Amplitude Modulation

Normal Amplitude Modulation
Time and Frequency Domain Characteristics
Lecture Outline

Lecture 1:

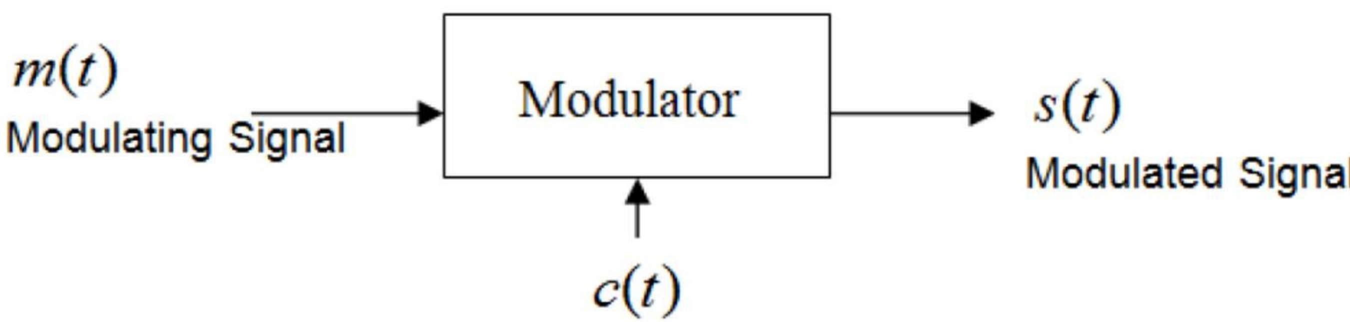
- Why do we need modulation?
- Define the normal AM signal
- The normal AM in the time and frequency domains
- Power efficiency
- Effect of the modulation index

Normal Amplitude Modulation

Modulation: is the process by which some characteristic of a high frequency signal $c(t)$, called the carrier, is varied in accordance with a message signal $m(t)$. A common form of the *carrier*, in the case of continuous wave modulation, is a sinusoidal signal

$$c(t) = A_C \cos(2\pi f_C t + \varphi)$$

The three parameters of c(t), amplitude, phase, and frequency may be varied in accordance of the message signal resulting in amplitude modulation, phase modulation, and frequency modulation.



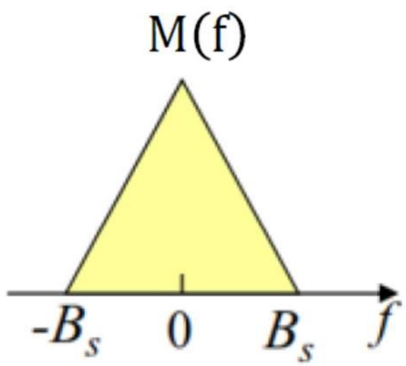
Why Modulation?

- There are several reasons why modulation is needed in a communication system.
- **Physical antenna size**: For efficient transmission of a signal, the antenna length should be about λ/4, where λ is the wavelength.
- For example, let the frequency of the message be 3KHz (**audio signal**)
- The wavelength $\lambda = \frac{c}{f} = \frac{3.0X10^8}{3.0X10^3} = 10^5 m = 100km.$
- Hence, the size of the antenna should be around (**λ/4 =25 km**), which is not at all practical.
- Now, let us find the antenna length in the GSM band (1000 MHz):
- $\lambda = \frac{c}{f} = \frac{3.0X10^8}{1000X10^6} = 0.3m$
- Hence, the size of the antenna should be around (**λ/4 = 7.5 cm**), which can easily fit into a mobile device. This is a challenging design issue in modern mobile technology.

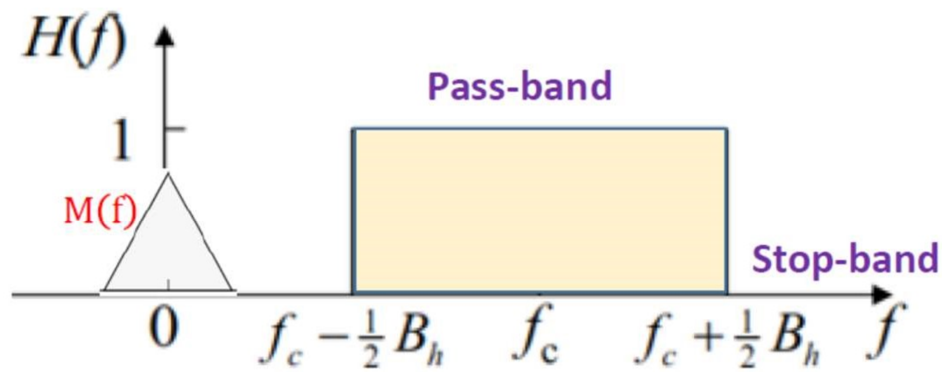
Why Modulation?

- **Band-pass channels**: Most, if not all, channels over which messages are transmitted are band-pass, while messages are low-pass signals. Hence, direct transmission of messages over band-pass channels would result in high attenuation (essentially no received signal). This necessitates shifting the message spectrum to coincide with the channel bandwidth.

$$Y(f) = M(f)H(f)$$



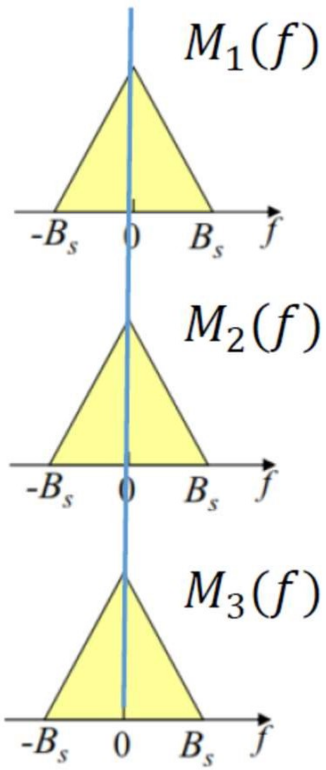
Baseband Message

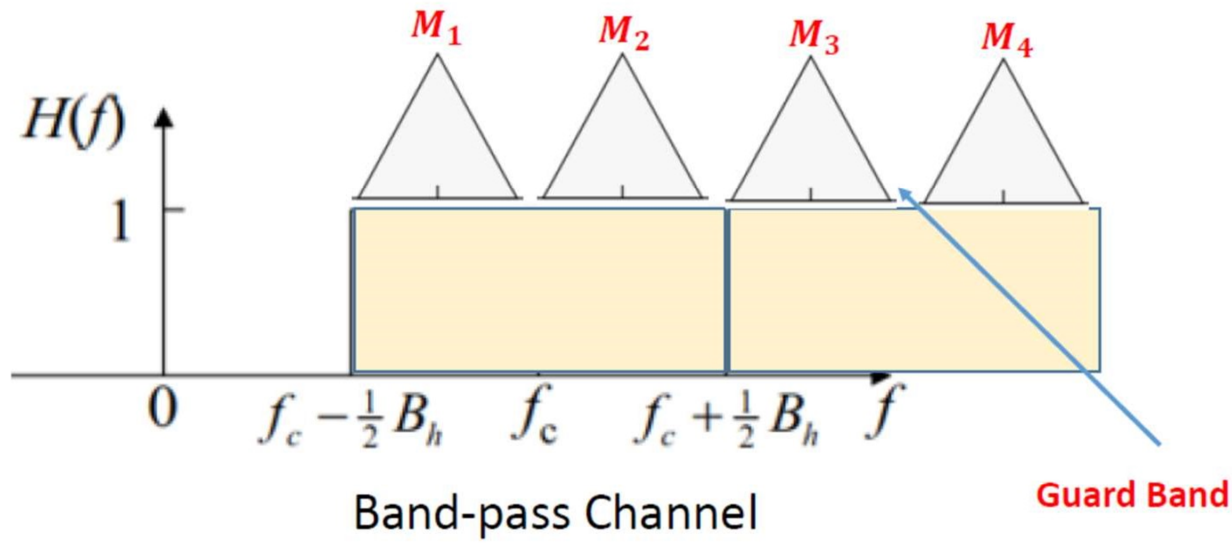


Band-pass Channel

Why Modulation?

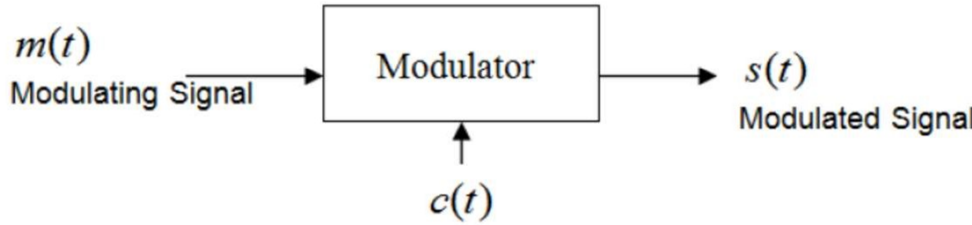
- **Multiplexing**: Modulation allows multiple users to use the same channel by assigning each user a portion of the available bandwidth without interfering with other users.





Amplitude modulation

- **Amplitude modulation (AM)** is defined as the process in which the amplitude of the carrier $c(t)$ is varied linearly with $m(t)$.
- Three types of amplitude modulation will be considered in detail. These are
  - Normal amplitude modulation
  - Double sideband suppressed carrier modulation (DSB-SC)
  - Single sideband modulation (SSB-SC)



- The baseband (message) signal $m(t)$ is referred to as the **modulating signal** and the result of the modulation process is referred to as the **modulated signal** $s(t)$.
- **Modulation** is performed at the transmitter
- **Demodulation**, which is the process of extracting $m(t)$ from $s(t)$, is performed at the receiver.

Normal Amplitude modulation

A *normal AM* signal is defined as: $s(t) = A_c(1 + k_a m(t)) \cos(2\pi f_c t)$

where, k_a is the sensitivity of the AM modulator (units in 1/volt).

$s(t)$ can be also be written in the form: $s(t) = A(t) \cos 2\pi f_C t$

The **envelope** of $s(t)$ is defined as

$$|A(t)| = A_C \, |1 + k_a m(t)|$$

Notice that the envelope of $s(t)$ has the same shape as $m(t)$ provided that:

1. $(1 + k_a m(t)) \geq 0$ Or equivalently, $|k_a m(t)| \leq 1$.
2. Over-modulation occurs when $|k_a m(t)| > 1$ resulting in envelope distortion
3. $f_C >> W$, where W is the bandwidth of $m(t)$. f_C has to be at least $10W$. This ensures the formation of an envelope, whose shape resembles the message signal.

Spectrum of the Normal AM Signal

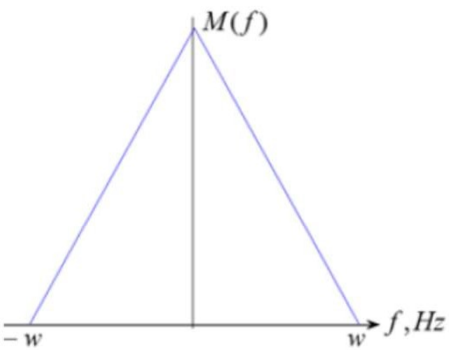
Let the Fourier transform of $m(t)$ be as shown (the B.W of $m(t) = W$ Hz).

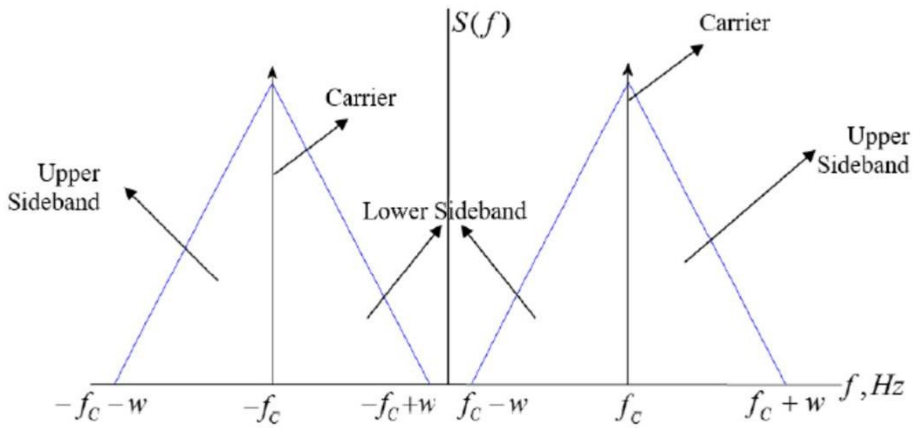
$$s(t) = A_C(1 + k_a m(t))\cos 2\pi f_C t \qquad \text{(dc + message)*carrier}$$

$$s(t) = A_C \cos 2\pi f_C t + A_C k_a m(t)\cos 2\pi f_C t \qquad \text{(carrier + message*carrier)}$$

Taking the Fourier transform, we get

$$S(f) = \frac{A_C}{2}\delta(f - f_C) + \frac{A_C}{2}\delta(f + f_C) + \frac{A_C k_a}{2} M(f - f_C) + \frac{A_C k_a}{2} M(f + f_C)$$





Remarks

a. The baseband spectrum $M(f)$, of the message has been shifted to the bandpass region centered around the carrier frequency f_c.

b. The spectrum $S(f)$ consists of two sidebands (upper sideband and lower sideband) and a carrier.

c. The transmission bandwidth of $s(t)$ is:

$B.W. = (f_c + W) - (f_c - W) = \mathbf{2W}$ which is twice the message bandwidth.

Lecture 2

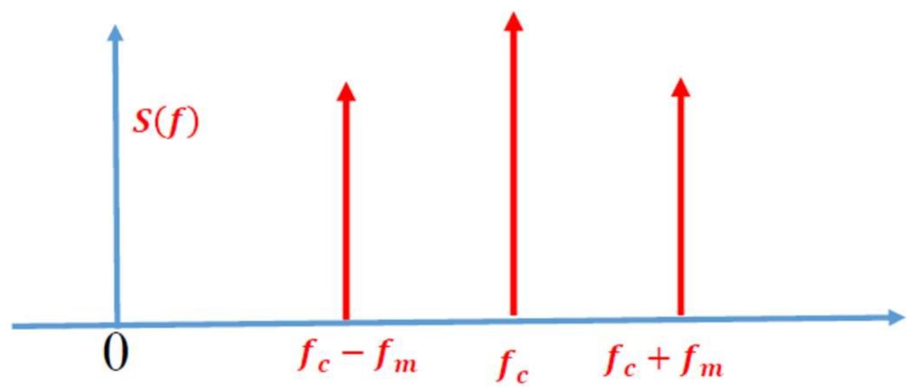
Spectrum of the Normal AM: Sinusoidal Modulation

Example: Consider the normal AM with sinusoidal modulation, where $c(t) = A_c\cos(2\pi f_c t)$; $m(t) = A_m\cos(2\pi f_m t)$; plot m(t), c(t), s(t) and find their spectrum.

Solution: $s(t) = A_c(1 + k_a m(t))\cos 2\pi(f_c)t$

- $s(t) = A_c cos(2\pi f_c t) + A_c k_a A_m\, cos(2\pi f_c t)\, cos(2\pi f_m t)$;
- $s(t) = A_c cos(2\pi f_c t) + \frac{A_c A_m k_a}{2} cos(2\pi(f_c + f_m)t) + \frac{A_c A_m k_a}{2} cos(2\pi(f_c - f_m)t)$
- $S(f) = \Im\{s(t)\}$
- $M(f) = \frac{A_m}{2}\delta(f - f_m) + \frac{A_m}{2}\delta(f + f_m)$
- The next figure shows all the plots

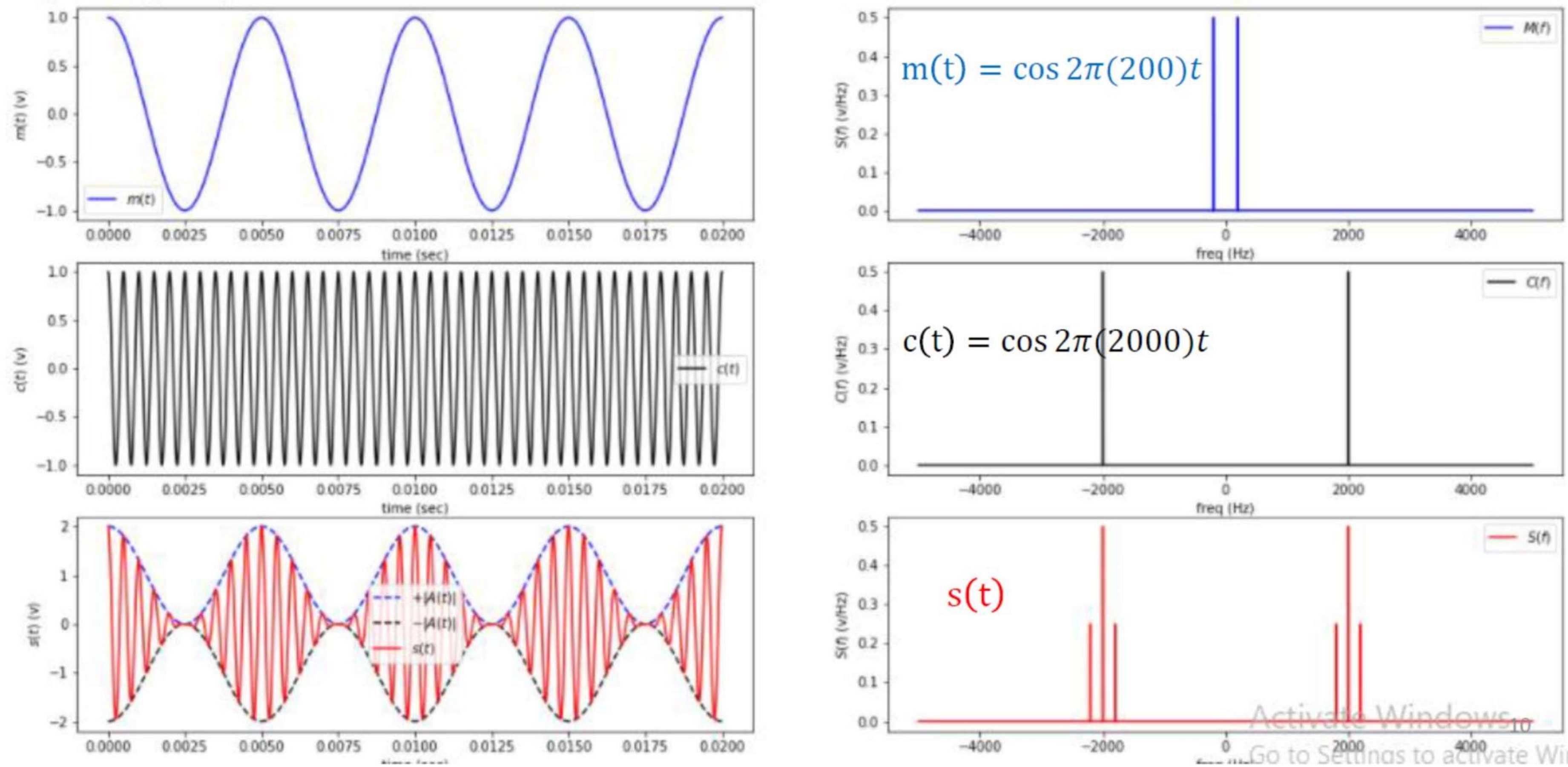
  when $f_m = 200$ Hz and $f_c = 000$ Hz
  2000



Spectrum of the Normal AM Signal

An AM signal in the time and frequency domains.

