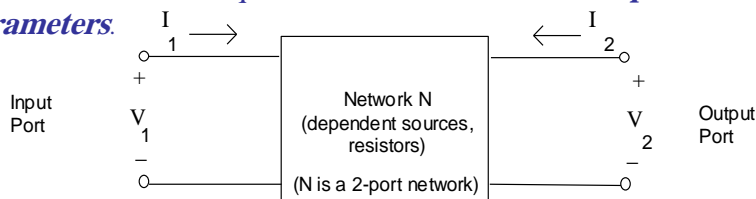


ENEE236

Two-port networks

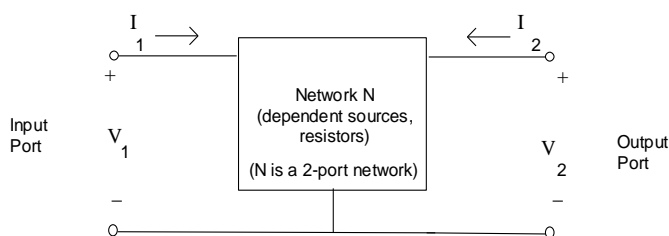
- Suppose that a network N has two ports as shown below. How could it be represented or modeled?
- A common way to represent such a network is to use one of 6 possible **two-port networks**.
- These networks are circuits that are based on one of 6 possible sets of **two-port equations**. These equations are simply different combinations of two equations that relate the variables V_1 , V_2 , I_1 , and I_2 to one another. The coefficients in these equations are referred to as **two-port parameters**.



4

ENEE234 – Circuit Analysis

Note that I_1 , I_2 , V_1 , and V_2 are labeled as shown by convention. Often there is a common negative terminal between the input and the output so the figure above could be redrawn as:



5

ENEE236

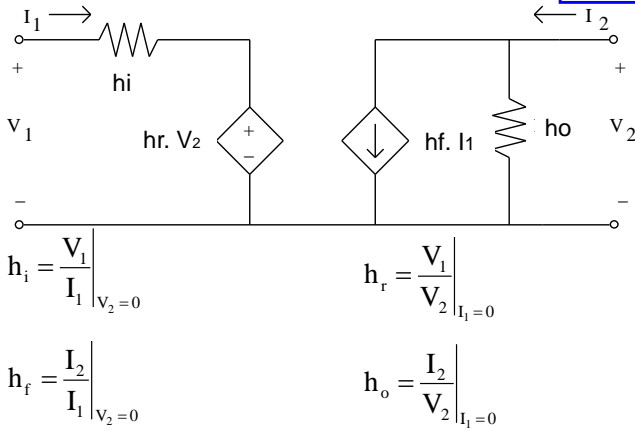
Development of the h-parameter model of BJT:

For A BJT the equivalent h parameter model can be described by the following equations:

h - parameter equations :

$$V_1 = h_i \cdot I_1 + h_r \cdot V_2$$

$$I_2 = h_f \cdot I_1 + h_o \cdot V_2$$

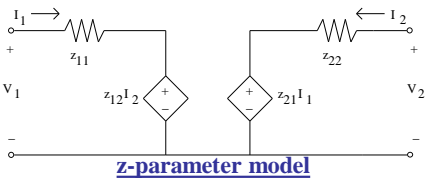


7

ENEE236

Summary:

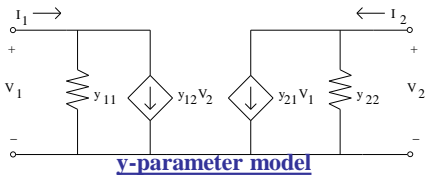
Note: This page is for information only



z - parameter equations :

$$V_1 = z_{11} \cdot I_1 + z_{12} \cdot I_2$$

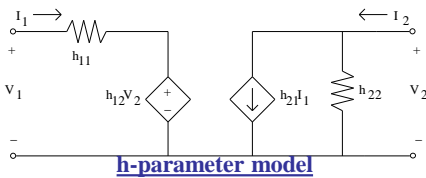
$$V_2 = z_{21} \cdot I_1 + z_{22} \cdot I_2$$



y - parameter equations :

$$I_1 = y_{11} \cdot V_1 + y_{12} \cdot V_2$$

$$I_2 = y_{21} \cdot V_1 + y_{22} \cdot V_2$$



h - parameter equations :

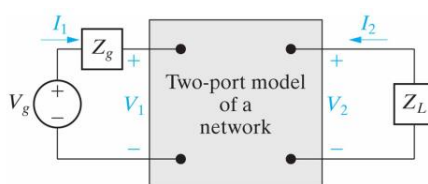
$$V_1 = h_{11} \cdot I_1 + h_{12} \cdot V_2$$

$$I_2 = h_{21} \cdot I_1 + h_{22} \cdot V_2$$

8

BJT Configurations

- Common Emitter
- Common Base
- Common Collector



Terminated Two port network
Includes source and load

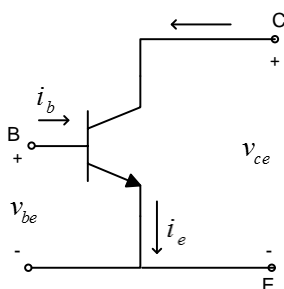
Common Emitter Configuration

(inverting configuration, provides voltage and current gain)

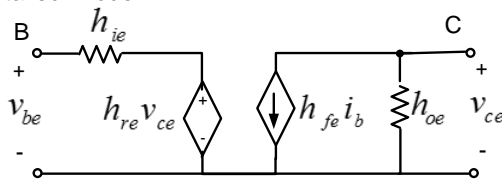
h - parameter equations :

$$V_{be} = h_{ie} \cdot I_b + h_{re} \cdot V_{ce}$$

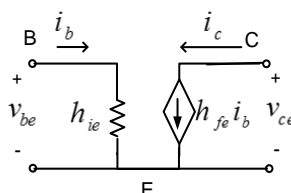
$$I_c = h_{fe} \cdot I_b + h_{oe} \cdot V_{ce}$$



