet is excessive since}$$

$$q_0 = \frac{Q}{B^2} = \frac{160 \text{ kips}}{20.25 \text{ sf}} = 7.9 \approx q_{all} = 13.9 \text{ ksf}$$

Assume  $B < 4$  ft, say  $B \sim 3.5$  ft, and use formula

$$q_{all} = \frac{N}{4} (K_D) \quad K_D = 1 + 0.33 D_f / B \quad \frac{56}{4} \left( 1 + \frac{0.33 D_f}{B} \right) \text{ and } D_f = 0 \quad q_{all} = 14 \text{ ksf}$$

$$\therefore q_0 = \frac{Q}{B^2} = \frac{160 \text{ kips}}{(3.5)^2} = 13.06 \approx 13 \text{ ksf} \leq 14 \text{ ksf} \quad \text{OK}$$

For footings in area of borings # 1 and #2, they will be deeper by 1-story (ie. for 5-story building). That places the shallow foundation at elevation 274 ft. This area will have bearing in the same strata.  $N = 32$  and using  $B = 3.50'$  and  $D_f = 4.5'$

$$q_{all} = \frac{N}{4} K_D \quad K_D = 1 + 0.33 D_f / B \leq 1.33 \quad K_D = \frac{32}{4} \left( 1 + \frac{0.33 \times 4.5}{3.50} \right) = 1.33$$

$$q_{all} = 10.64 \text{ ksf} < 13 \text{ ksf} \quad \text{NOT GOOD}$$

$$\text{Let's use } B = 3.90 \text{ feet} \quad q_{all} = 10.64 \text{ ksf} \quad q_0 = \left( \frac{Q}{B^2} \right) = 10.51 \text{ ksf} \leq 10.64 \text{ ksf}$$

Use  **$B = 3.90$  feet.**

# Chapter 18

## Shallow Foundations

***Symbols for Shallow Foundations***

*Properties of Reinforcing Steel (British and SI units).*

<b>ASTM STANDARD REINFORCING BARS</b>				
<b>BAR SIZE DESIGNATION</b>	<b>WEIGHT POUNDS PER FOOT</b>	<b>NOMINAL DIMENSIONS – ROUND SECTIONS</b>		
		<b>DIAMETER INCHES</b>	<b>CROSS SECTIONAL AREA - SQ. INCHES</b>	<b>PERIMETER INCHES</b>
<b>#3</b>	<b>.376</b>	<b>.375</b>	<b>.11</b>	<b>1.178</b>
<b>#4</b>	<b>.668</b>	<b>.500</b>	<b>.20</b>	<b>1.571</b>
<b>#5</b>	<b>1.043</b>	<b>.625</b>	<b>.31</b>	<b>1.963</b>
<b>#6</b>	<b>1.502</b>	<b>.750</b>	<b>.44</b>	<b>2.356</b>
<b>#7</b>	<b>2.044</b>	<b>.875</b>	<b>.60</b>	<b>2.749</b>
<b>#8</b>	<b>2.670</b>	<b>1.000</b>	<b>.79</b>	<b>3.142</b>
<b>#9</b>	<b>3.400</b>	<b>1.128</b>	<b>1.00</b>	<b>3.544</b>
<b>#10</b>	<b>4.303</b>	<b>1.270</b>	<b>1.27</b>	<b>3.990</b>
<b>#11</b>	<b>5.313</b>	<b>1.410</b>	<b>1.56</b>	<b>4.430</b>
<b>#14</b>	<b>7.65</b>	<b>1.693</b>	<b>2.25</b>	<b>5.32</b>
<b>#18</b>	<b>13.60</b>	<b>2.257</b>	<b>4.00</b>	<b>7.09</b>

<b>METRIC REINFORCING BARS</b>				
<b>BAR DESIG - NATION</b>	<b>MASS WEIGHT kg/m</b>	<b>NOMINAL DIMENSIONS</b>		
		<b>DIAMETER mm</b>	<b>CROSS SECTIONAL AREA mm<sup>2</sup></b>	<b>PERIMETER mm</b>
<b>10M</b>	<b>0.785</b>	<b>11.3</b>	<b>100</b>	<b>35.5</b>
<b>15M</b>	<b>1.570</b>	<b>16.0</b>	<b>200</b>	<b>50.1</b>
<b>20M</b>	<b>2.355</b>	<b>19.5</b>	<b>300</b>	<b>61.3</b>
<b>25M</b>	<b>3.925</b>	<b>25.2</b>	<b>500</b>	<b>79.2</b>
<b>30M</b>	<b>5.495</b>	<b>29.9</b>	<b>700</b>	<b>93.9</b>
<b>35M</b>	<b>7.850</b>	<b>35.7</b>	<b>1000</b>	<b>112.2</b>
<b>45M</b>	<b>11.775</b>	<b>43.7</b>	<b>1500</b>	<b>137.3</b>
<b>55M</b>	<b>19.625</b>	<b>56.4</b>	<b>2500</b>	<b>177.2</b>

The nominal dimensions of a deformed bar are equivalent to those of a plain round bar having the same mass per metre as the deformed bar.

### \*Footings–01: Analyze a simple square footing.

(Revision: Sept-08)

Design a square reinforced concrete footing for a column 15"x15" with 4 # 8 rebars, and,

$$D_L = 100 \text{ kips} \quad f'_c = 3,000 \text{ psi}$$

$$L_L = 120 \text{ kips} \quad f_y = 60,000 \text{ psi}$$

$$q_{allowable} = 4 \text{ ksf} \text{ from } q_{ult} = 10 \text{ ksf and } FS = 2.5$$

#### Solution:

1) Footing size for service loads:

$$B = \sqrt{\frac{Q}{q_a}} = \sqrt{\frac{220 \text{ kips}}{4 \text{ ksf}}} = 7.42 \text{ feet} \text{ therefore use } B = 7.5 \text{ feet.}$$

2) Check ultimate parameters: that is the actual soil pressure  $q_o$  under  $Q_{ult}$ ,

$$Q_{ULT} = 1.2D_L + 1.6L_L = 1.2(100) + 1.6(120) = 120 + 192 = 312 \text{ kips}$$

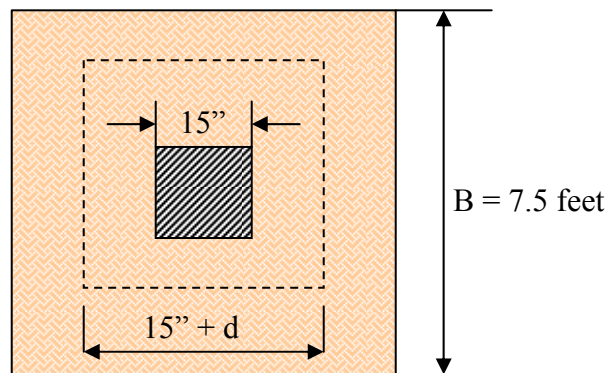
$$\therefore q_o = \frac{Q_U}{B^2} = \frac{312}{(7.5 \text{ ft})^2} = 5.5 \text{ ksf} < 10 \text{ ksf for } q_{ult} \dots \text{GOOD}$$

3) Compute the allowable concrete shear strength  $v_c$  (allowable)

$$v_{all} = 4\phi\sqrt{f'_c} \text{ where } \phi = 0.75 \text{ for shear and torsion,}$$

$$v_c = v_{all} = 4(0.75)\sqrt{3,000 \text{ psi}} = 164 \text{ psi} \approx 23.7 \text{ ksf}$$

4) Find  $d$ , the effective depth, (in this case two-way shear governs) in feet.



$$d^2 \left( v_c + \frac{q_o}{4} \right) + d \left( v_c + \frac{q_o}{4} \right) w - (B^2 - w^2) \left( \frac{q_o}{4} \right) = 0$$

$$d^2 \left( 23.7 + \frac{5.5}{4} \right) + d \left( 23.7 + \frac{5.5}{4} \right) \left( \frac{15}{12} \right) - \left[ 7.5^2 - \left( \frac{15}{12} \right)^2 \right] \left( \frac{5.5}{4} \right) = 0$$

$$d^2 + 1.25d - 3.0 = 0$$

Which yield to two solutions for  $d = \begin{cases} +1.22 \text{ ft} \\ -2.47 \text{ ft} \end{cases}$  and using the modified formula equation 2a:

When the column has a rectangular area  $bxc$ , the formula is,

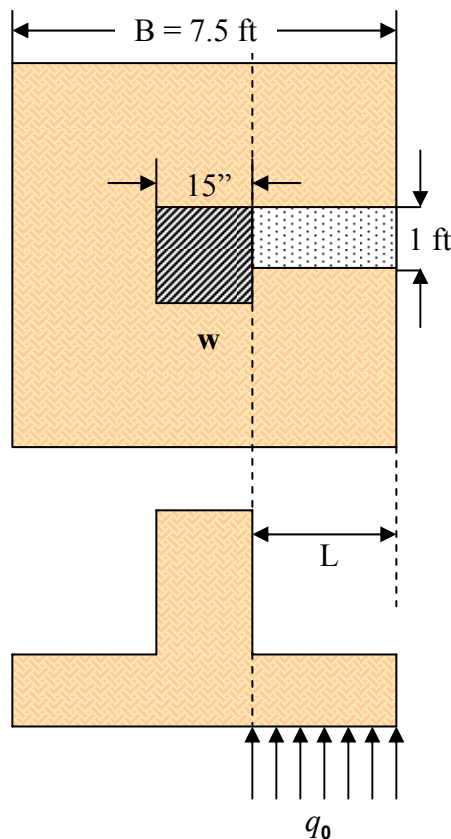
$$4d^2 + 2(b+c)d - \left( \frac{BLq_o}{v_c} \right) = 4d^2 + 2 \left( \frac{15}{12} + \frac{15}{12} \right) d - \left[ \frac{(7.5)^2 (5.5)}{23.7} \right] = 0$$

$$d^2 + 1.25d - 3.26 = 0 \text{ which yields } d = +0.39 \text{ ft}$$

Use the largest  $d = 1.22 \text{ feet} = 14.6 \text{ inches}$ ; round-out to  $d = 15 \text{ inches}$ . It is not necessary to check for wide-beam shear on a square footing.

5) Compute the area of steel  $A_s$  for flexure.

$$\text{Unit strip of the cantilever arm} = \left( \frac{B-w}{2} \right) = L \quad \therefore \quad L = \frac{\left( 7.5 - \left( \frac{15}{12} \right) \right)}{2} = 3.13 \text{ ft}$$



The cantilever moment:

$$M_U = \left( \frac{q_o L^2}{2} \right) = \frac{\left[ \left( 5.5 \frac{k}{ft^2} \right) (3.13 \text{ ft})^2 (12 \text{ in}) \right]}{2} = 359 \text{ in} - \text{kips}$$

$$M_U = \phi A_s f_y \left( \frac{d-a}{2} \right) \quad \text{where} \quad \phi = 0.9 \text{ for tension}$$

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{A_s (60)}{0.85 (3) (12 \text{ in})} = 1.96 A_s$$

Substituting  $a$  into the  $M_U$  equation above,

$$M_U = (0.9) A_s f_y \left( \frac{d-a}{2} \right) \quad \therefore \quad \frac{M_U}{(0.9) f_y} = A_s \left( \frac{d-a}{2} \right)$$

$$\frac{359 \text{ in} - \text{kip}}{(0.9)(60 \text{ ksi})} = A_s \left( 15 \text{ in} - 1.96 \frac{A_s}{2} \right)$$

$$15 A_s - 0.98 A_s^2 - 5.99 = 0$$

$$A_s^2 - 15.3 A_s + 6.11 = 0$$

$$A_s = 0.41 \text{ in}^2 \text{ per foot of footing}$$

The total steel required across the footing is  $A_s = 7.5 \text{ ft} (0.41 \text{ in}^2/\text{ft}) = 3.08 \text{ in}^2$  (Check ACI 10.5.1) for minimum steel and ACI 7.12 for temperature and shrinkage,

$$\rho = \frac{A_s}{bd} = \frac{0.41}{12(15)} = 0.0023 > 0.0018$$

$$\rho_{\min} = \frac{3\sqrt{f'_c}}{f_y} = 0.0018bh \text{ or}$$

$$\rho_{\min} = \frac{200}{f_y} = 0.0018$$

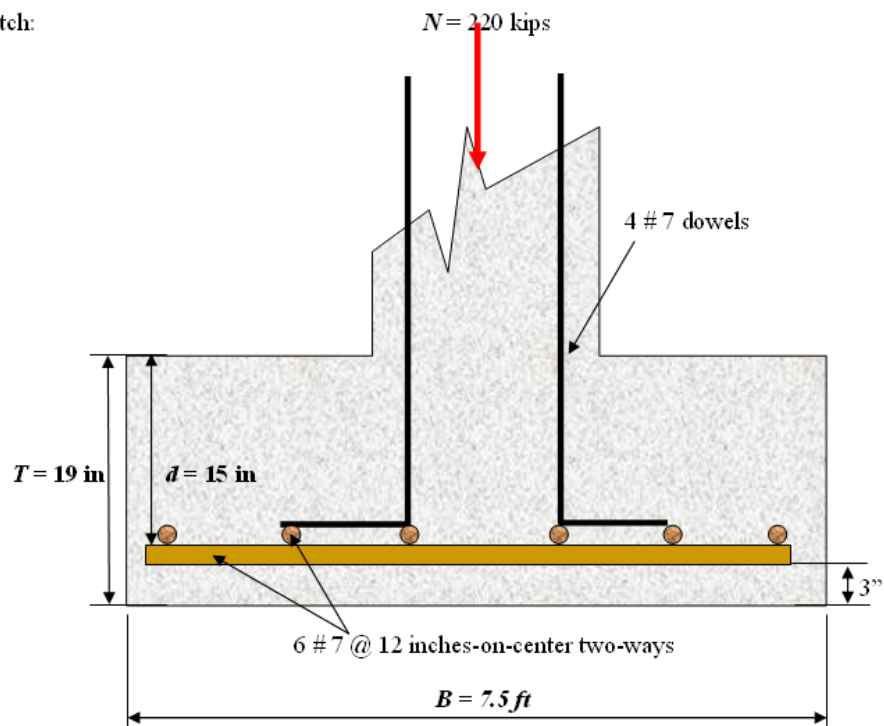
Therefore,  $A_s = 7.5 (0.0023)(12)(15) = 3.1 \text{ in}^2$  or  $0.59 \text{ in}^2$  per foot of footing

For  $B = 90'' (7.5')$  use 6 # 7 bars ( $A_s = 3.18 \text{ in}^2$ ) @ 12 inches on center

or 5 # 8 bars ( $A_s = 3.95 \text{ in}^2$ ) @ 12 inches on center

Check for Development length  $L_d$  (ACI-318-08.12.3), and the embedment length of the dowels.

Sketch:

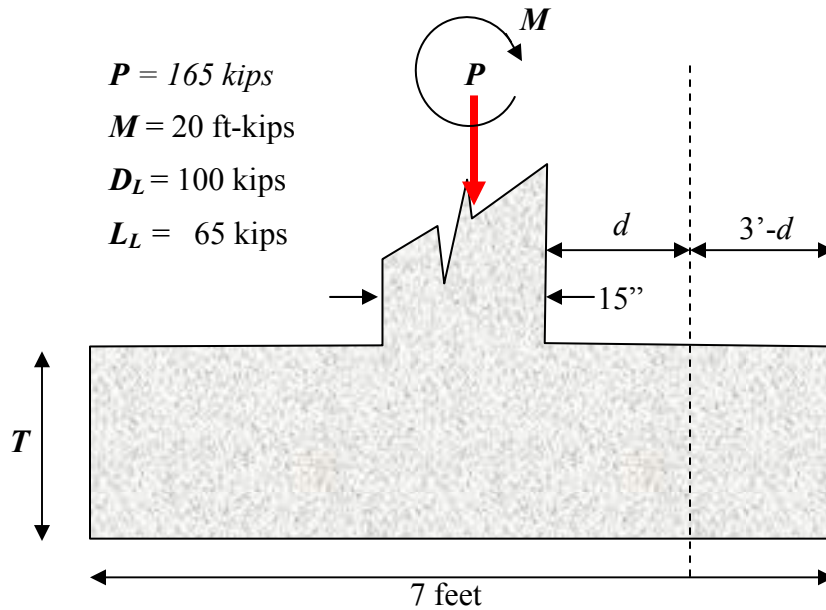


### \*Footings–02: Add a moment to the load on a footing.

(Revision: Sept-08)

The footing shown below is a square footing with the dimensions and loads as shown.

- Compute the load's eccentricity  $e$ .
- Check the bearing pressure at ultimate load.
- Calculate the wide beam shear.



- Determine the flexural moment for a strip of footing 1 foot wide.

$$q_{all} = 4 \text{ ksf (with a FS = 1.5)}, \quad f'_c = 3 \text{ ksi}, \quad f_y = 60 \text{ ksi}. \quad P = D_L + L_L$$

#### Solution:

$$\text{a) Compute the load eccentricity: } e = \frac{M}{P} = \frac{20}{165} = 0.121 \text{ ft}$$

$$q = \left( \frac{P}{A} \right) \left( 1 \pm \frac{6e}{B} \right) = \left( \frac{165}{7' \times 7'} \right) \left( 1 \pm \frac{6(0.121')}{7'} \right) =$$

$$q_{\max} = 3.72 \text{ ksf} \quad \text{and} \quad q_{\min} = 3.02 \text{ ksf}$$

$$\text{b) Check bearing pressure at ultimate load: } Q_u = 1.2(100) + 1.6(65) = 224 \text{ kips}$$

$$q_{all} = q_{\max} \left( \frac{Q_u}{P} \right) = 3.72 \left( \frac{224}{165} \right) = 5.05 \text{ ksf} \quad \text{and} \quad q_{all} = q_{\min} \left( \frac{Q_u}{P} \right) = 3.02 \left( \frac{224}{165} \right) = 4.1 \text{ ksf}$$

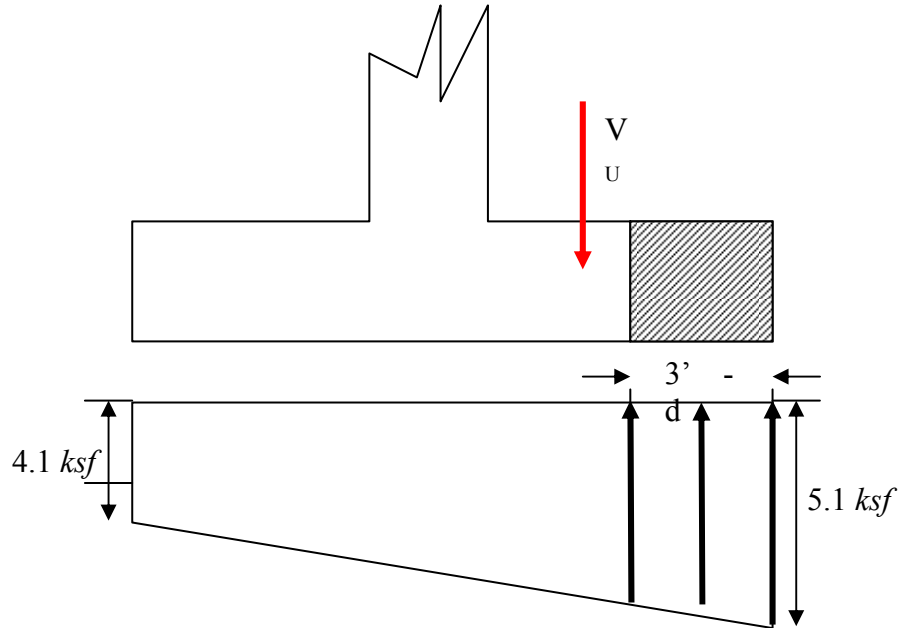
Then consider  $q_{all} = 5.1 \text{ ksf} < q_{all} (FS) = 4 (1.5) = 6 \text{ ksf}$ , GOOD.