$s(t) = (1 + k_a m(t))\cos 2\pi(2000)t$ $m(t) = \cos 2\pi(200)t$ $k_a = 1.0$ $\mu = A_m k_a = 1.0$



Power Efficiency of Normal AM

The ***power efficiency*** of a normal AM signal is defined as:

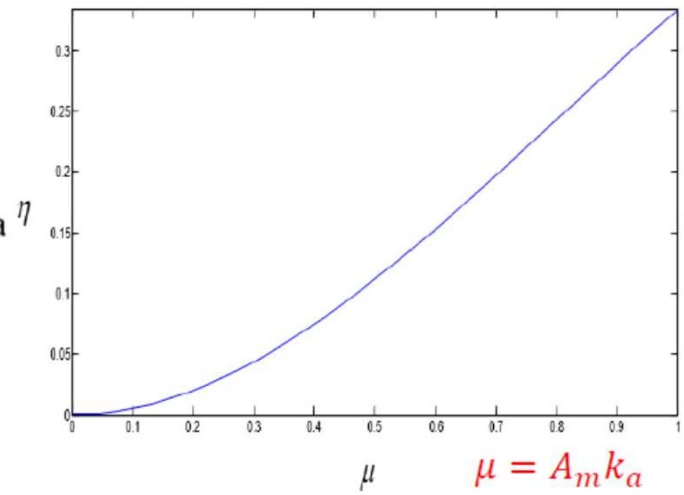
$$\eta = \frac{power\ \ in\ \ the\ \ sidebands}{power\ in\ the\ sidebands + power\ in\ the\ carrier}$$

Now, we find the power efficiency of the AM signal for the single-tone modulating signa $m(t) = A_m \cos(2\pi f_m t)$. Let $\mu = A_m k_a$, then $s(t)$ can be expressed as

$$s(t) = A_C(1 + \mu\cos 2\pi f_m t)\cos 2\pi f_C t$$

$$s(t) = A_C \cos 2\pi f_C t + A_C \mu \cos 2\pi f_C t \cos 2\pi f_m t$$

$$s(t) = A_C \cos 2\pi f_C t + \frac{A_C \mu}{2}\cos 2\pi (f_C + f_m)t + \frac{A_C \mu}{2}\cos 2\pi (f_C - f_m)t$$



$\mu = A_m k_a$

$$\text{Power in carrier } = \frac{A_C^{\ 2}}{2}$$

$$\text{Power in sidebands } = \frac{1}{2}\left(\frac{A_C \mu}{2}\right)^2 + \frac{1}{2}\left(\frac{A_C \mu}{2}\right)^2 = \frac{1}{4} A_C^{\ 2}\mu^2$$

Therefore,

$$\eta = \frac{\frac{1}{4}A_C^{\ 2}\mu^2}{\frac{A_C^{\ 2}}{2} + \frac{1}{4}A_C^{\ 2}\mu^2} = \frac{\mu^2}{2+\mu^2} \qquad ; \quad 1 \geq \mu \geq 0$$

- **The maximum efficiency occurs when μ=1, i.e. for a 100% modulation index. The corresponding maximum efficiency is only η = 1/3. As a result, 2/3 of the transmitted power is wasted in the carrier**
- **Remark: Normal AM is not an efficient modulation scheme in terms of the utilization of the transmitted power.**

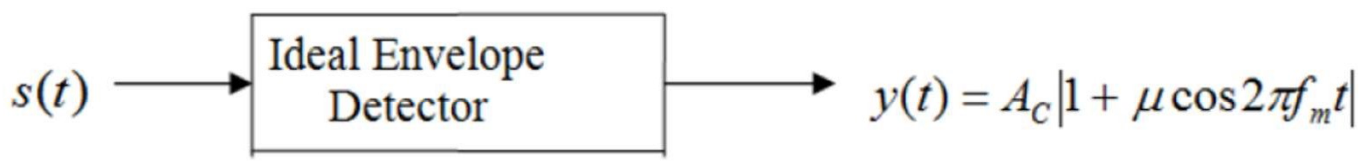
Amplitude Modulation: AM Modulation Index

Consider the AM signal: $s(t) = A_C\left(1 + k_a m(t)\right)\cos 2\pi f_C t = A(t)\cos 2\pi f_C t$

The envelope of $s(t)$ is:

$$|A(t)| = A_C\left|1 + k_a m(t)\right|$$

The following block diagram illustrate the envelope detection process for a sinusoidal message signal.



$$\mu = A_m k_a$$

To avoid distortion, the following condition must hold

$$\left(1 + k_a m(t) \geq 0 \qquad \text{or} \qquad \left|k_a m(t)\right| \leq 1\right.$$

The modulation index of an AM signal is defined as:

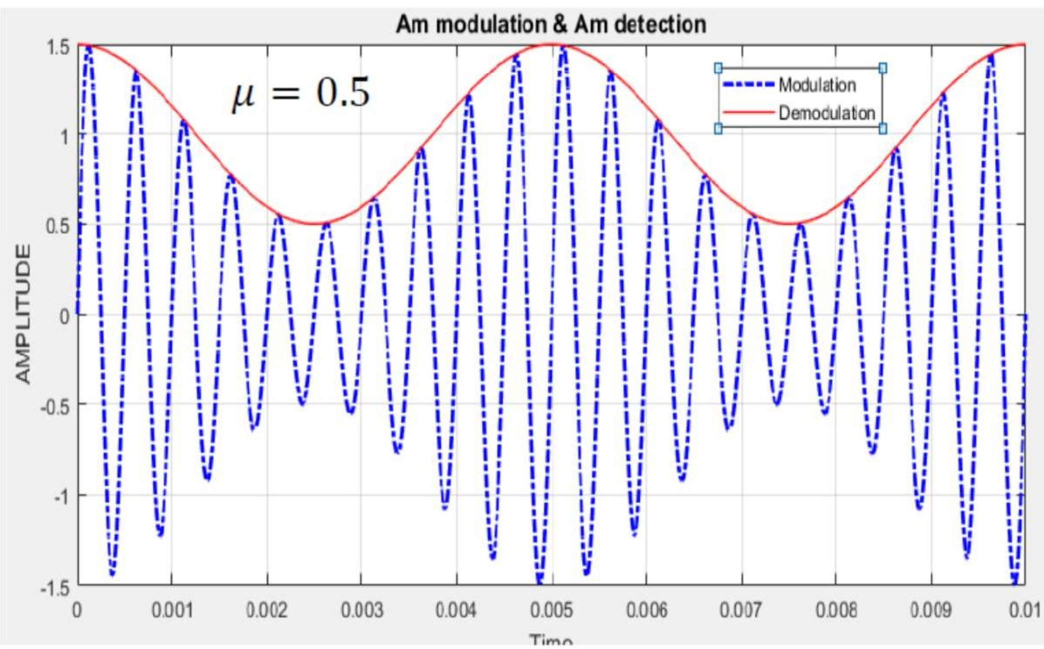
$$Modulation\ \ Indcx\ \ (M.I) = \frac{|A(t)|_{max} - |A(t)|_{min}}{|A(t)|_{max} + |A(t)|_{min}}$$

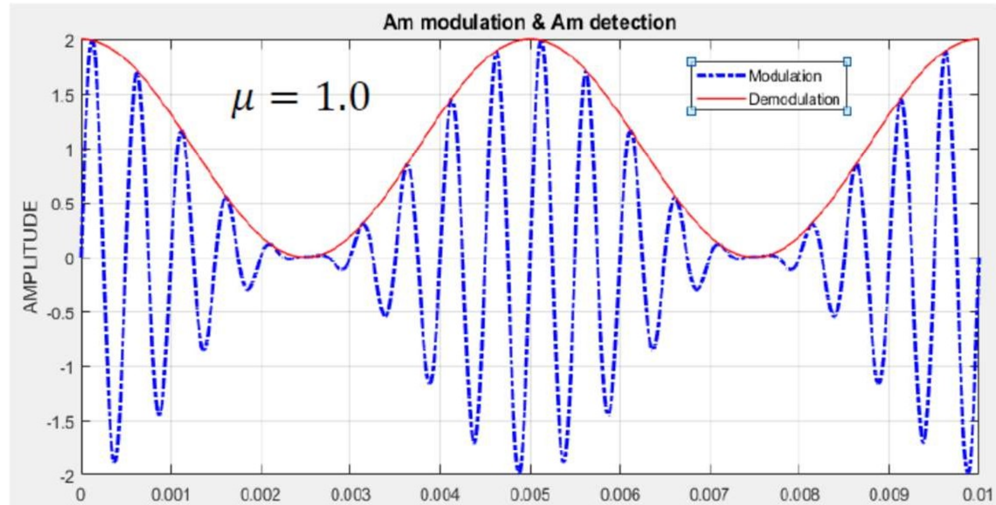
The modulation index μ (modulation depth) of an amplitude modulated signal is defined as the measure or extent of amplitude variation about an un-modulated carrier. In other words the amplitude modulation index describes the amount by which the modulated carrier envelope varies about the static level.

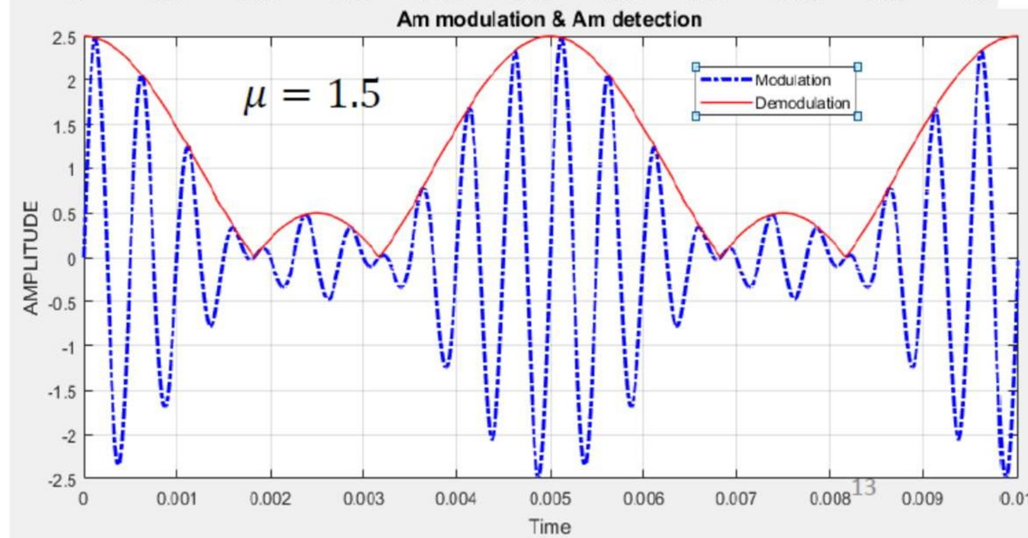
Amplitude Modulation: Effect of the Modulation Index

$$s(t) = (1 + \mu m(t))\cos 2\pi(2000)t$$

$$m(t) = \cos 2\pi(200)t \qquad \mu = k_a|m(t)|$$







Amplitude Modulation: Multi-tone Modulation

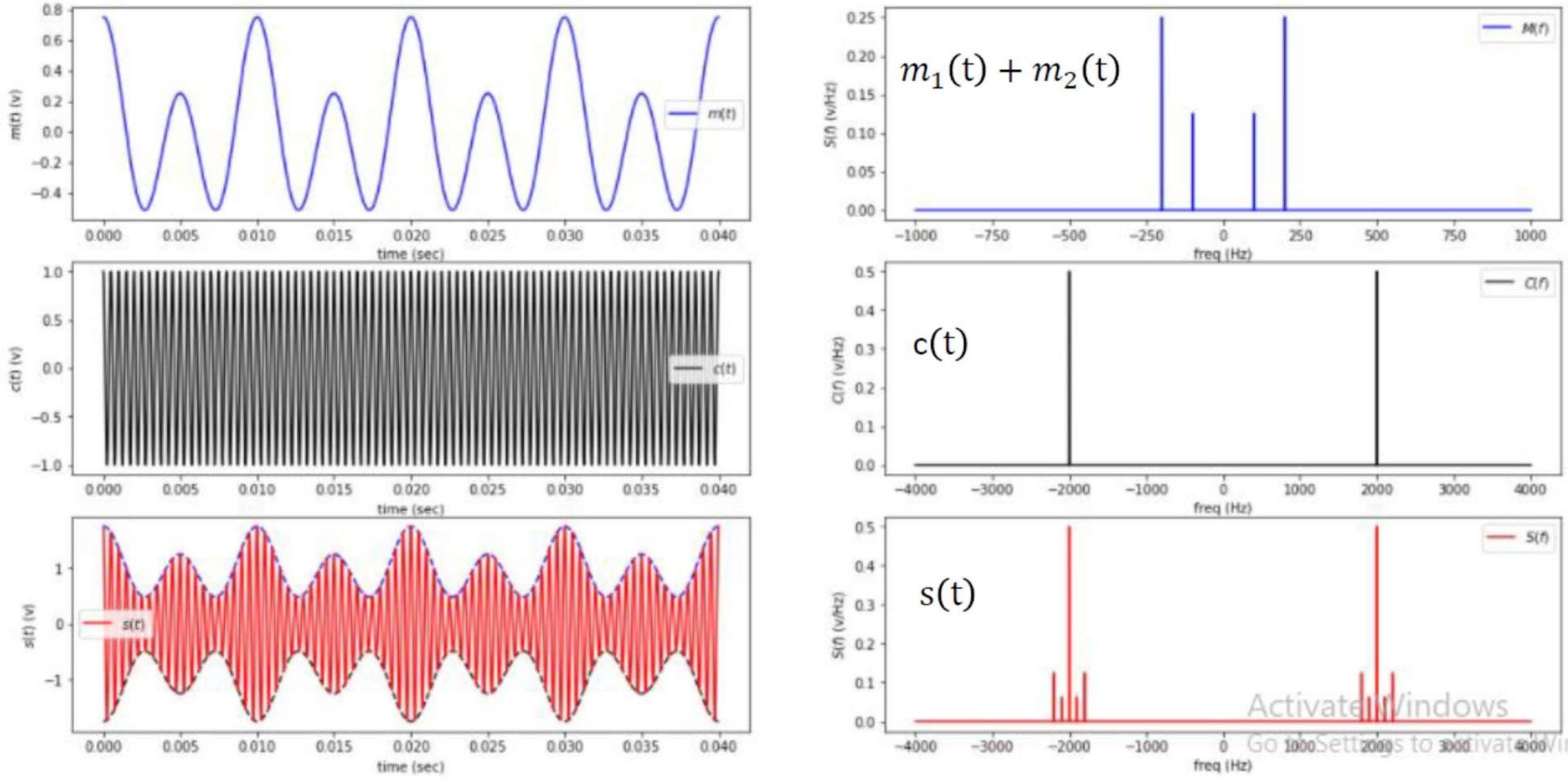
$s(\text{t}) = (1 + m_1(\text{t}) + m_2(\text{t}))\cos 2\pi(2000)t$

$k_a = 1; A_c = 1$

$m_1(\text{t}) = 0.5\cos 2\pi(200)t$

$m_2(\text{t}) = 0.25\cos 2\pi(100)t$

A non-envelope distortion case for multitoned transmission



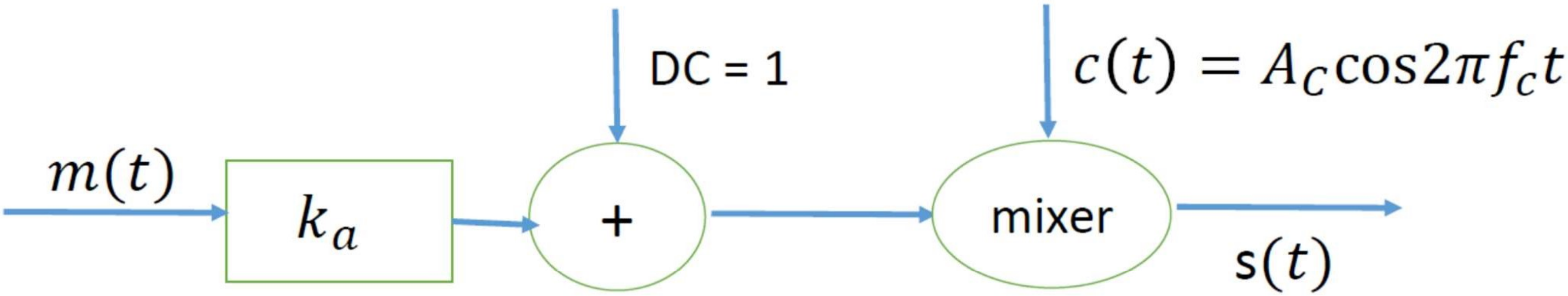
Normal Amplitude Modulation Generation and Demodulation Lecture Outline

Lecture 3

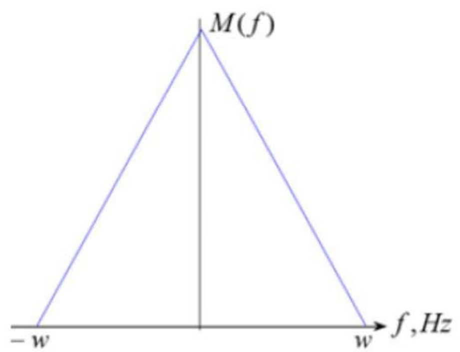
- Last Lecture:
  - Why do we need modulation?
  - Define the normal AM signal
  - The normal AM in the time and frequency domains
  - Power efficiency
  - Effect of the modulation index
- This Lecture:
  - AM generation techniques: the switching modulator
  - The envelope detector

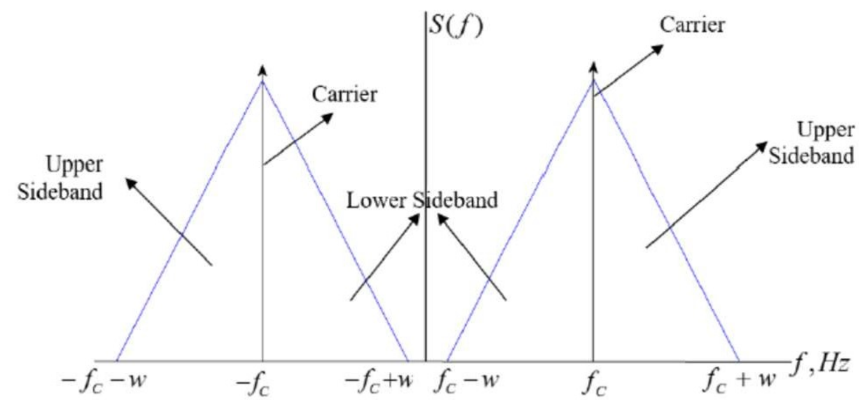
Normal Amplitude Modulation: Standard Form

$$s(t) = A_C(1 + k_a m(t))\cos 2\pi f_c t$$



$$S(f) = \frac{A_C}{2}\delta(f - f_C) + \frac{A_C}{2}\delta(f + f_C) + \frac{A_C k_a}{2}M(f - f_C) + \frac{A_C k_a}{2}M(f + f_C)$$





Generation of a Normal Amplitude Modulation: the Switching Modulator

Assume that the carrier $c(t)$ is large in amplitude so that the diode –shown in the figure below- acts like an ideal switch.