Detailed Model



Simplified Model



Typical Data sheet parameter values

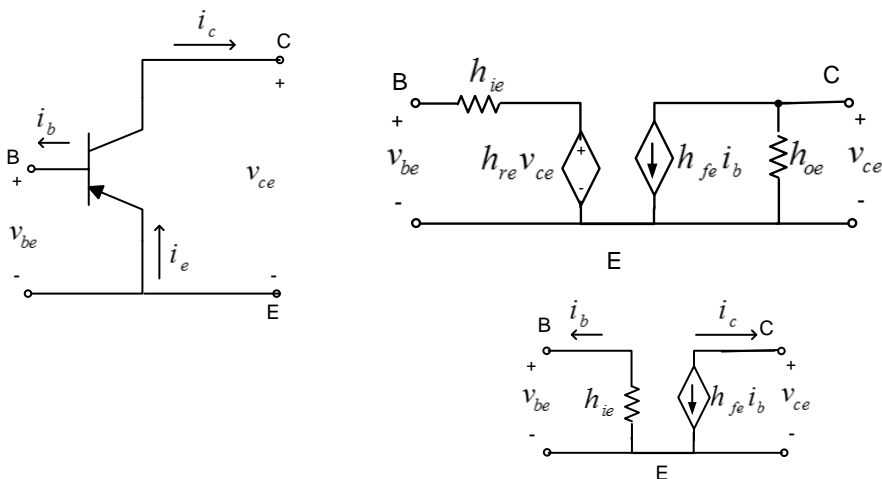
$$h_{ie} \approx 1600 \, \Omega$$

$$h_{re} \approx 0.0002$$

$$h_{fe} \approx 80$$

$$h_{oe} \approx 20 \cdot 10^{-6} \text{ Siemens}$$

Common Emitter and Common Collector Configuration



Value of h_{ie}

Base Emitter is a pn junction similar to a diode
 h_{ie} is the dynamic resistance of the pn junction

In a diode:

$$r_d = \frac{V_T}{I_{DQ}} \Rightarrow$$

$$h_{ie} = \frac{V_T}{I_{BQ}} = \frac{V_T}{\frac{I_{CQ}}{h_{fe}}} = \frac{h_{fe} V_T}{I_{CQ}}$$

I_{BQ} dc value of base current

I_{CQ} dc value of collector current

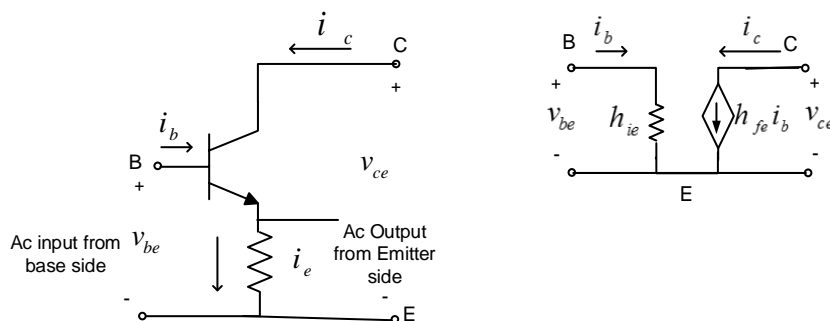
$$h_{fe} = \beta$$

$$V_T = 25.69 \text{ mV @ } 25^\circ\text{C}$$

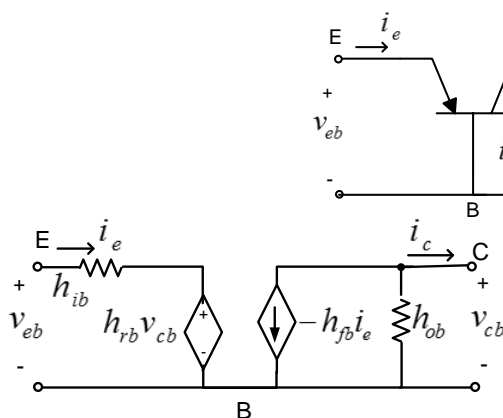
Common Collector

(provides current gain and no voltage gain)

Same Model of Common Emitter will be used due to the similarities between them and for simplicity



Common-Base Configuration



h - parameter equations :

$$V_{eb} = h_{ib} \cdot I_e + h_{rb} \cdot V_{cb}$$

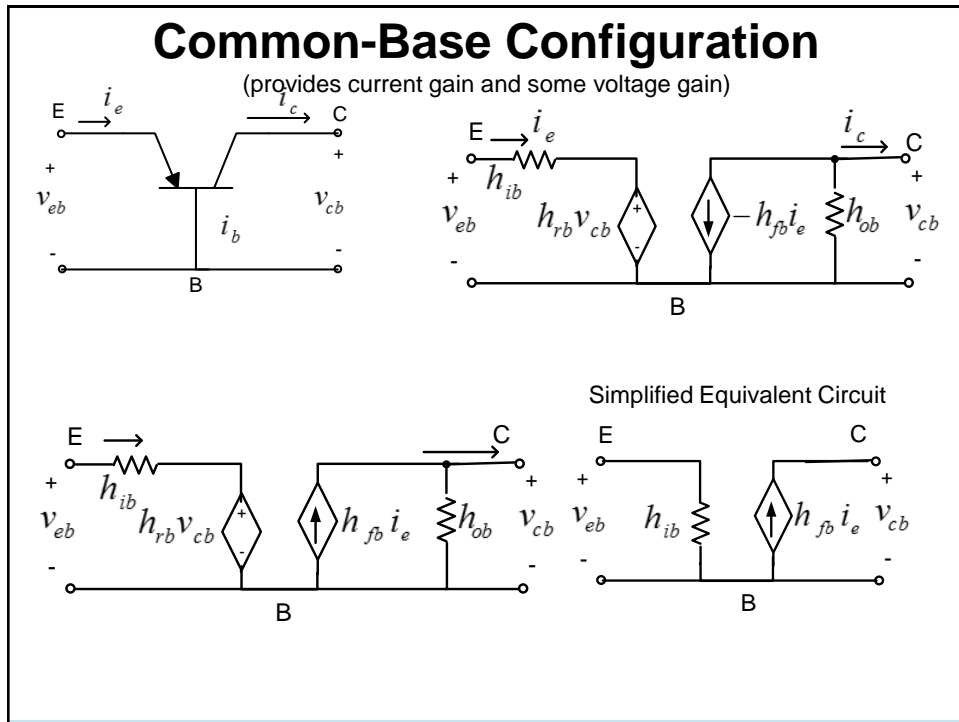
$$I_c = h_{fb} \cdot I_e + h_{ob} \cdot V_{cb}$$

$$h_{ib} = \left. \frac{V_{EB}}{I_E} \right|_{V_{CB}=0}$$

$$h_{fb} = \alpha = \left. \frac{I_C}{I_E} \right|_{V_{CB}=0}$$

$$h_{rb} = \left. \frac{V_{EB}}{V_{CB}} \right|_{I_E=0}$$

$$h_{ob} = \left. \frac{I_C}{V_{CB}} \right|_{I_E=0}$$



Common-Base Configuration

$$h_{ib} = \frac{V_T}{I_{EQ}}$$

$$h_{fb} = \alpha$$

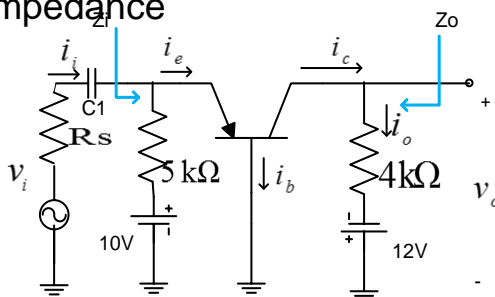
$$V_T = 25.69 \text{ mV @ } 25^\circ\text{C}$$

$$h_{ie} > h_{ib}$$

BJT Amplifier Analysis

When Analyzing Amplifier Circuits, we usually want to find some or all of the following quantities **with and without R_s** :