c) Calculate the allowable one-way (wide beam) shear; assume a  $d = 1$  ft;  $\bar{q} = \frac{5.1+4.1}{2} = 4.6$

$$V_u = \bar{q}\left(\frac{B}{2} - T - \frac{c}{2}\right) = 4.6\left(\frac{7}{2} - 1 - \frac{1}{2}\right) = 9.2 \text{ kips} \quad \text{Equation (1)}$$

$$\text{The allowable shear} = 2\phi\sqrt{f'_c} = 110 \text{ psi}$$

$$\text{The required } d = \frac{V_u}{2\phi\sqrt{f'_c}b} = \frac{(9.2)(1000)}{(0.75)(110)(12)} = 9.3 \text{ inches} \approx 12 \text{ in assumed} \quad \text{Equation (2)}$$



d) Determine the flexural moment for a strip 1 foot wide:

$$M = \int_0^x V dx = \int_0^3 \left( \frac{5.9x - 0.157x^2}{2} \right) dx = 25.9 \frac{\text{ft} - \text{kip}}{\text{ft}}$$

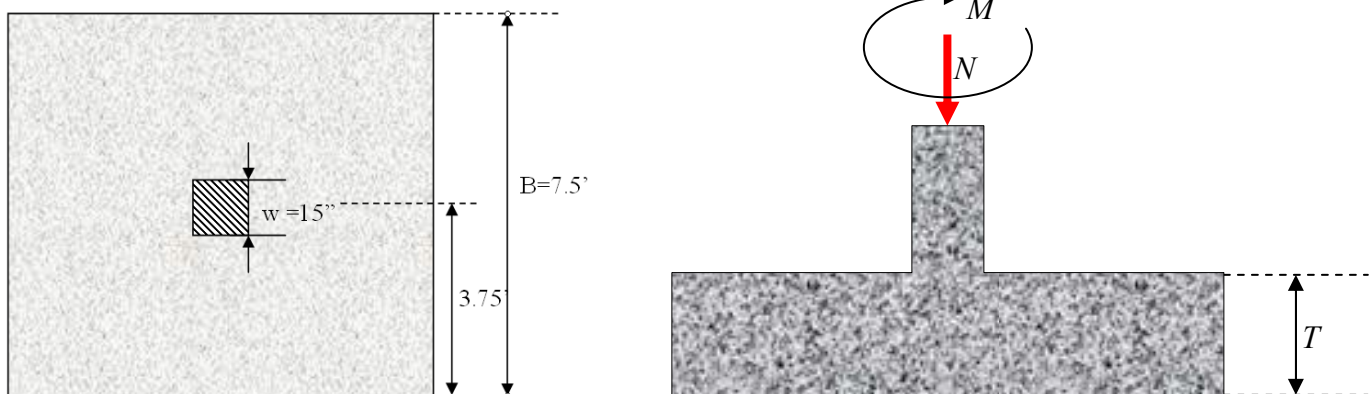
**\*Footings–03: Find the thickness  $T$  and the  $A_s$  of the previous problem.**

(Review:

Find, 1) The soil pressure under the footing for the given loads,

2) The footing thickness  $T$ , and

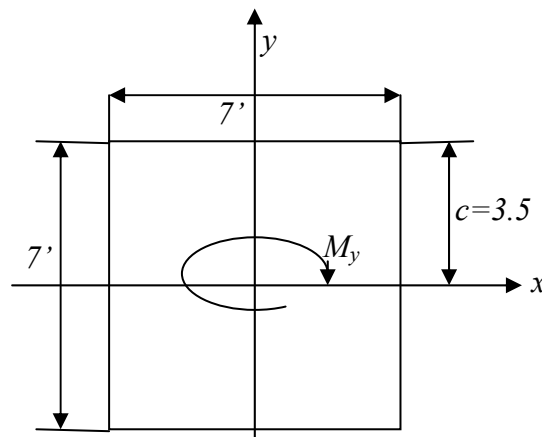
3) The flexural steel reinforcement  $A_s$ .



Given:  $f_y = 60 \text{ ksi}$ ,  $f'_c = 3 \text{ ksi}$ ,  $q_a = 4 \text{ ksf}$ ,  $D_L = 65 \text{ kips}$ ,  $L_L = 100 \text{ kips}$ ,  $M = 20 \text{ ft-kips}$

$$N = D_L + L_L$$

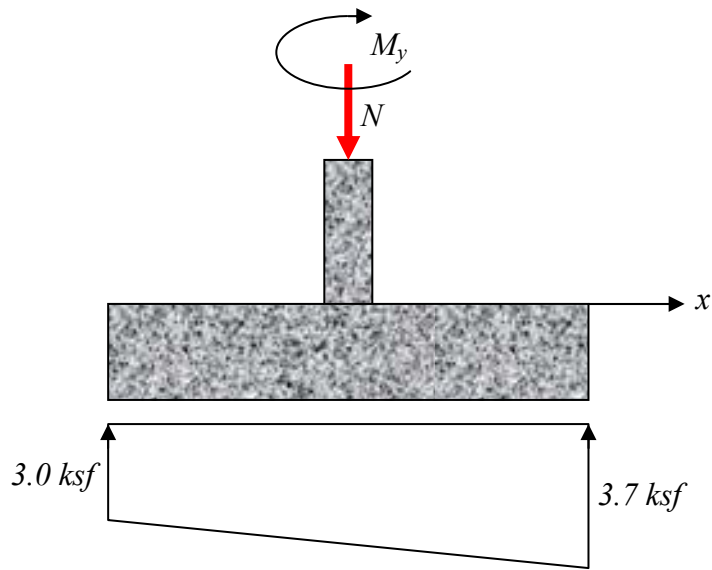
The actual soil pressure  $q_o$  (versus the allowable soil bearing capacity):



$$q_o = \frac{N}{A} \pm \frac{Mc}{I} \quad \text{where} \quad I = \frac{bd^3}{12} = \frac{(7')(7')^3}{12} = 200 \text{ ft}^4$$

$$q_o = \frac{165k}{(7' \times 7')} \pm \frac{(20k \cdot \text{ft})(3.5 \text{ ft})}{200 \text{ ft}^4} =$$

$$q_{o \text{ max}} = 3.7 \text{ ksf} \quad \text{and} \quad q_{o \text{ min}} = 3.0 \text{ ksf} < 4 \text{ ksf allowable} \quad \text{GOOD}$$



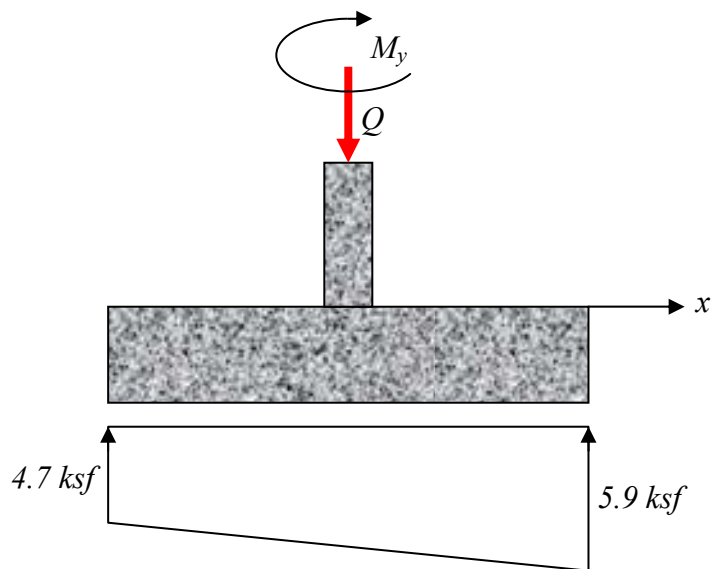
b) The ultimate load on the soil:

$$Q_u = 1.2D_L + 1.6L_L = 1.2(65 \text{ kips}) + 1.6(100 \text{ kips}) = 238 \text{ kips}$$

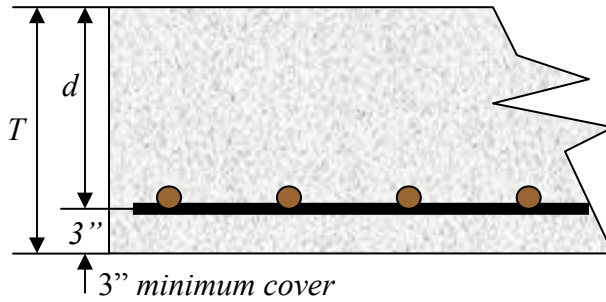
$$M_u = 1.6M_{LL} = 1.6(20 \text{ ft.kips}) = 32 \text{ ft.kips}$$

$$q_u = \frac{Q_u}{A} \pm \frac{M_u c}{I} = \frac{238 \text{ kips}}{(7')(7')} \pm \frac{(32 \text{ ft.kips})(3.5 \text{ ft})}{200 \text{ ft}^4} =$$

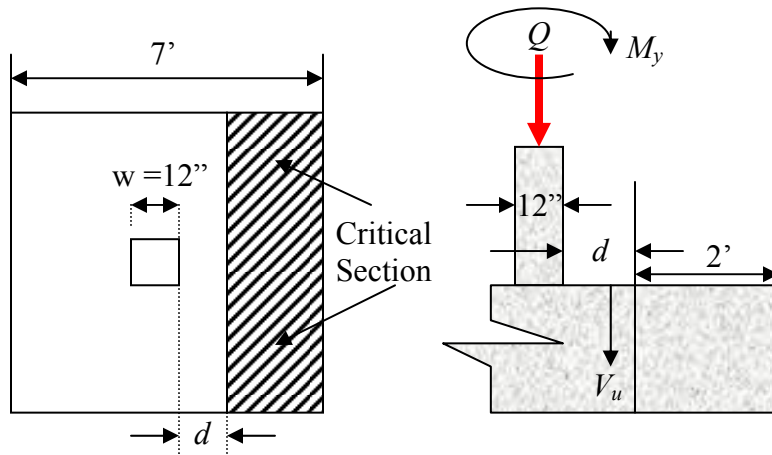
$$q_{u \text{ max}} = 5.9 \text{ ksf} \quad \text{and} \quad q_{u \text{ min}} = 4.7 \text{ ksf}$$



a) Determine the thickness of the footing  $T$ .



Check wide-beam shear: Assume  $d = 12''$ , (controls for rectangular footing).



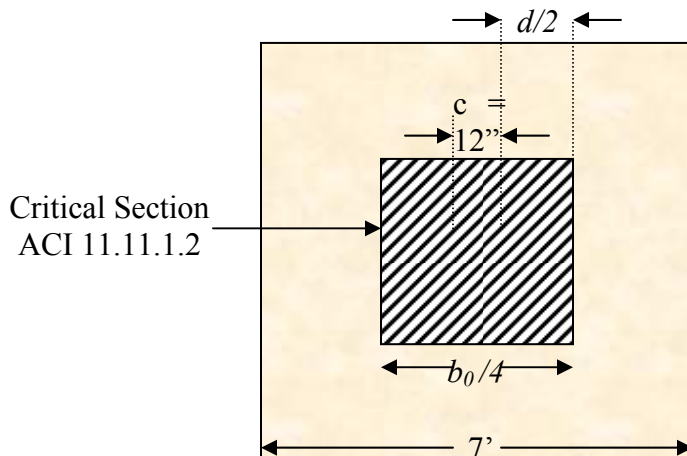
From

$\sum f_y = 0$ , the ultimate shear is

$$V_u = 82,600 \text{ lbs}, \text{ but } V_u = \phi 2 \sqrt{f'_c} b d,$$

$$d = \frac{V_u}{\phi 2 \sqrt{f'_c} b} = \frac{82,600 \text{ lbs}}{(0.85)(2) \sqrt{3000} (84 \text{ in})} = 10.6 \text{ in} < 12 \text{ in assumed}.$$

Check for punching shear (controls for square footings, such as this one).



$$V_u = \phi 4 \sqrt{f'_c} b_o d, \text{ where } b_o \text{ is the perimeter of the critical section; } b_o = 4 \left( c + 2 \left( \frac{d}{2} \right) \right)$$

$$\therefore d = \frac{V_u}{\phi 4 \sqrt{f'_c} b} = \frac{(5.9 \text{ ksf})(7 \text{ ft} \times 7 \text{ ft} - 2 \text{ ft} \times 2 \text{ ft})1000}{(0.85)(4)(\sqrt{3000})(4)(24 \text{ in})} = 14.8 \text{ in assumed;}$$

must increase  $d$ .

Recalculate by trying  $d = 14.0 \text{ in}$ .

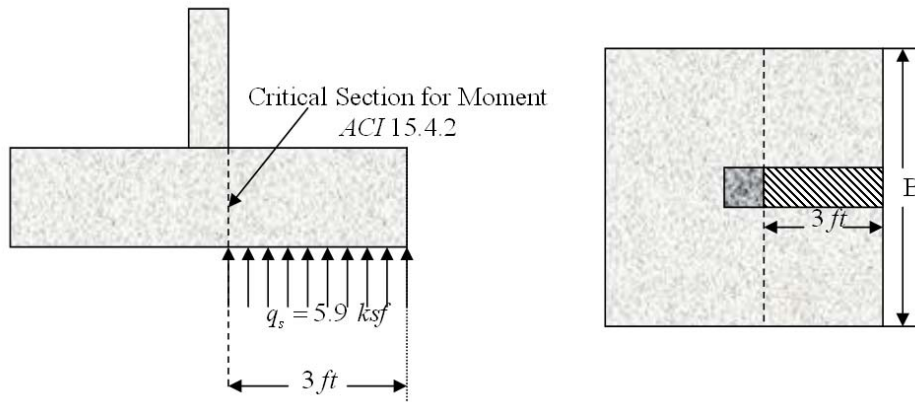
$$\therefore d = \frac{V_u}{\phi 4 \sqrt{f'_c} b} = \frac{(5.9 \text{ ksf})(7 \text{ ft} \times 7 \text{ ft} - 2.17 \text{ ft} \times 2.17 \text{ ft})1000}{(0.85)(4)(\sqrt{3000})(4)(26 \text{ in})} = 13.5 \text{ in}$$

$d = 13.5 \text{ in} < 14.0 \text{ in}$ , therefore is Good!!

The footing thickness  $T = 14 \text{ in} + 1 \text{ bar diameter (1")} + 3 \text{ in (cover)} = 18 \text{ in}$ .

### **Footing dimensions: 7.5 feet x 7.5 feet x 18 in.**

Finding the flexural reinforcement  $A_s$ :



$$M_u = \frac{q_s B l^2}{2} = \frac{(5.9 \text{ ksf})(7 \text{ ft})(3 \text{ ft})^2}{2} = 186 \text{ kip}\cdot\text{ft}$$

$$a = \frac{A_s f_y}{0.85 f'_c b} \quad (\text{where } b = B) \quad \text{then} \quad A_s = \frac{0.85 f'_c B a}{f_y} = \frac{0.85(3000)(7 \text{ ft})(a)}{60,000} = 0.3a$$

The ultimate moment  $M_u$  is given by:  $M_u = \phi A_s f_y \left( d - \frac{a}{2} \right)$  with  $M_u = 186 \text{ kip}\cdot\text{ft}$ , and  $\phi = 0.9$ .

$$\therefore 186 \text{ kip}\cdot\text{ft} = 0.9(0.3a \text{ ft}^2) \left( 60 \frac{\text{kip}}{\text{in}^2} \right) \left( 144 \frac{\text{in}^2}{\text{ft}^2} \right) \left( 1.16 - \frac{a}{2} \right) \text{ ft} =$$

$$a = 0.07 \text{ ft} \quad \text{and} \quad \therefore A_s = 0.3a = 0.021$$

$$\text{Percentage of steel } p = \frac{A_s}{bd} = \frac{A_s}{Bd} = \frac{0.021 \text{ ft}^2}{(7 \text{ ft})(1.167 \text{ ft})} = 0.00257 > p_{\min} = 0.0018$$

$$A_{s \min} = 0.021 ft^2 = 3.024 \text{ in}^2 \quad \text{Therefore, use 6 \# 7 bars at 12 inches.}$$

e) Check development length,  $L_d$  (ACI 12.2.2)  $L_d = 0.04 A_b \left( \frac{f_y}{\sqrt{f'_c}} \right) > 0.0004 d_b f_y$

$$L_d = 0.04 A_b \left( \frac{0.44 (60,000)}{\sqrt{3000}} \right) > 0.0004 (0.75) (60,000)$$

$$L_d = 19.3 \text{ in} > 18 \text{ in}$$

$$\text{Actual } L_{d \text{ (provided)}} = B - \frac{c}{2} - (\text{cover}) = \left( 7 \text{ ft } \frac{12 \text{ in}}{1 \text{ ft}} \right) - 6 \text{ in} - 3 \text{ in} = 75 \text{ in}$$

f) Check bearing strength on concrete (ACI 318-02 10.15)

$$\text{Bearing strength} = 0.85 \phi f'_c A_g \sqrt{\left( \frac{A_2}{A_g} \right)} \quad \text{where} \quad \sqrt{\left( \frac{A_2}{A_g} \right)} \leq 2$$

$A_g$  is the area of the column and  $A_2$  is the area of the footing.

$$\text{Then } 0.85 \phi f'_c A_g \sqrt{\left( \frac{A_2}{A_g} \right)} = 0.85 (0.70) \left( \frac{3 \text{ kips}}{\text{in}^2} \right) (144 \text{ in}^2) (2) = 514 \text{ kips}$$

But, the factored column load  $U = 261 \text{ kips} < 514 \text{ kips}$ . Good !

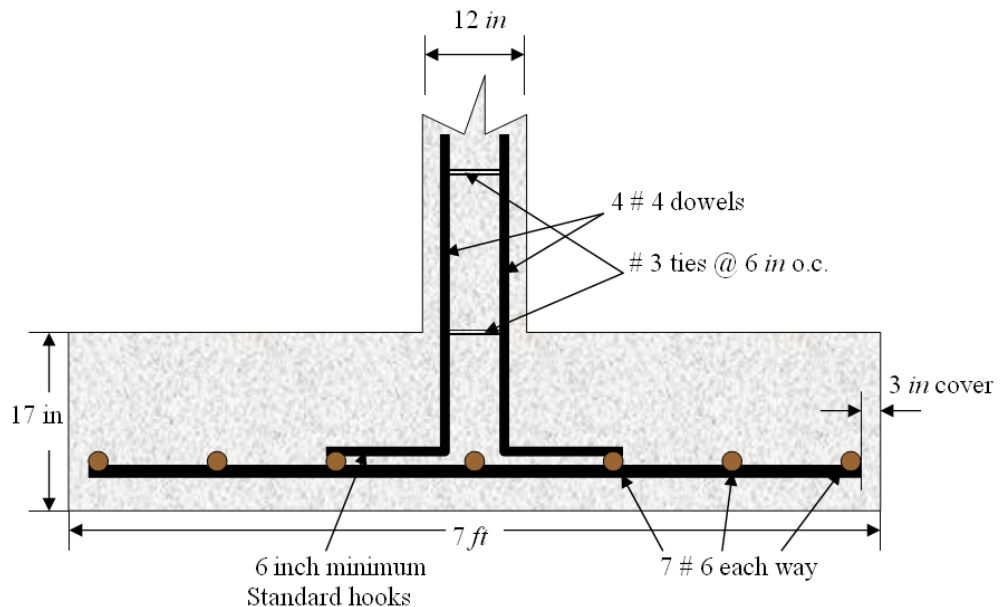
g) Dowels to column:

Since bearing strength is adequate, a minimum area of dowels should be provided across the interference of the column and footing (ACI 318-02 15.8.2.1)

$$\text{Minimum area of steel} = 0.005 (\text{area of column}) = 0.005 (144 \text{ in}^2) = 0.72 \text{ in}^2.$$

$$\text{Use 4 \# 4 bars } A_s = 0.20 \text{ in}^2 (4) = 0.80 \text{ in}^2$$

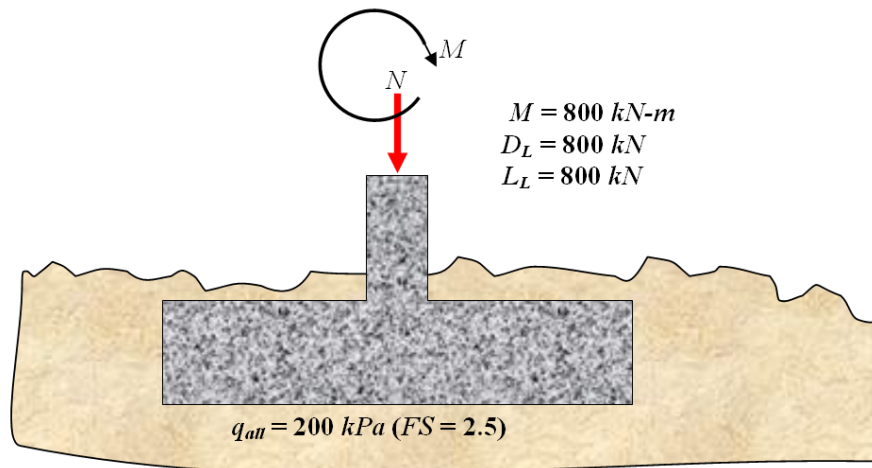
h) Final design



**\*Footings–04: Find the dimensions  $B \times L$  of a rectangular footing.**

(Revision: Sept-08)

Find the footing dimensions  $B \times L$  to carry a moment induced by winds of  $800 \text{ kN-m}$ .