When $m(t)$ is small compared to $|c(t)|$,

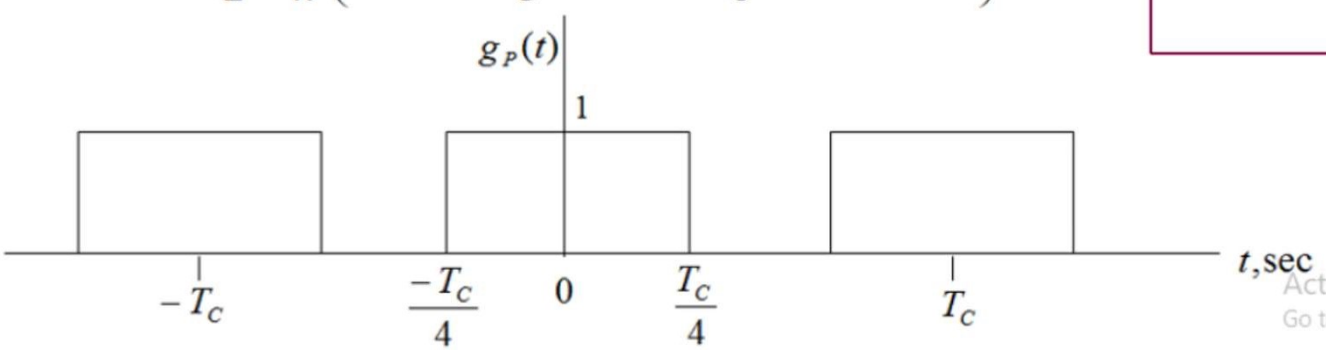
$$V_2(t) = \begin{cases} m(t) + A_C \cos \omega_C t & ; \quad c(t) > 0 \\ 0 & ; \quad c(t) < 0 \end{cases}$$

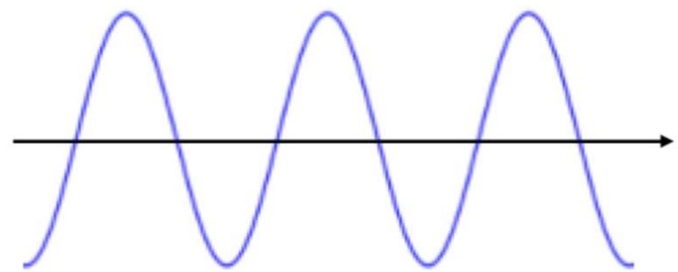
Here, the diode opens and closes at a rate equals to the carrier frequency f_C. This switching mechanism can be modeled as:

$$V_2(t) = [A_C \cos \omega_C t + m(t)] g_P(t)$$

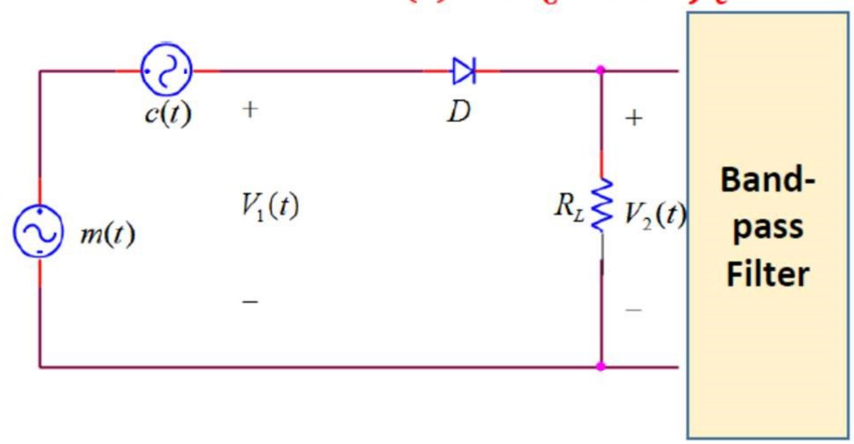
where $g_P(t)$ is the periodic square function, expanded in a Fourier series as

$$g_P(t) = \frac{1}{2} + \frac{2}{\pi}\left(\cos \omega_C t - \frac{1}{3}\cos 3\omega_C t + \frac{1}{5}\cos 5\omega_C t + \ldots \right)$$





$c(t) = A_C cos 2\pi f_c t$



Generation of a Normal Amplitude Modulation: the Switching Modulator

$$V_2(t) = [A_C \cos\omega_C t + m(t)]\left(\frac{1}{2}\right) + \left(\frac{2}{\pi}\cos\omega_C t\right)(A_C \cos\omega_C t + m(t)) - \left(\frac{2}{3\pi}\cos 3\omega_C t\right)$$
$$(m(t) + A_C \cos\omega_C t) + \ldots$$

$$V_2(t) = \frac{m(t)}{2} + \frac{A_C}{2}\cos\omega_C t + \frac{2}{\pi}m(t)\cos\omega_C t + \frac{A_C}{\pi} + \frac{A_C}{\pi}\cos 2\omega_C t + \frac{2}{3\pi}m(t)\cos 3\omega_C t$$
$$+\frac{2}{3\pi}A_C \cos 2\omega_C t + \ldots$$

$$\boldsymbol{V_2(t) = [A_C \cos\omega_C t + m(t)]\, g_P(t)}$$

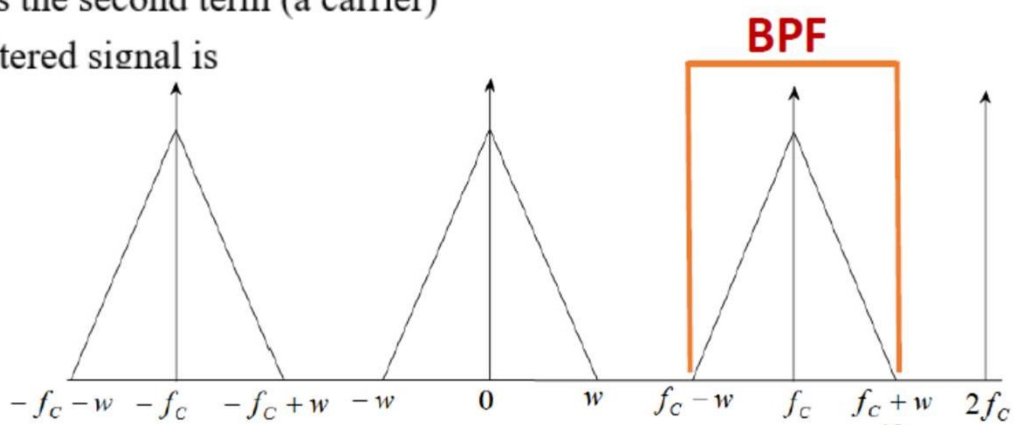
$$\boldsymbol{g_P(t) = \frac{1}{2} + \frac{2}{\pi}\left(\cos\omega_C t - \frac{1}{3}\cos 3\omega_C t + \frac{1}{5}\cos 5\omega_C t + \ldots\right)}$$

A band-pass filter with a bandwidth $2w$, centered at f_C, passes the second term (a carrier) and the third term (a carrier multiplied by the message). The filtered signal is

$$s(t) = \frac{A_C}{2}\cos\omega_C t + \frac{2}{\pi}m(t)\cos\omega_C t$$

$$s(t) = \frac{A_C}{2}\left(1 + \frac{4}{\pi A_C}m(t)\right)\cos\omega_C t \quad \text{; Desired AM signal.}$$

Modulation Index $= M.I = \dfrac{4}{\pi A_C}|m(t)|_{\max}$

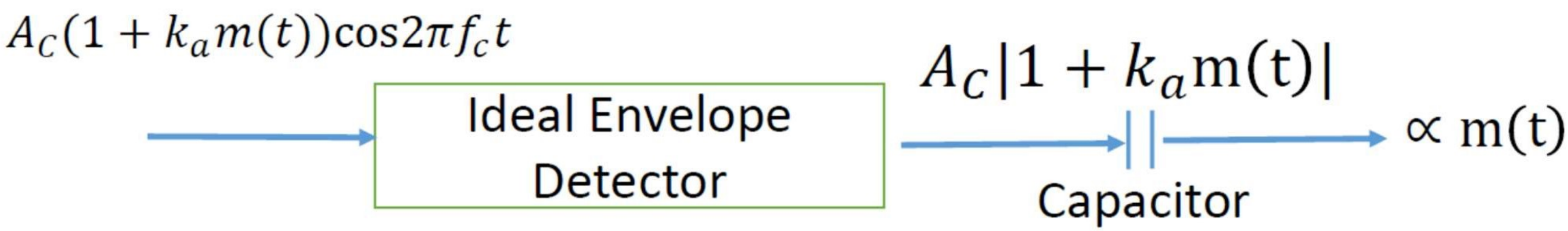


Demodulation of a Normal Amplitude Modulation: Envelope Detection

The Ideal Envelope Detector: The ideal envelope detector responds to the envelope of the signal, but is insensitive to phase variation. If

$$s(t) = A_C\left(1 + k_a m(t)\right)\cos 2\pi f_C t$$

then, the output of the ideal envelope detector is $y(t) = A_C\left|1 + k_a m(t)\right|$



To avoid envelope distortion,
$|1 + k_a \mathrm{m(t)}|$ should equal $(1 + k_a m(t))$
That is, $(1 + k_a m(t)) \geq 0$ for all time

Example: single tone modulation (under-modulation)

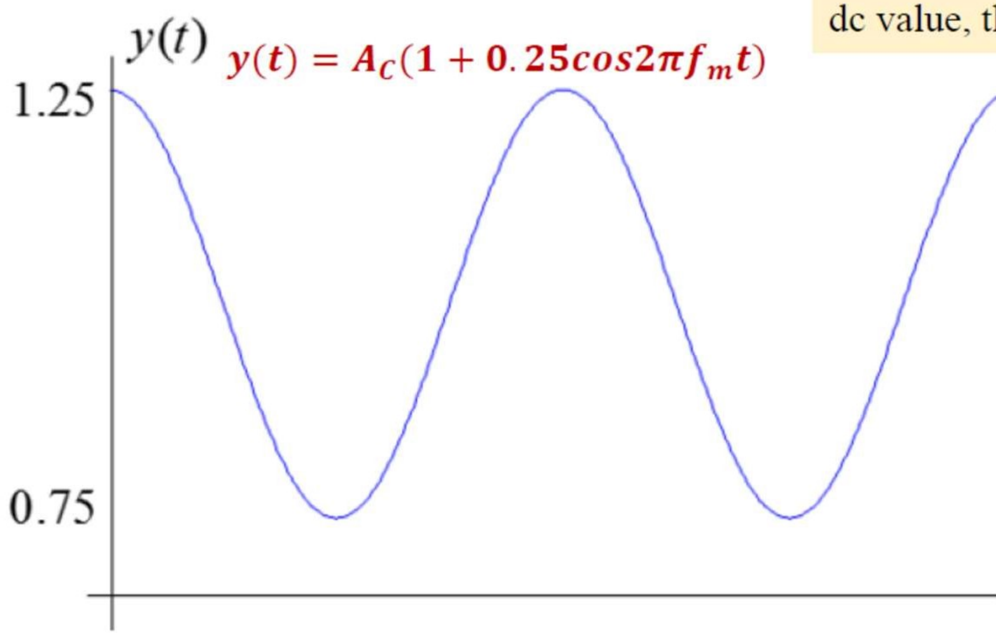
Example: Let $s(t) = A_C(1+\mu\cos 2\pi f_m t)\cos 2\pi f_C t$ be applied to an ideal envelope detector, sketch the demodulated signal for $\mu = 0.25, 1.0, \mathit{and}\ 1.25$.

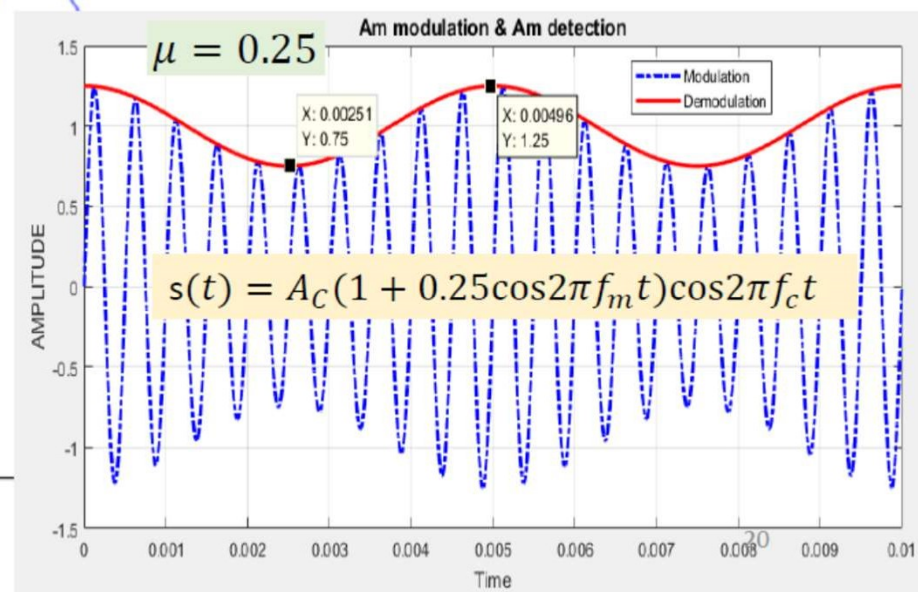
As was mentioned before, the output of the envelope detector is $y(t) = A_C\left|1+\mu\cos 2\pi f_m t\right|$

<u>Case1 :</u> $(\mu = 0.25)$

$$y(t) = A_C\left|1+0.25\cos 2\pi f_m t\right|$$

Here, m(t) can be extracted without distortion. $(1+k_a m(t)) \geq 0$ for all time. $|1+k_a m(t)| = (1+k_a m(t))$. By removing the dc value, the output will be proportional to the message

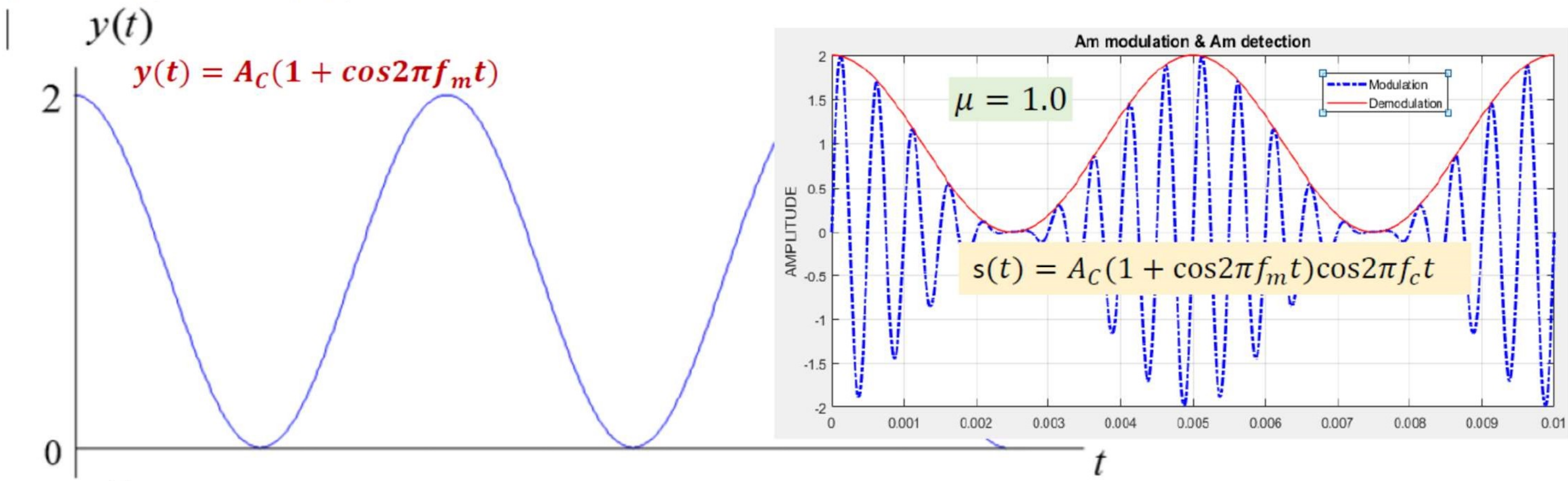




Example: single tone modulation (100% - modulation)

Case2: (μ = 1.0)

$$y(t) = A_C|1 + \cos 2\pi f_m t|$$

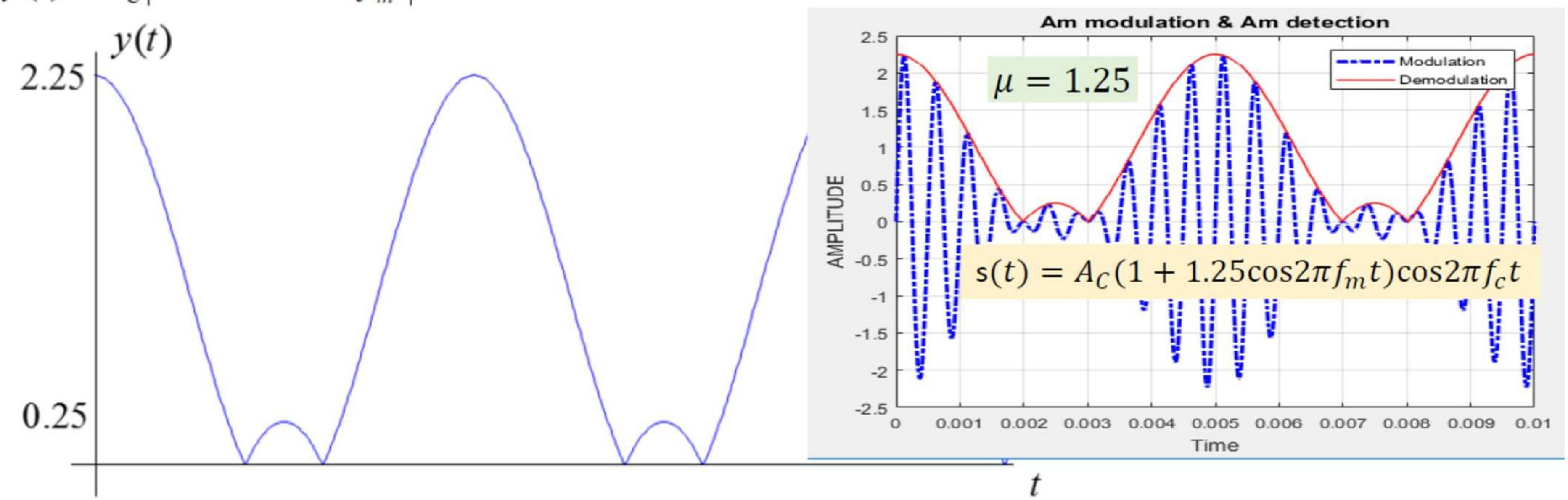


Here, m(t) can be extracted without distortion. $(1 + k_a m(t)) \geq 0$ for all time. That is, $|1 + k_a \mathrm{m(t)}| = (1 + k_a m(t))$. By removing the dc value, the output will be proportional to the message