- 1) $A_v = V_o/V_i$, small signal voltage gain
- 2) $A_i = i_o/i_i$, small signal current gain
- 3) Z_i Input Impedance
- 4) Z_o Output Impedance



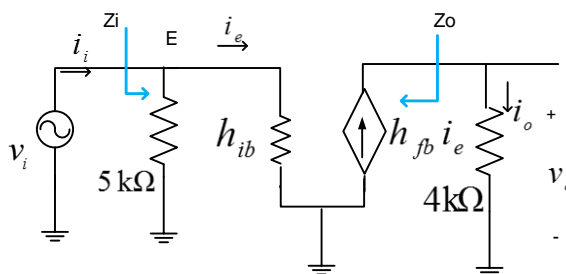
BJT Amplifier Analysis

Solution: (with $R_s=0$)

We draw the ac small signal equivalent circuit

Capacitors \Rightarrow replaced by short circuit

DC sources are killed ,



$$h_{ib} = \frac{V_T}{I_{EQ}}$$

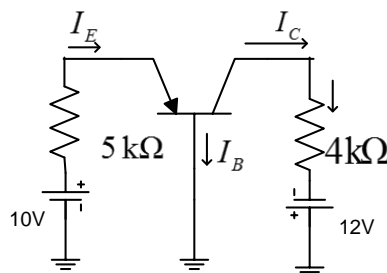
$$h_{fb} = \alpha \cong 1$$

I_{EQ} must be calculated from DC analysis

DC Analysis

DC Equivalent Circuit:

- Cap ==> open
- Kill ac sources ==>



$$10 = 5 \text{ k}\Omega \cdot I_{EQ} + V_{EB}$$

$$I_{EQ} = \frac{10 - 0.7}{5 \text{ k}\Omega} = 1.86 \text{ mA}$$

$$h_{ib} = \frac{V_T}{I_{EQ}} = \frac{25.69 \text{ mV}}{1.86 \text{ mA}} = 13.98 \Omega$$

Ac ss equivalent circuit

$$1) A_v = \frac{v_o}{v_i}$$

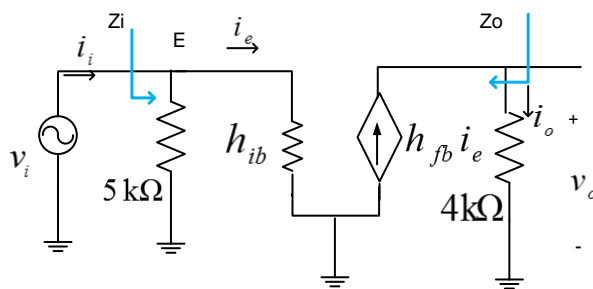
$$v_o = i_o \cdot 4 \text{ k}\Omega$$

$$i_o = h_{fb} \cdot i_e$$

$$i_e = \frac{v_i}{h_{ib}}$$

$$A_v = \frac{v_o}{v_i} = \frac{v_o}{i_o} \cdot \frac{i_o}{i_e} \cdot \frac{i_e}{v_i} \Rightarrow A_v = (4 \text{ k}\Omega) \cdot (h_{fb}) \cdot \left(\frac{1}{h_{ib}} \right)$$

$$= (4 \text{ k}\Omega) \cdot (1) \cdot \left(\frac{1}{13.98} \right) = 286 > 1$$

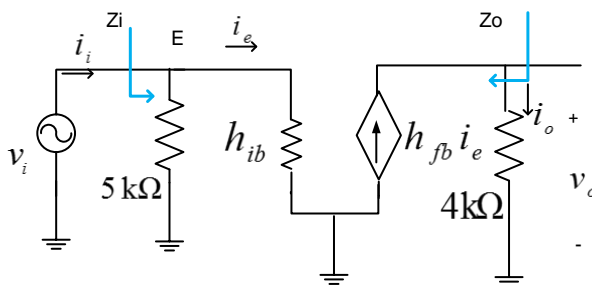


Current Gain Ai

$$2) A_i = \frac{i_o}{i_i}$$

$$i_o = h_{fb} i_e$$

$$i_e = i_i \frac{5 \text{ k}\Omega}{5 \text{ k}\Omega + h_{ib}}$$

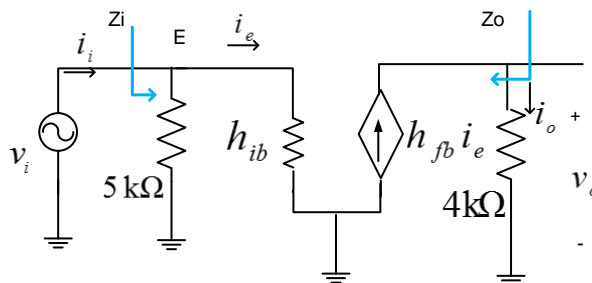


$$\Rightarrow A_i = \frac{i_o}{i_i} = \frac{i_o}{i_e} \cdot \frac{i_e}{i_i}$$

$$\Rightarrow A_i = (h_{fb}) \left(\frac{5 \text{ k}\Omega}{5 \text{ k}\Omega + h_{ib}} \right)$$

$$= (1) \left(\frac{5 \text{ k}\Omega}{5 \text{ k}\Omega + 13.98} \right) < 1$$

Zi & Zo



3) Input Impedance

$$Z_i = (h_{ib} // 5 \text{ k}\Omega) = \left(\frac{h_{ib} \cdot 5 \text{ k}\Omega}{5 \text{ k}\Omega + h_{ib}} \right)$$

4) Output Impedance

$$Z_o \Big|_{\text{all independent sources killed (i.e. } V_i=0 \text{ or short)}} = 4 \text{ k}\Omega$$

With Presence of R_s

with R_s

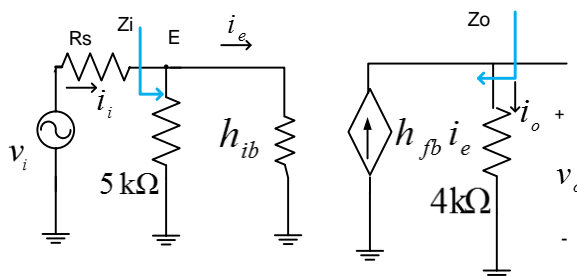
$$i_i = \frac{v_i}{Z_i + R_s}$$

For $R_s = 50 \Omega$

$$A_v = 62.5$$

For $R_s = 10 \text{ k}\Omega$

$$A_v = 0.4$$



Example: Common Emitter (CE)

