شعاره الصبر ، وراحته التعب

* يشمل تلخيص + حل أسئلة Suggested

التلخيص يحتمل الخطأ والنقص :)

① $h[n] \rightsquigarrow$ impulse Response

$$x[n] \longrightarrow \boxed{LTI} \longrightarrow y[n]$$

② $H(e^{j\omega}) \rightsquigarrow$ frequency Response

$$x[n] \longrightarrow \boxed{LTI} \longrightarrow y[n]$$

③ $H(z) \rightsquigarrow$ transfer function

$$x[n] \longrightarrow \boxed{LTI} \longrightarrow y[n]$$

$$Y[Z] = X[Z] \cdot H[Z]$$

④ linear Constant Coefficient Diffrence equation

$$y[n] - y[n-1] = x[n]$$

ideal frequeny selective filter

ideal low pass filter :-

$$H_{LP}(e^{jw}) = \begin{cases} 1 , & |w| < w_c \\ 0 , & w_c < |w| \le \pi \end{cases} , \quad h_{LP}(n) = \frac{\sin w_c n}{\pi n} , -\infty < n < \infty$$

Ideal high pass filter :-

$$H_{hp}(e^{jw}) = \begin{cases} 0 , & |w| < w_c \\ 1 , & w_c < |w| \le \pi \end{cases} \qquad h_{HP}(n) = \delta[n] - \frac{\sin w_c n}{\pi n}$$

However, the non ideal filters only shift it.

$$h_{LP}[n] = \frac{\sin w_c(n-n_0)}{\pi(n-n_0)}$$

ideal delay system :-

$h_{id}(n) = \delta(n-n_d)$

$H_{id}(e^{jw}) = 1 e^{-jwn_d}$

$\theta = -w\, n_d$

$\frac{d\theta}{dw} = -n_d \longrightarrow \frac{-d\theta}{dw} = n_d$ → Called : group delay

means :- → output الـ delay يعني nd samples بمقدار

*Note :- $A e^{j\theta}$ (A → magnitude, θ → phase)

Example : $h[n] = \delta[n-7]$, find $\tau(w)$ (group delay)

$H(e^{jw}) = e^{-jw7}$

$\frac{d\theta}{dw} = -7 \rightarrow \tau(w) = 7 \rightsquigarrow$ 7 samples.

Note that :-

$\theta = -7w \rightsquigarrow$ which is linear (linear phase system)

and the group delay is constant (-7), which means it gonna delay all the system with same phase

Example: $h[n] = \frac{1}{5}\{\overset{n=0}{\underset{\uparrow}{1}},1,1,1,1\}$, 5 point moving avarage.

1. Find transfer function $H[z]$
2. $H(e^{j\omega})$
3. group delay

1. $h[z] = \frac{1}{5}\left(z^{0} + z^{-1} + z^{-2} + z^{-3} + z^{-4}\right)$

2. $H(e^{j\omega}) = \frac{1}{5}\left(1 + e^{-j\omega} + e^{-j2\omega} + e^{-j3\omega} + e^{-j4\omega}\right)$ بدنا نختصرهم ، بس ما بنقدر نجمعهم لأنه كل حد فيهم إله قيمة phase مختلفة

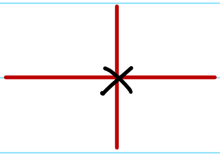
$$= \frac{e^{-j2\omega}}{5}\left(e^{j2\omega} + e^{j\omega} + 1 + e^{-j\omega} + e^{-j2\omega}\right)$$

$$= \frac{e^{-j2\omega}}{5}\left[1 + 2\cos\omega + 2\cos(2\omega)\right]$$

$$= \underbrace{\left\{\frac{[1+2\cos\omega+2\cos 2\omega]}{5}\right\}}_{\text{magnitude}} \cdot e^{\overbrace{-j2\omega}^{\text{phase}}} \Rightarrow Ae^{j\theta}$$ خليناها على شكل

3. $\tau(\omega) = \frac{-d\theta}{d\omega} = -(-2) = 2$ samples $\rightarrow$ delay the output by 2 sampels.