Example: single tone modulation (over-modulation)

Case3: ($\mu = 1.25$)

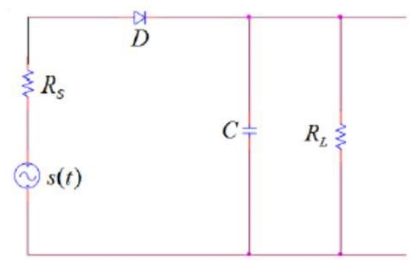
$$y(t) = A_C\left|1 + 1.25\cos 2\pi f_m t\right|$$

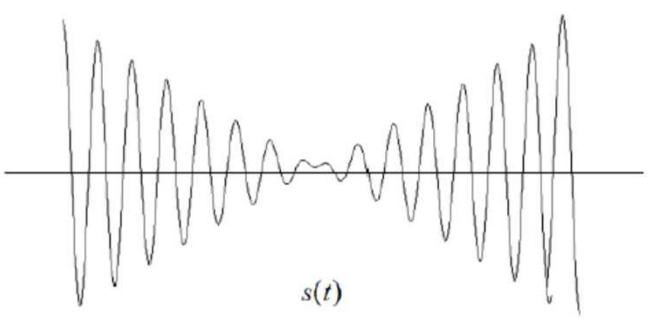


Here, m(t) cannot be extracted without distortion. The shape of the envelope is not the same as the shape of the message. $(1 + k_a m(t))$ fails to remain positive for all time. $|1 + k_a \mathrm{m(t)}| \neq (1 + k_a m(t))$

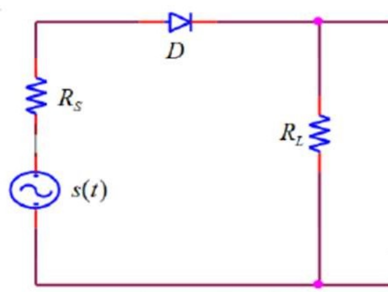
A Simple Practical Envelope Detector

- A practical envelope detector consists of a diode followed by an RC circuit that forms a low pass filter.
- During the positive half cycle of the input, the diode is forward biased and C charges rapidly to the peak value of the input.
- When s(t) falls below the maximum value, the diode becomes reverse biased and C discharges slowly through R_L.
- To follow the envelope of s(t), the circuit time constant should be chosen such that : $\frac{1}{f_C} << R_L C << \frac{1}{W}$ where W is the message B.W and f_C is the carrier frequency.
- When a capacitor C is added to a half wave rectifier circuit, the output follows the envelope of s(t). The circuit output (with C connected) follows a curve that connects the tips of the positive half cycles, which is the envelope of the AM signal.

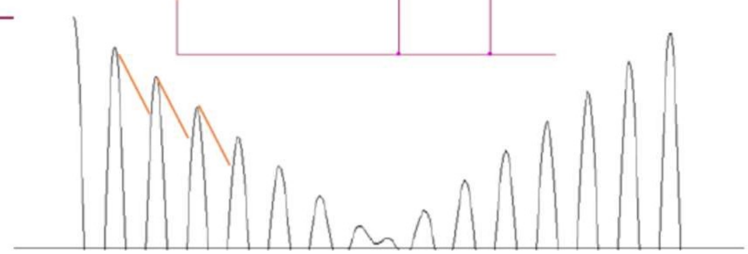




$s(t)$



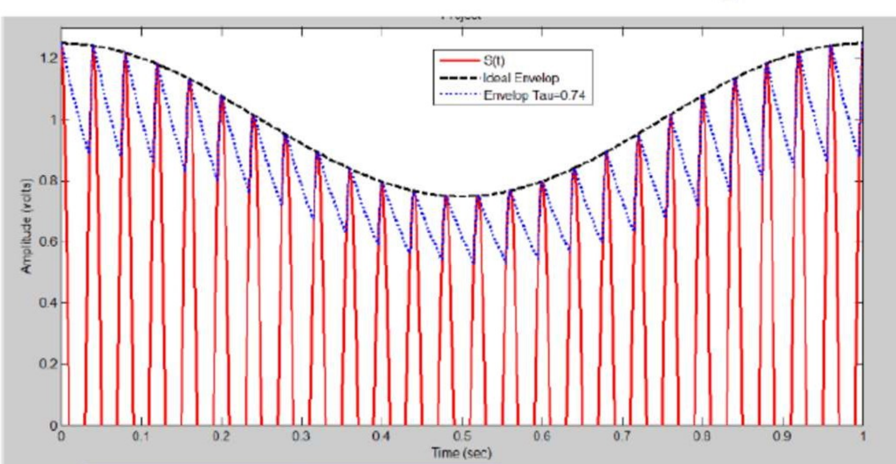
Half – Wave Rectifier



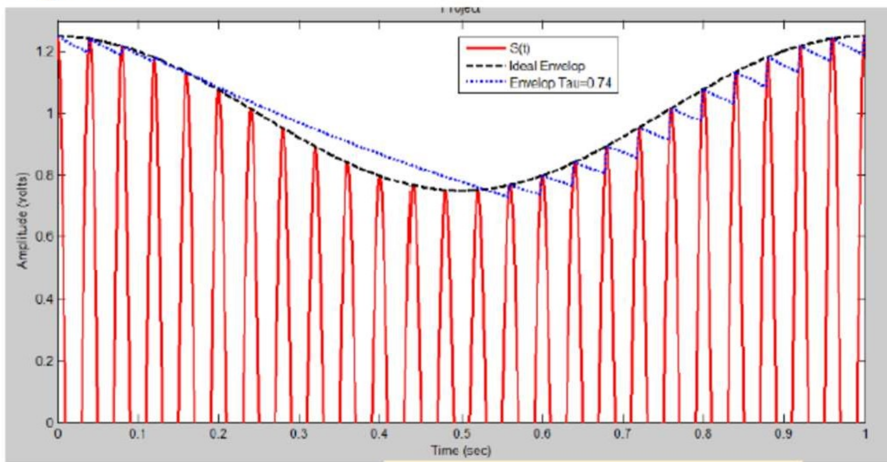
Output of half wave rectifier (without C)

A Simple Practical Envelope Detector: Effect of the Time Constant

Consider the AM signal $s(t) = A_c[1 + \mu\cos(2\pi f_m t)]\cos(2\pi f_c t)$ that is demodulated using the envelope detector. Assume $R_s = 0$, $\mu = 0.25$, $A_c = 1$, $f_m = 1\text{Hz}$, $f_c = 25\text{Hz}$. We show the effect of the time constant $\tau = R_L C$ on the detected signal

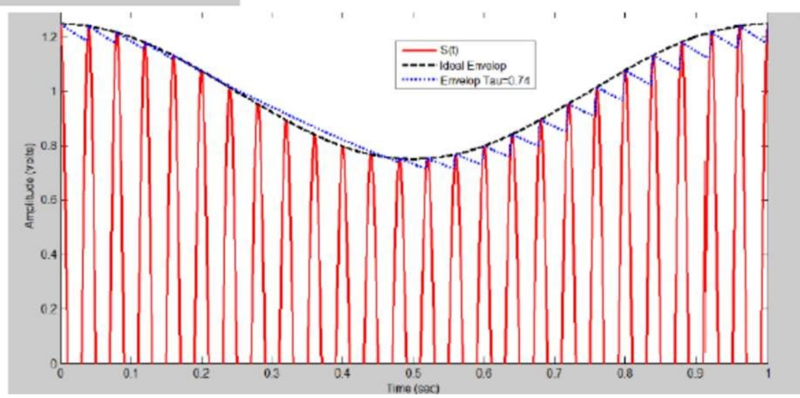


RC output when tau 0.1



RC output when tau 0.9

$T_C << R_L C << T_m$



RC optimum tau 0.74

Double Sideband Suppressed Carrier (DSB-SC) Modulation: Lecture Outline

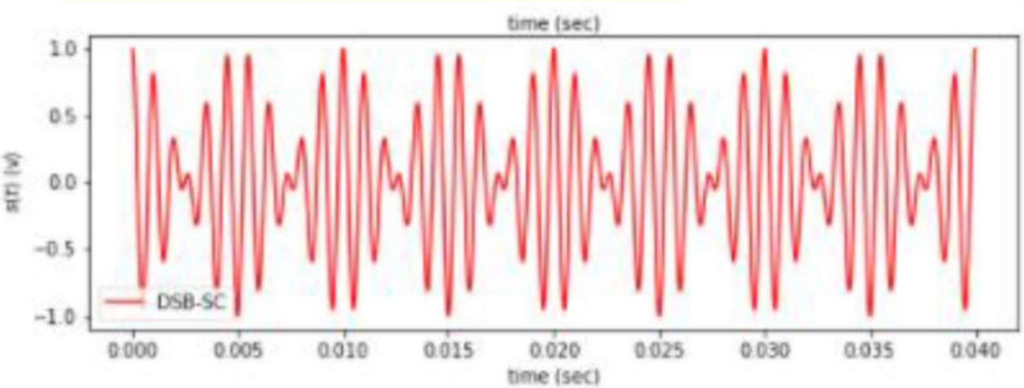
Lecture 4

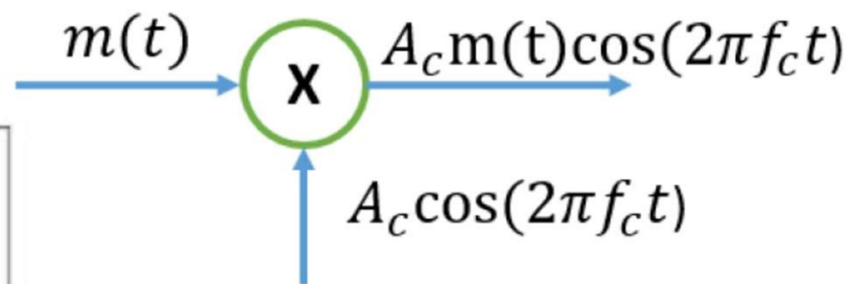
- In this lecture, we consider a second type of AM modulation called DSB-SC.
- We analyze this modulation technique in the time and frequency domains.
- Consider the generation and demodulation techniques.
- Study the effect of non-coherence in the phase and frequency of the locally generated carrier at the receiver on the demodulated signal.

Double Sideband Suppressed Carrier (DSB-SC) Modulation

- A DSB-SC signal is an amplitude-modulated signal that has the form
- $\mathbf{s(t) = A_c m(t) \cos(2\pi f_c t)}$, where
- $c(t) = A_c \cos(2\pi f_c t)$: is the carrier signal
- $m(t)$: is the baseband message signal
- $f_c >> W$, W is the bandwidth of the baseband message signal $m(t)$

FIGURE: $\mathbf{m(t)c(t)}$
$\mathbf{m(t) = cos(2\pi(100)t)}$
$\mathbf{c(t) = cos(2\pi(1000)t)}$;





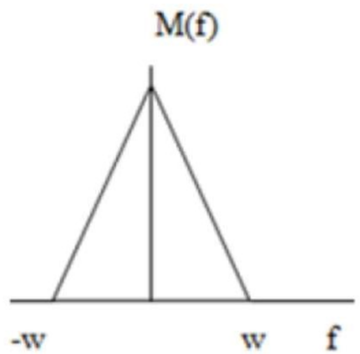
Generation of a DSB-SC Signal

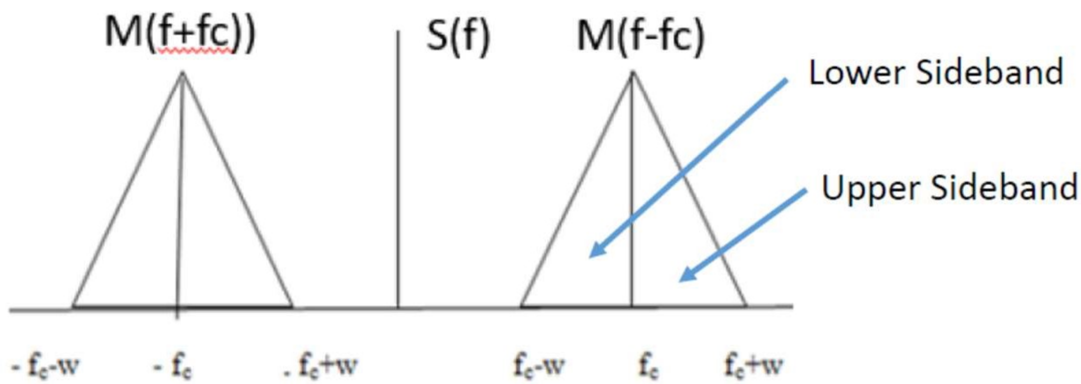
Spectrum of the Double Sideband Suppressed Carrier (DSB-SC)

- **DSB-SC:** $s(t) = A_c m(t)\cos(2\pi f_c t)$
- $S(f) = \Im\{A_c m(t)\cos(2\pi f_c t)\} = \frac{A_c}{2}[M(f - f_c) + M(f + f_c)]$

Remarks: *Similarities and Differences with Normal AM*

- 1. No impulses are present in the spectrum at $\pm f_c$, i.e., no carrier is transmitted as in the case of AM
- 2. The transmission B.W of s(t) = 2W; twice the message bandwidth (same as that of normal AM).
- 3. Power efficiency = $\frac{power\ in\ the\ side\ bands}{total\ transmitted\ power}$ = 100%. This is a power efficient modulation scheme.
- 4. Coherent detector is required to extract m(t) from s(t), as we shall demonstrate shortly.
- 5. Envelope detection cannot be used for this type of modulation.



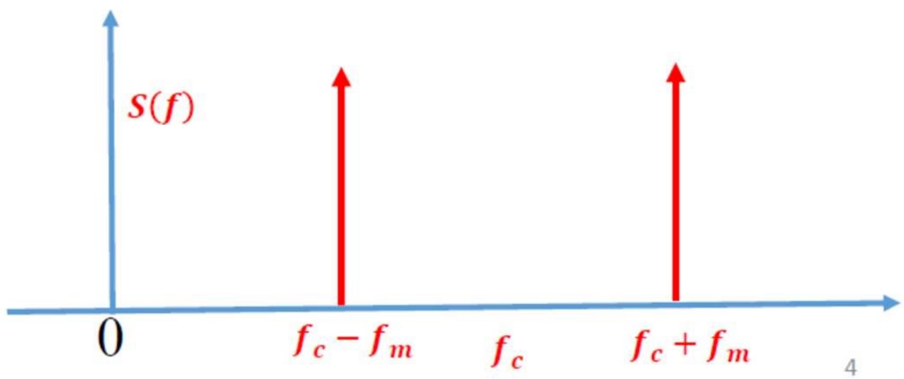


Spectrum of DSB-SC: Sinusoidal Modulation

- **Example**: Consider the sinusoidal modulation case where $\mathrm{c(t)} = A_c \cos(2\pi f_c t)$; $\mathrm{m(t)} = A_m \cos(2\pi f_m t)$; plot m(t), c(t), s(t) and find their spectrum.

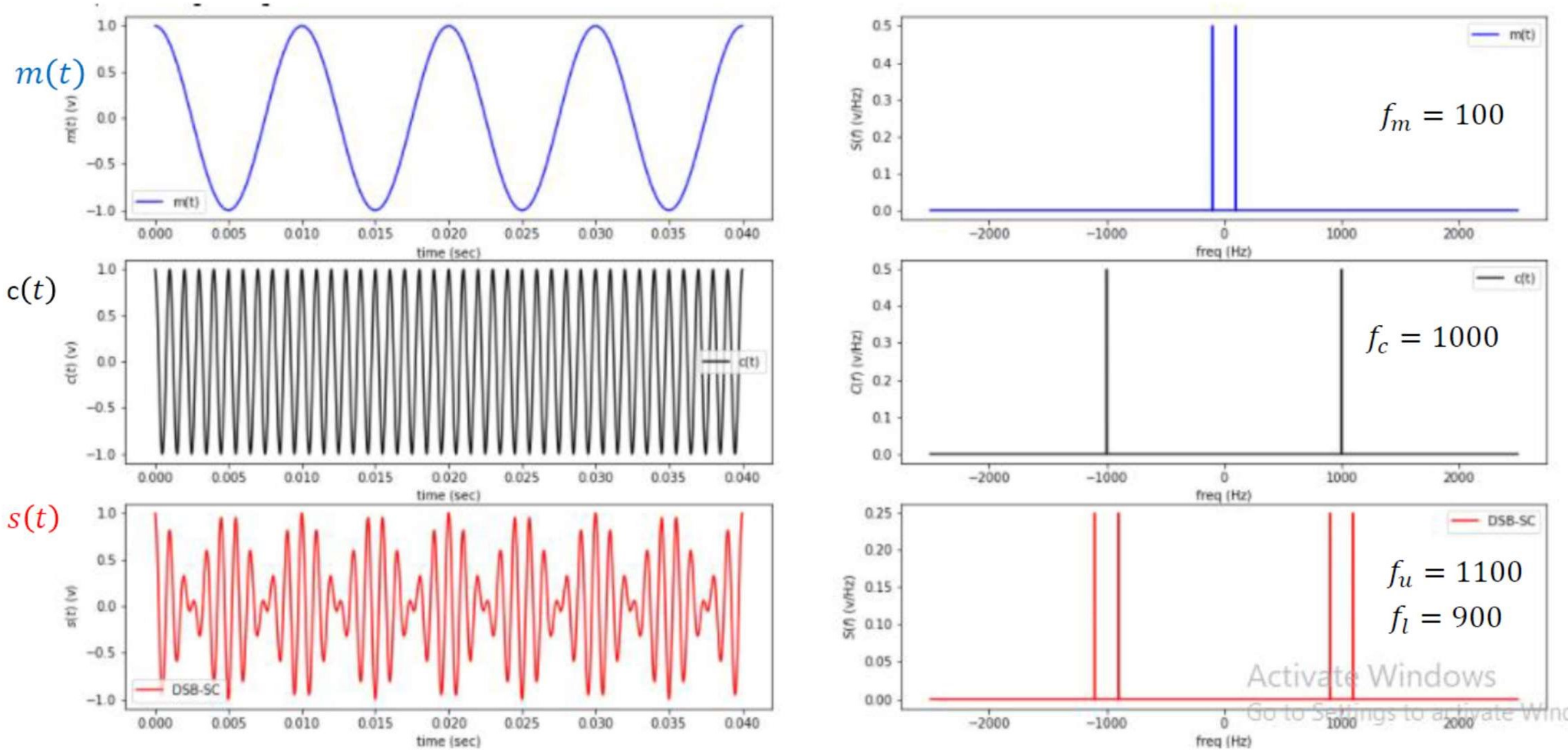
Solution:

- $s(t) = A_c m(t) cos(2\pi f_c t) = A_c cos(2\pi f_c t)\, A_m cos(2\pi f_m t)$;
- $= \frac{A_c A_m}{2} cos(2\pi(f_c + f_m)t) + \frac{A_c A_m}{2} cos(2\pi(f_c - f_m)t)$
- $S(f) = \Im\{A_c m(t) cos(2\pi f_c t)\} = \frac{A_c}{2}[M(f - f_c) + M(f + f_c)]$
- $\mathrm{M(f)} = \frac{A_m}{2}\delta(f - f_m) + \frac{A_m}{2}\delta(f + f_m)$
- The next figure shows all the plots when $f_m = 100$ Hz and $f_c = 1000$ Hz



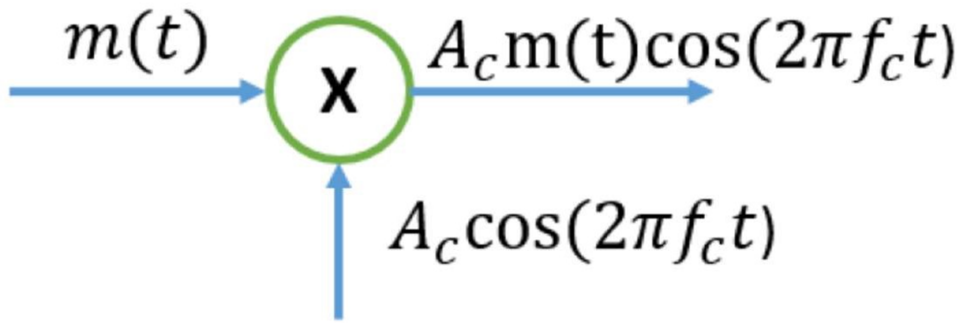
Spectrum of the DSB-SC Signal: Sinusoidal Modulation

$m(t) = A_m cos(2\pi(100)t);$ $c(t) = cos(2\pi(1000)t);$ $s(t) = A_c m(t) cos(2\pi f_c t);$



Generation of DSB-SC: The Product Modulator

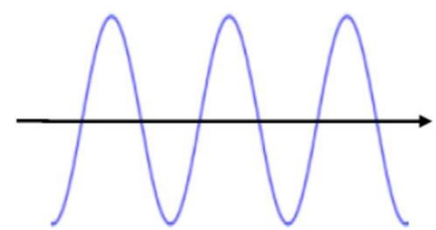
- **Product Modulator**: It multiplies the message signal m(t) with the carrier c(t). This technique is usually applicable when low power levels are possible and over a limited carrier frequency range.

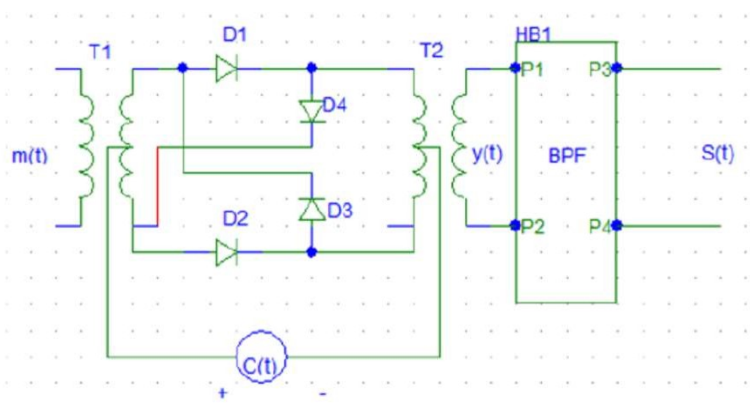


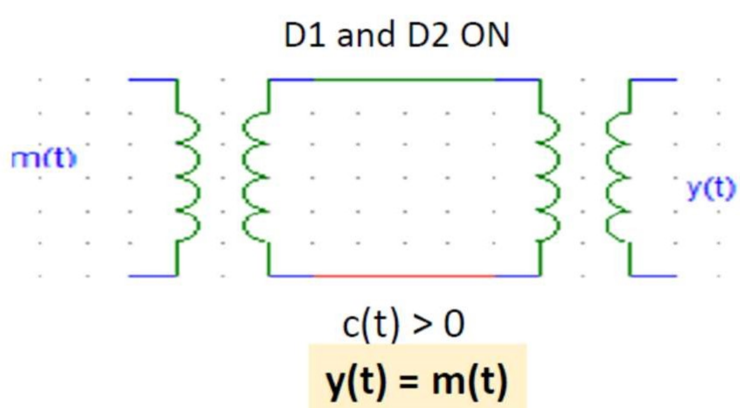
Generation of a DSB-SC Signal

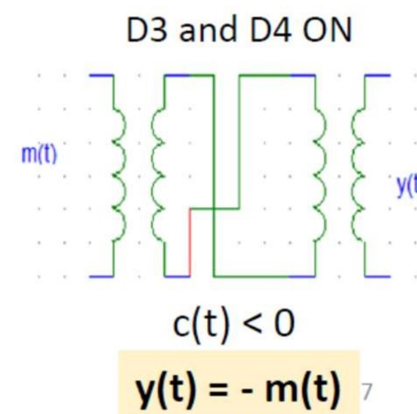
Generation of DSB-SC: The Ring Modulator

- Consider the scheme shown in the figure.
- Let c(t) >> m(t). Here the carrier c(t) controls the behavior of the diodes .
  - During the positive half cycle of c(t), c(t) > 0, and D1 and D2 are ON while D3 and D4 are OFF. Here, **y(t) = m(t).**
  - During the negative half cycle of c(t), c(t) < 0 and D3 and D4 are ON while and D1 and D2 are OFF. Here, **y(t) = - m(t).**
  - So m(t) is multiplied by +1 during the +ve half cycle of c(t) and m(t) is multiplied by -1 during the -ve half cycle of c(t)





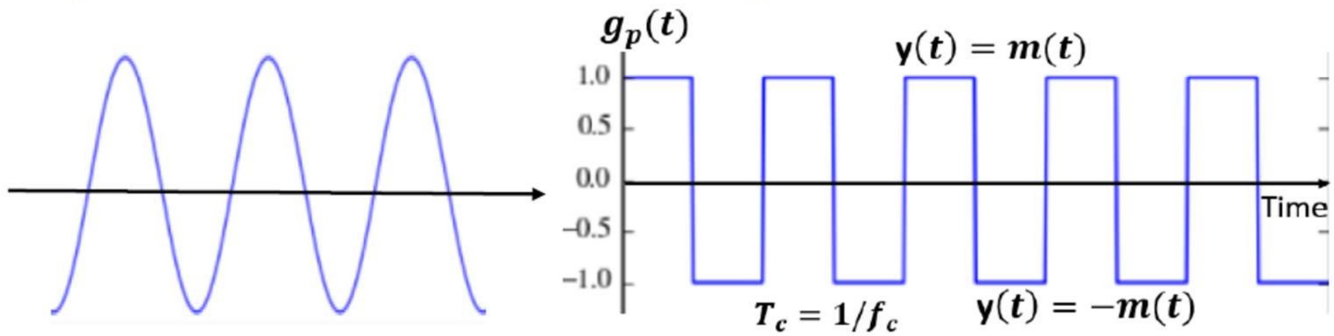




Generation of DSB-SC: The Ring Modulator

- So m(t) is multiplied by +1 during the +ve half cycle of c(t) and m(t) is multiplied by -1 during the -ve half cycle.
- Mathematically, y(t) behaves as if m(t) is multiplied by the switching function $g_p(t)$ where $g_p(t)$ is the square periodic function with period $T_c = \frac{1}{fc}$; T_c the period of c(t). By expanding $g_p(t)$ in a Fourier series, we get
- **$y(t) = m(t)\, g_p(t)$** $= m(t)[\frac{4}{\pi}\cos 2\pi f_c t - \frac{4}{3\pi}\cos 3(2\pi f_c t) + \frac{4}{5\pi}\cos 5(2\pi f_c t)]$
- $= m(t)\frac{4}{\pi}\cos 2\pi f_c t - m(t)\frac{4}{3\pi}\cos 3(2\pi f_c t) + m(t)\frac{4}{5\pi}\cos 5(2\pi f_c t)$
- When y(t) passes through the BPF with center frequency f_c, and bandwidth = 2W, the only component that appears at the output is the desired DSB-SC signal, which is

$$s(t) = \frac{4}{\pi} m(t) cos 2\pi f_c t$$

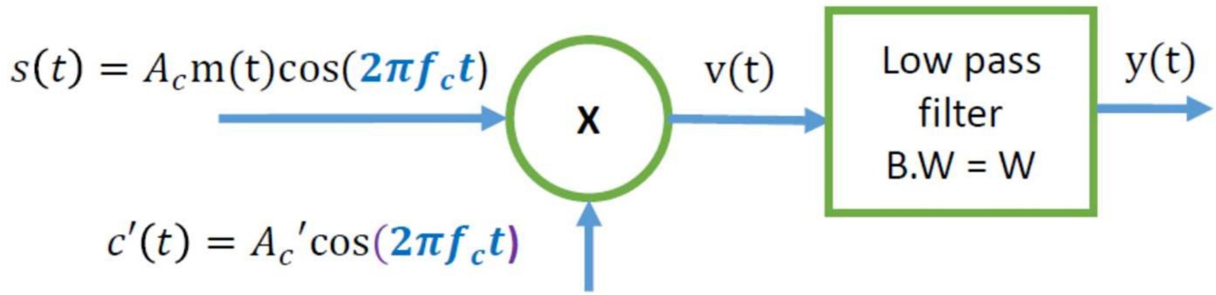


Demodulation of DSB-SC

- A DSB-SC signal is demodulated using what is known as ***coherent demodulation***. This means that the modulated signal s(t) is multiplied by a locally generated signal at the receiver which has the same frequency and phase as that of the carrier c(t) at the transmitting side

Perfect Coherent Demodulation

- Let $c(t) = A_c \cos(2\pi f_c t)$
- $c'(t) = A_c{}' \cos(2\pi f_c t)$

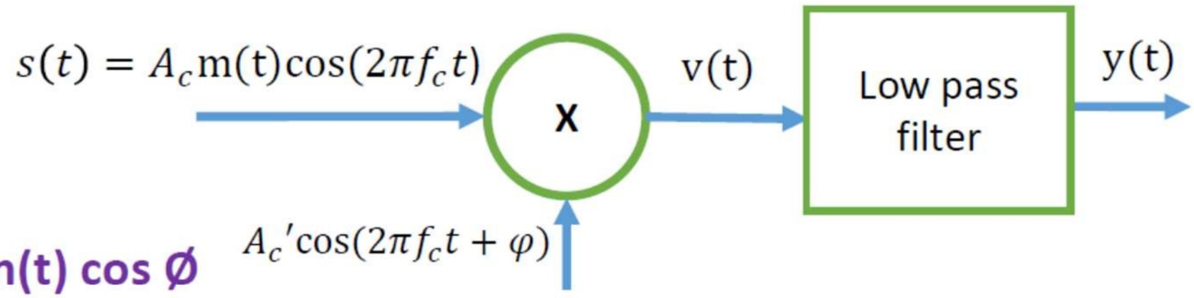


- Mixing the received signal with the version of the carrier at the receiving side, we get
- $\mathrm{v(t)} = s(t) A_c{}' \cos(2\pi f_c t) = A_c A_c{}' \, m(t) \cos^2 2\pi f_c t$
- $= \frac{A_c A_c{}'}{2} m(t) \, [1 + \cos 2\,(2\pi f_c t)] = \frac{A_c A_c{}'}{2} \mathbf{m(t)} + \frac{A_c A_c{}'}{2} m(t) \cos 2(2\pi f_c t)$
- The first term on the RHS is proportional to $m(t)$, while the second term is a DSB signal modulated on a carrier with frequency $2f_c$. The high frequency component can be eliminated using a LPF with B.W = W. The output is $\boldsymbol{y(t) = \frac{A_c A_c'}{2} m(t)}$
- Therefore, $m(t)$ has been recovered from $s(t)$ without distortion, i.e., the whole modulation-demodulation process is distortion-less.

Effect of Carrier Non-Coherence on Demodulated Signal: Constant Phase Shift

A constant phase difference between c(t) and $c'(t)$

- Let $c(t) = A_c \cos 2\pi f_c t$, $c'(t) = A_c' \cos(2\pi f_c t + \varphi)$
- We use the same demodulator
- $v(t) = A_c m(t) \cos 2\pi f_c t \,.\, A_c' \cos(2\pi f_c t + \emptyset)$
- $= \frac{A_c A_c'}{2} m(t)[\cos(4\pi f_c t + \emptyset) + \cos \emptyset]$
- $= \frac{A_c A_c'}{2} m(t) \cos(4\pi f_c t + \emptyset) + \frac{A_c A_c'}{2} m(t) \cos \emptyset$



- The low pass filter suppresses the first high frequency term and admits only the second low frequency term. The output is $y(t) = \frac{A_c A_c'}{2} m(t) \cos \emptyset$
- For $0 < \emptyset < \frac{\pi}{2}$, $0 < \cos \emptyset < 1$, y(t) suffers from an attenuation due to Ø.
- However, **for $\emptyset = \frac{\pi}{2}$, $\cos \emptyset = 0$ and y(t) = 0, i.e., receiver loses the signal.**
- The disappearance of a message component at the demodulator output is called ***quadrature null effect***. This highlights the importance of maintaining synchronism between the transmitting and receiving carrier signals $c'(t)$ and c(t).

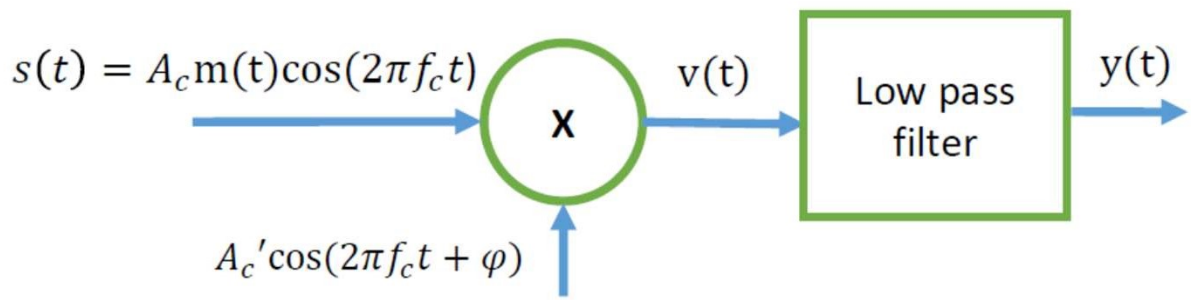
Effect of Carrier Non-Coherence on Demodulated Signal: Constant Phase Shift

Example: Let $m_1(t) = \cos 2\pi(1000)t$; $m_2(t) = \cos 2\pi(2000)t$; $m(t) = m_1(t) + m_2(t)$

$c(t) = \cos 2\pi(10000)t$ and let ϕ = 50 degrees.

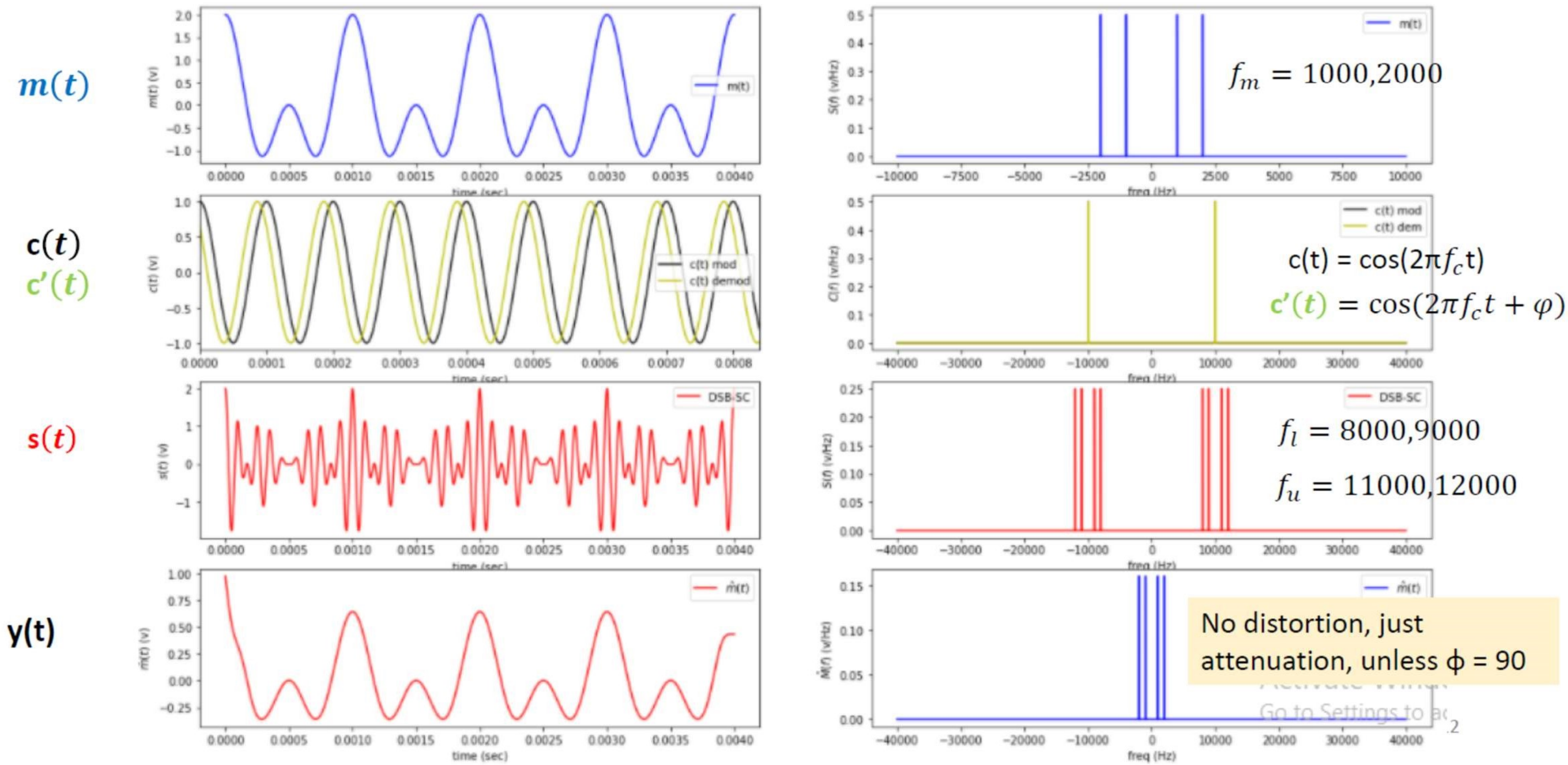
Solution: From the analysis above,

- $$y(t) = \frac{A_c A_c'}{2} m(t) \cos\emptyset$$
- The next figure shows the input message, carrier, modulated, and demodulated signals in the time and frequency domains.