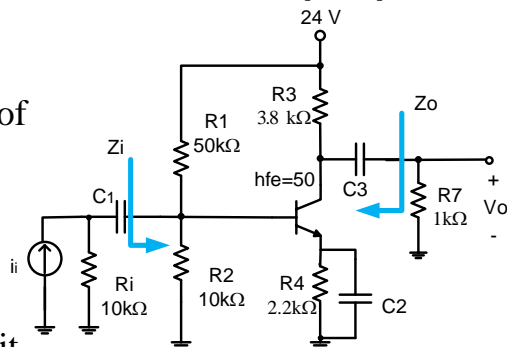
1) From DC Analysis,
we find Q - point and value of

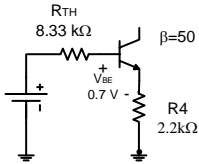
$$h_{ie} = \frac{V_T}{I_{BQ}}$$

Thevenin's equivalent circuit
as seen from the base

$$V_{TH} = \frac{10 \text{ k}\Omega}{10 \text{ k}\Omega + 50 \text{ k}\Omega} \cdot 24 \text{ V} = 4 \text{ V}$$

$$R_{TH} = 10 \text{ k}\Omega // 50 \text{ k}\Omega = 8.33 \text{ k}\Omega$$





$R_{TH} = 8.33 \text{ k}\Omega$
 $\beta = 50$
 $V_{BE} = 0.7 \text{ V}$
 $R_4 = 2.2 \text{ k}\Omega$

$$4 = 8.33 \text{ k}\Omega \cdot I_B + V_{BE} + 2.2 \text{ k}\Omega \cdot I_E$$

But,

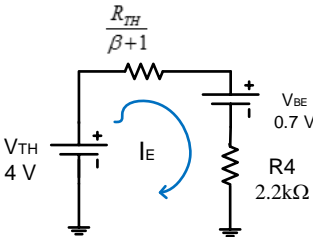
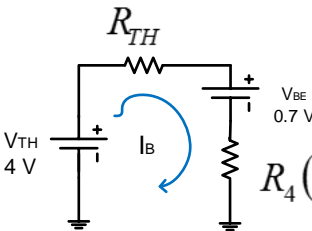
$$I_E = (1 + \beta) I_B$$

Solve for $I_E = \frac{4 - 0.7}{\frac{8.33 \text{ k}\Omega}{(1 + 50)} + 2.2 \text{ k}\Omega} = 1.4 \text{ mA}$

$$h_{ie} = \frac{V_T}{I_{BQ}} = \frac{25.69 \text{ mV}}{\frac{1.4 \text{ mA}}{51}} = 928 \Omega$$

Here we have base reflected to emitter

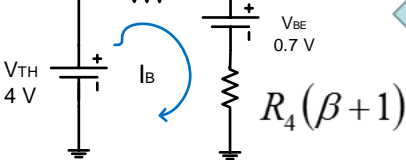
$$I_B \Rightarrow I_E = (\beta + 1) I_B$$

$$R_B \Rightarrow \frac{R_B}{\beta + 1}$$



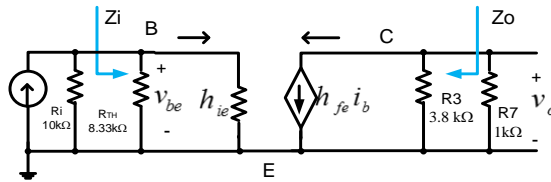
R_{TH}
 $V_{TH} = 4 \text{ V}$
 $V_{BE} = 0.7 \text{ V}$
 R_4

Here we have emitter reflected to base

$$I_E \Rightarrow I_B = \frac{I_E}{(\beta + 1)}$$

$$R_E \Rightarrow R_E (\beta + 1)$$


AC small signal Equivalent Circuit



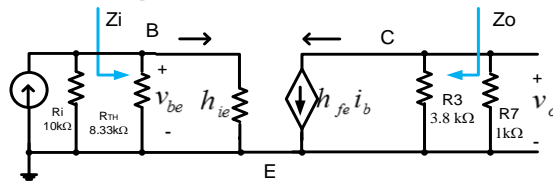
$$1) A_v = \frac{v_o}{v_i}$$

$$A_v = \frac{v_o}{v_i} = \frac{v_o}{i_b} \cdot \frac{i_b}{v_i}$$

$$v_o = -h_{fe} i_b \cdot (R_3 // R_7) \Rightarrow -h_{fe} \cdot (R_3 // R_7) \cdot \left(\frac{1}{h_{ie}} \right)$$

$$i_b = \frac{v_i}{h_{ie}} \Rightarrow -50 \cdot (3.8 \text{ k}\Omega // 1 \text{ k}\Omega) \cdot \left(\frac{1}{928 \Omega} \right) = -42.7$$

AC small signal Equivalent Circuit



$$2) Z_i = R_{TH} // h_{ie} \\ = 8.33 \text{ k}\Omega // 928 \Omega$$

only elements to the right of arrow are considered
according to the given direction of the arrow

$$3) Z_o \Big|_{\text{all independent sources killed (i.e. } v_i=0 \text{ or short)}} = 3.8 \text{ k}\Omega$$

here $h_{fe} \cdot i_b = 0$ since $i_b = 0$ ($v_i = 0$ - killed)

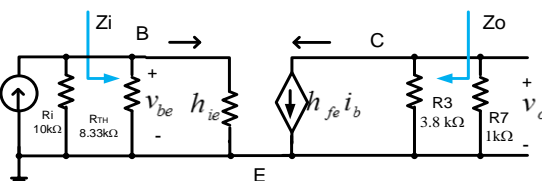
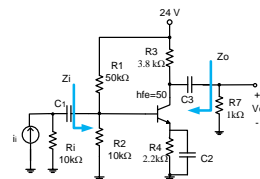
AC small signal Equivalent Circuit