**Solution:**

Select a test value for  $B \times L$ . Set  $B \times L = B^2$  and check the increase in soil pressure due to wind

$$B^2 = \frac{N}{q_a} = \frac{(800 \text{ kN} + 800 \text{ kN})}{\left(200 \frac{\text{kN}}{\text{m}^2}\right)} = 8 \text{ m}^2 \quad \therefore \quad B = 2.82 \text{ m}$$

$$e = \frac{M}{N} = \frac{800}{1600} = 0.5 \text{ m} \quad \therefore \quad L \geq 6(0.5) = 3 \text{ m}$$

If  $L = 3 \text{ m}$ , try for  $q_{\max} \cong 2q_{\text{avg}}$  iterate by trying footing:  $2.5 \text{ m} \times 4 \text{ m} = B \times L$ .

$$q_{\text{avg}} = \frac{N}{A} = \frac{1600 \text{ kN}}{10 \text{ m}^2} = 160 \text{ kPa}$$

$$q_{\max} = \frac{N}{BL \left(1 + \frac{6e}{L}\right)} = \frac{1600}{10 \left[1 + \frac{6(0.5)}{4}\right]} = 280 \text{ kPa}$$

Note that  $q_{\max}$  exceeds  $q_{\text{avg}}$  by 33 %, therefore increase the area.

Try using  $2.75 \text{ m} \times 4.5 \text{ m}$  dimensions;

$$q_{avg} = \frac{N}{A} = \frac{1600 \text{ kN}}{2.75(4.5) \text{ m}^2} = 130 \text{ kPa}$$

$$q_{max} = \frac{N}{BL \left(1 + \frac{6e}{L}\right)} = 130 \left[1 + \frac{6(0.5)}{4.5}\right] = 217 \text{ kPa}$$

Iterate again, with  $B = 3.0 \text{ m}$  and  $L = 5.0 \text{ m}$

$$q_{avg} = \frac{N}{A} = \frac{1600 \text{ kN}}{3 \text{ m}(5 \text{ m})} = 107 \text{ kPa}$$

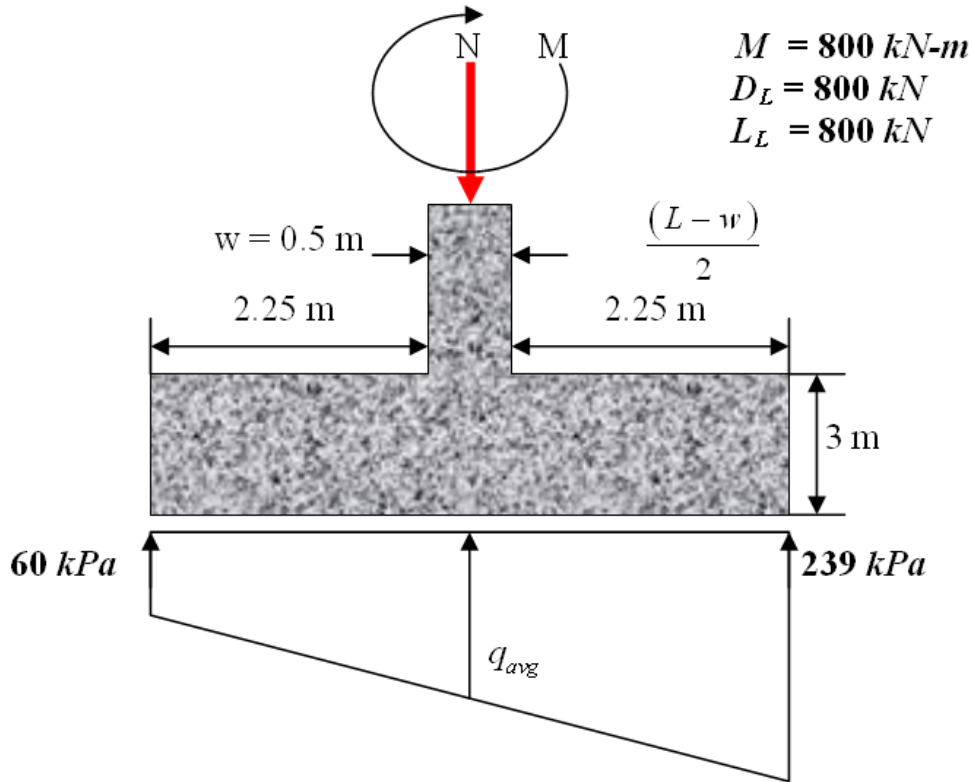
$$q_{max} = \frac{N}{BL \left(1 + \frac{6e}{L}\right)} = \frac{1600}{15} \left[1 + \frac{6(0.5)}{5}\right] = 171 \text{ kPa} \quad \text{GOOD}$$

**The footing dimensions are  $B = 3 \text{ m}$  by  $L = 5 \text{ m}$ .**

**\*Footings–05: Design the steel for the previous problem.**

(Revision: Sept-08)

Design the previous footing using  $f'_c = 21 \text{ MPa}$  and  $f_y = 415 \text{ MPa}$ .



**Solution.**

1) Check the ultimate pressures:

$$N_u = 1.2(D_L) + 1.6(L_L) = 1.2(800) + 1.6(800) = 2240 \text{ kN}$$

$$e = \frac{M}{N} = \frac{800}{1600} = 0.5 \text{ m}$$

$$q_{\max} = \frac{N_u}{A \left( 1 + \frac{6e}{L} \right)} = \left( \frac{2240}{15} \right) \left[ 1 + \frac{6(0.5)}{5} \right] = 239 \text{ kPa}$$

$$q_{\min} = \frac{N}{A \left( 1 - \frac{6e}{L} \right)} = \left( \frac{2240}{15} \right) \left[ 1 - \frac{6(0.5)}{5} \right] = 60 \text{ kPa}$$

$$q_{\max} = 239 \text{ kPa} \quad \text{and} \quad q_{\min} = 60 \text{ kPa}$$

$$q_{avg} = \frac{2240 \text{ kN}}{15 \text{ m}^2} = 149 \text{ kPa} < q_{all} = 200 \text{ kPa} \quad \text{Good}$$

2a) Calculate footing depth  $T$  based on punching shear, for a  $f'_c = 21 \text{ MPa}$ , and

$$V_c = 1.29 \text{ MPa}$$

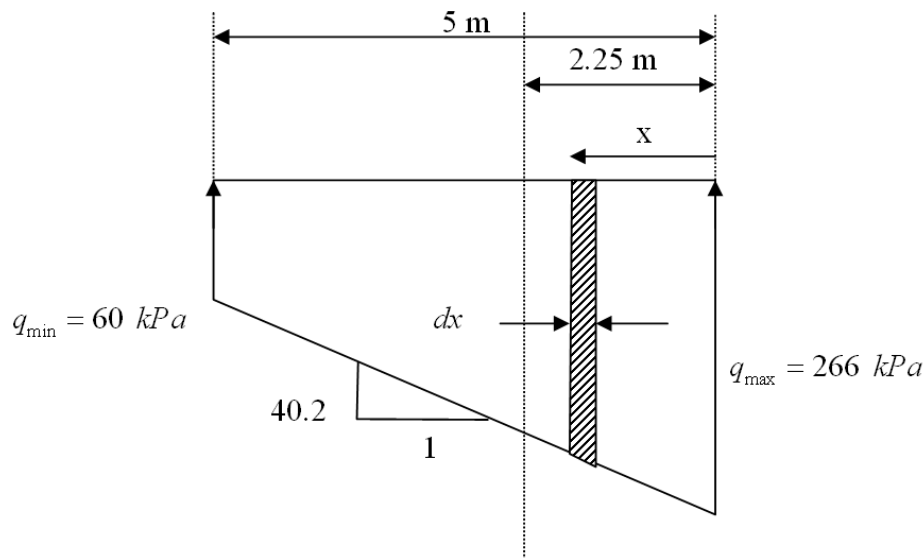
Using the simplified equation for a square footing:

$$4T^2 + 2(0.5 \text{ m} + 0.5 \text{ m})T - \frac{(15 \times 149)}{2240} = 0$$

$$T^2 + 0.5T - 0.25 = 0 \quad \therefore T \cong 0.50 \text{ m}$$

2b) Calculate footing depth  $T$  based on wide beam shear:

The shear is calculated from the outer edge of footing ( $x = 0$ ) inwards towards a distance  $d$  from the column ( $x = 2.225 - d$ ):



$$dv = qdx$$

$$V = \int_0^x q dx = \int_0^x (266 - 40.2x) dx = \left\{ 266x - \left( \frac{40.2x^2}{2} \right) \right\}_0^{2.25-T}$$

$$V = 598 - 266T - 20.1(2.25 - T)^2 =$$

$$V = \frac{V_c}{2T} = \frac{1290}{2T} \quad \therefore T^2 + 40.8T - 24.7 = 0 \quad T = 0.60 \text{ m}$$

We will use the highest of (2a) or (2b), therefore  $T = 0.60 \text{ m}$ .

3) a) Find  $A_s$  (Longitudinal): 
$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{415 A_s}{0.85(21)(1)} = 23.3 A_s$$

$$M_u = \int_0^{2.25} V dx = \int_0^{2.25} \left( 266x - \frac{40.2x^2}{2} \right) dx = \left[ \frac{266x^2}{2} - \frac{40.2x^3}{6} \right]_0^{2.25} = 597 \text{ kN.m}$$

$$M_u = \phi A_s f_y \left( d - \frac{a}{2} \right) \quad \therefore \quad A_s \left( d - \frac{a}{2} \right) = \frac{M_u}{\phi f_y}$$

$$A_s \left( 0.6 - \frac{23.3}{2} \right) = \frac{597}{0.9(415)} = A_s^2 - 0.0515 A_s + 1.37 \times 10^{-2} = 0$$

$$A_s = 28.2 \frac{\text{cm}^2}{\text{m}}$$

Check  $p = \frac{A_s}{bd} = \frac{0.00282}{1(0.6)} = 0.0047 > 0.002 \text{ GOOD}$

3) b) Find  $A_s$  (transverse): Use the high average.

$$q = \sqrt{(q_{ave} + q_{max})} = \sqrt{(149 + 266)} = 20.37 \text{ kPa}$$

$$M_u = \frac{wl^2}{2} = \frac{q \left[ \frac{(B-0.5)}{2} \right]^2}{2} = \frac{\left( 215 \left( \frac{(3-0.5)}{2} \right)^2 \right)}{2} = 168 \text{ kN.m}$$

$$\text{and } A_s \left( d - \frac{a}{2} \right) = \frac{M_u}{\phi f_y} \quad \therefore \quad A_s^2 - 0.0515 A_s + 0.000386 = 0 \therefore A_s = 7.61 \frac{\text{cm}^2}{\text{m}}$$

Check for  $\rho = \frac{A_s}{bT} = \frac{7.61 \times 10^{-4} \text{ m}^2}{1(0.6)} = 0.00126 < 0.002$

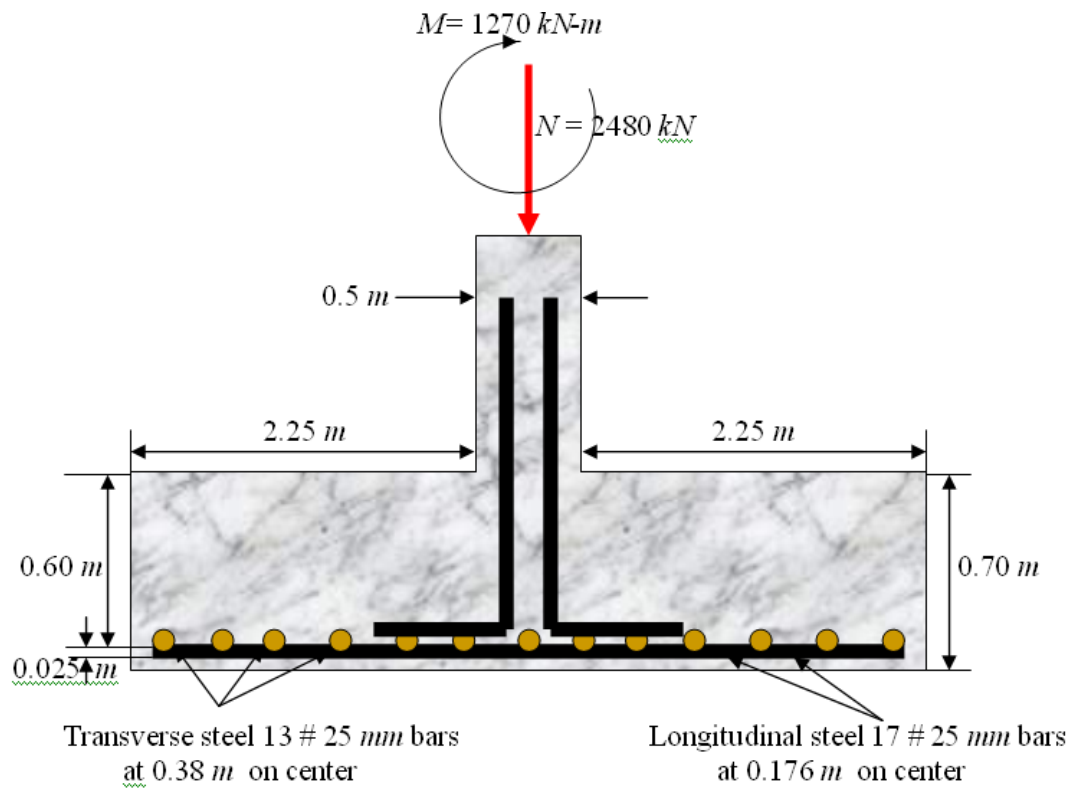
$$\therefore \text{ Use minimum } A_s = 0.002(b)(T) = 0.002(1)(0.6) = 0.0012 \frac{\text{m}^2}{\text{m}} = 12 \frac{\text{cm}^2}{\text{m}}$$

4) Check footing:

Longitudinal steel:  $28.2 \frac{\text{cm}^2}{\text{m}} (3 \text{ m}) = 84.6 \text{ cm}^2 \quad \frac{1 \text{ m}^2}{10,000 \text{ cm}^2} = 8.46 \times 10^{-3} \text{ m}$

Therefore use 17 # 25 mm bars at 17.6 cms o - c

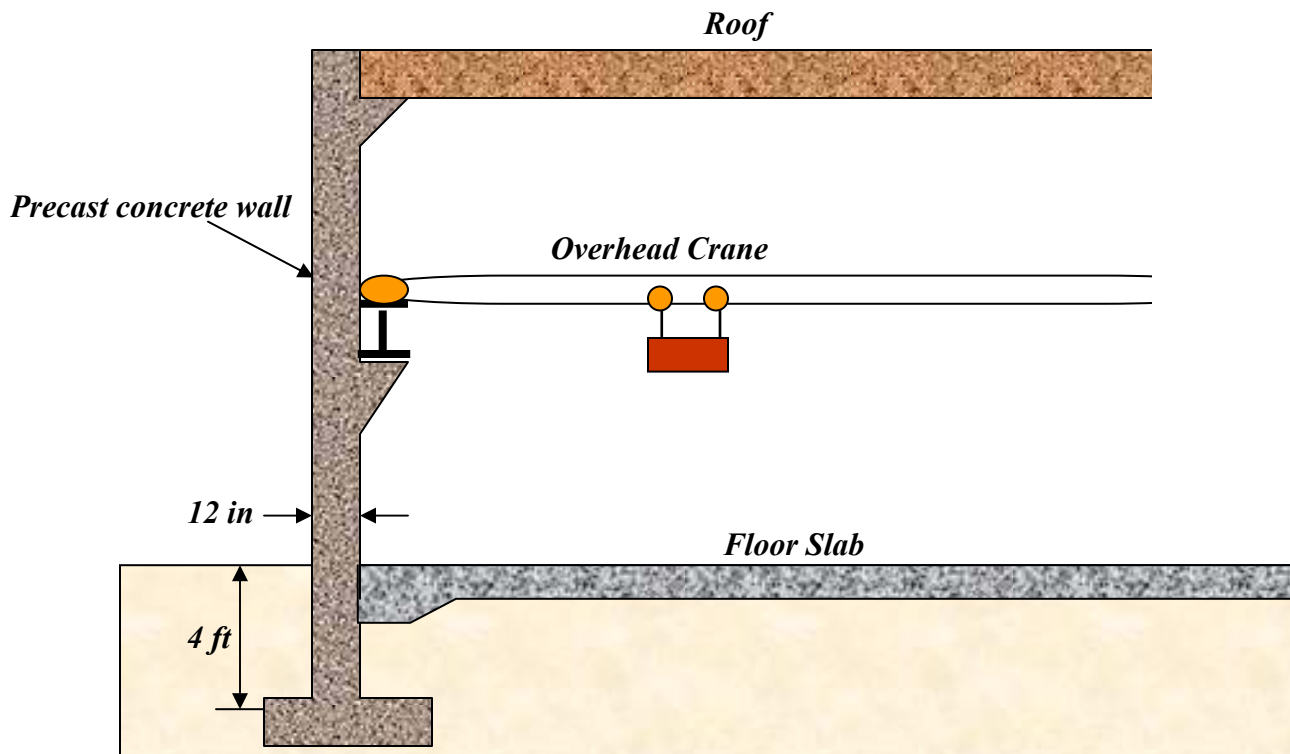
Sketch:



**\*Footings–06: Design a continuous footing for a pre-cast warehouse wall.**

(Revision: Sept-08)

Design a continuous footing for the warehouse wall with the loads shown below:



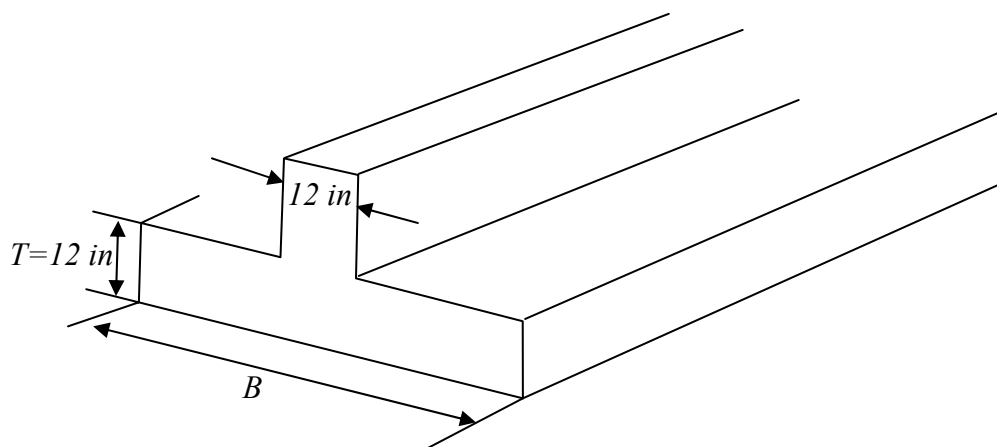
**Solution:**

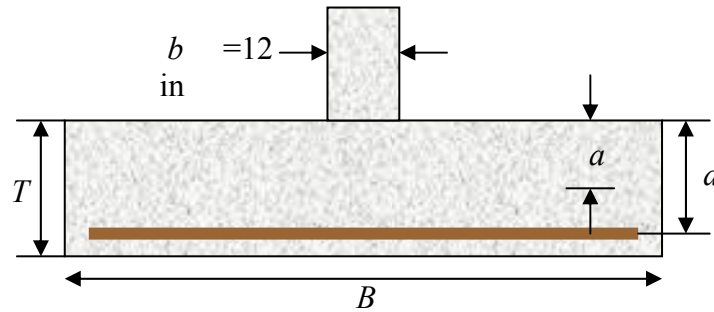
- 1) Assume an initial footing thickness  $T = 12''$ :

From ACI 7.7.2, the minimum cover is 3" from the steel to the footing invert.