* this System is Causal, stable (finite)
* ROC :- all values of z except 0.



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linear Constant Coefficient diffrence equation

$$\sum_{k=0}^{N} a_k\, y(n-k) = \sum_{k=0}^{M} b_k\, x(n-k)$$

$$\sum_{k=0}^{N} a_k\, Y(z)\, z^{-k} = \sum_{k=0}^{M} b_k X[z]\, z^{-k}$$

$$\frac{Y[z]}{X[z]} = \frac{\sum_{k=0}^{M} b_k z^{-k}}{\sum_{k=0}^{N} a_k z^{-k}} = \left(\frac{b_0}{a_0}\right) \frac{\prod_{k=1}^{M} \overbrace{(1-c_k z^{-1})}^{\text{Zeros}}}{\prod_{k=1}^{N} \underbrace{(1-d_k z^{-1})}_{\text{Poles}}}$$

معاملات x

معاملات y

Example :- $H(z) = \dfrac{(1+z^{-1})^2}{(1-\frac{1}{2}z^{-1})(1+\frac{3}{4}z^{-1})}$, find the DE ?

$$H(z) = \frac{1+2z^{-1}+z^{-2}}{1+\frac{1}{2}z^{-1}-\frac{3}{8}z^{-2}}$$

$$y[n] = x[n] + 2x[n-1] + x[n-2] - \tfrac{1}{2}y[n-1] + \tfrac{3}{8}y[n-2].$$

h[n] must be Right sided

Stability and Causality :-

should include the unit Circle

Example :- Determine ROC of the system described by $y[n] = \dfrac{1}{1-\frac{5}{2}z^{-1}+z^{-2}}$

$$y[n] = \frac{A}{(1-2z^{-1})} + \frac{B}{(1-\frac{1}{2}z^{-1})}$$

$$A(1-\tfrac{1}{2}z^{-1}) + B(1-2z^{-1}) = 1$$

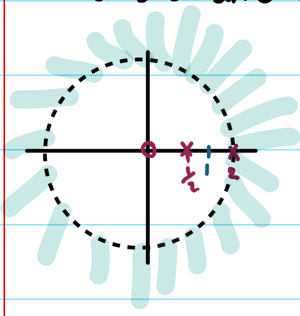
if $z^{-1}=2 \rightarrow B(-3)=1 \rightarrow B=-\frac{1}{3}$

if $z^{-1}=\frac{1}{2} \rightarrow A(\frac{3}{4})=1 \rightarrow A=\frac{4}{3}$

Poles: $z=2$, $z=\frac{1}{2}$

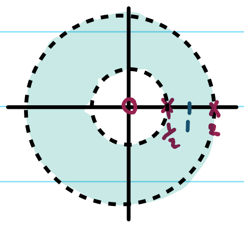
Zeros: $z=0$

1. Causal and Stable



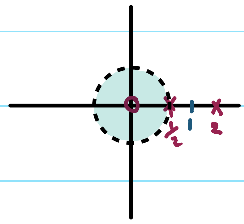
$|z|>2$

2. non Causal and stable



$\frac{1}{2}<|z|<2$

3. non Causal and unstable



$\frac{1}{2}>|z|$

Inverse System :-

$$x(n) \rightarrow \boxed{h_1(n)} \rightarrow \boxed{h_2(n)} \rightarrow y[n]$$

$$h_1(n) * h_2(n) = \delta[n]$$

$$H(z) \cdot H_i(z) = 1$$

$$h(n) * h_i(n) = \delta(n)$$

$$H_i(z) = \frac{1}{H(z)} \quad \text{and} \quad H_i(e^{j\omega}) = \frac{1}{H(e^{j\omega})}$$

Note :-

- Poles of $H(z)$ are Zeros for $H_i(z)$ and vice versa.
- ROC :- ROC system $\cap$ ROC inverse

* ROC of $H_i(z)$ must overlap with ROC of $H(z)$

يعني Roc لل Hi(z) لازم يكون متداخل مع Roc لل H(z)

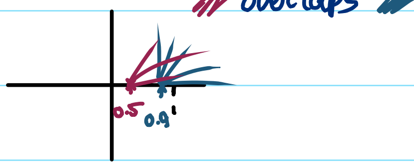
* if ROC of $H_i(z)$ is not overlapping with ROC of $H(z)$ then $H_i(z)$ does not exist (the system doesn't have any inverse).