$$4) A_i = \frac{i_o}{i_i}$$

$$i_b = (i_i) \left(\frac{R_I // R_{TH}}{(R_I // R_{TH}) + h_{ie}} \right)$$

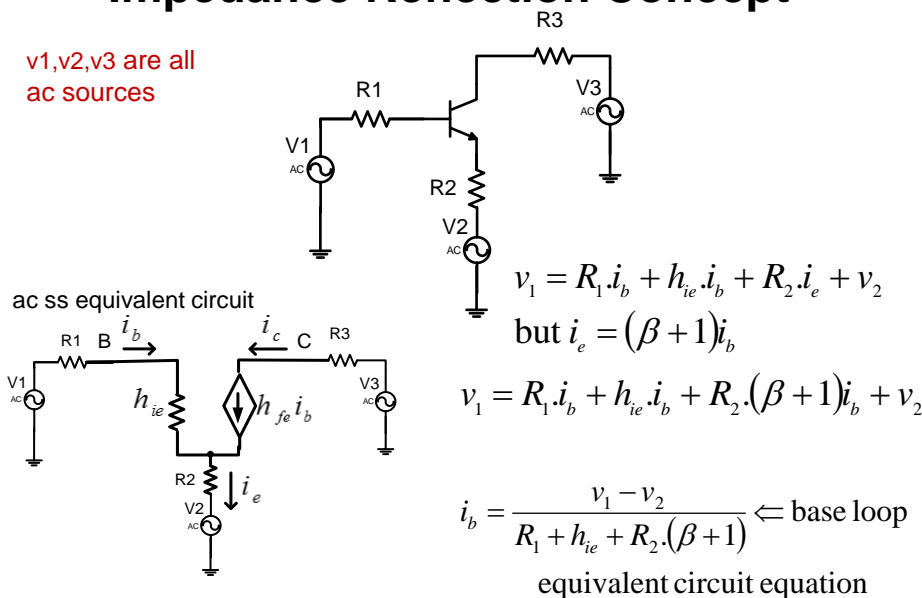
$$i_o = -h_{fe} i_b \left(\frac{R_3}{R_3 + R_7} \right)$$

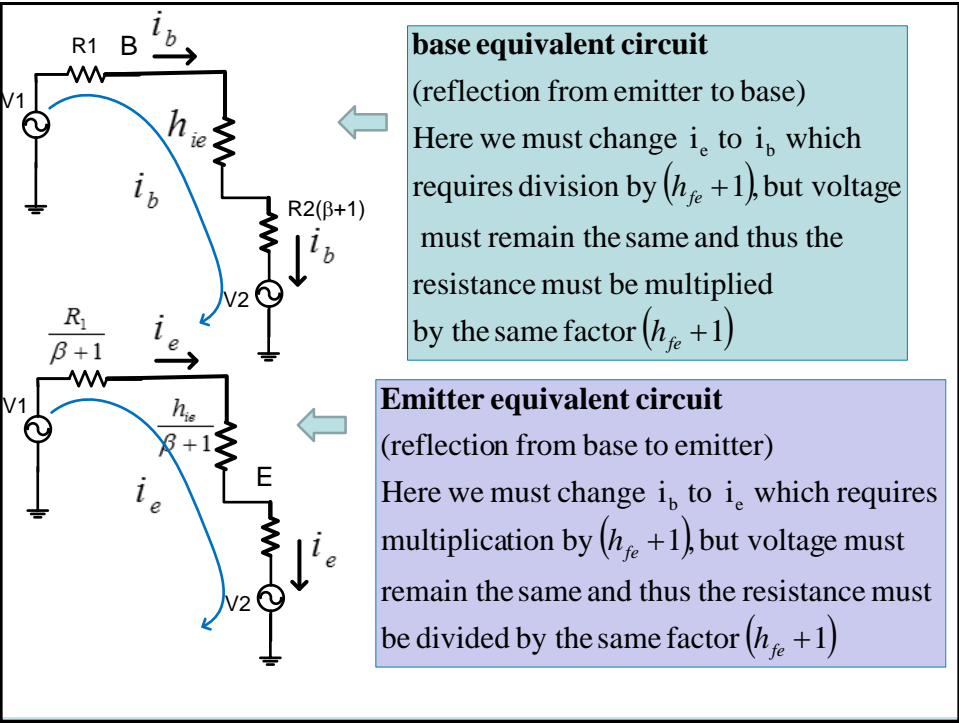
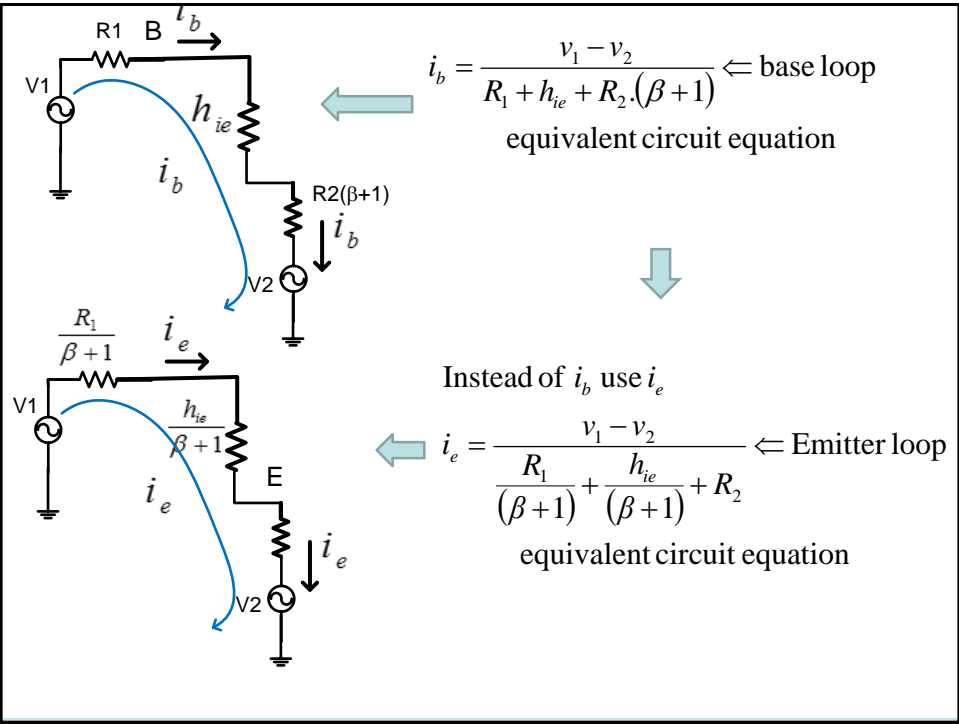
$$A_i = \frac{i_o}{i_i} = \frac{i_o}{i_b} \cdot \frac{i_b}{i_i} = -h_{fe} \left(\frac{R_3}{R_3 + R_7} \right) \left(\frac{R_I // R_{TH}}{(R_I // R_{TH}) + h_{ie}} \right) = -33$$



