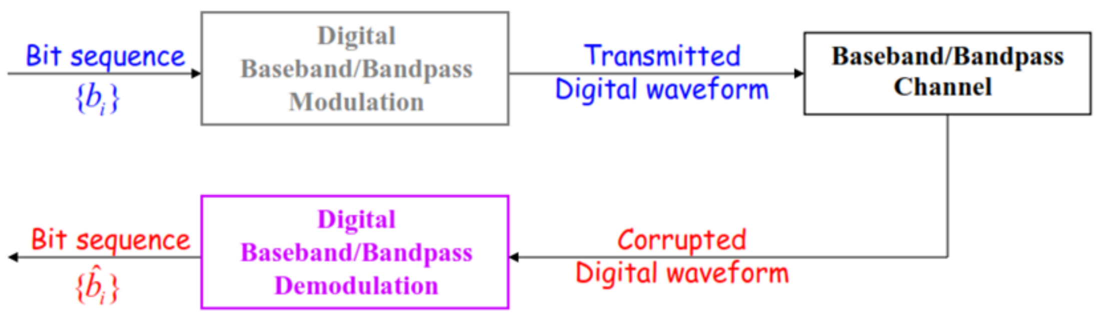
Problem Set 5

Digital Data Transmission

1. An Additive white Gaussian noise (AWGN) $n(t)$ with a power spectral density $N_0/2$ is applied to an ideal low pass filter with bandwidth B Hz. Let $y(t)$ denotes the filter output
   a. Find and sketch the power spectral density at the filter output.
   b. Find the total noise power at the filter output.
   c. Find the probability density function of $y(t_0)$ at some time t_0.
2. Find the average probability of error P_b of a digital communication system given that $P(b_i) = 0.3$, $P(\hat{b_i} = 1|b_i = 0) = 0.05$, and $P(\hat{b_i} = 0|b_i = 1) = 0.01$. Here, b_i refers to the transmitted bit and $\hat{b_i}$ refers to the received bit



3. Consider a digital communication system, corrupted by AWGN with power spectral density $N_0/2$, that uses $s_1(t)$ to represent digit 1 and $s_2(t) = -s_1(t)$ to represent digit 0, where

$$s_1(t) = \begin{cases} A & 0 \le t \le \tau/2 \\ -A & \tau/2 \le t \le \tau \end{cases}$$

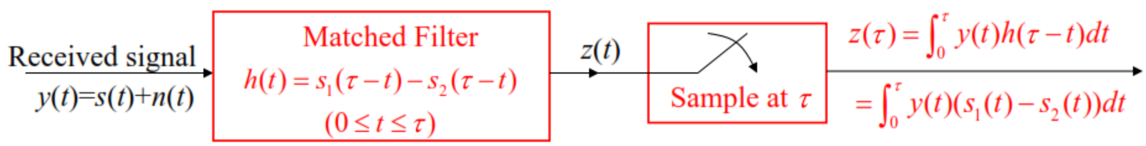
   a. Find and sketch the impulse response of the matched filter
   b. Find the optimum threshold used by the receiver when deciding between digits 0 and 1
   c. Find the system probability of error when the receiver employs the threshold of Part b.

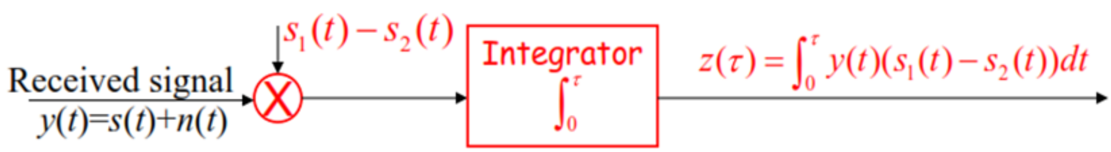
4. Consider a digital communication system, corrupted by AWGN with power spectral density $N_0/2$, that uses $s_1(t)$ to represent digit 1 and $s_2(t)$ to represent digit 0, where

$$s_1(t) = A, \qquad 0 \le t \le \tau$$
$$s_2(t) = 0, \qquad 0 \le t \le \tau$$

   a. Find and sketch the impulse response of the matched filter
   b. Find and sketch the output of the matched filter when $s_1(t)$ is applied at its input. At which time will the output be maximum?
   c. Find the output of the matched filter at $t = \tau$
   d. Find the output of the correlator at $t = \tau$

   Parts c and d should have the same answer. That is, the following two receiver structures are equivalent in terms of the output at time $t = \tau$.