Assume # 4 bars.

Therefore,  $d = 12'' - 3'' - 0.5'' / 2 = 8.75''$





Find the ultimate soil pressure  $q_{ult}$ .

The actual soil pressure from the structure is,

$$q_o = q_{all} - q_{concrete} - q_{soil-above footing}$$

$$q_o = 2 \text{ ksf} - (1 \text{ ft})(0.15 \text{ k/ft}^3) - (3 \text{ ft})(0.110 \text{ k/ft}^3) = 2 - 0.15 - 0.33 = 1.52 \text{ ksf}$$

Estimate  $B = Q / q_o = 4.2 \text{ k} / 1.52 \text{ ksf} = 2.76 \text{ ft.}$  per unit length. Therefore, assume  $B = 3 \text{ ft.}$

$$\text{Ultimate load, } U = 1.2 \text{ DL} + 1.6 \text{ LL} = 1.2(3) + 1.6(1.2) = 4.0 + 2.0 = 6.0 \text{ kips}$$

Therefore, the soil pressure at ultimate loads  $q_u$  is:

$$q_u = U / (B)(1) = 6.0 \text{ kips} / (3 \text{ ft})(1 \text{ ft}) = 2.0 \text{ ksf} < q_{ult} = 4 \text{ ksf} \rightarrow \text{GOOD}$$

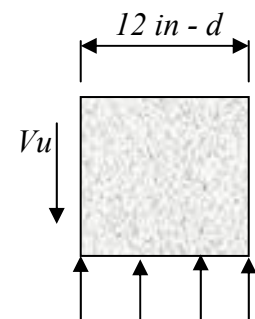
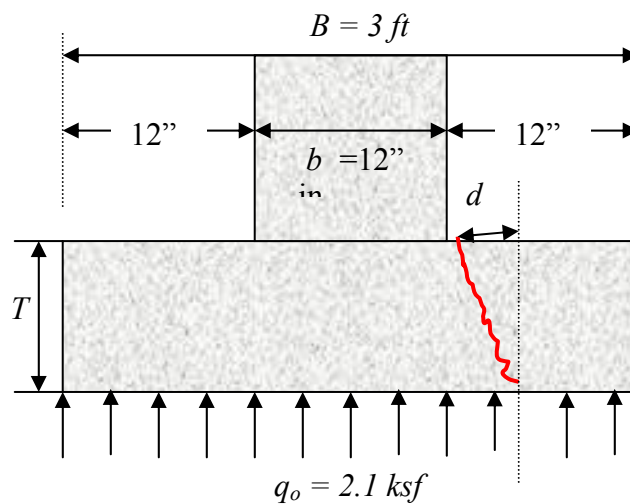
## 2) Check the shear strength of the footing:

The critical section for shear section occurs at a distance  $d$  from the face of the wall.

(ACI 15.5 & ACI 11.11.1).

$$q_o = 2.1 \text{ ksf}$$

$$V_u = (12 - d) q_o (1)$$



The ultimate shear  $V_u = (12'' - 8.75'') (1 \text{ ft} / 12 \text{ in}) \times 2.1 \text{ k} / \text{ft}^2 = 0.57 \text{ kips} / \text{ft of wall}$

The concrete shear strength must be:  $V_u \leq \phi V_c = \phi 2 \sqrt{f'_c} b d$

$$V_u = 0.85 \cdot 2 \cdot \sqrt{3000} \cdot 12 \text{ in} \cdot 8.75 \text{ in} = 9.8 \text{ kips} / \text{ft of wall} \gg 0.57 \text{ kips} / \text{ft}$$

Since  $V_u \ll \phi V_c$ , we can reduce the thickness of the footing  $T$  from 1.0 ft to say 0.85 ft (~10'').

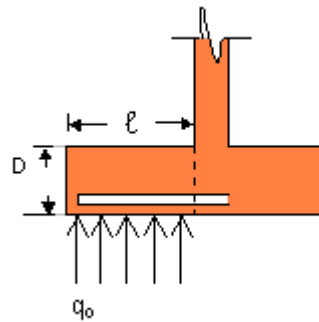
Therefore,  $d = 10'' - 3'' - 0.5 / 2 = 6.5'' > 6'' = d_{\min}$  from ACI 15.4

Rechecking,  $V_u = 0.85 \cdot 2 \cdot \sqrt{3000} \cdot 12 \text{ in} \cdot 6.5 \text{ in} = 7.3 \text{ kips} / \text{ft of wall} \rightarrow \text{GOOD}$

Therefore, the total thickness  $T = d + \text{bar diameter} + 3'' = 10'' + 1'' + 3'' = 14''$

### 3) Design the flexural reinforcement, $A_s$ (ACI 15.4):

$$M_u = q_o \ell^2 / 2 \quad \text{where } \ell = 12''$$



$$M_u = (2.1 \text{ ksf} \times 1 \text{ ft}^2) / 2 = 1.05 \text{ kip-ft} / \text{ft of wall}$$

$$\text{But, } a = A_s f_y / [0.85 f'_c (b)] = [(60 \text{ ksi}) A_s] / [0.85 (3 \text{ ksi}) (12 \text{ in})] = 1.96 A_s \text{ (inches)}$$

$$\text{and, } M_n = A_s f_y (d - a/2) = A_s (60 \text{ k} / \text{in}^2) (d - a/2) = 60 A_s (6.5 \text{ in} - 9.8 A_s)$$

$$\text{But, } M_u = \phi M_n = 0.9 M_n$$

$$\text{Therefore, } 1.05 \text{ kips-in} / \text{in} = 0.9 (60 \text{ k} / \text{in}^2) A_s (6.5 \text{ in} - 9.8 A_s)$$

$$53 A_s^2 - 351 A_s - 1.05 = 0$$

$$\text{Two possible answers: } A_s (1) = 6.6 \text{ in}^2 \text{ per ft. of wall}$$

$$A_s (2) = 0.003 \text{ in}^2 \text{ per ft. of wall}$$

The percentages of steel with  $A_s (1)$  &  $A_s (2)$  (ACI 7.12.2.1)

$$\rho_1 = A_{s(1)} / bd = 6.6 \text{ in}^2 / (12 \text{ in})(6.5 \text{ in}) = 0.085$$

$$\rho_2 = A_{s(2)} / bd = 0.003 \text{ in}^2 / (12 \text{ in})(6.5 \text{ in}) = 0.0004 < 0.0018 \text{ minimum steel}$$

The maximum steel percentages allowed  $\rho_{\max} = 0.75 \rho_b$ , where

$$\rho_b = (0.85 f'_c / f_y) \beta (87,000 / (87,000 + f_y))$$

$$= (0.85 (3) / 60) 0.85 (87,000 / (87,000 + 60,000)) = 0.021$$

therefore,

$$\rho_{\max} = 0.75 \rho_b = 0.75 (0.021) = 0.016$$

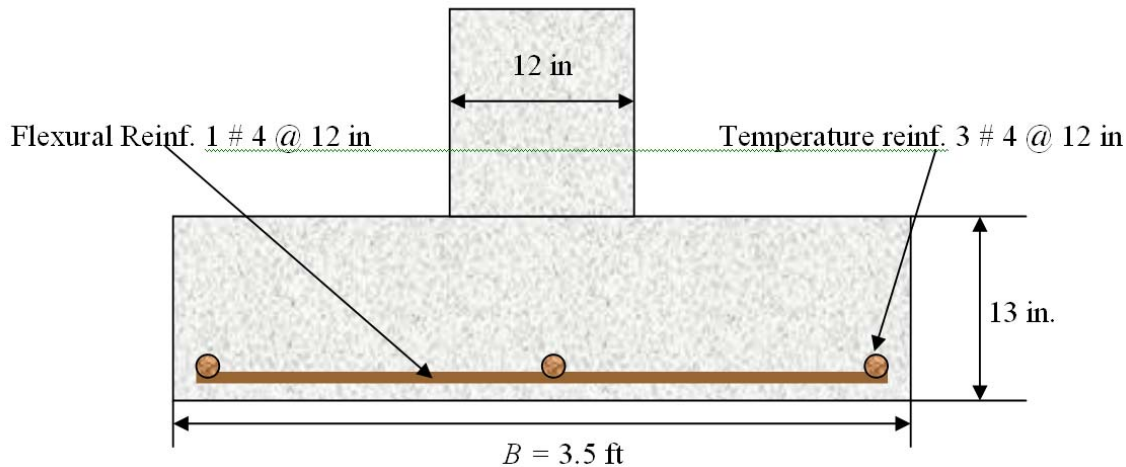
Note that  $\rho_1 = 0.085 > \rho_{\max} = 0.016$  therefore, use  $\rho_{\min} = 0.0018$

Therefore,

$$A_s = \rho_{\min} b d = (0.0018)(12 \text{ in})(6.5 \text{ in})$$

$$= 0.14 \text{ in}^2 \text{ per ft. of wall} \rightarrow \boxed{\text{use 1 \# 4 every ft. of wall (} A_s = 0.20 \text{ in}^2 \text{)}}$$

4) Check the development length,  $L_d$  (ACI 12.2 ):



$$L_d = 0.04 A_b \frac{f_y}{\sqrt{f'_c}} \text{ (But not less than } 0.0004 d_b f_y) = 0.04 \cdot 0.20 \cdot \frac{60,000}{\sqrt{3000}} = 8.8''$$

$L_d = 8.8''$  or  $12'' \rightarrow$  Clearly,  $12''$  controls.

Presently we have  $12'' - 3''$  cover  $= 9'' < 12''$ . Therefore, we are missing  $3''$  on each side.

Increase the footing width  $B$  by  $6''$  to  **$B = 3.5 \text{ ft.}$**

(Note that increasing  $B$ , reduces  $q_0$ , and the design could be further optimized.)

Therefore,

$$\begin{aligned} B &= 3.5 \text{ ft} \\ T &= 13'' \\ A_s &= 1 \text{ \# 4 @ 12'' along the wall} \end{aligned}$$

Use minimum steel in longitudinal direction, to offset shrinkage and temperature effects ( ACI 7.12 ):

$$A_s = ( 0.0018 )( b )( d ) = 0.0018 ( 42 \text{ in } )( 6.5 \text{ in } ) = 0.49 \text{ in}^2$$

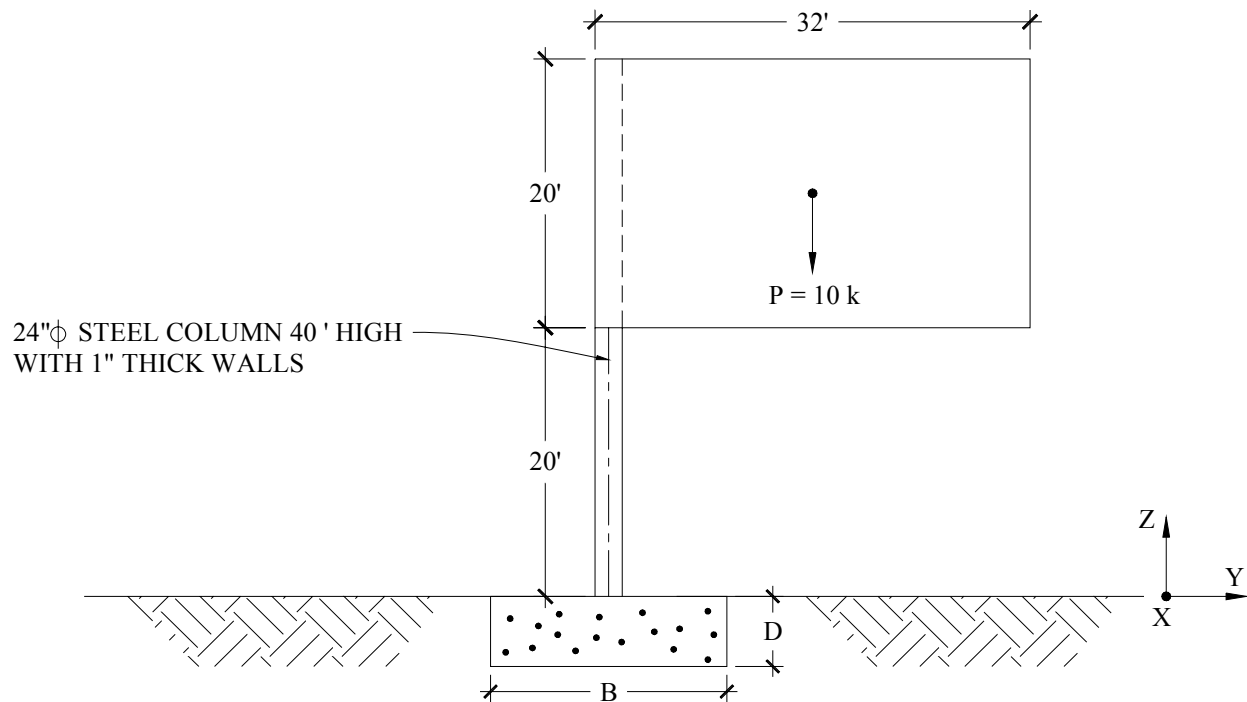
Provide 3 \# 4 bars at 12'' o.c. (  $A_s = 0.60 \text{ in}^2$  ).

## **\*\*Footings–07: Design the footings of a large billboard sign.**

(Revised Oct-09)

Design a spread footing for the billboard sign shown below using FBC-2004 and ASCE 7- 02. Ignore the torsion and the wind load on the column, and the water table.

Given:  $\gamma = 150$  pcf     $c = 150$  pcf     $\phi = 20^\circ$      $V = 146$  mph



### **solution:**

**STEP #1: Find the wind load as per ASCE 7-02, assuming an Exposure C, Category I.**

$$q_z = 0.00256 K_z (IV)^2$$

$$K_z = 0.98$$

$$I = 1.05$$

$$V = 146 \text{ mph}$$

$$G_h = 1.26$$

$$C_f = 1.2$$

$$F = q_z G_h C_f A_f \quad \therefore q_z = 52 \text{ psf}$$

$$\text{The sign shape factor is } \frac{M}{N} = \frac{32}{20} = 1.6$$

$$\text{Therefore: } F = \frac{(34 \text{ psf}) (1.26) (1.2) (32 \text{ ft} \times 20 \text{ ft})}{1000} = 32.4 \text{ kips}$$

**Step #2: Calculate loads on footing**

$$\text{Weight of steel column} = \gamma_s L A = 0.49$$

$$M_x = 10 \text{ kips} \times 15' = 150 \text{ k-ft}$$

$$M_y = 32.4 \text{ k} \times 30' = 972 \text{ k-ft}$$

$$M_z = 32.4 \text{ k} \times 15' = 486 \text{ k-ft}$$

$$\text{Total (normal) load } N = 10 \text{ k} + 5 \text{ k} = 15 \text{ kips}$$

**Step #3: Calculate the footing's bearing capacity using Hansen's formula.**

$$c \text{ (cohesion)} = 0.150 \text{ ksf}$$

$$q = \gamma D_f = (\text{embedment pressure}) = (0.130 \text{ ksf})(3 \text{ ft}) = 0.39 \text{ ksf}$$

$$B = (\text{footing width} - \text{initial assumptions}) = 5 \text{ ft}$$

$$L = (\text{footing length} - \text{initial assumptions}) = 15 \text{ ft}$$

$$N_q \text{ (factor for embedment at } \phi = 20^\circ) = e^{\pi \tan \phi} \tan^2(45 + \phi/2) = 6.40$$

$$N_c \text{ (factor for cohesion at } \phi = 20^\circ) = (N_q - 1) \cot \phi = 14.83$$

$$N_\gamma \text{ (factor for width at } \phi = 20^\circ) = 1.5 (N_q - 1) \tan \phi = 2.95$$

$$F_{sq} = (\text{shape factor for embedment}) = 1.0 + (B/L) \sin \phi = 1.11$$

$$F_{sc} = (\text{shape factor for cohesion}) = 1.0 + (N_q / N_c) (B/L) = 1.14$$

$$F_{s\gamma} = (\text{shape factor for width}) = 1.0 - 0.4 (B/L) = 0.867$$

$$F_{dq} = (\text{depth factor for embedment}) = 1 + 2 \tan \phi (1 - \sin \phi)^2 (D_f/B) = 1.19$$

$$F_{dc} = (\text{depth factor for cohesion}) = 1.0 + 0.4 (D_f/B) = 1.24$$

$$F_{d\gamma} = (\text{depth factor for width}) = 1.0$$

$$F_{ic} = (\text{inclination factor}) = 0.5 - \sqrt{\left( \frac{1 - H}{(A_f C_a)} \right)}$$

$$\text{where } c_a = (0.6 \text{ to } 1.0) c$$

$$F_{iq} = [1 - (0.5 H) / (V + A_f C_a \cot \phi)]^d \text{ where } 2 \leq d \leq 5$$

$$F_{i\gamma} = [1 - (0.7 H) / (V + A_f C_a \cot \phi)]^\alpha$$

$$q_{ult} = c' N_c F_{sc} F_{dc} F_{ic} + \bar{q} N_q F_{sq} F_{dq} F_{iq} + 0.5 \gamma B' N_\gamma F_{s\gamma} F_{d\gamma} F_{i\gamma}$$

**Step #4: Assume:**

$$B = 10'$$

$$L = 50'$$

$$D = 3'$$

$$B/L = 0.2$$

$$FS = 3.0$$

$$SC = 1.0 + (0.431 \times 0.2) = 1.09$$

$$DC = 1.0 + (0.4)(3/10) = 1.12$$

$$Q = (130) \times 3 = 390$$

$$SQ = 1.0 + 0.2 \sin 20 = 1.07$$

$$SJ = 1.0 - 0.4(0.2) = 0.92$$

$$DQ = 1 + (0.315)(3/10) = 1.09$$

$$q_{\max, \min} = \frac{P}{BL} + \left( \frac{-6P e_y}{B^2 L} \right) + \left( \frac{-6P e_x}{BL^2} \right) \text{ where } P = 15 \text{ kips} + \frac{((3 \times 10 \times 50 \times 0.150))}{\text{FTG WT}} = 240 \text{ kips}$$

$$e_x = \frac{M_y}{P} = \frac{972}{240} = 4.05'$$

$$e_y = \frac{M_x}{P} = \frac{150}{240} = 0.625'$$

$$q_{\max, \min} = \left( \frac{240 \text{ kips}}{500} \right) + \left( \frac{-6(240)(0.625)}{102 \times 50} \right) + \left( \frac{-6(240)(4.05)}{10 \times 502} \right) = 0.89 \text{ ksf} < 2.5 \text{ ksf} \text{ GOOD}$$

**Step #5: Check out (long direction).**

$$\text{MOT} = 32.4 \text{ Kips} (30+3) = 1069 \text{ kip-ft}$$

$$\text{MR} = 5 \text{ Kips} \times 25' + 225 \text{ Kips} \times 25' = 5750 \text{ kip-ft}$$

$$\text{F.S.} = 5750 / 1069 = 5.4 >> 1.5 \quad \text{GOOD}$$

$$\text{Check Sliding RS} = \Sigma V \tan f + CB = 240 \tan f + 150 (10) = 1587 \text{ kips} >> 32.4 \text{ kips}$$

#3

$$\text{LOAD COMBINATION} = 0.75 (1.2D + 1.6L + 1.7W)$$

$$\text{FACTORED LOADS:} \quad \mathbf{Pu} = 1.05 \times 240 \text{ Kips} = 252 \text{ kips}$$

$$\mathbf{Mu-x} = 1.05 \times 150 \text{ kip-ft} = 158 \text{ kip-ft}$$

$$\mathbf{Mu-y} = 1.275 \times 972 \text{ kip-ft} = 1239 \text{ kip-ft}$$

$$\therefore Q_{\max, \min} = \left( \frac{336}{10 \times 50} \right) \pm \left( \frac{6 \times 336 \times 0.625}{10^2 \times 50} \right) \pm \left( \frac{6 \times 336 \times 4.9}{10 \times 50^2} \right) = 0.989 \text{ ksf}, 0.019 \text{ ksf}$$

$$\text{Check beam shear: } D = 36'' - 4'' = 32''$$

$$V_U = 0.019 \times (50 - 21.33) + 0.15(50 - 21.33)(0.989 - 0.019) = 14.4 \text{ kip per feet}$$

Punching shear will not govern by observation.