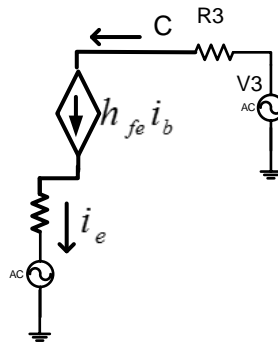
Impedance Reflection Concept

v_1, v_2, v_3 are all
ac sources



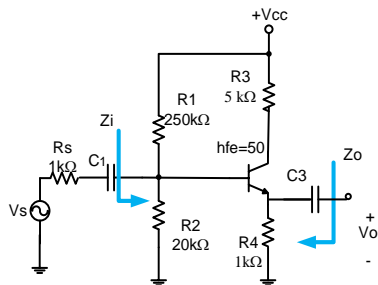


Collector Equivalent Circuit



Note: there is no reflection from emitter to collector or vice versa since the i_e and i_c are almost the same

Common Collector Amplifier



$$1) A_v = \frac{v_o}{v_s}$$

$$v_o = 1k\Omega \cdot i_e$$

$$i_e = i_b (h_{fe} + 1)$$

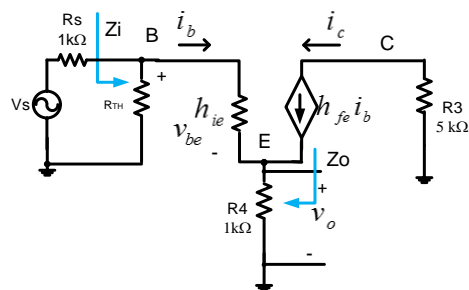
Given

$$h_{ie} = 1k\Omega$$

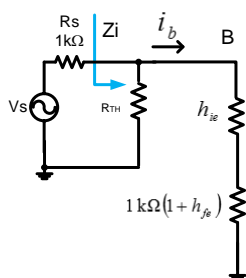
$$h_{fe} = \beta = 50$$

Find A_v, A_i, Z_i, Z_o

AC small signal Equivalent Circuit



i_b can be found from base equivalent circuit



$$R_{TH} = 20 \text{ k}\Omega // 250 \text{ k}\Omega$$

$$i_b = i_i \frac{R_{TH}}{(R_{TH}) + (h_{ie} + 1 \text{ k}\Omega(h_{fe} + 1))}$$

$$i_i = \frac{V_S}{R_S + (R_{TH} // (h_{ie} + 1 \text{ k}\Omega(h_{fe} + 1)))}$$

$$\therefore A_v = \frac{v_o}{v_s} = \frac{v_o}{i_e} \cdot \frac{i_e}{i_b} \cdot \frac{i_b}{i_i} \cdot \frac{i_i}{v_s}$$

$$= (1 \text{ k}\Omega) \cdot (h_{fe} + 1) \left(\frac{R_{TH}}{(R_{TH}) + (h_{ie} + 1 \text{ k}\Omega(h_{fe} + 1))} \right) \left(\frac{1}{R_S + (R_{TH} // (h_{ie} + 1 \text{ k}\Omega(h_{fe} + 1)))} \right)$$

$$= 0.915 < 1$$

$$2) A_i = \frac{i_o}{i_i}$$

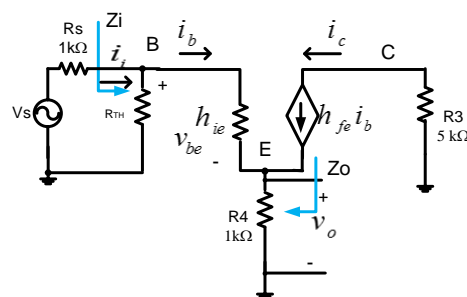
$$i_o = \frac{v_o}{1 \text{ k}\Omega}$$

$$i_o = i_e = i_b (h_{fe} + 1)$$

$$i_b = i_i \frac{R_{TH}}{(R_{TH}) + (h_{ie} + 1 \text{ k}\Omega(h_{fe} + 1))}$$

$$A_i = \frac{i_o}{i_i} = \frac{i_o}{i_e} \cdot \frac{i_e}{i_b} \cdot \frac{i_b}{i_i}$$

$$= 1(h_{fe} + 1) \left(\frac{R_{TH}}{R_{TH} + [h_{ie} + 1 \text{ k}\Omega(h_{fe} + 1)]} \right) = 13.39 > 1$$



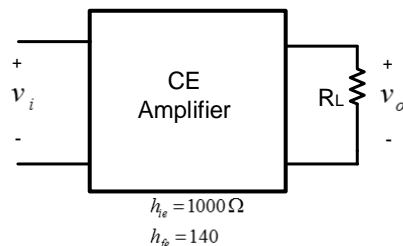
3) $Z_i = (R_{TH} // (h_{ie} + 1 \text{ k}\Omega(h_{fe} + 1)))$
 $= 13.66 \text{ k}\Omega$ (high)

Emitter Equivalent Circuit
 & $V_S = 0$

$Z_o|_{V_S=0} = \left(\frac{(R_S // R_{TH}) + h_{ie} // 1 \text{ k}\Omega}{(h_{fe} + 1)} \right)$
 $= \left(\left(\left(\frac{R_S}{(h_{fe} + 1)} // \frac{R_{TH}}{(h_{fe} + 1)} \right) + \frac{h_{ie}}{(h_{fe} + 1)} \right) // 1 \text{ k}\Omega \right)$
 $= 36.8 \Omega$ (low)

CC Amplifier as a Buffer

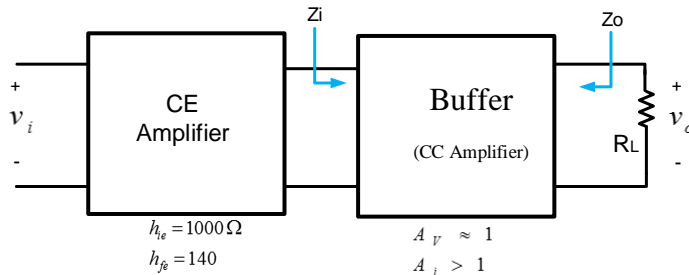
- The value of load resistor R_L affects the voltage gain A_v ,
- This effect is called loading effect and can be substantial



- A buffer (interface) can be used between the amplifier and the load to reduce this loading effect and keep the high gain
- CC Amplifier is also known as Emitter Follower

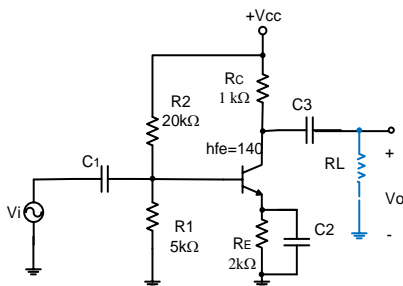
CC Amplifier as a Buffer

- The buffer must have the following characteristic:
 - $A_v \approx 1$
 - $A_i > 1$
 - $Z_i \gg \text{high}$
 - $Z_o \ll \text{low}$
- The above characteristic are present in the CC amplifier the load to reduce this loading effect and keep the high gain



Example

- First we consider effect of load (R_L) on amplifier voltage gain
- Then we use a buffer and see its effect on reducing effect of R_L