5. Show that the two diagrams given in Problem 4 have the same output at $t = \tau$, for any two arbitrary signals $s_1(t)$ and $s_2(t)$. Consider only the signal part.
6. Find the power spectral density of the unipolar non-return to zero waveform where

$$s_1(t) = A, \qquad 0 \le t \le \tau$$
$$s_2(t) = 0, \qquad 0 \le t \le \tau$$

   and $P(1) = P(0) = 0.5$

   You can use the following formula, given in the notes

$$G_s(f) = \frac{1}{\tau}|V(f)|^2 \cdot \left(\sigma_Z^2 + \frac{\mu_Z^2}{\tau} \sum_{m=-\infty}^{\infty} \delta\left(f - \frac{m}{\tau} \right) \right)$$

7. The binary sequence 11100101 is applied to an ASK modulator. The bit duration is $1\ \mu s$ and the sinusoidal carrier wave used to represent symbol 1 has a frequency equal to 5 MHz.
   a. Find the transmission bandwidth of the transmitted signal.

b. Plot the waveform of the transmitted ASK signal.

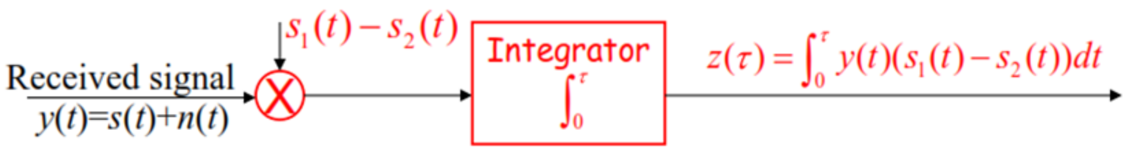
8. The binary sequence 11100101 is applied to a PSK modulator. The bit duration is $1\,\mu s$ and the sinusoidal carrier wave used to represent symbol 1 has a frequency equal to 5 MHz.
   a. Find the transmission bandwidth of the transmitted signal.
   b. Plot the waveform of the transmitted PSK signal.

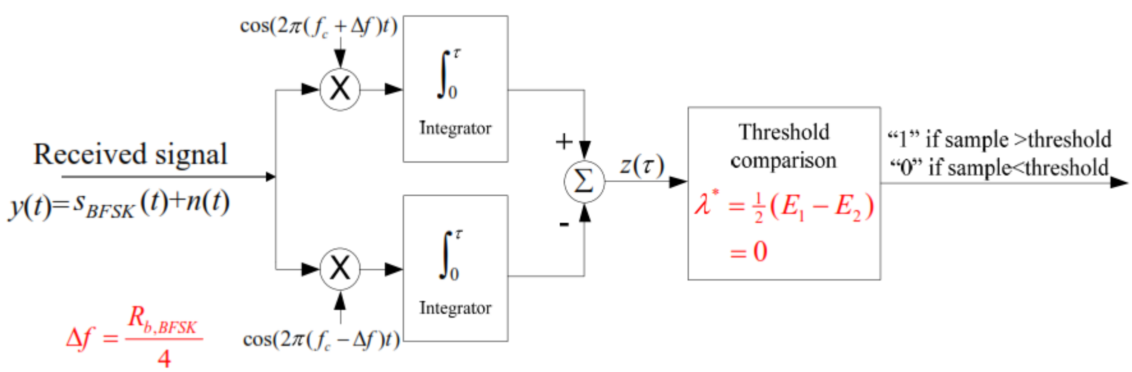
9. The binary sequence 11100101 is applied to a QPSK modulator. The bit duration is $1\,\mu s$ and the sinusoidal carrier frequency is 6 MHz.
   a. Calculate the transmission bandwidth of the QPSK signal
   b. Plot the waveform of the QPSK signal

10. Consider a binary ASK modulator where the bit duration is $1\,\mu s$ and the sinusoidal carrier wave used to represent symbol 1 has a frequency equal to 5 MHz.
   a. Draw the block diagram of the optimum coherent demodulator.
   b. Draw the block diagram of a noncoherent demodulator. Here, the receiver does not know the exact value of the frequency of the received signal
11. Consider an FSK system that uses the signals $s_1(t) = Acos(2\pi f_1 t)$ and $s_2(t) = Acos(2\pi f_2 t)$. Show that $s_1(t)$ and $s_2(t)$ are orthogonal when $f_1 = nR_b$ and $f_2 = mR_b$ where n and m are integers, $n \neq m$, i.e., show that

$$\int_0^{1/R_b} s_1(t)s_2(t)dt = 0$$

12. Show that the following two configurations of the optimum FSK receiver are equivalent





where, $s_1(t) = Acos(2\pi(f_c + \Delta f))t)$ and $s_2(t) = Acos(2\pi(f_c - \Delta f))t)$.

13. Find the probability of error of an FSK system that uses the signals $s_1(t) = Acos(2\pi f_1 t)$ and $s_2(t) = Acos(2\pi f_2 t)$, where $f_1 = nR_b$ and $f_2 = mR_b$ and n and m are integers, $n \neq m$.
14. Find the bandwidth of the FSK system in Problem 13.