Example :- Find the inverse and ROC of the following system :-

$$H(z) = \frac{1-0.5z^{-1}}{1-0.9z^{-1}}, \quad |z|>0.9 \rightarrow \text{causal and stable}$$

$$H_i(z) = \frac{1-0.9z^{-1}}{1-0.5z^{-1}}, \quad |z|>0.5$$

overlaps



* Since ROC for $H_i(z)$ overlaps $H(z)$

$H_i(z)$ is also causal and stable.

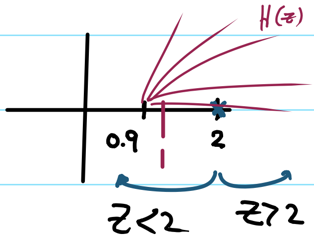
$$h_i(n) = (\tfrac{1}{2})^n u[n] - 0.9(\tfrac{1}{2})^{n-1} u[n-1]$$

* **Note :-**

if $H(z)$ is Causal and stable and it's inverse is also Causal and stable, this system called minimum phase System

* Poles and zeros inside the unit circil.
* Phase Response of such such system

Example :- $H(z) = \dfrac{z^{-1} - 0.5}{1 - 0.9z^{-1}}$, $|z| > 0.9$



$$H_i(z) = \frac{1 - 0.9z^{-1}}{z^{-1} - 0.5} \cdot \frac{-2}{-2} = \frac{1.8z^{-1} - 2}{1 - 2z^{-1}} ,$$

Here, we have two of Roc for $H_i(z)$ to be overlapped with Roc of $H(z)$

① $|z| > 2$

$$H_i(z) = \frac{1.8z^{-1}}{1 - 2z^{-1}} - \frac{2}{1 - 2z^{-1}}$$

$$h_i(n) = 1.8\,(2)^{n-1} u(n-1) - 2\,(2)^{n-1} u[n]$$

$$= 1.8\,(2)^{n-1} u(n-1) - 2^{n} u(n)$$

② $|z| < 2$

$$H_i(z) = \frac{1.8z^{-1}}{1 - 2z^{-1}} - \frac{2}{1 - 2z^{-1}}$$

$$= -1.8\,(2)^{n-1} u[-n] + 2^{n} u[-n-1]$$

*

$$h[n] = \sum_{r=0}^{M-N} B_r \delta[n-r] + \sum_{k=1}^{N} A_k (d_k)^n u[n]$$

Finite Impulse Response (FIR)

- $H(z)$ has no poles except at $z=0$

$$H(z) = \sum_{k=0}^{M} b_k \delta[n-k]$$

- $y[n] = \sum_{k=0}^{M} b_k z^{-k}$

Infinite impulse Response (IIR)

- has at least one pole is not canceled with zero
- feed back

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types of system :-

- → linear and phase system
- → non linear and phase system.
- → minimum phase system
- → all pass system

* all pass system :- passes all signals, has no effect on the magnitude, but it has a large impact on phase

$$H(z) = \frac{z^{-1} - c^*}{1 - c z^{-1}}$$

* minimum phase :- affects the magnitude, dont affect phase.

* minimum phase and all pass system decomposition :-

any arbitrary system can be factorised as a cascade of minimum phase - and - all pass system.