1) with $R_L = \infty$

$$v_o = -h_{fe} i_b (R_C)$$

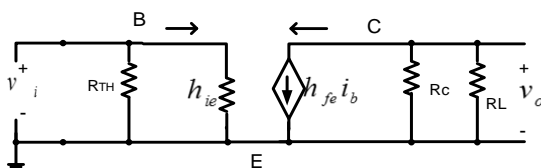
$$i_b = \frac{v_i}{h_{ie}}$$

$$A_v = \frac{v_o}{v_i} = (-h_{fe} R_C) \frac{1}{h_{ie}} = -140$$

2) with $R_L = 50 \Omega$

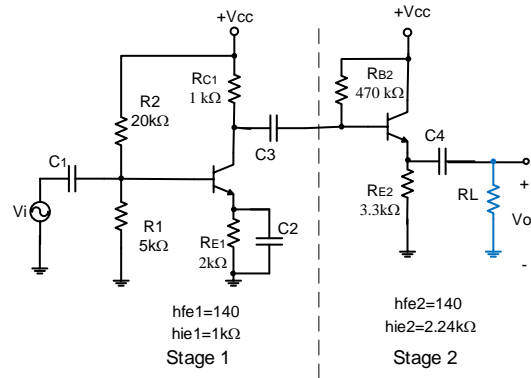
$$A_v = \frac{v_o}{v_i} = (-h_{fe} R_C // R_L) \frac{1}{h_{ie}} = -6.87$$

A_v have been reduced from -140 to -6.87

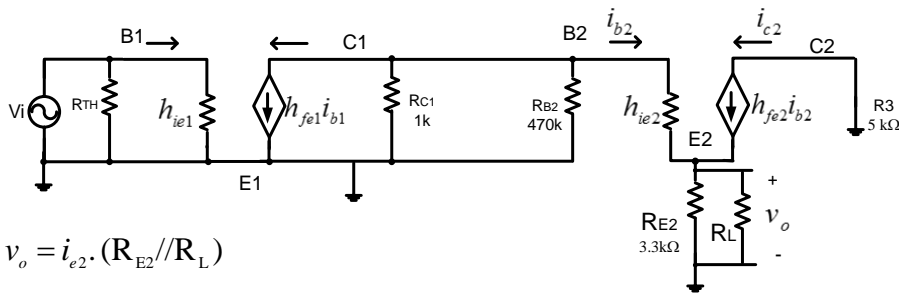


Amplifier + Buffer + Load

Now let us look at the new circuit with the buffer



ac ss equivalent Circuit



$$v_o = i_{e2} \cdot (R_{E2} // R_L)$$

$$i_{e2} = i_{b2} (1 + h_{fe2})$$

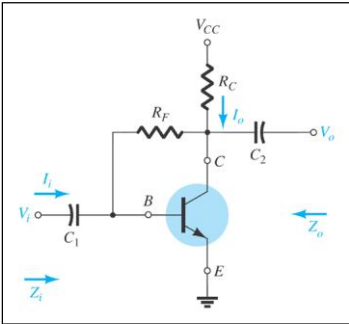
$$i_{b2} = -h_{fe1} \cdot i_{b1} \cdot \frac{(R_{C1} // R_{B2})}{((R_{C1} // R_{B2}) + (h_{ie2} + (R_{E2} // R_L)(1 + h_{fe2})))}$$

$$i_{b1} = \frac{V_i}{h_{ie1}}$$

$$\Rightarrow Av = \frac{v_o}{v_i} = \frac{v_o}{i_{e2}} \cdot \frac{i_{e2}}{i_{b2}} \cdot \frac{i_{b2}}{i_{b1}} \cdot \frac{i_{b1}}{v_i} = -95.6$$

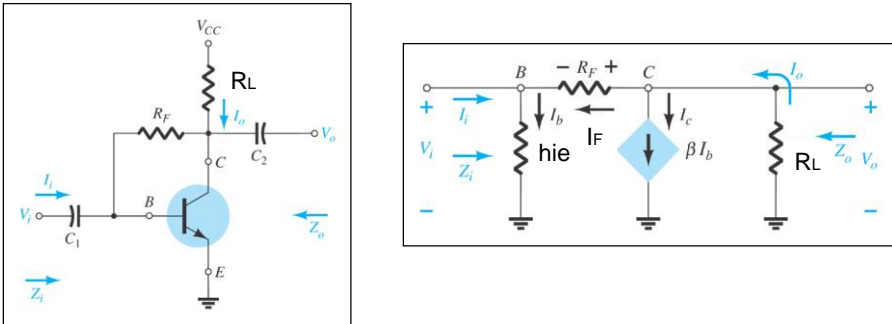
This is much better than the case without buffer

Base To Collector Feedback



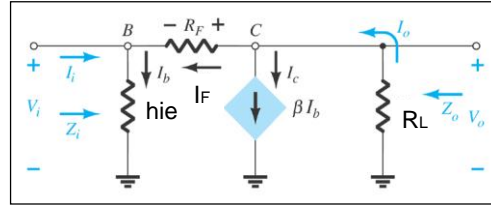
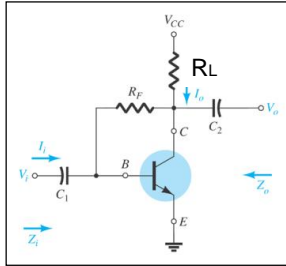
Exercise : Find A_v , Z_i and Z_o

Base To Collector Feedback



Exercise : Find A_v , Z_i and Z_o

Base To Collector Feedback



$$v_o = -i_o R_L$$

$$i_o = h_{fe} i_b + i_F$$

$$i_F = \frac{v_o - v_i}{R_F}$$

$$i_b = \frac{v_i}{h_{ie}}$$

$$v_o = -\left(h_{fe} \cdot \frac{v_i}{h_{ie}} + \frac{v_o - v_i}{R_F}\right) R_L$$

$$v_o = -R_L h_{fe} \cdot \frac{v_i}{h_{ie}} - \frac{v_o R_L}{R_F} + \frac{v_i R_L}{R_F}$$

$$v_o \left(1 + \frac{R_L}{R_F}\right) = v_i \left(\frac{R_L}{R_F} - R_L \cdot \frac{h_{fe}}{h_{ie}}\right)$$

$$A_v = \frac{\left(\frac{R_L}{R_F} - R_L \cdot \frac{h_{fe}}{h_{ie}}\right)}{\left(1 + \frac{R_L}{R_F}\right)}$$