Effect of Carrier Non-Coherence on Demodulated Signal: Constant Phase Shift

$m_1(t) = \cos 2\pi(1000)t$; $m_2(t) = \cos 2\pi(2000)t$; **$m(t) = m_1(t) + m_2(t)$** $c(t) = \cos 2\pi(10000)t$ and let ϕ = 50 degrees.

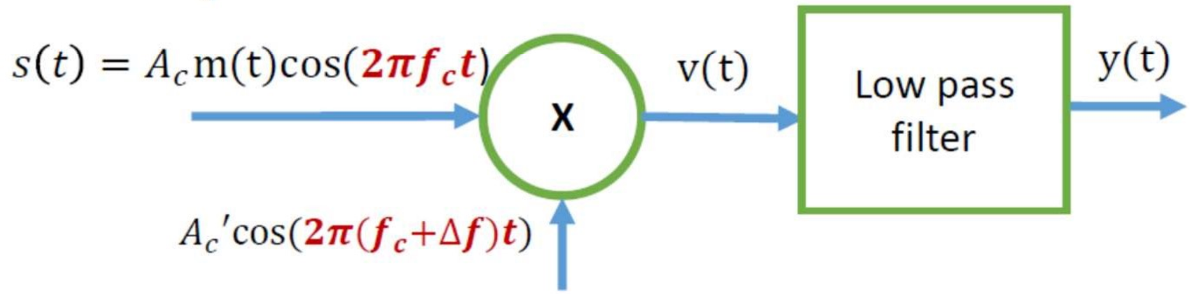


Effect of Carrier Non-Coherence on Demodulated Signal: Constant Frequency Difference

Constant Frequency Difference between c(t) and $c'(t)$

- Let c(t) = $A_c \cos 2\pi f_c t$, $c'(t) = A_c{}' \cos(2\pi(f_c+\Delta f)t)$
- Again, we use the same receiver structure as before.
- $v(t) = A_c \mathrm{m(t)} \cos(2\pi f_c t).\, A_c{}' \cos(2\pi(f_c+\Delta f)t)$
- $= \frac{A_c A_c{}'}{2} m(t)[\cos(4\pi f_c t + 2\pi\Delta f t) + \cos 2\pi\Delta f t]$
- After low-pass filtering,

$$y(t) = \frac{A_c A_c'}{2} m(t) \cos(2\pi\Delta f t)$$



- As you can see, $y(t) \neq km(t)$, but rather $m(t)$ is multiplied by a time function. Hence, the system is not distortion-less.
- In addition, y(t) appears as a **double side band modulated signal with a carrier with magnitude Δf**. The next example illustrates this case more.

Effect of Carrier Non-Coherence on Demodulated Signal: Constant Frequency Difference

Example: Let m(t) = cos2π(1000)t; c(t) = cos2π(10000)t and let Δf =500 Hz

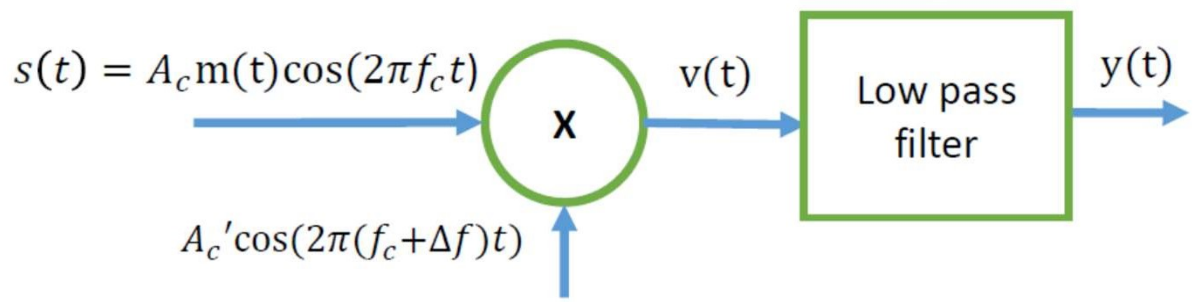
Solution: From the analysis in case 2 above,

$$y(t) = \frac{A_c A_c'}{2} m(t) cos(2\pi \Delta f t)$$

$$y(t) = \frac{A_c A_c'}{2} \cos 2\pi(1000)t \cos 2\pi(500)t$$

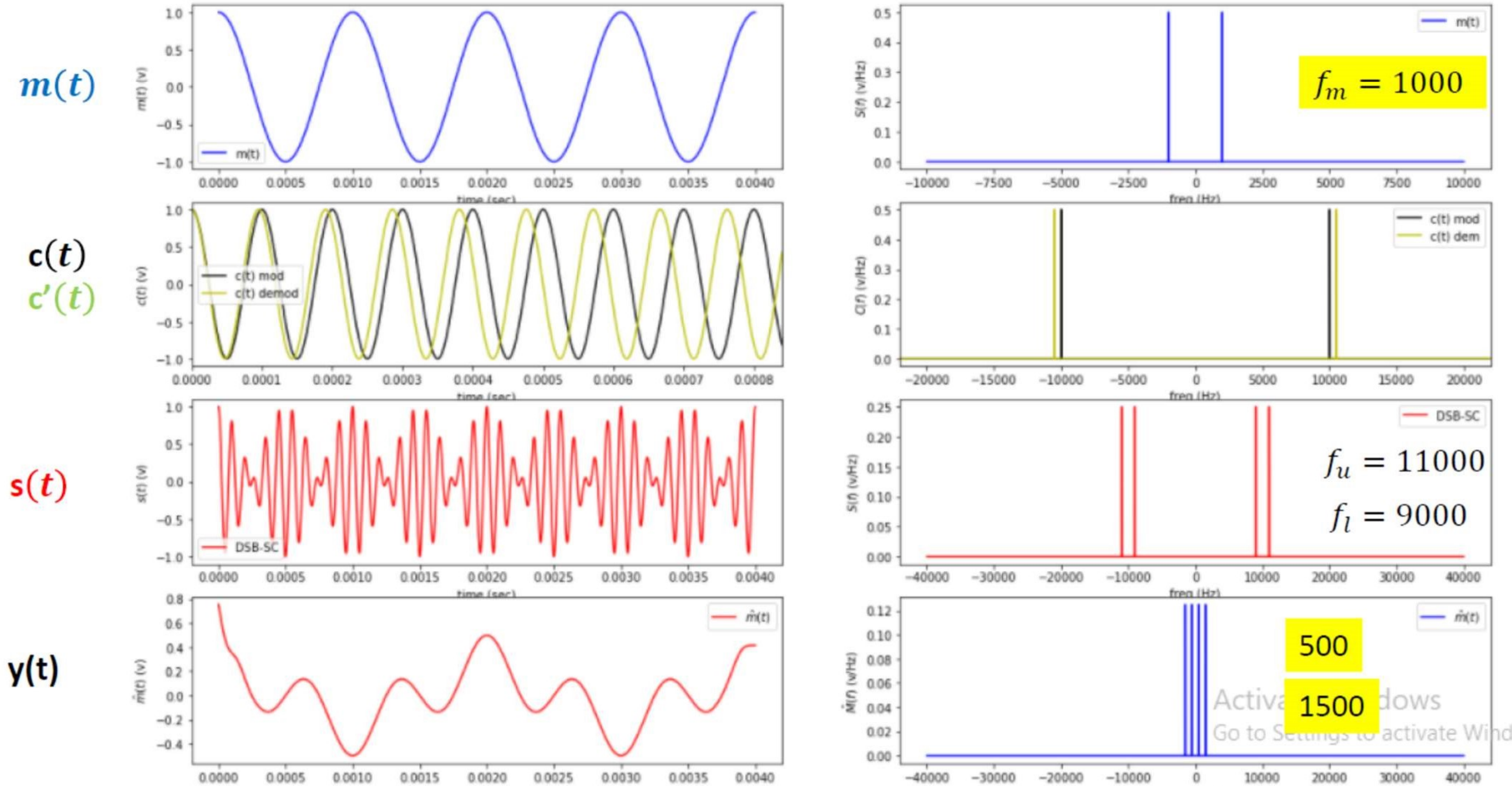
$$= \frac{A_c A_c'}{4}[\cos 2\pi(1500)t + \cos 2\pi(500)t]$$

- The original message is a signal with a single frequency of 1000 Hz, while the output consists of a signal with two frequencies at f_1 =1500 Hz and f_2 = 500 Hz
- $\Rightarrow$ ***Distortion***)



Effect of Carrier Non-Coherence on Demodulated Signal: Constant Frequency Difference

$m(t) = A_c cos(2\pi(1000)t)$; $c(t) = cos(2\pi(10000)t)$; $c'(t) = cos(2\pi(10500)t)$; $\Delta f = 500$



Single Sideband Suppressed Carrier (SSB-SC) Modulation: Lecture Outline

Lecture 5

- In this lecture, we consider another type of AM modulation called SSB-SC.
- We analyze this modulation technique in the time and frequency domains.
- Consider the generation and demodulation techniques.
- Study the effect of non-coherence in the phase and frequency of the locally generated carrier at the receiver, on the demodulated signal.

Normal AM Signal

Let the Fourier transform of $m(t)$ be as shown (the B.W of $m(t) = W$ Hz).

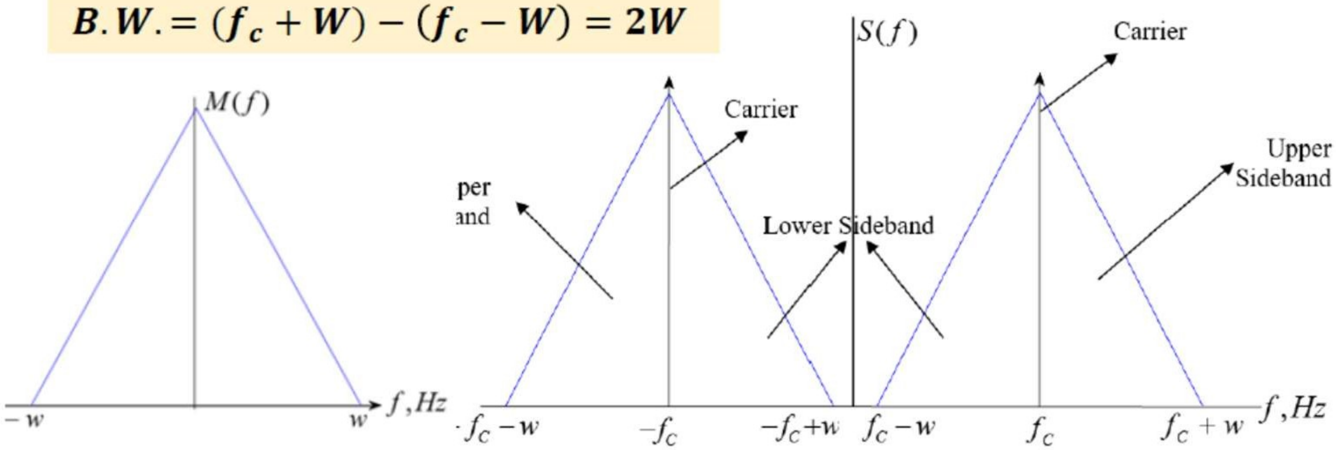
$$s(t) = A_C\left(1 + k_a m(t)\right)\cos 2\pi f_C t \qquad \text{(dc + message)*carrier}$$

$$s(t) = A_C \cos 2\pi f_C t + A_C k_a m(t)\cos 2\pi f_C t \qquad \text{(carrier + message*carrier)}$$

Taking the Fourier transform, we get

$$S(f) = \frac{A_C}{2}\delta(f - f_C) + \frac{A_C}{2}\delta(f + f_C) + \frac{A_C k_a}{2}M(f - f_C) + \frac{A_C k_a}{2}M(f + f_C)$$

$$\boldsymbol{B.W. = (f_c + W) - (f_c - W) = 2W}$$



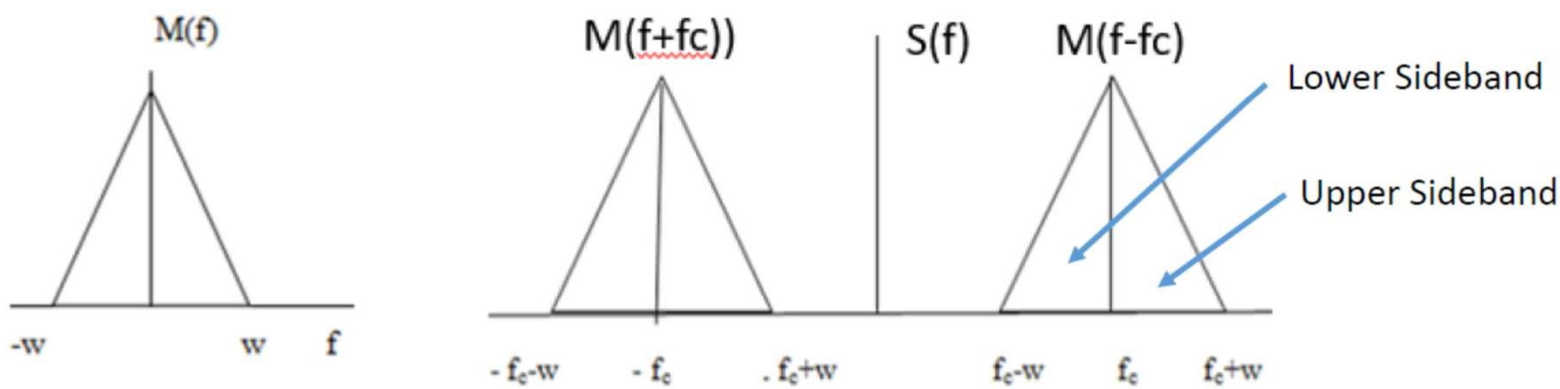
- **Two impulses are present in the spectrum at $\pm f_c$,**
- **2. The transmission B.W of s(t) = 2W; twice the message bandwidth**
- **Poor power efficiency.**
- **Envelope detection is used for this type of modulation.**

Double Sideband Suppressed Carrier (DSB-SC)

- **DSB-SC:** $\boldsymbol{s(t) = A_c m(t)\cos(2\pi f_c t)}$
- $S(f) = \Im\{A_c m(t)\cos(2\pi f_c t)\} = \frac{A_c}{2}[M(f - f_c) + M(f + f_c)]$

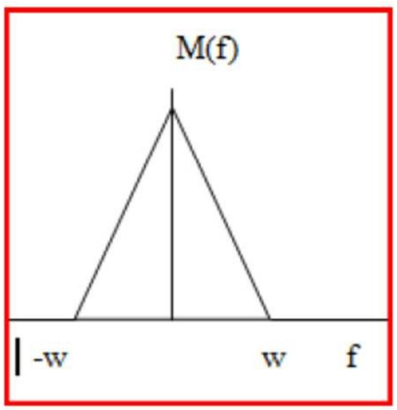
Remarks:

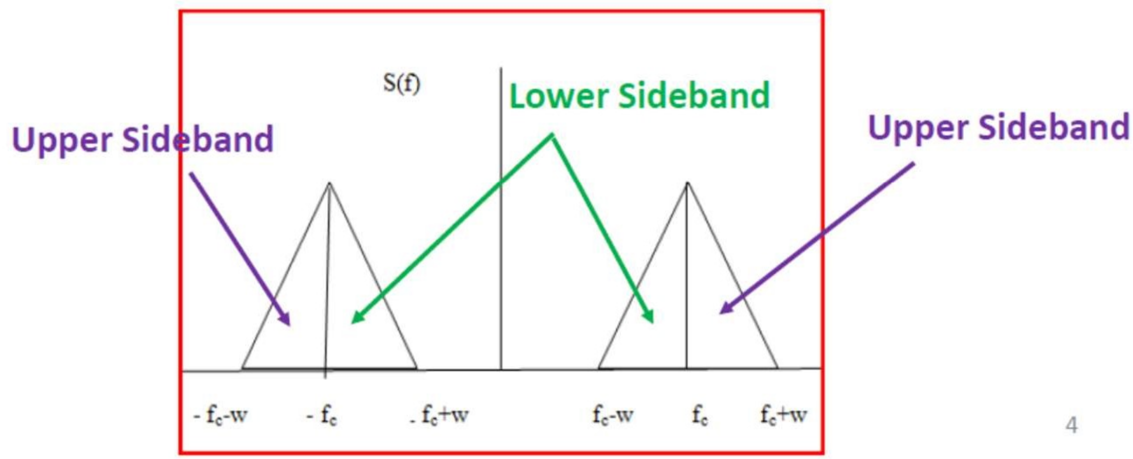
- 1. No impulses are present in the spectrum at $\pm f_c$, i.e., no carrier is transmitted as in the case of AM
- 2. The transmission B.W of s(t) = 2W; twice the message bandwidth (same as that of normal AM).
- 3. Power efficiency = $\frac{power\ in\ the\ side\ bands}{total\ transmitted\ power}$ = 100%. This is a power efficient modulation scheme.
- 4. Coherent detector is required to extract m(t) from s(t)
- 5. Envelope detection cannot be used for this type of modulation.



Single Sideband Modulation

- **Rationale:** The information representing the modulating waveform is contained in both the upper and the lower sidebands of the DSB signal. ⇒ Redundant Transmission.
- Therefore, it is not essential to transmit both side-bands. The transmission of one sideband will suffice in reconstructing the message signal at the receiver.
- In SSB-SC the carrier is suppressed and one of the two sideband is transmitted.
- Hence, power saving and bandwidth saving are achieved
- Sometimes, an attenuated part of the carrier is transmitted that will ease the process of demodulation called residual carrier SSB signal, but this will not be addressed in this lecture.