* $H(z)$ should be written as follow :

$$H(z) = H_{min}(z) \cdot H_{ap}(z)$$

$H(z)$ should be written as follow

$$H(z) = H_{min}(z) \cdot H_{ap}(z)$$

1. Take zeros that lie outside $|z|=1$ (unit circle) and move to $H_{ap}(z)$
2. add poles to $H_{ap}(z)$ in conjugate reciprocal location of zero
3. put zero $H_{min}(z)$ to cancel poles added to $H_{ap}(z)$

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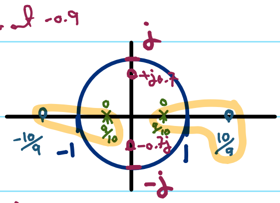
Example:- decompose $H(z)$ into min phase, and all pass :-

$$H(z) = \left(1-\frac{1}{0.9}z^{-1}\right)\left(1+\frac{1}{0.9}z^{-1}\right)\left(1-j0.7\,z^{-1}\right)\left(1+j0.7\,z^{-1}\right)$$

$$z = \mp\frac{10}{9} \quad , \mp j0.7$$

$$= \left(1-\frac{1}{0.9}z^{-1}\right)\left(1+\frac{1}{0.9}z^{-1}\right)\left(1-j0.7\,z^{-1}\right)\left(1+j0.7\,z^{-1}\right) \cdot \frac{\overbrace{(1-0.9z^{-1})}^{\text{zero at } 0.9}\overbrace{(1+0.9z^{-1})}^{\text{zero at } -0.9}}{\underbrace{(1-0.9z^{-1})}_{\text{pole at } 0.9}\underbrace{(1+0.9z^{-1})}_{\text{pole at } -0.9}}$$



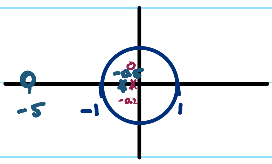
$$= \underbrace{(1-0.7jz^{-1})(1+j0.7z^{-1})(1-0.9z^{-1})(1+0.9z^{-1})}_{\text{min phase}} \cdot \left(\frac{1-\frac{1}{0.9}z^{-1}}{1-0.9z^{-1}}\right) \cdot \left(\frac{1+\frac{1}{0.9}z^{-1}}{1+0.9z^{-1}}\right)$$

$$\frac{z^{-1}-c^{*}}{1-\frac{1}{c}z^{-1}}$$

$$= \underbrace{\left(\frac{-1}{0.9}\right)\left(\frac{1}{0.9}\right)(1-0.7jz^{-1})(1+j0.7z^{-1})(1-0.9z^{-1})(1+0.9z^{-1})}_{\text{minimum phase}} \cdot \underbrace{\frac{z^{-1}-0.9}{1-0.9z^{-1}} \cdot \frac{z^{-1}+0.9}{1+0.9z^{-1}}}_{\text{all pass}}$$

Example:- $H(z) = \dfrac{1+5z^{-1}}{1+\frac{1}{2}z^{-1}}$ decompose $H(z)$ into H_{min} and H_{ap}?

$$H(z) = \frac{1+5z^{-1}}{1+\frac{1}{2}z^{-1}} \cdot \frac{1+0.2z^{-1}}{1+0.2z^{-1}}$$



$$= (5)\underbrace{\frac{1+0.2z^{-1}}{1+0.5z^{-1}}}_{\text{min phase}} \cdot \underbrace{\frac{z^{-1}+0.2}{1+0.2z^{-1}}}_{\text{all pass}}$$

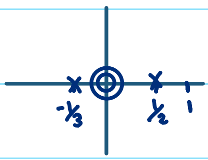
from Dr. Qadri Notes

Example:- The system function $H(z)$ for a Causal LTI system, if $H(1) = \frac{3}{4}$