#3 Design for flexure in long direction

$$f'_c = 3000 \text{ PSI} \quad \text{and} \quad f_y = 60000 \text{ PSI}$$

$$A = A_s \times F_y / 0.85 F'_c B \quad A_s = M_u \times 12 / F_y (D - A/2)$$

$$A = 1.64 \text{ in.} \quad A_s = 0.83 \text{ in}^2$$

$$R = \frac{A_s}{BD} = \left( \frac{0.83}{12 \times 32} \right) = 0.0022 > 0.0018 \text{ OK}$$

$$\text{USE \# 7 @ 8" O/C,} \quad A_s = 0.90 \text{ in}^2 \text{ OK}$$

$$\text{Therefore; } M_u = (0.9)(60) \left( \frac{132 - \frac{1.64}{2}}{12} \right) = 126 \text{ kip-ft} > 116 \text{ kip-ft} \text{ GOOD}$$

**Use a footing 10 feet x 50 feet x 3 feet thick with # 7 bars @ 8" on-center top and bottom, each way.**

**NOTE:** In lieu of such a large and expensive footing, a short drilled shaft would be an efficient and inexpensive foundation. That alternative will be covered in the drilled shaft section later in this course.

# Chapter 19

## Combined Footings

***Symbols for Combined Footings***

# Chapter 20

## Mat Foundations

***Symbols for Mat Foundations***

**\*Mat Foundations–01: Ultimate bearing capacity in a pure cohesive soil.**

(Revision: Sept-08)

Determine the ultimate bearing capacity of a mat foundation measuring 45 feet long by 30 feet wide placed 6.5 feet below the surface and resting upon a saturated clay stratum with  $c_u = 1,950 \text{ lb/ft}^2$  and  $\phi = 0^\circ$ .

**Solution:**

Mat foundations in purely cohesive soils have the following ultimate bearing capacity:

$$q_{\text{ult}(\text{net})} = 5.14 c_u \left(1 + \frac{0.195B}{L}\right) \left(1 + \frac{0.4D_f}{B}\right)$$

$$q_{\text{ult}(\text{net})} = 5.14(1.95 \text{ ksf}) \left[1 + \frac{0.195(30 \text{ ft})}{(45 \text{ ft})}\right] \left[1 + \frac{0.40(6.5 \text{ ft})}{(30 \text{ ft})}\right] = 12 \text{ ksf}$$

# Chapter 21

## Deep Foundations - Single Piles

***Symbols for Single Piles of Deep Foundations***

### \*Single-Pile–01: Pile capacity in a cohesive soil.

(Revision: Oct.-08)

A concrete pile 20 m long with a cross section of 381 mm x 381 mm is fully embedded in a saturated clay stratum. The clay has  $\gamma_{sat} = 18.5 \text{ kN/m}^3$ ,  $\phi = 0^\circ$  and  $c_u = 70 \text{ kN/m}^2$ . The water table lies below the tip of the pile. Determine the allowable capacity of the pile for a FS = 3 using the  $\alpha$ -method.

#### Solution:

The ultimate capacity of the pile  $Q_{ult}$  is given by the simple formula,

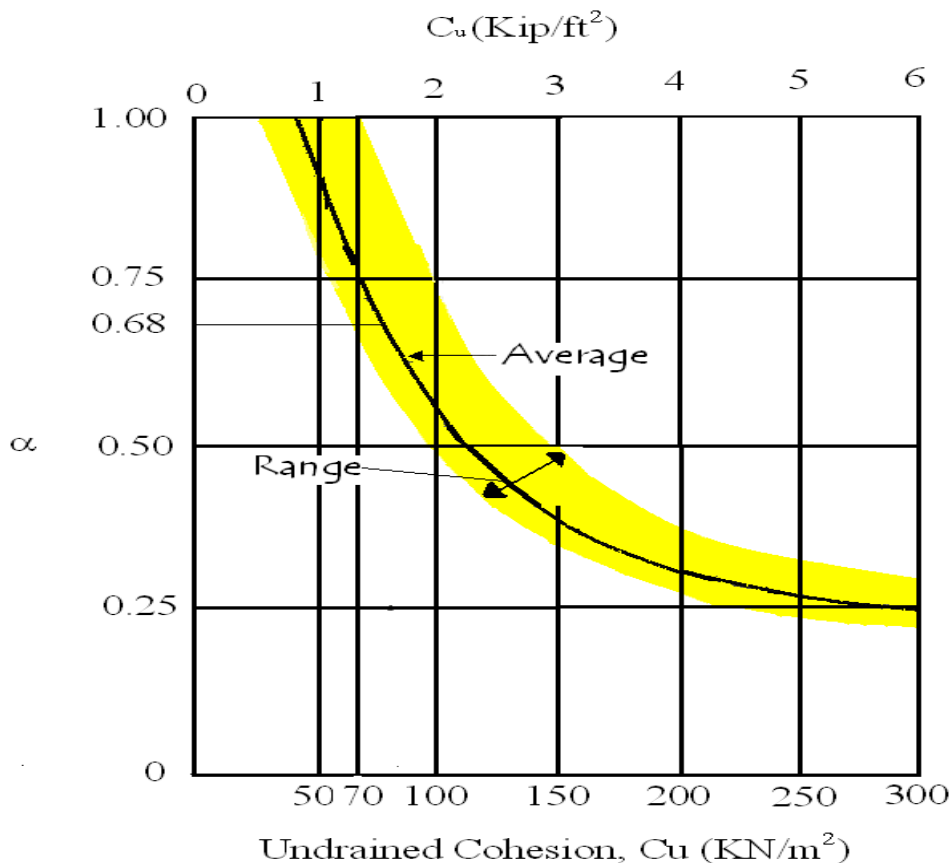
$$Q_{ult} = Q_{\text{point}} + Q_{\text{shaft}} = A_p q_p + (c_u)(\text{perimeter})L = A_p (c_u) N_c + \alpha (c_u)(\text{perimeter})L$$

Notice that the value of the cohesion is reduced by the " $\alpha$ " factor found in the graph below,

$$\therefore Q_{ult} = (0.38\text{m})^2 (70 \text{ kN/m}^2)(10.97) + (0.75)(70 \text{ kN/m}^2)4(0.38\text{m})(20\text{m}) = 1,890 \text{ kN}$$

The allowable capacity is,

$$Q_{all} = \frac{Q_{ult}}{3} = \frac{1,890\text{kN}}{3} = 630 \text{ kN}$$



# Chapter 22

## Deep Foundations - Pile Groups and Caps

*Symbols for Pile Groups and Caps of Deep Foundations*

## **\*\*Pile-caps-01: Design a pile cap for a 9-pile cluster.**

(Revision: Oct-08)

Design a pile cap footing to support an 18" square column subjected to a live load reaction of 180 kips and a dead load reaction of 160 kips at service loads. The testing laboratory recommends an ultimate pile load of 70 kips per pile, and a service pile load of 42 kips per pile. The vertical steel in the column consists of 12 No.7 bars. Use  $f'_c = 3000$  psi,  $f_y = 40,000$  psi, and 12" diameter piles.

### **Solution.**

Since the footing weight will be about 3 kips/pile, the net service load per pile is  $42.0 - 3.0 = 39.0$  kips/pile. The number of piles required in  $N = W/P = 340/39 = 8.7$ , or 9 piles. Use a pile pattern as shown in Fig. 1. The net ultimate load is used to design the footing; thus  $W_u = (1.4)(160) + (1.7)(180) = 530$  kips, and the load per pile is  $P_u = 530/9 = 58.9$ , say 59.0 kips/pile, which is less than the maximum ultimate load, 70 kips/pile. Punching shear around a single pile often governs the footing depth determination, except in cases in which the loads are small. In this case, it will be shown that beam shear governs. Referring to Fig. 2, we calculate the punching shear stress. After several trials, assume  $d = 19.5"$ . The shear perimeter is  $b_o = \pi(12 + d) = 99.0"$ . The permissible shear force around the pile will be,

$$V_c = 4\sqrt{f'_c} b_o d = 4\sqrt{3000} (99) (19.5) / 1000 = 423 \text{ kips}$$

Since the actual shear force is the nominal pile reaction,  $P_n = P_u/\phi = 59.0/0.85 = 69.4 \text{ kips} < 423 \text{ kips}$ , the pile will not punch through the pile cap (footing).

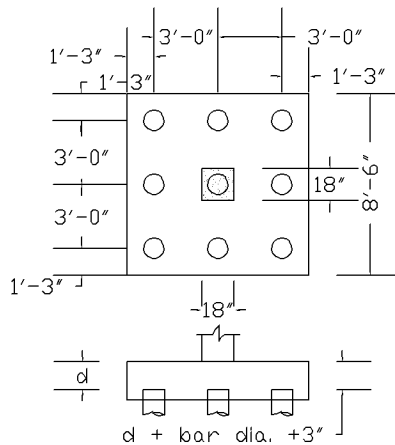


Figure 1

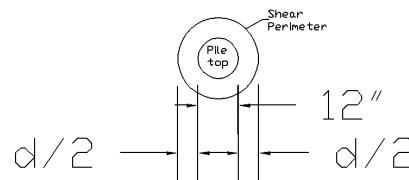


Figure 2

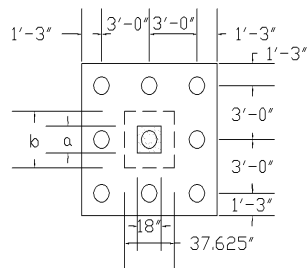


Figure 3

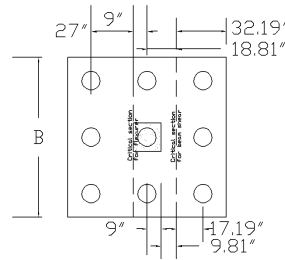


Figure 4

Perimeter shear (punching shear) must now be checked around the column in a similar manner. In this case, all of the nominal pile reactions outside of the critical section plus any partial reactions outside of the critical section will contribute to punching shear for the column. Refer to Figure 3. Assuming No. 6 bars will be used, clearance above the pile butts will be 3" and embedment of the piles will be 6". The total depth required will be 28.75". For practical reasons use 29"; this furnishes an effective depth  $d = 19.625"$ . Thus  $c = a + b = 18.0 + 19.625 = 37.625"$  and  $b_o = 4(37.625) = 150.0"$ . Hence,  $V_{ou} = 472$  kips on 8 piles outside of the critical section as shown on Fig. 3. The permissible punching shear force ( $\beta_c = 18/18 < 2$ ) is given by (6.12) as

$$V_c = 4 \sqrt{3000} (150) (19.625) / 1000 = 644.9 \text{ kips}$$

The force to be resisted is  $V_n = V_{ou} / \phi = 472 / 0.85 = 555.3$  kips; therefore the pile cap (footing) is satisfactory for punching shear. Beam shear must now be checked. Refer to Fig. 4. Three piles exist beyond the critical section, so  $V_u = (3) (59.0) = 177.0$  kips. Since  $b = B = 8'-6" = 102"$ , the permissible beam shear (one-way shear) force on the critical section is

$$V_c = 2 \sqrt{f'_c} b d = 2 \sqrt{3000} (102) (19.625) / 1000 = 219.3 \text{ kips}$$

The force to be resisted is the nominal shear force,  $V_n = V_u / \phi = 177 / 0.85 = 208.2$  kips. Hence the footing is satisfactory for beam shear. The bending moment about the face of the column must now be investigated. Refer to Fig. 4

$$M_u = (177) (27/12) = 398.3 \text{ ft-kips}$$

$$\phi R_u = M_u / b d^2 = 398.3 \times 12,000 / (102)(19.625)^2 = 121.67 \text{ psi}$$

Table 5.2 for  $f'_c = 3000$  psi and  $f_y = 40,000$  psi, discloses the fact that the steel ration required is less than the minimum steel ration,  $\rho_{min} = 200 / f_y = 0.005$ . Further, if the steel ration required is increased by 1/3, it will still be less than  $\rho_{min}$ . It would appear that 4/3 times the required steel ration would satisfy the 1983 ACI Code. However, the Code does not permit un-reinforced (plain concrete) pile caps. Since any section having less than minimum reinforcement is usually considered to be un-reinforced, the minimum are of steel will be provided. Thus,

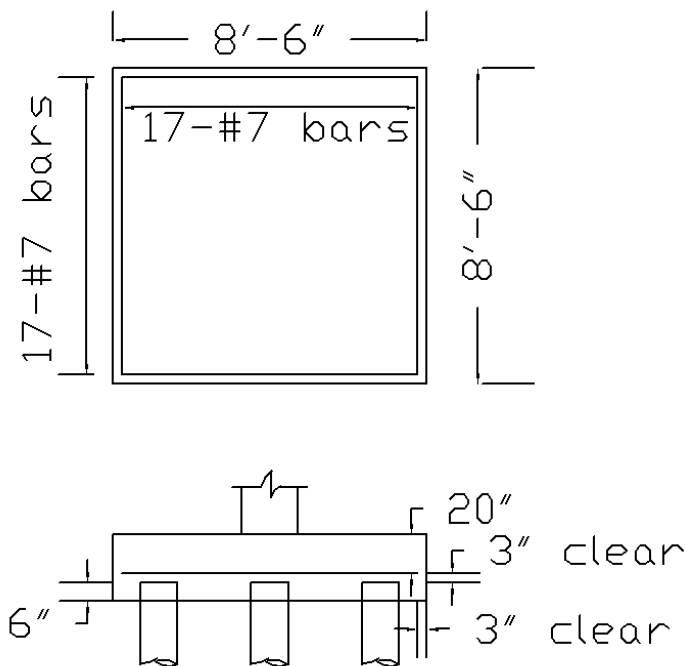
$$A_s = (200/f_y) bd = (200/40,000) (102) (19.925) = 10.0 \text{ in.}^2$$

Use seventeen No. 7 bars ( $A_s = 10.2 \text{ in.}^2$ ).

The 1983 ACI Code is not explicit concerning minimum steel for footings. Hence, some structural engineers use  $0.002bh$  for minimum steel area if  $f_y = 40,000$  psi and  $0.0018bh$  if  $f_y = 60,000$  psi. This corresponds to temperature and shrinkage reinforcement requirements. The assumed footing weight must finally be checked. The total weight is

$$W_F = (8.5) (8.5) (29) (12.5) / 1000 = 26.2 \text{ kips}$$

And the weight per pile is  $26.2 / 9 = 2.91$  kips / pile. The assumed weight of 3.0 kips / pile is most satisfactory. The final details are shown below



# Chapter 23

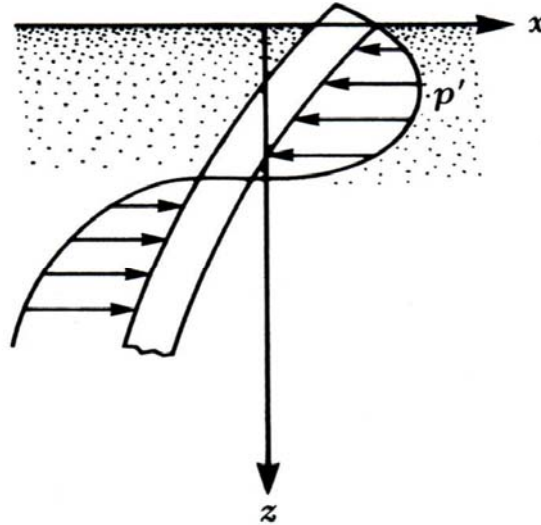
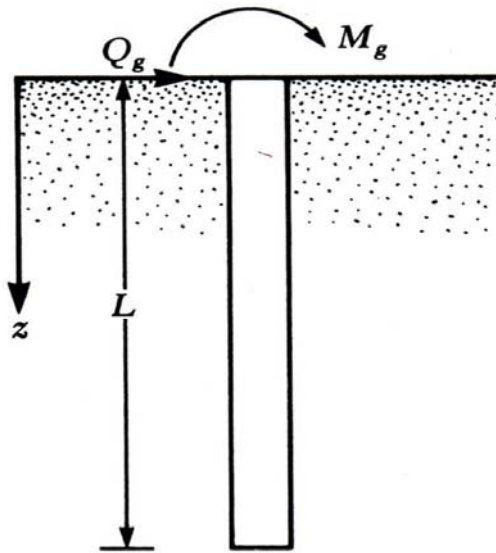
## Deep Foundations: Lateral Loads

*Symbols for Lateral Loads on Deep Foundations*

**\*\*Lateral loads on piles-01: Find the lateral load capacity of a steel pile.**

(Revision: Oct-08)

Determine the lateral load capacity  $Q_g$  of a steel H-pile (HP 250 x 0.834) fully embedded to a depth of 25 m in very dense submerged sand. The top end of the pile is allowed to deflect laterally 8 mm. For simplicity assume that there is no moment applied to the top of the pile (that is,  $M_g = 0$ ).