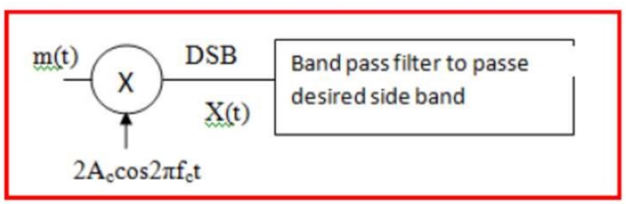


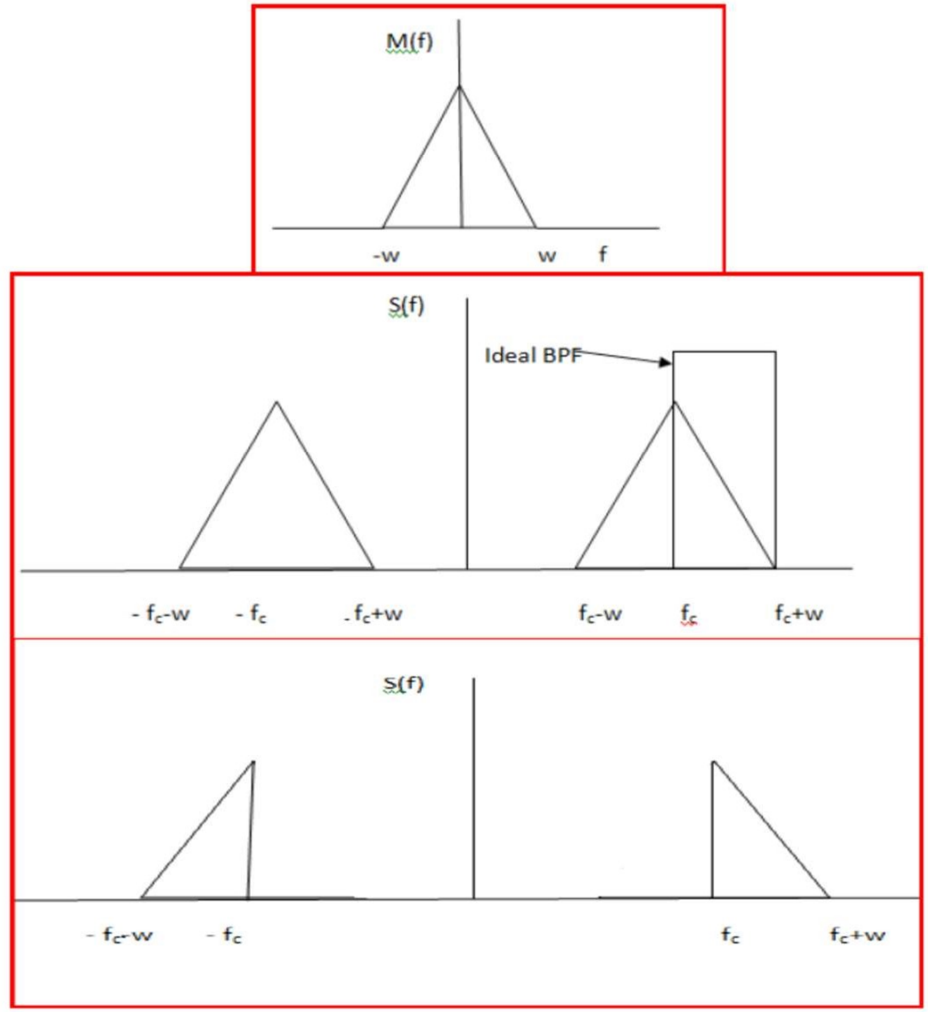
Single Sideband Suppressed Carrier (DSB-SC) Modulation

- In this type of modulation, only one of the two sidebands of a DSB-SC is retained while the other sideband is suppressed. This means that the B.W of the SSB signal is one half that of DSB-SC. The saving in the bandwidth comes at the expense of increasing modulation/demodulation complexity.
- The time-domain representation of a SSB signal is
- $$\boldsymbol{s(t) = A_C m(t) cos\omega_c t \pm A_C \widehat{m}(t) sin\omega_c t}$$
- $m(t)$: is the baseband message signal with bandwidth W.
- $\widehat{m}(t)$: Hilbert transform of $m(t)$ obtained by passing $m(t)$ through a 90-degrees phase shifter.
- \- sign: upper sideband is retained.
- \+ sign: lower sideband is retained.
- $c(t) = A_c \cos(2\pi f_c t)$: is the high frequency carrier signal; $f_c >> W$.

Generation of SSB: Filtering Method

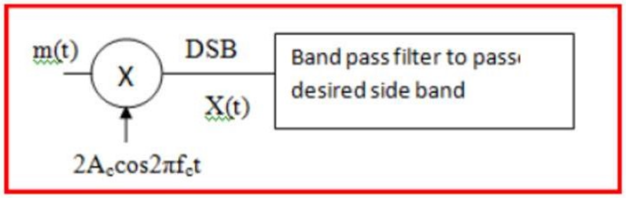
- A DSB-SC signal $x(t) = 2A_C m(t) cos\omega_c t$ is generated first. A band pass filter with appropriate B.W and center frequency is used to pass the desired side band only and suppress the other sideband.
- The pass band of the filter must occupy the same frequency range as the desired sideband.
- **Remark**: Ideal filter do not exist in practice meaning that a complete elimination of the undesired side band is not possible. The consequence of this is that either part of the undesired side band is passed or the desired one will be highly attenuated. SSB modulation is suitable for signals with low frequency components that are not rich in terms of their power content, as we shall see next.

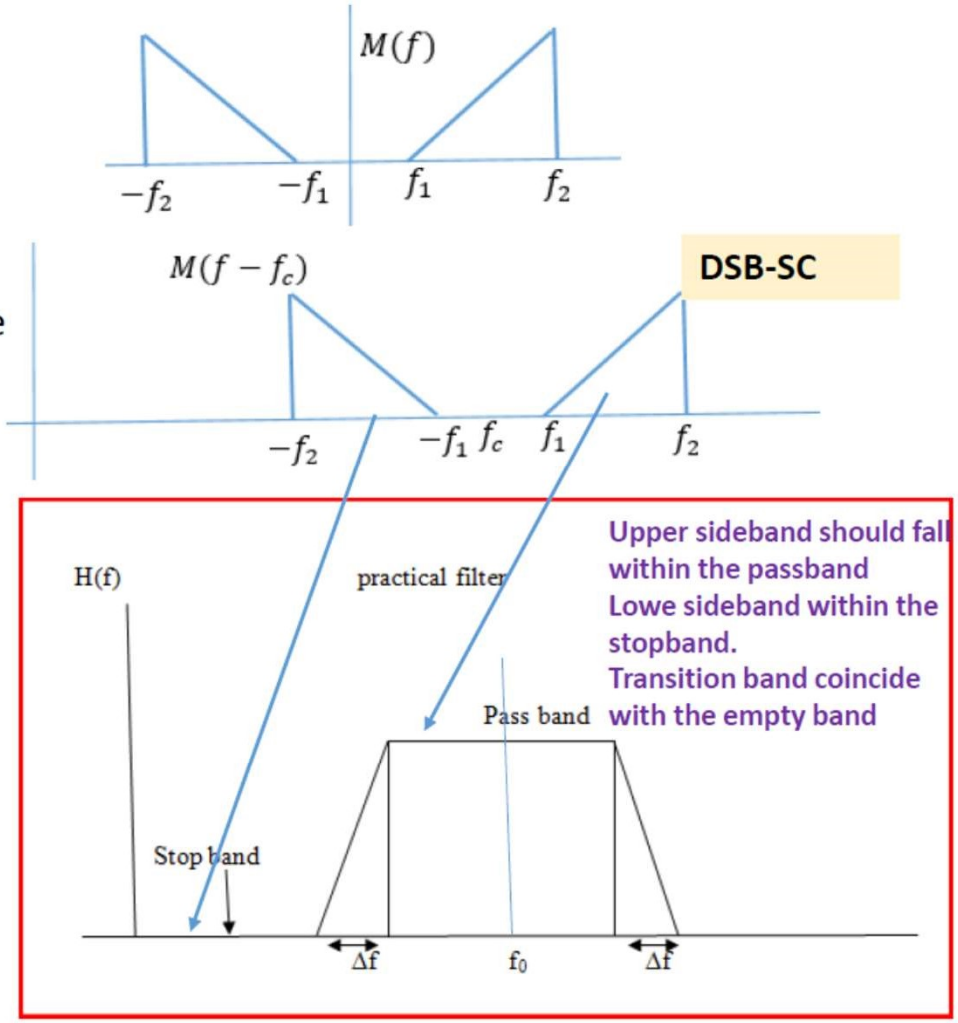




Generation of SSB: Practical Consideration on the Filtering Method

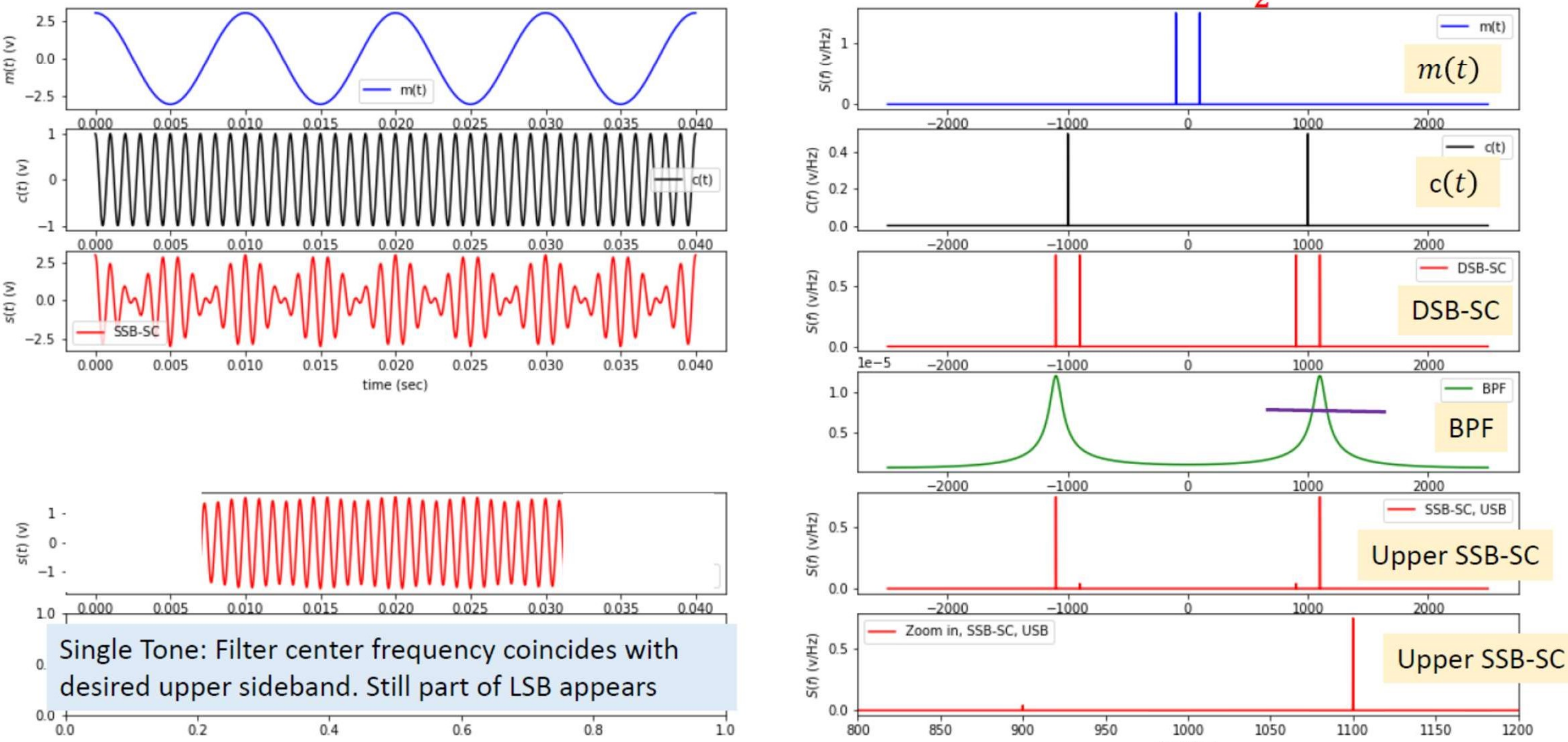
- The following practical considerations must be taken into account:
  - The pass band of the filter must occupy the same frequency band as the desired sideband.
  - The width of the transition band of the filter separating the pass band and the stop band must be at least 1% of the center frequency of the filter. i.e., **$0.01f_0 \le \Delta f$**. This is sort of a rule of thumb for realizable filters on the relationship between the transition band and the center frequency.
  - The width of the transition band of the filter should be at most twice the lowest frequency components of the message signal so that a reasonable separation of the two side band is possible. If the message significant frequency components extends between (f_1, f_2), then $\mathbf{2f_1 \ge \Delta f}$.





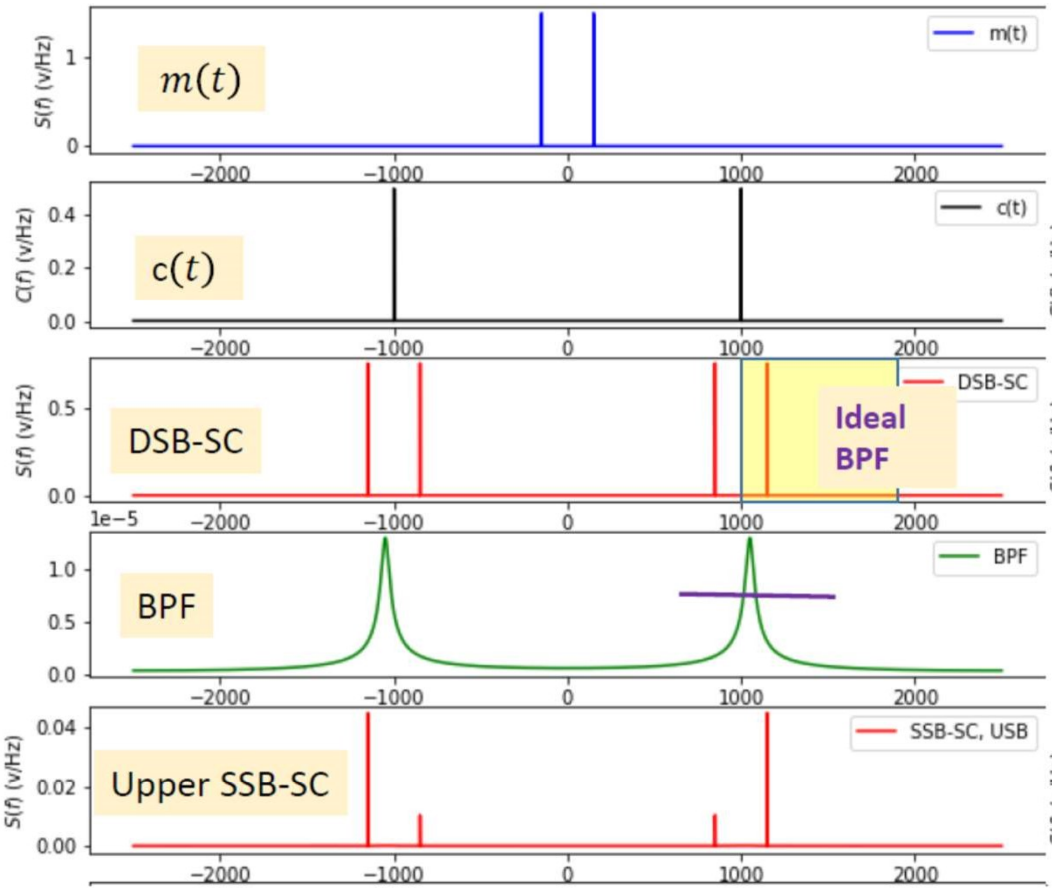
Upper SSB Signal in the Time and Frequency Domains: Single Tone Message

- Example: $m(t) = 3\cos(2\pi 100t)$; Let $\mathrm{c}(t) = \cos(2\pi 1000t)$; $\mathbf{s(t) = \frac{A_c A_m}{2} cos(2\pi 1100 t)}$



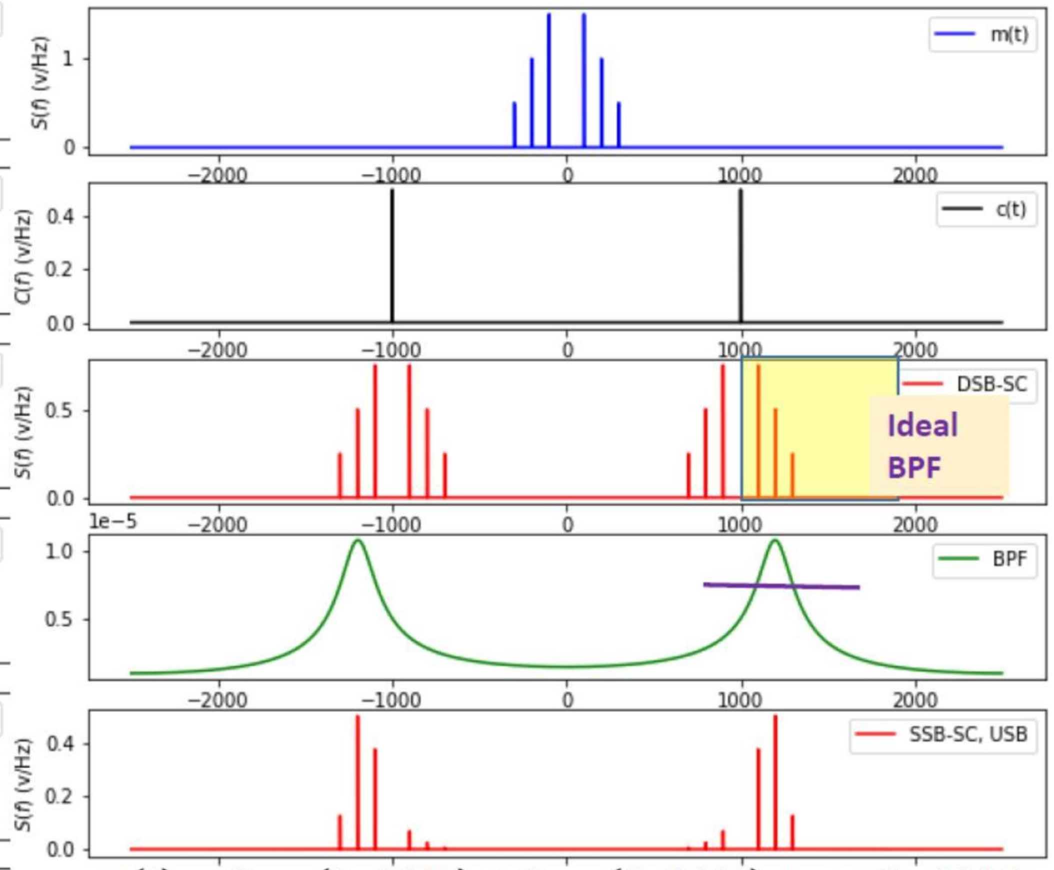
Single Tone: Filter center frequency coincides with desired upper sideband. Still part of LSB appears

Filtering Issues in SSB Modulation



$$m(t) = 3\cos(2\pi 150 t); \quad c(t) = \cos(2\pi 1000 t);$$

Single Tone: Filter center frequency does not coincide with desired USB. USB attenuates, Part of LSB appears