Find :- 1) $H(z)$, $H(e^{j\omega})$

2) Impulse Response

3) Determine DE



1) since the system is Causal then $|z| > \frac{1}{2}$

مهمة جداً
لإنو مش شرط يكون معاملها 1 ممكن يكون أي رقم، و بنوجده من خلال initial condition

$$H(z) = \frac{Kz^{2}}{(z+\frac{1}{3})(z-\frac{1}{2})} \quad , \quad H(e^{j\omega}) = \frac{Ke^{j\omega}}{(e^{j\omega}+\frac{1}{3})(e^{j\omega}-\frac{1}{2})}$$

$$H(1) = \frac{K}{(\frac{4}{3})(\frac{1}{2})} = \frac{3}{4}$$

$$K = \frac{1}{2}$$

2) $H(z) = \dfrac{\frac{1}{2} z^2}{(z+\frac{1}{3})(z-\frac{1}{2})} = \dfrac{\frac{1}{2}}{(1+\frac{1}{3}z^{-1})(1-\frac{1}{2}z^{-1})}$

$$= \frac{A}{1+\frac{1}{3}z^{-1}} + \frac{B}{1-\frac{1}{2}z^{-1}}$$

$$A(1-\tfrac{1}{2}z^{-1}) + B(1+\tfrac{1}{3}z^{-1}) = \tfrac{1}{2}$$

if $z^{-1} = 2$, $B = 0.3$

if $z^{-1} = -3$, $A = 0.2$

$$= \frac{0.2}{1+\frac{1}{3}z^{-1}} + \frac{0.3}{1-\frac{1}{2}z^{-1}}$$

$$h[n] = 0.2\,(-\tfrac{1}{3})^n u[n] + 0.3\,(\tfrac{1}{2})^n u[n]$$

from Dr. Qadri Notes.

Example 8- Consider LTI system, Assume $y[n]$ and $x[n]$ are stable.

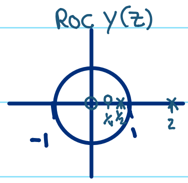
1) ROC of $Y(z)$ $\frac{1}{2} < |z| < 2$, two sided, non Causal

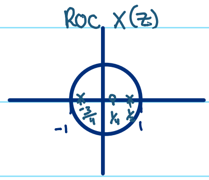
2) ROC of $X(z)$ $\frac{3}{4} < |z|$, Right sided, Causal

3) draw pole-zero diagram of $H(z)$

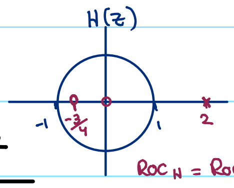
Find $H(z)$

Find $h[n]$





$$H(z) = \frac{Y(z)}{X(z)}$$



(1) $H(z) = \dfrac{(z+\frac{3}{4})(z)}{(z-2)} = \dfrac{(z^2 + \frac{3}{4}z)K}{z-2} \cdot \dfrac{z^{-2}}{z^{-2}}$

assum K=1 since there's no initial condition

$|z| > 2$

$2 > |z|$

$ROC_H = ROC_Y \cap ROC_X$

$= \frac{1}{2} < z < 2 \cap \frac{1}{2} < |z|$

$=$ $|z| < 2$
left sided

(2) $H(z) = \dfrac{1 + \frac{3}{4}z^{-1}}{z^{-1} - 2z^{-2}} = \dfrac{1 + \frac{3}{4}z^{-1}}{z^{-1}(1-2z^{-1})}$

$$= \frac{A}{z^{-1}} + \frac{B}{1-2z^{-1}}$$

$$A(1-2z^{-1}) + B(z^{-1}) = z^{-1}(1-2z^{-1})$$

if $z^{-1} = 0 \rightarrow -A = 0 \rightarrow A = 0$

if $z^{-1} = 0.5 \rightarrow \frac{1}{2}B = \frac{1}{2} - 1 \rightarrow \boxed{B = -1}$

$$= \frac{-1}{1 - 2z^{-1}}$$

$$h[n] = (2)^n u[-n-1]$$

ربنا تقبل منا إنك

أنت السميع العليم ..

Suggested Problems

5.4 , 5.5 , 5.6 , 5.8 , 5.9 , 5.10 , 5.11

5.12 , 5.22 , ~~5.28~~ , ~~5.36~~

5.36 $H(z) = \dfrac{(1-1.5z^{-1}-z^{-2})(1+0.9z^{-1})}{(1-z^{-1})(1+0.7jz^{-1})(1-0.7jz^{-1})} = \dfrac{(1-2z^{-1})(1+\frac{1}{2}z^{-1})(1+0.9z^{-1})}{(1-z^{-1})(1+0.7jz^{-1})(1-0.7jz^{-1})}$

$1+0.7jz^{-1}-z^{-1}-0.7jz^{-2}$

(1) DE?

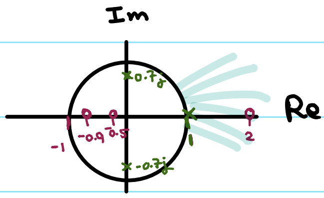
$$= \frac{1+0.9z^{-1}-1.5z^{-1}-1.35z^{-2}-z^{-2}-0.9z^{-3}}{1-\cancel{0.7jz^{-1}}+\cancel{0.7jz^{-1}}+0.49z^{-2}-z^{-1}+\cancel{0.7jz^{-2}}-\cancel{0.7jz^{-2}}-0.49z^{-3}}$$

$$= \frac{1-0.6z^{-1}-2.35z^{-2}-0.9z^{-3}}{1-z^{-1}+0.49z^{-2}-0.49z^{-3}}$$

$$y[n] = x[n]-0.6x[n-1]-2.35x[n-2]-0.9x[n-3]+y[n-1]-0.49y[n-2] + 0.49y[n-3]$$

(2) Plote the pole-zero diagram and indicate the ROC.

Since $H(z)$ is Causal so, Roc $|z|>1$



(3) sketch $H(e^{j\omega})$

Roc 1 ??

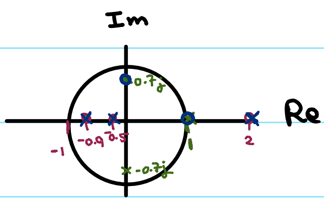
o the system is unstable, since Roc doesn't include 1.

o the inverse of the system

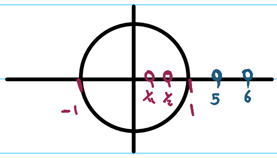
We have two Roc

① $|z|>2$ → Causal and unstable

② $\frac{1}{2}<|z|<2$ → non Causal and stable



5.37 $X(z) = \dfrac{(1-\frac{1}{2}z^{-1})(1-\frac{1}{4}z^{-1})(1-\frac{1}{5}z)}{(1-\frac{1}{6}z)}$



5.37. Consider a causal sequence $x[n]$ with the z-transform

$$X(z) = \frac{\left(1 - \frac{1}{2}z^{-1}\right)\left(1 - \frac{1}{4}z^{-1}\right)\left(1 - \frac{1}{5}z\right)}{\left(1 - \frac{1}{6}z\right)}.$$

For what values of α is $\alpha^n x[n]$ a real, minimum-phase sequence?

5.28. The system function $H(z)$ of a causal linear time-invariant system has the pole-zero configuration shown in Figure P5.28-1. It is also known that $H(z) = 6$ when $z = 1$.

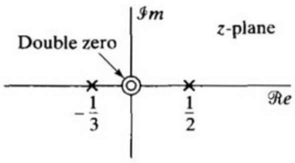


Figure P5.28-1

(a) Determine $H(z)$.
(b) Determine the impulse response $h[n]$ of the system.
(c) Determine the response of the system to the following input signals:
(i) $x[n] = u[n] - \frac{1}{2}u[n-1]$
(ii) The sequence $x[n]$ obtained from sampling the continuous-time signal

$$x(t) = 50 + 10\cos 20\pi t + 30\cos 40\pi t$$

at a sampling frequency $\Omega_s = 2\pi(40)$ rad/s

1) $$H(z) = \frac{Kz^2}{(z+\frac{1}{3})(z-\frac{1}{2})} = \frac{K}{(1+\frac{1}{3}z^{-1})(1-\frac{1}{2}z^{-1})}$$