**Solution:**

The subgrade modulus  $k_s$  is a description of the reaction of the soil mass to vertical loads. The modulus of horizontal subgrade reaction  $n_h$  is a function of  $k_s$  at any depth  $z$ ,

$$k_z = n_h z$$

Type of soil	$n_h$ modulus of horizontal subgrade reaction	
	lb/in <sup>3</sup>	kN/m <sup>3</sup>
<b>Dry or moist sand</b>		
- loose	6.5 to 8.0	1,800 to 2,200
- medium	20 to 25	5,500 to 7,000
- dense	55 to 65	15,000 to 18,000
<b>Submerged sand</b>		
- loose	3.5 to 5.0	1,000 to 1,400
- medium	12 to 18	3,500 to 4,500
- dense	32 to 45	9,000 to 12,000

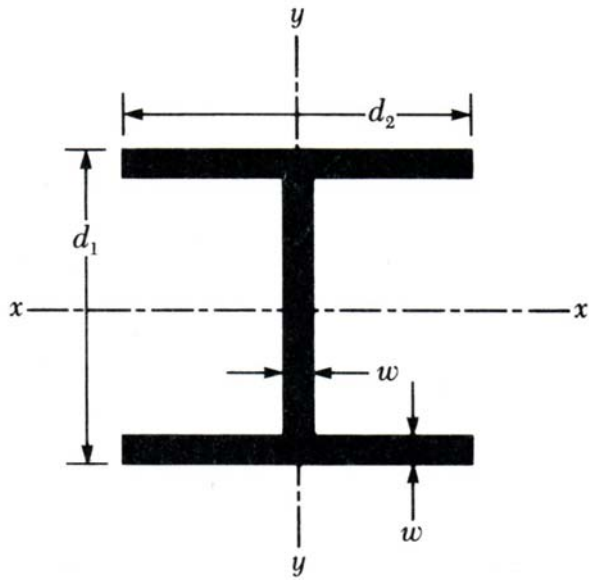
From this table and the soil conditions noted above, choose  $n_h = 12 \text{ MN/m}^3$  for the modulus. Now choose the parameters for the steel H-pile,

Designation, size (mm) $\times$ weight (kN/m)	Depth $d_1$ (mm)	Section area ( $\text{m}^2 \times 10^{-3}$ )	Flange and web thickness $w$ (mm)	Flange width $d_2$ (mm)	Moment of inertia ( $\text{m}^4 \times 10^{-6}$ )	
					$I_{xx}$	$I_{yy}$
HP 200 $\times$ 0.52	204	6.84	11.3	207	49.4	16.8
HP 250 $\times$ 0.834	254	10.8	14.4	260	123	42
$\times$ 0.608	246	8.0	10.6	256	87.5	24
HP 310 $\times$ 1.226	312	15.9	17.5	312	271	89
$\times$ 1.079	308	14.1	15.49	310	237	77.5
$\times$ 0.912	303	11.9	13.1	308	197	63.7
$\times$ 0.775	299	10.0	11.05	306	164	62.9
HP 330 $\times$ 1.462	334	19.0	19.45	335	370	123
$\times$ 1.264	329	16.5	16.9	333	314	104
$\times$ 1.069	324	13.9	14.5	330	263	86
$\times$ 0.873	319	11.3	11.7	328	210	69
HP 360 $\times$ 1.707	361	22.2	20.45	378	508	184
$\times$ 1.491	356	19.4	17.91	376	437	158
$\times$ 1.295	351	16.8	15.62	373	374	136
$\times$ 1.060	346	13.8	12.82	371	303	109

For future reference, this is the same table in British units,

Designation size (in.) $\times$ weight (lb/ft)	Depth $d_1$ (in.)	Section area ( $\text{in}^2$ )	Flange and web thickness $w$ (in.)	Flange width $d_2$ (in.)	Moment of inertia ( $\text{in}^4$ )	
					$I_{xx}$	$I_{yy}$
HP 8 $\times$ 36	8.02	10.6	0.445	8.155	119	40.3
HP 10 $\times$ 57	9.99	16.8	0.565	10.225	294	101
$\times$ 42	9.70	12.4	0.420	10.075	210	71.7
HP 12 $\times$ 84	12.28	24.6	0.685	12.295	650	213
$\times$ 74	12.13	21.8	0.610	12.215	570	186
$\times$ 63	11.94	18.4	0.515	12.125	472	153
$\times$ 53	11.78	15.5	0.435	12.045	394	127
HP 13 $\times$ 100	13.15	29.4	0.766	13.21	886	294
$\times$ 87	12.95	25.5	0.665	13.11	755	250
$\times$ 73	12.74	21.6	0.565	13.01	630	207
$\times$ 60	12.54	17.5	0.460	12.90	503	165
HP 14 $\times$ 117	14.21	34.4	0.805	14.89	1220	443
$\times$ 102	14.01	30.0	0.705	14.78	1050	380
$\times$ 89	13.84	26.1	0.615	14.70	904	326
$\times$ 73	13.61	21.4	0.505	14.59	729	262

For the steel HP 250  $\times$  0.834 pile, the moment of inertia about the strong axis is  $I_p = 123 \times 10^{-6} \text{ m}^4$ , its modulus of elasticity is  $E_p = 207 \times 10^6 \text{ kN/m}^2$ , the steel's yield strength is  $F_y = 248 \text{ MN/m}^2$  and the pile depth  $d_1 = 0.254 \text{ m}$ .



The characteristic length  $T$  of a pile-soil system is given by,

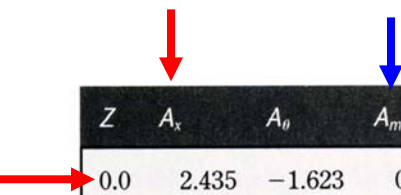
$$T = \sqrt[5]{\frac{E_p I_p}{n_h}} = \sqrt[5]{\frac{(207 \times 10^6)(123 \times 10^{-6})}{(12,000)}} = 1.16 \text{ m}$$

Therefore, the ratio  $L / T = 25 \text{ m} / 1.16 \text{ m} = 21.6 > 5$ , so this is a long pile.

The formula for the pile's top end lateral deflection  $\Delta$  at any depth  $z$  is given by,

$$\Delta = A_z \frac{Q_g T^3}{E_p I_p} + B_z \frac{M_g T^2}{E_p I_p} \text{ but } M_g = 0$$

In this problem we are given this value of  $\Delta = 8 \text{ mm}$ , and we want to find the allowable lateral load  $Q_g$ , at a depth  $z = 0$ , where the coefficient  $A_z$  is taken from a table of coefficients  $k_z = n_h z$  for long piles.



Z	$A_x$	$A_\theta$	$A_m$	$A_v$	$A'_p$	$B_x$	$B_\theta$	$B_m$	$B_v$	$B'_p$
0.0	2.435	-1.623	0.000	1.000	0.000	1.623	-1.750	1.000	0.000	0.000
0.1	2.273	-1.618	0.100	0.989	-0.227	1.453	-1.650	1.000	-0.007	-0.145
0.2	2.112	-1.603	0.198	0.956	-0.422	1.293	-1.550	0.999	-0.028	-0.259
0.3	1.952	-1.578	0.291	0.906	-0.586	1.143	-1.450	0.994	-0.058	-0.343
0.4	1.796	-1.545	0.379	0.840	-0.718	1.003	-1.351	0.987	-0.095	-0.401
0.5	1.644	-1.503	0.459	0.764	-0.822	0.873	-1.253	0.976	-0.137	-0.436
0.6	1.496	-1.454	0.532	0.677	-0.897	0.752	-1.156	0.960	-0.181	-0.451
0.7	1.353	-1.397	0.595	0.585	-0.947	0.642	-1.061	0.939	-0.226	-0.449
0.8	1.216	-1.335	0.649	0.489	-0.973	0.540	-0.968	0.914	-0.270	-0.432
0.9	1.086	-1.268	0.693	0.392	-0.977	0.448	-0.878	0.885	-0.312	-0.403
1.0	0.962	-1.197	0.727	0.295	-0.962	0.364	-0.792	0.852	-0.350	-0.364
1.2	0.738	-1.047	0.767	0.109	-0.885	0.223	-0.629	0.775	-0.414	-0.268
1.4	0.544	-0.893	0.772	-0.056	-0.761	0.112	-0.482	0.688	-0.456	-0.157
1.6	0.381	-0.741	0.746	-0.193	-0.609	0.029	-0.354	0.594	-0.477	-0.047
1.8	0.247	-0.596	0.696	-0.298	-0.445	-0.030	-0.245	0.498	-0.476	0.054
2.0	0.142	-0.464	0.628	-0.371	-0.283	-0.070	-0.155	0.404	-0.456	0.140
3.0	-0.075	-0.040	0.225	-0.349	0.226	-0.089	0.057	0.059	-0.213	0.268
4.0	-0.050	0.052	0.000	-0.106	0.201	-0.028	0.049	-0.042	0.017	0.112
5.0	-0.009	0.025	-0.033	0.015	0.046	0.000	-0.011	-0.026	0.029	-0.002

From *Drilled Pier Foundations*, by R. J. Woodwood, W. S. Gardner, and D. M. Greer. Copyright 1972 by McGraw-Hill. Used with the permission of McGraw-Hill Book Company.

The magnitude of the lateral load  $Q_g$  limited by the displacement condition only is,

$$\therefore Q_g = \frac{\Delta(E_p I_p)}{A_z T^3} = \frac{(0.008 \text{ m})(207 \times 10^6 \text{ kN/m}^2)(123 \times 10^{-6} \text{ m}^4)}{(2.435)(1.16)^3} = 54 \text{ kN}$$

Since the value of the allowable lateral load  $Q_g$  found above is based on the limiting displacement conditions only, and ignores that the pile has a moment capacity, that moment capacity at any depth  $z$  is found through,

$$M_z = A_m Q_g T$$

The table above shows that the maximum value of  $A_m$  at any depth is 0.772. The maximum allowable moment that the pile can carry is,

$$M_{max} = F_y \frac{I_p}{d_1/2} = (248 \text{ MN/m}^2) \frac{(123 \times 10^{-6} \text{ m}^4)}{(0.254 \text{ m}/2)} = 240 \text{ kN-m}$$

$$\therefore Q_g = \frac{M_{max}}{A_m T} = \frac{(240 \text{ kN-m})}{(0.772)(1.16 \text{ m})} = 268 \text{ kN}$$

This last value of  $Q_g$  emanating from the moment capacity is much larger than the value of  $Q_g = 54 \text{ kN}$  found for the deflection criterion.

Therefore use,  $Q_g = 54 \text{ kN}$ .

# **Chapter 24**

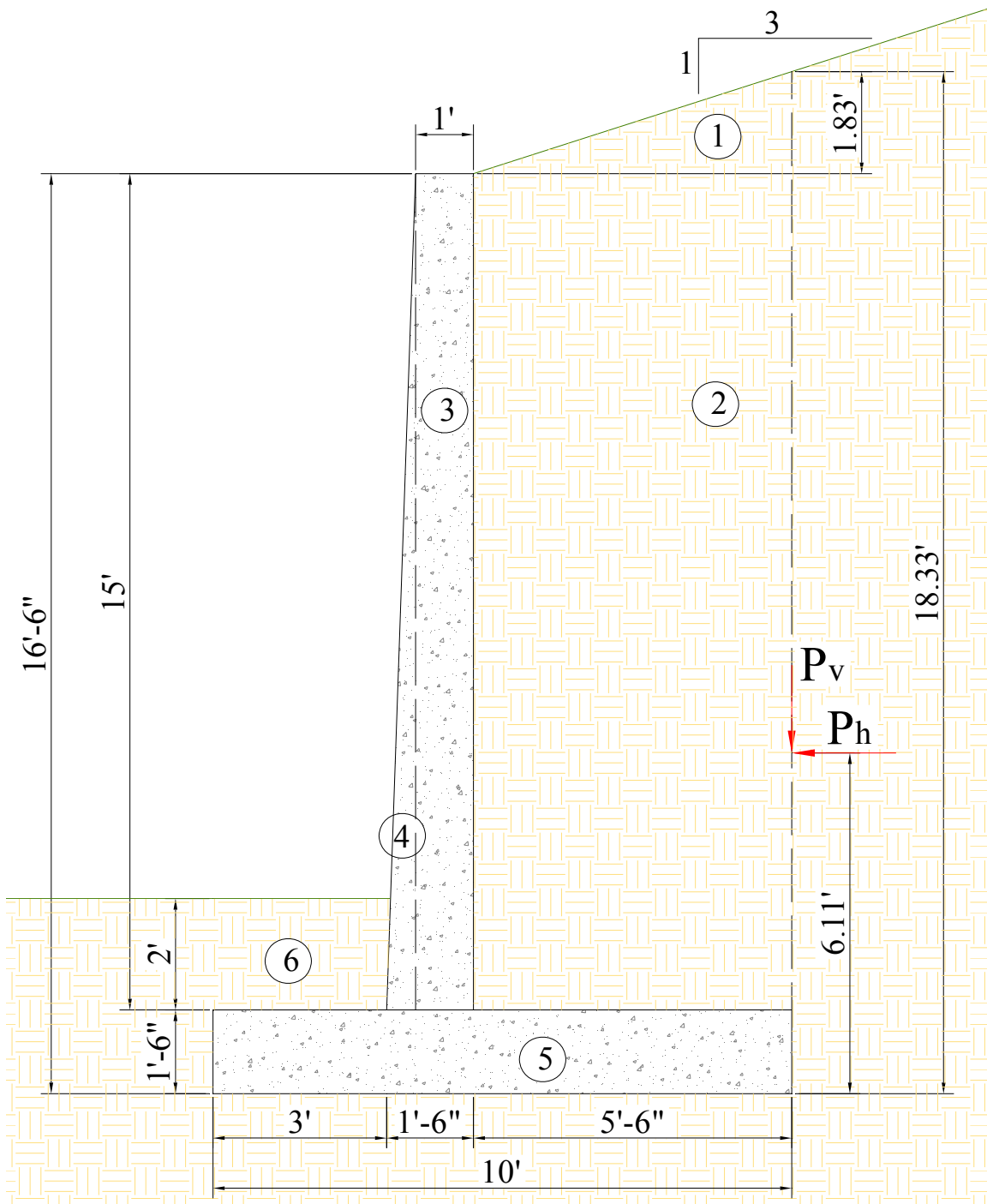
## **Reinforced Concrete Retaining Walls and Bridge Abutments**

***Symbols for Reinforced Concrete Retaining Walls***

## **\*\*RC Retaining Walls–01: Design a RC wall for a sloped backfill.**

(Revision: Oct-08)

Design a reinforced concrete wall with a backfill  $\gamma = 125$  pcf, an allowable soil bearing capacity of  $q_{all} = 3$  ksf, and a friction at the base of  $\phi = 30^\circ$ . Design the wall and check for its stability under working loads. (Note: All loads, shears and moments are per linear ft. of retaining wall).



Area

Area

Force

Arm

Moment

**Solutio**

**n:**

Step 1: Find

The active la

$$K_a = \tan^2 \left( 45^\circ - \frac{\phi}{2} \right)$$

The presure

$$p_b = \gamma H K_a =$$

The forces o

$$P_v = \frac{1}{2}(12)(18.33)$$

$$P_h = \frac{1}{2}(39)(18.33)$$

Step 2: Stabi

Moment  
About A

			(kip)	(ft)	(kip-ft)
1	$\frac{1}{2} \times 5.5 \times 1.83$	$= 5.03 \times 0.125 =$	0.63	1.83	1.2
and 2	$5.5 \times 15.0$	$= 82.5 \times 0.125 =$	10.31	2.75	28.4
3	$1.0 \times 15.0$	$= 15.0 \times 0.150 =$	2.25	6.00	13.5
4	$\frac{1}{2} \times 0.5 \times 15.0$	$= 3.75 \times 0.150 =$	0.56	6.67	3.7
5	$10.0 \times 1.5$	$= 15.0 \times 0.150 =$	2.25	5.00	11.3
6	$3.0 \times 2.0$	$= 6.0 \times 0.125 =$	0.75	8.50	6.4
$P_v$			2.00		
$P_h$	$\Sigma H = 6.60$			6.11	<u>40.4</u>
			<u><math>\Sigma V = 18.75</math></u>		<u><math>\Sigma M = 104.9</math></u>

#### Location of Resultant

From point A,  $\frac{104.9}{18.75} = 5.6 \text{ ft}$

$$18.75$$

$$\text{then } e = 5.6 - \frac{10}{2} = 0.6 \text{ ft o.k. } < \frac{B}{6}$$

#### Soil Pressure at Toe of Base

$$q_{\max} = \frac{18.75}{10} \left( 1 + \frac{6 \times 0.6}{10} \right) = 1.875 (1 + 0.36) = 2.55 \text{ ksf OK } < 3 \text{ ksf}$$

#### Check F against Sliding

$$\text{Shear available along base} = 18.75 \text{ kips} \times 0.58 = 10.9 \text{ kips}$$

Passive force at toe

$$\text{Use } S = 2/3 (30^\circ) = 20^\circ, P_p = \left( \frac{\cos \delta}{\frac{1}{2} \gamma H^2} \right) = 5.8 \text{ kips}$$

$$P_p = \frac{5.8 (0.125) (3.5)^2}{2 (0.940)} = 4.7 \text{ kips}$$

$$\text{Min. } F = \frac{10.9}{6.6} = 1.7 \text{ kips}, \text{ Max } F = \frac{10.9 + 4.7}{6.6} = \frac{15.6}{6.6} = 2.4 \text{ kips OK without Key}$$

## Step 2: Design parameters.

### Load Factors

Stem – Use  $1.7 P_h$

Base (toe and heel) – distribute  $\Sigma V$  uniformly over front  $B/3$

### Concrete and Steel Data

Capacity reduction factors: 0.90(flexure); 0.85(shear)

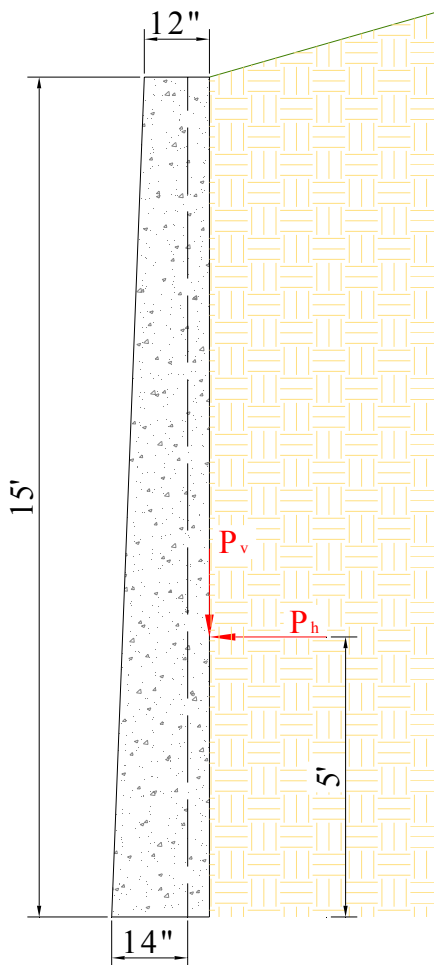
$$F'_c = 3,000 \text{ psi} \times 0.85 = 2,550 \text{ psi (for stress block)}$$

$$V_c = 2 \sqrt{3,000} = 110 \text{ psi}$$

$$f_y = 40,000 \text{ psi} ; \rho_{\min} = 0.005 ; \rho_{\max} = 0.0278 ; \rho_{\text{shrinkage}} = 0.002$$

$$l_d = \frac{0.04 A_b (40,000)}{\sqrt{3,000}} = 29.2 A_b \text{ (bottom bars)} \times 1.4 = 40.9 \text{ (top bars)}$$

## Step 3: Design the stem of the wall.



### Vertical Reinforcement

$$P_h = 1.7 \frac{1}{2} 39 (15)^2 = 7.46$$

$$M = 7.46 \times 5 \times 12 = 448 \text{ kip-in}$$

$$\text{Use } 6'' \text{ batter on front, then } t = 12 + 6 = 18''$$

$$\text{Use } d = 18 - 4 = 14''$$

$$\text{Assume arm} = d - a/2 = 13''$$

$$T = 448/13 = 34.5 \text{ kip}$$

$$A_s = 34.5 / (40 \times 0.90) = 0.96 \text{ in}^2/\text{ft}$$

At bottom of wall Use #6 @ 5'' ctrs

$$A_s = 1.06 \text{ in}^2/\text{ft}$$

$$\rho = 1.06 / (14 \times 12) = 0.0063 > 0.005 \text{ and } < 0.0278 \text{ OK}$$

Check compressive stress block:

$$C = T = 34.5 / (2.55 \times 12 \times 0.90) = \alpha_{\min} = \underline{1.25 \text{ in OK} < 2.0}$$

Check Shear:

$$7.46 / (14 \times 12 \times 0.85) = \underline{52 \text{ psi OK} < 110 \text{ psi}}$$

Moment computations for steel cutoff:

At bottom :  $M = 448 \text{ kips}$

At 10' level :  $M = 448 (10/15)^3 = 133 \text{ kip-in}$

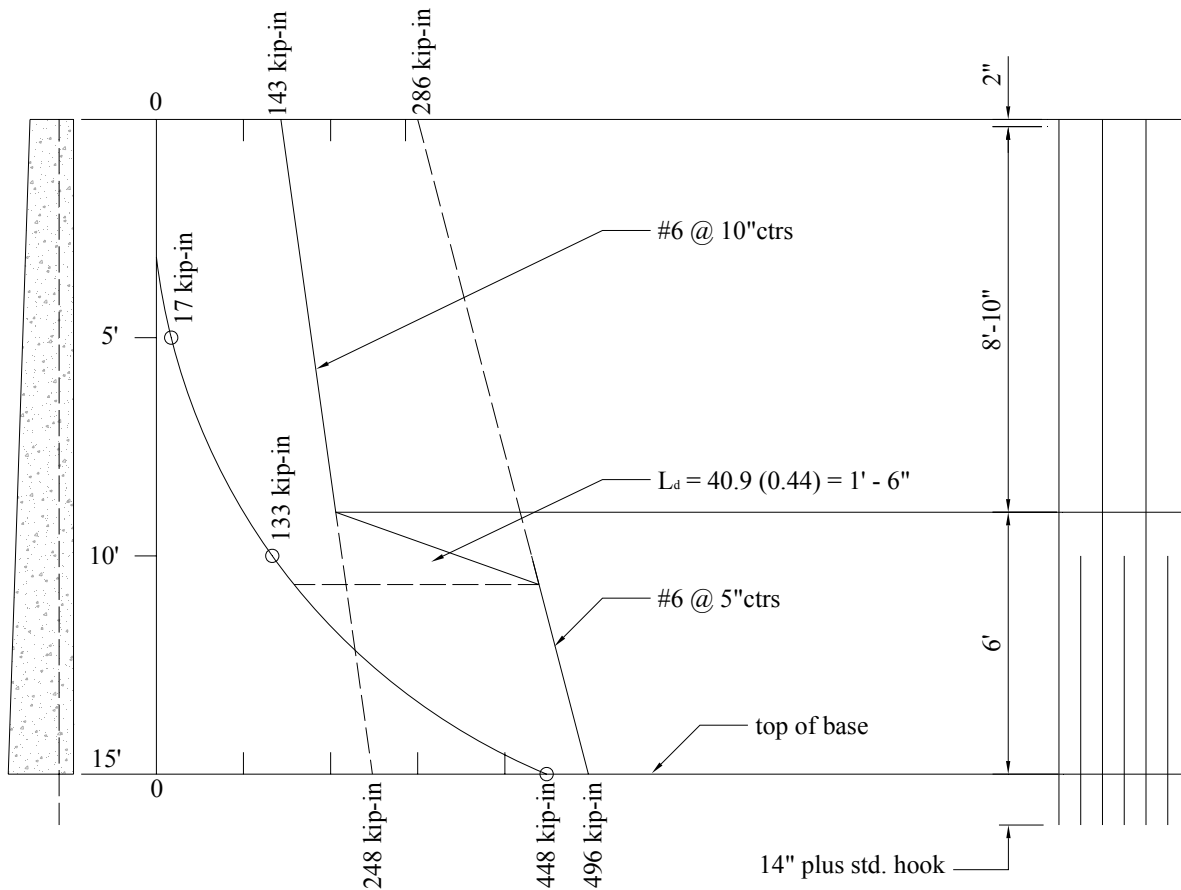
At 5' level :  $M = 448 (5/15)^3 = 17 \text{ kip-in}$

Resisting moment of steel :

#6 @ 5" ctrs. ;  $A_s = 1.06 \text{ in}^2/\text{ft}$

At bottom :  $M_r = 1.06 (40) (0.90) (13) = 496 \text{ kip-in}$

At top :  $M_r = 106 (40) (0.90) (7.5) = 286 \text{ kip-in}$



Compute level of cutoff for #6 @ 10" ctrs. :

$$A_s = 0.53 \text{ in}^2/\text{ft} ; \rho = 0.53 / (\pm 11 \times 12) = 0.004 > 0.002 < 0.005$$

Therefore, cutoff and develop bars above level where:

$$M = \frac{0.53 (40) (\pm 11) (0.90)}{1.33} = 158 \text{ kip-in}$$

$$M = 158 \text{ kip-ft @ } \pm 10'-6'' \text{ level}$$

$$\text{Developmental length} = 1'-6''$$

$$\underline{\text{Level of Cutoff} = 9'-0''}$$

#### **Step 4: Design the toe of the base of the wall.**

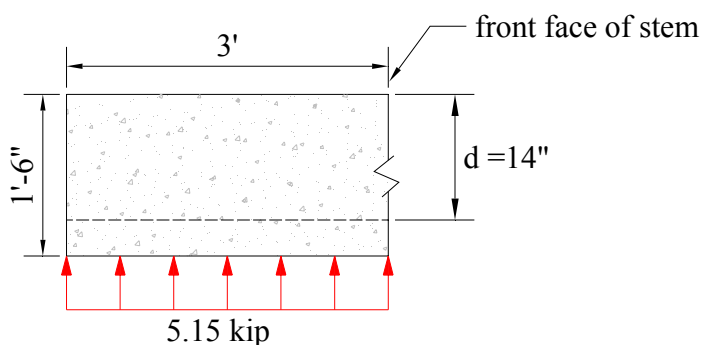
Distribute  $\Sigma V$  over front B/3. Assume  $t = 18''$ ,  $d = 14''$

$$\Sigma V = \frac{18.75 \text{ kips}}{(10 / 3)} = q_{\max} = 5.63 \text{ ksf}$$

$$\text{wt. of soil over toe} = 2.0 (0.125) = 0.25 \text{ ksf}$$

$$\text{wt of concrete base} = 1.5 (0.150) = 0.23 \text{ ksf}$$

$$\begin{aligned} \underline{\text{Net toe pressure for design}} &= q_{\max} - \text{wt. of soil over toe} - \text{wt of concrete base} \\ &= 5.63 \text{ ksf} - 0.25 \text{ ksf} - 0.23 \text{ ksf} = \underline{5.15 \text{ ksf}} \end{aligned}$$



$$V = 5.15 \times (3.0) = 15.45 \text{ kips}$$

$$M = 15.45 \times (3/2) \times 12 = 278 \text{ kip-in}$$

Assume arm = 13 in

$$T = 278/13 = \underline{21.4 \text{ kips}} = 0.59 \text{ in}^2/\text{ft}$$

$$(40 \times 0.9)$$

$$\text{Check } \rho = \underline{0.59} = 0.0035 > 0.002, \text{ shrinkage OK}$$

14 x 12

$P < 0.005$ , so increase  $A_s$  by  $1/3$ , then  $A_s = 0.59 \times 1.33 = 0.80 \text{ in}^2/\text{ft}$

use #6 @ 6" ctrs.  $A_s = 0.88 \text{ in}^2/\text{ft}$

Compressive stress block and shear OK by inspection after stem computations

Development length : Extend full base width, therefore  $l_d$  OK

### Step 5: Design the heel of the base of the wall.

#### At Stem:

Weight of soil above heel at back face of stem =  $15.0 (0.125) = 1.88 \text{ ksf}$

Weight of concrete base =  $1.5 (0.150) = 0.23 \text{ ksf}$

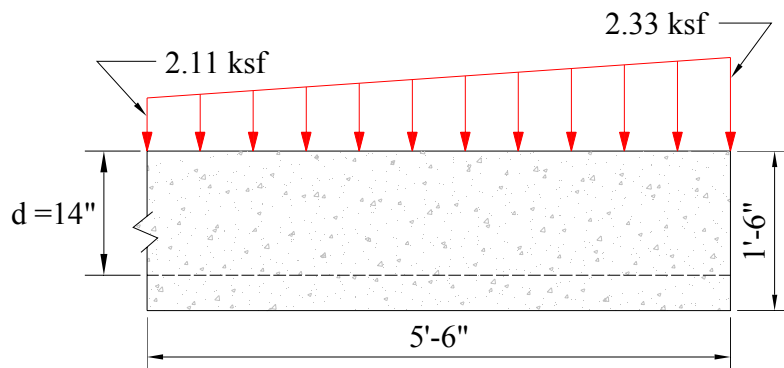
Net pressure for design =  $1.88 + 0.23 = 2.11 \text{ ksf}$

#### At Back:

Weight of soil above hell at back =  $16.83 (0.125) = 2.10 \text{ ksf}$

Weight of concrete base =  $0.23 \text{ ksf}$

Net pressure for design =  $2.33 \text{ ksf}$



$$V_1 = \frac{1}{2} (2.11) (5.5) = 5.80 \text{ kips}$$

$$V_2 = \frac{1}{2} (2.33) (5.5) = 6.41 \text{ kips}$$

$$\text{Total } V = 5.80 + 6.41 = 12.21 \text{ kips}$$

$$M_1 = 5.8 (1/3) (5.5) (12) = 128 \text{ kip-in}$$

$$M_2 = 6.41 (2/3) (5.5) (12) = 282 \text{ kip-in}$$

$$\text{Total } M = 128 + 282 = 410 \text{ kip-in}$$

Assume arm = 13"

$$T = 410/13 = 31.5 \text{ kip}$$

$$A_s = \frac{31.5 \text{ kip}}{(40 \times 0.9)} = 0.88 \text{ in}^2/\text{ft}$$

Use #6 @ 6" ctrs.  $A_s = 0.88 \text{ in}^2/\text{ft}$   $\rho > 0.002$  and  $> 0.005$  and  $< 0.0278$  OK

Compressive stress block and shear OK

Development length: Extend full base width, therefore  $l_d$  OK

Horizontal shrinkage steel in stem:

Required:  $0.002 (\pm 15) (12) = 0.36 \text{ in}^2/\text{ft}$  of height

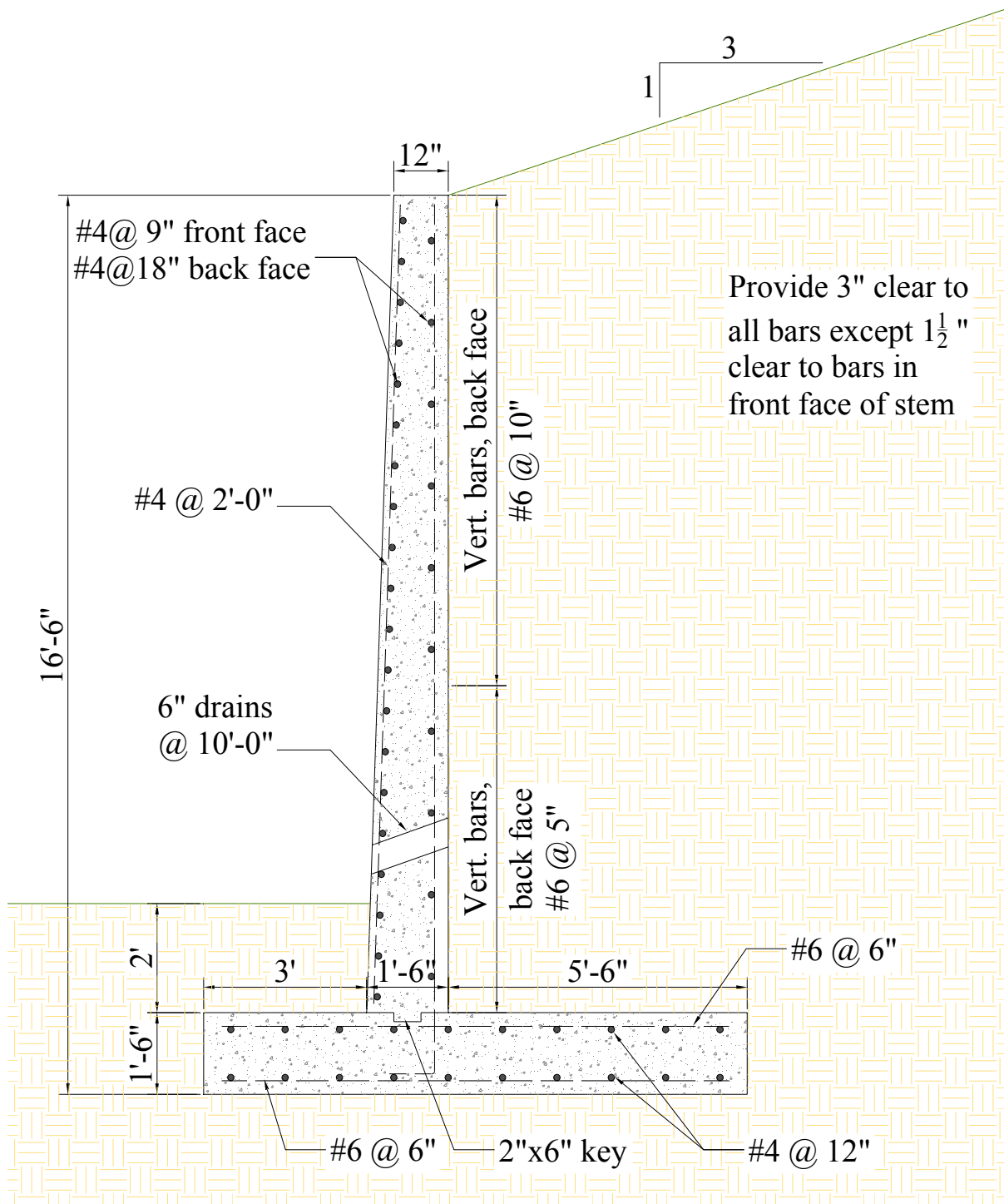
Use #4 @ 9" ctrs. front  $A_s = 0.27 \text{ in}^2/\text{ft}$

Use #4 @ 18" ctrs back  $A_s = 0.13 \text{ in}^2/\text{ft}$

Total  $A_s = 0.27 + 0.13 = 0.40 \text{ in}^2/\text{ft}$

Horizontal shrinkage steel in base: Use #4 @ 12 ctrs. top and bottom  $A_s = 0.40 \text{ in}^2/\text{ft}$

**Step 6: Finished sketch of the wall.**



# Chapter 25

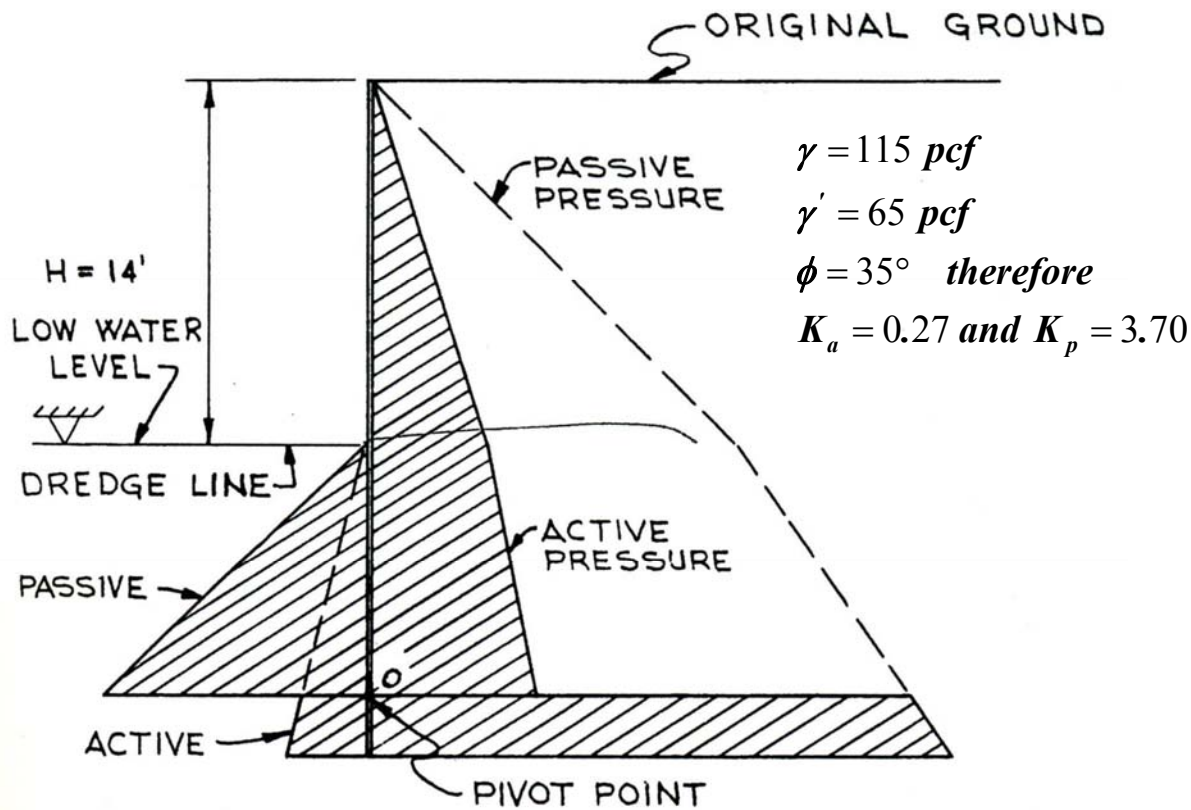
## Steel Sheet Pile Retaining Walls

***Symbols for Steel Sheet Pile Retaining Walls***

**\*\*Sheet-pile Wall-01: Free-Earth for cantilevered walls in granular soils.**

(Updated: 9 April 2008)

Select an appropriate steel sheet-pile section and its total length  $L$  to retain a medium sand backfill for the conditions shown below without an anchoring system.