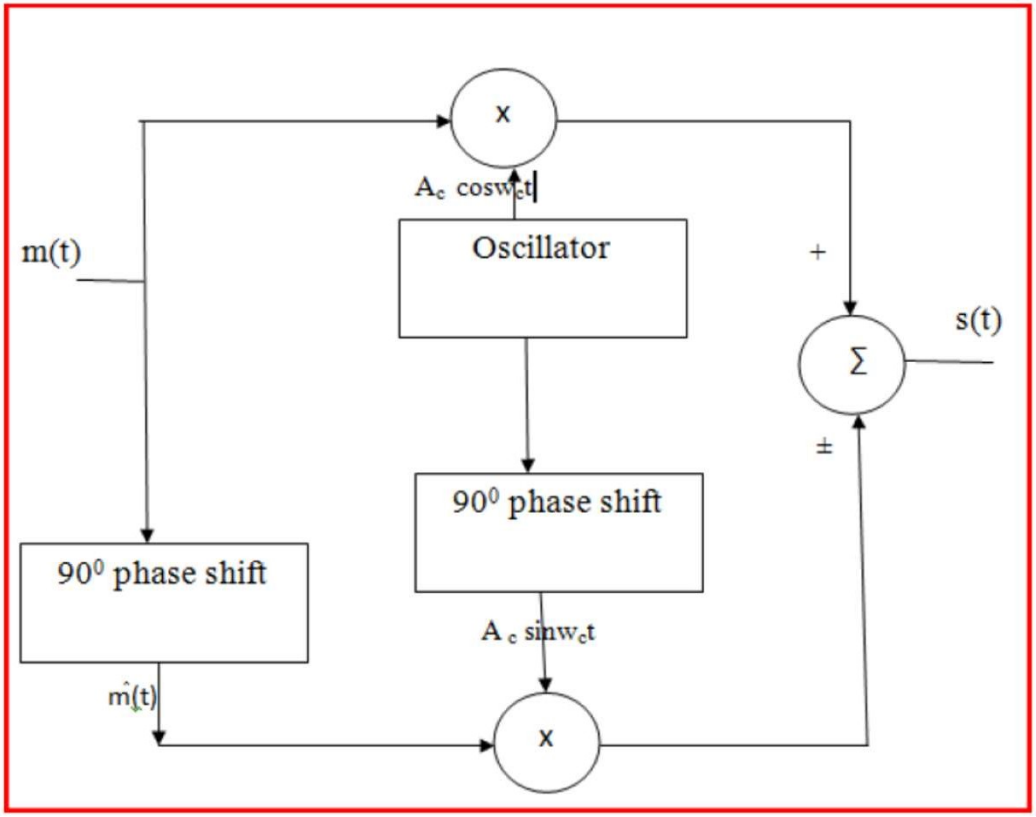


$$m(t) = 3\cos(2\pi 100 t) + 2\cos(2\pi 200 t) + \cos(2\pi 300 t)$$

Multi-tone: Filter center frequency does not coincide with desired USB. Components in USB are not passed proportionately. Part of LSB appears

Generation of SSB Signal: Phase Shift Method

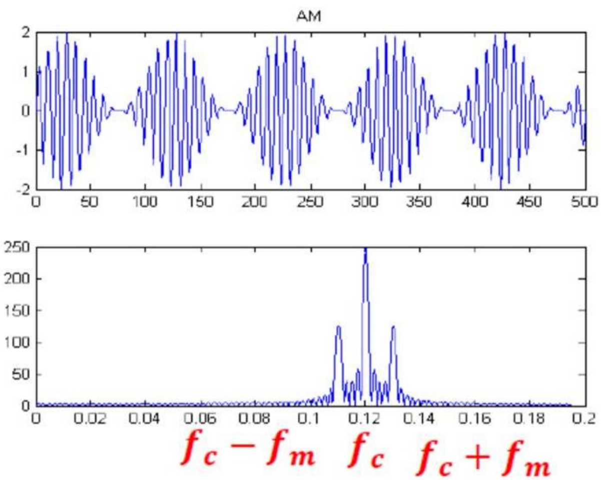
- The method is based on the time –domain representation of the SSB signal
- $$s(t) = A_C m(t) cos\omega_c t \pm A_C \hat{m}(t) sin\omega_c t$$



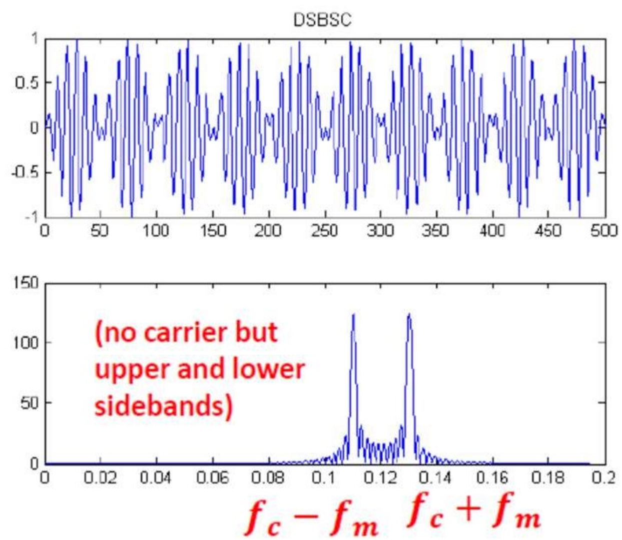
Comparison of the three types of AM modulation

Here, we show all three types of AM modulation in the time and frequency domains when $m(t) = A_m \cos(2\pi f_m t);$ $c(t) = A_c \cos(2\pi f_c t);$

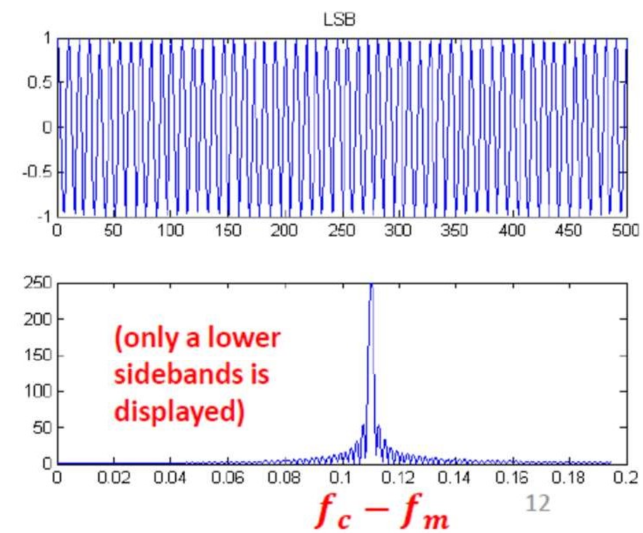
AM in time and frequency domains
(carrier + two sidebands)



DSB in time and frequency domains

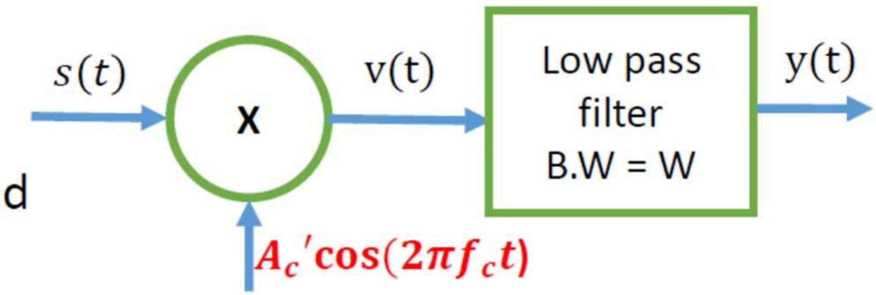


SSB in time and frequency domains



Demodulation of SSB-SC

- A SSB-SC signal is demodulated using what is known as ***coherent demodulation***. This means that the modulated signal s(t) is multiplied by a locally generated signal at the receiver which has the same frequency and phase as that of the carrier c(t) at the transmitting side

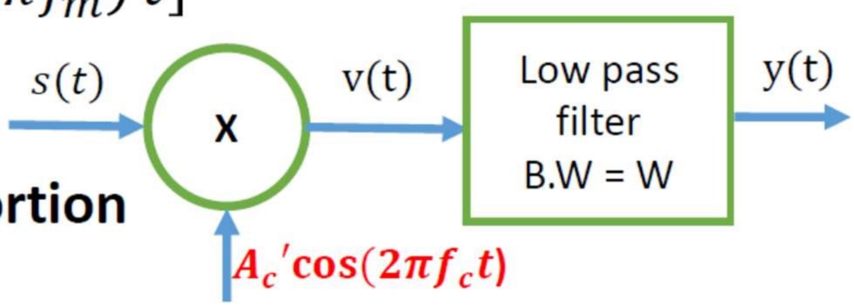


Perfect Coherent Demodulation

- Let the received signal be the upper single sideband
- $s(t) = A_C m(t) cos\omega_c t - A_C \hat{m}(t) sin\omega_c t$
- At the receiver, $s(t)$ is mixed with the carrier signal. The result is
- $v(t) = s(t) A_C' cos\omega_c t$
- $= A_C' [A_C m(t) cos\omega_c t - A_C \hat{m}(t) sin\omega_c t] cos\omega_c t$
- $= A_C A_C' m(t)(cos\omega_c t)^2 - A_C A_C' \hat{m}(t)\, sin\omega_c t\, cos\omega_c t$
- $= \frac{A_C A_C'}{2} m(t) + \frac{A_C A_C'}{2} m(t) cos2\omega_c t - \frac{A_C A_C'}{2} \hat{m}(t)\, sin2\omega_c t$
- The low pass filter admits only the first terms. The output is: $\boldsymbol{y(t) = \frac{A_C A_C'}{2} m(t)}$

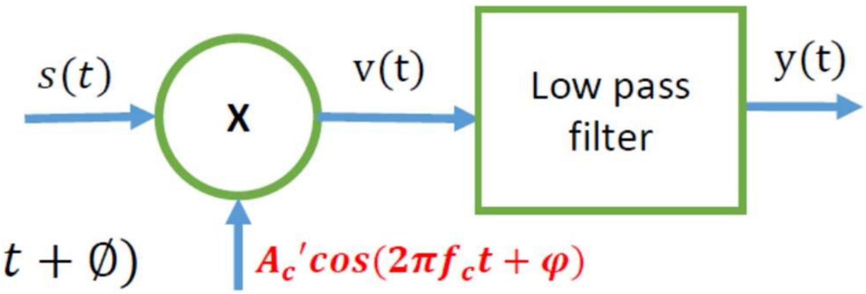
Single Tone Modulation: Why One Sideband is Sufficient

- **Example:** When $m(t) = A_m\cos(2\pi f_m t)$; $c(t) = A_c\cos(2\pi f_c t)$;
- $s_{DSB}(t) = A_c A_m \cos(2\pi f_c t)\cos(2\pi f_m t)$; DSB-SC modulation
- $s_{DSB}(t) = \frac{A_c A_m}{2}\cos(2\pi(f_c+f_m)t) + \frac{A_c A_m}{2}\cos(2\pi(f_c-f_m)t)$;
- The upper SSB signal is: $\boldsymbol{s_{SSB}(t) = \frac{A_c A_m}{2} cos(2\pi(f_c+f_m)t)}$;
- $v(t) = s(t)A_c'\cos(2\pi f_c t) = \frac{A_c A_m A_c'}{2} cos\ (2\pi f_c t) cos(2\pi(f_c+f_m)t)$;
- $v(t) = \frac{A_c A_m A_c'}{4}[cos(2\pi(2f_c+f_m)t + \cos(2\pi f_m)\ t]$
- $\mathbf{y}\boldsymbol{(t) = \frac{A_c A_m A_c'}{4}[cos(2\pi f_m)\ t]}$
- **Message has been recovered without distortion**



Effect of Carrier Non-Coherence on Demodulated Signal: Constant Phase Shift

A constant phase difference between c(t) and $c'(t)$

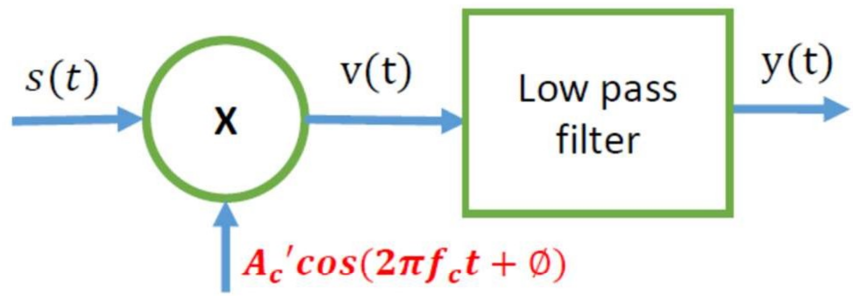


- The local oscillator takes the form
- $\acute{c}(t) = \acute{A}_c \cos(\omega_c t + \emptyset)$;
- $v(t) = [A_c m(t) \cos \omega_c t - A_c \hat{m}(t) \sin \omega_c t] \acute{A}_c \cos(\omega_c t + \emptyset)$
- $= A_c \acute{A}_c m(t) \cos \omega_c t \cos(\omega_c t + \emptyset) - A_c \acute{A}_c \hat{m}(t) \sin \omega_c t \cos(\omega_c t + \emptyset)$
- $= \frac{A_c \acute{A}_c}{2} m(t) \cos(2\omega_c t + \emptyset) + \frac{A_c \acute{A}_c}{2} m(t) \cos(\emptyset)$
- $- \frac{A_c \acute{A}_c}{2} \hat{m}(t) \cos(2\omega_c t + \emptyset) + \frac{A_c \acute{A}_c}{2} \hat{m}(t) \sin(\emptyset)$
- $\boldsymbol{y(t) = \frac{A_c \acute{A}_c}{2} m(t) \cos(\emptyset) + \frac{A_c \acute{A}_c}{2} \hat{m}(t) \sin(\emptyset)}$
- Note that there is a distortion due to the appearance of the Hilbert transform of the message signal at the output.

Single Tone Modulation: Effect of a Constant Phase Shift of the Carrier at Receiver

- **Example**: When $m(t) = A_m \cos(2\pi f_m t)$; $c(t) = A_c \cos(2\pi f_c t)$;
- The upper SSB signal is: $\boldsymbol{s_{SSB}(t) = \frac{A_c A_m}{2} cos(2\pi(f_c + f_m)t)}$;
- $v(t) = s(t) A_c' \cos(2\pi f_c t + \emptyset) = \frac{A_c A_m A_c'}{2} cos\ (2\pi f_c t + \emptyset) cos(2\pi(f_c + f_m)t)$;
- $v(t) = \frac{A_c A_m A_c'}{4} [cos\ (4\pi f_c t + 2\pi f_m t + \emptyset) + \cos(2\pi f_m t - \emptyset)]$
- $\boldsymbol{y(t) = \frac{A_c A_m A_c'}{4} [cos(2\pi f_m t - \emptyset)]}$
- If the message consists of multiple tones
- $y(t) = \frac{A_c A_1 A_c'}{4} [cos(2\pi f_1 (t - \emptyset / 2\pi f_1)]$

$$+ \frac{A_c A_2 A_c'}{4} [cos(2\pi f_2 (t - \emptyset / 2\pi f_2)] + \frac{A_c A_3 A_c'}{4} [cos(2\pi f_3 (t - \emptyset / 2\pi f_3)]$$

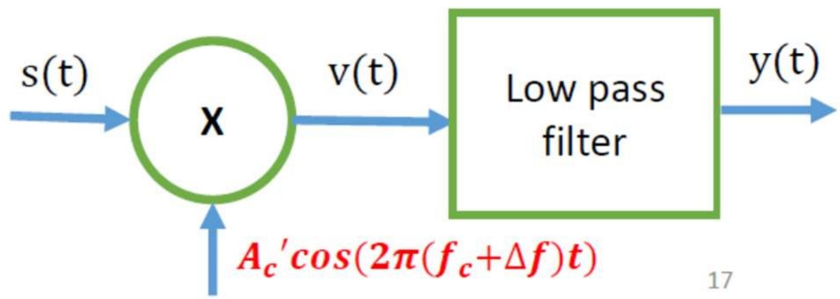


Here, phase distortion becomes more apparent since we **cannot** write $y(t) = kx(t - t_d)$

Effect of Carrier Non-Coherence on Demodulated Signal: Constant Frequency Difference

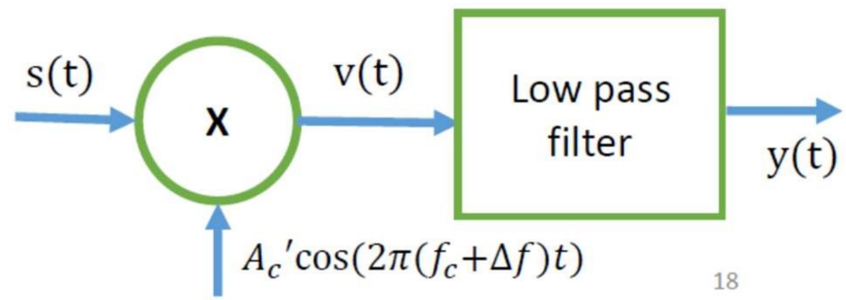
Constant Frequency Difference between c(t) and $c'(t)$

- $\acute{c}(t) = A'_c \cos\ 2\pi(f_c + \Delta f)t;$ Constant frequency shift
- $v(t) = [A_c m(t) \cos\omega_c t - A_c \hat{m}(t) \sin\omega_c t]\acute{A}_c \cos\ 2\pi(f_c + \Delta f)t$
- $= \frac{A_c \acute{A}_c}{2} m(t)[\cos(2\omega_c + \Delta\omega)t + \cos 2\pi\Delta f t]$
- $-\frac{A_c \acute{A}_c}{2} \hat{m}(t)[\sin(2\omega_c + \Delta\omega)t - \sin 2\pi\Delta f t]$
- $\boldsymbol{y(t) = \frac{A_c \acute{A}_c}{2} m(t) \cos 2\pi\Delta f t + \frac{A_c \acute{A}_c}{2} \hat{m}(t) \sin 2\pi\Delta f t}$
- Once again we have distortion and m(t) appears as if single sideband modulated on a carrier frequency = Δf



Single Tone Modulation: Effect of a Constant Frequency of the Carrier at Receiver

- **Example**: When $m(t) = A_m \cos(2\pi f_m t)$; $c(t) = A_c \cos(2\pi f_c t)$;
- The upper SSB signal is: $\boldsymbol{s_{SSB}(t) = \frac{A_c A_m}{2} cos(2\pi(f_c + f_m)t)}$;
- $v(t) = s(t) A_c{'} \cos(2\pi f_c t + 2\pi \Delta f t)$
- $v(t) = \frac{A_c A_m A_{c'}}{2} cos\ (2\pi f_c t + 2\pi \Delta f t) cos(2\pi(f_c + f_m)t)$;
- $v(t) = \frac{A_c A_m A_{c'}}{4} [cos\ (4\pi f_c t + 2\pi f_m t + 2\pi \Delta f t) + \cos(2\pi f_m t - 2\pi \Delta f t)]$
- $\boldsymbol{y(t) = \frac{A_c A_m A_{c'}}{4} [cos(2\pi(f_m - \Delta f)t)]}$
- So, when $\Delta f = 100$, a message component with **f = 1000Hz appears as a 900Hz** component at the demodulator output. Again, distortion occurs because of failing to synchronize the transmitter and receiver carrier frequencies.



Single Tone Modulation: Effect of a Constant Frequency of the Carrier at Receiver

$m(t) = 3\cos(2\pi 1000t)$

$c(t) = \cos(2\pi 10000t)$

$c'(t) = \cos(2\pi 10100t)$

$\Delta f = 100$