$$H(1) = \frac{k}{(1+\frac{1}{3})(\frac{1}{2})} = 6 \quad , \boxed{K=4}$$

$$H(z) = \frac{A}{1+\frac{1}{3}z^{-1}} + \frac{B}{1-\frac{1}{2}z^{-1}}$$

$$A(1-\tfrac{1}{2}z^{-1}) + B(1+\tfrac{1}{3}z^{-1}) = 4$$

if $z^{-1} = 2 \rightarrow \frac{5}{3}B = 4 \rightarrow B = 12/5$

if $z^{-1} = -3 \rightarrow \frac{5}{2}A = 4 \rightarrow A = 8/5$

$$H(z) = \frac{8/5}{1+\frac{1}{3}z^{-1}} + \frac{12/5}{1-\frac{1}{2}z^{-1}} \quad , \quad |z| > \tfrac{1}{2}$$

2) $$h[n] = 8/5\,(-\tfrac{1}{3})^n u[n] + \frac{12}{5}(\tfrac{1}{2})^n u[n]$$

3) $$Y(z) = H(z) \cdot X(z)$$

$$= \left[\frac{4}{(1+\frac{1}{3}z^{-1})\cancel{(1-\frac{1}{2}z^{-1})}}\right] \cdot \frac{\cancel{(1-\frac{1}{2}z^{-1})}}{(1-z^{-1})}$$

$$= \frac{4}{(1+\frac{1}{3}z^{-1})(1-z^{-1})}$$

$$X(z) = \frac{1}{1-z^{-1}} + \frac{-\frac{1}{2}z^{-1}}{1-z^{-1}}$$

$$X(z) = \frac{1-\frac{1}{2}z^{-1}}{1-z^{-1}} \quad , |z| > 1$$

$$= \frac{A}{(1+\frac{1}{3}z^{-1})} + \frac{B}{(1-z^{-1})}$$

$$= A(1-z^{-1}) + B(1+\frac{1}{3}z^{-1}) = Y$$

if $z^{-1}=1 \rightarrow B=3$

if $z^{-1}=-3 \rightarrow A=1$

$$Y[z] = \frac{1}{1+\frac{1}{3}z^{-1}} + \frac{3}{1-z^{-1}}$$

$$y[n] = (-\tfrac{1}{3})^n u[n] + 3\,u[n] \quad , \quad |z|>1$$

5.22

5.22. Consider a causal linear time-invariant system with system function

$$H(z) = \frac{1 - a^{-1}z^{-1}}{1 - az^{-1}},$$

where a is real.

(a) Write the difference equation that relates the input and the output of this system.

(b) For what range of values of a is the system stable?

(c) For $a = \frac{1}{2}$, plot the pole–zero diagram and shade the region of convergence.

(d) Find the impulse response $h[n]$ for the system.

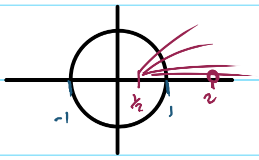
(e) Show that the system is an all-pass system, i.e., that the magnitude of the frequency response is a constant. Also, specify the value of the constant.

1) $y[n] = x[n] - a^{-1}x[n-1] + ay[n-1]$

2) $\dfrac{1-a^{-1}z^{-1}}{1-az^{-1}} \cdot \dfrac{z}{z} = \dfrac{z - a^{-1}}{z-a}$, $z = a \rightarrow$ Pole

$z = a^{-1} \rightarrow$ zero

if $a = \frac{1}{2}$



, if $a<1 \longrightarrow |z| > a \rightsquigarrow$ to be stable

if $a \geq 1 \longrightarrow \underline{|z| < a}$ $\rightsquigarrow$ couldn't be stable since the system is causal

$\underline{\text{not ROC}}$

$$\frac{z - c^*}{1 - \frac{1}{c}z^{-1}}$$

$$H(z) = \frac{1}{1-az^{-1}} - \frac{a^{-1}z^{-1}}{1-az^{-1}}$$

$$h[n] = a^n u[n] - a^{-1}(a)^{n-1}u[n-1]$$

$$= a^n u[n] - (a)^{n-2}u[n-1]$$