$$Z_0 \Big|_{v_i=0} = R_F // R_L$$

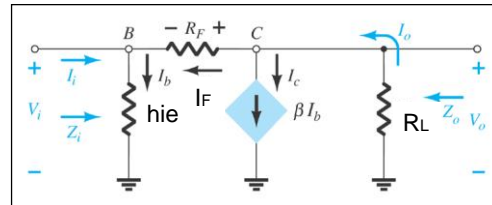
$$Z_i = \frac{v_i}{i_i}$$

$$i_i = i_b - i_F = \left(\frac{v_i}{h_{ie}} - \frac{v_o - v_i}{R_F}\right)$$

$$Z_i = \frac{v_i}{i_i} = \frac{v_i}{\left(\frac{v_i}{h_{ie}} - \frac{v_o - v_i}{R_F}\right)}$$

$$= \frac{v_i}{\left(\frac{R_F v_i - h_{ie} (v_o - v_i)}{R_F h_{ie}}\right)}$$

$$= \frac{v_i R_F h_{ie}}{(R_F v_i - h_{ie} (v_o - v_i))}$$



$$= \frac{v_i R_F h_{ie}}{((R_F + h_{ie}) v_i - h_{ie} v_o)}$$

$$= \frac{R_F h_{ie}}{\left((R_F + h_{ie}) - h_{ie} \frac{v_o}{v_i}\right)}$$

$$= \frac{R_F h_{ie}}{((R_F + h_{ie}) - h_{ie} A_v)}$$

The common emitter amplifier design:

Design a common emitter amplifier using a transistor having

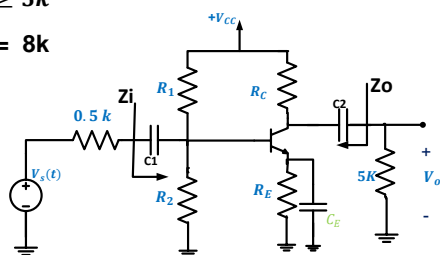
$$\beta(\min) = 480, \quad \beta(\max) = 1500$$

To provide a voltage gain $\left| \frac{V_o}{V_s} \right| \geq 200$, between a small signal voltage source having a resistance 500Ω and load $R_L = 5k$

Its specified that $Z_{in} \geq 5k$

Its specified that $Z_o = 8k$

Solution :



Solution:

Ac small signal equivalent circuit:

$$V_o = -(R_C \parallel R_L) h_{fe} i_b$$

$$i_b = \frac{V_i}{h_{ie}}$$

$$V_i = \frac{Z_i}{Z_i + R_s} V_s$$

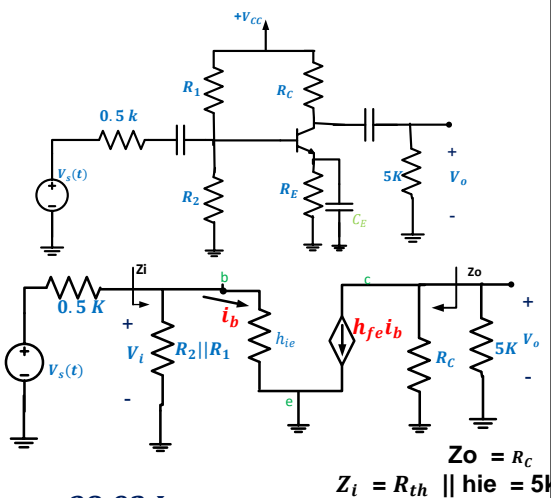
$$\diamond |A_v| = \frac{h_{fe}}{h_{ie}} \frac{Z_i}{Z_i + R_s} (R_C \parallel R_L)$$

$$1 > \frac{Z_i}{Z_i + R_s} > 0.9$$

$$\diamond |A_v| = \frac{h_{fe}}{h_{ie}} (0.9) (R_C \parallel 5k)$$

$$\text{Let } g_m = \frac{h_{fe}}{h_{ie}} = \frac{I_{CQ}}{V_T} = \frac{1}{25.69m} = 38.92 I_{CQ}$$

$$h_{ie} = \frac{\beta V_T}{I_C} = \frac{h_{fe} V_T}{I_C}$$



$$\diamond |A_v| = (g_m)(0.9)(R_C \parallel 5k) \geq 200$$

Its specified that $Z_o = 8k$

$$\therefore R_C = 8k, \quad \text{then} \quad g_m \geq \frac{200}{0.9(3k \parallel 8k)} = 72.2$$

Let $g_m = 77.86$, then $I_{CQ} = 2mA$

Since $V_{RC} = 16V$; let $V_{CC} = 30V$

$$\text{Let } V_{RE} = \frac{V_{CC}}{5} = 6\text{volt}$$

$$R_E = \frac{V_{RE}}{I_E} = 3k\Omega$$

$$Z_i = R_{th} \parallel h_{ie} = 5k$$

$$h_{ie} = \frac{\beta V_T}{I_C} = 6.165K \quad \therefore R_{th} = 26.45k$$

$$\text{From : } I_E = \frac{V_{th} - V_{BE}}{\frac{R_{th}}{\beta + 1} + R_E}$$

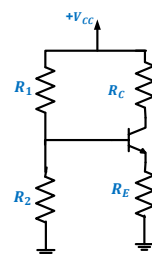
$$V_{th} = 6.81\text{ volt}$$

$$V_{th} = \frac{R_2}{R_1 + R_2} V_{CC}$$

$$R_{th} = \frac{R_1 R_2}{R_1 + R_2}$$

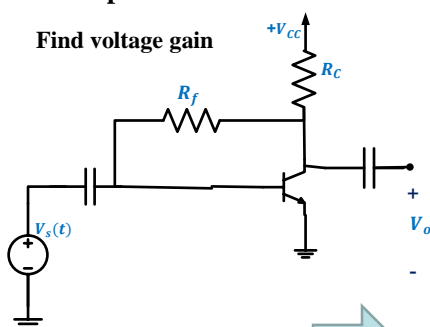
$$\triangleright R_1 = 34.22k$$

$$\triangleright R_2 = 116.5.6k$$



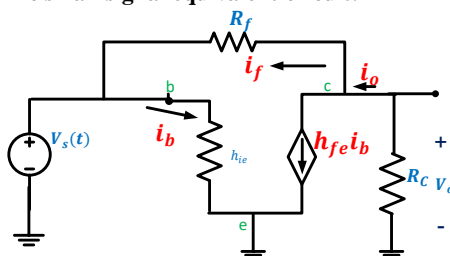
Example :

Find voltage gain



$$\begin{aligned} V_o &= -R_C i_o \\ i_o &= h_{fe} i_b + i_f \\ i_f &= \frac{V_o - V_s}{R_f} \end{aligned}$$

Ac small signal equivalent circuit:



$$i_b = \frac{V_s}{h_{ie}}$$

$$A_v = - \frac{\frac{R_C}{R_E} - R_C \frac{h_{fe}}{h_{ie}}}{1 + \frac{R_C}{R_E}}$$