Solution:

Step 1. Determine the pressures on the cantilevered wall.



$$(1) \quad \Sigma F_H = 0$$

$$Area(BAA_1) + Area(AA_1A_2F) + Area(ECJ) - Area(EA_1A_2) = 0$$

$$\frac{1}{2}Hp_{A1} + (p_{A1} + p_{A2})\left(\frac{D}{2}\right) + (p_E + p_J)\left(\frac{z}{2}\right) - (p_E + p_{A2})\left(\frac{D}{2}\right) = 0$$

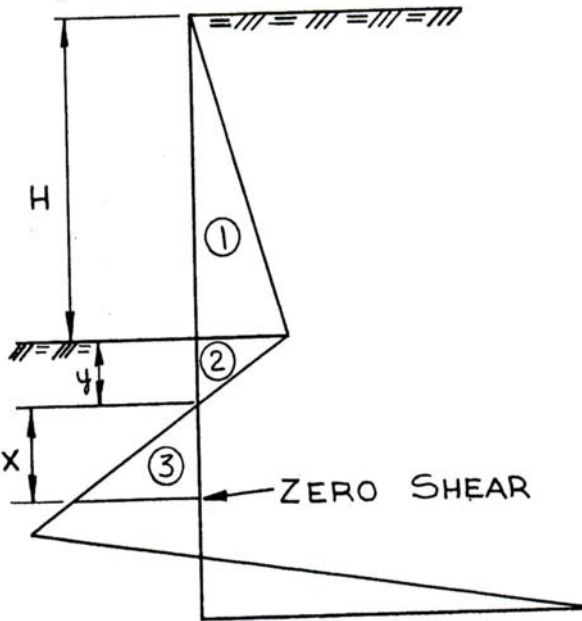
Solving for  $z$ ,

$$z = \frac{(p_E - p_{A1})D - Hp_1}{p_E + p_J}$$

$$(2) \quad \Sigma M = 0 \text{ about any point, say } F,$$

$$\frac{1}{2}(H)p_{A1}\left(D + \frac{H}{3}\right) + (p_{A1})\left(\frac{D^2}{2}\right) + (p_E + p_J)\left(\frac{z^2}{6}\right) - (p_E + p_{A2})\left(\frac{D^2}{6}\right) + (p_{A2} - p_{A1})\left(\frac{D^2}{6}\right) = 0$$

Step 3. Determine the maximum moment  $M_{max}$  (point of zero shear).



$$y = \frac{p_{A1}}{\gamma'(K_p - K_a)} = \frac{435}{(65)(6.29)} = 1.06 \text{ feet}$$

$$P_1 = \frac{1}{2}p_{A1}H = \frac{1}{2}(435)(14) = 3,040 \text{ lb}$$

$$P_2 = \frac{1}{2}p_{A1}y = \frac{1}{2}(435)(1.0) = 218 \text{ lb}$$

also,

$$\frac{1}{2}\gamma'(K_p - K_a)x^2 = P_1 + P_2$$

$$\therefore x^2 = \frac{2(P_1 + P_2)}{\gamma'(K_p - K_a)} = \frac{2(3,040 + 218)}{(65)(6.29)} = 16 \quad \therefore x = 4.0 \text{ feet}$$

$$P_3 = \frac{1}{2}\gamma'(K_p - K_a)(4)^2 = 3,280 \text{ lb}$$

$$M_{max} = P_1l_1 + P_2l_2 - P_3l_3$$

where,

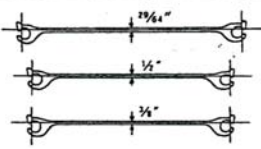
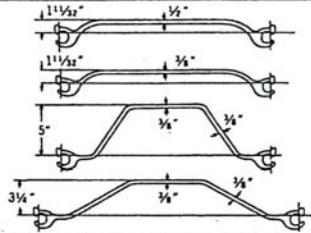
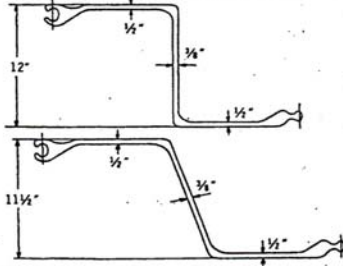
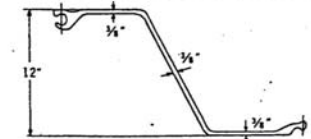
$$l_1 = \frac{H}{3} + y + x$$

$$l_2 = \frac{2y}{3} + x$$

$$l_3 = \frac{x}{3}$$

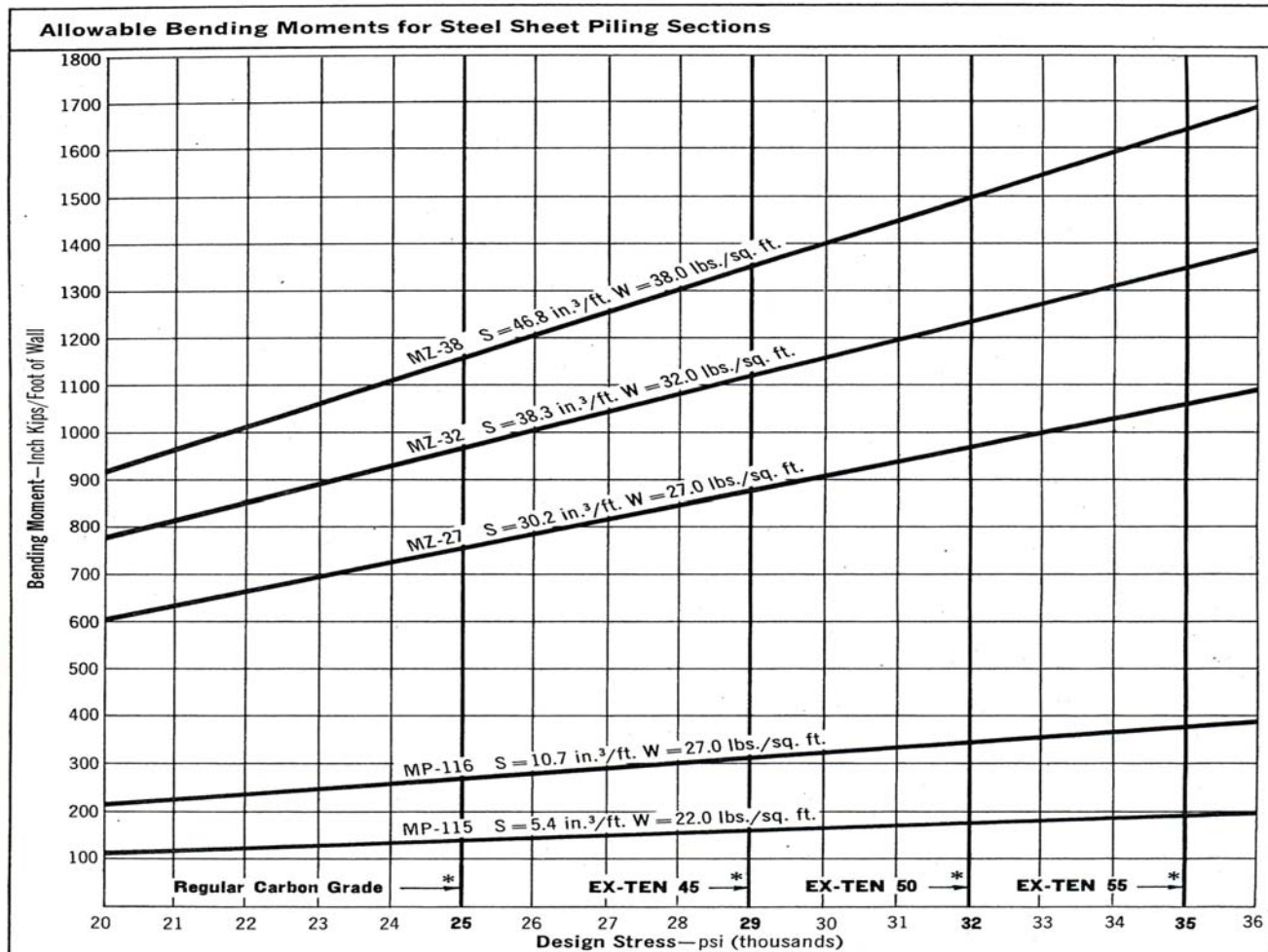
$$M_{max} = P_1l_1 + P_2l_2 - P_3l_3 = (3,040)\left(\frac{14}{3} + 1.0 + 4.0\right) + (218)\left(\frac{2(1.0)}{3} + 4.0\right) - (3,280)\left(\frac{4.0}{3}\right)$$

$$M_{max} = 29,300 + 1,030 - 4,360 = 26,000 \text{ ft-lb} = 312 \text{ in-kips}$$

Steel Sheet Piling Sections												
Profile	Section Index		District Rolled	Driving Distance per Pile	Weight		Web Thickness	Section Modulus		Area	Moment of Inertia	
					Per Foot	Per Square Foot of Wall		Per Pile	Per Foot of Wall	Per Pile	Per Pile	Per Foot of Wall
					In.	Lbs.	Lbs.	In.	In. <sup>3</sup>	In. <sup>3</sup>	In. <sup>2</sup>	In. <sup>4</sup>
	Interlock with Each Other	MP 103	P.	16 1/2	44.0	32.0	29/64	3.3	2.4	12.94	5.1	3.7
		MP 102*	P.C.	15	40.0	32.0	1/2	2.4	1.9	11.77	3.6	2.9
		MP 101	P.C.	15	35.0	28.0	3/8	2.4	1.9	10.30	3.5	2.8
	Interlock with Each Other	MP 113*	P.	16	37.3	28.0	1/2	3.3	2.5	10.98	6.0	4.5
		MP 112	P.C.	16	30.7	23.0	3/8	3.2	2.4	8.99	5.5	4.1
		MP 116	P.C.	16	36.0	27.0	3/8	14.3	10.7	10.59	53.0	39.8
		MP 115	P.C.	19 3/8	36.0	22.0	3/8	8.8	5.4	10.59	22.4	13.7
	Interlock with Each Other and with MP 112 or MP 113	MZ 38	P.	18	57.0	38.0	3/8	70.2	46.8	16.77	421.2	280.8
		MZ 32	P.	21	56.0	32.0	3/8	67.0	38.3	16.47	385.7	220.4
	Interlocks with Itself and with MP 112 or MP 113	MZ 27	P.	18	40.5	27.0	3/8	45.3	30.2	11.91	276.3	184.2

Suggested Allowable Design Stresses—Sheet Piling		
Steel Brand or Grade	Minimum Yield Point, psi	Allowable Design Stress, psi*
USS EX-TEN 55	55,000	35,000
USS EX-TEN 50	50,000	32,000
USS EX-TEN 45	45,000	29,000
Regular Carbon Grade	38,500	25,000

\*Based on 65% of minimum yield point. Some increase for temporary over stresses generally permissible.



*If instead, an Ex-Ten-45 steel section is used,  $f_s = 29$  ksi*

*The required section modulus  $S = \frac{M_{max}}{f_s} = \frac{(26,000)(12)}{(29,000)} = 10.8 \text{ in}^3$*

*$\therefore$  Use a MP-116 that provides a  $S = 10.7 \text{ in}^3$  at a lower cost.*

*This is shown plotted with the red lines.*

Step 5. Determine the sheet-pile length  $L$ .

The total sheet-pile length  $L = H + D = 14.0 + 13.5 = 27.5$  feet

# Chapter 26

## MSE (Mechanically Stabilized Earth) Walls

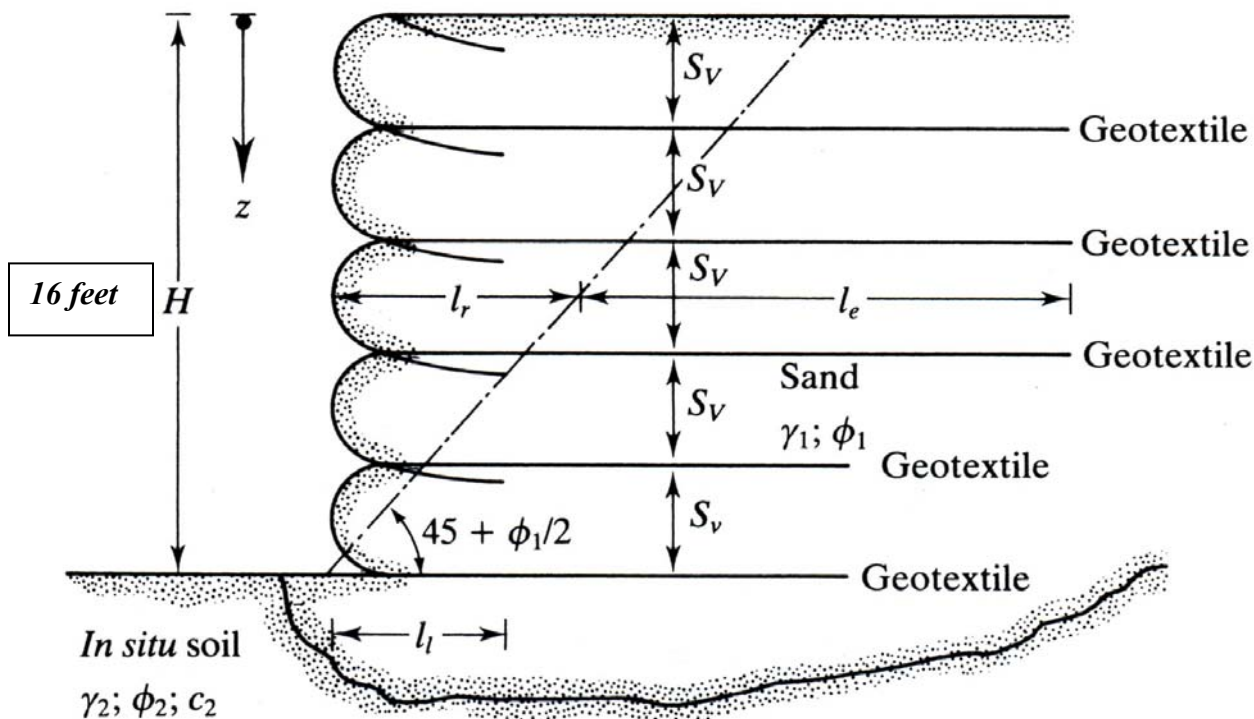
*Symbols for Mechanically Stabilized Earth Walls*

**\*\*MSE Walls-01: Design the length  $L$  of geotextiles for a 16 ft wall.**

(Revision: Oct-08)

Determine the length  $L$  of a geotextile-reinforcing for the 16 foot high temporary retaining wall shown below. Also determine the required vertical spacing of the reinforcing layers  $S_v$  and the required lap length  $l_l$ . The geotextile chosen has an allowable strength of  $\sigma_G$  of 80 lb/inch, and the available granular backfill has a unit weight of  $\gamma = 110 \text{ lb/ft}^3$  and the angle of internal friction is  $\phi = 36^\circ$ .

**Comment:** These geotextile walls are usually used for temporary civil works, such as detour roadways, temporary abutments or excavation walls, etc. If the wall must become permanent, then the face is stabilized and the plastic geotextile is protected from ultra-violet light through a layer of shotcrete.



**Solution:**

The active pressure on the wall is,

$$\sigma_a = K_a \sigma_v = K_a (\gamma z) \quad \text{where } K_a = \tan^2 (45^\circ - \phi / 2) = \tan^2 (45^\circ - 36^\circ / 2) = 0.26$$

Step 1: Find the vertical spacing  $S_V$  of each layer of geotextile, where the *Factor of Safety (FS)* is generally chosen between 1.3 to 1.5 for temporary walls. Permanent walls should use at least  $FS \geq 2$ . For this problem choose  $FS = 1.5$ .

$$S_V = \frac{\sigma_G}{\sigma_a FS} = \frac{\sigma_G}{(\gamma z K_a) FS}$$

At a depth of  $z = 8$  feet from the top,

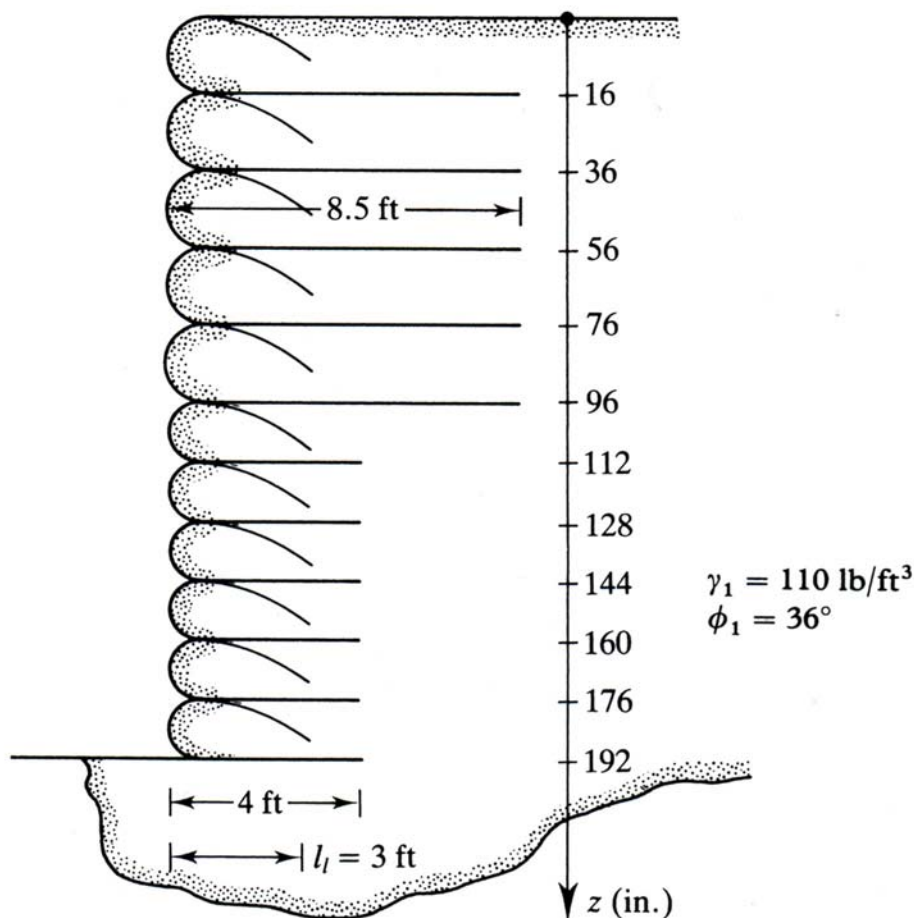
$$S_V = \frac{\sigma_G}{(\gamma z K_a) FS} = \frac{(80 \times 12 \text{ lb/ft})}{(110)(8)(0.26)(1.5)} = 2.8 \text{ feet} \approx 34 \text{ inches}$$

At a depth of  $z = 12$  feet from the top,

$$S_V = \frac{\sigma_G}{(\gamma z K_a) FS} = \frac{(80 \times 12 \text{ lb/ft})}{(110)(12)(0.26)(1.5)} = 1.9 \text{ feet} \approx 22 \text{ inches}$$

At a depth of  $z = 16$  feet from the top,

$$S_V = \frac{\sigma_G}{(\gamma z K_a) FS} = \frac{(80 \times 12 \text{ lb/ft})}{(110)(16)(0.26)(1.5)} = 1.4 \text{ feet} \approx 17 \text{ inches}$$



Notice how the spacing becomes denser the deeper we go below the surface; choose  $S_V = 20$  inches from  $z = 0$  to  $z = 8$  feet; below  $z = 8$  feet use  $S_V = 16$  inches throughout, as shown in the figure.

Step 2: Find the length of each layer of geotextile  $L$ , which is composed of two parts,  $l_R$  which is the length of the geotextile within the Rankine failure zone, and  $l_e$  which is the effective length of the geotextile beyond the failure zone (see the first figure, page 408). Again use  $FS = 1.5$ , and the angle  $\phi_F$  is the soil-to-geotextile angle of friction, which is usually assumed to be  $2/3 \phi$  of the soil. Other values can be used, and a few are shown in this table,

Type	$\phi_f / \phi_1$
Woven — monofilament/concrete sand	0.87
Woven — silt film/concrete sand	0.8
Woven — silt film/rounded sand	0.86
Woven — silt film/silty sand	0.92
Nonwoven — melt-bonded/concrete sand	0.87
Nonwoven — needle-punched/concrete sand	1.0
Nonwoven — needle-punched/rounded sand	0.93
Nonwoven — needle-punched/silty sand	0.91

Therefore, the length  $L$  of the geotextile layer is,

$$L = l_R + l_e = \frac{(H - z)}{\tan(45^\circ - \phi / 2)} + \frac{S_v K_a (FS)}{2 \tan \phi_F} = \frac{(H - z)}{\tan(45^\circ - 36^\circ / 2)} + \frac{S_v (0.26)(1.5)}{2(0.445)}$$

$$L = (0.51)(H - z) + 0.438 S_v$$

From this equation, we can now prepare a table with the required lengths.

$z$		$S_v$	$(0.51)(H - z)$	$(0.438 S_v)$	$L$
inches	feet	feet	feet	feet	feet
16	1.33	1.67	7.48	0.731	8.21
56	4.67	1.67	5.78	0.731	6.51
76	6.34	1.67	4.93	0.731	5.66
96	8.00	1.67	4.08	0.731	4.81
112	9.34	1.33	3.40	0.582	3.99
144	12.00	1.33	2.04	0.582	2.66
176	14.67	1.33	0.68	0.582	1.26

Based on this table, use  $L = 8.5$  feet for  $z \leq 8$  feet and  $L = 4.0$  feet for  $z > 8$  feet.

Step 3: Find lap length  $l_l$  for the geotextile, but never smaller than 3 feet,

$$l_i = \frac{S_v \sigma_a (FS)}{4 \sigma_v \tan \phi_F} = \frac{S_v (0.26)(1.5)}{4 \tan \left[ \left( \frac{2}{3} \right) (36^\circ) \right]} = 0.219 S_v$$

For a depth of  $z=16$  inches,

$$l_i = 0.219 S_v = (0.219) \left( \frac{20 \text{ inches}}{12 \text{ in / ft}} \right) = 0.365 \text{ feet} < 3 \text{ feet}$$

Therefore, use  $l_i = 3 \text{ feet}$

Comment:

These MSE problems commonly use Rankine's active pressure coefficient. However, the actual value of  $K$  must depend on the degree of restraint of the type of reinforcement, as shown in this figure:

